A Novel Array Configuration Metric: Imaging Bandwidth

Tobia Carozzi

Onsala Space Observatory
Chalmers University, Sweden

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How to assess an imaging interferometer?

Standard telescope metrics (and their limitations) are:

- Effective collecting area
  - Neglects array geometry
- Resolution
  - Only considers maximum baseline
- Dynamic range
  - Does not give how many sources can be imaged
- Sensitivity
  - Formula for arrays $\sqrt{n(n-1)}G/kT$ has no array geometry
- Dirty beam quality
  - Does not include deconvolution

Suggested new metric

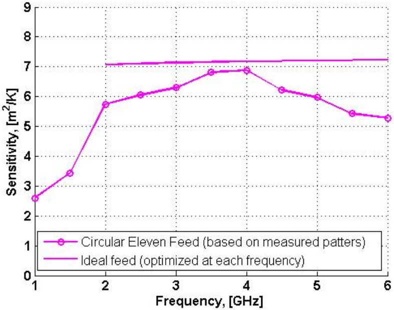
Let us view the interferometer from the perspective of information theory. Interferometers collects a subset of information from sky to make an image. But how much desired information can it extract? In information theory this is known as channel capacity and colloquially as effective bandwidth.

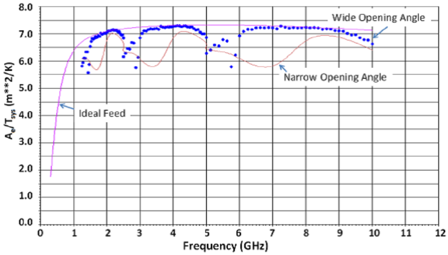
Definition

Imaging bandwidth or interferometer channel capacity is the maximum amount of useful image information that can be extracted with an interferometer (measured in bits per spectral band and per observation)

The intuition is that an interferometer with a *large* imaging bandwidth is *better* than one with a *small* imaging bandwidth. Crude analogy with digital cameras one could say how many mega-pixels one has combined with dynamic range (pixel word-size).

Example: what constitutes a good filter?





Filter spectrum

The output $\{Y_i\}_1^n$ of a filter $\{a_i\}_1^n$ on an input $\{Y_i\}_1^n$ can be approximated in the time domain as the convolution

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_i \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} a_1 & \cdots & a_i & \cdots & a_n \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ a_{-i} & \ddots & a_1 & \ddots & a_i \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ a_{-n} & \cdots & a_{-i} & \cdots & a_1 \end{bmatrix} \begin{bmatrix} X_1 \\ \vdots \\ X_i \\ \vdots \\ X_n \end{bmatrix} \iff \mathbf{Y} = \mathbf{AX}$$

This can be diagonalized by a Fourier transform (an unitary matrix) but, for reasons that will be revealed later, we use the singular value decomposition (SVD)

$$\mathbf{Y} = \mathbf{U\Sigma V}^H\mathbf{VX} \iff \tilde{\mathbf{Y}} = \mathbf{\Sigma}\tilde{\mathbf{X}}$$

where $\mathbf{\Sigma}$ is a diagonal matrix with diagonal entries $\{\sigma_i\}_1^n$ a.k.a the *singular values* and in this case is equivalent the square-root of the power

Filter channel capacity

The channel capacity of the filter for a unit spectral denisty white noise signal input and additive white noise with spectral density N is the mutual information/entropy

$$C = \sum_{i=1}^{n} \log_2 \left(1 + \frac{\sigma_i}{N}\right)$$

in units of bits per n samples.

Suggestion

The more information transfered through the filter (channel capacity), the better the filter

Interferometer as a spatial frequency filter

We are familiar with the interpretation of an interferometer as a spatial frequency filter. We now try to compute its channel capacity.
Simple model of MEq: for $N = n(n-1)$ scalar visibilities and m point sources and equal gains G, then

$$\begin{bmatrix} \mathscr{V}_{12} \\ \vdots \\ \mathscr{V}_{pq} \\ \vdots \\ \mathscr{V}_{(n-1)n} \end{bmatrix} = G \begin{bmatrix} \mathrm{e}^{\mathrm{i}u_{12}l_1} & \dots & \mathrm{e}^{\mathrm{i}u_{12}l_s} & \dots & \mathrm{e}^{\mathrm{i}u_{12}l_m} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathrm{e}^{\mathrm{i}u_{pq}l_1} & \dots & \mathrm{e}^{\mathrm{i}u_{pq}l_s} & \dots & \mathrm{e}^{\mathrm{i}u_{pq}l_m} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \mathrm{e}^{\mathrm{i}u_{(n-1)n}l_1} & \dots & \mathrm{e}^{\mathrm{i}u_{(n-1)n}l_s} & \dots & \mathrm{e}^{\mathrm{i}u_{(n-1)n}l_m} \end{bmatrix} \begin{bmatrix} B_1 \\ \vdots \\ B_s \\ \vdots \\ B_m \end{bmatrix}$$

$$\mathscr{V} = G\mathbf{MB}$$

Interferometer Channel Capacity

The SVD of interferometer spatial frequency filter matrix $\mathbf{M}$

$$\mathbf{M} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$$

gives the singular values $\mathrm{diag}(\mathbf{\Sigma})$. The singular values can be used to compute the interferometer channel capacity

$$C_{\mathrm{intferometer}} = \sum_{i=1}^{n(n-1)/2} \log_2\left(1 + \frac{\sigma_i}{N}\right)$$

with unit sky temperature, and noise power given by the system temperature kT.

One can view the σ_i as a gain in some direction, and for $\sigma_{\max}/N > 1$ the formula can compared to the sensitivity:

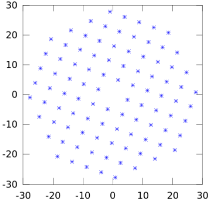
$$S = \frac{G_{\max}}{kT}$$

Example: LOFAR LBA & HBA

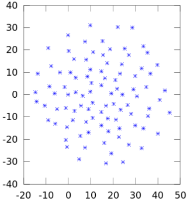
Let's take a single LOFAR station as a good example of an AA configuration. An international station has 96 LB antennas and 96 HB antennas. Imaging with LBA or HBA only entails 96*95/2=4560 unique correlations. If we grid the sky into 4560 pixels, then the MEq will be a 4560x4560 matrix, for which we wish to compute the (4590) singular values. Calculations for both arrys done at the same hypothetical frequency 100 MHz.

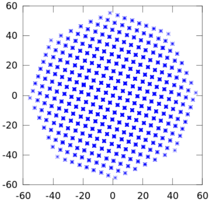
LBA & HBA array configurations (SE607)

HBA highly uniform

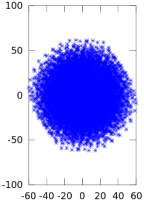


LBA pseudo random



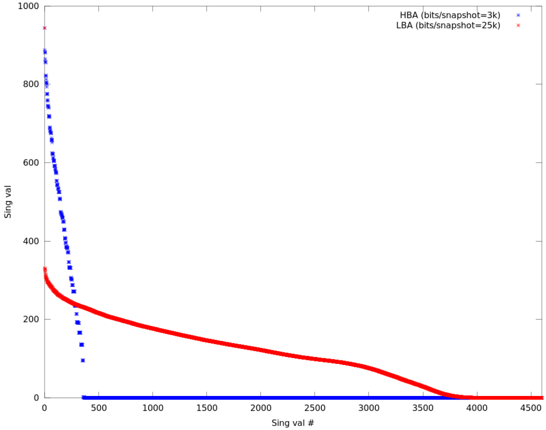


large redundancy



little redundancy

Pseudo spatial frequency spectrum @ 100 MHz



A good spectrum has large number of large singular values.

Discussion on pseudo spatial frequency spectrum

The singular values of a MEq is a useful function. It can be used to say:

- the dynamic range of an interferometer
- and the number of effective pixels
- number of calibrators that can be used for a calibration

Conclusions

I have suggested the novel metrics for interferometers

- singular value (pseudo-spatial frequency) spectrum
- imaging bandwidth (or spatial frequency channel capacity)

for assessing the quality of array configurations. It can be used with or without a given sky model.

I hope to implement them in interferometer simulators, such as *MeqTrees* and *OSKAR* to assess SKA designs