System Requirements Derived from Calibratability

Stefan J. Wijnholds
e-mail: wijnholds@astron.nl

AACal 2012
Amsterdam (NL), 13 July 2012

Overview of system calibration

ASTRON

- **~10 s, ~5% fractional BW: bootstrapping**
  - ionospheric scintillation
  - removal of strong sources in side lobes
  - clock drifts
- **~10 min, full BW: refinement of DDEs**
  - deviations from predicted station beam
  - pointing errors
- **full observation**
  - DDE correction, absolute flux calibration

Calibratability
Wijnholds, Bregman & Van Ardenne, RS, 2011
Van der Tol, Jeffs & Van der Veen, TSP, 2007

- **Calibratability: ability to bootstrap calibration**
  - limited sensitivity (~10 s, ~5% fractional BW)
  - gain factors needed per source per station
  - only 8 – 10 sources can be dealt with
  - 2-D parabolic phase screen requires 5 points
- **Consequences**
  - up to 5 sources required in main beam
  - only 3 – 5 sources allowed in side lobes
  - beam size should be matched to patch size

Beam requirements
Wijnholds, AA Station Beam Specs, 2012, in prep.

- **Beam size / station size**
  - match to patch size at lowest frequency
  - single band: too large stations at highest freq?
- **Aperture efficiency**
  - detect sufficient number of calibrators
- **Side lobe level**
  - limit side lobe detections to 3 – 5 sources
- **Beam stability / accuracy**
  - allow forward modeling to ~10 min, full BW

Required station size (1)

Wijnholds, AA Station Beam Specs, 2012, in prep.

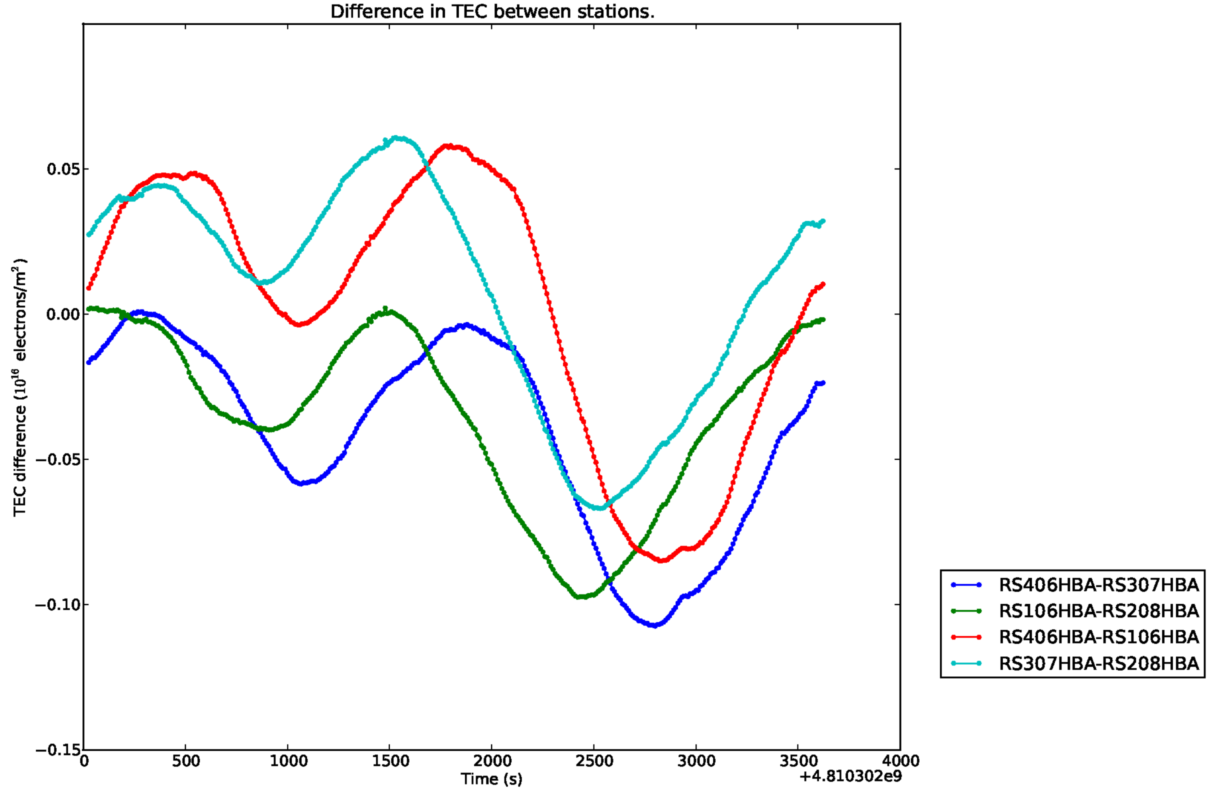
ASTRON

- ionospheric phase: $\Delta\varphi = 28.2 \cdot \lambda \cdot \Delta TEC$
- dominant effect: TID, approximated by sine wave

TID seen by LOFAR

- 20 km in 300 s
- $\lambda_{TID} \approx 120$ km

(Courtesy Mark Aartsen and Ilse van Bemmel)



Required station size (2)

Wijnholds, AA Station Beam Specs, 2012, in prep.

- Taylor expansion of sine wave gives
- $\Delta\varphi_{RMS} = \Delta\varphi \{(\int_{-\beta\pi}^{\beta\pi} \theta^3/(3!)^2 \, d\theta) / (\int_{-\beta\pi}^{\beta\pi} d\theta)\}^{1/2}$
- log N – log S with power 1 with 5 sources: $SNR_5 = 5 \rightarrow SNR_1 = 25 \rightarrow \Delta\varphi_{RMS} = 0.04$ rad
- TID amplitude gives $\Delta\varphi = 12.1$ rad
- This gives $\beta = 0.12$
- **Hence D = 81.8 α meters** (HPBW = α λ / D)
- accurate to first null (α = 2.4): D = 196.3 m

Station size on short baselines

Wijnholds, AA Station Beam Specs, 2012, in prep.

ASTRON

- $D = 81.8\ \alpha$ assumes non-overlapping FoV
- in core of array
  - 5 sources needed per patch
  - requirement $A_{beam} / A_{patch} \leq N_{station} / 5$
  - $D \geq h\ \alpha\ \lambda / ((N_{station} / 5)^{1/2}\ \beta\ \lambda_{TID} - D_{core})$
  - $h = 200$ km, $\alpha = 1.2$, $\lambda_{TID} = 120$ km,

    $D_{core} = 5$ km, $N_{station} = 100$ and $\beta = 0.2$ gives

    D ≥ 10 m → no restriction for science case

Required aperture efficiency (1)

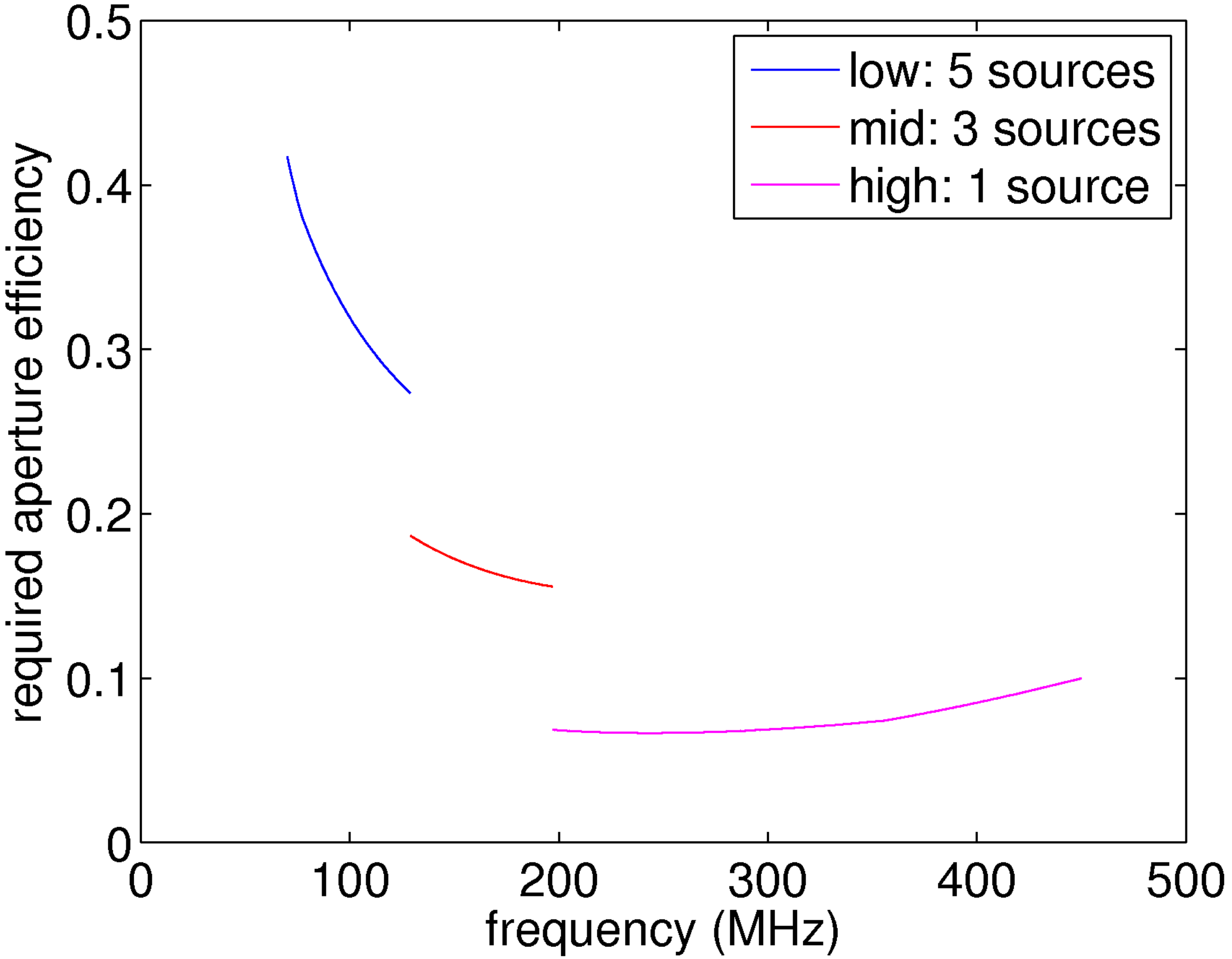
Wijnholds, AA Station Beam Specs, 2012, in prep.

ASTRON

- 2-D parabolic screen requires 5 sources
- five 5σ sources required at 70 MHz
- HPBW ~ λ, patch size ~ 1 / λ
- @129 MHz 2-D gradient provides sufficient accuracy
- only three 5σ sources required > 129 MHz
- @197 MHz a single phase factor is sufficient
- only one 5σ source required > 197 MHz

Required aperture efficiency (2)
Wijnholds, AA Station Beam Specs, 2012, in prep.

ASTRON



Consequences

- $A_e \sim \lambda^2$
- 10% @450 MHz gives 100% near 140 MHz
- A_e/T_{sys} will drop sharply towards 70 MHz
- Science case
  - dark ages / EoR is challenging
  - we should not compromise on 70 – 150 MHz
  - 300 – 450 "only" for continuous freq. coverage
  - focus on low freqs., best effort at high freqs?
- 8.5% @ 400 MHz gives 100% near 117 MHz

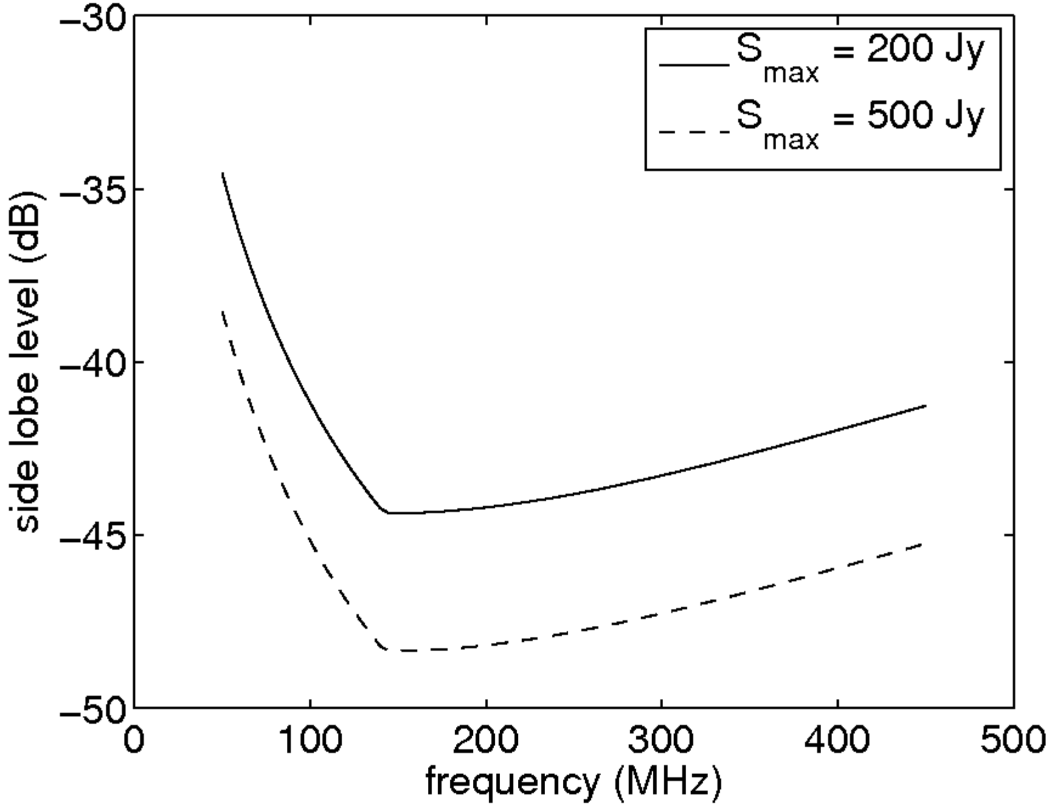
Required side lobe level (1)
Wijnholds, AA Station Beam Specs, 2012, in prep.

ASTRON

- Only 3 – 5 sources in side lobes in (10 s, 5% frac. BW)
- Assumed values
  - $D = 81.8\,\alpha$ (with $\alpha = 1.2$, i.e. $D = 98.2$ m)
  - $A_{eff} = \min\{\, 0.1\,(\lambda/0.66\mathrm{m})^2\, A_{phys},\, A_{phys}\}$
  - $T_{sys} = T_{sky} + 40$ K
  - $B = 0.05\, f$ (5% frac. BW)
  - $\tau = 10$ s
  - sources have synchrotron spectrum

Required side lobe level (2)
Wijnholds, AA Station Beam Specs, 2012, in prep.

- strongest suppressed source 200 or 500 Jy @ 200 MHz
- requirement on ***average*** pattern → helpful “tricks”:
  - station rotation
  - randomization
  - freq. averaging
- crucial for far sidelobes of random array
- first sidelobe needs careful consideration



Required station beam accuracy (1)

Wijnholds, AA Station Beam Specs, 2012, in prep.
Wijnholds, AAVP Workshop 2011, Dec. 2011

ASTRON

- problem: integration to ~ 10 minutes
- from there: SAGEcal (Yatawatta), diff. gains (Smirnov)
- Requires
  - forward calculation of beam pattern
  - correction for short term variations

What accuracy is required in this step?

Required station beam accuracy (2)

Wijnholds, AA Station Beam Specs, 2012, in prep.
Wijnholds, AAVP Workshop 2011, Dec. 2011

ASTRON

- Data model: $\mathbf{v} = \mathbf{M}(\boldsymbol{\theta})\, \boldsymbol{\sigma}$
- Req. on model errors $||\Delta\mathbf{M}|| \,/\, ||\mathbf{M}|| \leq ||\Delta\mathbf{v}|| \,/\, ||\mathbf{v}||$
- Two derivations
  - using 2-norm / spectral norm of **M**
  - using Frobenius norm of **M**
- Similar result:

$$\varepsilon_{stat} \approx \Delta\sigma \,/\, S_{tot}$$

with $\Delta\sigma = \Delta S_0 \,/\, (B\, \tau)^{1/2}$ (flux limit)

S_{tot} total flux in the FoV

Conclusions

- **Station size**
  - long baselines: D = 81.8 ɑ
  - short baselines: D driven by science (EoR)
- **Aperture efficiency**: 10% @450 MHz (8.5% @400MHz?)
- **Side lobe level**
  - depends on frequency and sky model
  - first side lobe needs more analysis
- **Beam accuracy**: $\varepsilon_{stat} \approx \Delta\sigma / S_{tot}$
- **Calibratability requirements narrow design space**