An important lesson learned from redundancy calibration

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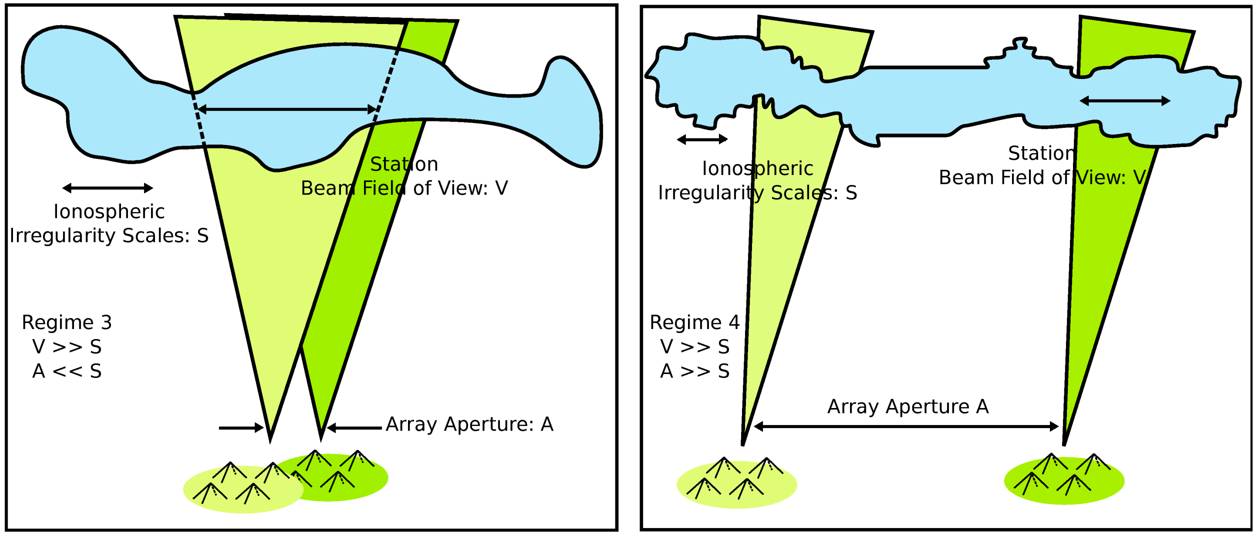
The lesson is

Self-cal can be optimized "one step" using the constrains learned from the redundancy calibration for an array with arbitrary geometry.

The noise floor reduces accordingly.

Problem statement: ionosphere as a DDE



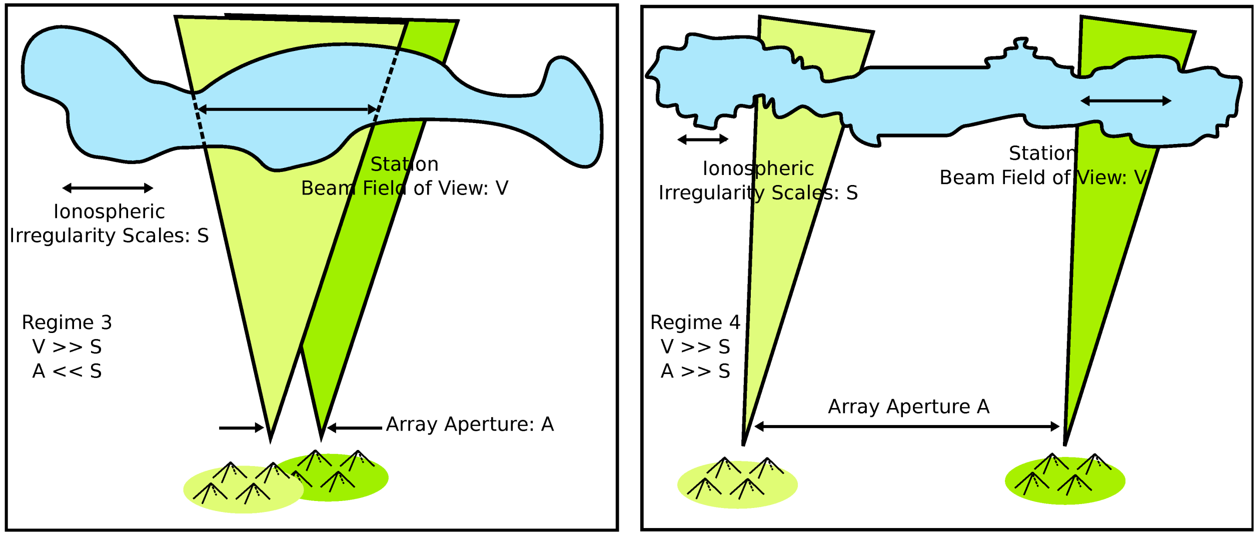
$$\boldsymbol{\theta} = \mathrm{argmin}_{\boldsymbol{\theta}} \left\| \hat{\mathbf{R}} - \mathbf{R}(\boldsymbol{\theta}) \right\|_F^2$$

$$\boldsymbol{\theta} = [\gamma_1, \gamma_1, ..., \gamma_p, \phi_2, \phi_3, ..., \phi_p, \sigma_1^2, \sigma_2^2, ..., \sigma_p^2]^T$$

$$\mathbf{R}(\boldsymbol{\theta}) = \mathbf{G}\mathbf{A}\boldsymbol{\Sigma}_{\mathrm{s}}\mathbf{A}^H\mathbf{G}^H + \boldsymbol{\Sigma}_{\mathrm{n}}$$

Acknowledged by Boonstra & van der Veen (2003), van der Tol et al. (2007), Yatawatta et al. (2008), Wijnholds & van der Veen (2009), Smirnov (2011), Kazemi et al. (2011), and many more.

Ionosphere: source position shift & a phase slope



$$\mathbf{A}' = \begin{bmatrix} e^{-j2\pi\frac{\mathbf{r}_1\cdot(\mathbf{s}_1+\Delta\mathbf{s}_{11})}{\lambda}} & e^{-j2\pi\frac{\mathbf{r}_1\cdot(\mathbf{s}_2+\Delta\mathbf{s}_{21})}{\lambda}} & \cdots & e^{-j2\pi\frac{\mathbf{r}_1\cdot(\mathbf{s}_q+\Delta\mathbf{s}_{q1})}{\lambda}} \\ e^{-j2\pi\frac{\mathbf{r}_2\cdot(\mathbf{s}_1+\Delta\mathbf{s}_{12})}{\lambda}} & e^{-j2\pi\frac{\mathbf{r}_2\cdot(\mathbf{s}_2+\Delta\mathbf{s}_{22})}{\lambda}} & \cdots & e^{-j2\pi\frac{\mathbf{r}_2\cdot(\mathbf{s}_q+\Delta\mathbf{s}_{q2})}{\lambda}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j2\pi\frac{\mathbf{r}_p\cdot(\mathbf{s}_1+\Delta\mathbf{s}_{1p})}{\lambda}} & e^{-j2\pi\frac{\mathbf{r}_p\cdot(\mathbf{s}_2+\Delta\mathbf{s}_{2p})}{\lambda}} & \cdots & e^{-j2\pi\frac{\mathbf{r}_p\cdot(\mathbf{s}_q+\Delta\mathbf{s}_{qp})}{\lambda}} \end{bmatrix}$$

$$\mathbf{A}' = \begin{bmatrix} e^{-j2\pi\frac{\mathbf{r}_1\cdot\Delta\mathbf{s}}{\lambda}} & 0 & \cdots & 0 \\ 0 & e^{-j2\pi\frac{\mathbf{r}_2\cdot\Delta\mathbf{s}}{\lambda}} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-j2\pi\frac{\mathbf{r}_p\cdot\Delta\mathbf{s}}{\lambda}} \end{bmatrix} \mathbf{A}$$

$$\mathbf{R}(\boldsymbol{\theta}) = \mathbf{G}\mathbf{A}\boldsymbol{\Sigma}_{\mathrm{s}}\mathbf{A}^H\mathbf{G}^H + \boldsymbol{\Sigma}_{\mathrm{n}}$$

A solution: using the deterministic constrains

$$\mathbf{R}(\boldsymbol{\theta}) = \mathbf{G}\boxed{\mathbf{A}\boldsymbol{\Sigma}_{\mathrm{s}}\mathbf{A}^H}\mathbf{G}^H + \boldsymbol{\Sigma}_{\mathrm{n}}$$

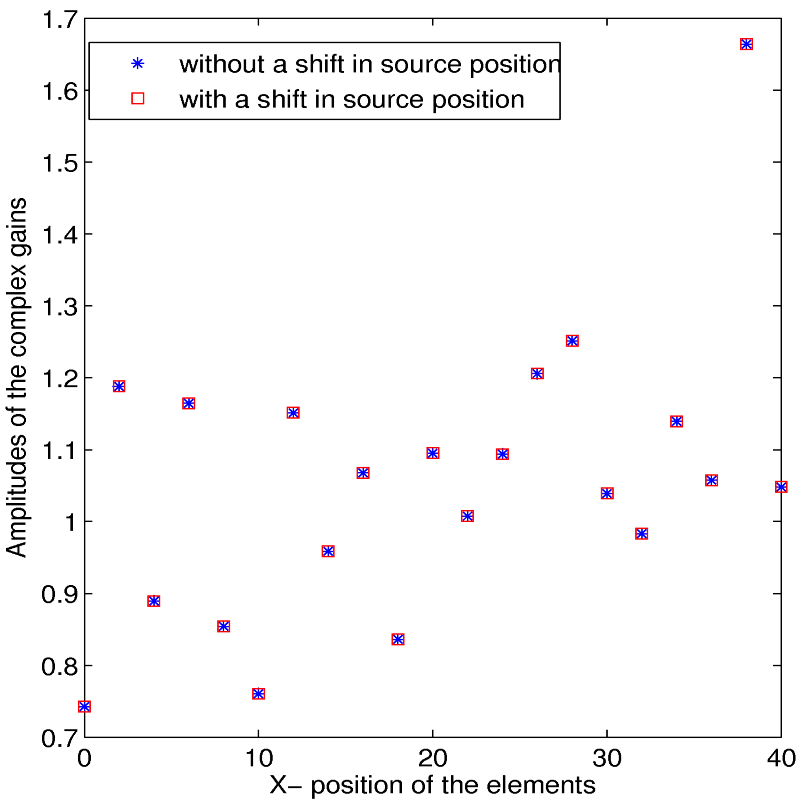
$$\mathbf{G} = \begin{bmatrix} \gamma_1 e^{j(\phi_1 - 2\pi\frac{\mathbf{r}_1.\Delta\mathbf{s}}{\lambda})} & 0 & \cdots & 0 \\ 0 & \gamma_2 e^{j(\phi_2 - 2\pi\frac{\mathbf{r}_2.\Delta\mathbf{s}}{\lambda})} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma_p e^{j(\phi_p - 2\pi\frac{\mathbf{r}_p.\Delta\mathbf{s}}{\lambda})} \end{bmatrix}$$

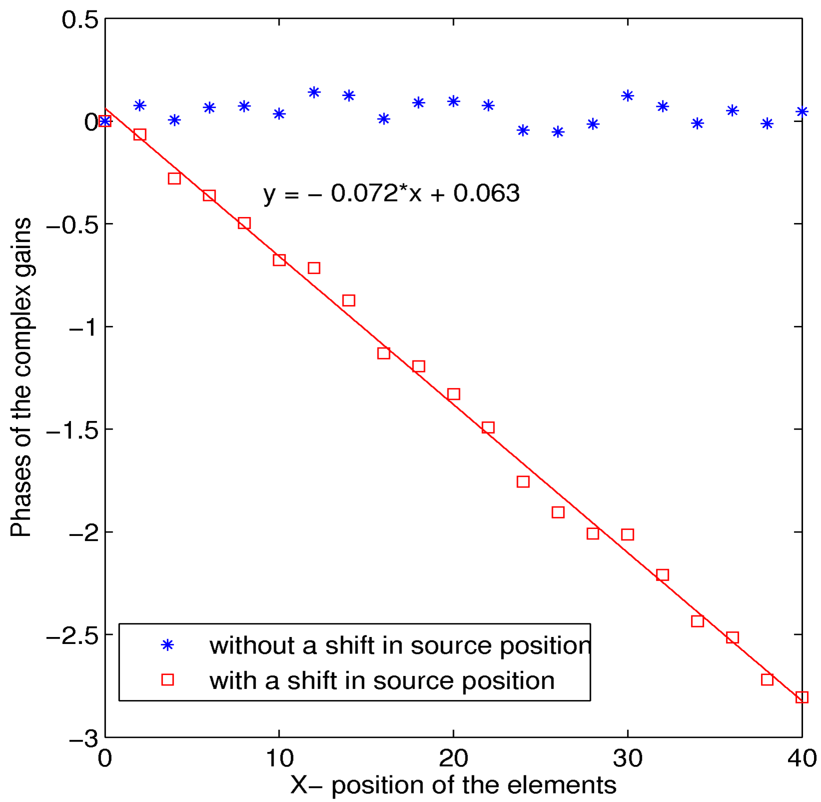
$$\mathbf{g}' = [\gamma_1 e^{j(\phi_1 \boxed{-2\frac{\pi}{\lambda}(\Delta l x_1 + \Delta m y_1))}}, \gamma_2 e^{j(\phi_2 - 2\frac{\pi}{\lambda}(\Delta l x_2 + \Delta m y_2))}, \ldots, \gamma_p e^{j(\phi_p - 2\frac{\pi}{\lambda}(\Delta l x_p + \Delta m y_p))}]^T$$

$$\mathbf{f}(\boldsymbol{\theta}) = \begin{bmatrix} \sum_{i=1}^{p} \phi_i x_i \\ \sum_{i=1}^{p} \phi_i y_i \end{bmatrix} = \begin{bmatrix} \phi \mathbf{x}^T \\ \phi \mathbf{y}^T \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \mathbf{0}_{2\times 1}$$

We always calibrate against a single source to reduce the parameters space (Kazemi et al. 2011). In this way, we do not have the unitary ambiguity either.

A simple example: self-cal results





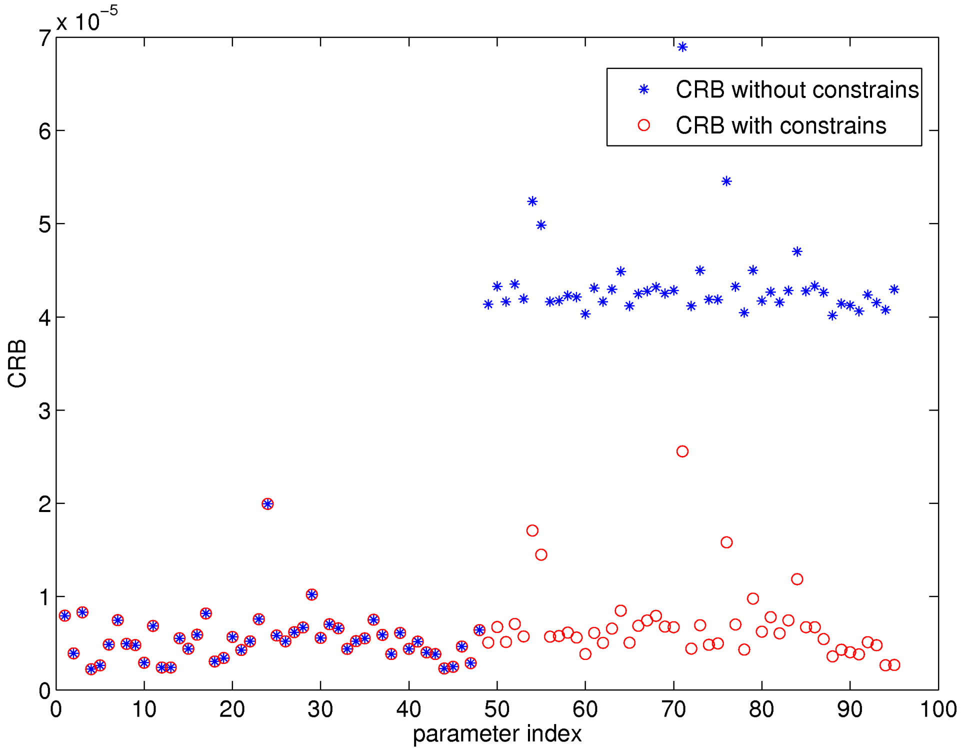
$$\Delta\phi = 0.2, \text{ where } \Delta\phi < 2\pi$$

$$\Delta l = 0.02$$

$$f = 170\text{MHz, thus } \lambda = 1.7635m$$

$$-2\pi\frac{\Delta l}{\lambda} = -0.0713$$

CRB analysis: the constrains indeed optimize the self-cal performance



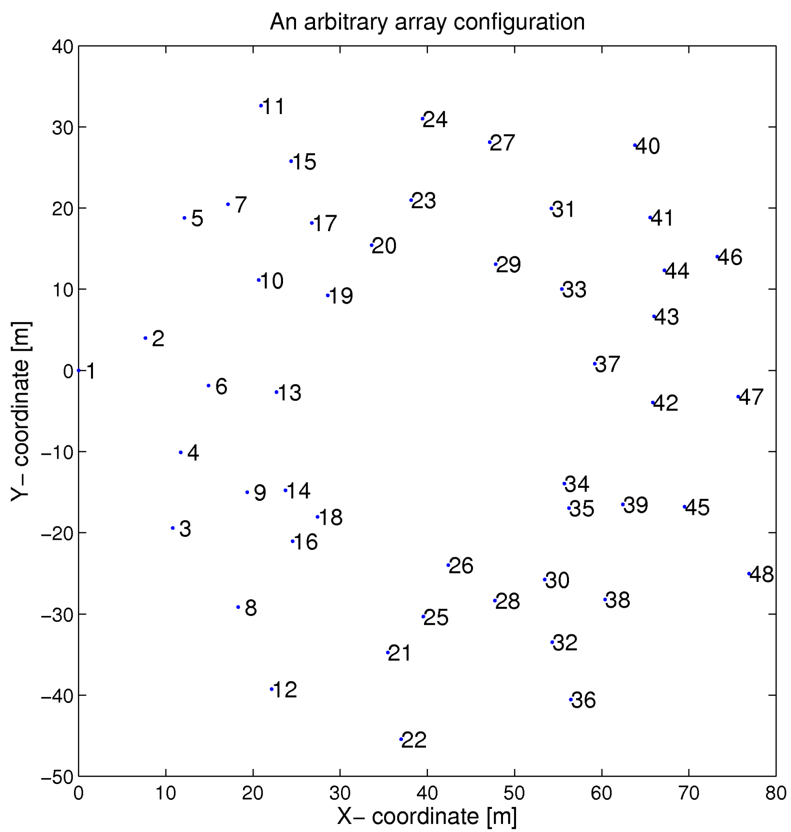
$$\mathbf{f}(\boldsymbol{\theta}) = \begin{bmatrix} \sum_{i=1}^{p} \phi_i x_i \\ \sum_{i=1}^{p} \phi_i y_i \end{bmatrix} = \begin{bmatrix} \boldsymbol{\phi}\mathbf{x}^T \\ \boldsymbol{\phi}\mathbf{y}^T \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \mathbf{0}_{2\times 1}$$

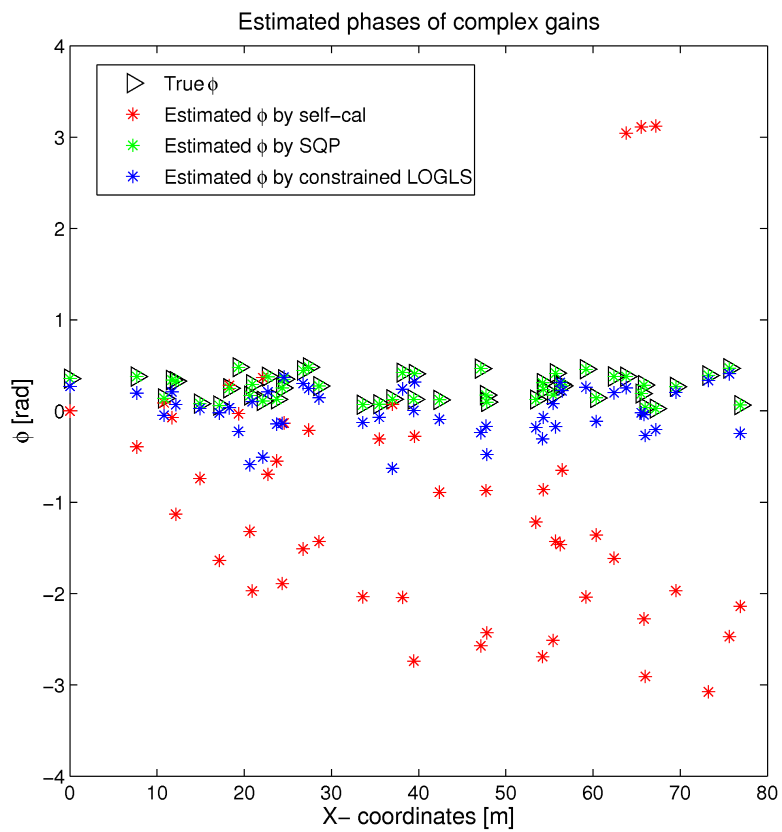
$$\mathbf{C} = E\left((\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^T\right) \geq \frac{1}{N}\mathbf{U}(\mathbf{U}^T\mathbf{J}\mathbf{U})^{-1}\mathbf{U}$$

Based on Wieringa (1991), Boonstra & van der Veen (2003), van der Tol et al. (2007), Wijnholds & van der Veen (2009), and Stoica & Ng (1998)

Solving a constrained non-linear LS

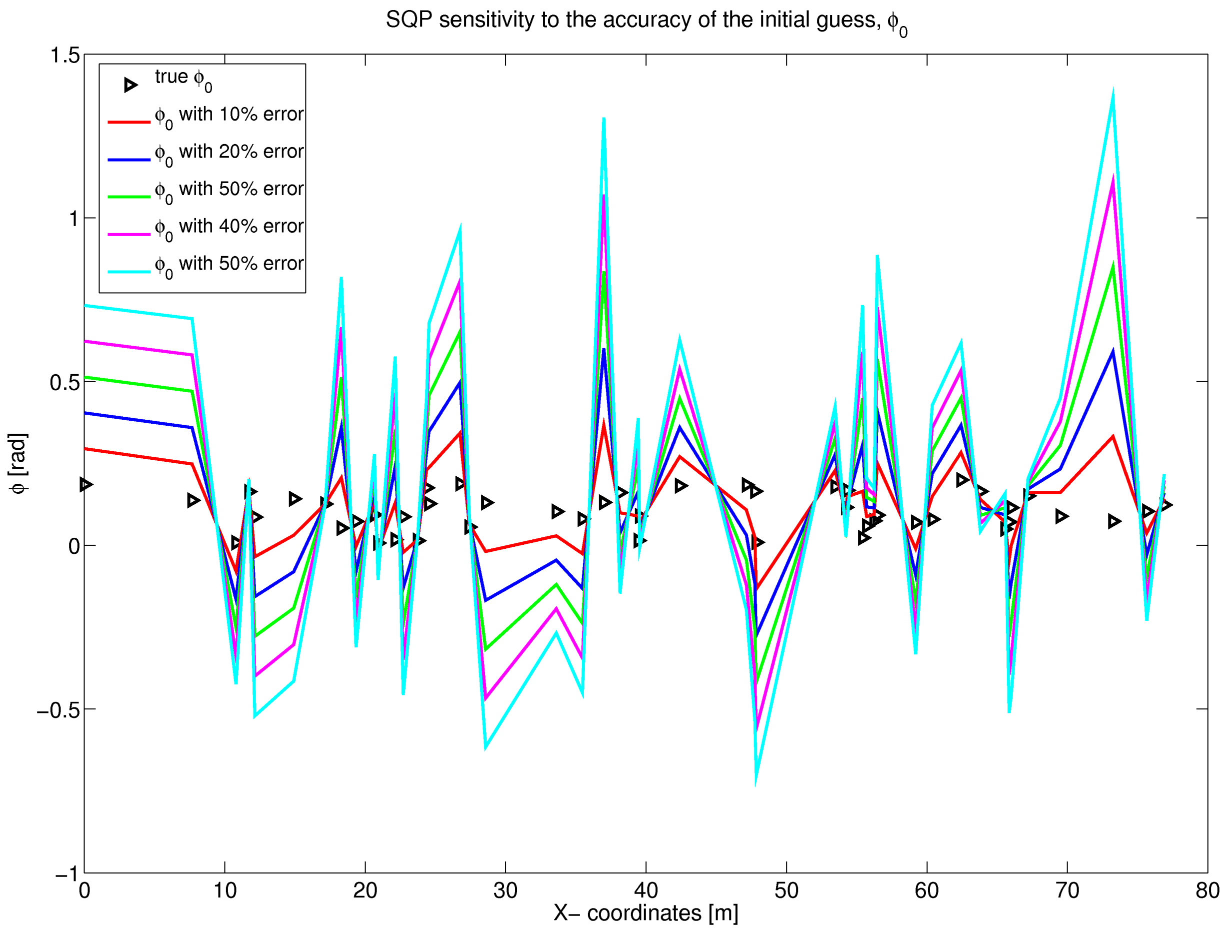
- NOTE: To see the slope over the array and remove it, element numbering is important.





Based on Boonstra & van der Veen (2003), Schittkowski (1985) and many more.

SQP and initial guess accuracy



- These results are after only one iteration.
- We hope to have an initial guess within 10% of accuracy.

Summary & conclusion

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- We suggested a solution for a more accurate self-cal (for an array with arbitrary geometry).
- Using a priori information e.g. constraining some of the parameters (to reduce the parameters degrees of freedom), instead of estimating so many parameters in the calibration process.
- To see the slope and correct for it, element number is important.
- This is a one step.
- **And I have one more un-discussed slide.**

One more slide in favor of redundancy

- There are variants of redundancy calibration that we have not exploited in radio astronomy.
- They seem to be efficient and computationally cheaper than our version of redundancy (Li & Er, 2006)
- All variants of redundancy determine the complex gains up to a scalar. Why don't we consider normalizing the gain of our array elements in their design (especially for the sake of constraining the amplitudes as well)?