



UNIVERSITY OF
CAMBRIDGE

LOW-ORDER BEAM MODELS FOR APERTURE ARRAYS

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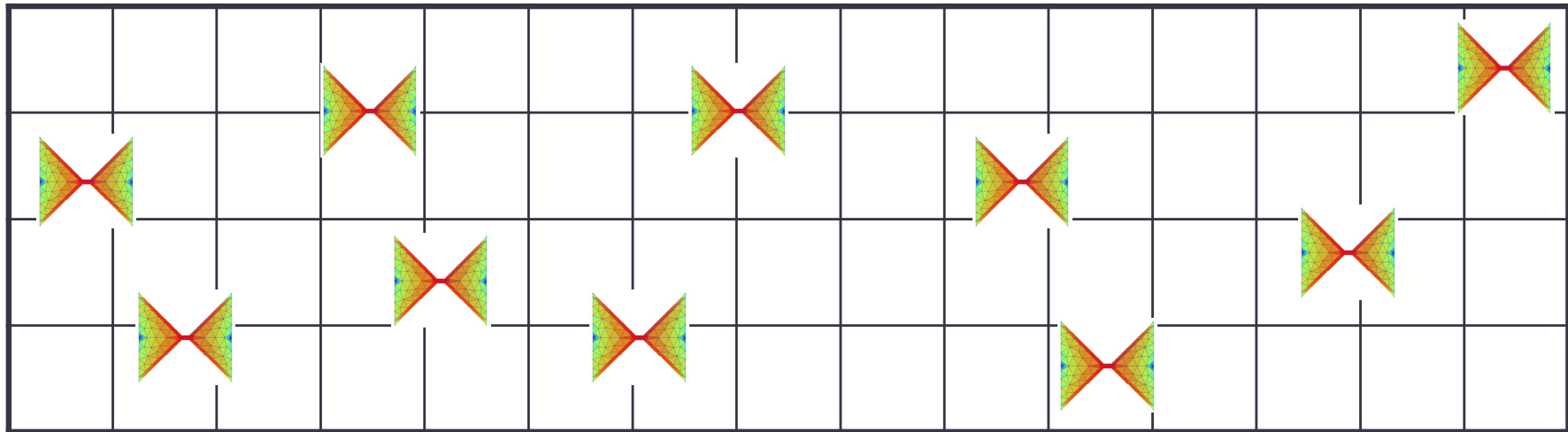
Christophe Craeye

Overview

- MoM-MBF simulations of large irregular arrays
 - We have a powerful simulation tool!
- Mutual coupling in irregular arrays
 - What have we learnt from that tool?
- Representation of mutual coupling effects
 - Initial ideas to model the MC effects
- Low-order models for station beams
 - Where are we right now?
- Conclusions
 - And where do we go from now...?

MoM-MBF simulation of large irregular arrays

- Based on **Method of Moments + MBFs** (CBFs) and the interpolation technique presented in [1], where the computation of interactions between MBFs is carried out by **interpolating exact data obtained on a simple grid.**



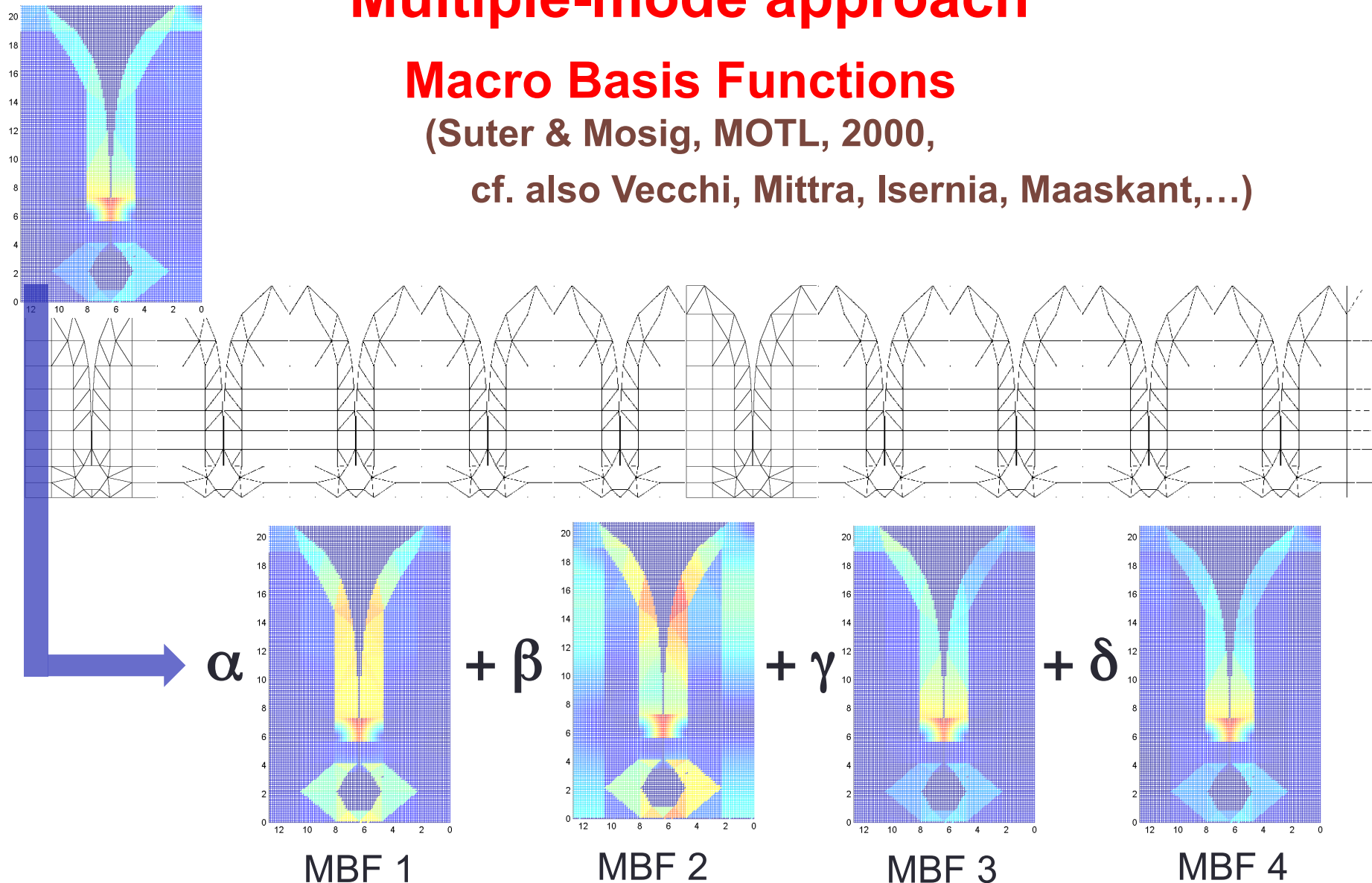
[1] D. Gonzalez-Ovejero and C. Craeye, "Fast computation of Macro Basis Functions interactions in non-uniform arrays," in *Proc. IEEE AP-S Soc. Int. Symp.*, San Diego, CA, Jul. 2008.

Multiple-mode approach

Macro Basis Functions

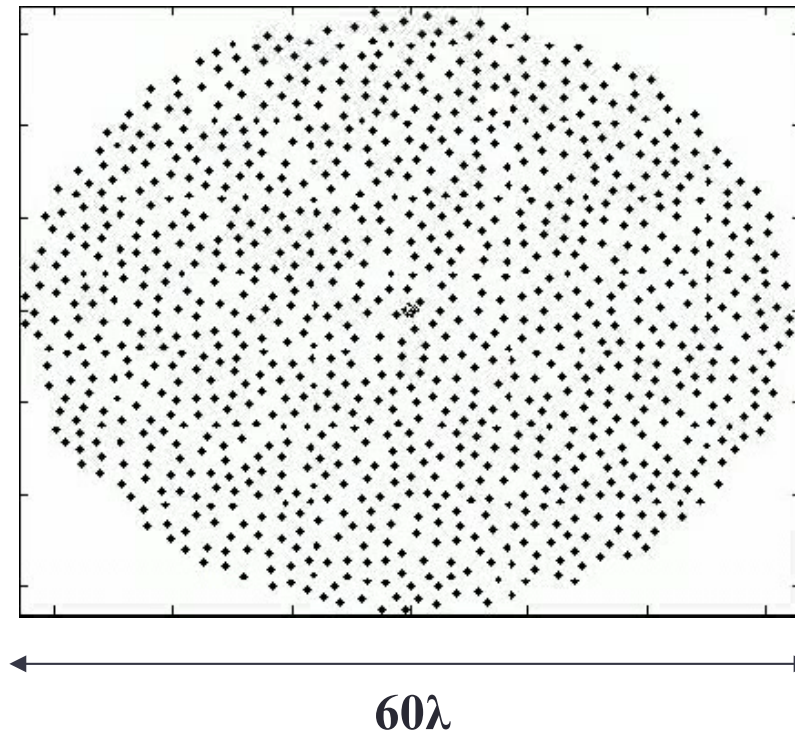
(Suter & Mosig, MOTL, 2000,

cf. also Vecchi, Mittra, Isernia, Maaskant,...)



Radius of Influence

A so-called “radius of influence” (RI) is defined for every antenna in the array.

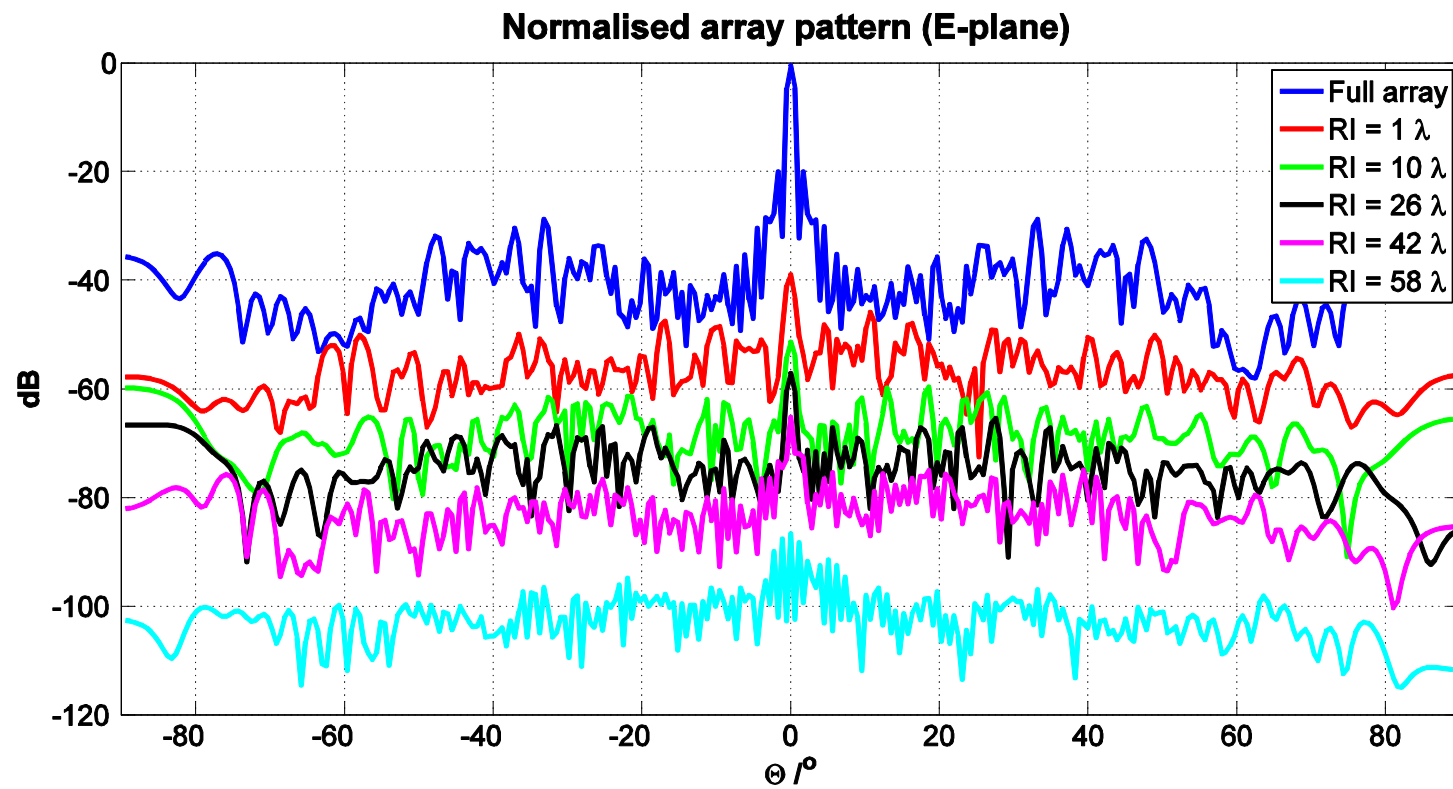


The interactions between MBFs are computed only within that region reducing drastically the number of unknowns.

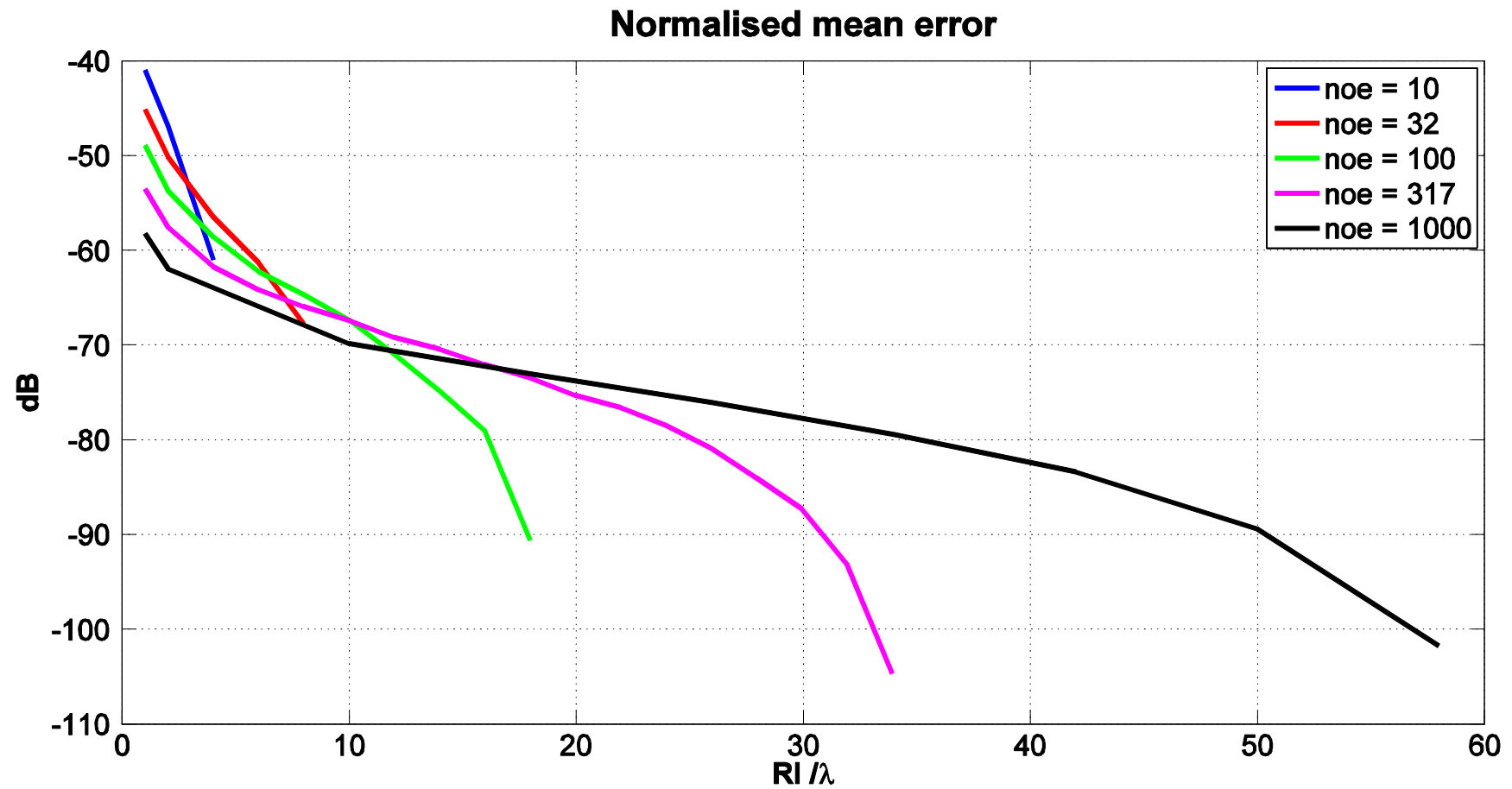
The system is solved for the whole array but now we have a sparse matrix.

$$e_i(\theta, \phi) = 10 \log_{10} \left(\frac{\left| E_{full}^h(\theta, \phi) - E_i^h(\theta, \phi) \right|^2 + \left| E_{full}^v(\theta, \phi) - E_i^v(\theta, \phi) \right|^2}{\max \left(\left| E_{full}^h(\theta, \phi) \right|^2 + \left| E_{full}^v(\theta, \phi) \right|^2 \right)} \right)$$

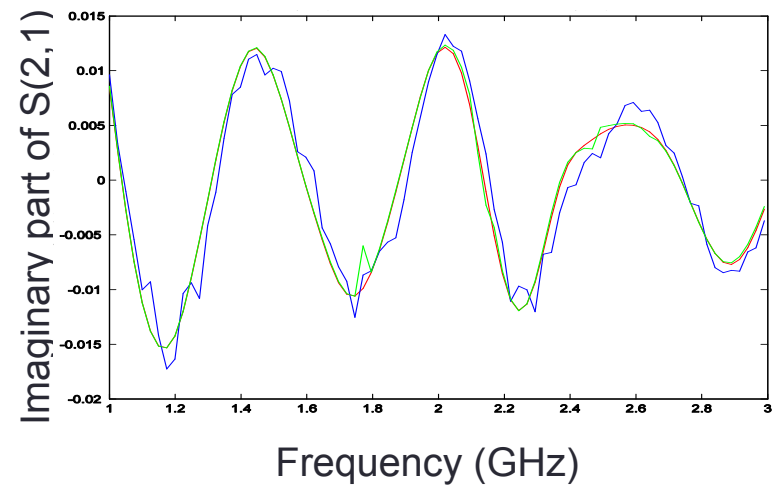
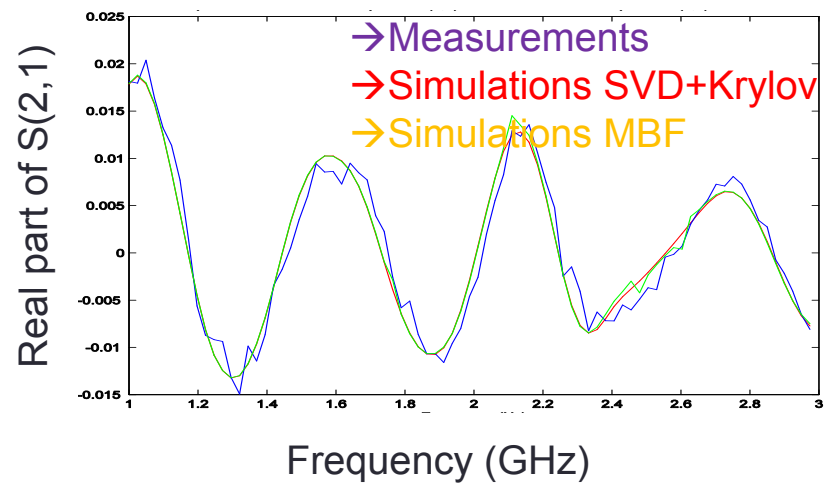
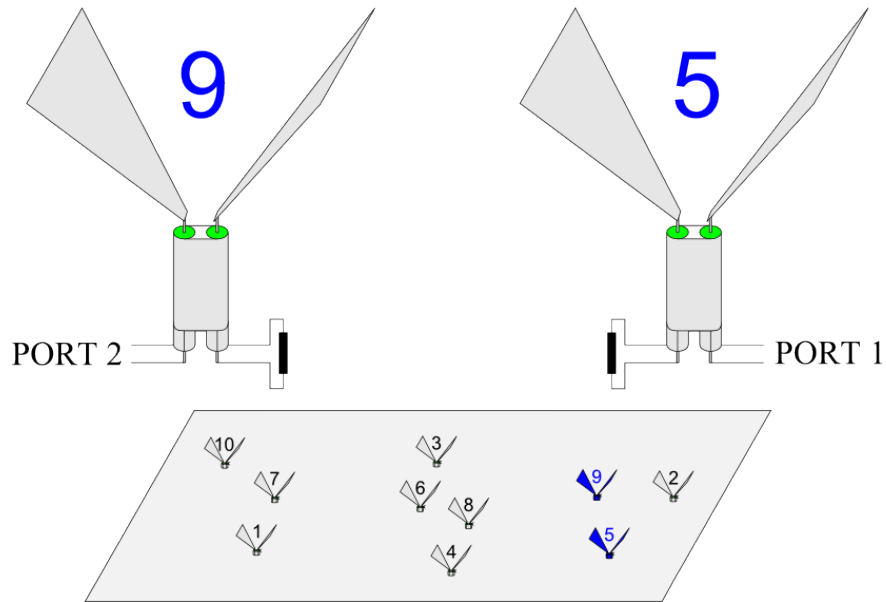
1000 elements



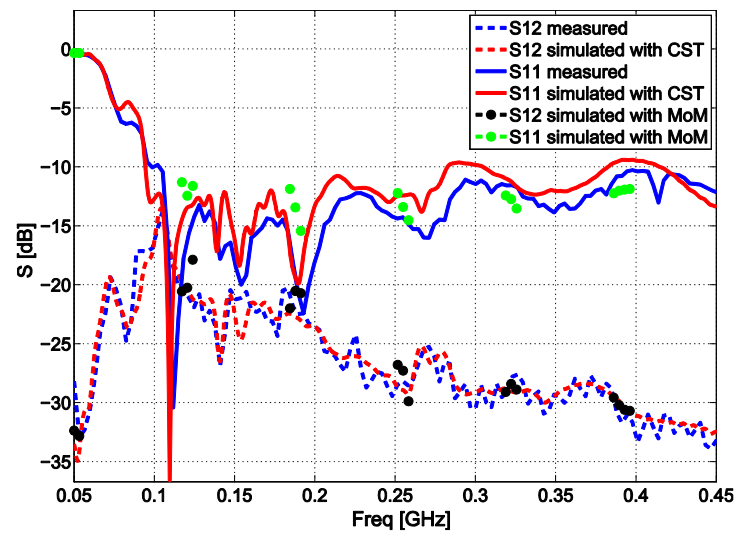
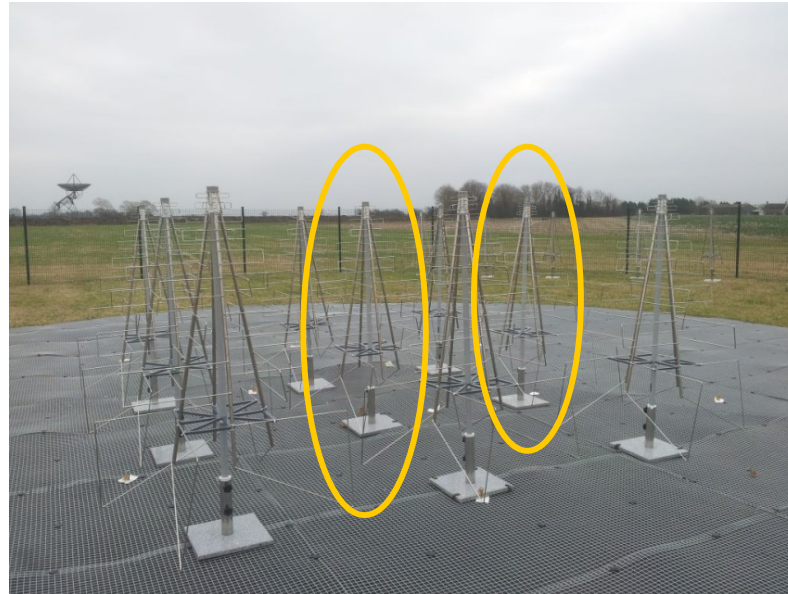
Edge effect



Validation



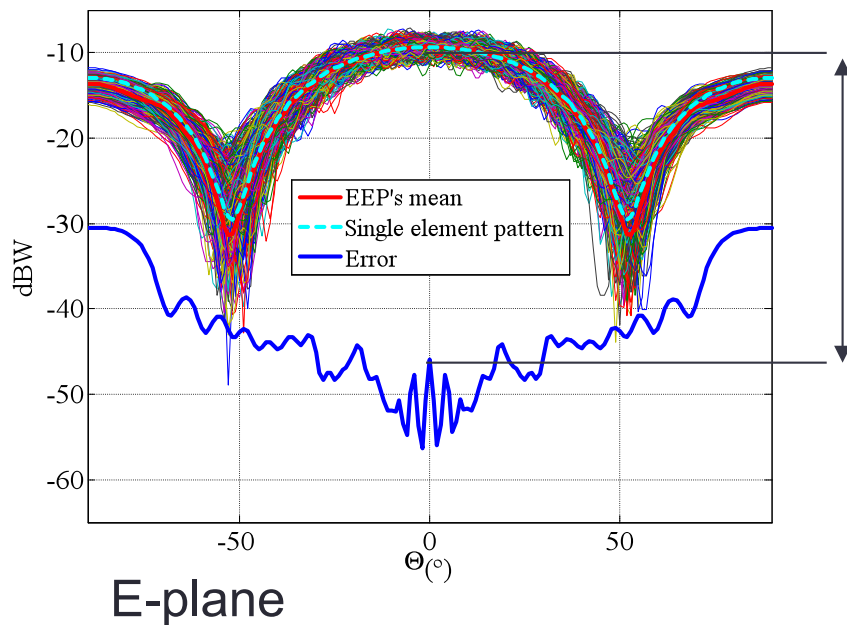
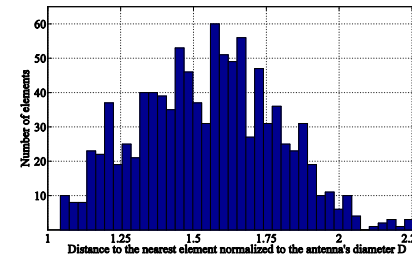
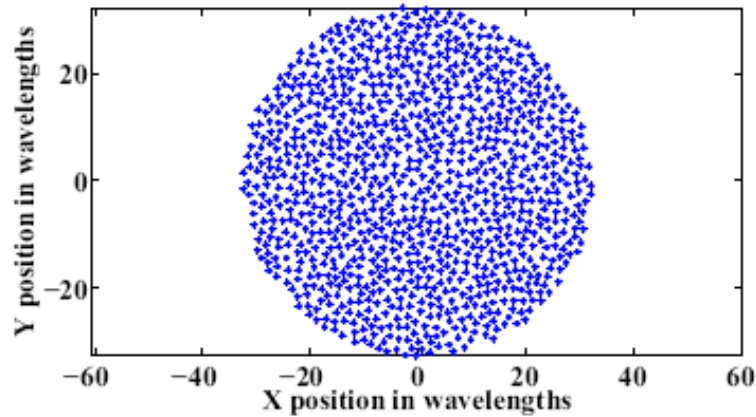
Validation AAV/S0 array



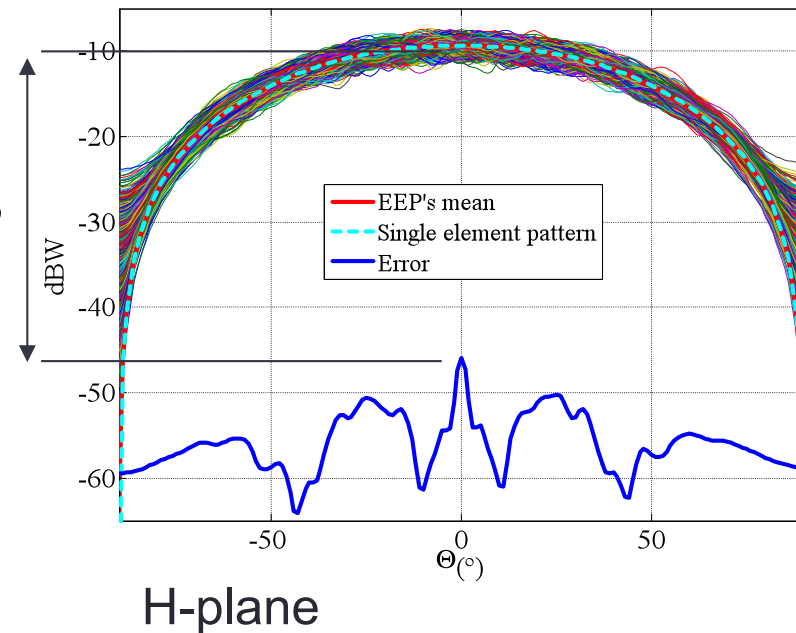
Mutual coupling in irregular arrays

$$e = 10 \log_{10} \left(\left| E_{mean}(\theta, \phi) - E_{single}(\theta, \phi) \right|^2 \right)$$

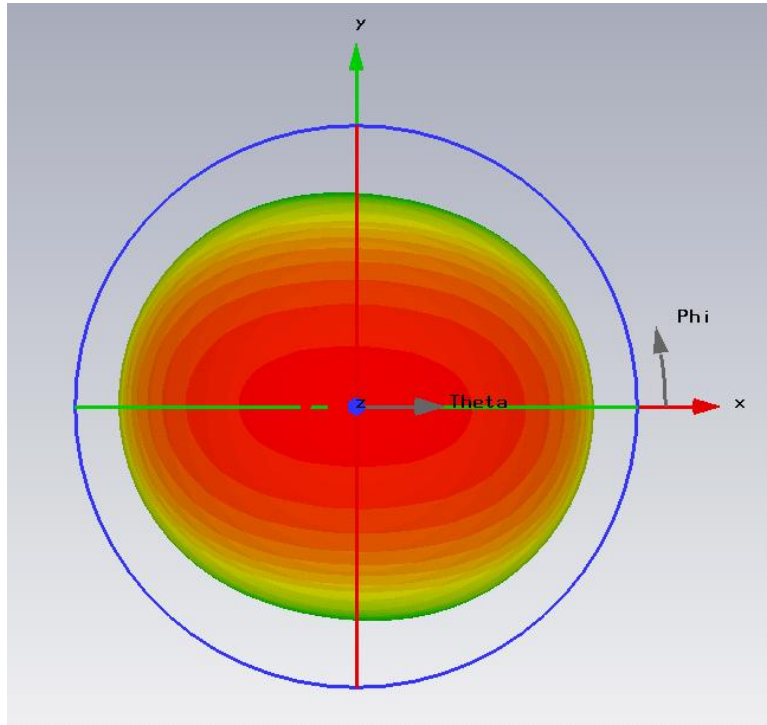
Random configuration



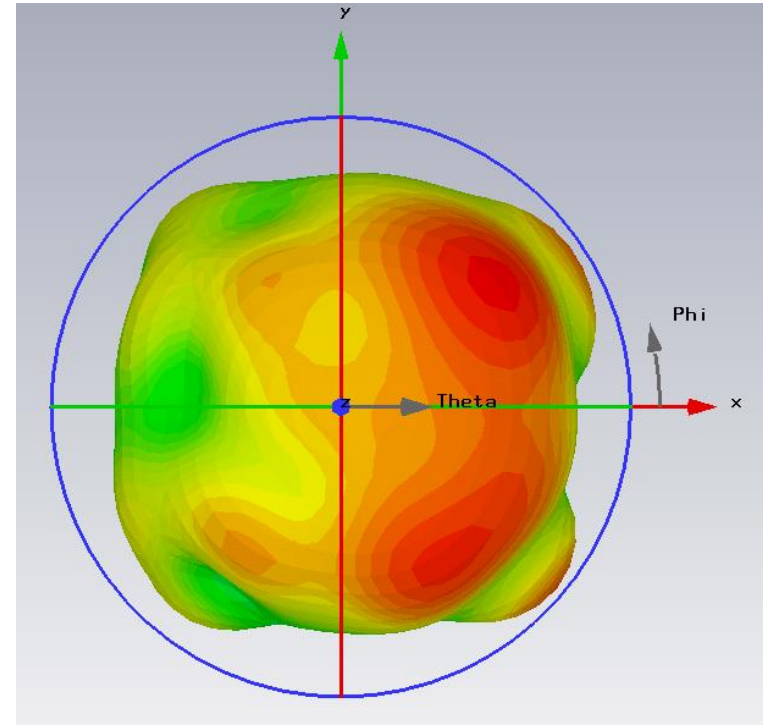
~ 35 dB



$$\text{Station_Beam} = \text{Array_Factor} \times \text{Average_EEP}$$

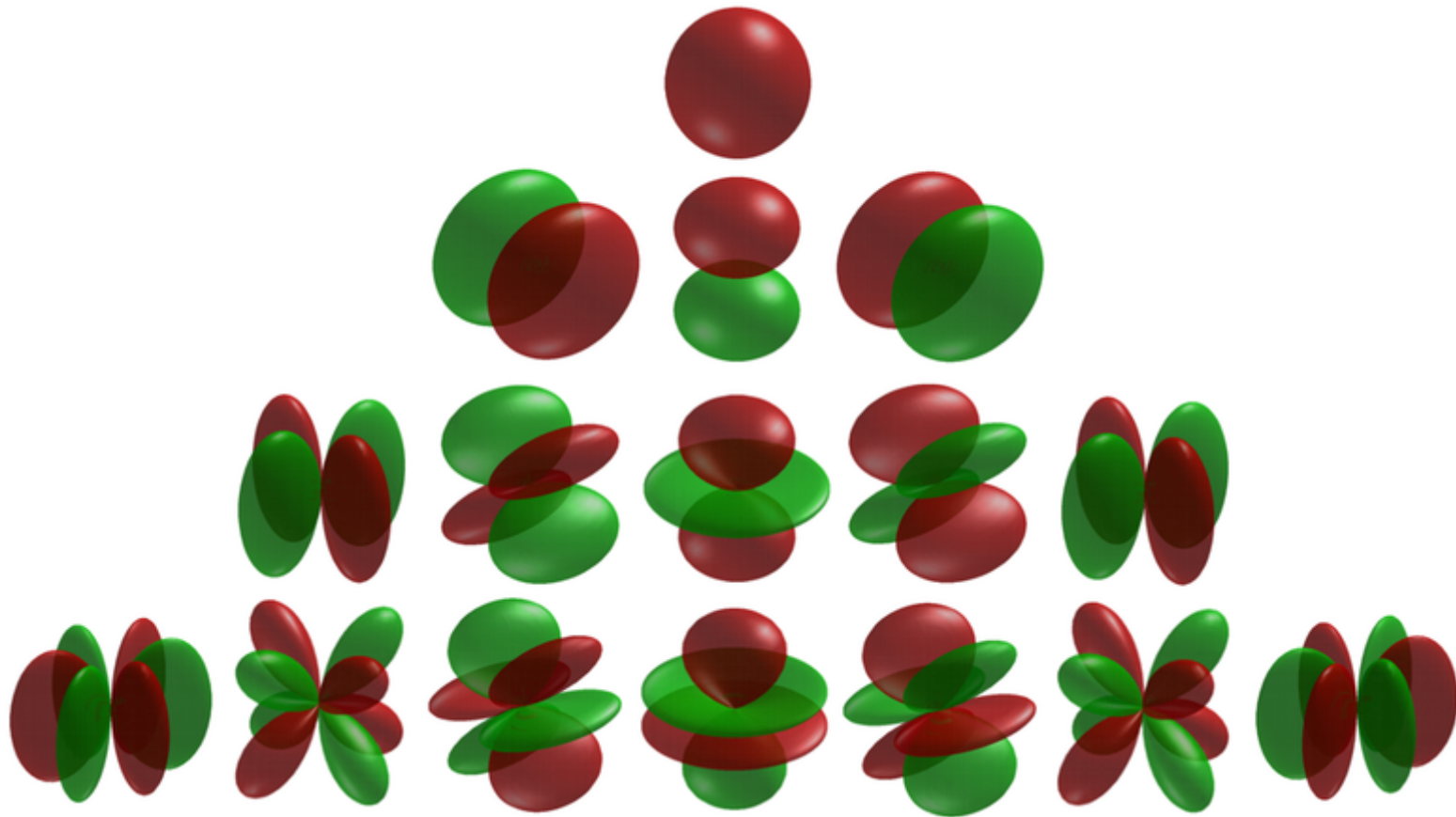


Average Embedded Element Pattern

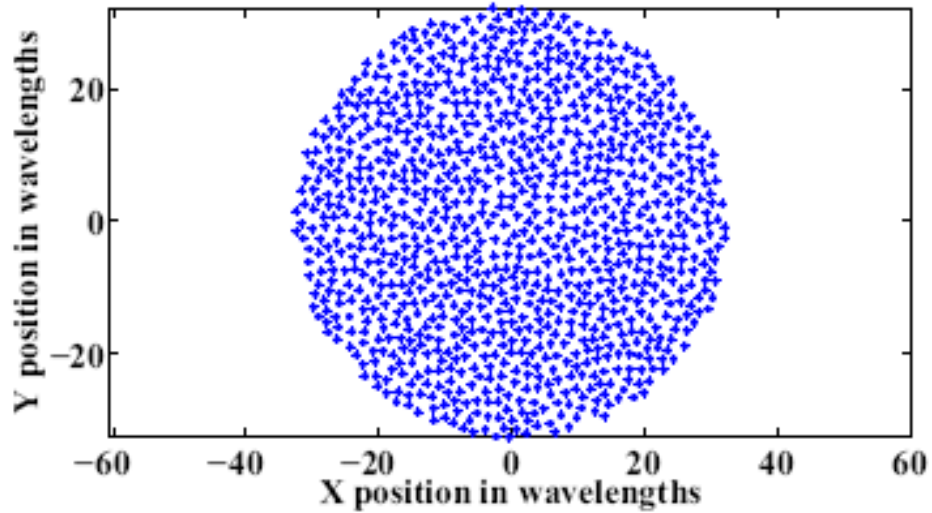


Pattern of a given element

Representation of mutual coupling effects

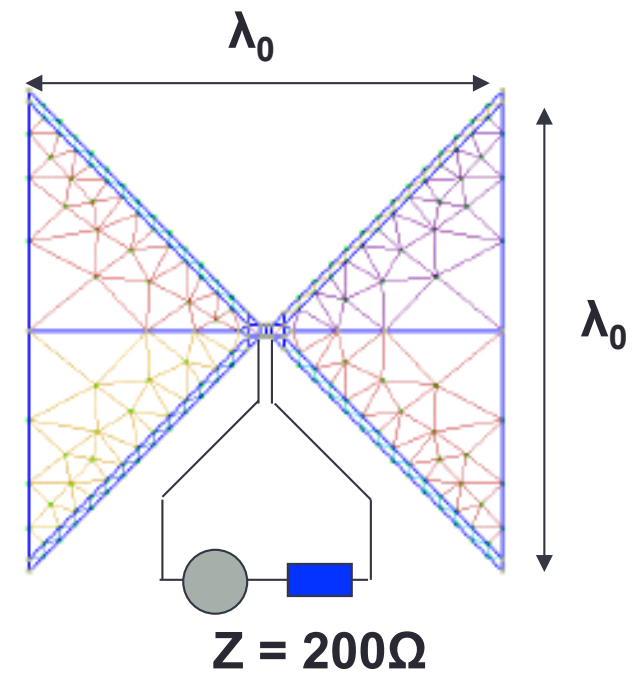
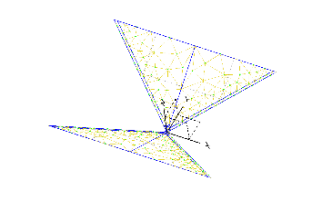


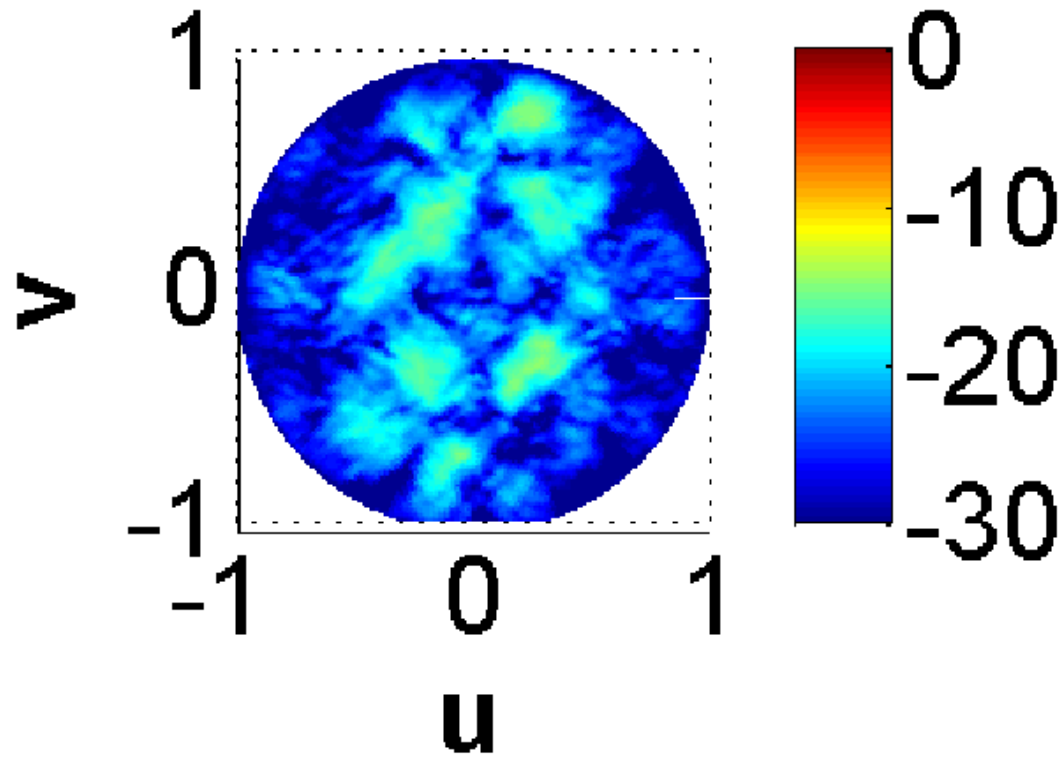
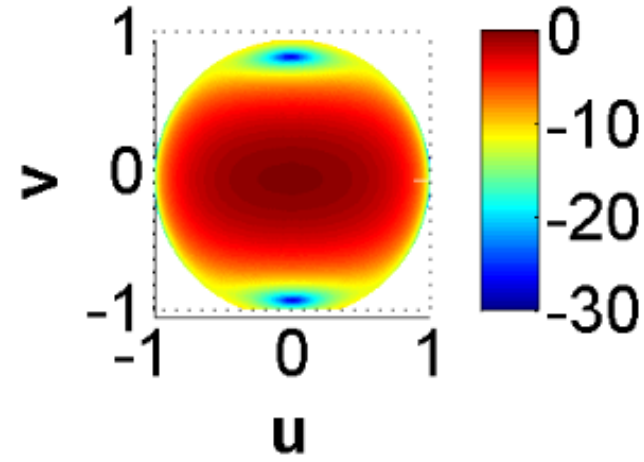
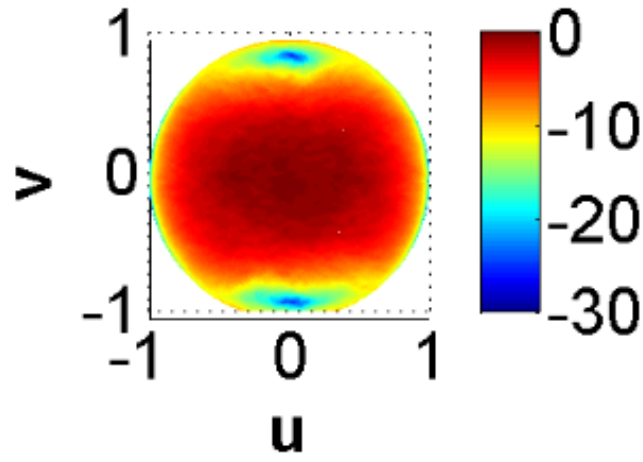
Spherical Harmonics



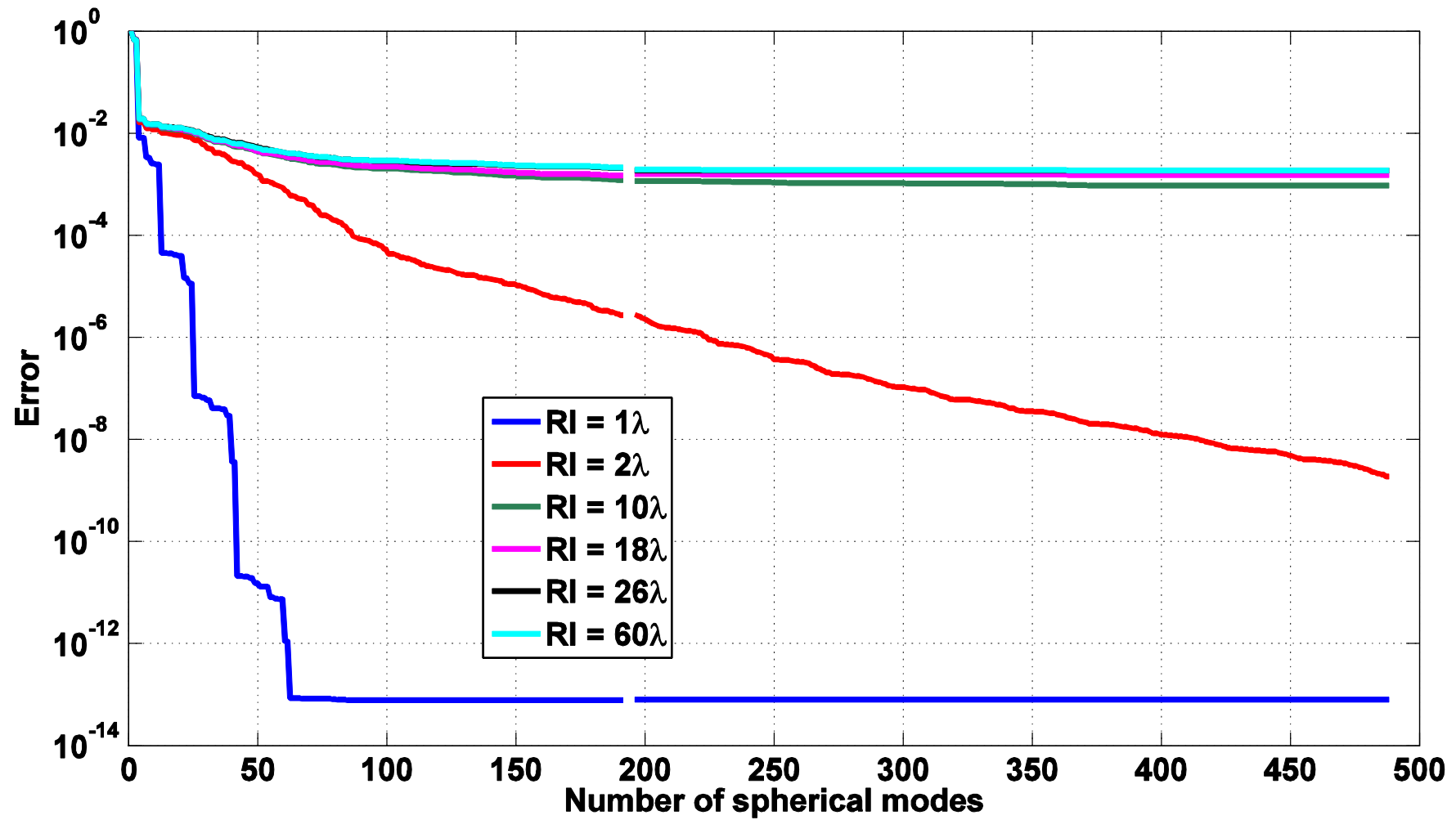
- Array radius = $30\lambda_0$.
- Number of elements = 1000.

- Distance to ground plane = $\lambda_0/4$.
- No dielectric.

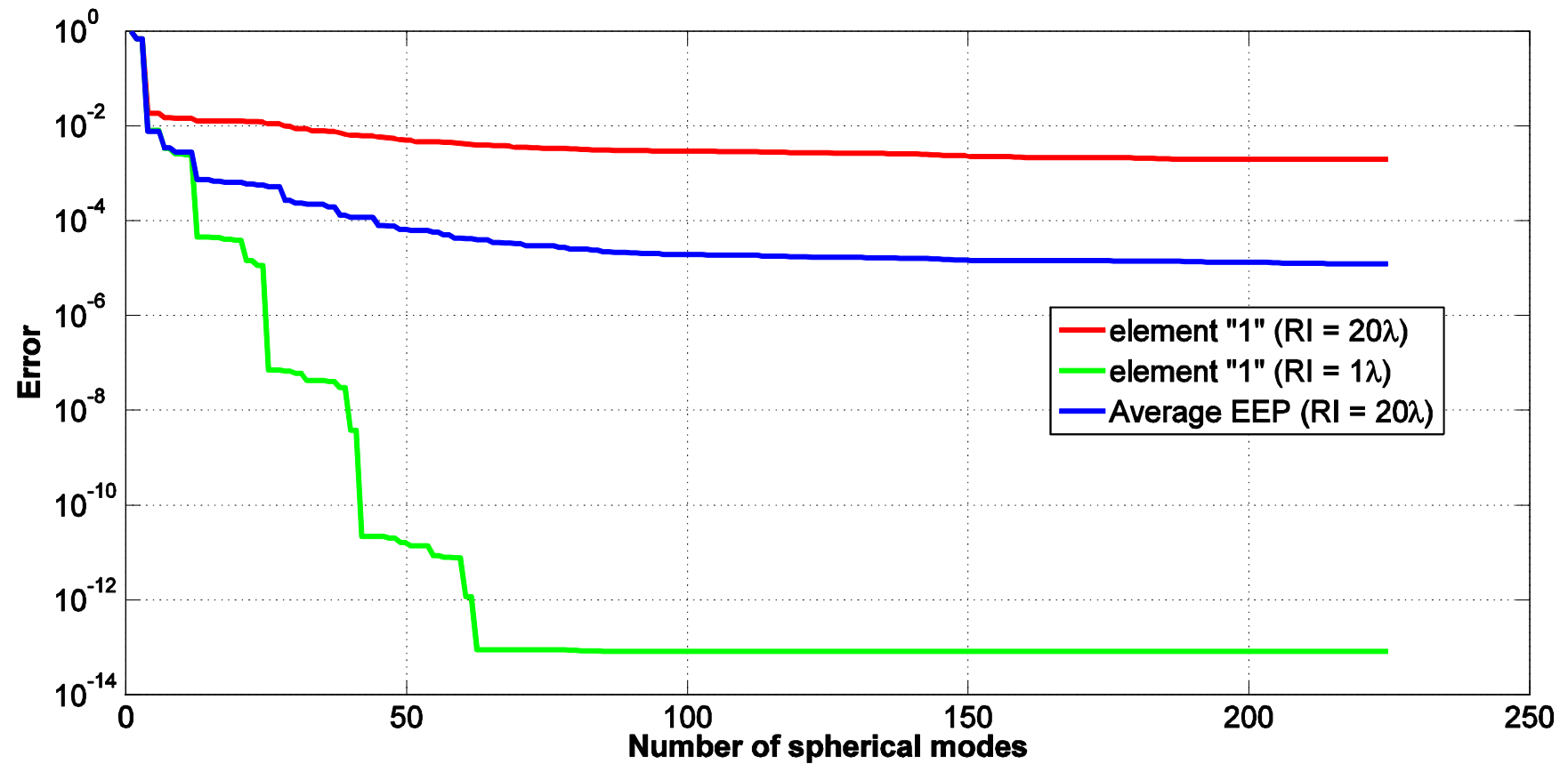




Convergence



Average Embedded Element Pattern



Low-order models for station beams

Goal: pattern representation for all modes of operation at station level.

Too many antennas vs. number of calibration sources

➔ **Calibrate the main beam and first few sidelobes**

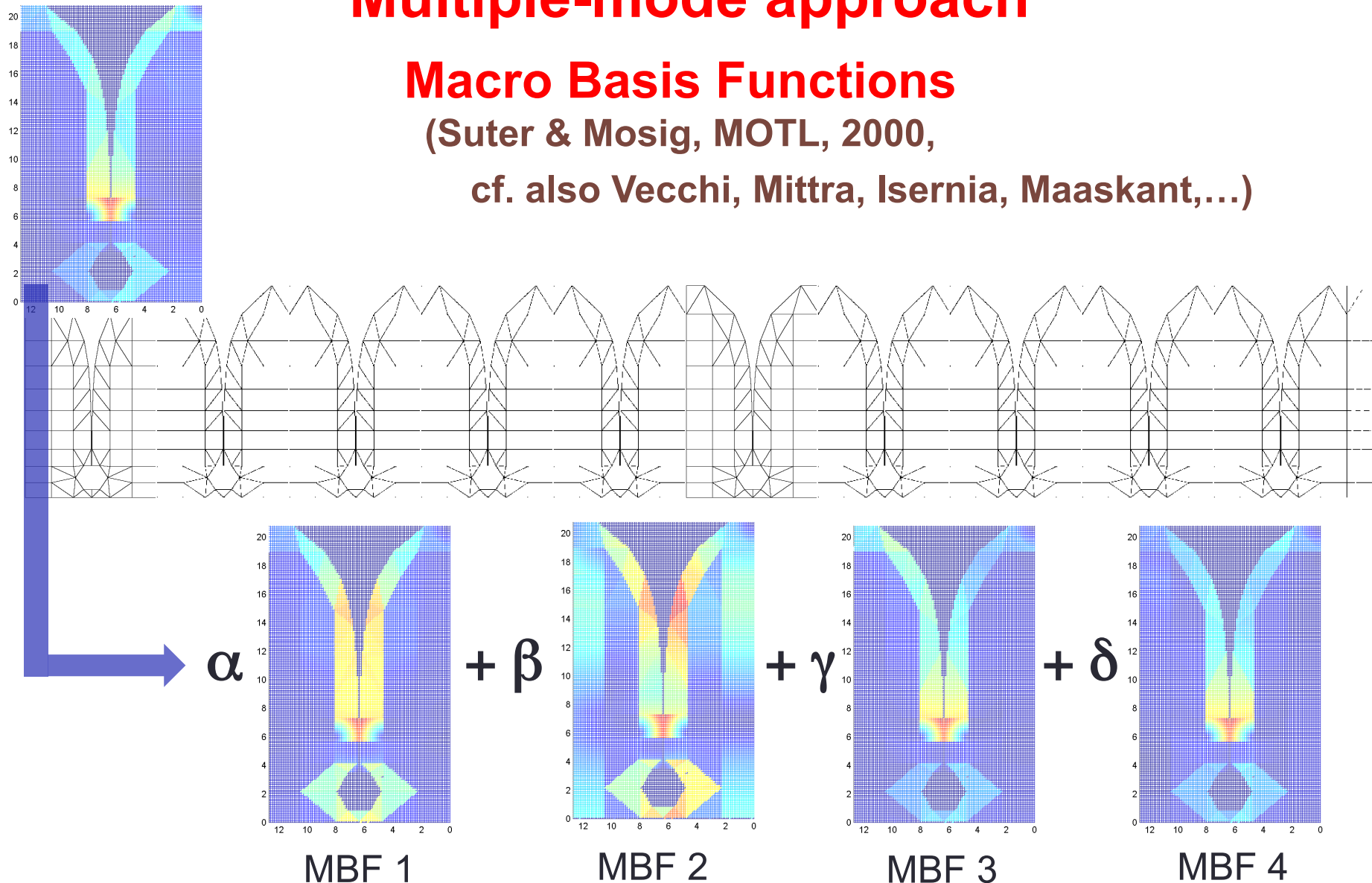
➔ **Compact** representation of patterns, inspired from radiation from apertures, including effects of mutual coupling:
Pattern \neq AF X element pattern !

Multiple-mode approach

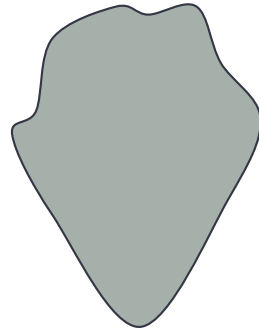
Macro Basis Functions

(Suter & Mosig, MOTL, 2000,

cf. also Vecchi, Mittra, Isernia, Maaskant,...)

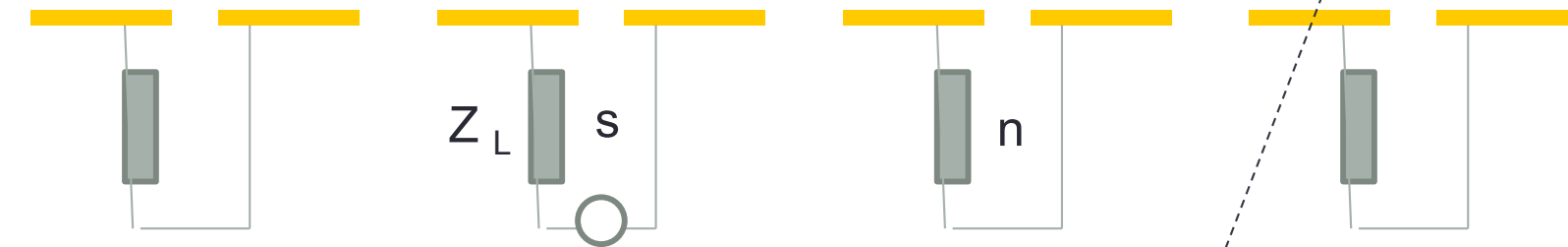


Array factorisation



Index of excited element

$$\vec{J}_{ns} \simeq \sum_{p=1}^P C_{nsp} \vec{J}_p^o$$



Antenna index

MBF 1

C_{111} C_{211} C_{311}

Coefficients for **IDENTICAL** current distribution

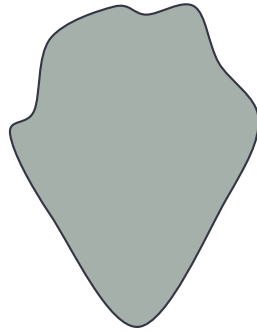
MBF 2

C_{112} C_{212} C_{312}

⋮

$$\vec{F}_s \simeq \sum_{p=1}^P \sum_{s=1}^N A_s \sum_{n=1}^N C_{nsp} e^{j k \hat{u} \cdot \vec{r}_n} \vec{F}_p^o$$

Array factorisation



$$\vec{J}_{ns} \simeq \sum_{p=1}^P C_{nsp} \vec{J}_p^o$$



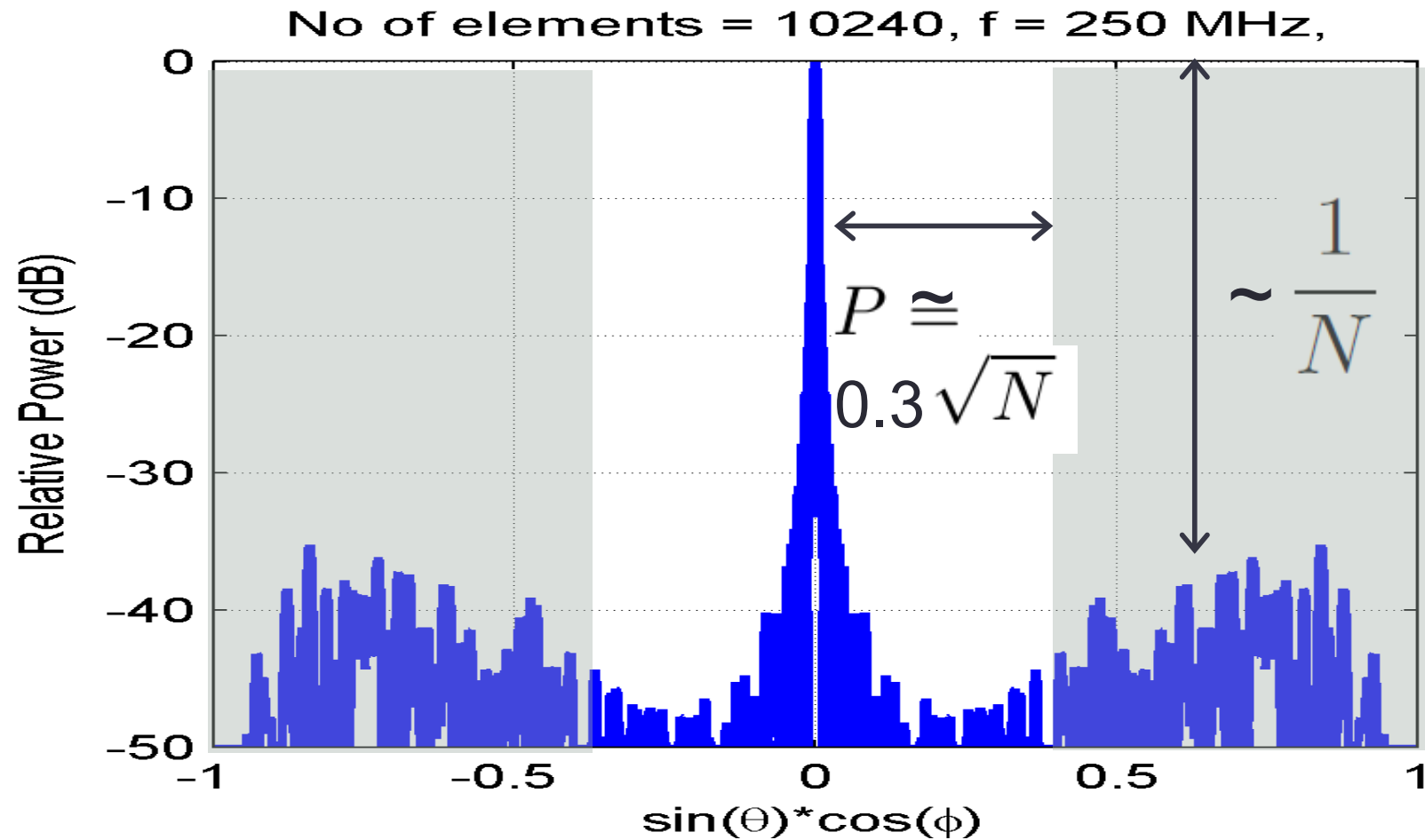
Antenna index

MBF 1	C_{111}	C_{211}	C_{311}	Coefficients for IDENTICAL current distribution
MBF 2	C_{112}	C_{212}	C_{322}	

$$\vec{F}_s \simeq \sum_{p=1}^P \sum_{n=1}^N \left(\sum_{s=1}^N A_s C_{nsp} \right) e^{j k \hat{u} \cdot \vec{r}_n} \vec{F}_p^o \rightarrow \mathbf{AF}_p$$

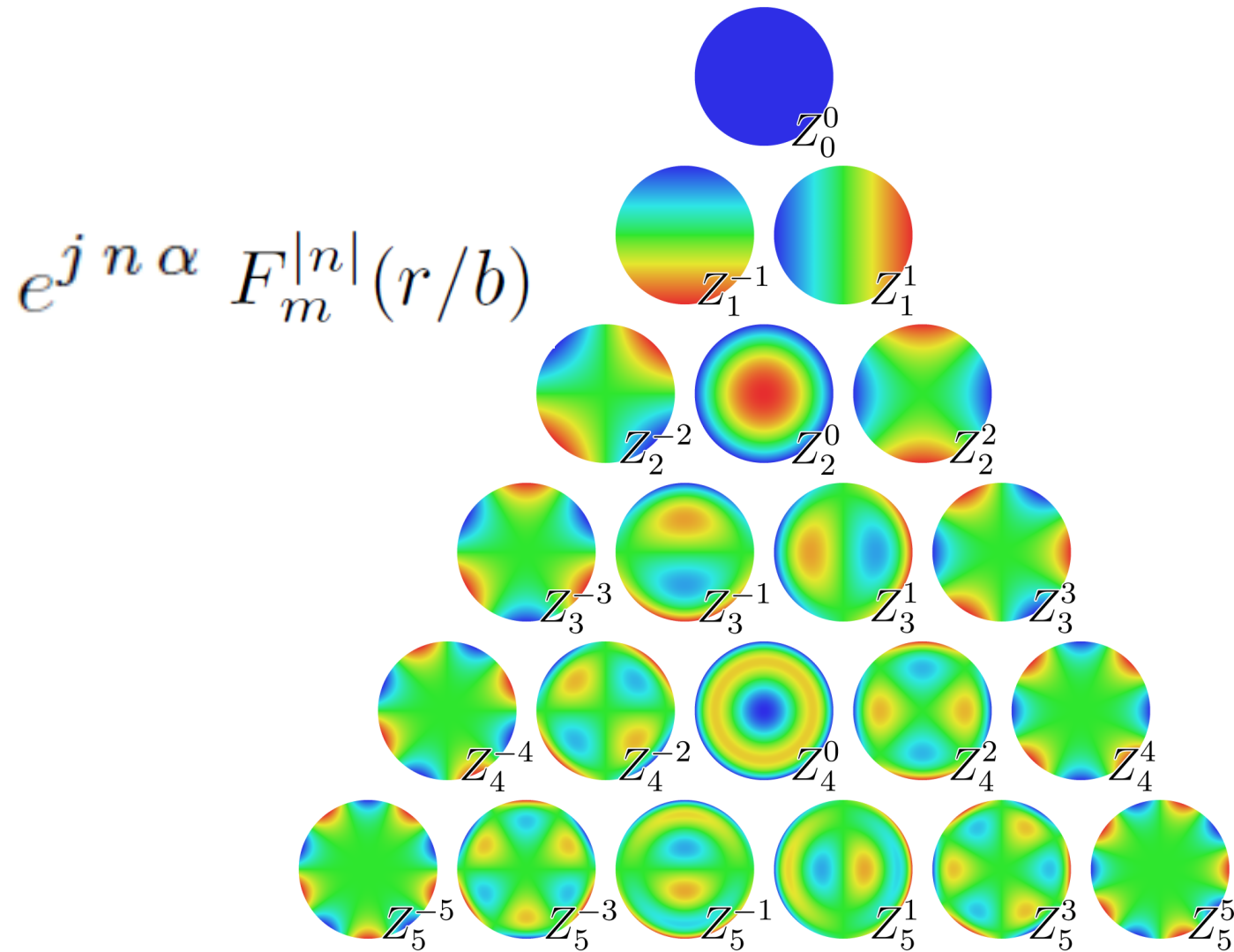
A'_p

Array factor



Continuous aperture regime

Zernike functions



Picture from Wikipedia

Zernike-Bessel decomposition

$$AF(\theta, \phi) = 2 \sum_{n=-N}^N \sum_{m=0}^M (|n| + 2m + 1) B_{mn} j^n e^{j n \phi} \\ (-1)^s \frac{J_{|n|+2m+1}(k b \sin \theta)}{k b \sin \theta}$$

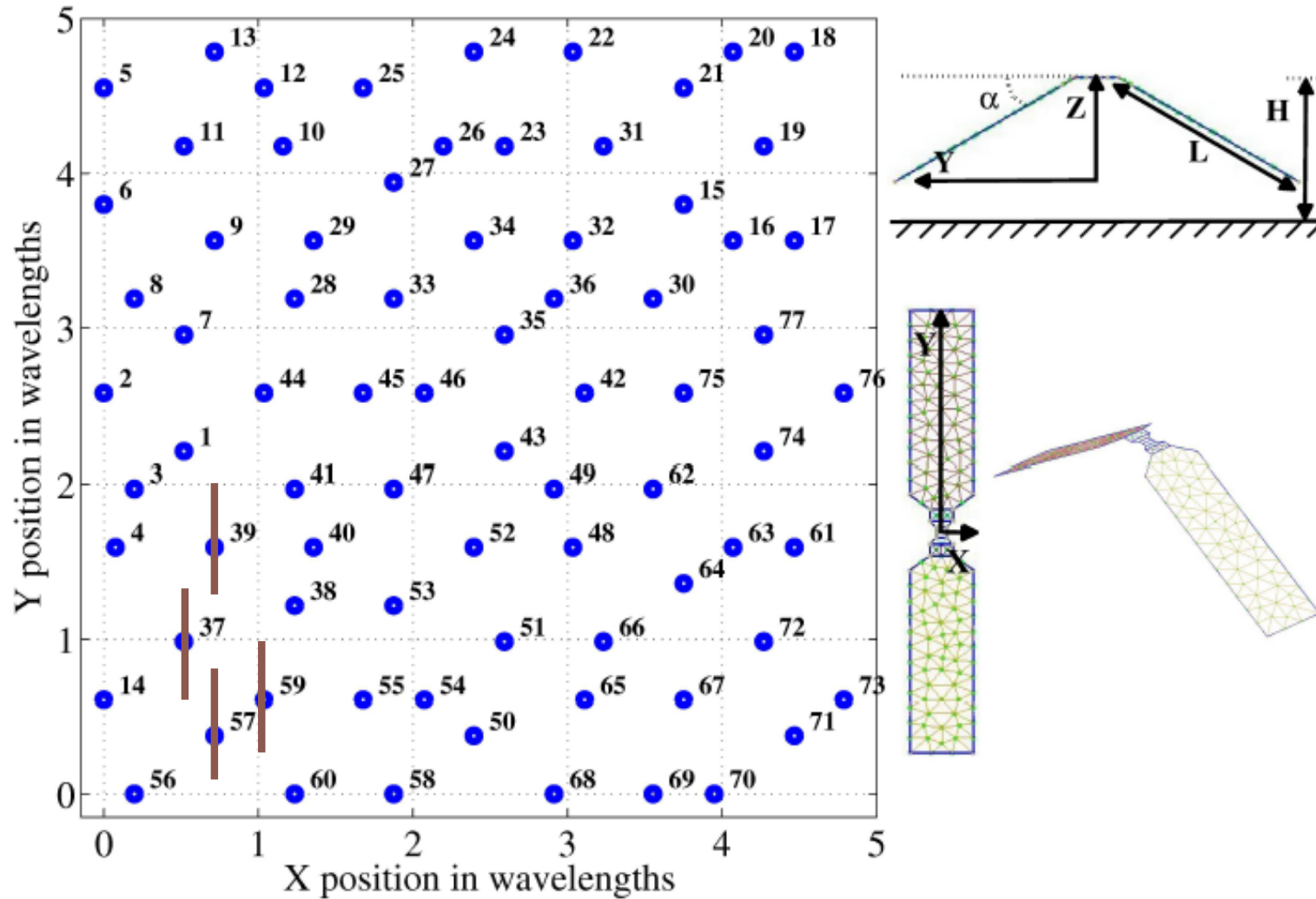
$$B_{mn} = \sum_i A'_i F_m^{|n|}(r/b) e^{-j n \alpha_i}$$

Similar to theory of ~circular apertures:

Y. Rahmat-Samii and V. Galindo-Israel, "Shaped reflector antenna analysis using the Jacobi-Bessel series," IEEE Trans.

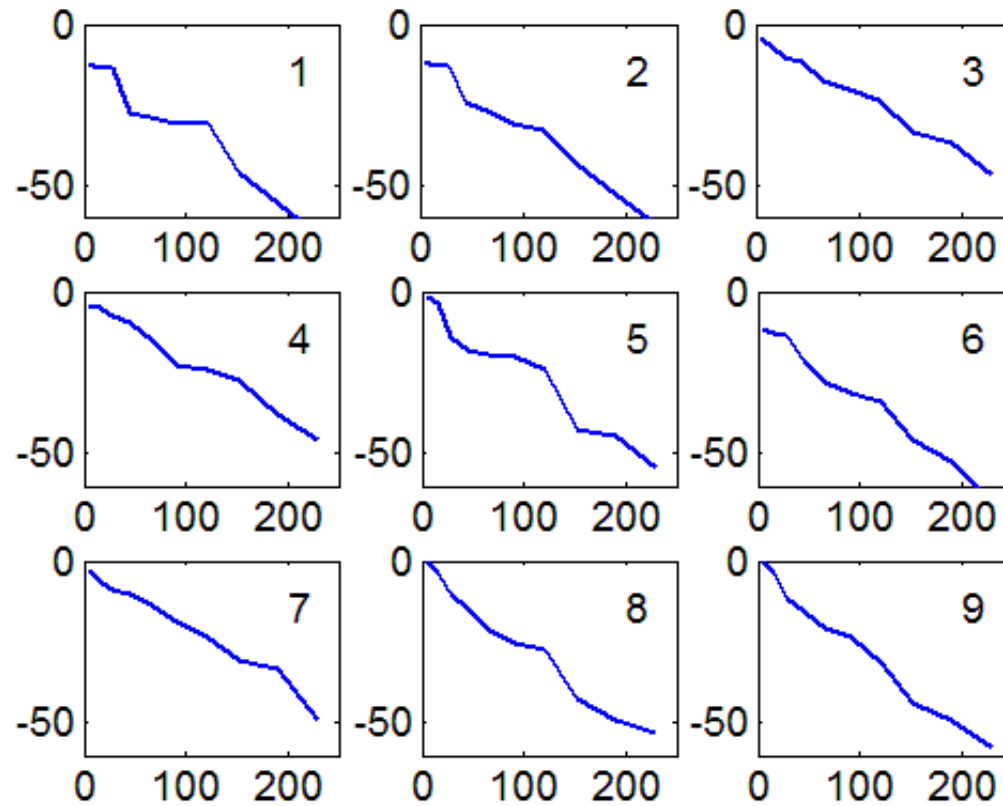
Antennas Propagat., Vol. 28, no.4, pp. 425-435, Jul. 1980.

Array of wideband dipoles



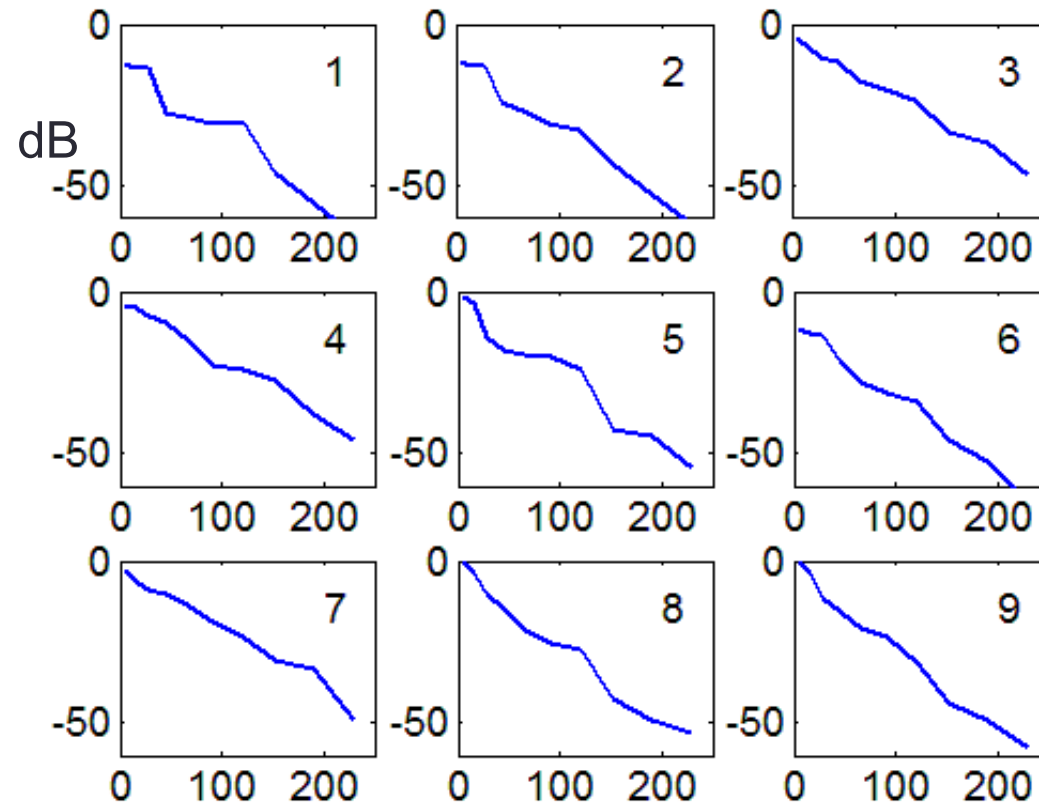
Modeled with 9 MBFs

AF convergence



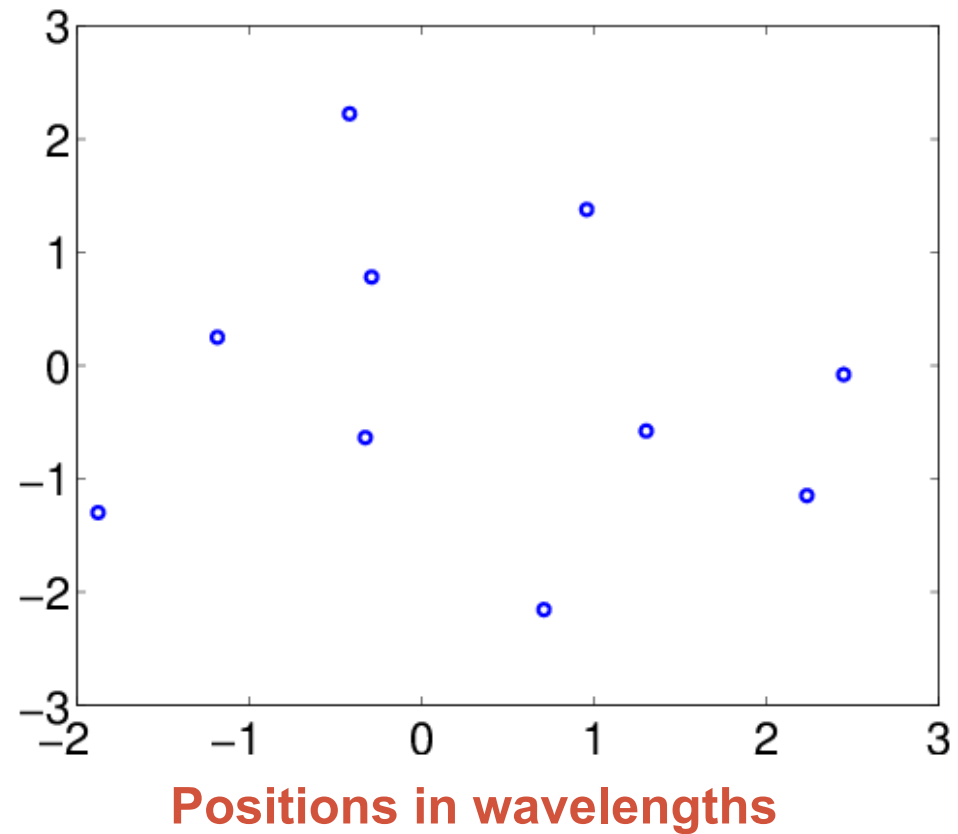
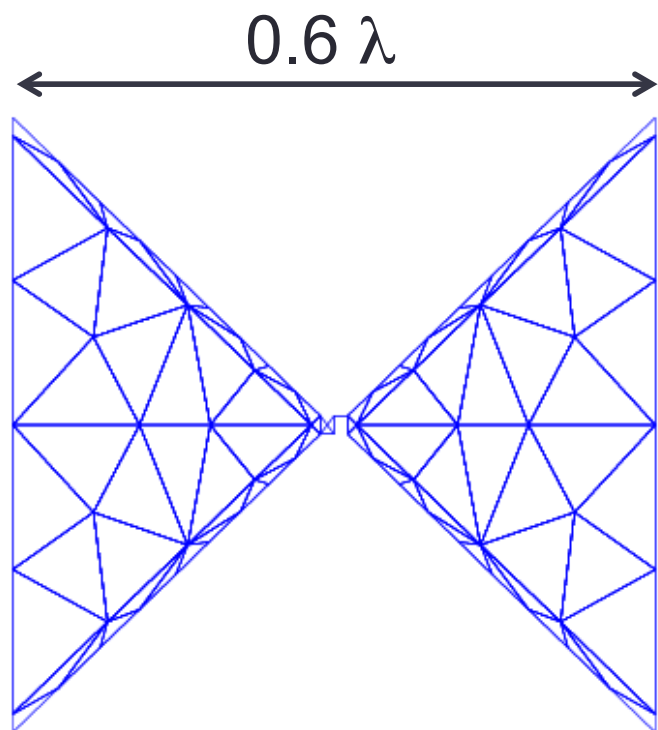
Main beam + 1st sidelobe

Reduce number of terms ?



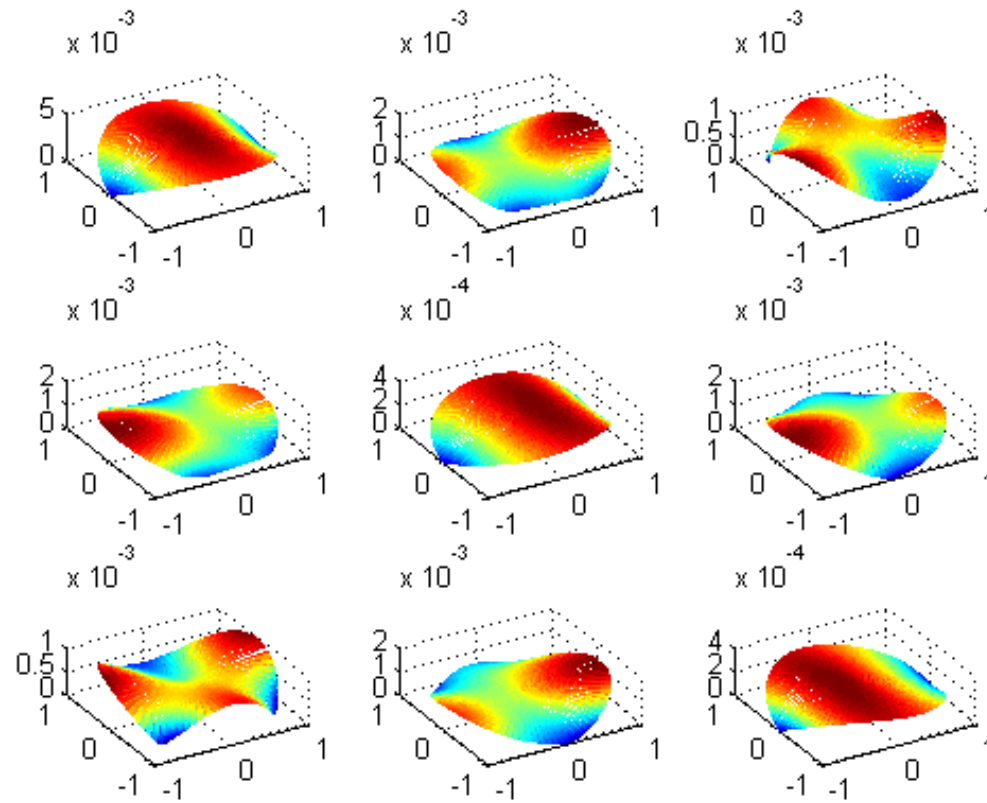
**9 is still too many:
Exploit similarity between MBF patterns**

Small array of over-moded elements

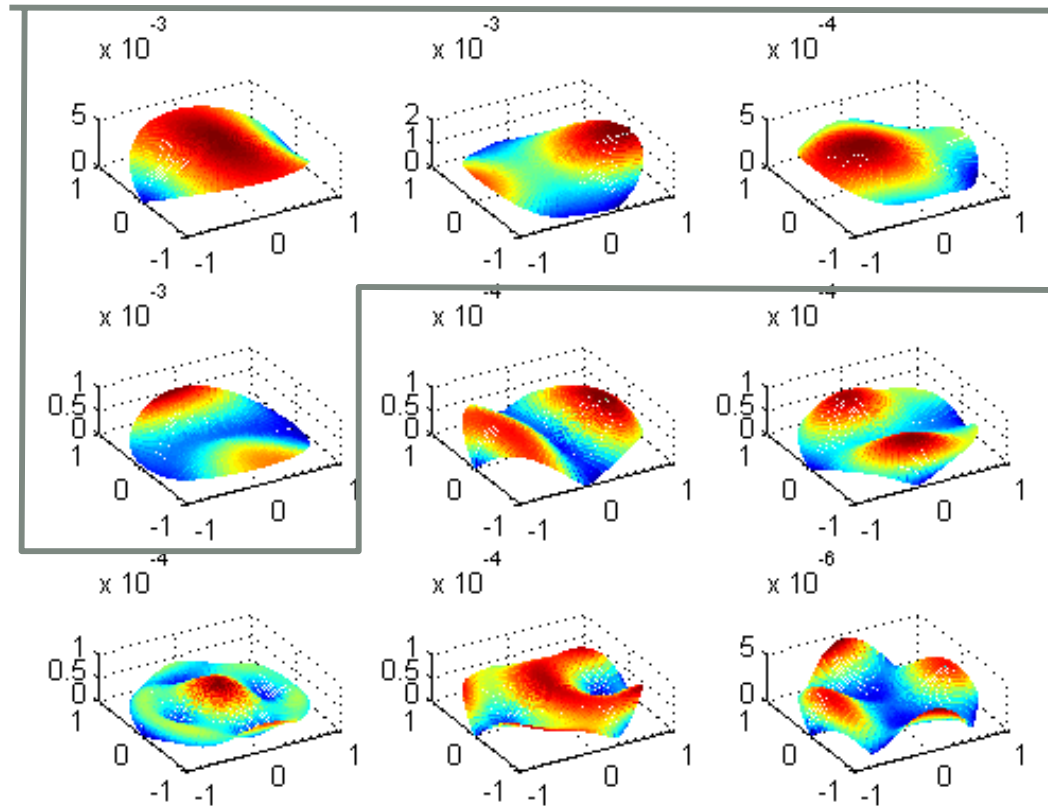


Elements are almost touching !

MBF patterns



Orthogonalized patterns



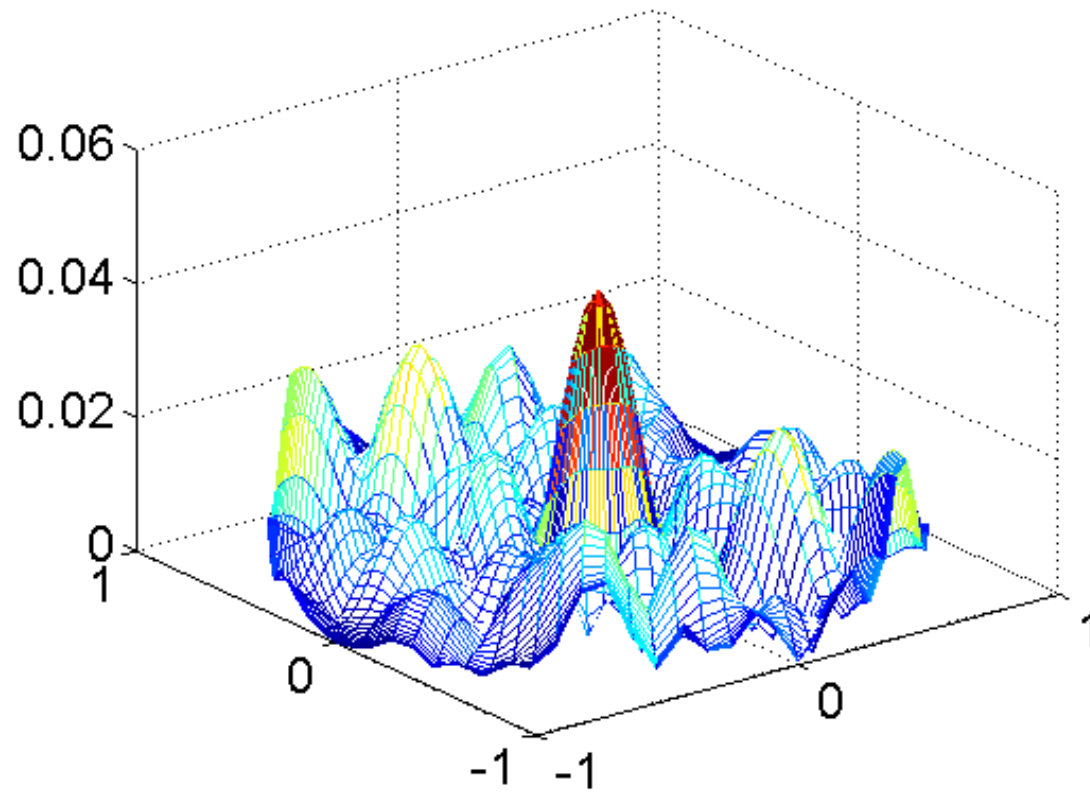
Pattern orthogonalized through Gram-Smidt procedure, 1st pattern is that of primary.

$$\vec{F}_p^{\circ} \simeq \sum_{q=1}^{Q < P} \alpha_{p,q} \vec{F}_q^{\circ, \text{orth}}$$

$$\sum_p^P AF_p \vec{F}_p^{\circ} \simeq \sum_q^{Q < P} \left(\sum_p^P \alpha_{p,q} AF_p \right) \vec{F}_q^{\circ, \text{orth}}$$

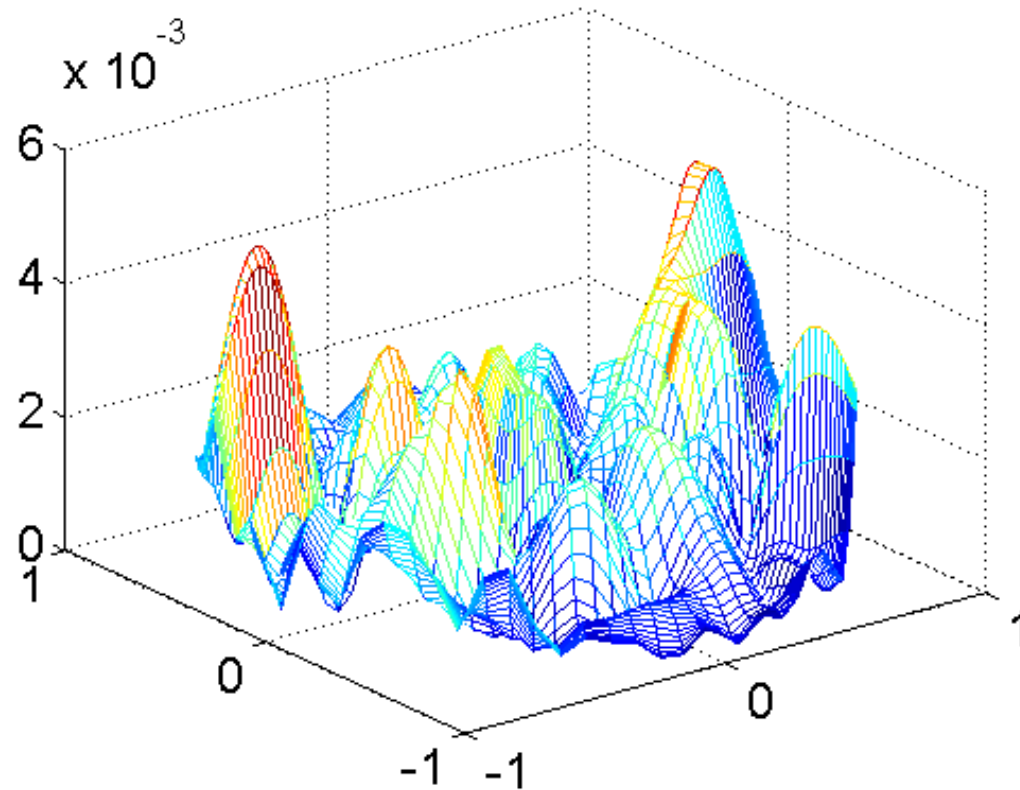
Project on Q=1, 2, 3, 4 patterns at most

Array pattern



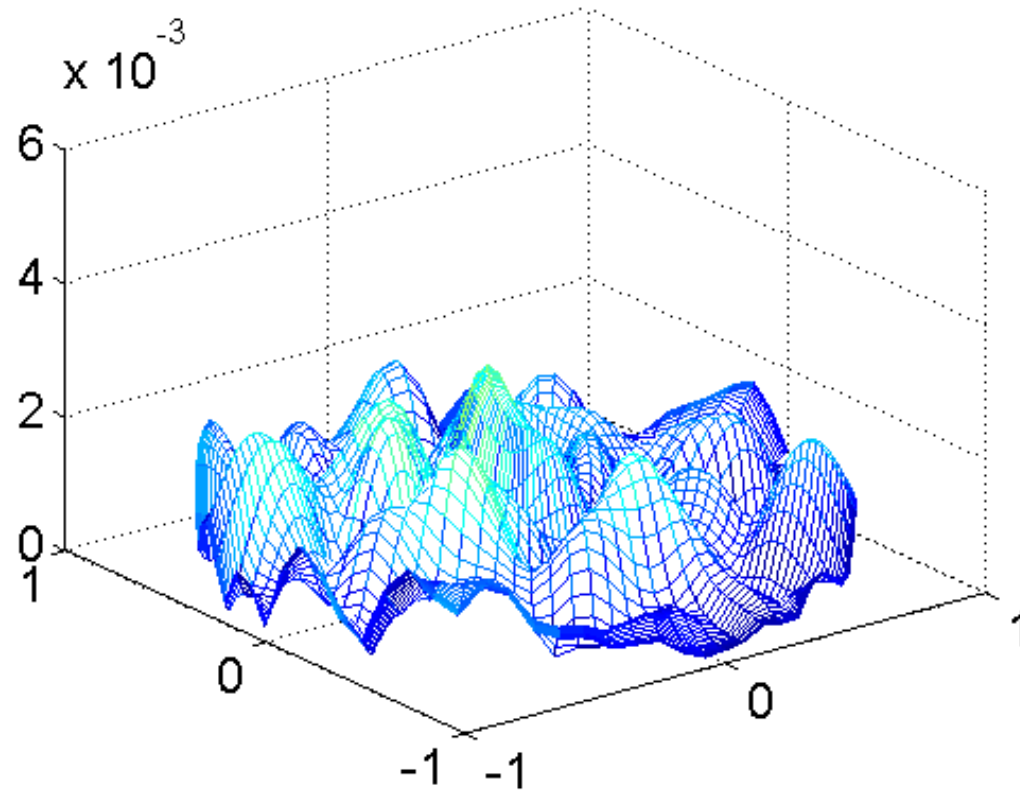
Array pattern $|F|$

Error after pattern projection



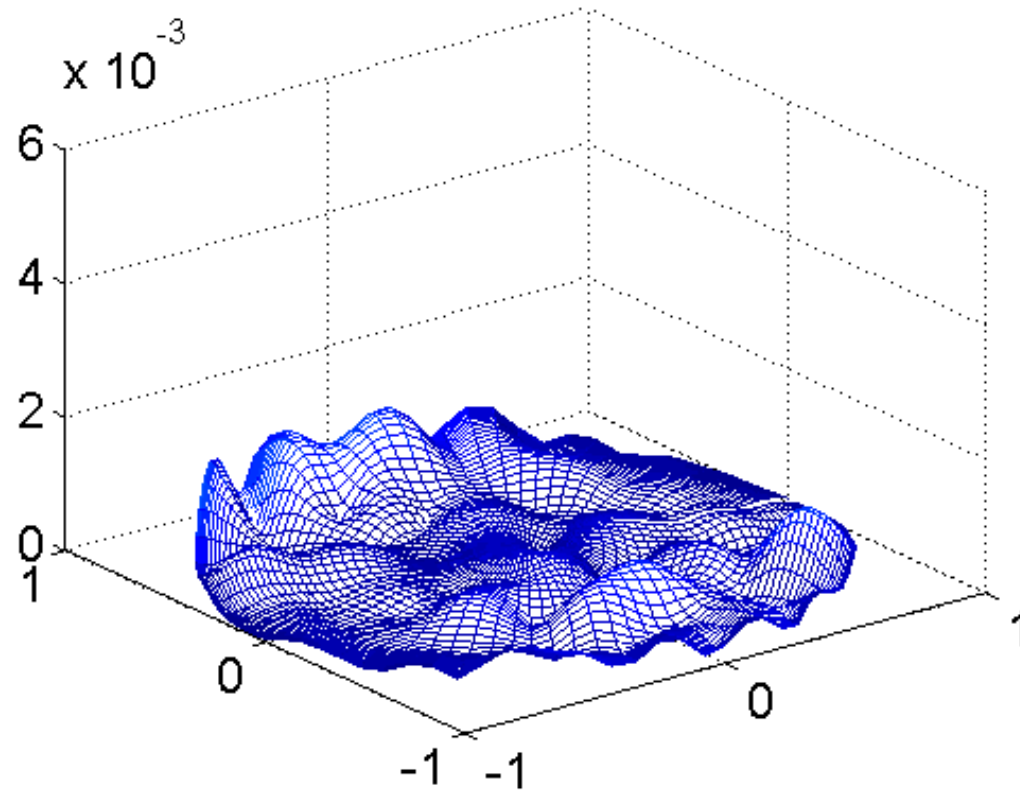
Error pattern $|F - F_{\text{approx}}|$ for $Q=1$

Error after pattern projection



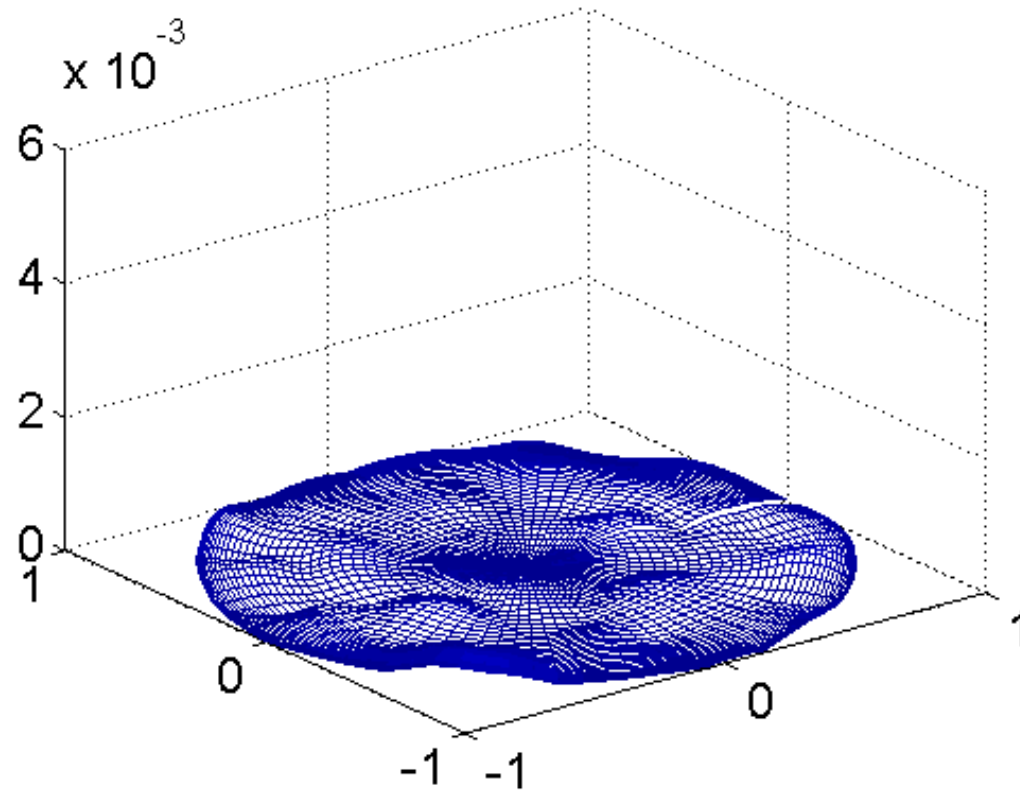
Error pattern $|F - F_{\text{approx}}|$ for $Q=2$

Error after pattern projection



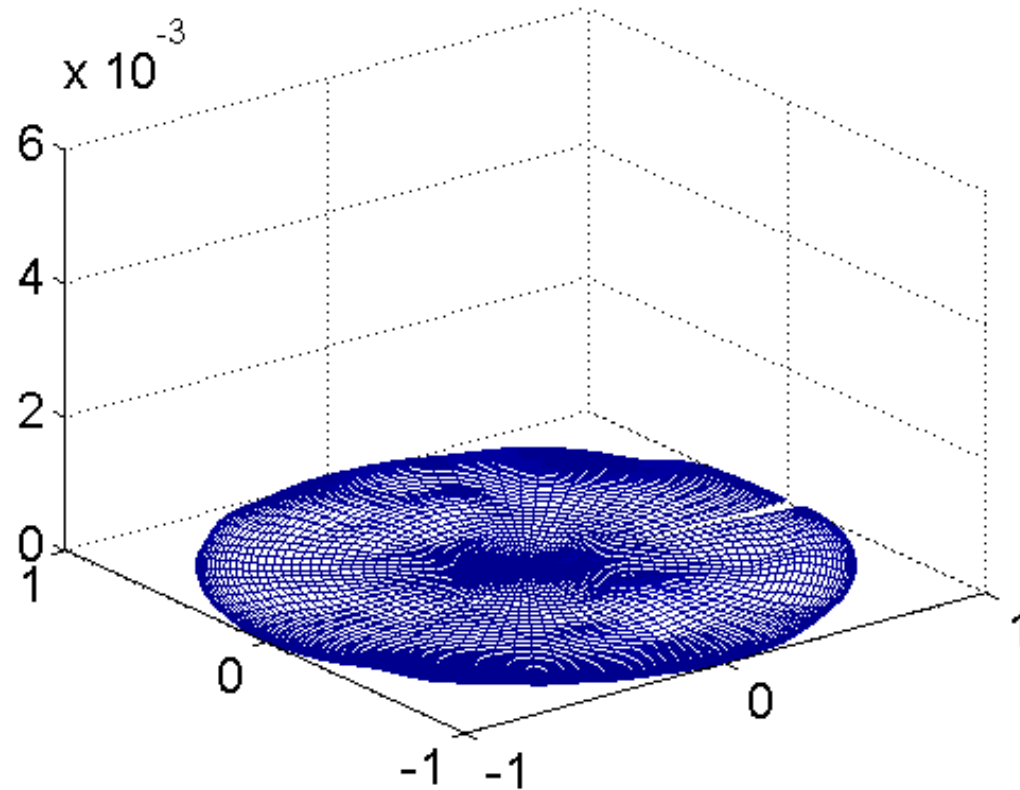
Error pattern $|F - F_{\text{approx}}|$ for $Q=3$

Error after pattern projection



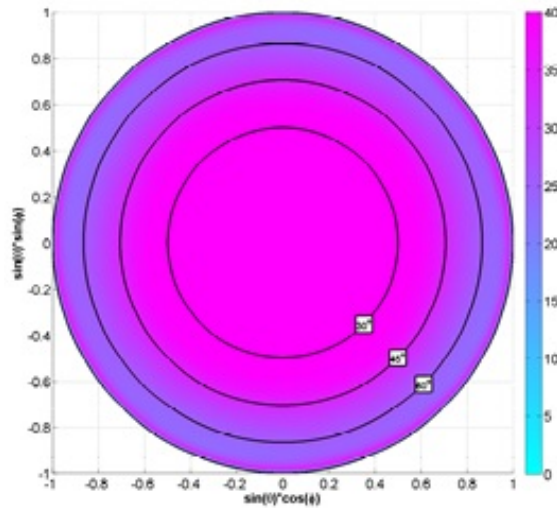
Error pattern $|F - F_{\text{approx}}|$ for $Q=4$

Error after pattern projection

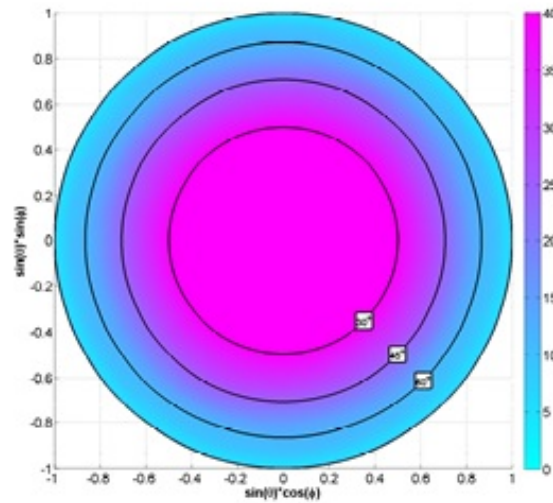


Error pattern $|F - F_{\text{approx}}|$ for $Q=5$

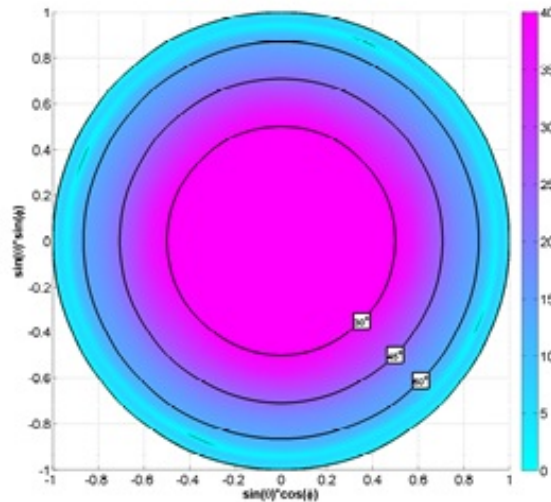
Before the end... *IXR* for SKALA element



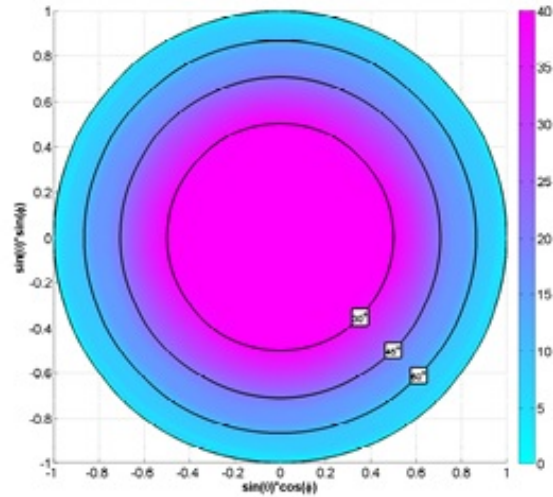
100 MHz



200 MHz



300 MHz



400 MHz

$$IXR_J = \left(\frac{\kappa(J) + 1}{\kappa(J) - 1} \right)^2$$



Conclusions

- Great tool available. Many lessons learnt about aperture array beams: average EEP, edge effect, etc.
- MBFs => Finite series of pattern multiplications
- Representation of array factors with functions used for ~circular apertures. Fast convergence.
- Further reduction through projection of MBF patterns on a few patterns. Fast convergence.
- Better selection of subset of orthogonal patterns: Average EEP?
- Need to link all this to the efforts of others...
- Science/calibration requirements must get to the lab/workshop sooner rather than later (IXR, etc.)



Thank you!

- Questions?