

Efficient Modeling of Beam Patterns Using Physics-Based Basis Functions

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- Small difference between actual and predicted beams
 - broken elements
 - beamformer errors
 - element displacement
- Limited number of calibration sources
 - few parameters to describe actual beam
 - proper basis functions needed
- Possible solutions
 - tapered aperture approximation (Craeye)
 - characteristic basis function patterns (this talk)

Current distribution over aperture

- determines far field radiation pattern
- superposition of characteristic basis functions (CBFs)
→ exploited in CBFM to reduce numerical complexity

Definition of CBFP

- far field radiation pattern associated with CBF

Total far field pattern is superposition of CBFPs

- Using EM-models for expected physical effects
 - primary CBFP: nominal beam (no errors)
 - secondary CBFPs: deviations from “nominal”
 - impedance matching errors
 - beamformer errors
- After instrument characterization: measured CBFPs?
- Efficient representation by superposition of analytic functions saves processing power

- **Beam model**

$$\mathbf{F}(\theta, \varphi) = \sum a_m \mathbf{G}_m(\theta, \varphi)$$

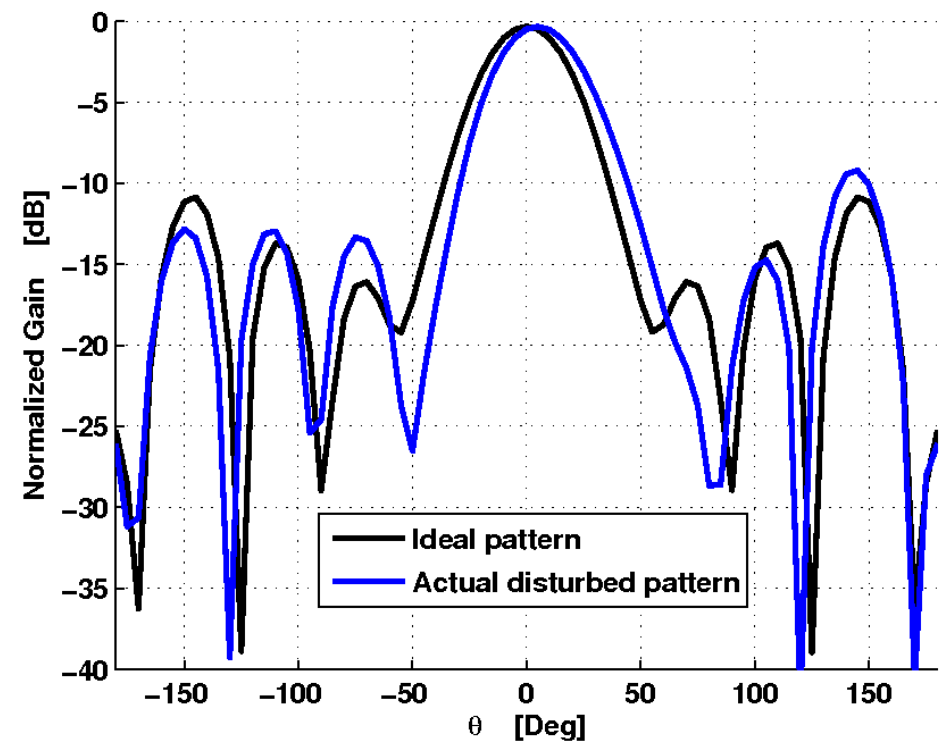
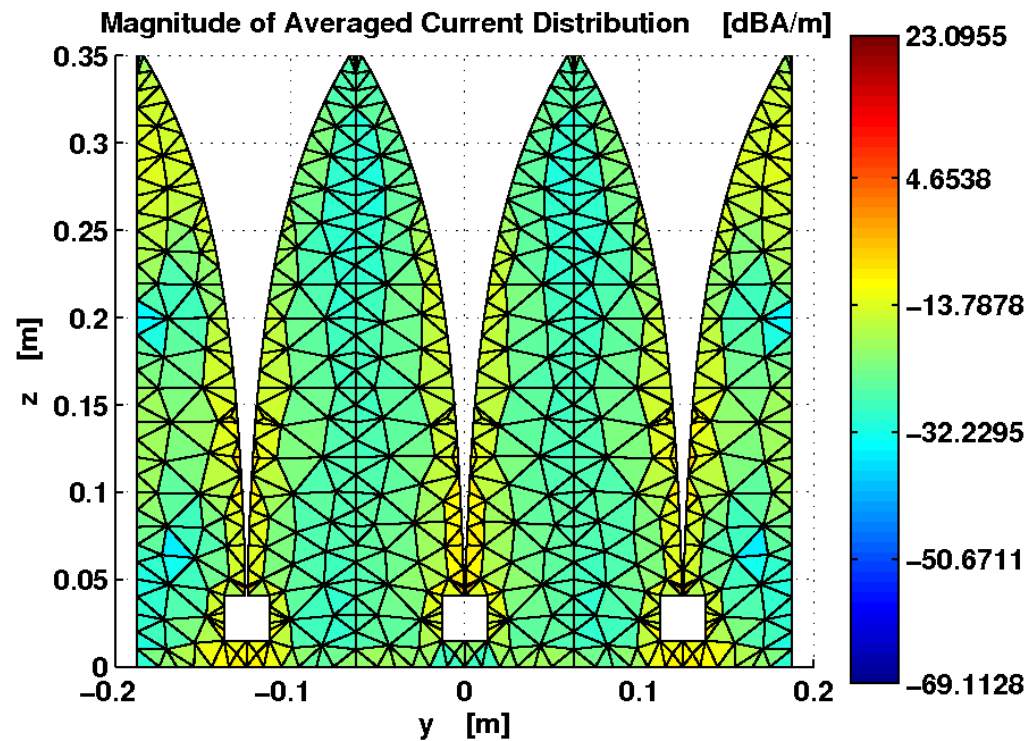
- **Voltage domain**

$$V_k = \mathbf{F}(\theta_k, \varphi_k) \cdot \mathbf{E}_i(\theta_k, \varphi_k)$$

- **Power domain:**

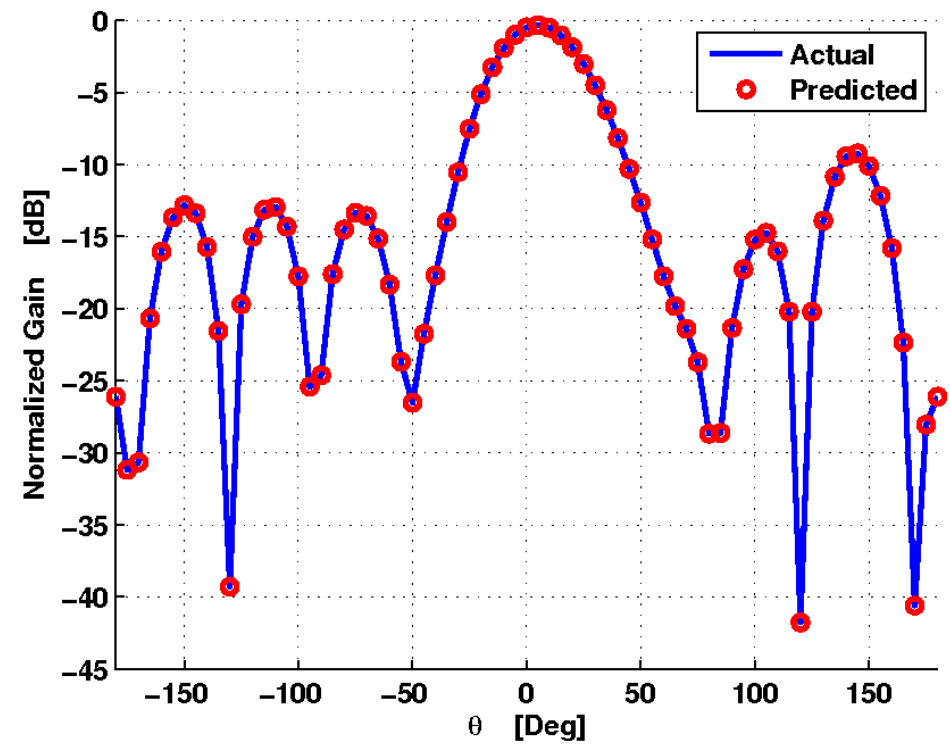
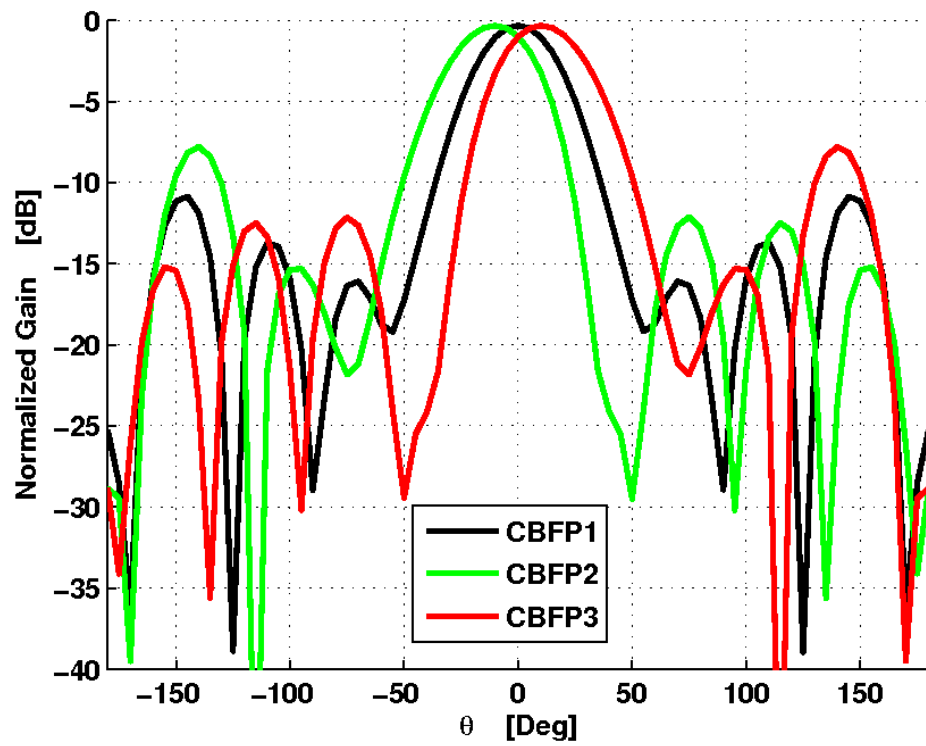
$$\begin{aligned} R_{pq} &= \langle V_p, V_q^* \rangle \\ &= \langle \int \sum a_{mp} \mathbf{G}_{mp}(\Omega) \cdot \mathbf{E}_i(\Omega) d\Omega, V_q^* \rangle \\ &= a_p^H \mathbf{A}_{pq} a_q \end{aligned}$$

Scenario: 3-element Vivaldi array with 5° mis-pointing



CBFPs: nominal beam and $\pm 10^\circ$ mis-pointed beams

CBFPs provide very accurate beam prediction



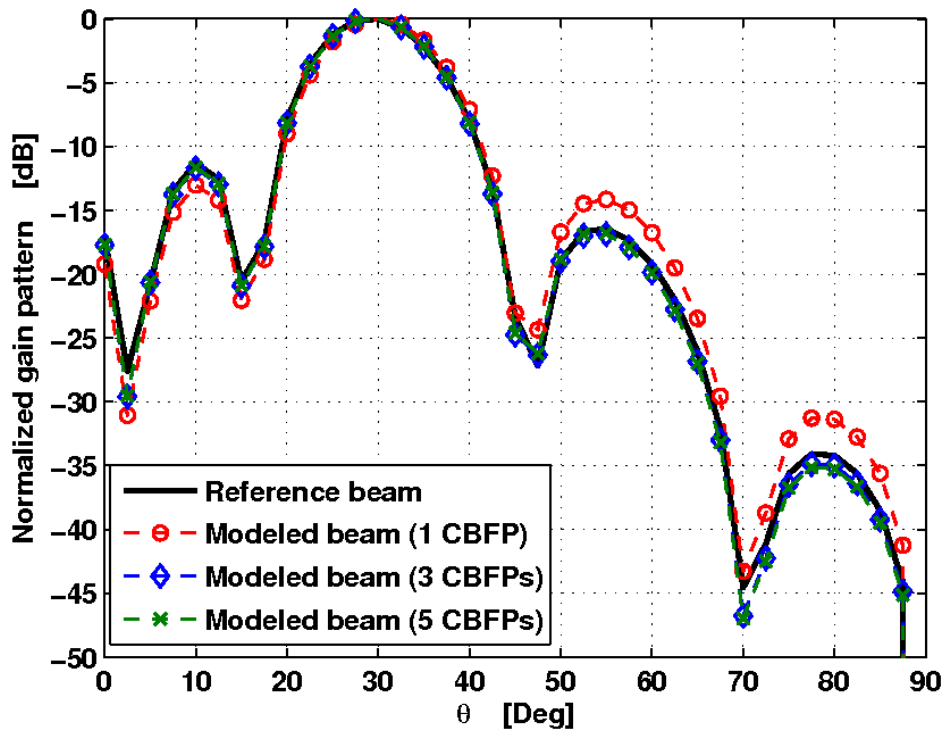
Example 2: impedance mismatch (1)

Maaskant et al., IEEE TAP, 2012, in press

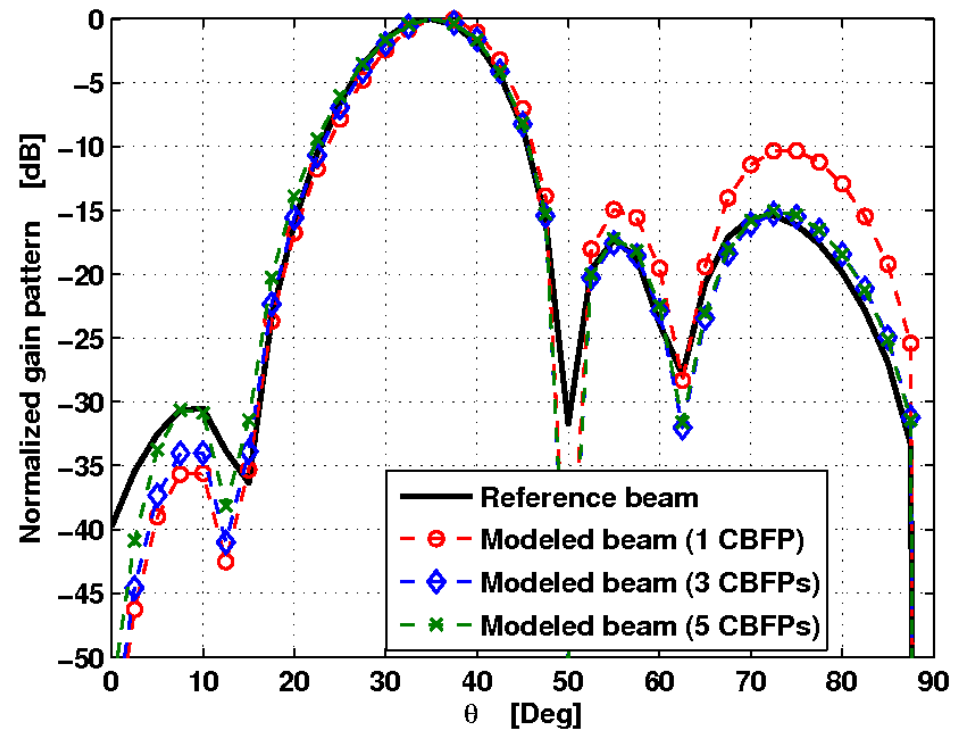
Scenario: 11-elem ULA with impedance mismatch

CBFPs: shifted copies of EEP neighboring elements

Beam scanned to $\theta_{\text{scan}} = 30^\circ, \phi_{\text{scan}} = 90^\circ$



Beam scanned to $\theta_{\text{scan}} = 30^\circ, \phi_{\text{scan}} = 90^\circ$

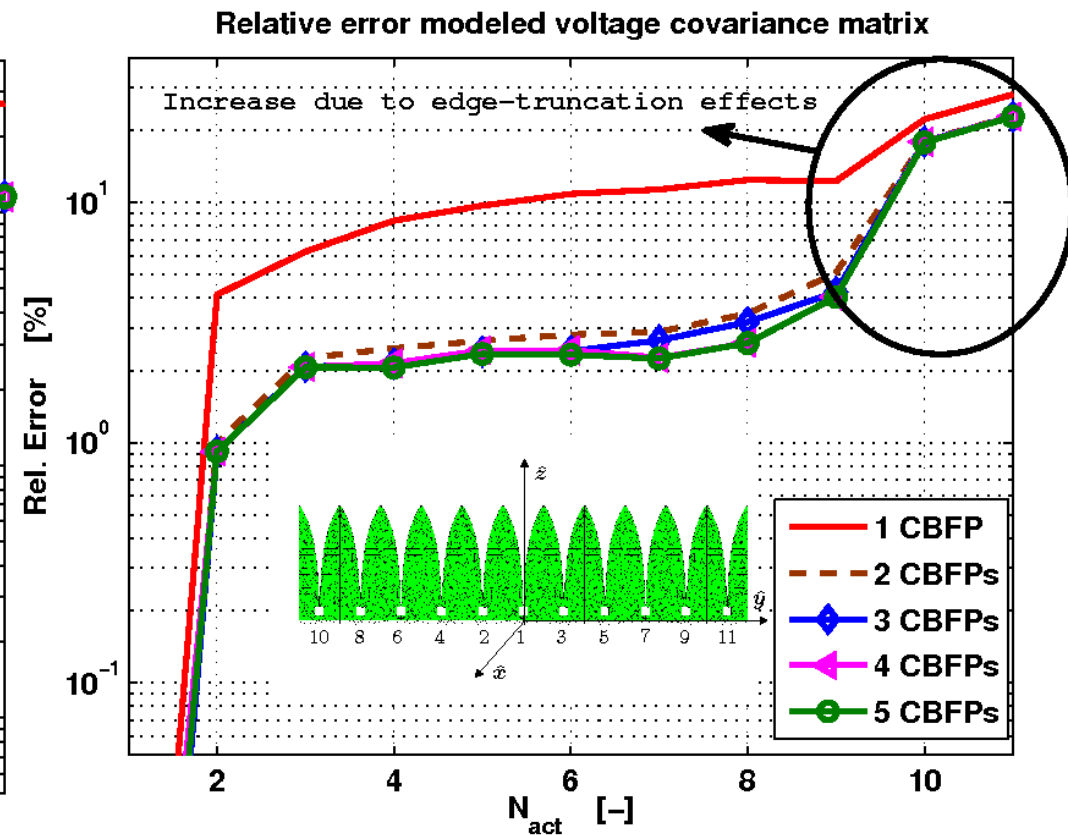
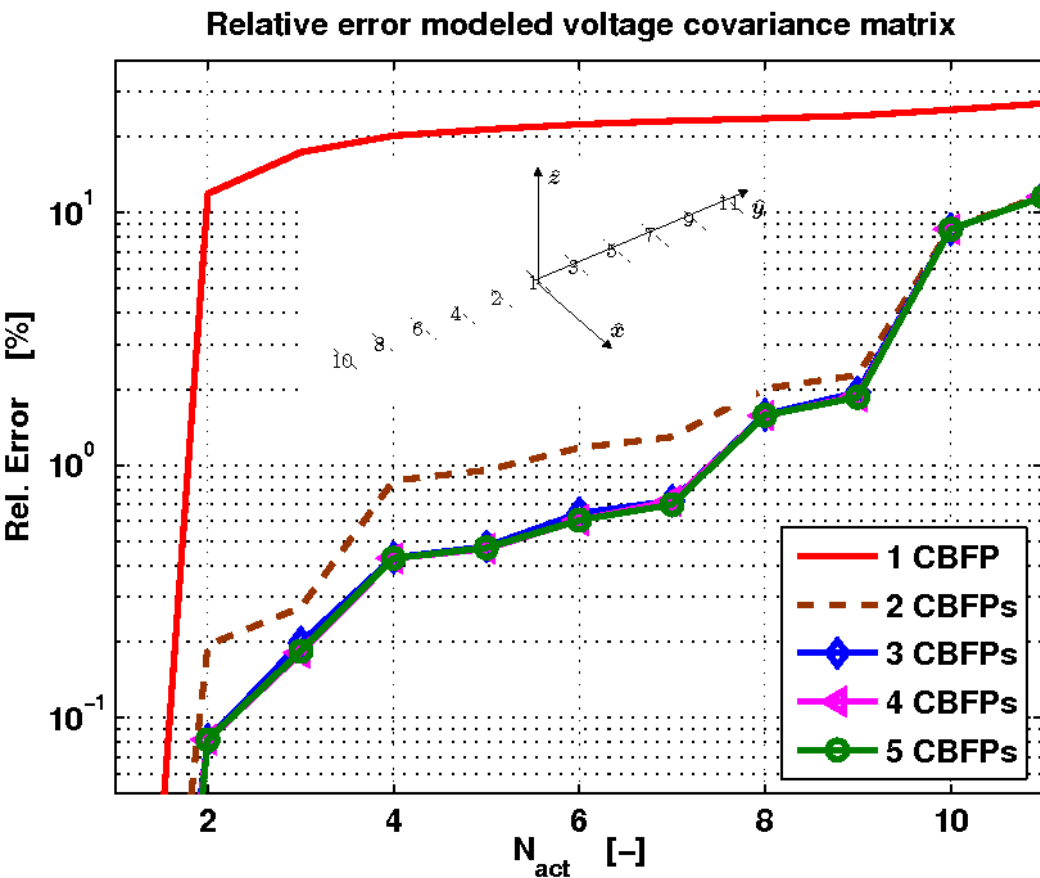


Example 2: impedance mismatch (2)

Maaskant et al., IEEE TAP, 2012, in press

Rel. error as function of #elements used for fitting

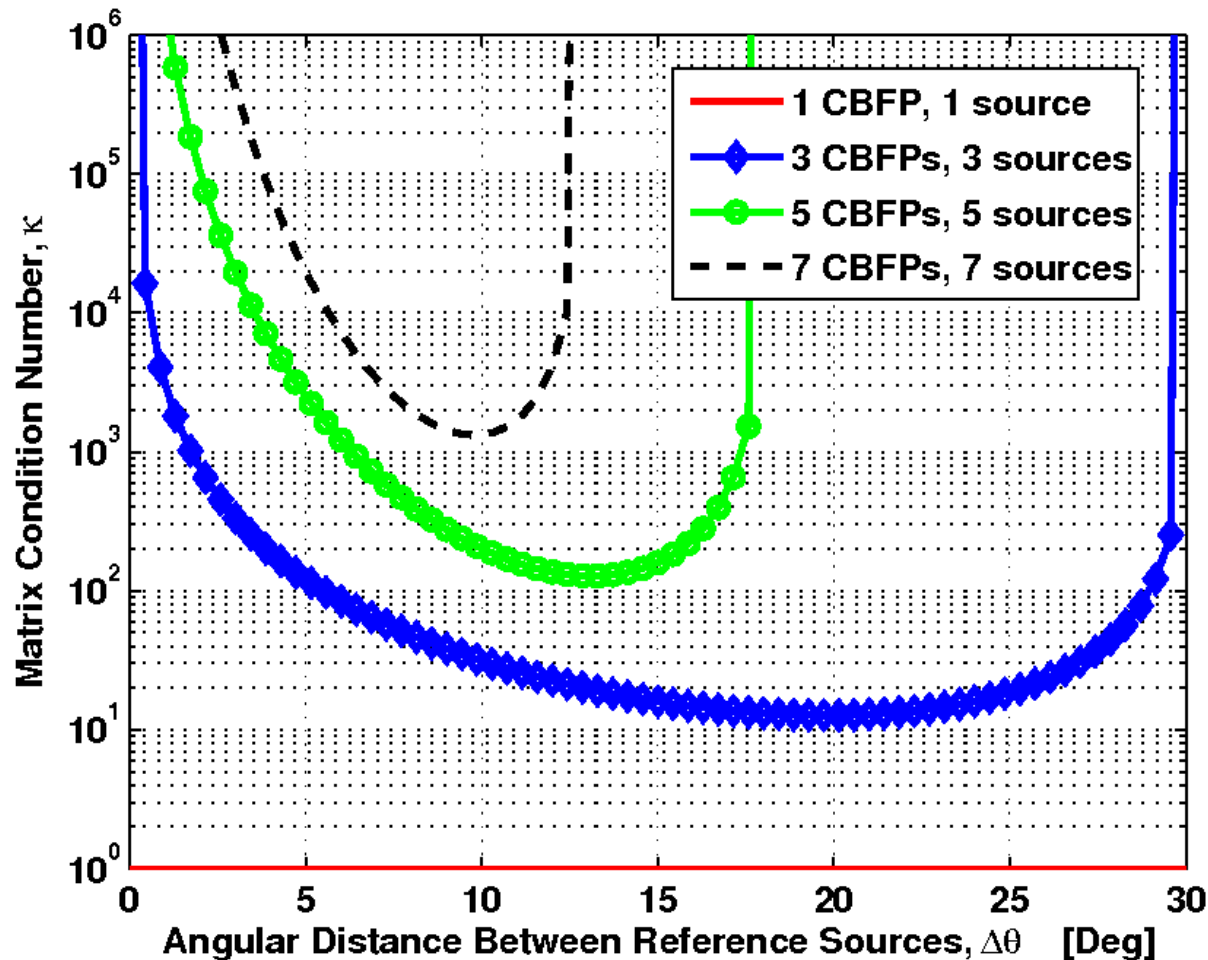
11-element dipole array (left) and Vivaldi array (right)



Example 2: impedance mismatch (3)

Maaskant et al., IEEE TAP, 2012, in press

Analysis of reconstructability: condition number as function of #CBFPs and angular separation



- **Definition of scenarios**
 - CBFs should describe deviations in array
- **Dealing with phase / unitary ambiguity**
 - non-unique solution for coefficients
 - use of reference antenna?
 - **however:** power patterns seem accurate
- **Algorithm optimization**
 - CBFs as superposition of basis functions (instead of tabulation)
 - sparse reconstruction with multiple dictionaries

Characteristic Basis Function Patterns

- radiation patterns **associated with CBFs**
- **precomputed** based on specific scenarios
- provide **efficient description** of beam pattern