Adaptive Selective Sidelobe Canceller Beamformer

Radio Imaging With Strong Interfering Sources

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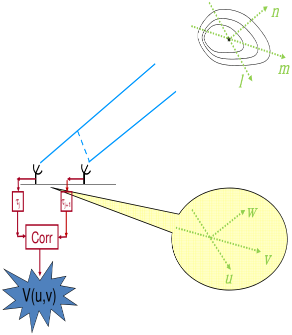
6. Summary

Visibility and the dirty image

- Radio telescope measures the correlation/visibility between antenna pairs.
- The classic dirty image is given by

$$\hat{I}(l, m) = \frac{1}{M} \sum_{q=1}^{M} V(u_q, v_q) e^{\frac{2\pi \jmath}{\lambda}(u_q l + u_q m)}$$

  - (u_q, v_q) are the baselines at time t_q.
  - M is the number of measurements.
  - $V(u_q, v_q)$ is the measured correlation (visibility) of the antenna pair.
  - λ is the wavelength.
  - (l, m) are the direction cosines.



Matrix Based Approach

Define

- $\mathbf{R}_k$ $(P \times P)$ the measured correlation matrix at epoch k

$$\mathbf{R}_k(i,j) \equiv V\left(u_k(i,j), v_k(i,j)\right)$$

- $\mathbf{a}_k(l,m)$ the array steering vector at epoch k

$$\mathbf{a}_k(l,m) \equiv \begin{pmatrix} e^{-\frac{2\pi \jmath}{\lambda}(u^k_{1,0} l + v^k_{1,0} m)} \\ \vdots \\ e^{\frac{-2\pi \jmath}{\lambda}(u^k_{P,0} l + v^k_{P,0} m)} \end{pmatrix}$$

where

- P - the number of antennas in the array.
- $(u^k_{i,0}, v^k_{i,0})$ - the location of antenna i at the k'Th epoch, relative to some convenient point (u_0, v_0)

Matrix Based Approach Cont.

- The classic dirty image

$$\hat{I}(l,m) = \frac{1}{M}\sum_{q=1}^{M} V(u_q,v_q)e^{\frac{2\pi\jmath}{\lambda}(u_q l + u_q m)}$$

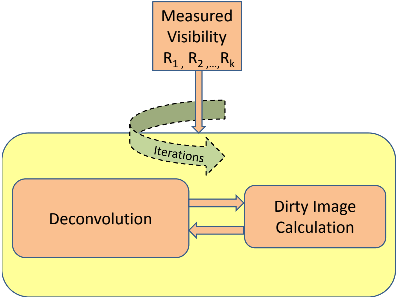
is mathematically equivalent to the classic (Bartlett) beamformer

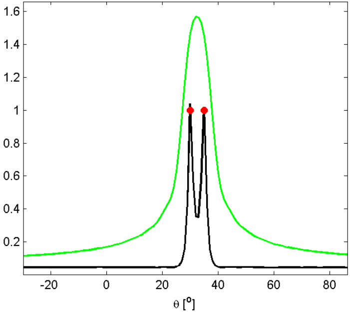
$$\begin{aligned}\hat{I}(l,m) &= \frac{1}{K}\sum_{k=1}^{K}\mathbf{w}_k^H(l,m)\mathbf{R}_k\mathbf{w}_k(l,m) = \frac{1}{K}\sum_{k=1}^{K}\hat{I}_k(l,m)\\ \mathbf{w}_k(l,m) &= \frac{1}{P}\mathbf{a}_k(l,m)\end{aligned}$$

- MVDR (Minimum Variance Distortionless Response) adaptive beamformer

$$\begin{aligned}\mathbf{w}_{\mathtt{MVDR}} &= \arg\min_{\mathbf{w}} \mathbf{w}^H\mathbf{R}\mathbf{w}\\ \mathbf{w}_{\mathtt{MVDR}}^H\mathbf{a} &= 1\end{aligned}$$

Imaging



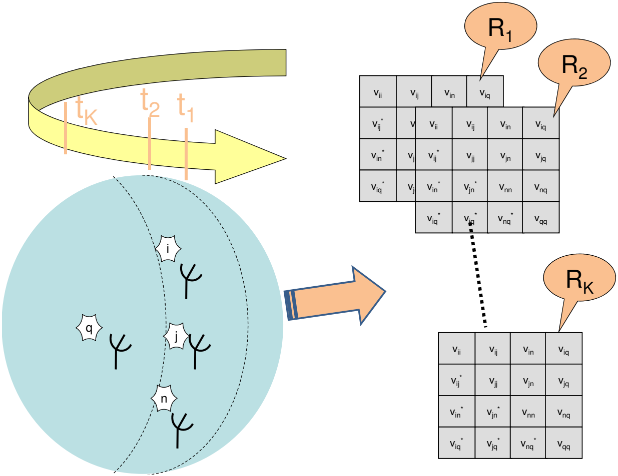


The spatial resolution and accuracy of the dirty image is a limiting factor in the deconvolution process.

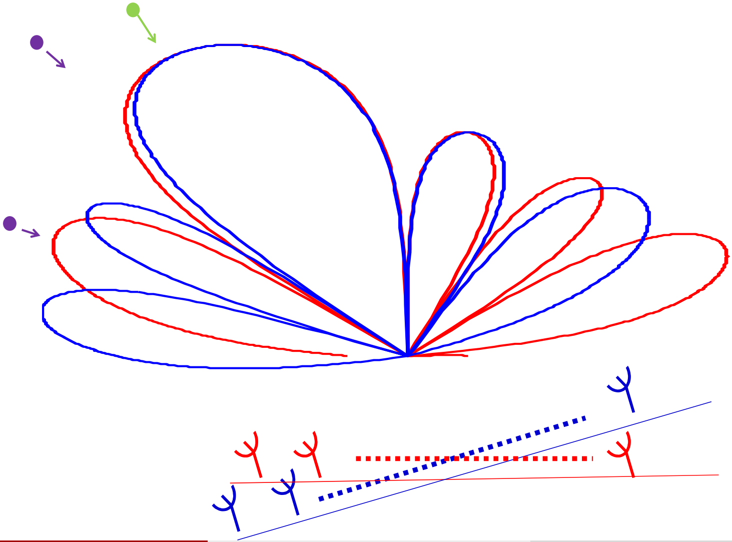
Producing a better dirty image will result in a more accurate reconstructed image.

ASSC Rational

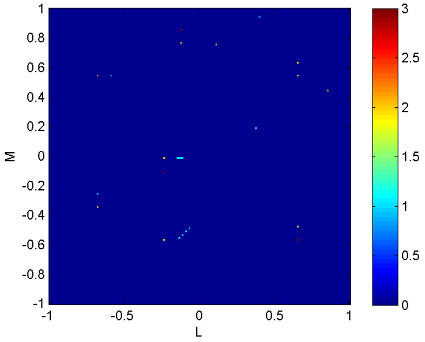
What Happens When the Array is Rotating



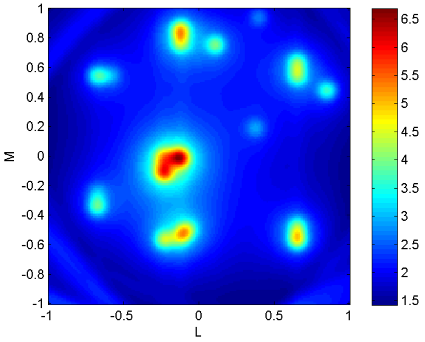
What Happens When the Array is Rotating Cont.



Illustrative Example



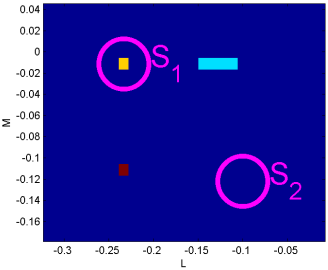
Original image

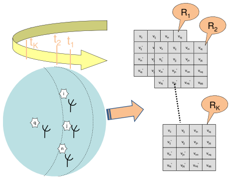


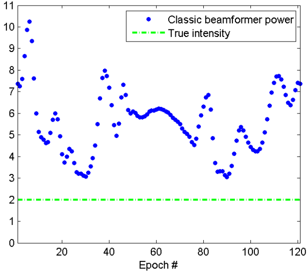
Classic dirty image

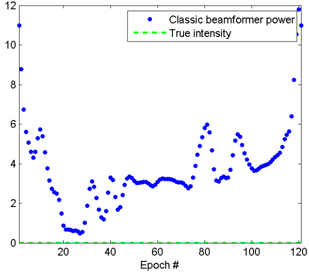
- East-West array with 20 antennas $\lambda/2$ spaced.
- Observation was done every 6 minutes for a 12-hour period.

Illustrative Example Cont.





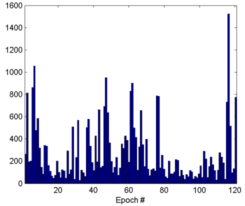




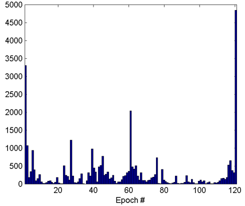
Illustrative Example Cont.

Number of pixels, a specific time epoch estimated the minimal power

Classic beamformer



MVDR beamformer



- Image contains 181×181 pixels
- For the classic beamformer
  - Most epochs (more than 90%) suffered from minimal interference for at least 661 pixels.
  - Over 65% of epochs suffered from minimal interference for 90% of the pixels.
- For the MVDR beamformer
  - Most epochs (more than 90%) suffered from minimal interference for at least 507 pixels.
  - Over 45% of epochs suffered from minimal interference for 90% of the pixels.

The Main Idea behind the ASSC

For a specific direction (l, m)

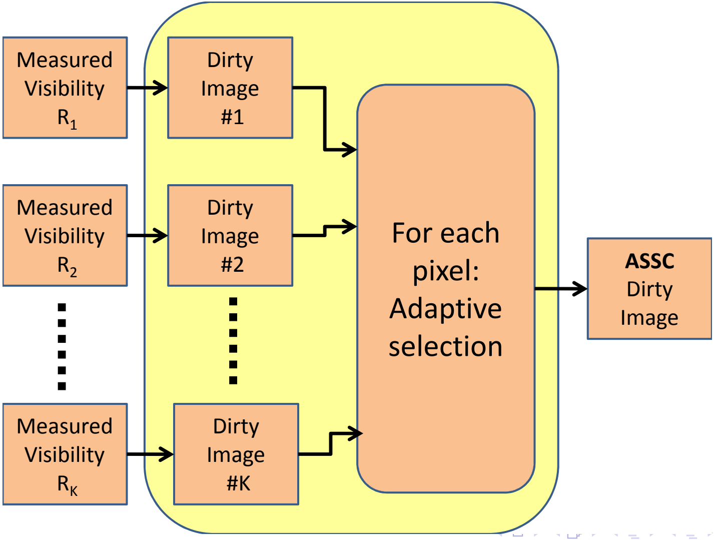
- The dirty image intensity measured by most time ticks, is significantly higher due to interfering sources.
- Only the few time ticks that benefit from minimal interference, yield an unbiased intensity estimation.

For each observation direction

Choosing the time epoch with the minimal power, we choose the correlation matrix with the smallest interfering power, that happens to best suppress the interference.

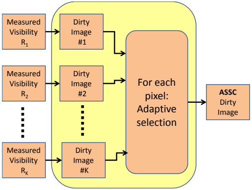
The ASSC Algorithm

ASSC Algorithm Flow



Calculate the dirty image for each time tick separately

$$\hat{I}_k(l,m) = \mathbf{w}_k^H(l,m)\mathbf{R_k}\mathbf{w}_k(l,m).$$



- For the classic beamformer

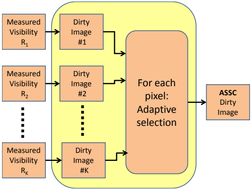
$$\mathbf{w}_k(l,m) = \frac{1}{P}\mathbf{a}_k(l,m)$$

- For the MVDR beamformer

$$\mathbf{w}_{\mathtt{MVDR}}^H(l,m) = \frac{\mathbf{a}_k^H(l,m)\mathbf{R}_k^{-1}}{\mathbf{a}_k^H(l,m)\mathbf{R}_k^{-1}\mathbf{a}_k(l,m)}.$$

Adaptive selection per pixel

- ASSC parameters
  - $\tilde{k}$ - the number of epochs to consider.
  - μ_k - their weight.



- Find the $\tilde{k}$ minimal values among $\left[\hat{I}_1(l,m),\ldots,\hat{I}_K(l,m)\right]$

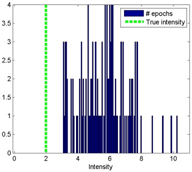
$$\left[\hat{I}_{(1)}(l,m),\ldots,\hat{I}_{(\tilde{k})}(l,m)\right]$$

- The ASSC dirty image is

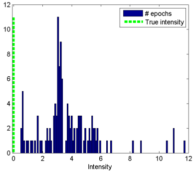
$$\hat{I}^{ASSC}(l,m) = \sum_{k=1}^{\tilde{k}} \mu_k \hat{I}_{(k)}(l,m)$$

ASSC Parameters

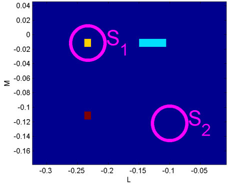
Classic beamformer for S_1



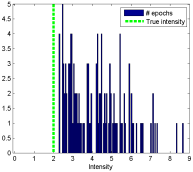
Classic beamformer for S_2



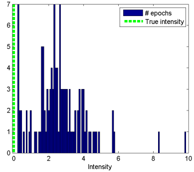
Original image - zoom in view



MVDR beamformer for S_1



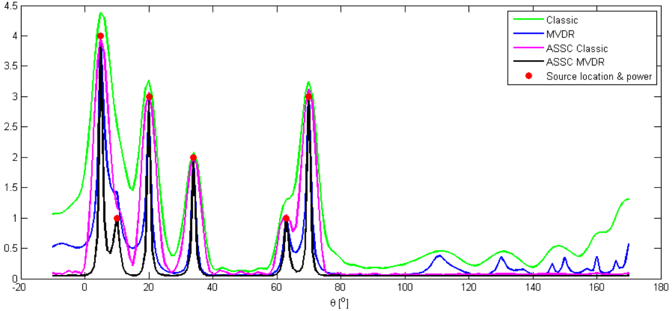
MVDR beamformer for S_2



- $\tilde{k}$ - number of epochs to consider
  - Typically, $\tilde{k} < 5\%$.
  - The stronger the interference, the smaller the $\tilde{k}$
- μ_k - epochs weight
  - $\mu_{k+1} \leq \mu_k$
  - $\sum_{k=1}^{\tilde{k}} \mu_k = 1$

Simulated Examples

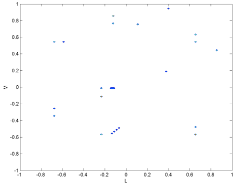
In FOV Interference - 1D example



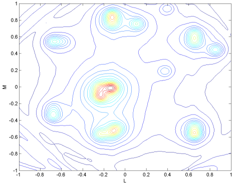
- Array with 20 antennas $\lambda/2$ spaced.
- Array rotation angle $0^o - 90^o$
- Measurement was done every 10^o
- $\tilde{k} = 1$, $\mu = 1$.

In FOV Interference

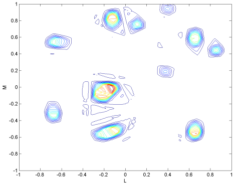
Original image



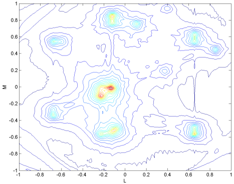
Classic dirty image



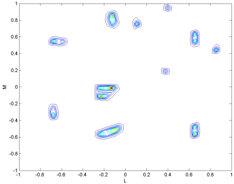
ASSC classic dirty image



MVDR dirty image

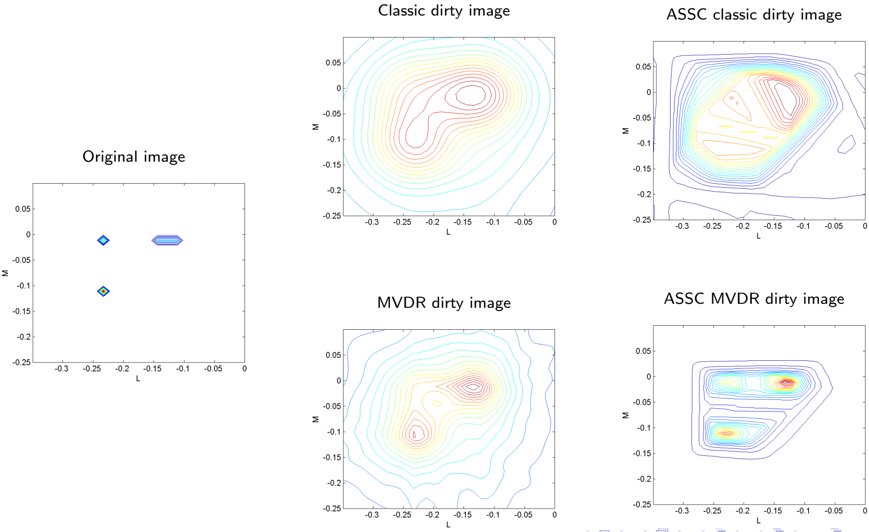


ASSC MVDR dirty image



- East-West array with 20 antennas $\lambda/2$ spaced.
- Measurement was done every 6 minutes for a 12-hour period.
- $\tilde{k} = 3,\ \mu_k = \frac{1}{3}$.

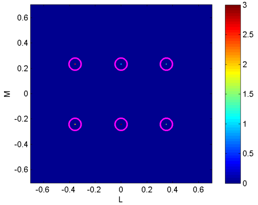
In FOV Interference - Zoom In View



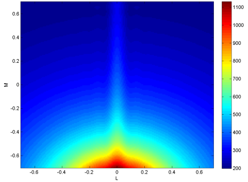
Out-of-FOV Interference

Interference stronger by 10, 000 than sources in the FOV

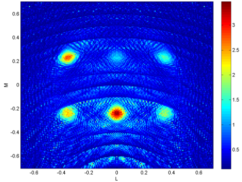
Original image



Classic dirty image

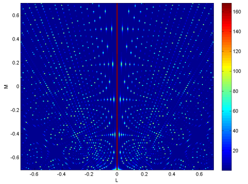


ASSC classic dirty image

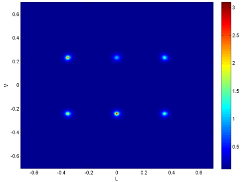


- East-West array with 20 antennas $\lambda/2$ spaced.
- Measurement was done every 6 minutes for a 12-hour period.
- Interference is stronger by 10, 000 than sources in FOV.
- $\tilde{k} = 3,\ \mu_k = \frac{1}{3}$.

MVDR dirty image



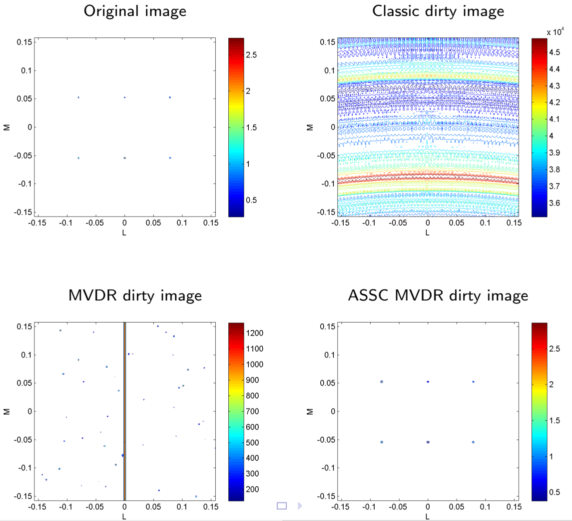
ASSC MVDR dirty image



Out-of-FOV Interference

Interference stronger by 10^6 than sources in the FOV

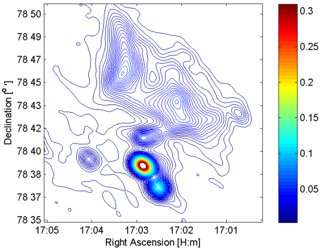
- East-West array with 20 antennas logarithmically spaced $0 - 200\lambda$.
- Measurement was done every minute for a 12-hour period.
- Interference is stronger by 10^6 than sources in FOV.
- $\tilde{k} = 5$, $\mu_k = \frac{1}{\tilde{k}}$.



Preliminary Results of Radio Astronomical Measurements

Abell 2256 Cleaned Images

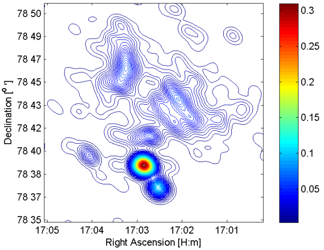
CLEAN using classic dirty image



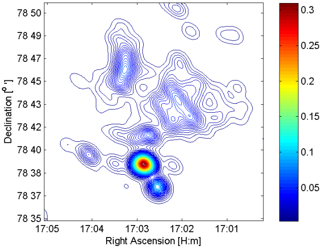
- Observed by the VLA*
- Single frequency bin around 1369 MHz

* T. Clarke and T. Ensslin, "Deep 1.4 GHz very large array observations of the radio halo and relic in Abell 2256."
Data was calibrated and provided by Prof. Huub Rottgering and Huib Intema

CLEAN using ASSC classic dirty image

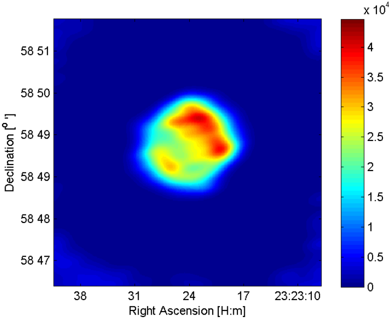


CLEAN using ASSC MVDR dirty image

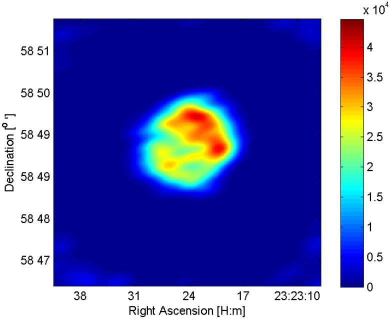


Cassiopeia A Cleaned Images

CLEAN using classic dirty image



CLEAN using ASSC classic dirty image



- Observed by the Allen Telescope Array*
- Single frequency bin around 4.2 GHz.

* Data was calibrated and provided by Gerald Harp

Summary

- We propose the *Adaptive Selective Sidelobe Canceller* beamformer - a novel method to produce a dirty image to be further processed by CLEAN or MEM.
- For interference dominant cases,the ASSC algorithm obtains images with higher spatial resolution and interference cancellation than either the classic and MVDR beamformers.
- The ASSC performance were demonstrated over simulated and radio astronomical data.

Thank you