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# **KLT-DETECTED TRANSIENT SIGNALS FROM RELATIVISITIC SPACESHIPS**

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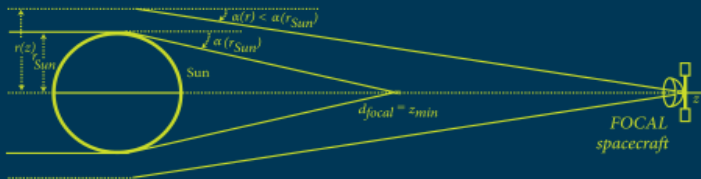
**Home page: <http://www.maccone.com/>**

# 700-pages BOOK about “Mathematical SETI” (2012)



This book introduces the Statistical Drake Equation where, from a simple product of seven positive numbers, the Drake Equation is turned into the product of seven positive random variables. The mathematical consequences of this transformation are demonstrated and it is proven that the new random variable  $N$  for the number of communicating civilizations in the Galaxy must follow the lognormal probability distribution when the number of factors in the Drake equation is allowed to increase at will.

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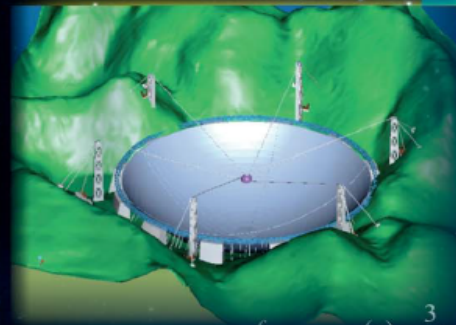
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Mathematical SETI

## Mathematical SETI

Statistics,  
Signal Processing,  
Space Missions



$$f_{ET\_Distance}(r) = \frac{3}{r} \cdot \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(\ln(n)-\mu)^2}{2\sigma^2}}$$

Claudio Maccone

$$X(t) = \sum_{n=1}^{\infty} Z_n \phi_n(t) \quad \text{with } 0 \leq t \leq T.$$

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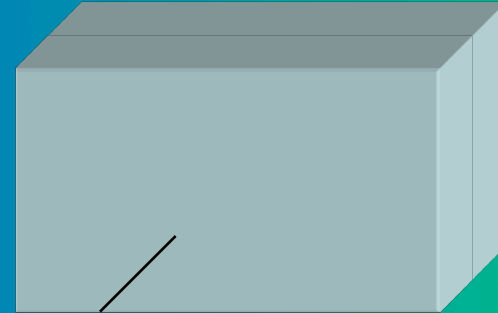
$$N = N_s \cdot f_p \cdot n_e \cdot f_l \cdot f_i \cdot f_c \cdot f_L$$

$$f_N(n) = \frac{1}{n} \cdot \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(\ln(n)-\mu)^2}{2\sigma^2}} \quad (n \geq 0)$$

$$\left( \ln \left[ \frac{6 R_{Galaxy}^2 h_{Galaxy}}{r^3} \right] - \mu \right)^2$$

# What is the KLT ?

- Example (a Newtonian analogy): consider a solid object, like a BOOK, described by its INERTIA MATRIX.
- Then there exist only one special reference frame where the Inertia Matrix is DIAGONAL. This is the reference frame spanned by the EIGENVECTORS of the Inertia matrix.



$$I = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{pmatrix}$$

# KLT mathematics



- If  $X(t)$  is a stochastic process (= input to the radio telescope) it can be expanded into an infinite series

$$X(t) = \sum_{n=1}^{\infty} Z_n \phi_n(t) \quad 0 \leq t \leq T$$

- then
  - $\phi_n(t)$  are orthonormalized functions of the time:

$$\int_0^T \phi_m(t) \phi_n(t) dt = \delta_{mn}$$

$Z_n$  are random variables, not changing in time, with the property

$$\langle Z_m Z_n \rangle = \lambda_n \delta_{mn}$$

- In conclusion, the KLT separates the radiotelescope input (= noise + signal(s)) into **UNCORRELATED** components.

# KLT mathematics



$$\int_0^T \langle X(t_1) X(t_2) \rangle \phi_n(t_1) dt_1 = \lambda_n \phi_n(t_2)$$

- This is the integral equation yielding the Karhunen-Loève's eigenfunctions  $\phi_n(t)$  and corresponding eigenvalues  $\lambda_n$ .
- The kernel of this integral equation is the autocorrelation.
- This is the best basis in the Hilbert space describing the (signal + noise). The KLT adapts itself to the shape of the radiotelescope input (signal+noise) by adopting, as a reference frame, the one spanned by the eigenfunctions of the autocorrelation. And this turns out to be just a LINEAR transformation of coordinates in the Hilbert space. Thus, the KLT is an easily INVERTIBLE TRANSFORMATION.

# KLT Filtering



- There is no degeneracy (i.e. each eigenvalue corresponds to just one eigenfunction only).
- The eigenvalues turn out to be the variances of the random variables  $Z_n$ , that is  $\sigma_{Z_n}^2 = \lambda_n = \langle Z_n^2 \rangle > 0$ .
- Since  $\langle Z_n \rangle = 0$  we can SORT in descending order of magnitude both the eigenvalues and the corresponding eigenfunctions. Then, if we decide to consider only the first few eigenfunctions as the “bulk” of the signal, and to apply the inverse KLT, that’s what KLT filtering is: we just declare the taken-off part as “noise”!
- The Galileo mission by NASA-JPL used the KLT...

# “Classical” KLT vs. FFT



KLT		FFT	
↑	Works well for both wide and narrow band signals	Rigorously true for narrow band signals only	↓
↑	Works for both stationary and non-stationary input stochastic processes	Works OK for stationary input stochastic processes only	↓
↑	Is defined for any finite time interval	Is plagued by the “windowing” problems	↓
↓	Needs high computational burden: no “fast” KLT	Fast algorithm FFT	↑

# NO Classical KLT in the 1990s



- If  $N$  is the size of the autocorrelation matrix (  $N$  may equal millions or more in SETI), the number of calculations requested to find the KLT is of the order of  $N$  square, while the same number for the FFT is much less: just  $N \ln(N)$ .
- This *COMPUTATIONAL BURDEN* prevented all SETI scientists from replacing the FFT by the KLT until 2007:
  - 1) François Biraud et al. at Nançay in the years after 1983.
  - 2) Bob Dixon et al. the Ohio State SETI Program after 1985.
  - 3) Stelio Montebugnoli et al. at Medicina, Italy, after 1990.



# BAM (=Bordered Autocorrelation Method) to EASILY find the KLT



- BAM is acronym for “BORDERED Autocorrelation Method”.
- The new key idea is to regard the autocorrelation as a NEW FUNCTION OF KLT FINAL INSTANT,  $T$ . That is, to add one more row and one more column (= bordering) to the autocorrelation matrix for each new positive, increasing  $T$ .
- For STATIONARY processes, this amounts to the matrix:

$$R_{Toeplitz} = \begin{bmatrix} R_{XX}(0) & R_{XX}(1) & R_{XX}(2) & \dots & \dots & R_{XX}(N) \\ R_{XX}(1) & R_{XX}(0) & R_{XX}(1) & \dots & \dots & R_{XX}(N-1) \\ R_{XX}(2) & R_{XX}(1) & R_{XX}(0) & \dots & \dots & R_{XX}(N-2) \\ \dots & \dots & \dots & R_{XX}(0) & \dots & \dots \\ R_{XX}(N) & R_{XX}(N-1) & \dots & \dots & R_{XX}(1) & R_{XX}(0) \end{bmatrix}$$

# 2007 Breakthrough in the KLT



- In the winter of 2006-7, the SETI-Italia Group at Medicina, (Stelio Montebugnoli, Francesco Schillirò, Salvo Pluchino and Claudio Maccone) discovered a way to CIRCUMVENT that big obstacle of the KLT  $N^2$  computational burden.
- The idea is to use the BAM to exploit the **dependence on the final instant**  $T$  in both sides of the relationship, firstly proved in 1994 by Maccone in his KLT book, p.12, eq. (1.13):

$$\sum_{n=1}^{\infty} \lambda_n(T) = \int_0^T \sigma_{X(t)}^2 dt$$

# BAM (Bordered Autocorrelation Method) to EASILY find the KLT



- Differentiating both sides wrt  $T$  yields the FINAL VARIANCE THEOREM (Maccone, Proc. of Science, 2007)

$$\sum_{n=1}^{\infty} \frac{\partial \lambda_n(T)}{\partial T} = \sigma_{X(T)}^2$$

- Confining ourselves to the FIRST EIGENVALUE (the “dominant one” = the largest) & STATIONARY  $X(t)$  ONLY:

$$\sum_{n=1}^{\infty} \frac{\partial \lambda_n(T)}{\partial T} \approx \frac{\partial \lambda_1(T)}{\partial T} = \sigma_X^2 = \text{a\_CONSTANT\_wrt\_T}$$

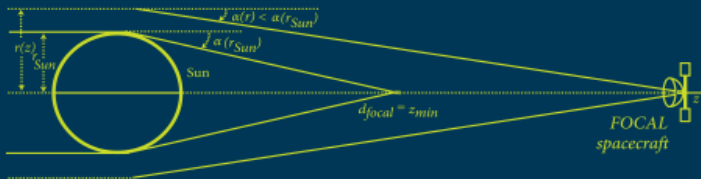
- The SETI-Italia Team discovered that *the Fourier transform of this constant is a peak (i.e. Dirac delta function) that is the FREQUENCY of the ET-SIGNAL !!!*

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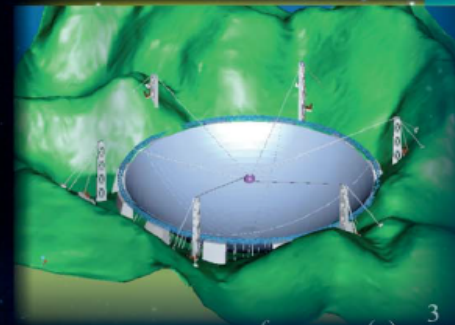
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*Thanks !*