Mathematical SETI

Statistics, Signal Processing, Space Missions

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Front cover: (Upper right) Some of the 42 six-meter dishes now making up the Allen Telescope Array (ATA) at Hot Creek Radio Observatory in northern California, U.S.A. (courtesy: SETI Institute). (Lower left) Artist concept of the 500-meter Arecibo-like FAST radiotelescope, now under construction in Guizhou Province, southwest China (courtesy: Professor Rendong Nan).

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Preface

SETI, the modern Search for Extra Terrestrial Intelligence, began in 1959 with the publication of the seminal paper entitled "Searching for Interstellar Communications" by Giuseppe Cocconi (1914–2008) and Philip Morrison (1915–2005) in *Nature*, Vol. 184, Number 4690, pp. 844–846, September 19, 1959. Just one year later, in 1960, Frank Drake started experimental radio SETI with Project Ozma, when he first looked for possible extraterrestrial (ET) signals near the 1.420 gigahertz marker frequency known as "the hydrogen line". Modern radio SETI was thus born, and so it continues today as a result of the enormous progress in electronic equipment and mathematical computing algorithms used to detect ET signals.

But, a further year later, at a meeting on SETI held at the National Radio Astronomy Observatory (NRAO) in Green Bank, West Virginia, Frank Drake gave another outstanding contribution, now called the Drake equation (*http://en.wikipedia.org/wiki/Drake_equation*). This is described in Chapter 1 of this book, together with its extension for the equation to encompass probability and statistics, as discovered by this author in 2007 and first presented in 2008. That analysis occupies the first 11 chapters of this book.

PART I—SETI STATISTICS. This first part of the book comprises 11 chapters.

Chapter 1—The statistical Drake equation. This chapter shows how the classical Drake equation, the product of seven positive numbers, may be replaced by the product of seven positive random variables, called the "statistical Drake equation". This way of doing things is scientifically more robust inasmuch as each input of the classical Drake equation now has an error bar around itself. In other words, the purely numeric inputs of the classical Drake equation now become the mean values of the corresponding random variables, around which a certain standard deviation (to be found experimentally) is subtracted or added, as is customary in serious scientific papers. The mathematical consequences of this transformation are demonstrated and it is proven that the new random variable N for the number of communicating civilizations in the Galaxy must follow the lognormal probability distribution when the number of factors in the Drake equation is allowed to increase at will. This result opens up the possibility of inserting increasingly more factors into the Drake equation letting it become more representative of physical reality: for example, the end of civilization as a result of asteroid impact is absent in the 1961 formulation by Frank Drake probably because it was not until 1980 that the dinosaur demise was accepted by the scientific community as the consequence of an asteroid impact. Chapter 1 also derives another probability distribution (called "Maccone distribution" by Paul Davies and others) yielding the probability density function (pdf) of the distance between any two nearby civilizations in the Galaxy. This is of paramount importance for SETI inasmuch as it explains that we should hardly expect any ET civilization to be located at distances from us smaller than, say, 500 light-years. Thus, the most natural explanation for the apparent failure of 50 years of SETI search (1960–2010) is we did not find them simply because our current radio telescopes do not reach out far enough, being confined to ranges up to 100 or 200 light-years at best.

Chapter 2—Letting Maxima do the calculations. This chapter introduces students and young researchers to the joy of getting rid of hand-written calculations by resorting to the freely downloadable "Maxima" computer algebra code. In other words, the reader will find in the appendixes to the various chapters all those Maxima codes that this author had to write himself in order to prove the many equations given in the book for the first time. This way of doing things for heavily mathematical books like this one is brand new: we are not ashamed to demonstrate the beauty of SETI, astrophysics, and signal processing to our readers, we also teach them how to derive relevant new results by virtue of both Maxima and Mathcad. A couple of examples demonstrate this, in Appendixes 2.A and 2.B we derive the statistical properties of the lognormal distribution, key to the statistical Drake equation as shown in Chapter 1, and, as a demo of the strong capabilities of Maxima in tensor calculus, we derive the closed Einstein universe of 1917 (basic to cosmology), the Friedman equations of 1924, and the ensuing number of protons in the universe, the famous 10^{80} found by Dirac in 1937 (Dirac cosmology).

Chapter 3—How many planets for man and aliens? This chapter introduces the reader to the Dole equation (1964). This equation is mathematically the same as the Drake equation, but applies to the number of habitable planets in the Galaxy, rather than the number of communicating civilizations in the Galaxy. Thus, we extend our study to the statistical Dole equation, rather than the classical Dole equation of 1964, and we reach the conclusion that there should be some 100 million habitable planets for man in the Galaxy, plus a standard deviation of about 200 million. Not bad for future human expansion into the Galaxy, if we survive the many hazards we will have to face in the centuries to come, such as physical adversities and alien opposition. Having found the probability distribution of the distance between two nearby ET civilizations in Chapter 1, so in Chapter 3 we find that the same probability distribution applies to the distance between two nearby habitable planets—after changing the numbers (but not the equations) as necessary, of course.

Chapter 4—Statistical Fermi paradox and Galactic travel. This chapter considers the possible expansion of a certain civilization throughout the Galaxy, whether it be human or ET. The key idea here is that the amount of time required for expansion into space is basically determined by two factors: (1) the speed of spaceships used for hopping from one habitable planet to the next, and (2) the time required to colonize a new planet from scratch and make it a new base for further space voyages. We assume that the first variable (spaceship speed) is essentially deterministic, and so does not require any statistical treatment. However, we also assume that the second variable (colonization time) follows the lognormal distribution, again as a result of the number of unknown factors being so large that it approaches infinity. The Central Limit Theorem of statistics—as used in Chapter 1 to find the lognormal distribution of N and in Chapter 3 for N_{Hab}, respectively—is used here. Having made this assumption, the coral model for the expansion of corals at sea and of a civilization in the Galaxy allows us to determine the probability distribution of the overall time necessary for a given civilization to expand throughout the whole Galaxy. The mathematics gets quite complicated, and only wise use of Maxima allowed us to find the relevant probability distributions. All this is of course a statistical enlargement of the famous Fermi paradox, so far only treated by other authors in naive purely deterministic settings.

Chapter 5—How long does a civilization live? This chapter tries to address the totally unknown value of the last term in the Drake equation: How long might a technological civilization possibly survive? Since nobody knows, for we ourselves are the only example we have, we confine discussion in this chapter to changes in the number N that would be caused by long-lived civilizations vs. short-lived ones. The numerical examples given in this chapter are a statistical extension of the corresponding deterministic values given by Carl Sagan in his book (and TV series) *Cosmos* (1980).

Chapter 6—Life-span modeling by finite *b*-lognormals. This chapter contains profoundly innovative material, regarded by the author as one of the best mathematical models he has devised so far in his 64-year lifetime. The idea is as follows. All living beings are born at a certain time ($t = b =$ birth), they then grow up through adolescence ($t = a =$ adolescence), reach their prime at the peak ($t = p =$ peak), then decline through senility ($t = s =$ senility), and finally die ($t = d =$ death). Is there a probability finite density function that has such a behavior in time? Yes, is our answer and that is the b-lognormal. What is a b-lognormal? It simply is an ordinary (μ, σ) lognormal starting at a certain positive time $t = b > 0$, rather than at $t = 0$. Its equation just requires shifting the origin to a new positive instant $t = b > 0$, which we call a "b-lognormal" since that probability density function does not seem to have a name yet. But, the other four points in time mentioned above do have an immediate mathematical meaning: (1) the adolescence time ($t = a$) is the abscissa on the ascending inflexion point, (2) the peak time ($t = p$) is obviously the abscissa of the maximum, (3) the senility time ($t = s$) is the abscissa of the descending inflexion point, and finally (4) the death time ($t = d$) is the abscissa of the intercept between the tangent to senility and the time axis, and this "mathematical trick" allows us to get rid of the infinite tail on the right, replacing it by an obvious finite point. Such are *b*-lognormals. Now, Chapter 6 is entirely devoted to discovering new mathematical equations expressing the two unknown parameters (μ, σ) as functions of some of the known inputs such as birth time $(t = b)$ plus two of the four remaining known inputs (a, p, s, d). The author was able to discover some finite equations of this type, and probably still unknown similar equations exist as well. But, what he was able to discover was enough to write Chapters 7 and 8, of key importance to "mathematical history" and "mathematical Darwinian evolution", respectively. Finally, the author also derived an expression for the finite probability density of *b*-lognormals to renormalize to 1 vs. the ordinary normalization constant of ordinary lognormals. The set of all these new results is a breakthrough in replacing the mountain of words used today to describe Darwinian evolution and mathematical history by a simple set of statistical distributions in agreement with the statistical Drake equation and SETI.

Chapter 7—Historic civilizations as finite *b*-lognormals. We apply the results of Chapter 6 to mathematical history. We compute and compare the finite *b*-lognormals of eight civilizations whose existence had the greatest effect on the history of the world in the last 3,000 years: Ancient Greece (600 BC–30 BC), Ancient Rome (753 BC–476 AD), Renaissance Italy (1250–1600), Portugal (1419–1974), Spain (1492–1898), France (1524–1962), Britain (1588–1974), and the U.S.A. (1898–2050 tentatively). It will be objected that all these civilizations belong to the so-called Western world: nevertheless, the West has been home to the most advanced civilizations in the last 3,000 years. Whether Asia will replace the West in leading humanity in the future is highly likely, but that is still in the balance right now in 2012. So, these eight *b*-lognormals were compared on the same plot and a sort of "upper envelope" clearly emerged: it is an exponential curve that, more or less, embraces all *b*-lognormals as the geometric locus of their peaks! The principal result here is that *b*-lognormals become increasingly narrow as time elapses (i.e., their peaks get higher and higher), and this reveals progress (i.e., an increasing degree of civilization). To make this result quantitative, rather than just qualitative, we need a new unit measuring the "amount of evolution" reached by a certain civilization at a certain time, just as meters measure lengths, seconds measure time, coulombs measure electric charge, and so on in physics. This new unit of evolution we call the "darwin" and introduce it in the next chapter which deals with Darwinian evolution. We do so because in science "to measure is to understand".

Chapter 8—A mathematical model for evolution and SETI. The "exponential envelope" that was looming in the background of the previous chapter fully comes to light as the link between Darwinian evolution and the family of *b*-lognormals which is constrained between the exponential and the time axis. In fact, we first define Darwinian evolution simply as the exponential increase in the number of living species on Earth that characterized the last 3.5 billion years of life on Earth. In other words, we assume that 3.5 billion years ago the first and only living organism appeared (RNA?) and draw an exponential curve linking that point to the current (about) 500,000 living species. This exponential curve is then the geometric locus of all maxima of a one-parameter family of *b*-lognormals (the family free parameter is the birth time *b* of any new species) accounting for cladistics (i.e., the modern theory of evolution rigorously based on when a new species appears in the course of evolution, and not on any other naive and simplistic taxonomy claims). In still other words, each new species is an exponential curve, either slightly increasing or slightly decreasing in time, which departs from the "main exponential" (the overall envelope) when that new species originates. As a further new result, we also derive the "NoEv" or "No-Evolution" probability distribution for a given species, namely the pdf applying when a given species does not undergo any change for ages (i.e., when its members are born, grow up, mate, age, and die for millions or billions of years without their number being subject to any significant increase or decrease). Surprisingly, this brand-new probability distribution which stems from our theory is no longer a lognormal or a *b*-lognormal: it is something new, like a static law of evolution, and the fact that the relevant "NoEv" paper was published in a journal like *OLEB* (*Origins of Life and Evolution of Biospheres*) means we are not talking garbage.

Chapter 9—Societal statistics by the statistical Drake equation. This chapter is devoted to a new possibility thrown up by the statistical Drake equation: mathematically deriving new statistical results about previously unknown topics from statistical data we already know about. The unknown topic here is the "societal part" of the Drake equation (i.e., the product of its last three terms $fi \cdot fc \cdot fL$). These three terms yield, respectively: (1) fi the probability that intelligent life (i.e., higher than monkeys) could arise on a planet already teeming with life as has been the case for the historic evolution of humankind since its appearance on Earth some 7 million years ago up until the discovery of radiowaves enabling communications between different ET civilizations in the Galaxy (the existence of radiowaves was first understood mathematically in 1864 by James Clerk Maxwell as sinusoidal solutions to his newly discovered Maxwell equations); (2) fc means the communicative phase of a civilization using radio, lasers, or even neutrinos, which historically started in 1864 for humans and is ongoing right now; (3) fL means the overall life-time of a civilization, from its inception until its death (e.g., as a result of asteroid impact, nearby supernova explosion, a rogue planet or star disrupting the gravitational stability of the stellar system of interest, or even because of nuclear wars among ETs), about which we know nothing at all. Having said this, Chapter 9 suggests we might know something (i.e., a statistical distribution) for the "societal part" $fi \cdot fc \cdot fL$ by rewriting it as the ratio $fi \cdot fc \cdot fL = N/(Ns \cdot fp \cdot ne \cdot fl) = N/N_{\text{Hab}}$. Since the probability distributions of both N and N_{Hab} are known (lognormals of the Drake and Dole equations, respectively), it all boils down to computing the new probability distribution of the ratio between two lognormals, which is not a lognormal but another more general distribution derived by us in Chapter 9.

Chapter 10—Cubics of historical recovery. Carl Sagan in his book (and TV series) *Cosmos* neatly points out the 1,000 years of lost progress by humanity in between the fall of the Western Roman Empire (476 AD) and the recovery of the

Italian Renaissance (about 1400 AD). Well, in Chapter 10 we cast this into a simple (perhaps simplistic) mathematical curve: a cubic (i.e., an algebraic equation of the third degree as a function of time). We show that the numeric values of this cubic match historic progress made in the following fields rather well: (1) astronomy from 1000 BC to 2000 AD, (2) SETI between 1450 and 2000, (3) search for exoplanets between 1950 and 2010, (4) unification of Europe between 1750 and 2010, (5) human life expectancy between 10000 BC and 2000 AD also extrapolated out to 3000 AD and 10000 AD. All these results are offered as simple mathematical models of what appears to be an "historical recovery law" of human civilizations, which might perhaps be extended to other ET civilizations as well, but only after SETI succeeds, of course.

Chapter 11—Exponential evolution in time as geometric Brownian motion. The statistical Drake equation described in Chapter 1 and the following chapters is static (i.e., it does not change in time). It was not until January 8, 2012 that this author came to realize that his static statistical Drake equation actually is a "snapshot" of a very important stochastic process called a "geometric Brownian motion" (GBM), which is more a "movie" than a "snapshot". But GBM is a very, very important stochastic process, probably the most important stochastic process of all: in fact, it was proven back in 1973 to be the key equation in the Black–Scholes model now used everyday in mathematical finance (*http://en.wikipedia.org/wiki/Black%E2%80%93Scholes*). Robert C. Merton was the first to publish a paper expanding the mathematical understanding of the options-pricing model and coined the term "Black–Scholes options-pricing model". Merton and Scholes received the 1997 Nobel Prize in Economics for their work and, though ineligible for the prize because of his death in 1995, Black was mentioned as a contributor by the Swedish academy. Having said this, we prove in Chapter 11 that GBM really is the same as the exponentially increasing number $N(t)$ of communicating civilizations in the Galaxy, subject, however, to uncertainty. In other words: as intelligence and technologies continue to evolve the overall number $N(t)$ of ET civilizations in the Galaxy increase exponentially, but this is subject to the risk of some civilizations suddenly disappearing because of asteroid impacts, nearby supernova explosions, rogue planets or stars disrupting the gravitational stability of the stellar system in question, or even because of nuclear wars among ETs. So, the mean value of $N(t)$ grows exponentially in time as $\langle N(t) \rangle = N_0\, e^{\mu t}$, but $N(t)$ itself is a random process with ups and downs, essentially given by $N(t) = N_0 \cdot e^{\left(\mu - \frac{\sigma^2}{2}\right)} \cdot e^{\sigma B(t)}$, which is what GBN is, $B(t)$ being the standard $(0, 1)$ Brownian motion. So far so good, but after having discovered this, we did more: we found the pdf of the distance stochastic process ("Maccone process"?) given by $D(t) = \dfrac{C}{\sqrt[3]{N(t)}} = \dfrac{\sqrt[3]{6R^2_{\text{Galaxy}} h_{\text{Galaxy}}}}{\sqrt[3]{N(t)}}$. This of course reduces to the "Maccone" distance distribution between any two nearby ET civilizations found in Chapter 1 for the static case, which is also the distance distribution between two nearby habitable planets (with different numbers) as proven in Chapter 3. Thus, in conclusion, we believe that Chapter 11 is the most important chapter in this book as it paves the way to future statistical considerations about ETs and their distances in the Galaxy.

PART II—SPACE MISSIONS TO EXPLOIT GRAVITATIONAL LENSING. This second part of the book comprises five chapters.

Chapter 12—So much gain at 550 AU. Gravitational lensing is the bending of light around massive celestial objects as predicted by Einstein's general theory of relativity. Although Einstein published his predictions in 1915 and in greater detail in 1936, it was not until 1978 that the first "twin-quasar image" (i.e., an image of the same quasar "doubled" by the gravitational lens of an intermediate galaxy) was indeed spotted by astronomers, proving the theory. But, what do we know about single spherical stars acting as lensing objects? Well, in Chapter 12 it is shown that around each star a "focal sphere" exists such that, if the observer is inside that sphere, no lensing effect occurs, but if the observer is outside that sphere, then an enormous magnification of all objects located on the opposite side of the star with respect to the observer's position occurs. In particular, this is true for the radio signals emitted by an ET civilization located so far away from the Sun that no antenna on Earth, however large, would be capable of detecting them. Thus, exploiting the Sun as a gravitational lens has become a must for all precursor interstellar missions reaching at least 550 AU from the Sun in any direction. The distance 550 AU (i.e., 3.17 light-days or 14 times the Sun-to-Pluto distance) is the radius of the focal sphere around the Sun predicted by general relativity. This author has given the name "FOCAL" to any future space mission getting to 550 AU and beyond to take advantage of the huge radio and visual magnifications provided by the gravitational lens of the Sun. These future space missions were described with plenty of technical details in the author's 2009 book entitled *Deep Space Flight and Communications: Exploiting the Sun as a Gravitational Lens*, published by Springer–Praxis. FOCAL is an acronym for "Fast Outgoing Cyclopean Astronomical Lens", which summarizes well what this new space mission is all about.

Chapter 13—FOCAL mission to 1,000 AU as an interstellar precursor. Future FOCAL space missions might have three different targets: (1) getting a magnified image of the black hole (called Sgr A*) located at the center of the Galaxy and of its surroundings. (2) Getting unprecedented details of the nearest triple-star system (Alpha Centauri A, B, and C) in view of the first truly interstellar mission that would probably have that stellar system as its target, simply because it is the nearest to the solar system. (3) Getting unprecedented details about any extrasolar planet that might possibly be discovered in the future to host life, whether in lower forms only or in the higher form of an extraterrestrial civilization. These topics are technically discussed in Chapter 13, keeping in mind that the gravitational lens of the Sun (or of any star) favors space missions getting farther away from their minimal focal sphere inasmuch as the farther they go the more they get rid of the coronal effects of that star, such as the electrons creating divergent lensing effects. This is the reason the real optimal distance that any future FOCAL spacecraft from humanity must reach is about 1,000 AU (i.e., about one light-week), rather than just 550 AU or half a light-week.

Chapter 14—Belt of FOCAL spheres between 550 and 17,000 AU. So far we have discussed the gravitational lens created by the Sun, but what about the gravitational lenses created by the planets? Well, the radius of each focal sphere is proportional to the ratio between the square of the body's physical radius and the body mass. Thus, it is shown in Chapter 14 that an actual belt of eight nearly concentric focal spheres exists between 550 and 17,000 AU, created by the gravitational lensing effect of the Sun and the six more massive planets. In increasing distance from the Sun, there are the focal spheres of the Naked Sun (at 550 AU), Coronal Sun (at 1,000 AU), Jupiter, Neptune, Saturn, Earth, Uranus, and Venus. It follows that any precursor interstellar space mission will have to cross all these focal spheres, taking advantage of each of them in order to magnify the images of celestial objects lying on the other side of the lensing body. In addition, since all planets move around the Sun, planet lenses will be moving lenses with respect to the Sun, thus making sweeping arcs of circles on the celestial sphere as seen from the FOCAL spacecraft. In must be added, however, that the magnification provided by each such planetary lens will be smaller, or much smaller, than the magnification provided by the Sun. Thus, the notion of magnification must be carefully considered. On February 2, 2010, this author received an e-mail from Professor Frank Drake in which he suggested that the Kraus gain used up to then should be replaced by another mathematical formula, called the "Drake gain" by the author of this book, proving much higher numbers for the gain, just as power gain is much higher than voltage gain. This topic is also discussed in Chapter 14, although it seems to the author of this book a matter open for debate (at least as of 2012).

Chapter 15—Galactic Internet by star gravitational lensing. Up to now, we have only considered gravitational lensing made by a single body, whether the Sun or a planet. In Chapter 15 we study the brand-new idea of a "radio bridge" between any two stars, with each star contributing the gravitational lensing focusing power due to its own radius and mass. The combined effect of the two stars would enormously improve radio communications between the two stellar systems inasmuch as the overall gain provided by the two stars would be so large that communications across huge distances would become feasible with modest input powers too. In other words, (in popular terms) the answer to the question "Could we talk to Alpha Centauri A with a cellphone?" would be "Yes, if we exploit the gravitational lenses of both stars." Of course, two FOCAL space missions would now be required to build the radio bridge: one in the solar system reaching the minimal focal distance of 550 AU or beyond, and one in the Alpha Centauri A system reaching the minimal focal distance of 749 AU and beyond. Better still, suppose for a moment that humanity was already capable of sending a FOCAL probe to α Cen A. The answer to the question "What would we do then?" would be "Put it on the other side of α Cen A with respect to the Sun direction and keep the perfect (or nearly perfect) alignment between the four points in space: the FOCAL probe on the opposite side of α Cen A, the center of α Cen A, the center of the Sun, and another FOCAL probe at, say, 600 AU from the Sun in the direction opposite to

α Cen A.'' Then, a perfect radio bridge exploiting both stars as gravitational lenses would have been constructed, and communications would be feasible at minimal input energy costs, although the required time for all messages to reach the destination would still be 4.37 years, of course. In Chapter 15 we also consider several more radio bridges, like those of the Sun–Barnard Star, the Sun–Sirius, the Sun–Sun_at_Galactic_Center, and even the Sun–Sun_at_Andromeda. In all cases, the radio bridge insures moderate power expense to keep the radio link. This is a wonderful new step ahead, inconceivable before the Sun as a gravitational lens was deeply studied for the first time.

Chapter 16—Extragalactic Internet by black hole gravitational lensing. SETI between galaxies has long been regarded as impossible to achieve because power attenuation across huge intergalactic distances would be fatal to any signal, however energetic. In Chapter 16 we prove that this assumption is incorrect if radio bridges (in the sense described in Chapter 15) between the two black holes located at the center of the two galaxies are exploited. In other words, consider, for instance, the big black hole Sgr A* located at the center of the Milky Way Galactic Bulge, which has a mass of about 4 million Suns, and the similar but 10 times more massive (i.e., a mass of 40 million solar masses) P2 black hole located at the center of the Andromeda Galaxy (M31) at a distance of 2.5 million light-years from the Milky Way. Quite surprisingly, it turns out that the Sgr A*–Andromeda (M31) P2 black hole radio bridge performs better than the ''small'' Sun–α Cen A radio bridge! This is to say that our calculations show that, to maintain the link between the two black holes at the two galaxies' centers the same transmission power is required as to maintain the link between the Sun and α Cen A. In fact, it is actually much less than that since only about 1/10,000 of the above-mentioned power would be required. This ''miracle'' is of course due to the huge masses of the two black holes, which more than compensates for the abysmal distance between the Milky Way and Andromeda. Similarly, it is proven in Chapter 16 that other radio bridges between supermassive black holes of other nearby galaxies work equally as well. The most unexpected case is provided by the radio bridge Sgr A*–M67 big black hole, having an estimated mass of 6.6 billion solar masses compensating for the 57 million light-year distance and yielding a performance about 10,000 times better than the ''small'' Sun–α Cen A radio bridge. All this sounds like science fiction, but it is science fact showing that the black hole at the center of each galaxy is indeed the most important part of the galaxy, where ET civilizations may even be fighting each other to keep control of this powerful supermassive black hole radio station. Human SETI scientists should carefully look at Sgr A* and its surroundings to see if ''something strange'', such as a ''star war'', is taking place there right now (i.e., about 25,000 years ago!).

PART III—KLT FOR OPTIMAL SIGNAL PROCESSING. The third part of this book comprises 12 chapters.

Chapter 17—A simple introduction to the KLT and BAM-KLT. The KLT (Karhunen–Loève Transform) is a mathematical algorithm to extract weak signals from thick noise much more powerfully than the Fast Fourier Transform (FFT). In essence, the KLT is the well-known Principal Components Analysis (PCA) applied to both filtering (as just said) and data compression, and it is widely used by scientists and engineers of all kinds, unfortunately under many different names and often without a clear understanding of the mathematics involved inasmuch as the mathematics is often regarded as too difficult to understand by engineers. In Chapter 17, the first of 12 chapters devoted to studying the KLT in different situations, we first provide an easy-to-understand description of the classical KLT (i.e., the KLT as usually taught in university graduate courses—Sections 17.1 through 17.6). The classical KLT may be summarized (in technical language) as a principal axes transformation in the Hilbert space spanned by eigenvectors of the autocorrelation of noise plus a possible embedded signal. Radio SETI, when regarded as the extraction of weak signals from thick noise, is of course the ideal application field of the KLT. Later in the chapter (Sections 17.7 through 17.14) the reader is introduced to the BAM-KLT (an acronym for Bordered Autocorrelation Matrix KLT), investigated and published by this author since 2008 as an advanced section of KLT theory. The basic BAM-KLT goal is to get around the main KLT difficulty of the $O(N^2)$ calculations required to compute the eigenvalues and eigenvectors of an $N \times N$ autocorrelation matrix, which is a much higher computational burden than the $O(N \ln(N))$ calculations required by the FFT. The BAM-KLT, however, is still a fresh new topic, needing more profound investigations. Finally, we would like to point out the recent and important new development that began with the publication of the paper by Arkadiusz Szumski, "Finding the interference: The Karhunen–Loève Transform as an instrument to detect weak RF signals," *InsideGNSS* (Working Papers section), May–June 2011 issue, pp. 56–63. In practice, the young Polish telecommunications engineer Szumski was tasked by the European Space Agency (ESA) with checking the validity of this author's new results about the KLT and the BAM-KLT by writing a numerical code enabling KLT to extract very weak and complicated (chirped, non-stationary, transient, etc.) signals from random noise across both narrow and wide bands and with signal-to-noise ratios (SNRs) like 10^{-3} or smaller (the FFT already fails with $\text{SNR} = 1$). The results of Szumski's simulations have convinced many that the KLT and BAM-KLT are well suited to be applied in GNSS (global navigation satellite systems), where huge investments are being made to improve old-fashioned FFT-based technologies, totally inadequate when compared with the KLT. The future will tell, and this book, originally intended for SETI scientists only, might become palatable to GNSS engineers as well.

Chapter 18—KLT of radio signals from relativistic spaceships in uniform and decelerated motion. By "relativistic spaceship (or probe)" in the following we mean a spacecraft that can reach speeds that are a significant fraction of the speed of light. For such spacecraft, Newtonian physics (nowadays successfully applied to nearly all spacecraft) would no longer hold good, and Einstein's special theory of relativity must be applied instead. No relativistic spaceship yet exists, and so the wealth of results, presented in this and the following chapters about optimal telecommunications between the Earth and a relativistic probe moving either away from Earth or approaching it, might superficially be dismissed. However, we are convinced that, sooner or later, relativistic spacecraft will indeed be built, and so Chapters 18 through 28 are hardly a waste of time for the reader to digest, even if mathematically difficult to understand at first sight. So, let us start by pointing out that the most striking feature of the theory of relativity is of course the existence of two times: (1) "coordinate time" (i.e. the time of guys remaining on Earth, say) and (2) "proper time" (i.e., the time of astronauts living inside the relativistic spacecraft). The key point is that proper time elapses more slowly than coordinate time (i.e., the twin paradox): if one of two twins stays on Earth and the other one flies around in space at relativistic speeds, when he comes back to Earth his brother would probably have long since died, inasmuch as his brother's time had elapsed much faster than the relativistic astronaut's time. With this in mind, the author (in his youth) developed a full mathematical theory of the KLT for time-rescaled Brownian motions for Gaussian stochastic processes (i.e., for the radio noise hampering telecommunications in space between a relativistic spaceship and the Earth). This mathematical theory is described in Chapters 21 ("Brownian motion and its time rescaling") and 22 ("Maccone First KLT Theorem: KLT of all time-rescaled Brownian motions"), as we shall see later. Essentially, the author proved that the KLT eigenfunctions of all time-rescaled Gaussian processes are Bessel functions of the first kind, while the corresponding eigenvalues are the zeros of such Bessel functions and their first derivative. In Chapter 18 these results are applied to two cases of relativistic motions: (1) uniform motion, for which time rescaling is rather obvious since Brownian motion KLT eigenfunctions are just sines, and (2) decelerated motion (i.e., the motion of a spaceship slowing down from high relativistic speed to practically zero speed—as done by alien spaceships in the movie *Independence Day*). Computation of the stochastic integral yielding the total energy of signals received on Earth completes the chapter.

Chapter 19—KLT of radio signals from relativistic spaceships in hyperbolic motion. When we think about interstellar flights to the nearest stars, special relativity shows that the best velocity profile to reach the target with minimal proper time is the so-called "hyperbolic motion": namely, (proper) uniformly accelerated motion from the start to midway and then, after turning the ship by 180 degrees, (proper) uniformly decelerated motion from midway to the target, finally getting there at zero speed. The optimal (proper) uniform acceleration would be $1\,g = 9.8\,\text{m/s}^2$, so as to let the people and equipment on board feel the same weight they have on Earth. At such $1\,g$ (proper) uniform acceleration, the center of the Galaxy would be reached in just 21 years of proper time despite it being 25,000 light-years away from us, but of course the energy requirements for the spacecraft to maintain $1\,g$ (proper) acceleration for 21 years would be huge, which is why the project is unfeasible by today's standards. Less demanding energy requirements would apply of course if the target is the nearest stellar system, Alpha Centauri, just 4.37 light-years away, but, whatever the target may be, telecommunications of some kind will have to be kept between the (proper) uniformly accelerated spacecraft and the Earth. This chapter proves that the KLT eigenfunctions for this hyperbolic motion are Bessel functions of the first kind and order zero. To be precise, this KLT is an asymptotically exact KLT for $t \to \infty$, but in reality the numbers are already acceptable about one year after departure from Earth, thus yielding an applicable result.

Chapter 20—KLT of radio signals from relativistic spaceships in arbitrary motion. The same mathematical techniques already applied in Chapter 19 to hyperbolic motion (i.e., to proper uniformly accelerated motion) are shown in this chapter to be appropriate to derive the KLT for signals emitted by a spacecraft in arbitrary radial motion away from the Earth or towards it (such as an alien spacecraft heading for the Earth would do, as in the *Independence Day* movie of 1996). The calculations are unfortunately long and, at first sight, may appear daunting. But the starting idea is easy: just as was the case for hyperbolic motion, it is possible to exactly integrate the relativistic arbitrary motion radial differential equation. This opens up the possibility of computing the relevant KLT by resorting to the theory of time-rescaled Brownian motions already used in Chapter 18 and described in detail in Chapters 21, 22, and 23. Thus, equations are provided not only for the KLT of the signals received on Earth from a receding or approaching spaceship at an arbitrary speed and acceleration profile, but also for the total amount of energy of the signals emitted by such a spaceship (i.e., for the stochastic integral of the square of the intensity of such radio signals). The time will come when these equations will become important to human expansion (or to human understanding of an alien) into the Galaxy, just as in *Star Trek*.

Chapter 21—Brownian motion and its time rescaling. This is the first of three chapters devoted to detailed mathematical description of the KLT for all time-rescaled Gaussian processes (as mathematicians call them) or time-rescaled Brownian motions (as physicists prefer to call them). This formed a large part of the author's work following his Ph.D. obtained at the Department of Mathematics of the University of London (U.K.) King's College in 1980. The key idea is that if one lets the time variable of ordinary Brownian motion elapse other than in a uniform way (i.e., if one allows time to be stretched, time dilation of special relativity, in an arbitrary fashion), the process still remains Gaussian, and its autocorrelation can be easily expressed in terms of the arbitrary time-rescaling function $f(t)$. Actually, it can easily be proven that the square of the time-rescaling function, $f^2(t)$, equals the well-known radical $\sqrt{1-\frac{v^2(t)}{c^2}}$ of special relativity, and this establishes a firm connection between time-rescaled Brownian motions and special relativity. It is as easy as that.

Chapter 22—Maccone First KLT Theorem: KLT of all time-rescaled Brownian motions. This is the second of three chapters devoted to the detailed mathematical description of the KLT for all time-rescaled Gaussian processes. Since the KLT of ordinary Brownian motion can be proven to be given exactly by sinusoidal functions (i.e., it is basically the same as a Fourier transform), then one may wonder what the KLT of time-rescaled Brownian motions might possibly be. The answer is provided by the "Maccone First KLT Theorem" and is that eigenfuctions are Bessel functions of the first kind suitably time-rescaled, while eigenvalues are the zeros of certain linear combinations of such Bessel functions and their derivatives. It is also possible to find an asymptotic version of this KLT for $t \to \infty$ where things are easier and ready for engineering applications. The mathematical proof of this "Maccone First KLT Theorem" is unfortunately lengthy and hinges on the Sturm–Liouville theory of boundary problems for ordinary differential equations of the second order with non-constant coefficients. The author discovered all this back in the early 1980s, and published these results in the Italian mathematical journal *Bollettino dell'Unione Matematica Italiana* ("Eigenfunctions and energy for time-rescaled Gaussian processes", Bollettino UMI, Series 6, Vol. 3A (1984), pp. 212–219). He was then seeking a university position somewhere, but that paper was regarded as "just a physical application" by pure mathematicians and "useless for particle physics" by physicists of the time. So, the author failed to get a position, and in 1985 he joined the space company Alenia Spazio S.p.A., where the engineers were even less interested in his work than the pure mathematicians and particle physicists. The truth is that nobody cared about the telecommunication theory for relativistic interstellar flights, with the exception of a few bright minds such as Dr. Leslie Shepherd and Dr. Giovanni Vulpetti of the Interstellar Space Exploration Committee (ISEC) of the International Academy of Astronautics (IAA), which this author eventually joined.

Chapter 23—KLT of the $B(t^{2H})$ time-rescaled Brownian motion. This is the third of three chapters devoted to detailed mathematical description of the KLT for all time-rescaled Gaussian processes. It deals with the KLT of a Brownian motion whose time variable does not elapse uniformly, but rather increases like power t^{2H} (time dilation). The H parameter is the Hurst exponent of Mandelbrot since the $B(t^{2H})$ is indeed an H self-similar stochastic process. This author discovered that this process has an exact KLT by solving the relevant integral equation back in 1979, when he was preparing his Ph.D. thesis in Mathematics at the University of London (U.K.) King's College, awarded to him on September 19, 1980. Afterwards, the author returned home to Turin, and finally published his exact KLT of $B(t^{2H})$ in both the *Bollettino dell'Unione Matematica Italiana*, Series 6, Vol. 4-C (1985), pp. 363–378, and *Il Nuovo Cimento* (the Italian physics journal), Series B, Vol. 100 (1987), pp. 329–342. He also found time to study the stochastic integral yielding the total energy of this stochastic process and generalize the Cameron–Martin formula that holds for standard Brownian motion to this process.

Chapter 24—Maccone Second KLT Theorem: KLT of all time-rescaled square Brownian motions. The content of this chapter was long regarded by this author as his best mathematical result ever about the KLT: it was published as "The Karhunen-Loève expansion of the zero-mean square process of a time-rescaled Gaussian process," *Bollettino dell'Unione Matematica Italiana*, Series 7, Vol. 2-A (1988), pp. 221–229. The author still remembers the reviewer's words *con una serie di passaggi ben condotti* ("by virtue of a series of well-conducted steps") it was indeed possible for him to exactly solve the KLT integral equation for the square of any time-rescaled Gaussian process (as described in detail in Chapter 24). The calculations are complicated, and the very few who read that 1988 paper probabaly uttered the usual question "What's the purpose of all this?" This author's paper was regarded as of little use for pure mathematicians or particle physicists, and no use for telecommunication engineers. But, all this simply stems from the fact that no relativistic interstellar flights have been studied, neither back in 1988 nor today. Looked at more generally and more positively, this and related KLT papers could become useful in the study of transient astrophysical phenomena, like supernova explosions, gamma ray bursts, and SETI signals that are just transient because of short-lasting gravitational lensing alignments among stars and/or other bodies. However, Chapter 24 contains a result that might already be of some use today: the exact KLT of the square of standard Brownian motion, which is even simulated numerically in the chapter just to check that everything is right.

Chapter 25—KLT of the $B^2(t^{2H})$ time-rescaled square Brownian motion. This chapter is with respect to Chapter 24 the same as Chapter 23 is with respect to Chapter 22. In other words, in Chapter 25 we prove that the KLT of the time-rescaled stochastic process $B^2(t^{2H})$ is the exact solution of the corresponding KLT integral equation, given by certain Bessel functions of the first kind and, similarly, the eigenvalues are the zeros of similar Bessel functions. The asymptotic version of these results for $t \to \infty$ is also obtained and is particularly simple inasmuch as the Bessel functions then downgrade to ordinary cosines whose time variable is time-rescaled like a power of t. In this chapter we were likewise able to check by numeric simulations that the results are indeed correct.

Chapter 26—Maccone Third KLT Theorem: Asymptotic KLT of GBM. The geometric Brownian motion (GBM) is today the most important stochastic process of all, since it is widely used in the mathematics of finance and, in particular, in the Black–Scholes models. It was not until January 8, 2012 that it dawned on this author that his statistical Drake equation (described in Chapter 1) is just the particular case $t = 1$ and $N_0 = e^{\frac{\sigma^2}{2}}$ of a GBM. In other words, all the equations derived in Chapter 1 are static particular cases of much more general corresponding equations of a GBM in which time t is not constrained to any particular numeric value. This was proven in Chapter 11, and the conclusion was that the $N(t)$ stochastic process ($t \geq 0$), yielding the number of communicating ET civilizations in the Galaxy at any positive time t, is exactly a GBM. We know that every stochastic process—and thus $N(t)$—has a KLT. So, the next natural question arises: What is the KLT of a GBM? The author is unable to answer this question at the moment. However, back in September 2007, he was able to derive the asymptotic KLT (i.e., valid for $t \to \infty$) of the exponential Brownian motion $e^{B(t)}$ which now appears to be the key stochastic (i.e., non-simply deterministic) part of the GBM definition. This asymptotic KLT of a GBM is published here for the first time. The starting point is the surprising result that the time-dependent order $\nu(t)$ of Bessel functions of the first kind described in Chapter 22 is practically equal to 1 for just moderate values of t a few instants after the initial time $t = 0$, and stays as such for ever (i.e., for all higher values of time). Thus, in practice it is possible to simply set $\nu(t) = 1$ as the order of Bessel functions order, and then keep going! The asymptotic KLT of the GBM for $t \to \infty$ is announced—a brand-new result that might have profound impacts on further developments. But, at the moment the author confines himself to giving the new equations.

Chapter 27—A Matlab code for KLT simulations. Mathematical books today should also provide numeric codes to simulate the new equations provided in analytical form. This is the goal of Chapter 27. In 2008, the author and his pupil Dr. Nicolò Antonietti set out to write a Matlab (or Octave) code capable of simulating at least the most important KLT expansions given in this book. The input was usually standard Brownian motion or, alternatively, a user-provided input stochastic process numerically assigned. The output was the KLT displayed in three forms: (1) the full KLT reconstruction based on all instants between 0 and T; (2) the KLT reconstructed by using only the first few eigenvalues and eigenvectors of the autocorrelation matrix, thus yielding—instead of a perfect reconstruction—a smooth curve interpolating the data of the input stochastic process (this we call "an empirical reconstruction" of the input stochastic process where the degree of approximation can be selected at will, of course—in other words, the KLT is also a data compression algorithm); (3) when available, we also provide the exact KLT or, if that is unavailable, we provide the asymptotic KLT for $t \to \infty$ based on the asymptotic expansion of Bessel functions of the first kind for $t \to \infty$.

Chapter 28—KLT applications (by *Stephane Dumas*). The author's new French Canadian pupil, Monsieur Stephane Dumas of Québec, kindly provided for free the whole content of Chapter 28. After an introduction to the many applications of the KLT under different names (PCA = Principal Component Analysis, SVD = Singular Value Decomposition, and so on), Dumas provides the Fortran code that he wrote in order to use the KLT in a number of applications. He also sought to optimize the computing time, which is of course the main obstacle still hampering real-time applications of the KLT instead of the much more customary (and much less profound) FFT. This chapter is of course intended for the applied sciences of all kinds, and not just SETI.

PART IV: THE UNITED NATIONS AND PROTECTION OF THE MOON'S FARSIDE. The fourth part of this book comprises a single chapter.

Chapter 29—The United Nations and protection of the Moon's farside. One of this author's teachers who made a profound impression upon him was the Frenchman, Jean Heidmann (1920–2000) (*http://www.universalis.fr/encyclopedie/jean-heidmann/*). In the 1990s, Heidmann was tasked by the International Academy of Astronautics (IAA) with studying how best the Moon farside could be protected by law against future human polluters of any kind (real estate, industry, military) that could damage the unique environment existing on the Moon's farside, which is free of man-made radio noise. This is more a political problem than an astronautical and scientific problem and needs to be resolved urgently by some international agreement among the main spacefaring nations before it is too late. Just before Heidmann passed away in 2000, he appointed this author as his deputy to lead the relevant IAA team, and so this author took over that difficult task. Although he published several papers on the subject (see, e.g., "Protected Antipode Circle on the farside of the Moon,'' *Acta Astronautica*, Vol. 63 (2008), pp. 110–118), this was just the beginning of his activity in this field. On June 10, 2010, this author delivered a speech at the United Nations Committee on the Peaceful Uses of Outer Space (COPUOS) in Vienna, Austria, to raise the issue at the United Nations level for the first time. Chapter 29 describes in detail the issues at stake, and how the United Nations could resolve this space problem for the future benefit of all humankind.

PART V: EPILOGUE. The fifth part of this book comprises a single chapter.

Chapter 30—Epilogue: Evolution, progress, and SETI. This book ends with an epilogue about SETI. What if SETI succeeds? It's hard to predict the consequences of the first contact between humankind and ETs. If we look back at past human history, we know that the most important sudden contact between two historic civilizations that had independently developed different levels of technology was the discovery of America by Europeans, which resulted in the violent subjugation of the lower-level civilization by the higher-level one. Yet, those were times when war was common practice to all civilizations on Earth, and so the resulting violence was perhaps inevitable. Perhaps more advanced civilizations than ours might come to put an end to wars and contact would thus be smoother. In any case, as of 2012 we know that an immense number of extrasolar planets do indeed exist in the Galaxy, and so thinking that we are alone is no longer realistic. The best we can do is to keep exploring, as astrobiology does nowadays, and keep ''Searching for Life Signatures'', as the conferences run by this author since 2008 under the aegis of the IAA are called. Only the future will tell.

Conclusion

This is a textbook for high-level students and research scientists—it is not popular stuff at all. The author feels that a book like this is really needed by the worldwide scientific community since it tries to bridge the gaps among branches of science that are still regarded as independent of each other—just to mention a few: astronomy, evolution of life on Earth and elsewhere, mathematical history (a brand-new discipline?), and of course astronautics (especially, future relativistic interstellar flights). To combine all this in some ''unified mathematical description'' is something that needed to be done. This book is a step in the right direction toward this new vision of humanity in the Galaxy that future decades and centuries will increasingly develop.

We would like to conclude this preface with the same sentence used by Johannes Kepler (1571–1630) when he disclosed to the world the discovery of his Third Law (1619), which he could only achieve by coupling astronomical observations to the intense use of mathematics. To correctly understand Kepler's sentence, however, one has to remember that, in Kepler's time, Bible reckoning had made scholars believe that the Universe had been created just about 6,000 years earlier. Keeping that in mind, we now report Kepler's sentence:

> *"The die is cast; the book is written, to be read either now or by posterity, I care not which. It may well wait a century for a reader, as God has waited six thousand years for an observer"*

from *Harmonice mundi* (1619), translated in Bill Swainson and Anne H. Soukhanov, *Encarta Book of Quotations* (2000), p. 514.

Claudio Maccone
Torino (Turin), Italy, February 29, 2012

Acknowledgments and dedication

This author is indebted to many individuals for inspiration and help over the 30-year period (1982–2012) of his involvement in the SETI field. To list them all, however, would simply be impossible. Thus, only two colleagues will be publicly acknowledged here because of their influence on the author's thoughts as well as on his career over the last three decades:

(1) Professor Frank Donald Drake, an American researcher, the first experimental SETI scientist ever and discoverer of the Drake equation studied in this book—quite simply this author's most inspiring and respected teacher.
(2) Dr. Jean-Michel Contant, a French researcher, Secretary General of the International Academy of Astronautics (IAA) and constant supporter of SETI activities within the IAA. Indeed, the IAA is responsible for:
    (a) creation of the IAA's SETI Permanent Committee and its two sessions (SETI I: science and technology and SETI II: societal aspects) held yearly at the International Astronautical Congress (IAC);
    (b) publication of the many important SETI papers in the IAA journal *Acta Astronautica* over the years;
    (c) the Moon Farside Protection action undertaken by this author at the United Nations (as described in Chapter 29).

To these two individuals in particular, as well as to all other SETI scientists from all over the world, this book is dedicated.

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Technical Director for Scientific Space Exploration
International Academy of Astronautics (IAA)
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Torino (Turin), Italy, March 19, 2012

(Left to right) Jean-Michel Contant, Frank Drake, and Claudio Maccone at the Royal Society in London, January 26, 2010.

Foreword

One of the most important discoveries in the history of astronomy—indeed, in all of science—was made in recent years: this was the detection of thousands of other planetary systems beyond our own. This is the result of the endeavors made by a small group of astronomers diligently observing, night after night, small deviations in the positions of lines in stellar spectra and observing faint occultations of stars by planets as revealed by the Kepler spacecraft. All of this has happened while observing less than 1% of the visible stars in the night sky. The detection technique used by Kepler can only detect about 1% of the planetary systems actually orbiting the stars observed. So these amazing results tell us that there must be millions of planetary systems orbiting the stars we can observe, and simple models put the number of planetary systems in our entire Milky Way Galaxy in the billions. What an exciting result this is, and the age-old question of whether there are other worlds like ours has been answered with a resounding "Yes!"

Equally exciting is the discovery that a large proportion of these systems have planets that may be inhabited by living creatures, just as here on planet Earth. These planets must be the right size and right distance from their star for life as we know it to exist. There are very many of these planets. If there is life and if it has evolved in much the same way as life has on Earth—which seems inevitable—then there must be a large number of planets supporting intelligent life probably with technologies like ours. Such technologies as radio transmitters and powerful lasers may be detectable by instruments that already exist on Earth or that soon may be developed.

All of this strengthens our resolve to extend the scope and speed with which the search for extraterrestrial intelligence, SETI, is undertaken, which has for the last 50 years or so been done with very limited resources. The knowledge that there really is life akin to ours deep in space justifies assigning far greater resources to the search than in the past—resources like telescope capability, telescope observing time, the time of dedicated scientists. To facilitate eventual discovery and to use such resources responsibly calls for the most careful planning.

Thus, it is imperative to analyze our search methods and procedures carefully so that we can create the best possible SETI program from the resources available. A giant step toward quantifying the power of our searches and, in so doing, optimizing our search programs has been made by Claudio Maccone in this book.

The author has created a powerful mathematical framework for quantifying the capability of SETI programs to succeed. This is based on the 50-year-old Drake equation, which can be used to predict the probability that a civilization might be encountered in a given search. In this book the author has applied statistical methods rigorously to application of this equation. He shows how his methods may be applied beneficially to other problems and, in so doing, enhance our understanding of the mathematical approaches he has developed.

Lastly, he points the way to a very powerful method to detect other civilizations from their faint signals, feeble radio transmissions, and dim lights of their cities at night. This is to use our Sun as a gravitational lens to create a giant telescope that concentrates all forms of electromagnetic radiation at a series of foci positioned many hundreds of astronomical units from the Sun. This, in effect, creates a lens that is larger than the diameter of the Sun, more than a million kilometers across, to create brilliant images with image detail of incredible resolution.

Although such a telescope is beyond our technical capabilities at present, it may be probable in the not-too-distant future, perhaps in a hundred years or so. It is important to consider it in detail, as the author does, because many, maybe most, older civilizations than ours will have long since implemented the lens provided by their star. If they use such lenses not only to search for life, but also to amplify signals very powerfully and focus them on distant civilizations, how might these signals appear? This is important because it could well be that such amplified signals will be what we can most easily detect. We need to know what forms they might take so that we are instrumentally and mentally prepared should such a signal be captured. The detection of such a signal would be one of the most monumental discoveries ever and, simultaneously, would demonstrate that many more discoveries are awaiting our attention. The window would be opened to an incredible wealth of beneficial information which would enrich us all.

Foreword by Giovanni Bignami

Dr. Claudio Maccone submitted a Proposal to the European Space Agency (ESA) about the FOCAL (Fast Outgoing Cyclopean Astronomical Lens) space mission to 550 AU and beyond in the year 2000, when I was Scientific Director of the Italian Space Agency (ASI). I immediately endorsed Maccone's Proposal acknowledging that ASI would support it if ESA approved it. Unfortunately, ESA failed to select Maccone's Proposal for further study and so his ideas had to be temporarily abandoned, but that did not mean they were bad ideas. In 2009, Dr. Maccone published a whole technical book on the subject, entitled *Deep Space Flight and Communications*, and more interest arose again in 2011 on the occasion of the DARPA-NASA 100 Year Starship Study. Quite simply, any future precursor interstellar mission will have to cross the minimal focal sphere at 550 AU beyond which it is possible to exploit the huge signal magnifications provided by the Sun as a gravitational lens. Thus, Maccone's FOCAL mission is destined to pop up again and again in the future, until favorable political conditions enable some important space agency to make it a reality. In this new book about mathematical SETI, five chapters (Chapter 12 through 16) are devoted to the physics and mathematics of the Sun as a gravitational lens with important updates with respect to the 2009 book by the author. For instance, it is proven that, even for the nearest interstellar mission to Alpha Centauri (4.37 light-years away), the ordinary radio telescopes we have on Earth would be inadequate to maintain the radio link between the Earth and the receding spacecraft unless the huge radio amplification provided by the Sun as a gravitational lens is invoked. In other words, Maccone points out that, before launching a truly interstellar spacecraft to even the nearest star, it would be necessary to send a large space antenna in the opposite direction of the target with respect to the Sun just in order to keep the radio link at moderate powers until the interstellar spacecraft reached Alpha Centauri. But, Maccone goes beyond that. Suppose for a moment that our spacecraft had already reached Alpha Centauri: What would we then do with it? Maccone's suggestion is to put the interstellar probe on the other side of, say,

α Cen A (the most massive of the three stars) and in a direction exactly opposite to the Sun. Then, these two stars and the two FOCAL probes on their outer sides, respectively, would form a huge "radio bridge" capable of reducing the link powers dramatically. In popular terms, Maccone answers the question "Would we be able to maintain the link to α Cen using a cellphone?" by saying "Yes, if we exploited the gravitational lenses of both stars by putting two FOCAL probes on their opposite sides." This notion of a "radio bridge made possible by star gravitational lensing" is then extended to other increasingly distant stars, showing that the links are still feasible even up to thousands of light years and more. Thus, even a sort of "Galactic Internet" could indeed be created, or perhaps it has already been created by more advanced civilizations in the Galaxy, but humans will be unable to join it until their spacecraft reach the minimal focal distance from our own star, the Sun, at 550 AU. Finally, Maccone extends these notions even to extragalactic distances, showing that the notion of a "radio bridge" still holds if one replaces the masses of the stars by the masses of the supermassive black holes located at the center of each galaxy. Thus, for the Milky Way, SgrA* (i.e., the 4,000,000 $M_{\odot}$ central black hole) would be the "magnifying gravitational lens" capable of keeping the Milky Way in touch with other galaxies. Just science fiction? No, just classical physics, but revealed by the new open-minded SETI vision of the universe that many still lack.

Chapters 17 through 28 of this book deal with a completely different mathematical topic: how to improve the ordinary noise-filtering techniques based on the FFT (Fast Fourier Transform) by the much more sophisticated KLT (Karhunen–Loève Transform). The basic idea seems to be that, while the FFT only uses sines and cosines as the set of orthonormal functions in the Hilbert space upon which the series expansion is made, the KLT uses the best set of orthonormal functions changing them according to the input, rather than keeping them fixed to sines and cosines. This optimizes the transform by adapting it to the input, whatever that might be. Unfortunately, KLT mathematics is much less simple than FFT mathematics, and that makes the topic difficult to follow for engineers not particularly familiar with eigenvalues, eigenvectors, and all that. To physicists, on the contrary, the KLT is immediately clear since it is just the ordinary Hilbert space apparatus of quantum physics translated into the language of signal processing. Maccone then finds explicit equations for KLT eigenvalues and eigenfuctions in a number of practical cases of interest to both SETI and astrophysics in general. For instance, the rotational period of the Vela pulsar was impossible to find by the ordinary FFT, but was indeed found by a team from the Asiago Observatory at the European Southern Observatory at La Silla (Chile) using the KLT. More generally, the theory of transient phenomena might greatly benefit by the adoption of the KLT instead of the FFT. It must be said, however, that the computational burden is much higher for the KLT than for the FFT: such as $O(N^2)$ for the KLT as compared with the much easier $(O(N \ln(N))$ for the FFT, which is why many researchers still prefer the FFT just to save computational time and money. Yet, the FFT utterly fails for signal-to-noise ratios (SNRs) lower than one, while the KLT succeeds in retrieving very weak signals in SNRs of even (say) 10^{-3} or smaller.

Thus, for the purpose of research, the KLT is vastly superior to the FFT. Moreover, it also applies to wide bands rather than just to narrow bands to which the FFT is confined. Actual numerical codes to compute the KLT are also provided in this book, making it an ideal tool for research.

But the most profound novelty put forward in this book is the statistical Drake equation and its far-reaching consequences, which are described in the first 11 chapters of the book. The basic idea here is to replace all numerical input variables in the classical Drake equation of 1961 by positive random variables whose probability distribution may be unknown at the outset. Maccone shows that, if the number of such input random variables is made to increase at will (meaning that increasingly more physical factors are taken into account), then the probability distribution of N, the number of communicating civilizations in the Galaxy, approaches the lognormal distribution. This is really a simple consequence of the central limit theorem of statistics, but was not realized as such until 2008, when Maccone first presented it at the SETI I Session of the International Astronautical Congress held in Glasgow, Scotland. After that, Maccone reformulated the Dole equation too, yielding the number of habitable planets in the Galaxy in just the same way as the Drake equation yields the number of ET civilizations. The most important new result of all, however, is the discovery of the probability distribution yielding the distance between any two nearby ET civilizations in the Galaxy. This is no longer a lognormal distribution, but rather a new equation dubbed "Maccone distribution" by Paul Davies (who first understood and popularized it after its discovery by Maccone). This really is at the heart of SETI inasmuch as it predicts that the probability of finding ETs at distances less than about 500 light-years is virtually zero. Since our radio telescopes on Earth cannot detect ET signals arriving from distances greater than 100 or 200 light-years, it is immediately apparent that SETI searches have so far failed to find ETs simply because they do not reach far enough out into the Galaxy! Indeed, this is even more obvious if one considers that the size of the Galaxy is about 100,000 light-years and we are struggling to explore a sphere of 500 to 1,000 light-years around the Sun. Put flippantly, it is as if Christopher Columbus had stopped at the Canary Islands and then "concluded" that America could not possibly exist!

Finally, one last chapter describes Maccone's activity on the United Nations Committee on the Peaceful Uses of Outer Space (COPUOS) in order to protect the farside of the Moon from wild future exploitation by anyone disrespectful of its unique radio-quiet environment. This is of course more politics than science. But, it is high time something was done before private entrepreneurs achieve the capability of flying to the Moon at their own private expense, thus making them believe that they can dispose at will of the Moon just because they have money.

In conclusion, this is a highly innovative research book, opening up new prospects for the future of humanity in space and I can only highly recommend it.

Giovanni F. Bignami
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Milan, Italy, March 2, 2012

Figures

Tables

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Abbreviations and acronyms

ALMA	Atacama Large Millimeter/submillimeter Array
ATA	Allen Telescope Array
BAM	Bordered Autocorrelation Method
BER	Bit Error Rate
BPP	Breakthrough Propulsion Physics (program)
CLT	Central Limit Theorem
CFHT	Canada–France–Hawaii Telescope
CLOE	Common Lisp Oriented Environment for PCs by Symbolics
CMB	Cosmic Microwave Background
DSN	Deep Space Network
DOE	Department Of Energy (U.S.)
FFT	Fast Fourier Transform
FT	Fourier Transform
FTL	Faster Than Light
GPL	General Public License
HARPS	High Accuracy Radial Velocity Planet Searcher
HPBW	Half Power Beam Width
KL	Karhunen–Loève
KLT	Karhunen–Loève Transform
IAC	International Astronautical Congress
LAPACK	Linear Algebra PACKage
LOFAR	LOw Frequency ARray
LU	LU factorization (decomposition) of a matrix
MACSYMA	Project MAC's SYmbolic MAnipulator
MIT	Massachusetts Institute of Technology
MKS	Meter Kilogram Second system of units
NESC	National Energy Software Center

NCP	North Celestial Pole
OHP	Observatoire de Haute Provence
PCA	Principal Components Analysis
pdf	probability density function
PA	Position Angle
SETI	Search for ExtraTerrestrial Intelligence
SKA	Square Kilometer Array
SIM	Space Interferometry Mission
SVD	Singular Value Decomposition
TPF	Terrestrial Planet Finder
VLSI	Very Large Scale Integration