Adaptive Selective Sidelobe Canceller Beamformer
Radio Imaging With Strong Interfering Sources

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Adaptive Selective Sidelobe Canceller Beamformer

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6. Summary
Visibility and the dirty image

- Radio telescope measures the correlation/visibility between antenna pairs.

- The classic dirty image is given by

\[
\hat{i}(l, m) = \frac{1}{M} \sum_{q=1}^{M} V(u_q, v_q) e^{\frac{2\pi i}{\lambda}(u_q l + u_q m)}
\]

- \((u_q, v_q)\) are the baselines at time \(t_q\).
- \(M\) is the number of measurements.
- \(V(u_q, v_q)\) is the measured correlation (visibility) of the antenna pair.
- \(\lambda\) is the wavelength.
- \((l, m)\) are the direction cosines.
Define

- $\mathbf{R}_k (P \times P)$ the measured correlation matrix at epoch $k$

$$\mathbf{R}_k(i,j) \equiv V(u_k(i,j), v_k(i,j))$$

- $\mathbf{a}_k(l, m)$ the array steering vector at epoch $k$

$$\mathbf{a}_k(l, m) \equiv \begin{pmatrix}
  e^{-\frac{2\pi j}{\lambda}(u_{1,0}^k l + v_{1,0}^k m)} \\
  \vdots \\
  e^{-\frac{2\pi j}{\lambda}(u_{P,0}^k l + v_{P,0}^k m)} 
\end{pmatrix}$$

where

- $P$ - the number of antennas in the array.
- $(u_{i,0}^k, v_{i,0}^k)$ - the location of antenna $i$ at the $k$’Th epoch, relative to some convenient point $(u_0, v_0)$
Matrix Based Approach Cont.

- The classic dirty image

\[ \hat{I}(l, m) = \frac{1}{M} \sum_{q=1}^{M} V(u_q, v_q) e^{\frac{2\pi j}{\lambda}(u_q l + u_q m)} \]

is mathematically equivalent to the classic (Bartlett) beamformer

\[ \hat{I}(l, m) = \frac{1}{K} \sum_{k=1}^{K} w_k^H(l, m) R_k w_k(l, m) = \frac{1}{K} \sum_{k=1}^{K} \hat{I}_k(l, m) \]

\[ w_k(l, m) = \frac{1}{P} a_k(l, m) \]

- MVDR (Minimum Variance Distortionless Response) adaptive beamformer

\[ w_{MVDR} = \arg \min_w w^H R w \]

\[ w_{MVDR}^H a = 1 \]
The spatial resolution and accuracy of the dirty image is a limiting factor in the deconvolution process.

Producing a better dirty image will result in a more accurate reconstructed image.
ASSC Rational
What Happens When the Array is Rotating
What Happens When the Array is Rotating Cont.
Illustrative Example

- East-West array with 20 antennas $\lambda/2$ spaced.
- Observation was done every 6 minutes for a 12-hour period.
Illustrative Example Cont.
Illustrative Example Cont.

Number of pixels, a specific time epoch estimated the minimal power

- Image contains $181 \times 181$ pixels
- For the classic beamformer
  - Most epochs (more than 90%) suffered from minimal interference for at least 661 pixels.
  - Over 65% of epochs suffered from minimal interference for 90% of the pixels.

- For the MVDR beamformer
  - Most epochs (more than 90%) suffered from minimal interference for at least 507 pixels.
  - Over 45% of epochs suffered from minimal interference for 90% of the pixels.
The Main Idea behind the ASSC

For a specific direction \((l, m)\)

- The dirty image intensity measured by most time ticks, is significantly higher due to interfering sources.
- Only the few time ticks that benefit from minimal interference, yield an unbiased intensity estimation.

For each observation direction

Choosing the time epoch with the minimal power, we choose the correlation matrix with the smallest interfering power, that happens to best suppress the interference.
The ASSC Algorithm
The ASSC Algorithm

**ASSC Algorithm Flow**

For each pixel: ASSC

Measured Visibility $R_1$

Measured Visibility $R_2$

...  

Measured Visibility $R_K$

Dirty Image #1

Dirty Image #2

...  

Dirty Image #K

For each pixel: Adaptive selection

ASSC Dirty Image
Calculate the dirty image for each time tick separately

\[ \hat{i}_k(l, m) = w_k^H(l, m)R_k w_k(l, m). \]

- For the classic beamformer
  \[ w_k(l, m) = \frac{1}{p} a_k(l, m) \]

- For the MVDR beamformer
  \[ w_{MVDR}^H(l, m) = \frac{a_k^H(l, m)R_k^{-1}}{a_k^H(l, m)R_k^{-1}a_k(l, m)}. \]
Adaptive selection per pixel

ASSC parameters
- $\tilde{k}$ - the number of epochs to consider.
- $\mu_k$ - their weight.

Find the $\tilde{k}$ minimal values among $\hat{I}_1(l, m), \ldots, \hat{I}_K(l, m)$

$$\hat{I}_{(1)}(l, m), \ldots, \hat{I}_{(\tilde{k})}(l, m)$$

The ASSC dirty image is

$$\hat{I}^{ASSC}(l, m) = \sum_{k=1}^{\tilde{k}} \mu_k \hat{I}(k)(l, m)$$
ASSC Parameters

\[ \tilde{k} \] - number of epochs to consider

- Typically, \( \tilde{k} < 5\% \).
- The stronger the interference, the smaller the \( \tilde{k} \).

\[ \mu_k \] - epochs weight

- \( \mu_{k+1} \leq \mu_k \)
- \( \sum_{k=1}^{\hat{k}} \mu_k = 1 \)
Simulated Examples
Array with 20 antennas $\lambda/2$ spaced.
- Array rotation angle $0^\circ - 90^\circ$
- Measurement was done every $10^\circ$
- $\tilde{k} = 1$, $\mu = 1$. 
Simulated Examples

In FOV Interference

Original image

- East-West array with 20 antennas $\lambda/2$ spaced.
- Measurement was done every 6 minutes for a 12-hour period.
- $\tilde{k} = 3$, $\mu_k = \frac{1}{3}$. 

Classic dirty image

ASSC classic dirty image

MVDR dirty image

ASSC MVDR dirty image

Ronny Levanda (BIU)
In FOV Interference - Zoom In View

Original image

Classic dirty image

ASSC classic dirty image

MVDR dirty image

ASSC MVDR dirty image
Out-of-FOV Interference

Interference stronger by 10,000 than sources in the FOV

- East-West array with 20 antennas $\lambda/2$ spaced.
- Measurement was done every 6 minutes for a 12-hour period.
- Interference is stronger by 10,000 than sources in FOV.
- $\tilde{k} = 3$, $\mu_k = \frac{1}{3}$. 
Out-of-FOV Interference

Interference stronger by $10^6$ than sources in the FOV

- East-West array with 20 antennas logarithmically spaced $0 - 200\lambda$.
- Measurement was done every minute for a 12-hour period.
- Interference is stronger by $10^6$ than sources in FOV.
- $\tilde{k} = 5$, $\mu_k = \frac{1}{k}$. 

MVDR dirty image

ASSC MVDR dirty image
Preliminary Results of Radio Astronomical Measurements
Abell 2256 Cleaned Images

- Observed by the VLA*
- Single frequency bin around 1369 MHz

* T. Clarke and T. Ensslin, "Deep 1.4 GHz very large array observations of the radio halo and relic in Abell 2256."

Data was calibrated and provided by Prof. Huub Rottgering and Huib Intema
Cassiopeia A Cleaned Images

- Observed by the Allen Telescope Array*
- Single frequency bin around 4.2 GHz.

* Data was calibrated and provided by Gerald Harp
Summary

- We propose the *Adaptive Selective Sidelobe Canceller* beamformer - a novel method to produce a dirty image to be further processed by CLEAN or MEM.

- For interference dominant cases, the ASSC algorithm obtains images with higher spatial resolution and interference cancellation than either the classic and MVDR beamformers.

- The ASSC performance were demonstrated over simulated and radio astronomical data.
Thank you