Polarimetric Beamforming Methods for PAF Systems. Numerical Results for the APERTIF Prototype

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APERTIF: Phased Array Feed System for the Westerbork Syntheses Radio Telescope

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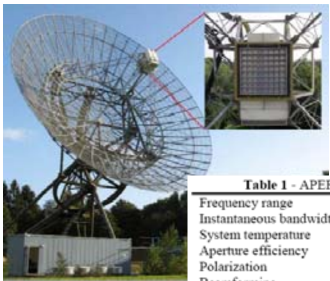


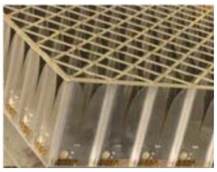


Table 1 - APERTIF Front-end requirements.

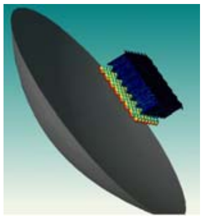
Frequency range	1000 – 1750 MHz
Instantaneous bandwidth	300 MHz
System temperature	55 K
Aperture efficiency	75%
Polarization	Dual linear
Beamforming	All digital
Simultaneous beams	37
Field-of-view	8 deg^2
Reflector diameter	25 m, f/D=0.35

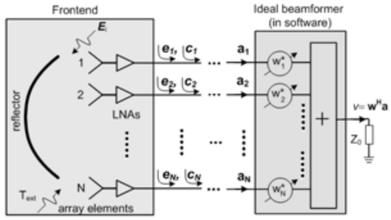
PAF is a dual-polarized array of144 Vivaldi elements (8x9x2). Numerical results are shown here for 1.42 GHz.

W. van Cappellen, L. Bakker, "APERTIF: Phased Array Feeds for the Westerbork Synthesis Radio Telescope", accepted for the 2010 IEEE International Symposium on Phased Array Systems and Technology.

Numerical methods and tools for PAF analysis and optimization:

2





Combined EM and MW numerical solver CAESAR* for phased array systems for Radio Astronomy

A newly developed num. toolbox** for FPA systems *extends the capabilities of CAESAR and GRASP9 to perform a system analysis and beamformer optimization.*

- *R. Maaskant, "Analysis of large antenna systems," Ph.D. dissertation, Eindhoven Univ. of Technology, The Netherlands, June 2010.
- **M.V. Ivashina, O. Iupikov, W. van Cappellen, "A New Numerical Toolbox of the CAESAR Software for Analysis and Optimization of Reflector Antennas Phased Array Feed Systems', ICEAA2010, Australia, Sept., 2010.
- ***M. V. Ivashina. O.Iupikov, R.Maaskant, W. van Cappellen, T. Oosterloo, 'An Optimal Beamforming Strategy for Wide-Field Surveys With Phased-Array-Fed Reflector Antennas', accepted by IEEE Trans. on AP.

Ongoing developments of beamforming methods

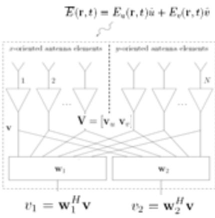
3

Optimal method: If the voltage response vectors $[\mathbf{v}_u, \mathbf{v}_v] = \mathbf{V}$ to two perfectly orthogonally polarized signals $\{\mathbf{E}_u, \mathbf{E}_v\}$ are known, the optimal weights $\mathbf{W}_{opt} = [\mathbf{w}_1, \mathbf{w}_2]$ for the beamformer that ensure minimization of the noise in the measurement and perfect reconstruction of the polarization properties of the source (MaxSNR Jones matrix diagonalization) can be found as [2]:

$$\mathbf{W}_{\mathrm{opt}} = \mathbf{R}_n^{-1}\mathbf{V}(\mathbf{V}^H\mathbf{R}_n^{-1}\mathbf{V})^{-1} = \mathbf{W}_{\mathrm{MaxSNR}}\mathbf{J}_{\mathrm{MaxSNR}}^{-1}$$

where $\mathbf{R}_n$ is the noise covariance matrix, and the two beamformer output voltages are given by

$$\begin{bmatrix} v_{1u} \\ v_{2u} \end{bmatrix} = \begin{bmatrix} \mathbf{w}_1^H\mathbf{v}_u \\ \mathbf{w}_2^H\mathbf{v}_u \end{bmatrix} \quad \begin{bmatrix} v_{1v} \\ v_{2v} \end{bmatrix} = \begin{bmatrix} \mathbf{w}_1^H\mathbf{v}_v \\ \mathbf{w}_2^H\mathbf{v}_v \end{bmatrix}$$



Eigenvector method: In practice **V** is not known, so the optimal method cannot be used, and a practical polarimetric array calibration procedure must be found. For a polarimetric array, neglecting estimation error, the signal covariance matrix $\mathbf{R}_s$ has rank 2 for a randomly polarized source. Bruce Veidt has proposed to use the two principal eigenvectors, $\{\mathbf{v}_{1,eig}, \mathbf{v}_{2,eig}\}$, to form the maximum SNR eigenvector beam former weight vectors [1], where $\mathbf{V}_{eig} = [\mathbf{v}_{1,eig}, \mathbf{v}_{2,eig}]$:

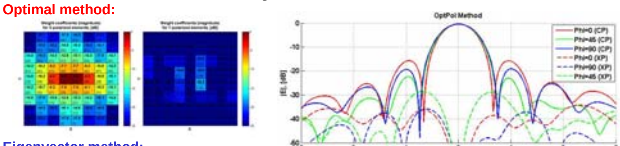
$$\mathbf{W}_{\mathrm{eig}} = \mathbf{R}_n^{-1}\mathbf{V}_{\mathrm{eig}}$$

[1] B. Veidt, G. Honey, T.Buergess, R. Smegal, R. Messing, A.G. Willis, A. Gray, P. Dewtney, 'Demonstration of Polarimetry with a Phaded Array Feed', submitted to the Special Issue on Antennas for Next Generation Radio Telescopes of the IEEE Trans. on AP.
[2] K. F. Warnick, M. V. Ivashina, S.J. Wijnholds, R. Maaskant, and B. D. Jeffs 'Polarimetric Performance of Radio Astronomical Phased Arrays', for submission to IEEE trans. on AP., 2010 AND Stefan J. Wijnholds, M. V. Ivashina, R. Maaskant, K.I F. Warnick and B.D. Jeffs, 'Polarimetric Calibration of Phased Array Feed Systems', for submission to IEEE trans. on AP., 2010.
[3] S. Wijnholds, M. V. Ivashina, Maaskant, K. F. Warnick, and B. D. Jeffs, 'Polarimetric calibration of PAF Systems', CALIM 2010, Dwingeloo, 24th Augustus, 2010.

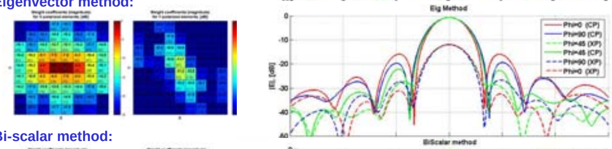
Beamformer weights and On-Axis Beams

4

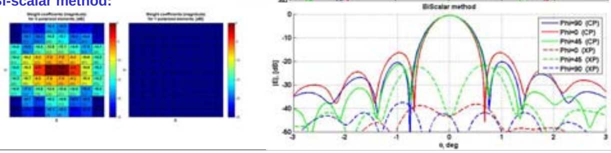
Optimal method:



Eigenvector method:



Bi-scalar method:



Results: Jones Matrix

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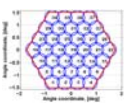
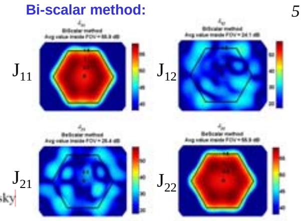
$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}\begin{bmatrix} E_u \\ E_v \end{bmatrix} \qquad \mathbf{J} = \begin{bmatrix} \mathbf{w}_1^H\mathbf{v}_u & \mathbf{w}_1^H\mathbf{v}_v \\ \mathbf{w}_2^H\mathbf{v}_u & \mathbf{w}_2^H\mathbf{v}_v \end{bmatrix}$$

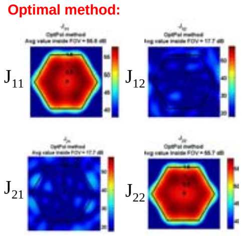


Numbering and arrangement of the main beams on the sky

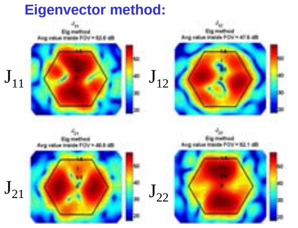
Bi-scalar method:



Optimal method:



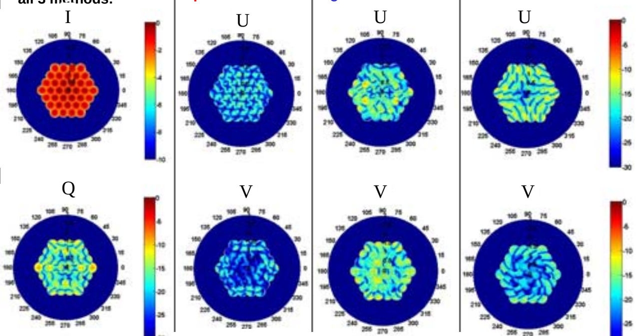
Eigenvector method:



Results: Stokes parameters

$$\mathbf{s} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} \sigma_{uu} + \sigma_{vv} \\ \sigma_{uu} - \sigma_{vv} \\ 2\,\mathrm{Re}(\sigma_{uv}) \\ 2\,\mathrm{Im}(\sigma_{uv}) \end{bmatrix} \qquad \begin{aligned} I &= \mathbf{w}_1^H\mathbf{R}_s\mathbf{w}_1 + \mathbf{w}_2^H\mathbf{R}_s\mathbf{w}_2, \\ Q &= \mathbf{w}_1^H\mathbf{R}_s\mathbf{w}_1 - \mathbf{w}_2^H\mathbf{R}_s\mathbf{w}_2, \\ U &= \mathbf{w}_1^H\mathbf{R}_s\mathbf{w}_2 + \mathbf{w}_2^H\mathbf{R}_s\mathbf{w}_1, \\ jV &= \mathbf{w}_1^H\mathbf{R}_s\mathbf{w}_2 - \mathbf{w}_2^H\mathbf{R}_s\mathbf{w}_1 \end{aligned}$$

Similar for all 3 methods: | **Optimal method:** | **Eigenvector method:** | **Bi-scalar method:**



The choice of the beamforming method has a significant impact on the reconstruction of the Stokes parameters. For a randomly polarized source, the residue in U and V (and Q) is lowest for the optimal beamforming method.

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