Radio Polarimetry

Michiel Brentjens

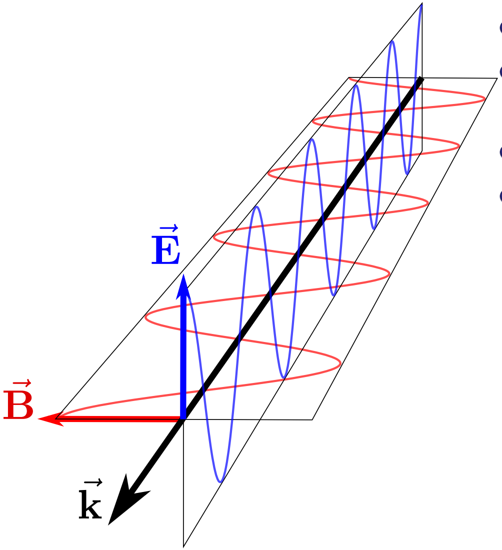
Radio Observatory
ASTRON, Dwingeloo, The Netherlands

September 9, 2013

Reading list

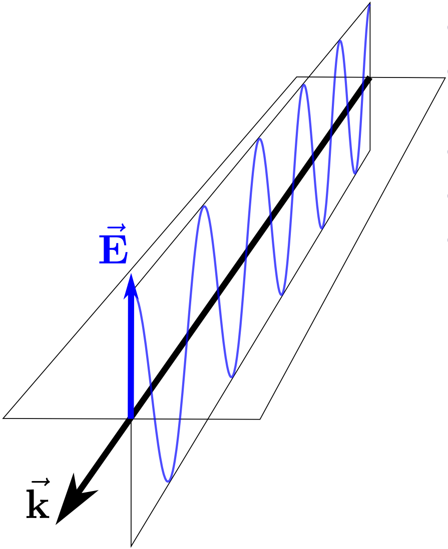
- Born & Wolf *Principles of optics*
- Thompson, Moran & Swenson *Interferometry and Synthesis in Radio Astronomy*
- Taylor, Carilli & Perley *Synthesis Imaging in Radio Astronomy II*
- Bracewell *The Fourier Transform & Its Applications*
- Hamaker, Bregman & Sault *Understanding radio polarimetry: paper I*(1996)
- Sault, Hamaker& Bregman *paper II*(1996)
- Hamaker & Bregman *paper III* (1996)
- Hamaker *paper IV* (2000)
- Hamaker *paper V* (2006)
- Brentjens & de Bruyn *Faraday rotation measure synthesis* (2005)

Outline



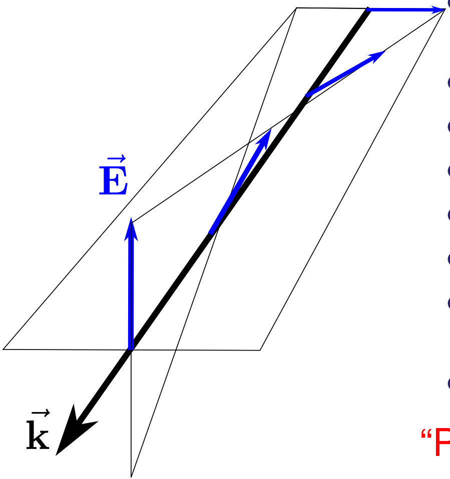
- Vector phenomenon
- From Maxwell's equations: $\hat{\mathbf{k}} = \hat{\mathbf{E}} \times \hat{\mathbf{B}}$
- We know **k** (yesterday)
- Measure either **E** or **B**

Electromagnetic (EM) wave



- Vector phenomenon
- From Maxwell's equations: $\hat{\mathbf{k}} = \hat{\mathbf{E}} \times \hat{\mathbf{B}}$
- We know **k** (yesterday)
- Measure either **E** or **B**
- **E** is easier (yesterday)

Electro-magnetic wave



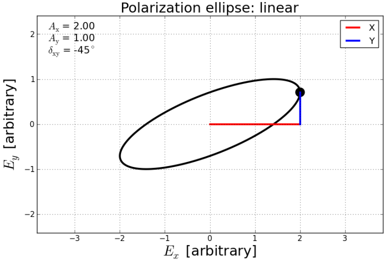
- Vector phenomenon
- From Maxwell's equations: $\hat{\mathbf{k}} = \hat{\mathbf{E}} \times \hat{\mathbf{B}}$
- We know **k** (yesterday)
- Measure either **E** or **B**
- **E** is easier (yesterday)
- But:
- E_x and E_y not equal
- **E** may rotate as function of **x** and *t*.
- **E** traces ellipse

"Polarization"

Outline

ASTRON

Geometry



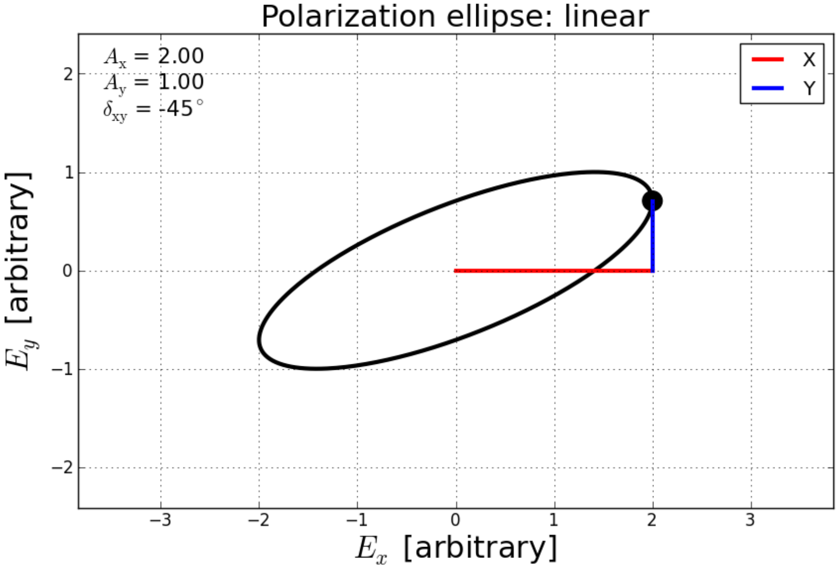
Viewing from antenna towards source, watching orientation and length of **E** vector on a plane at a fixed location in space.

$$\mathbf{E} = E_\mathrm{x}\hat{\mathbf{e}}_\mathrm{x} + E_\mathrm{y}\hat{\mathbf{e}}_\mathrm{y}$$
$$E_\mathrm{x} = A_\mathrm{x}\cos(2\pi\nu t + \delta_\mathrm{x})$$
$$E_\mathrm{y} = A_\mathrm{y}\cos(2\pi\nu t + \delta_\mathrm{y})$$

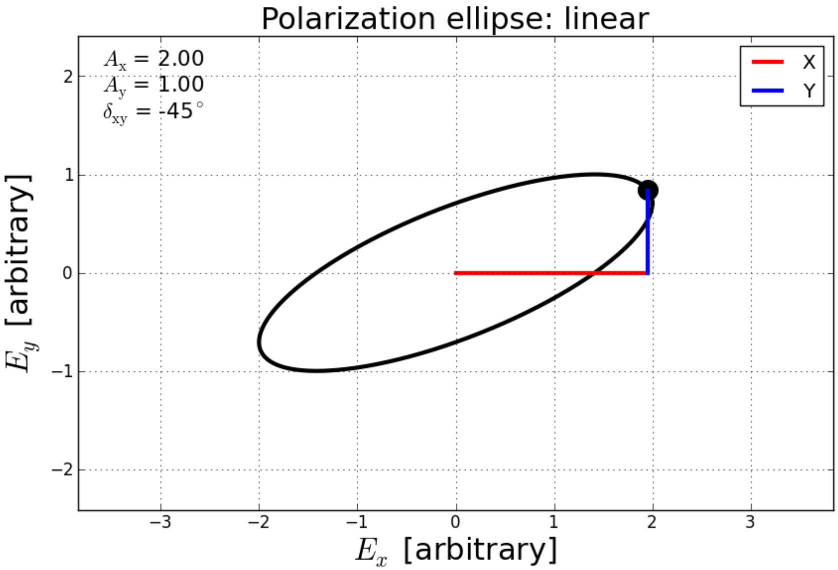
- A_x = x-amplitude
- A_y = y-amplitude
- $\delta_\mathrm{xy} = \delta_\mathrm{y} - \delta_\mathrm{x}$
- δ_xy = measure of ellipticity

- $\delta_\mathrm{xy} > 0$: CW rotation $\Rightarrow$ LEP
- $\delta_\mathrm{xy} = 0$: linear polarization
- $\delta_\mathrm{xy} < 0$: CCW rotation $\Rightarrow$ REP

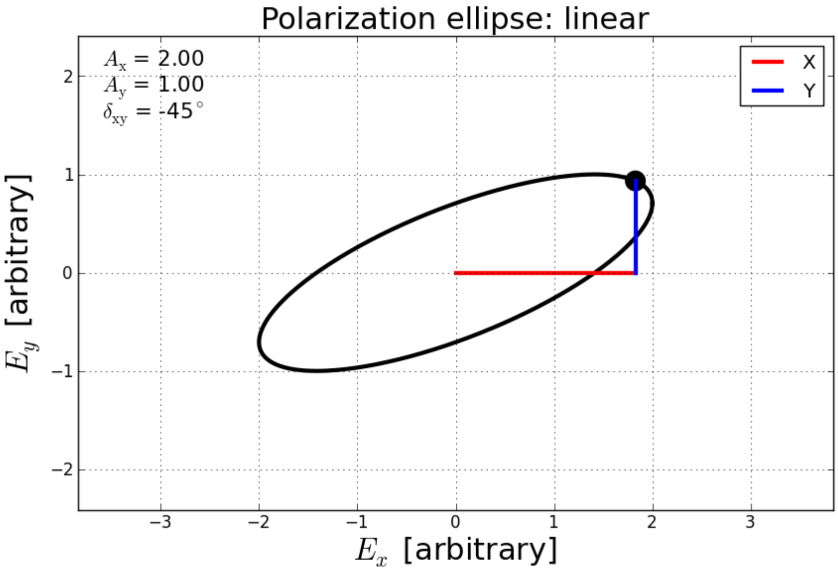
Polarization ellipse: *xy* 000



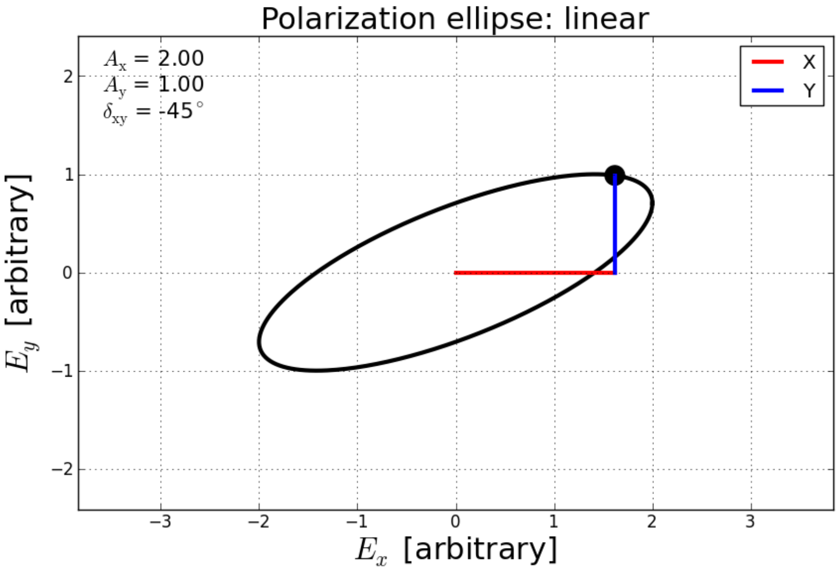
Polarization ellipse: *xy* 001

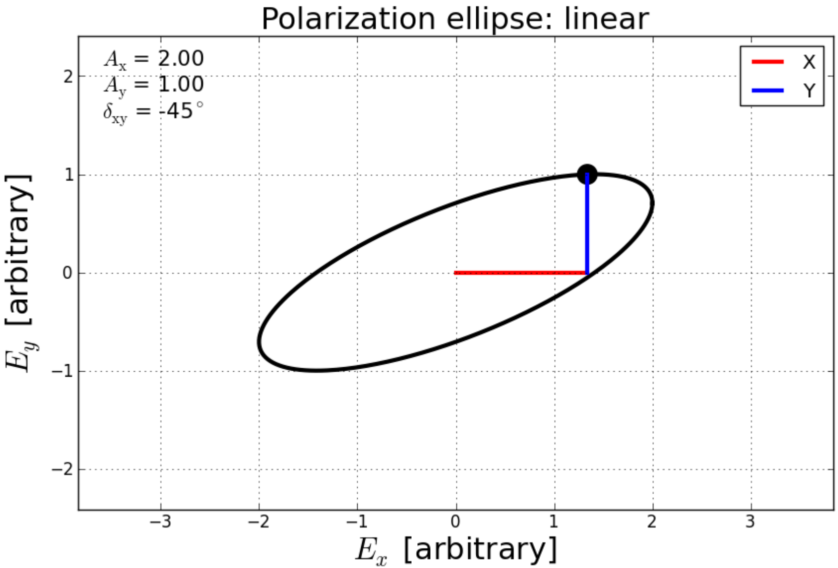


Polarization ellipse: *xy* 002



Polarization ellipse: *xy* 003




Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
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−3
−2
−1
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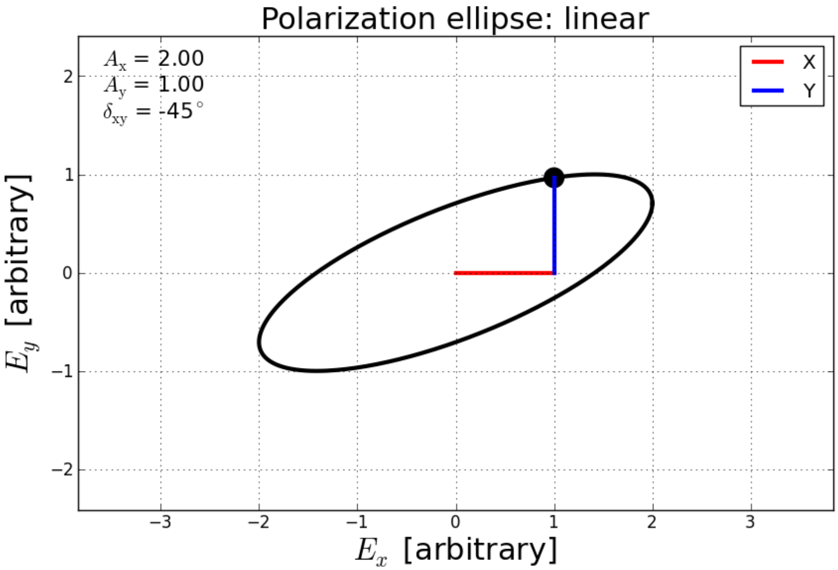
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Y
E_y [arbitrary]
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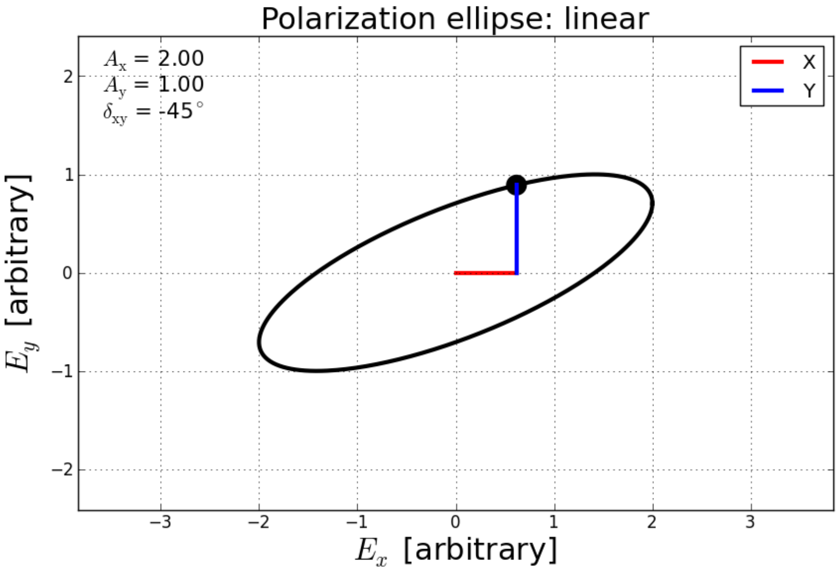
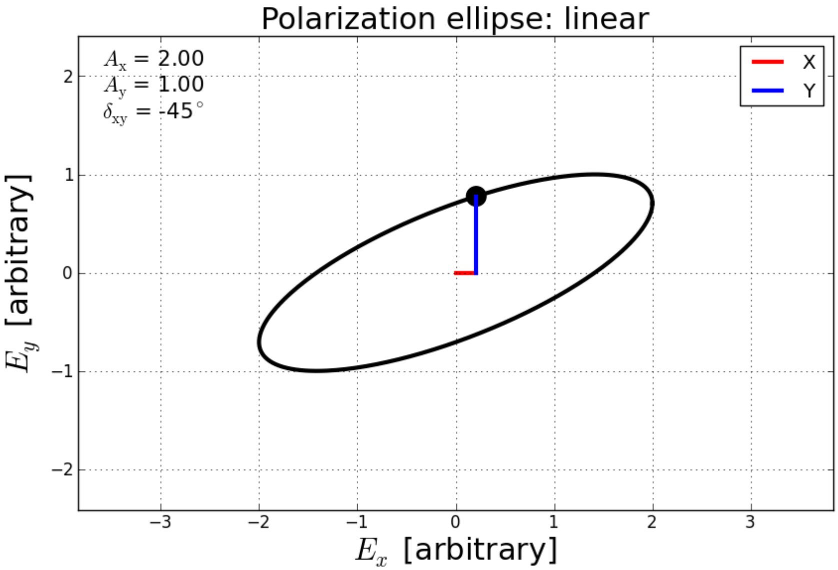
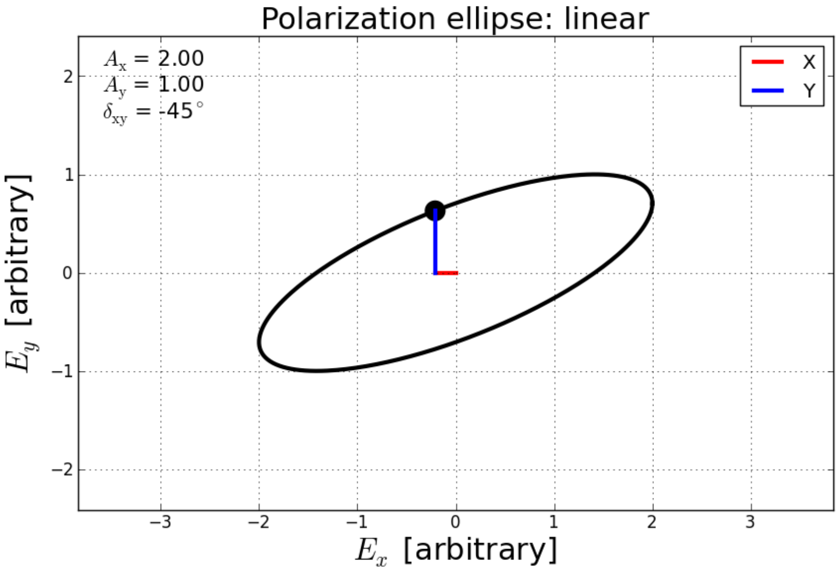
Polarization ellipse: *xy* 006



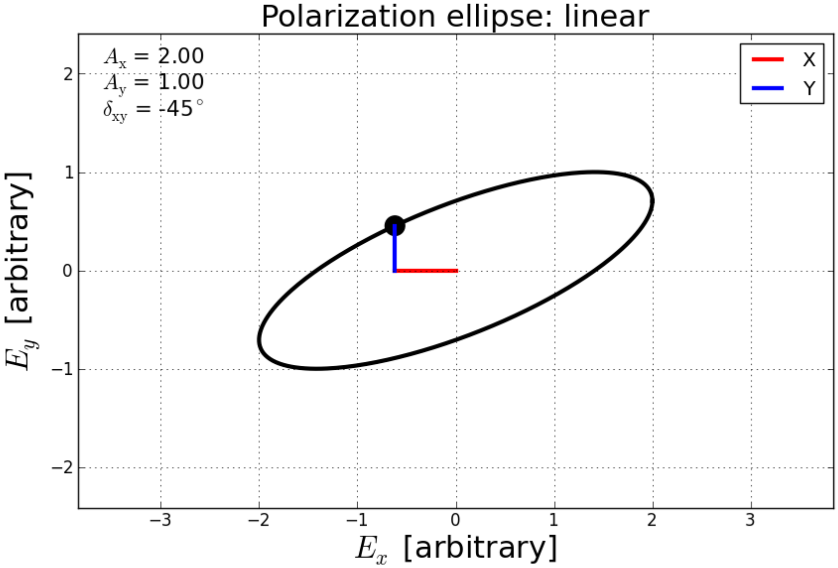
Polarization ellipse: *xy* 007

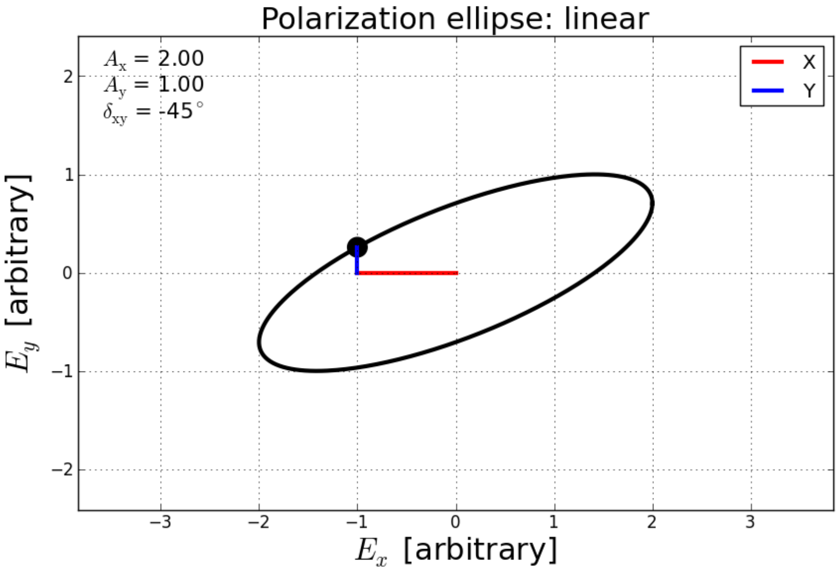


Polarization ellipse: *xy* 008



Polarization ellipse: *xy* 009




Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
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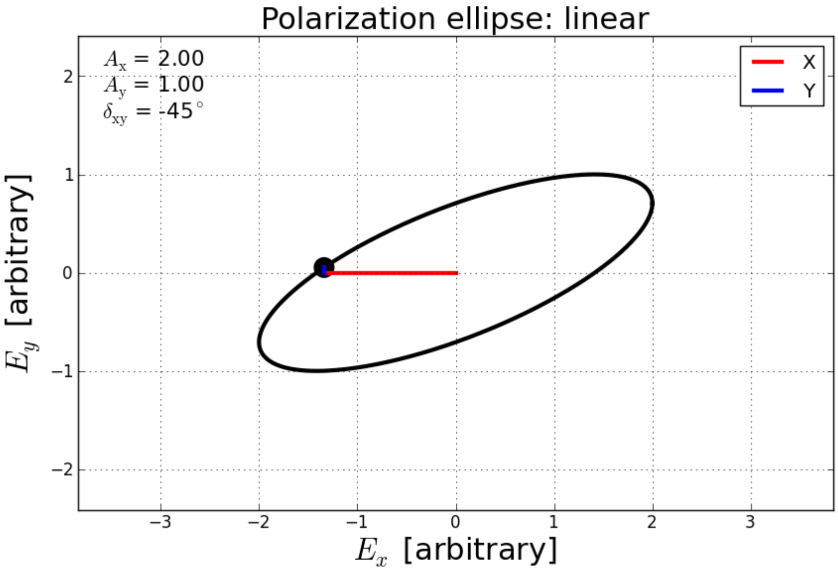
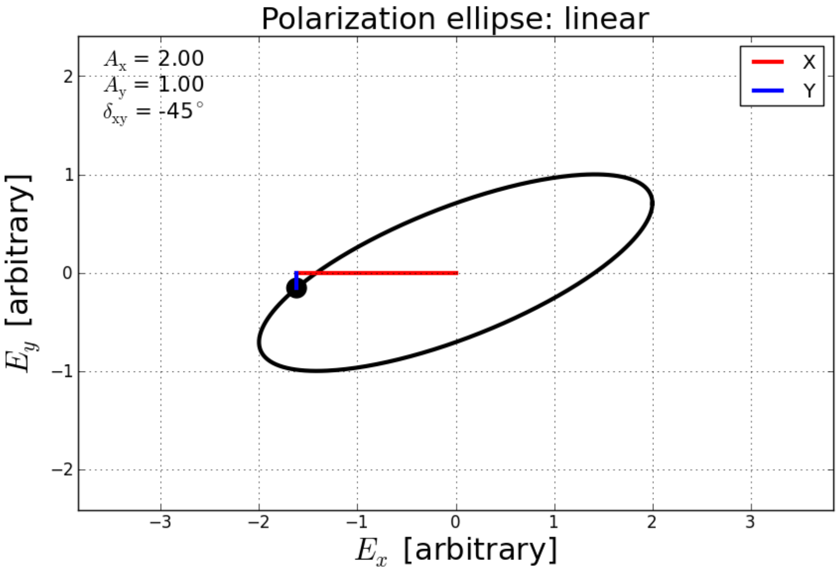
E_y [arbitrary]
E_x [arbitrary]

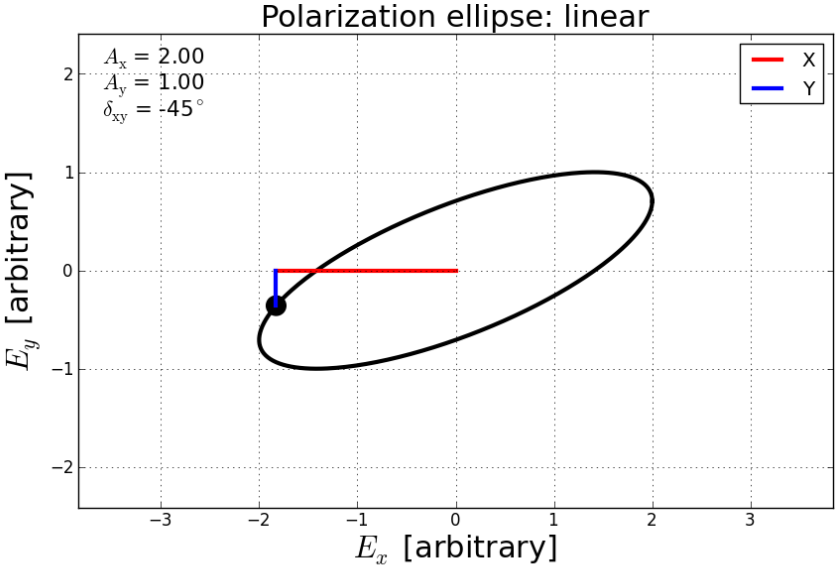
Polarization ellipse: *xy* 011



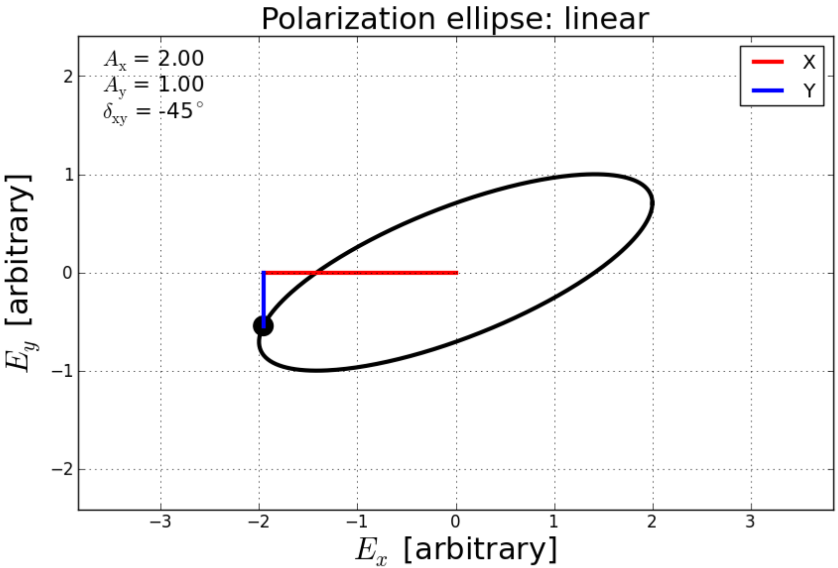
Polarization ellipse: *xy* 012



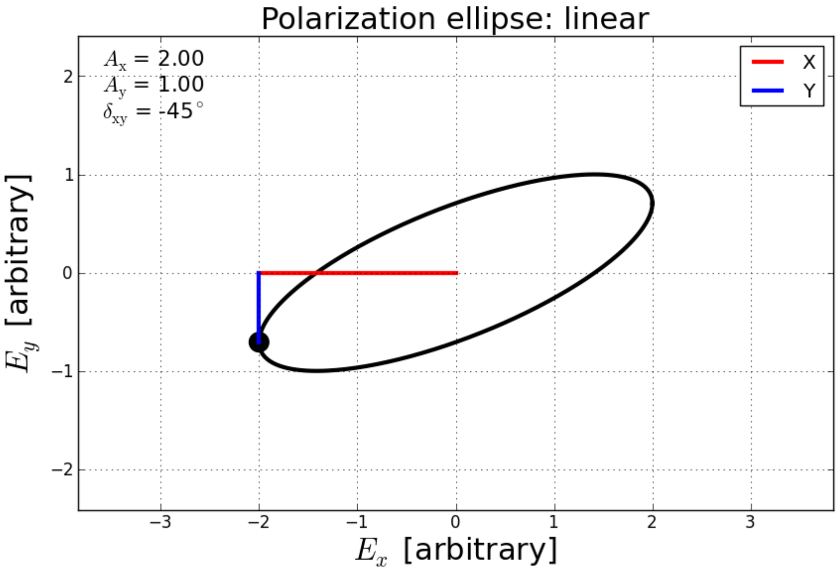
Polarization ellipse: *xy* 013



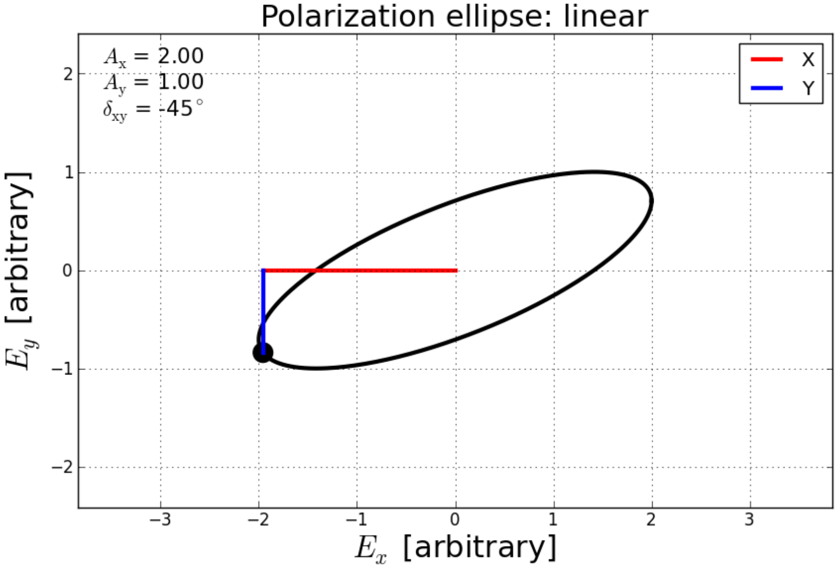
Polarization ellipse: *xy* 014



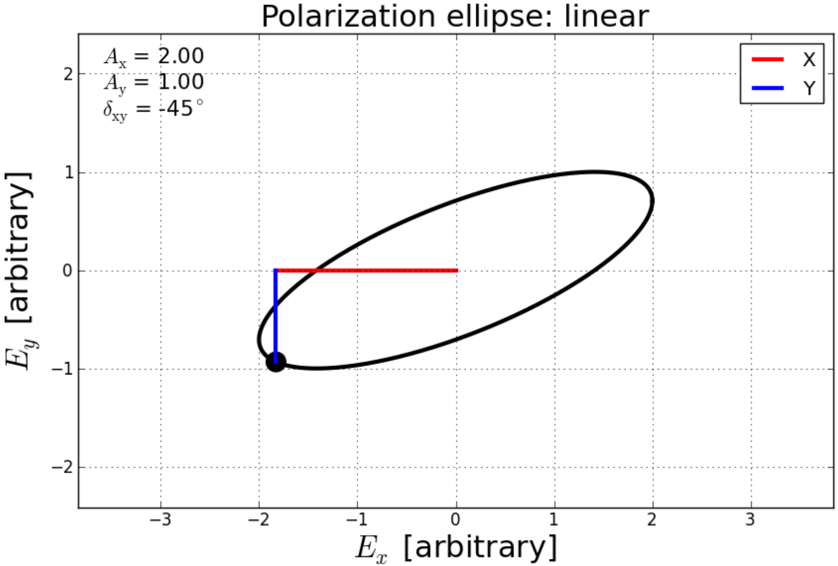
Polarization ellipse: *xy* 015

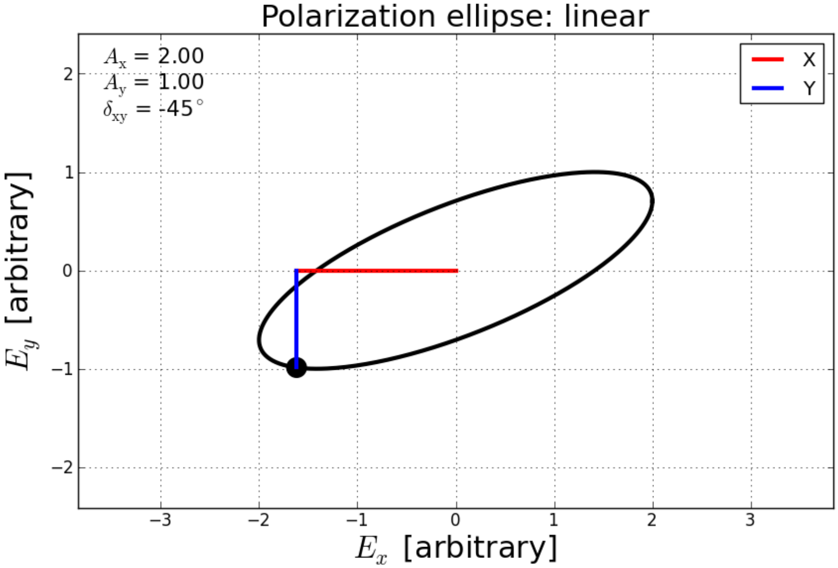


Polarization ellipse: *xy* 016



Polarization ellipse: *xy* 017




Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
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−1
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2
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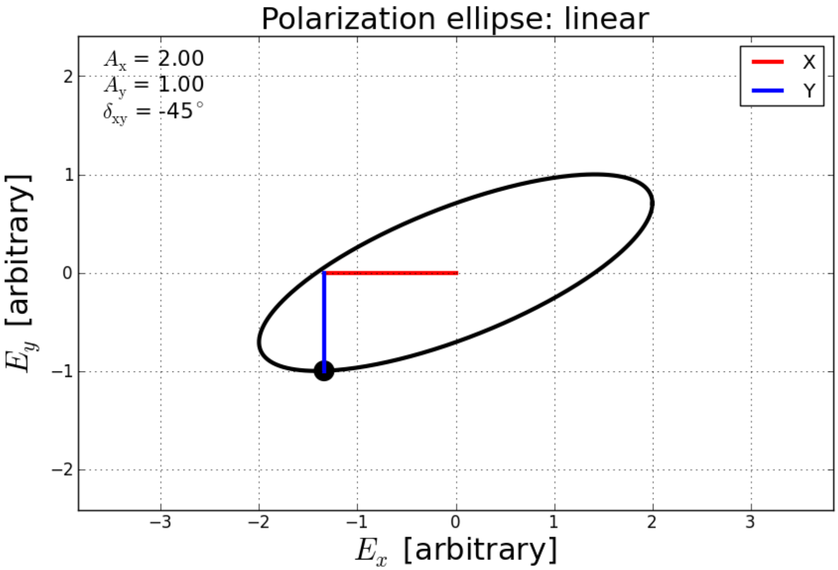
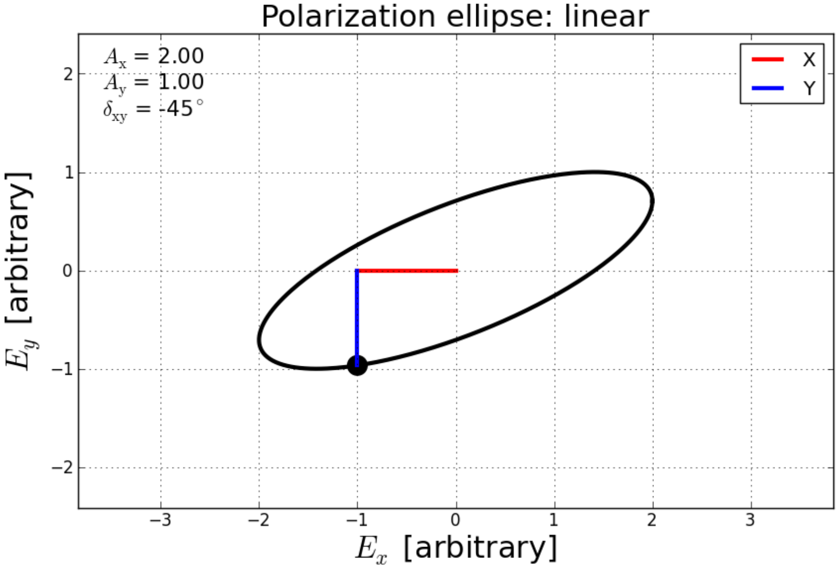
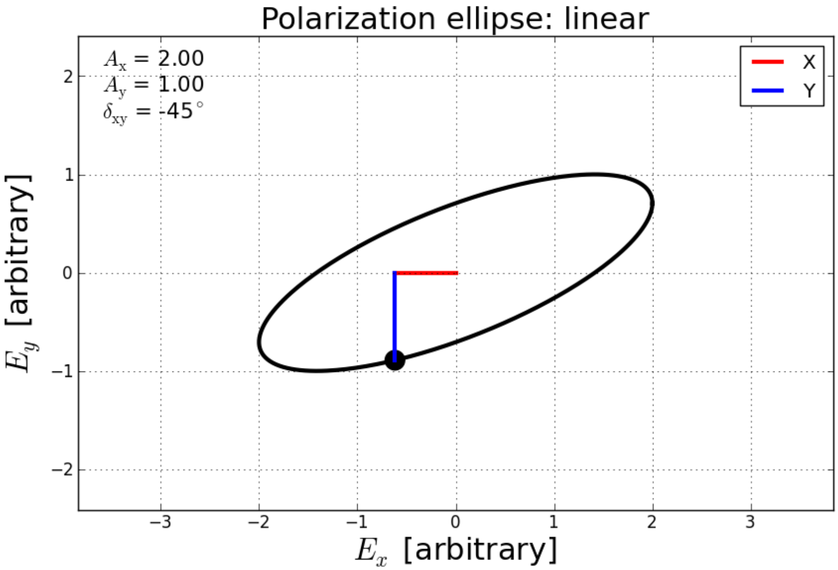
Polarization ellipse: *xy* 019

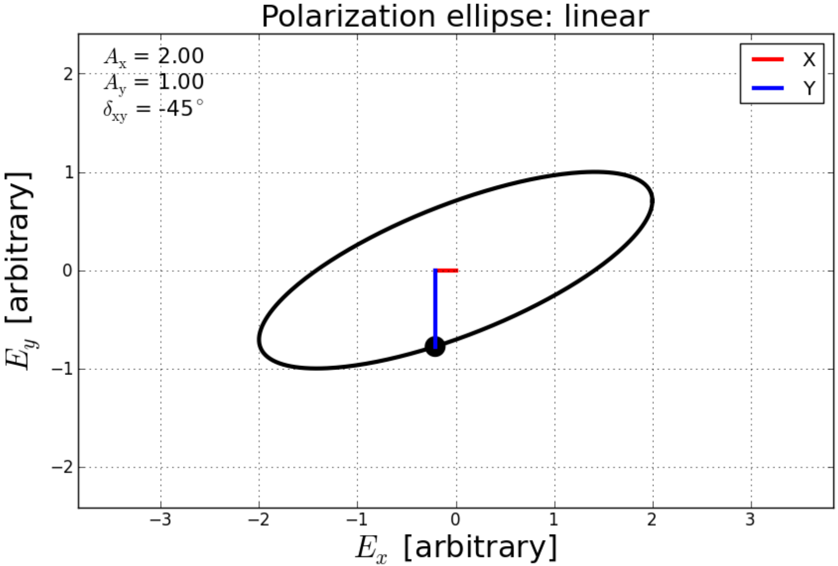


Polarization ellipse: *xy* 020

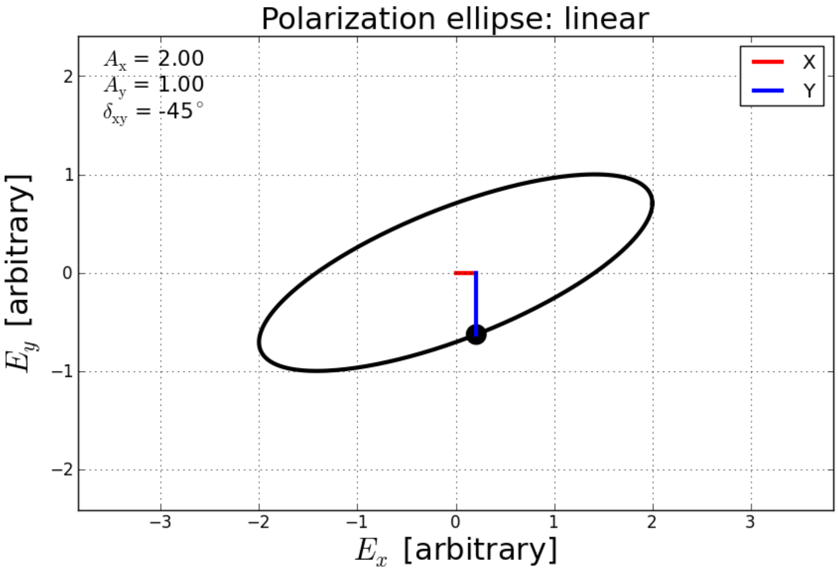


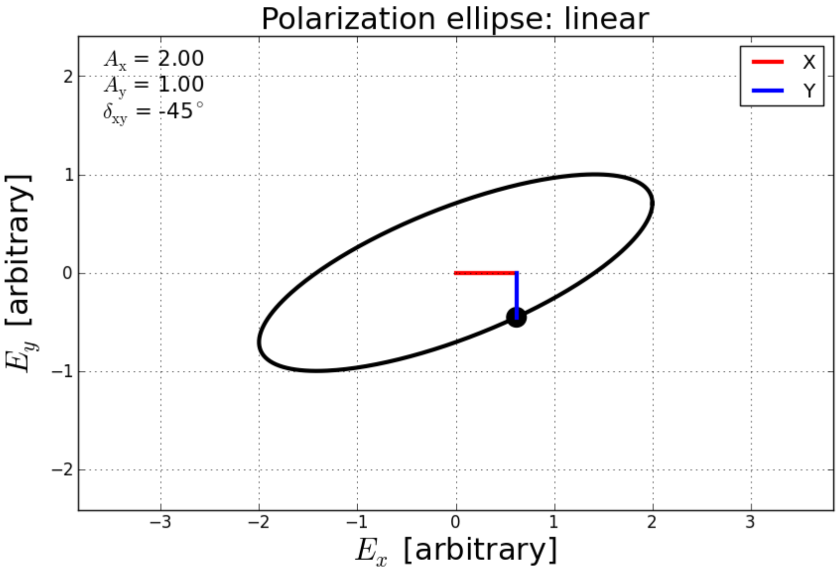
Polarization ellipse: *xy* 021




Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

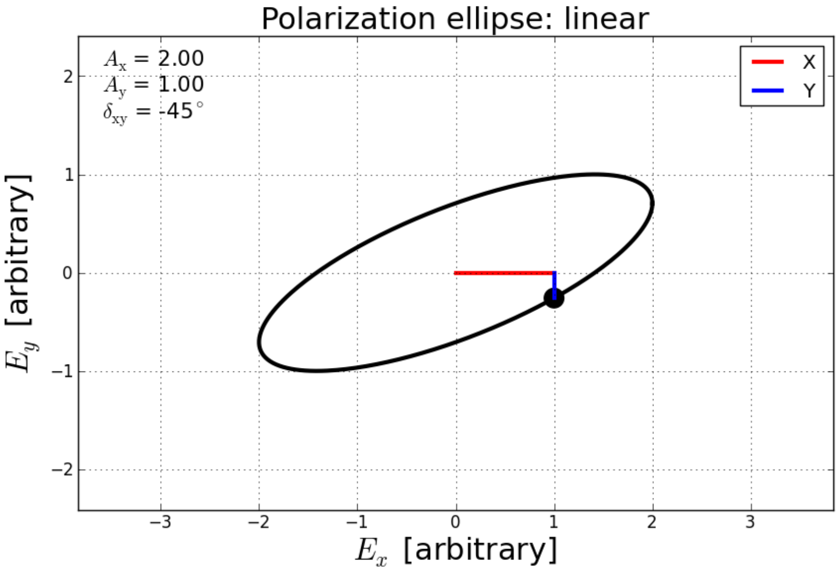
Polarization ellipse: *xy* 023

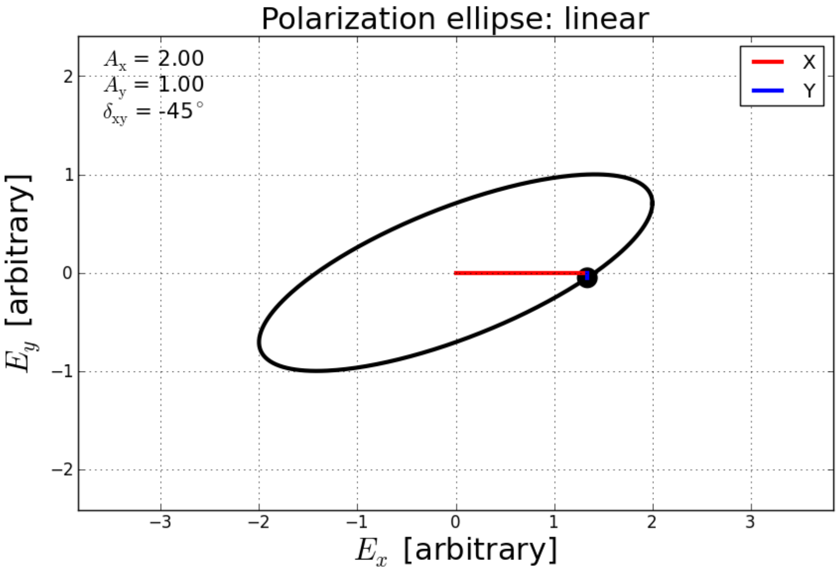




Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
E_y [arbitrary]
E_x [arbitrary]
−3
−2
−1
0
1
2
3

Polarization ellipse: *xy* 025




Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
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E_y [arbitrary]
E_x [arbitrary]
2
1
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−1
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−3
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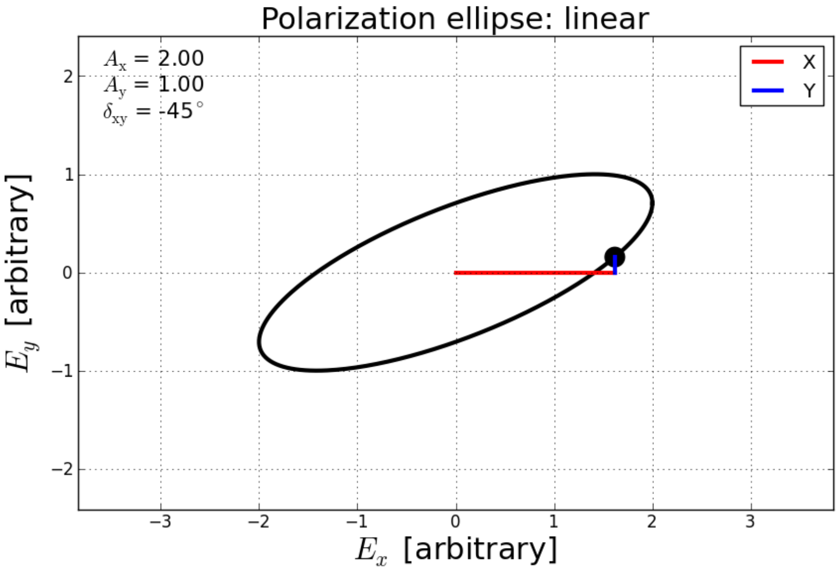
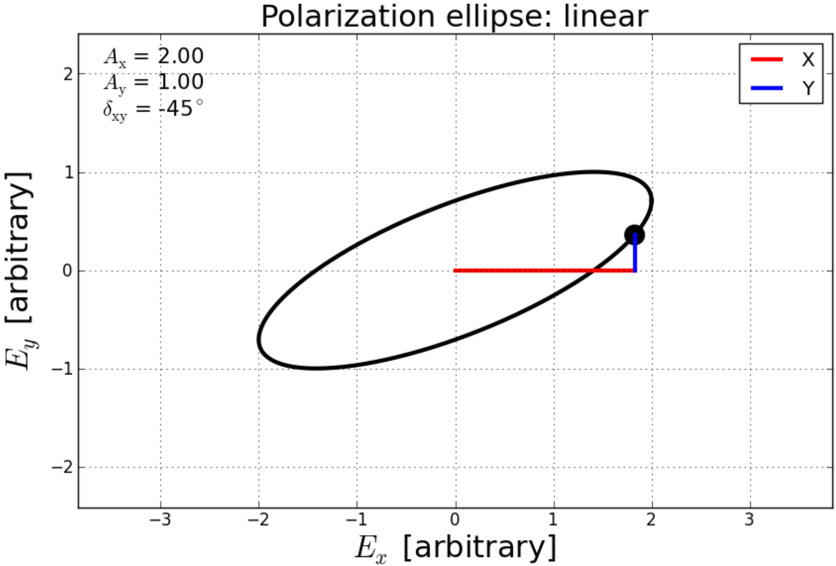
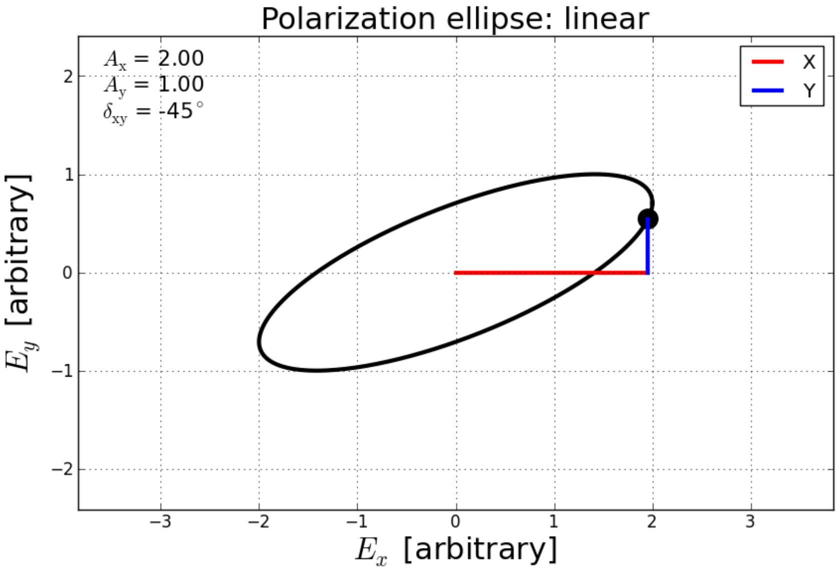
Polarization ellipse: *xy* 027

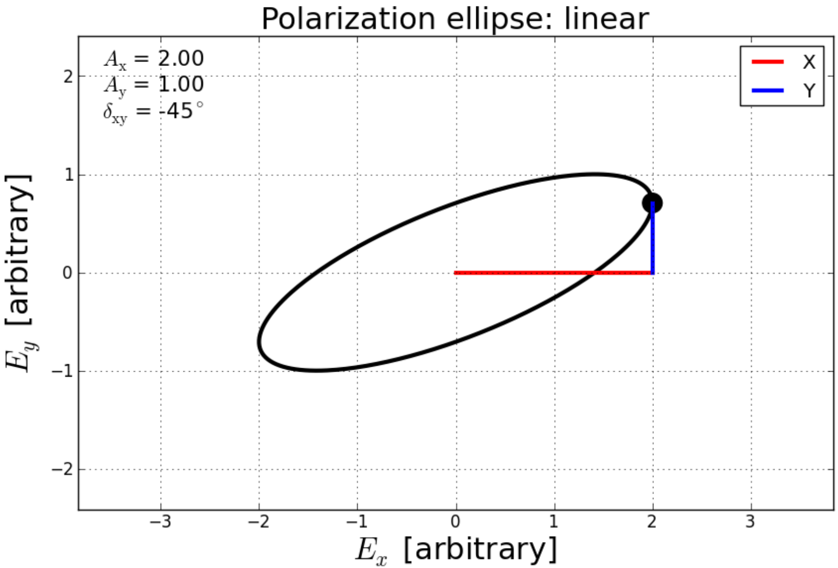


Polarization ellipse: *xy* 028



Polarization ellipse: *xy* 029





Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
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−2
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−2
−1
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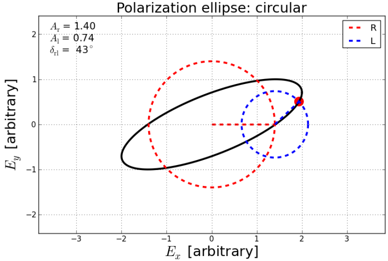
E_y [arbitrary]
E_x [arbitrary]

Geometry



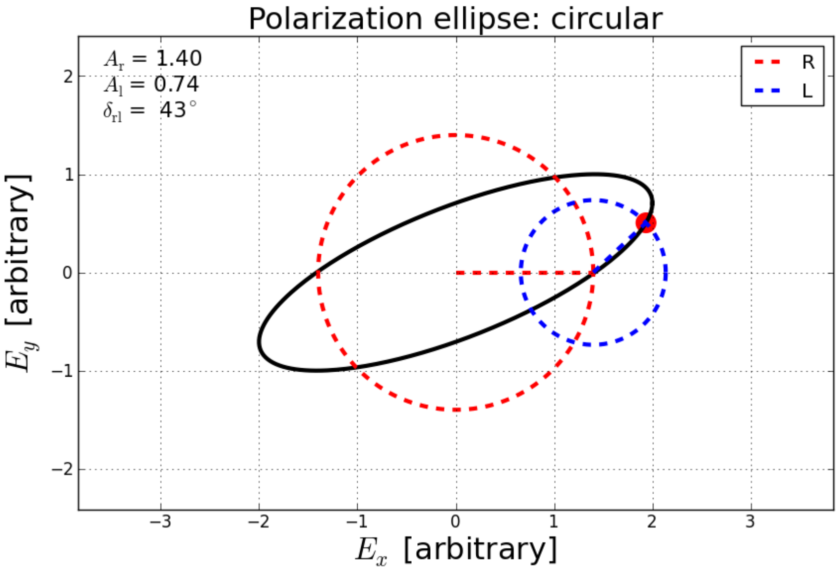
Viewing from antenna towards source, watching orientation and length of **E** vector on a plane at a fixed location in space.

$$\mathbf{E} = A_r \hat{\mathbf{e}}_r + A_l \hat{\mathbf{e}}_l$$

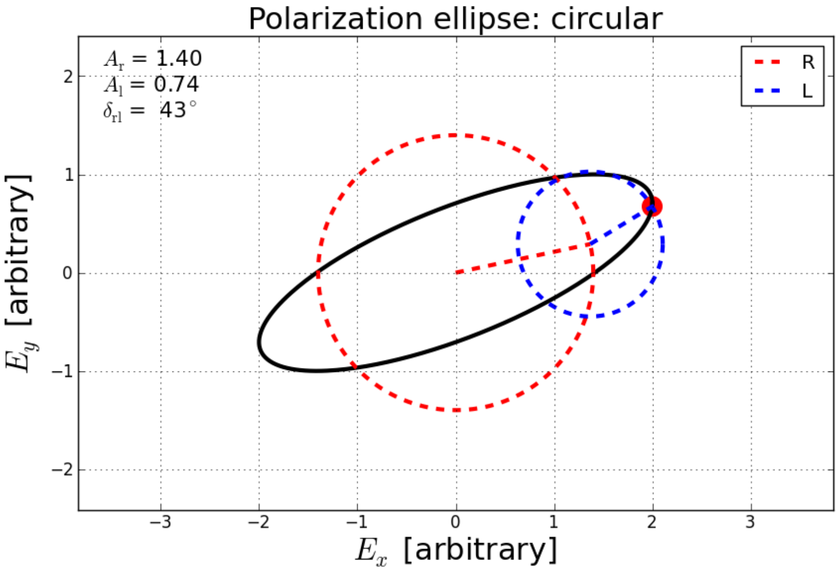
$$\hat{\mathbf{e}}_r = \begin{pmatrix} \cos(2\pi\nu t + \delta_r) \\ \sin(2\pi\nu t + \delta_r) \end{pmatrix}$$

$$\hat{\mathbf{e}}_l = \begin{pmatrix} \cos(2\pi\nu t + \delta_l) \\ -\sin(2\pi\nu t + \delta_l) \end{pmatrix}$$

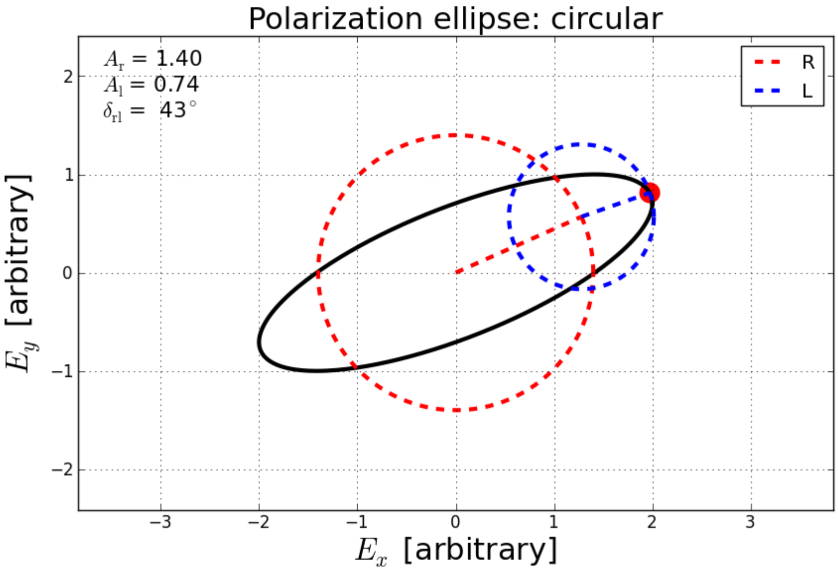
- $A_r + A_l$ = semi-major axis
- $\|A_r - A_l\|$ = semi-minor axis
- $\delta_{rl} = \delta_r - \delta_l$
- δ_{rl} = orientation of major axis
- $\delta_{rl} > 0$: MA rotated CCW
- $\delta_{rl} = 0$: MA along x-axis
- $\delta_{rl} < 0$: MA rotated CW


Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
E_y [arbitrary]
E_x [arbitrary]
−3
−2
−1
0
1
2
3
2
1
0
−1
−2

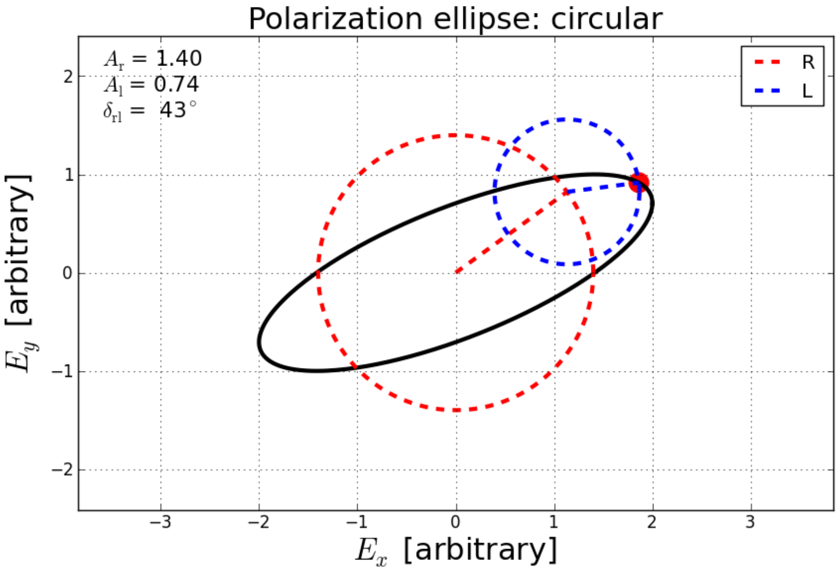
Polarization ellipse: circular 001



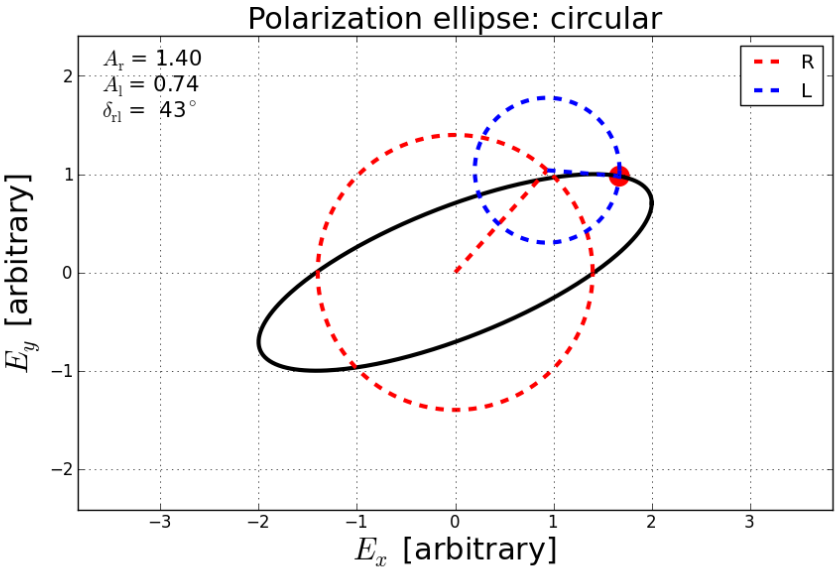
Polarization ellipse: circular 002



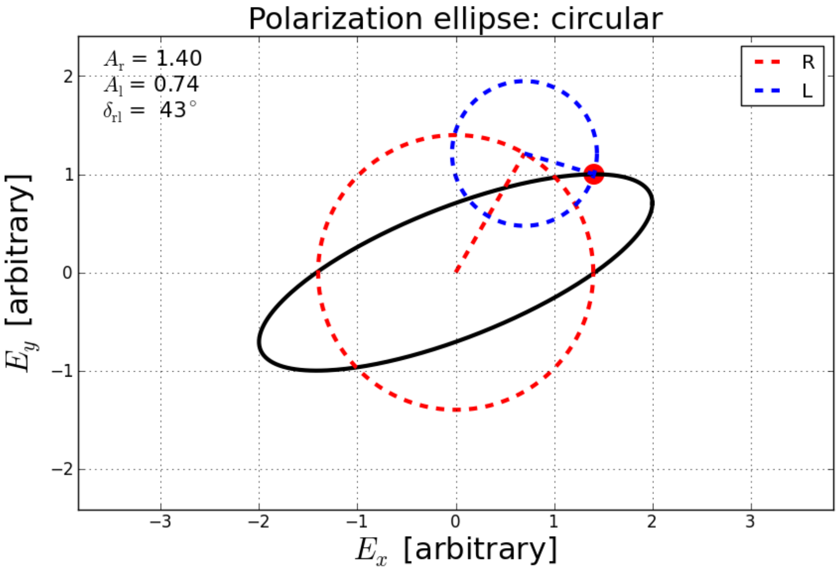
Polarization ellipse: circular 003



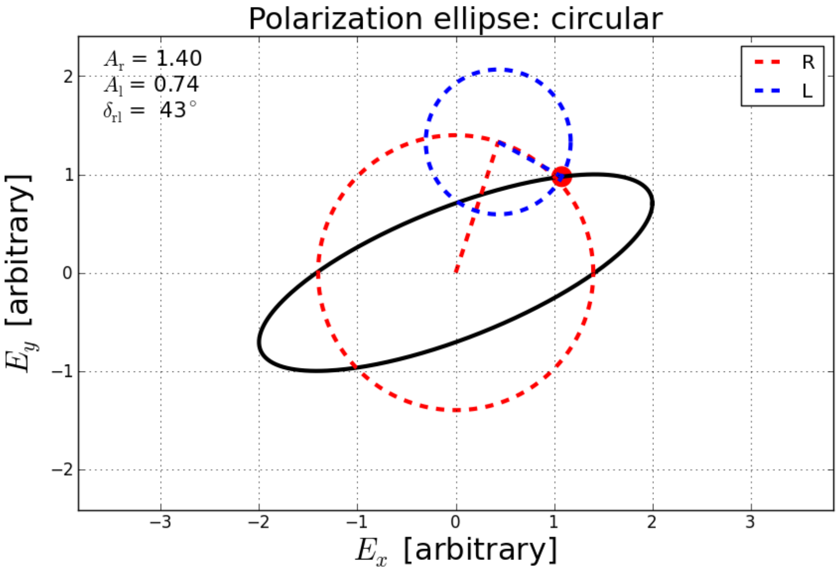
Polarization ellipse: circular 004



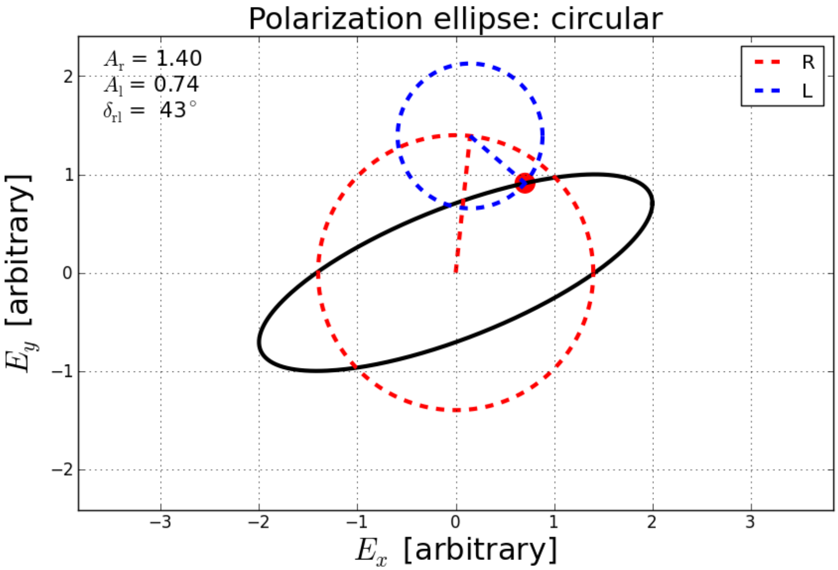
Polarization ellipse: circular 005



Polarization ellipse: circular 006



ASTRON


Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
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R
L
E_y [arbitrary]
E_x [arbitrary]
−3
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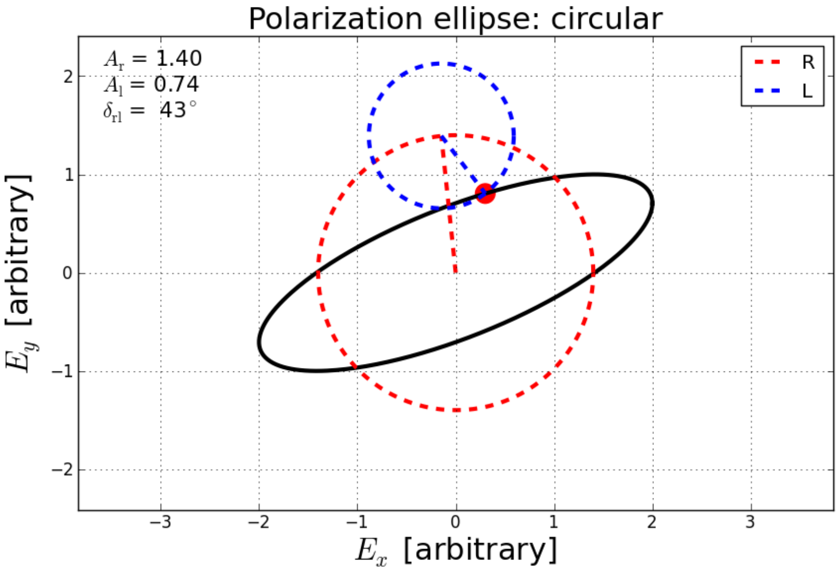
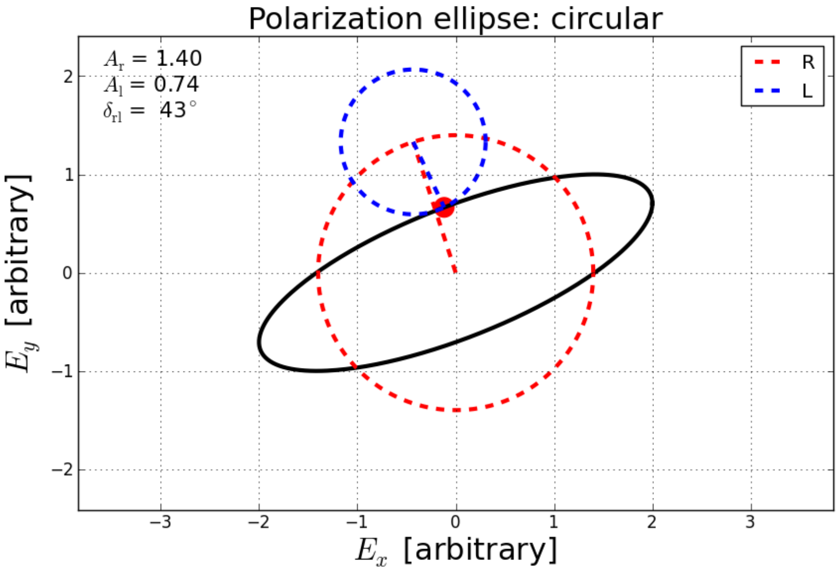
Polarization ellipse: circular 008

ASTRON




Polarization ellipse: circular
A_r = 1.40
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δ_{rl} = 43°
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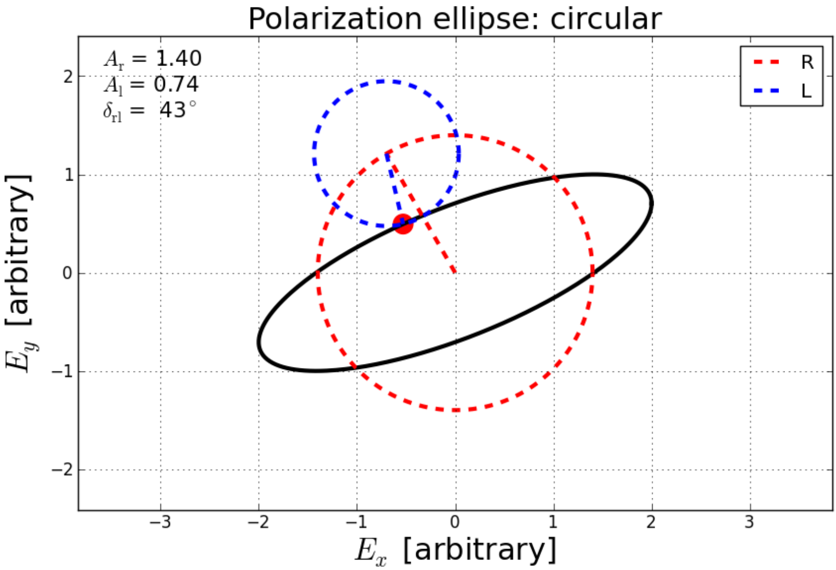
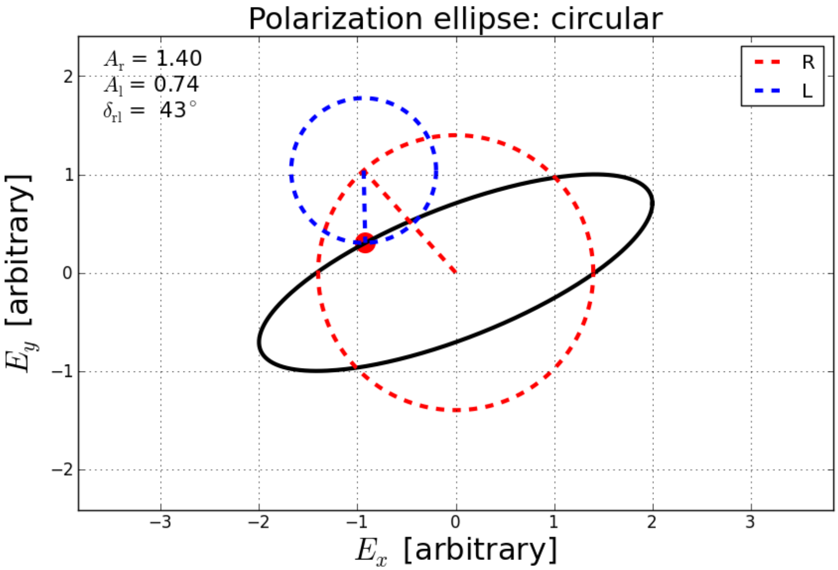
E_x [arbitrary]
E_y [arbitrary]

Polarization ellipse: circular 010





Polarization ellipse: circular
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δ_{rl} = 43°
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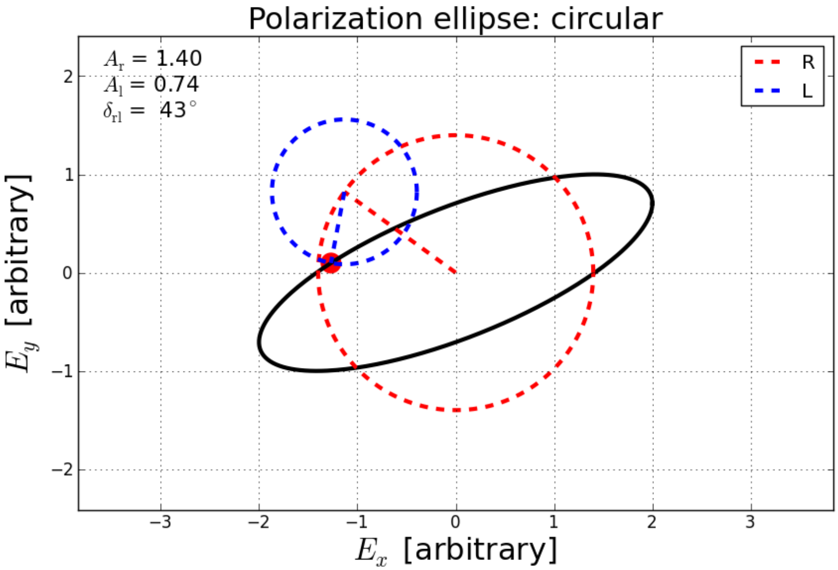
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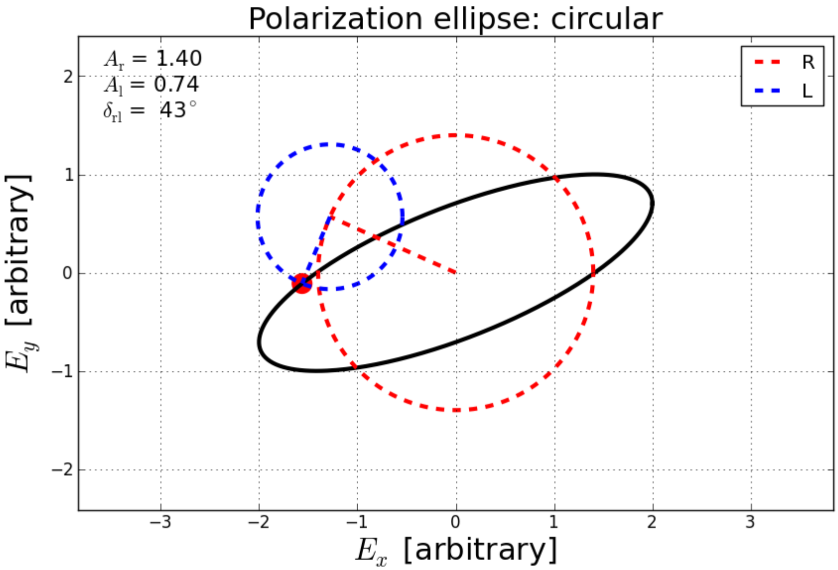

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A_l = 0.74
δ_{rl} = 43°
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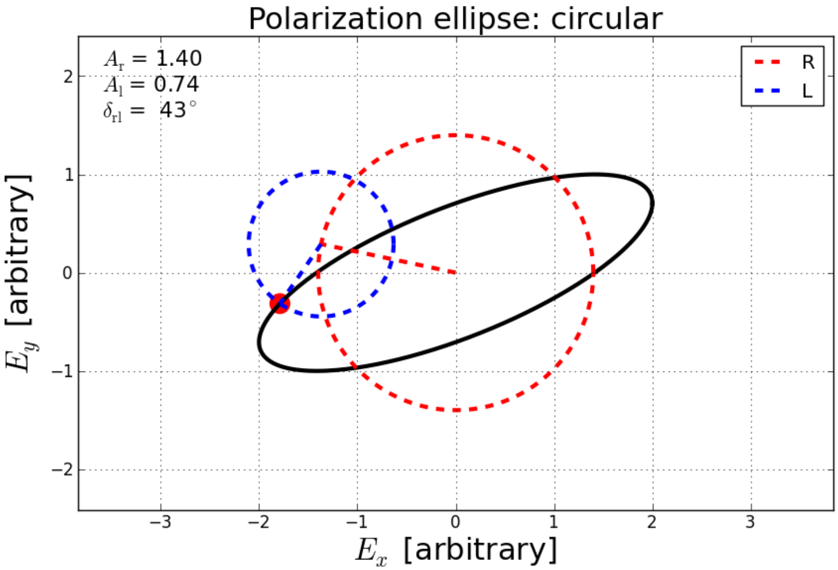
E_y [arbitrary]
E_x [arbitrary]

ASTRON


Polarization ellipse: circular
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A_l = 0.74
δ_{rl} = 43°
R
L
2
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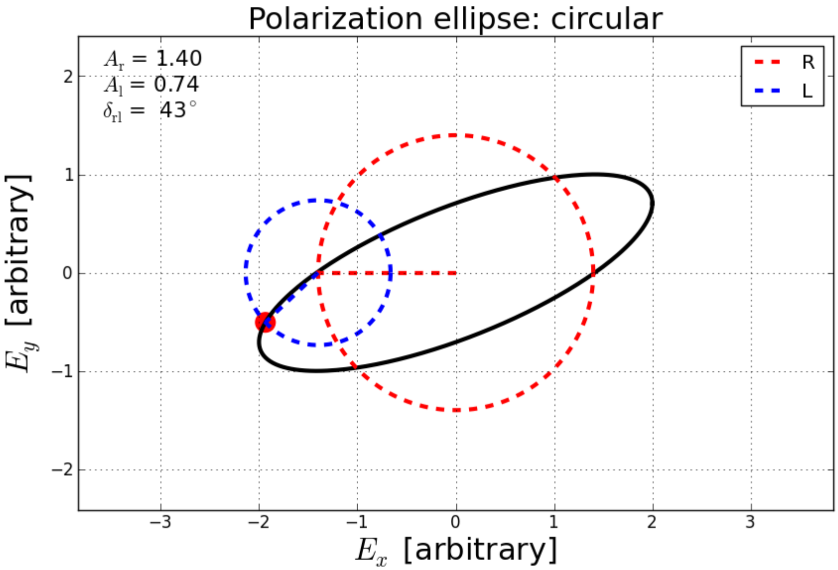
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

Polarization ellipse: circular 015



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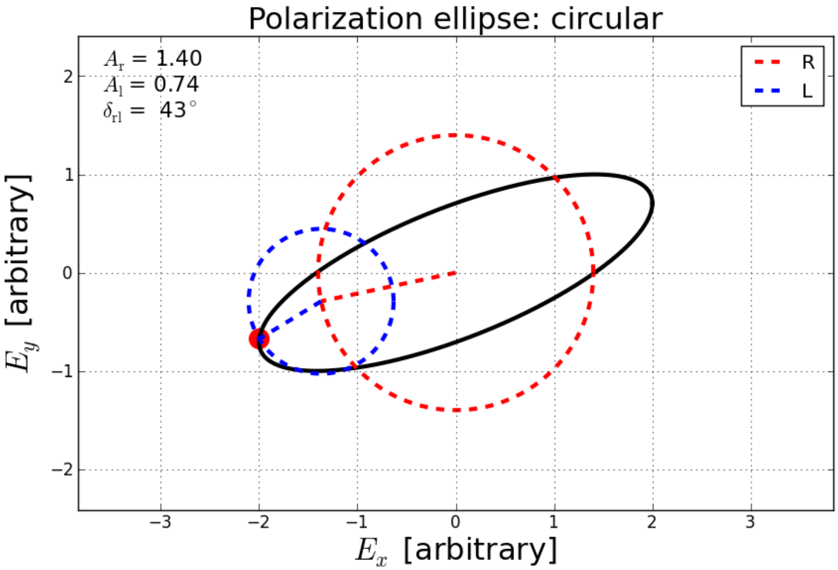
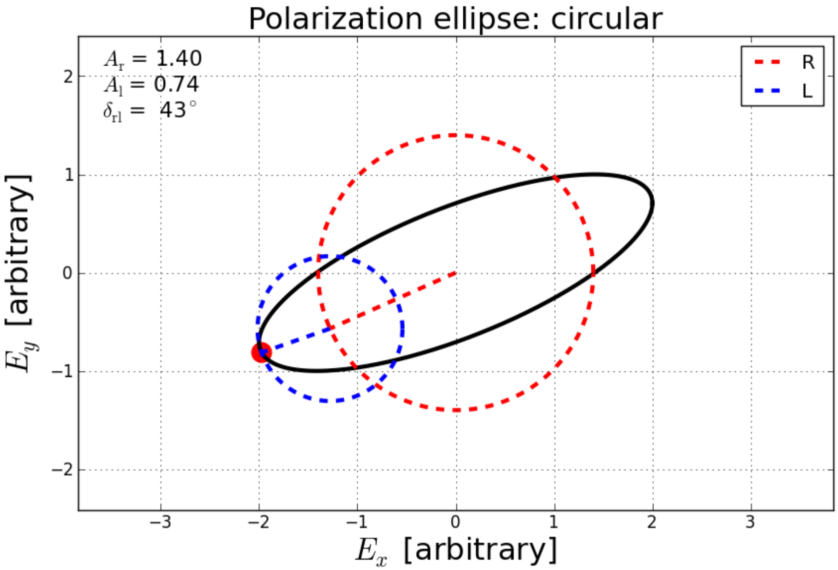
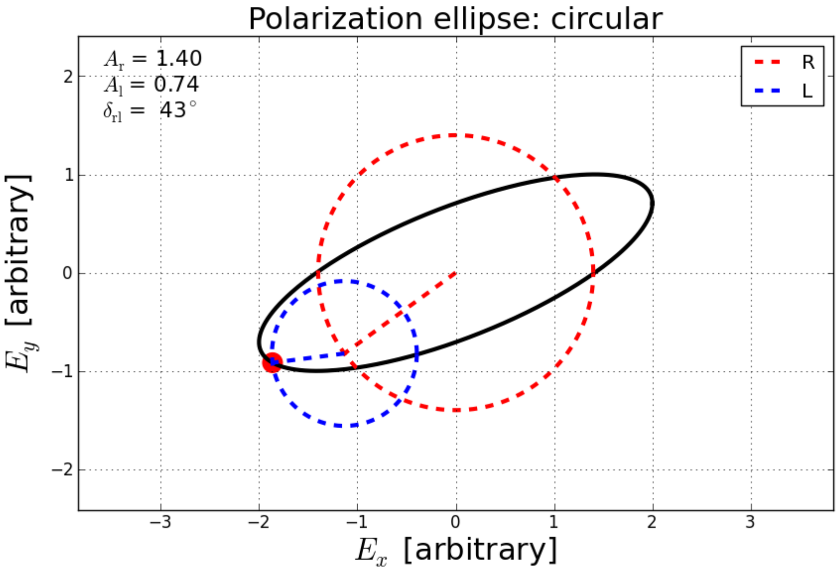

Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

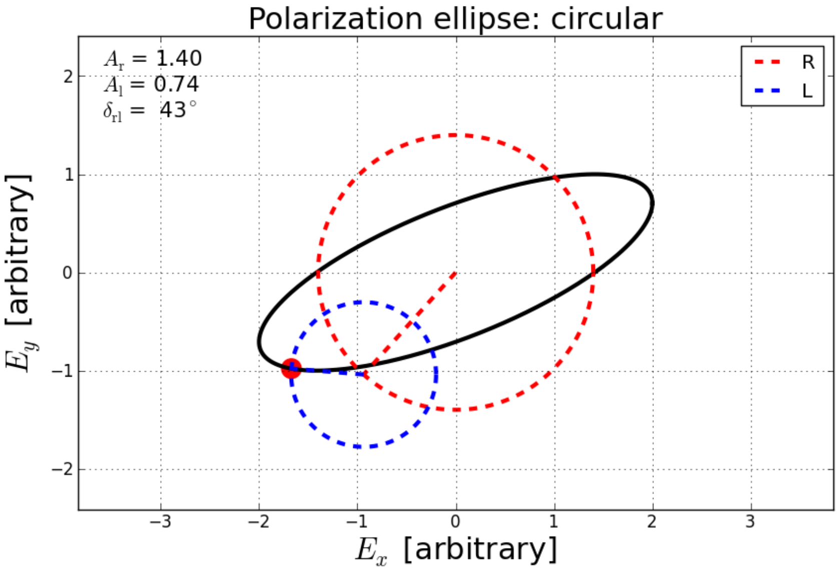
Polarization ellipse: circular 017



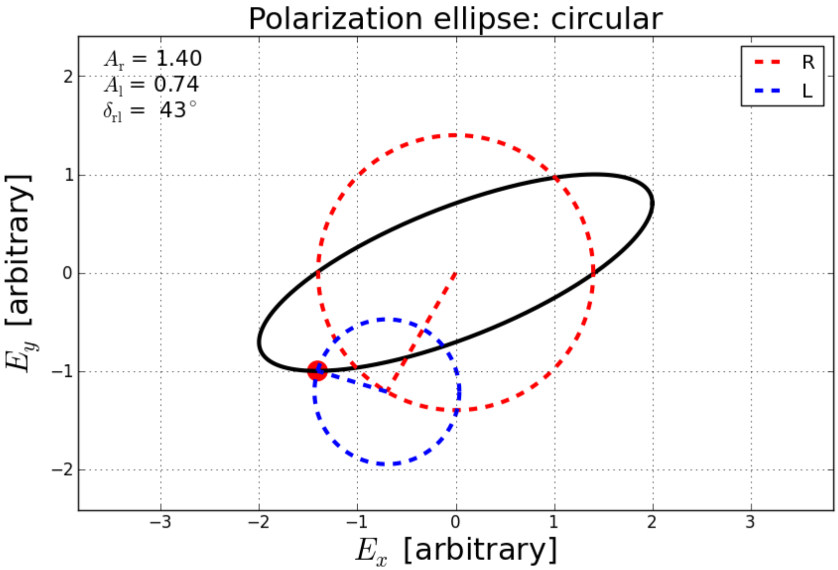
Polarization ellipse: circular 018

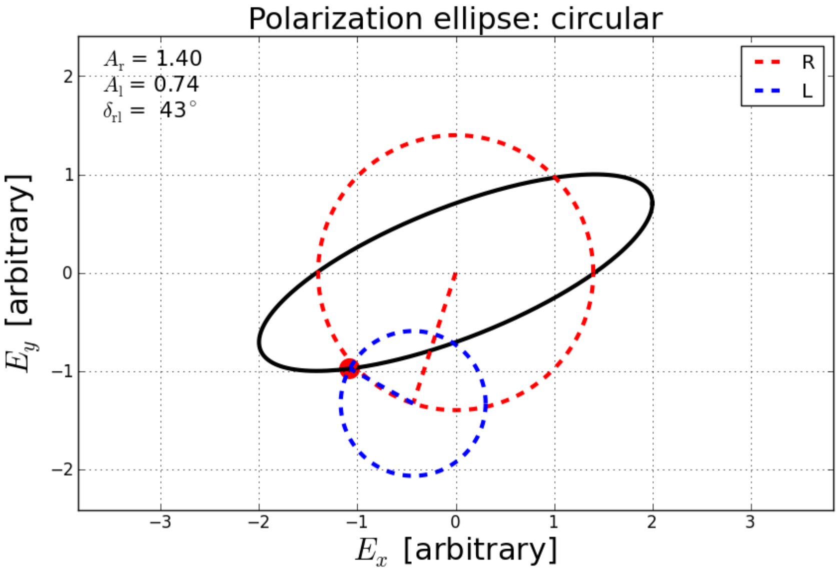


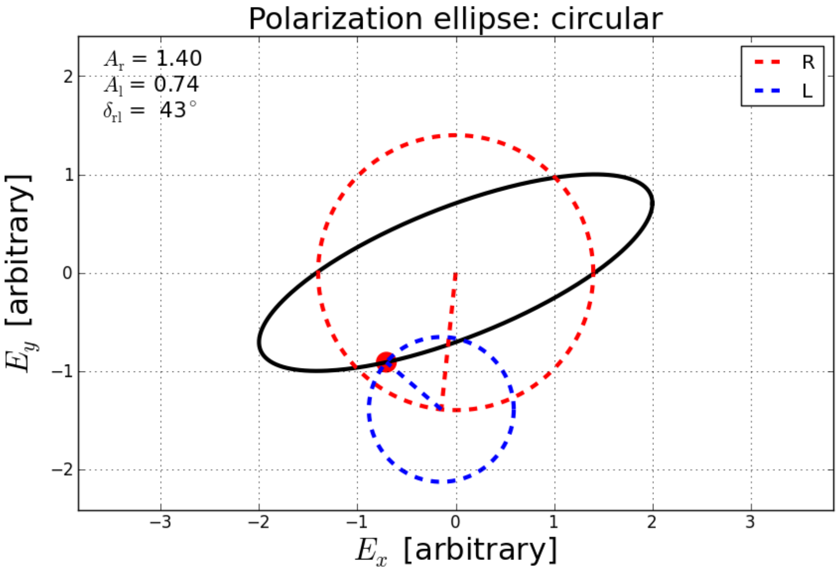
Polarization ellipse: circular 019



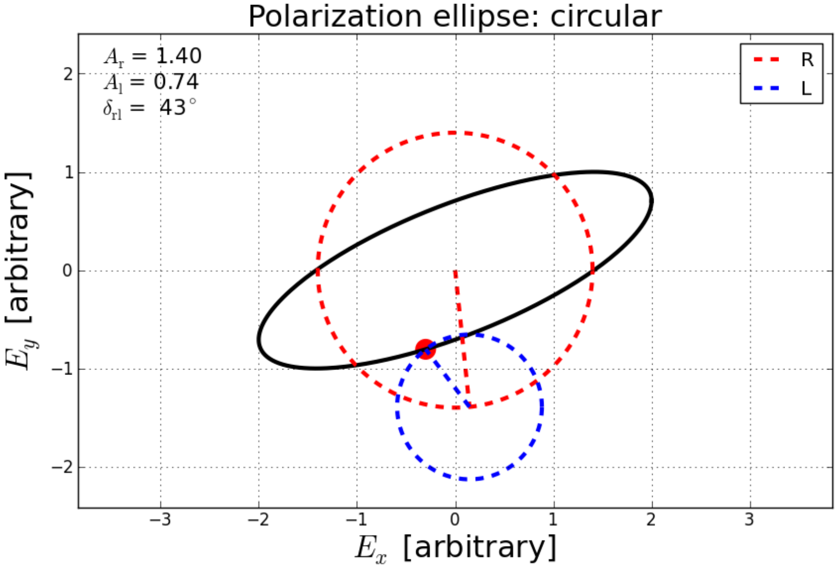
Polarization ellipse: circular 020

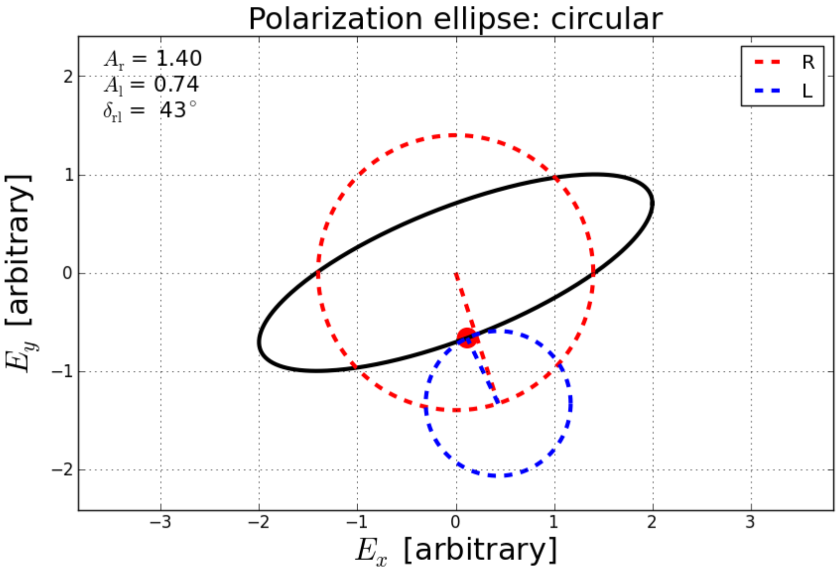



Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3

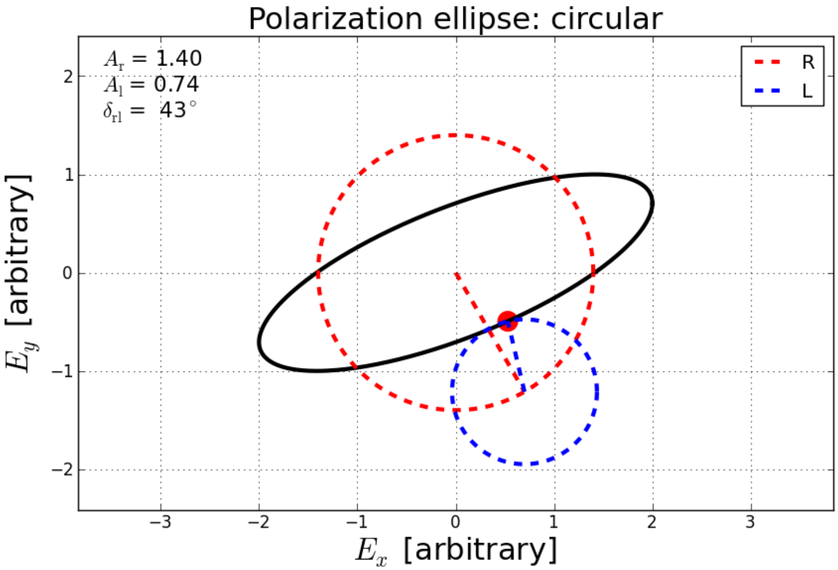

Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

Polarization ellipse: circular 023

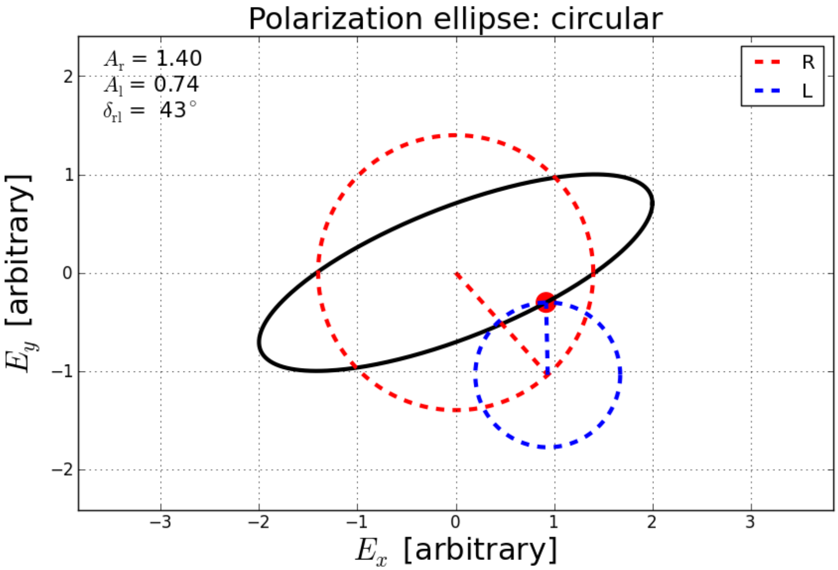




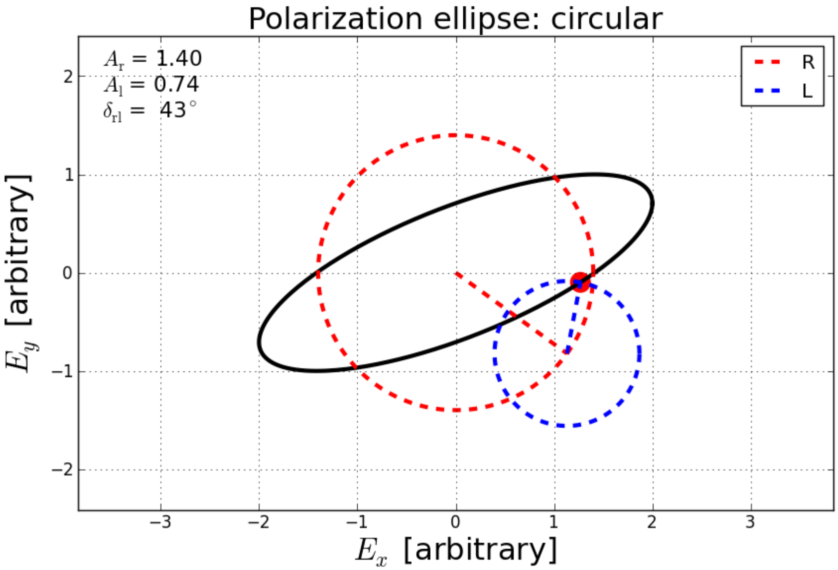
Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

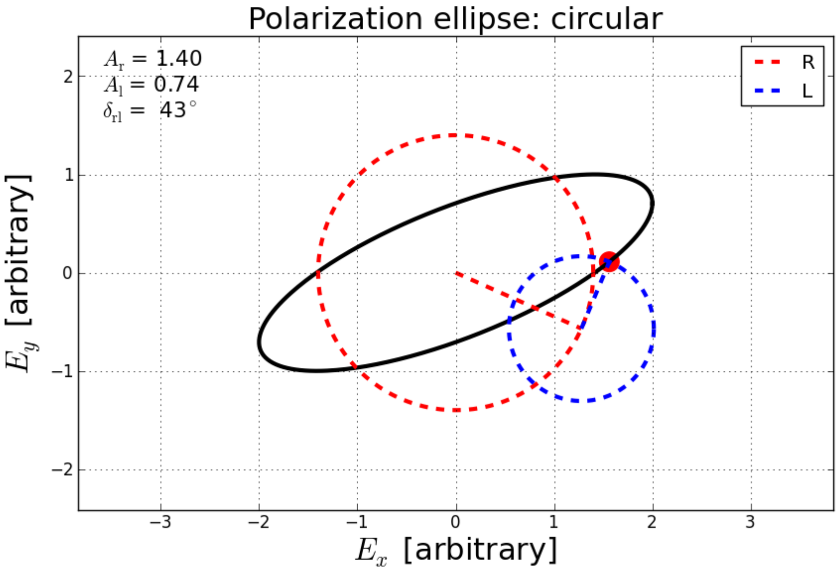

Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
E_x [arbitrary]
E_y [arbitrary]
−3
−2
−1
0
1
2
3
−2
−1
0
1
2

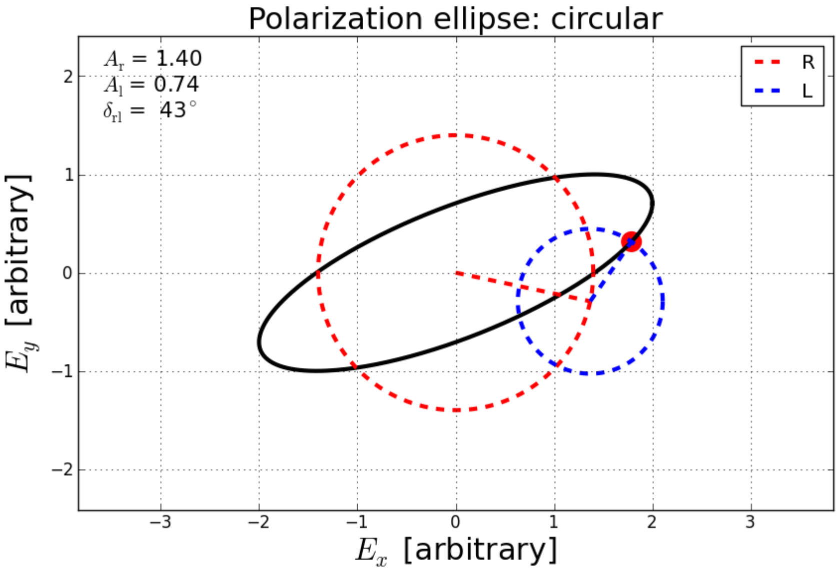
Polarization ellipse: circular 026



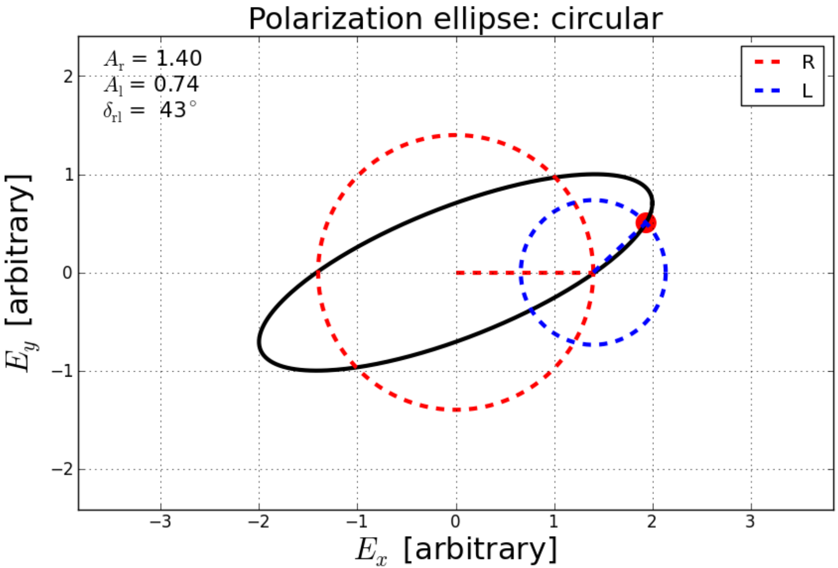
Polarization ellipse: circular 027




Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

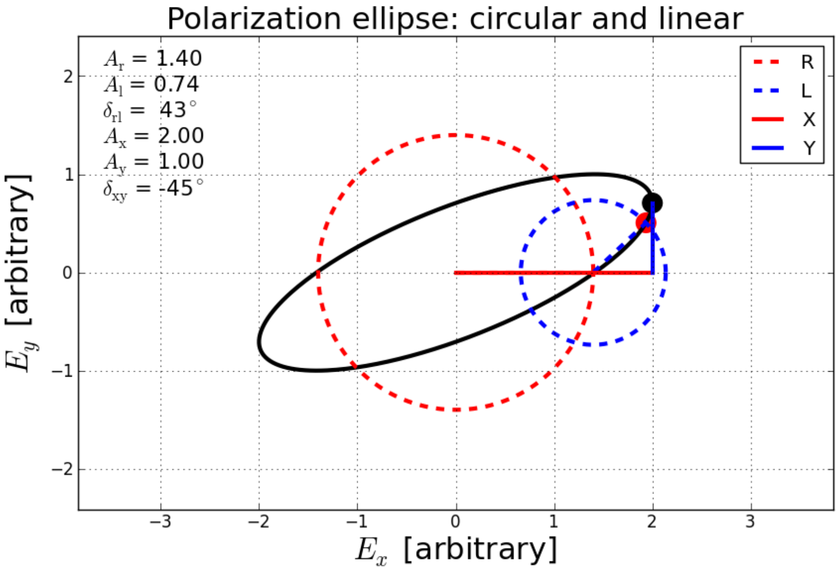

Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

Polarization ellipse: circular 030



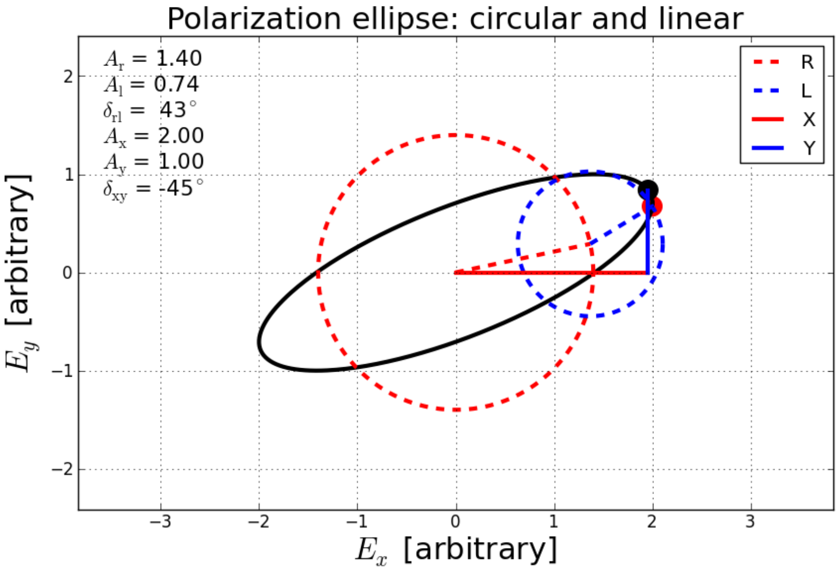
ASTRON

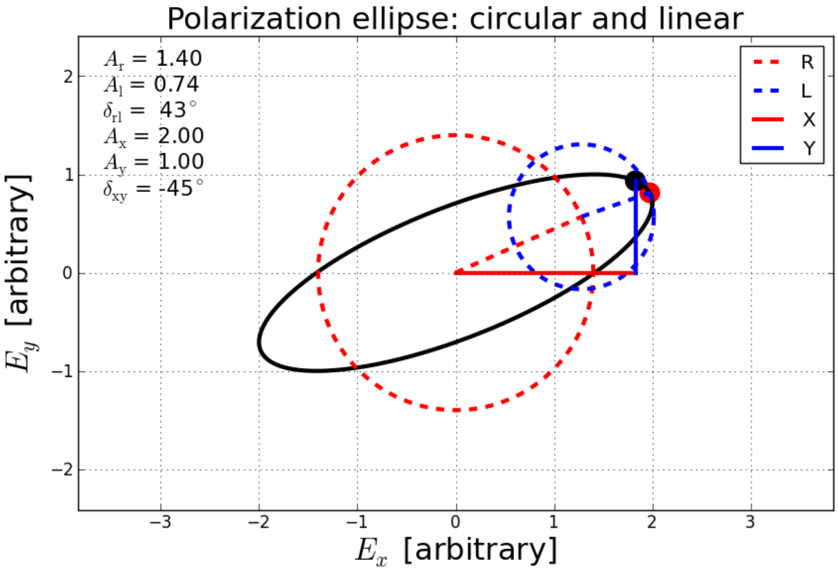
$$
\begin{aligned}
A_{\mathrm{r}} &= \frac{1}{2}\sqrt{A_{\mathrm{x}}^2 + A_{\mathrm{y}}^2 - 2A_{\mathrm{x}}A_{\mathrm{y}}\sin\delta_{\mathrm{xy}}} \\
A_{\mathrm{l}} &= \frac{1}{2}\sqrt{A_{\mathrm{x}}^2 + A_{\mathrm{y}}^2 + 2A_{\mathrm{x}}A_{\mathrm{y}}\sin\delta_{\mathrm{xy}}} \\
\tan\delta_{\mathrm{rl}} &= \frac{2A_{\mathrm{x}}A_{\mathrm{y}}\cos\delta_{\mathrm{xy}}}{A_{\mathrm{x}}^2 - A_{\mathrm{y}}^2}
\end{aligned}
$$



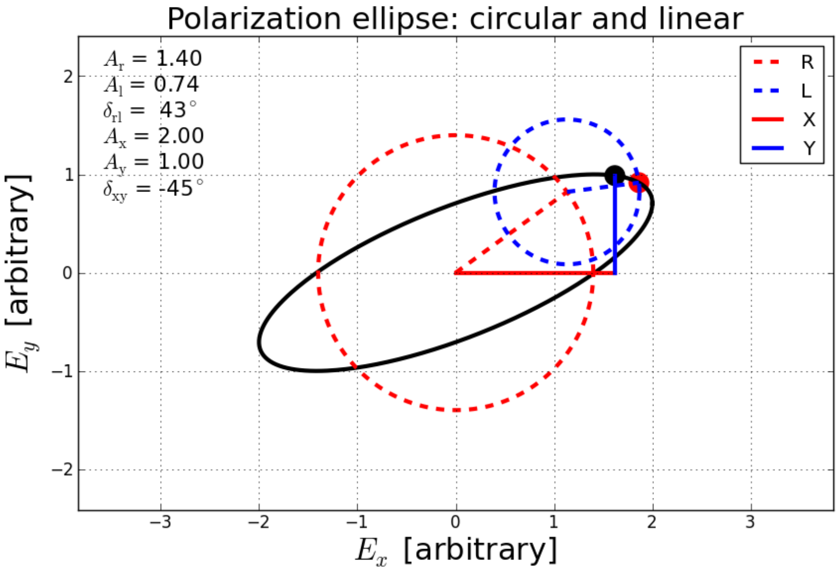
Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

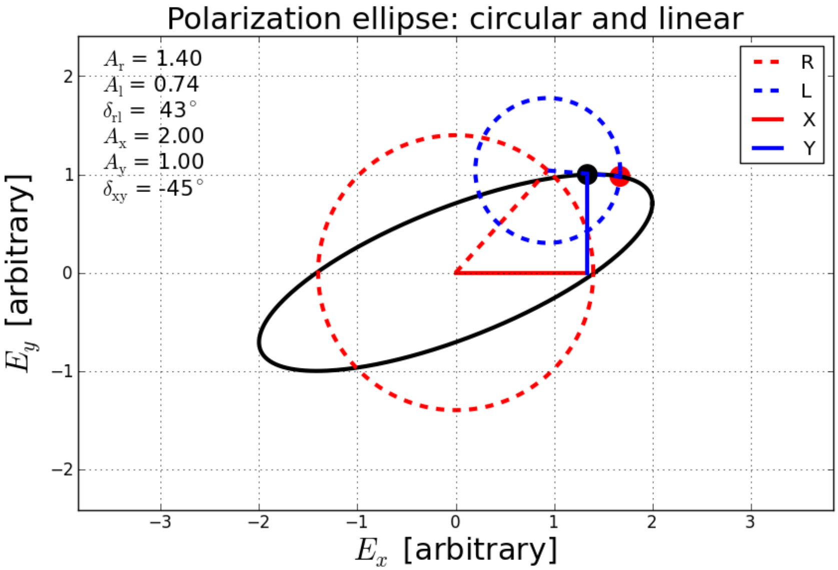
Polarization ellipse: circular and linear 001



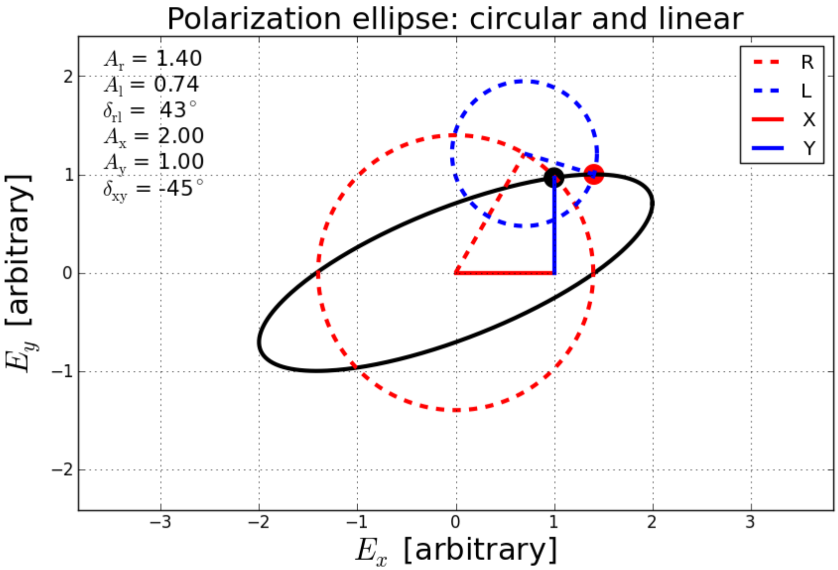

Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
−3
−2
−1
0
1
2
3
−2
−1
0
1
2

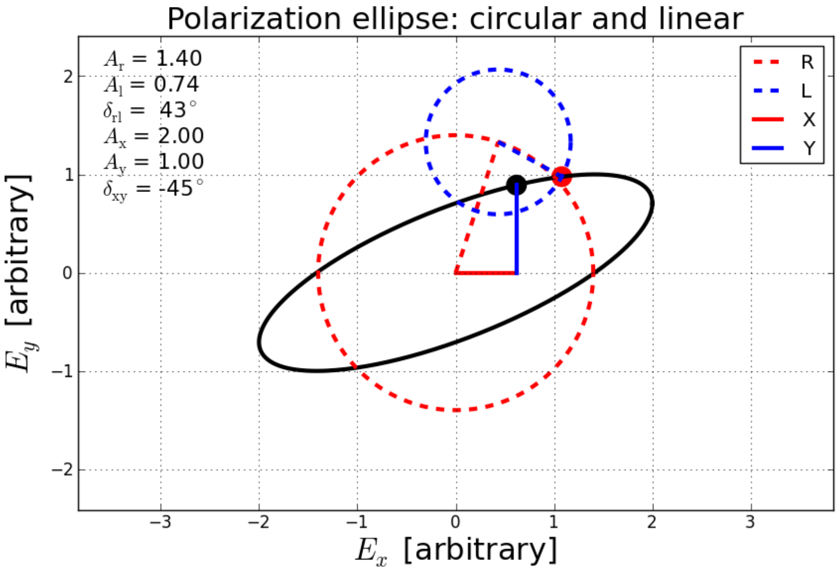
Polarization ellipse: circular and linear 003



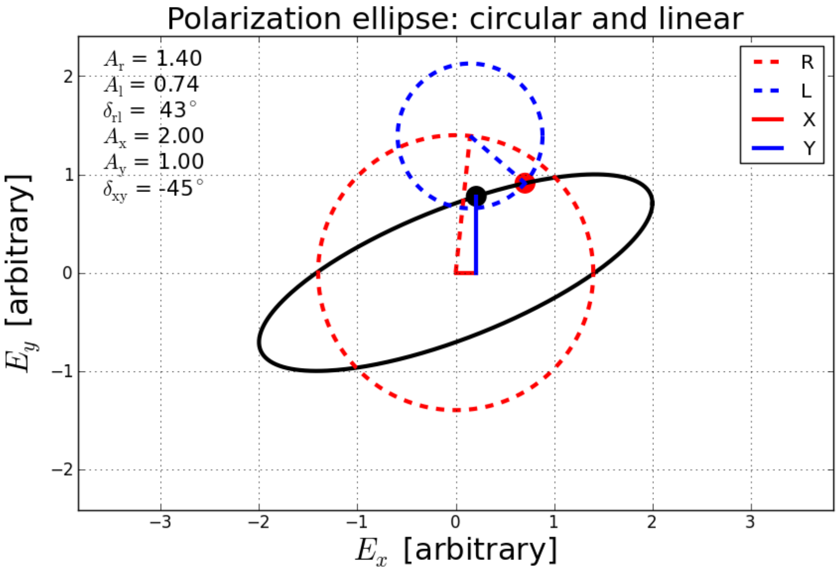

Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3

Polarization ellipse: circular and linear 005

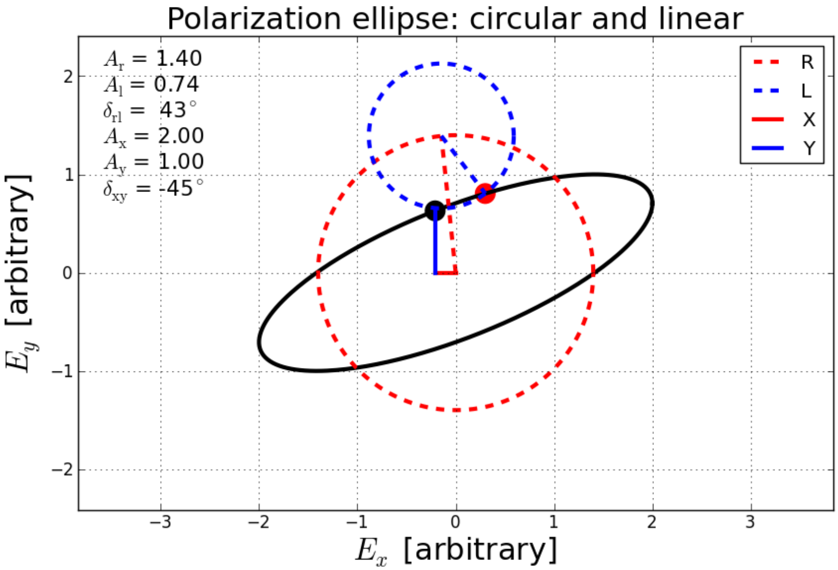


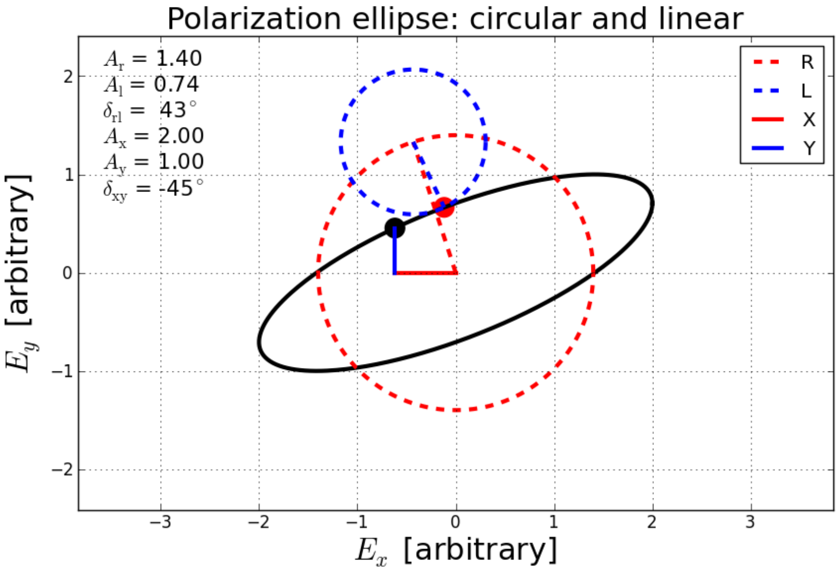


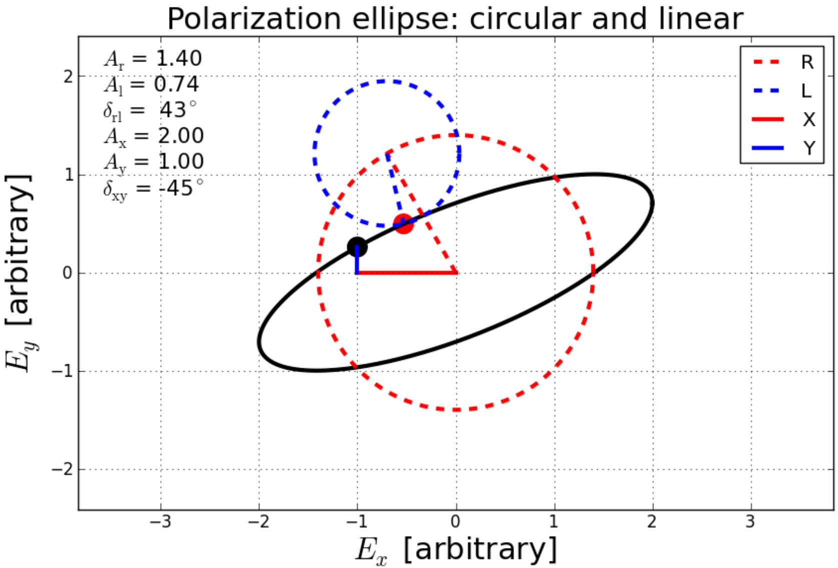
Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
3

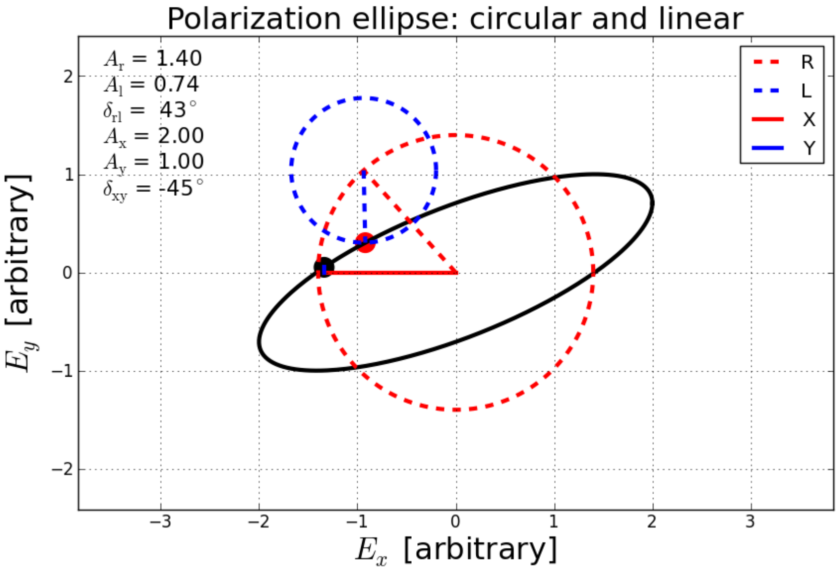


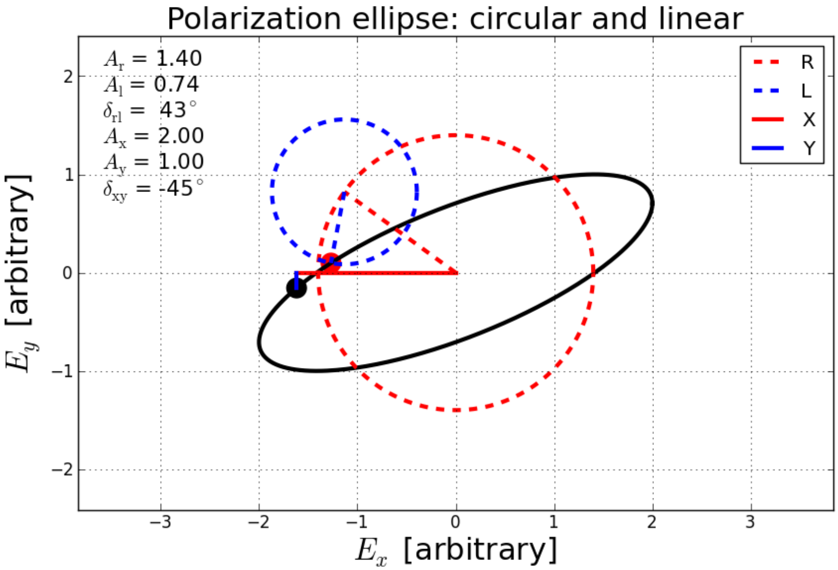
Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_x [arbitrary]
E_y [arbitrary]


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_x [arbitrary]
E_y [arbitrary]

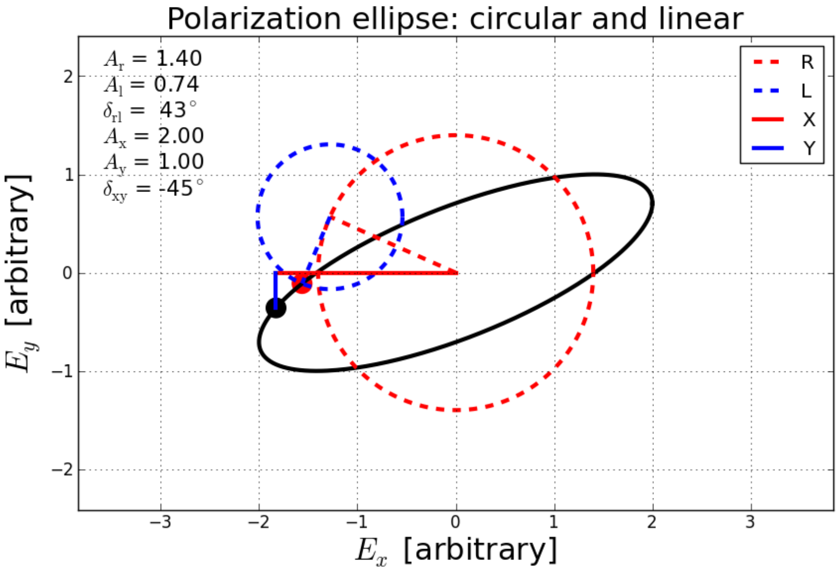

Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_x [arbitrary]
E_y [arbitrary]


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_x [arbitrary]
E_y [arbitrary]

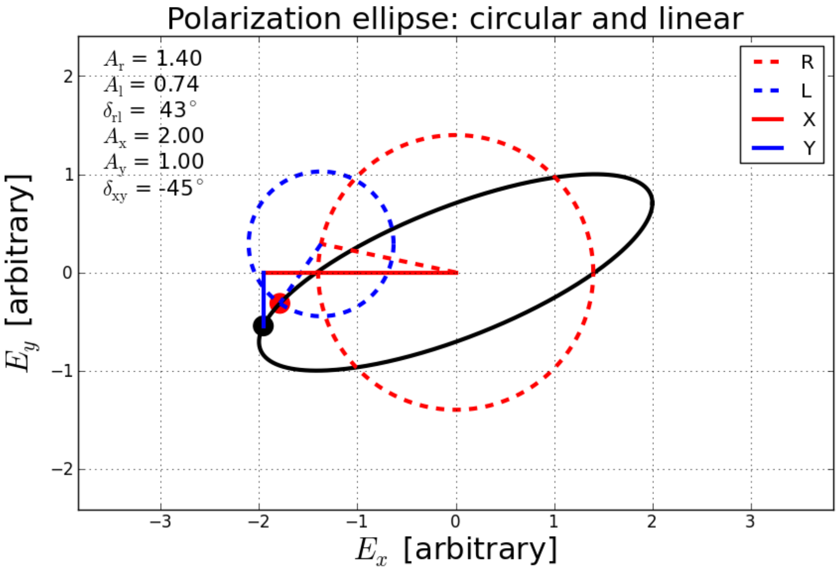


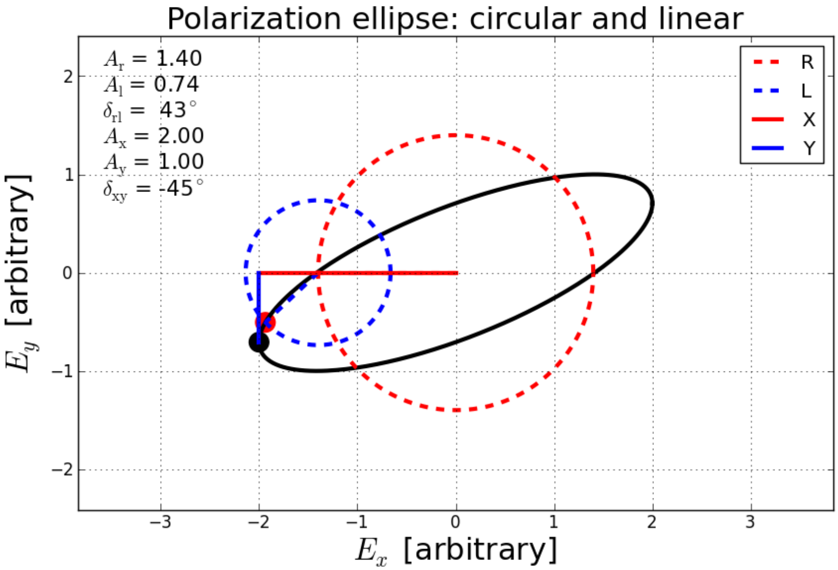
Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

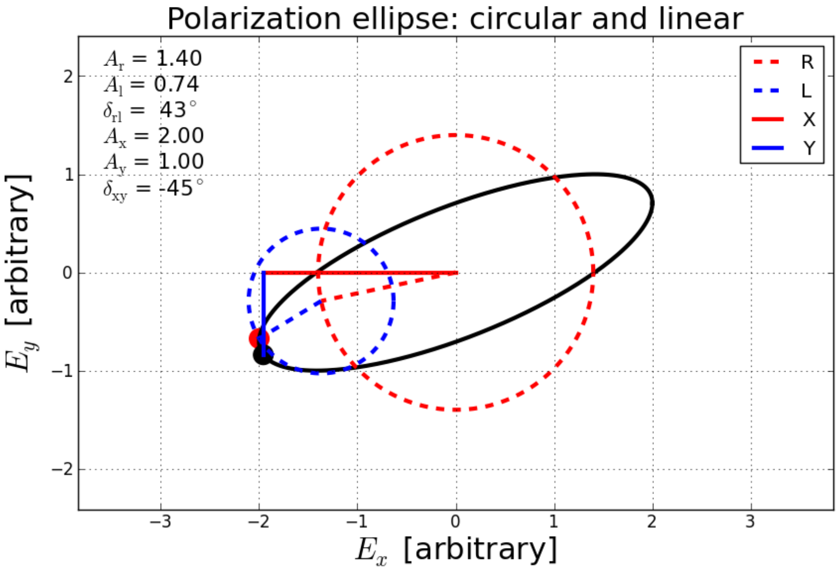
Polarization ellipse: circular and linear 013



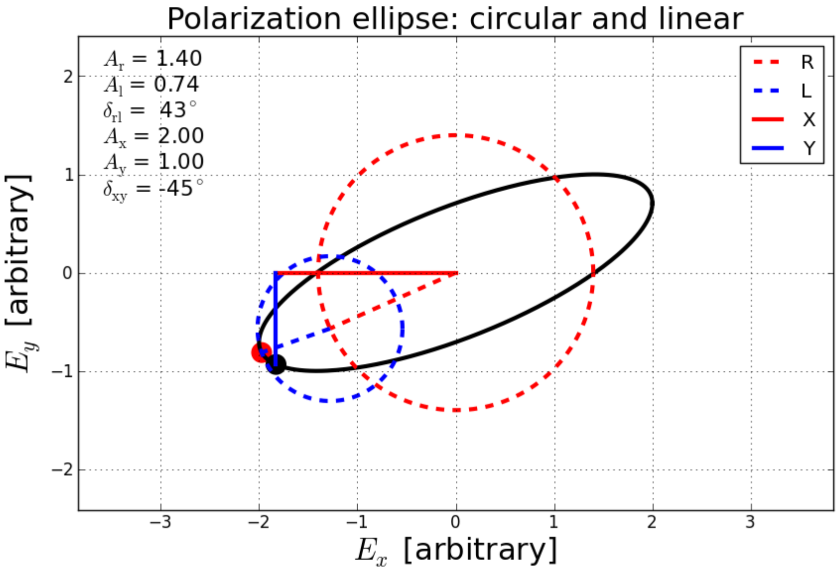
Polarization ellipse: circular and linear 014

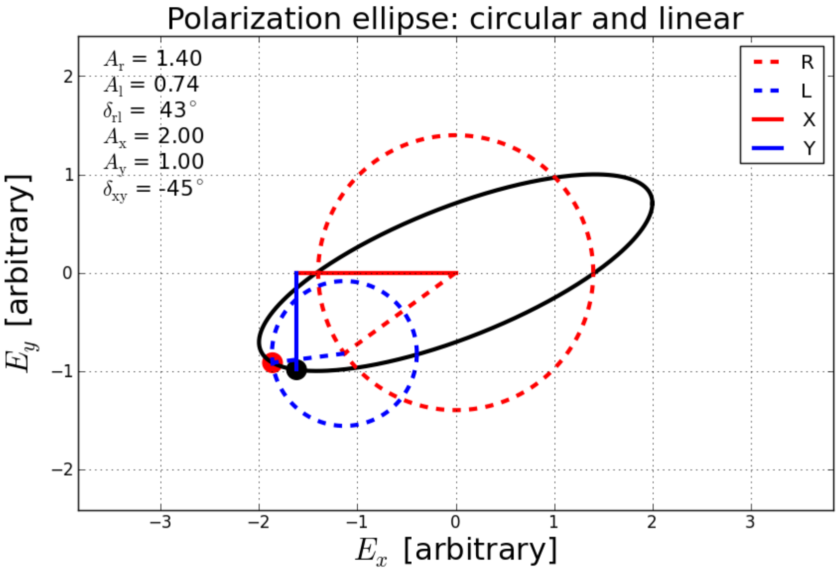


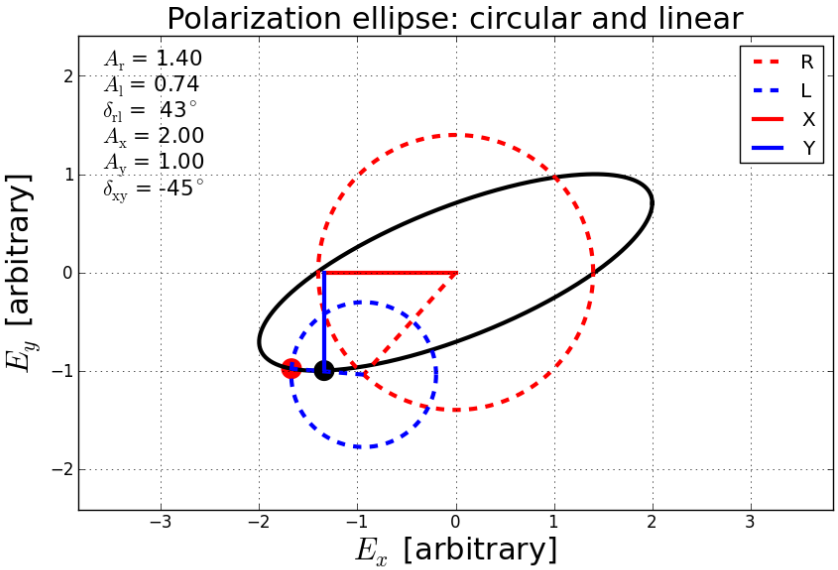

Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3

Polarization ellipse: circular and linear 017




Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

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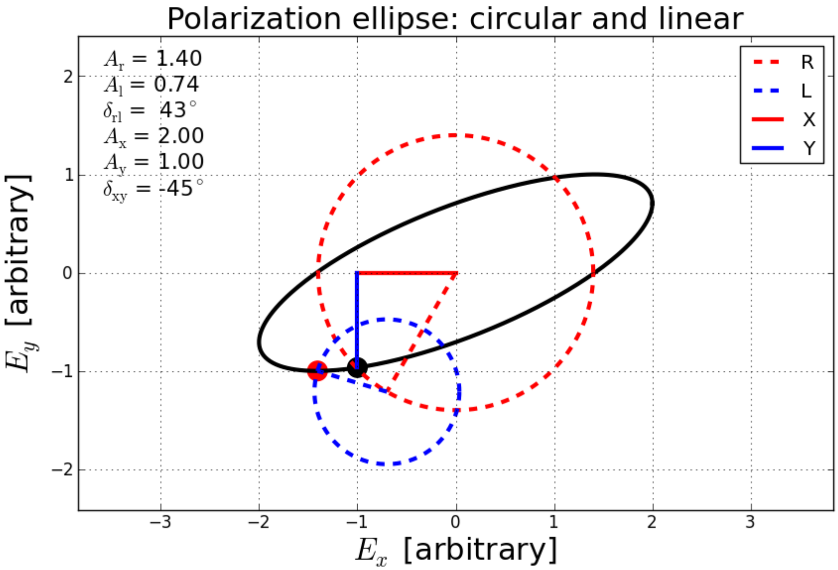
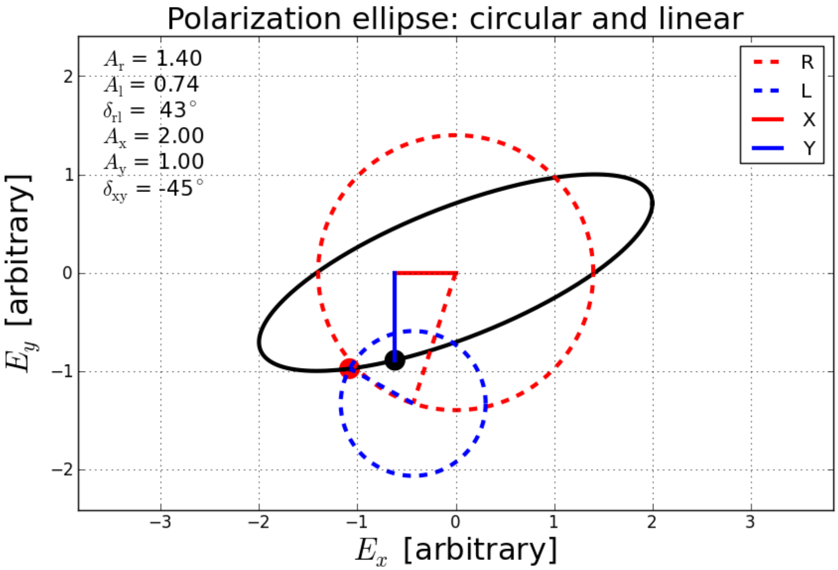
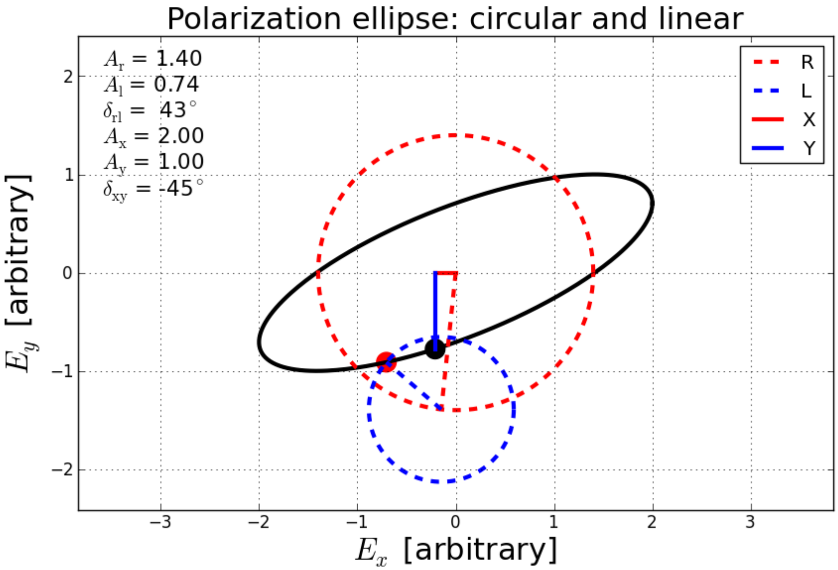
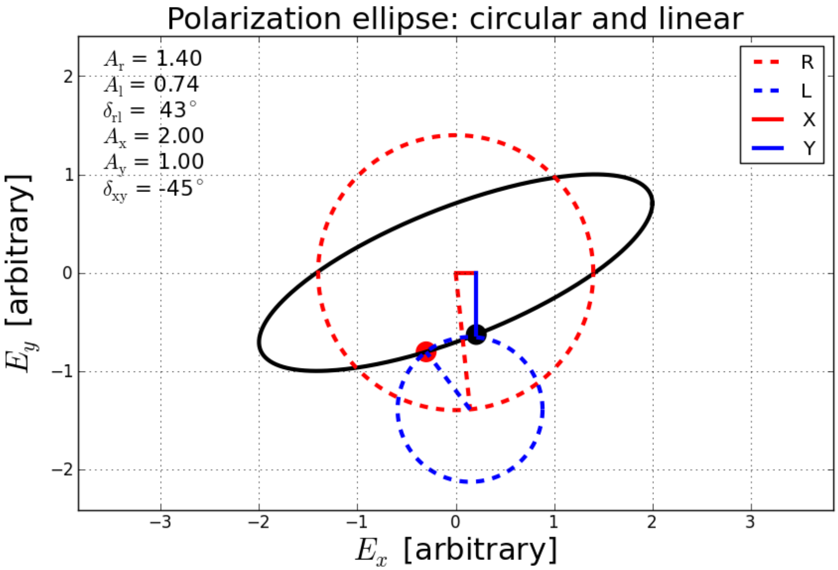

Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
−3
−2
−1
0
1
2
3
−2
−1
0
1
2

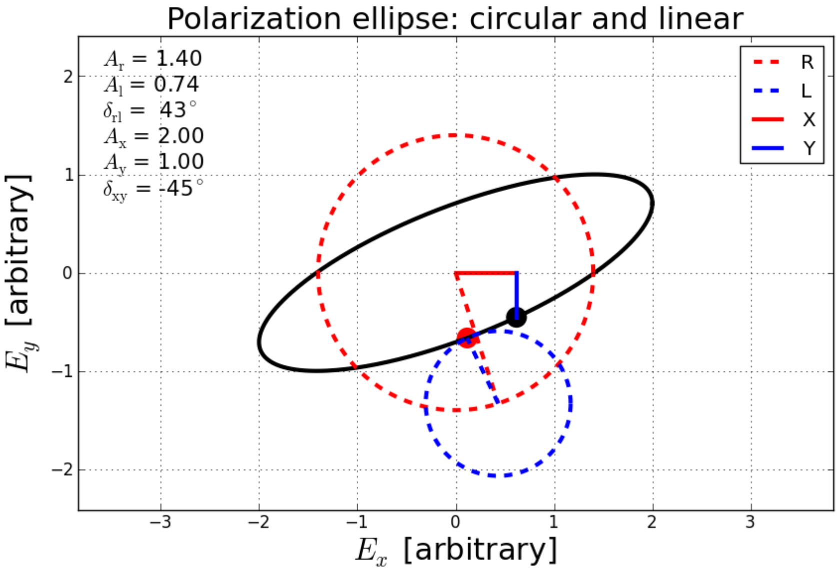

Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3



Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

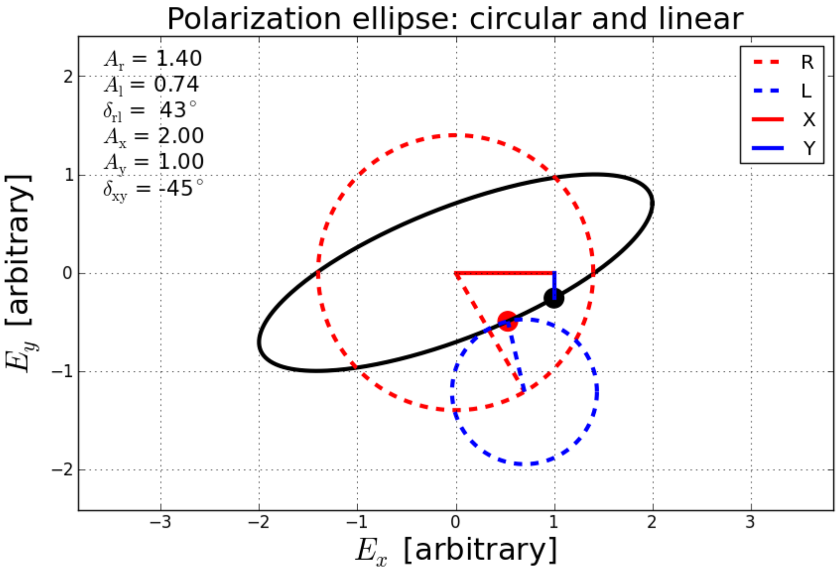

Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
−3
−2
−1
0
1
2
3
2
1
0
−1
−2

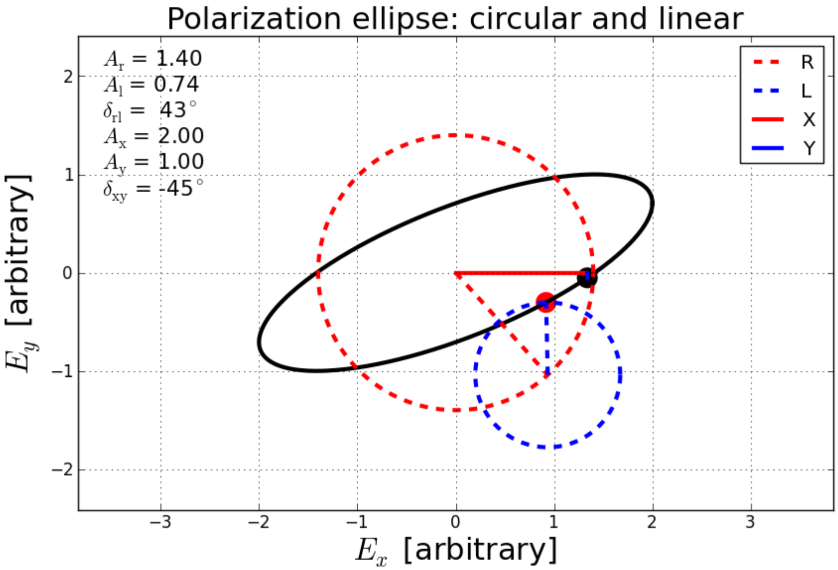
Polarization ellipse: circular and linear 024



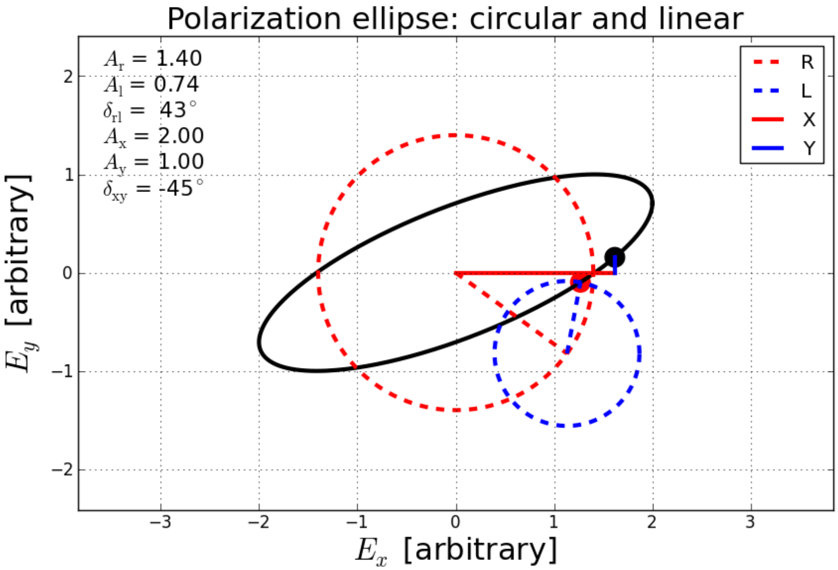
Polarization ellipse: circular and linear 025

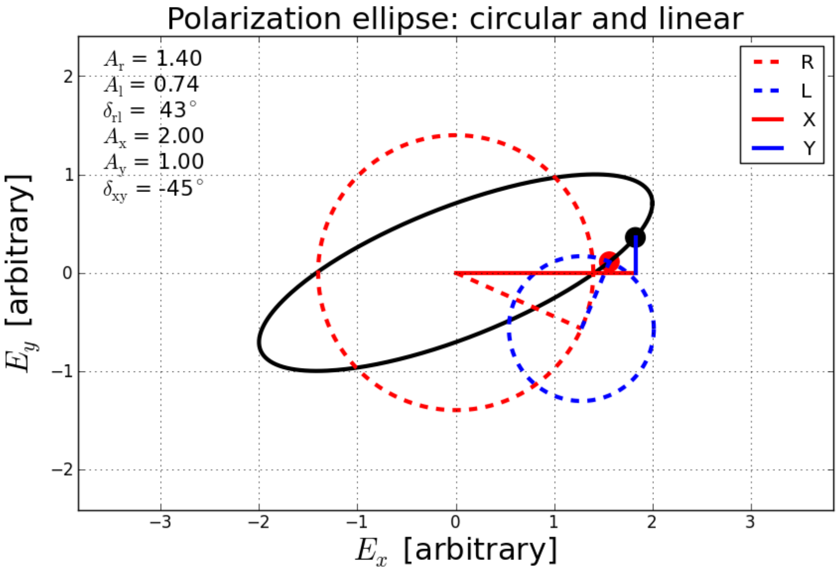
ASTRON

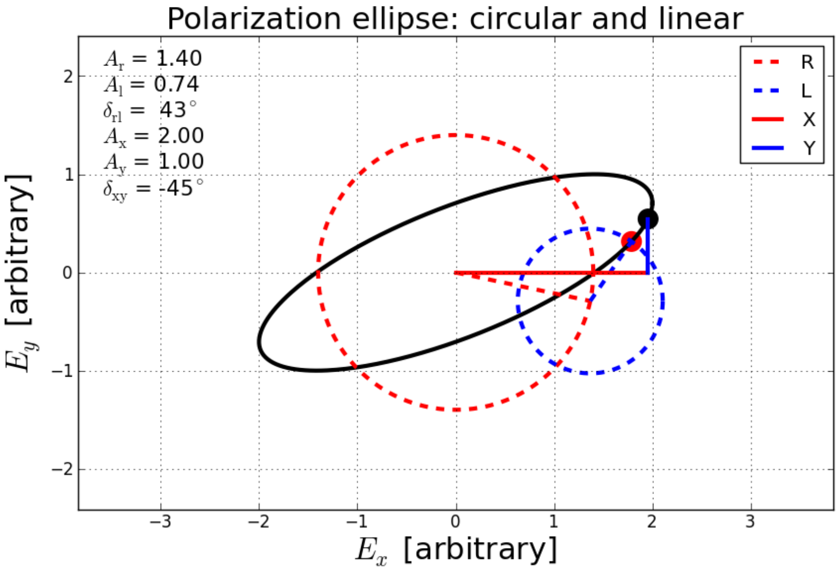


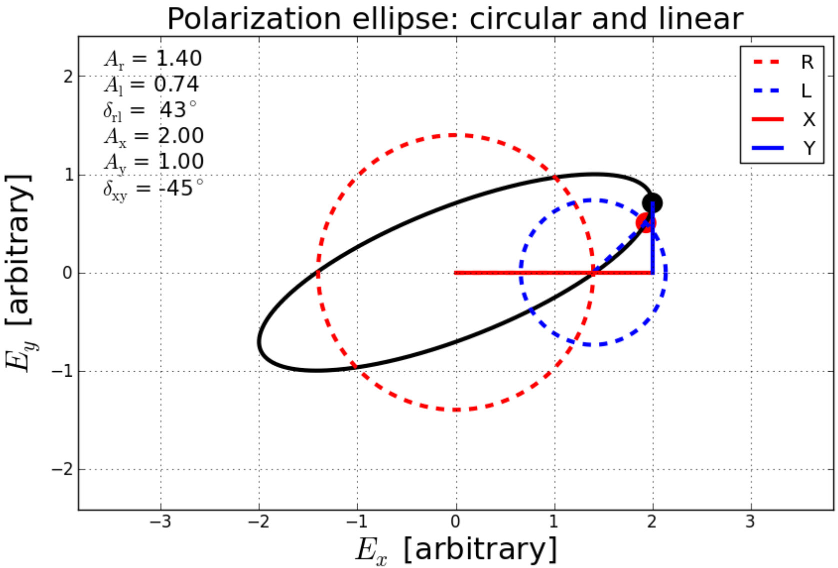

Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3

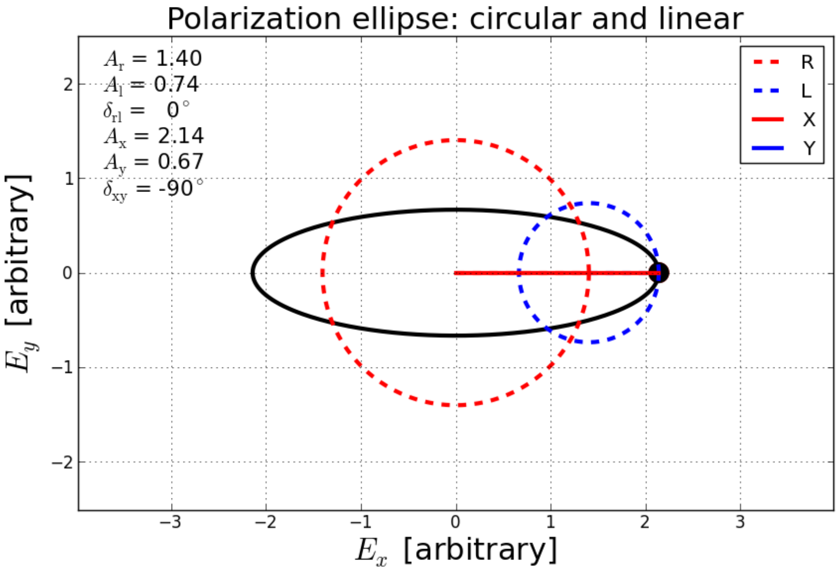
Polarization ellipse: circular and linear 027



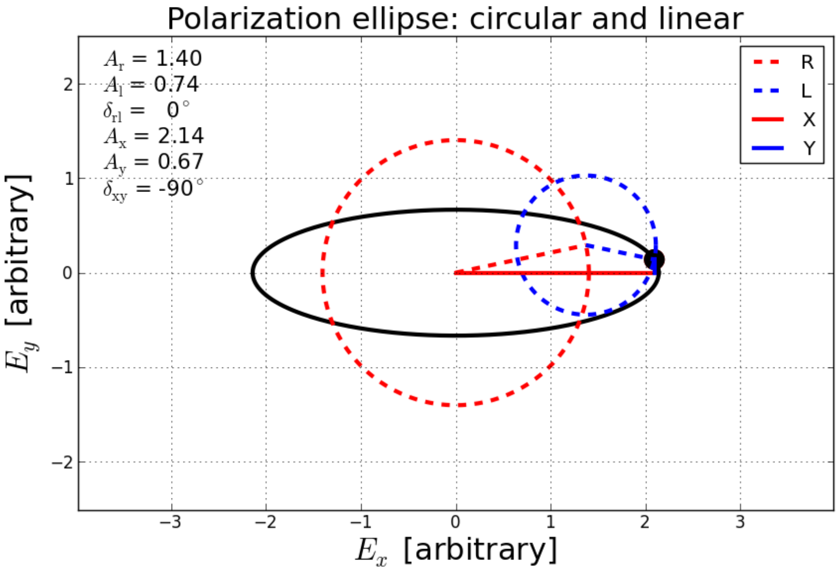

Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
−3
−2
−1
0
1
2
3
−2
−1
0
1
2


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
3

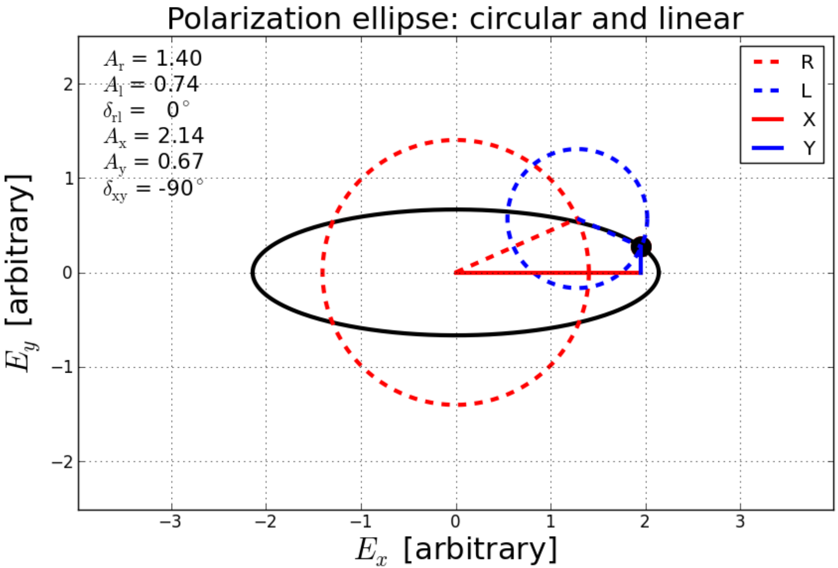

Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3

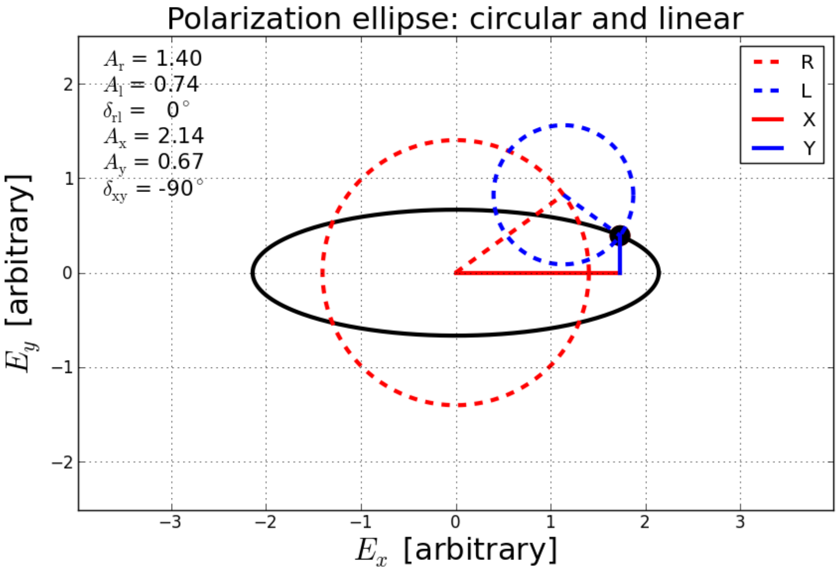
Polarization ellipse: circular and linear 001

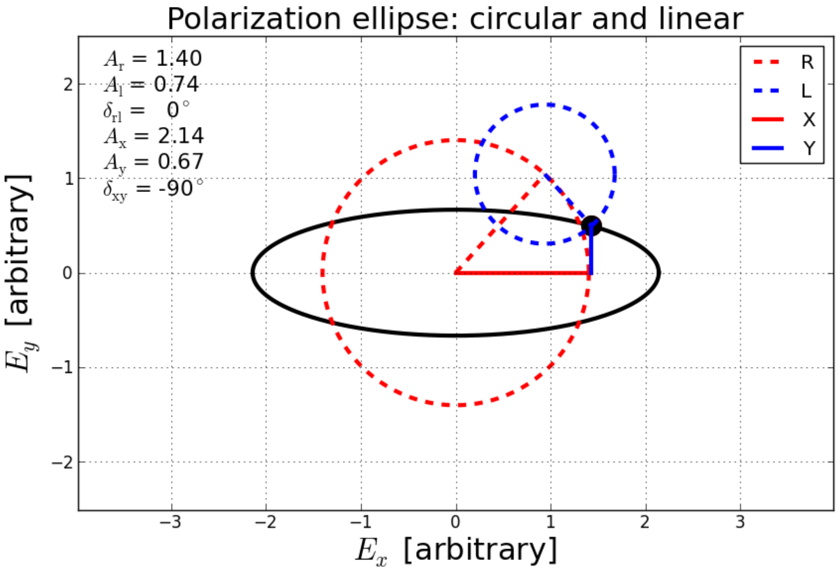


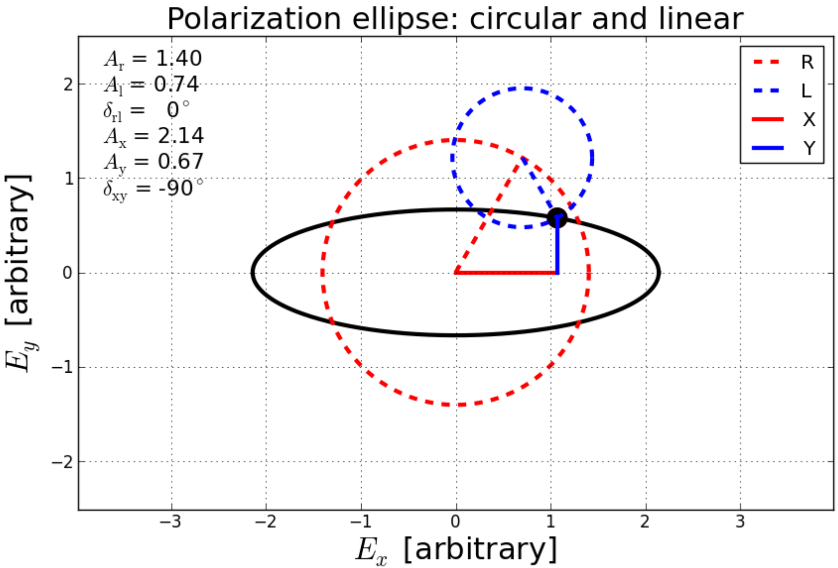
Polarization ellipse: circular and linear 002



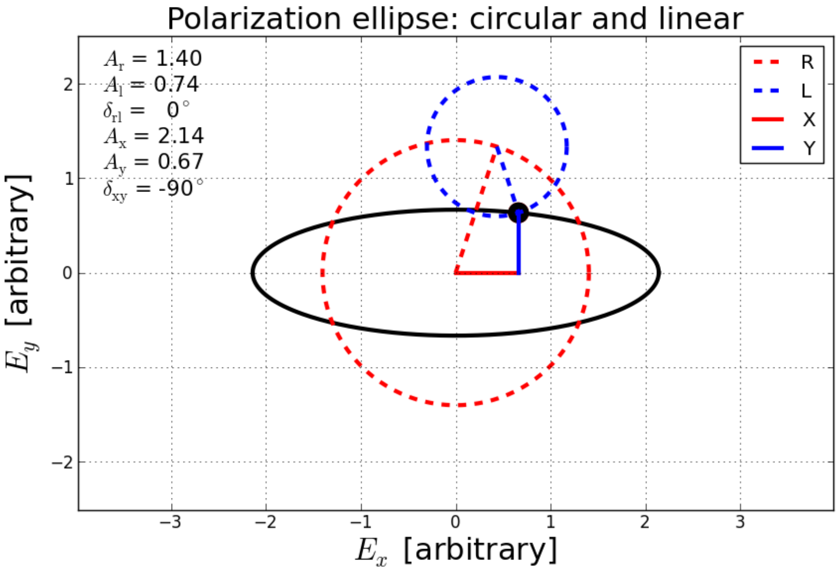
Polarization ellipse: circular and linear 003

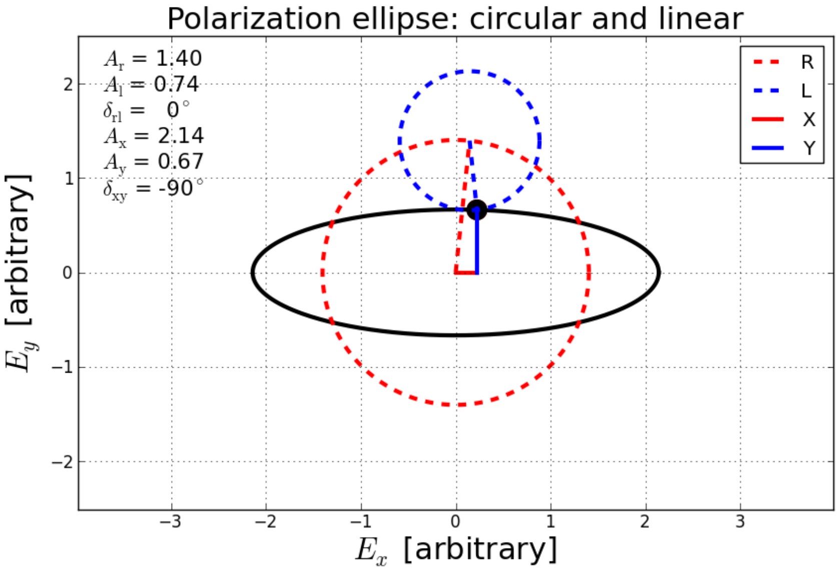


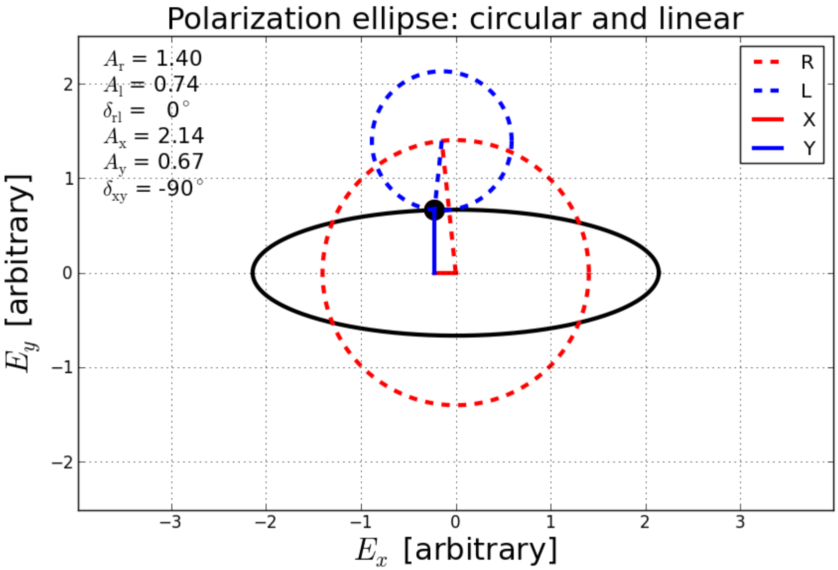

Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3

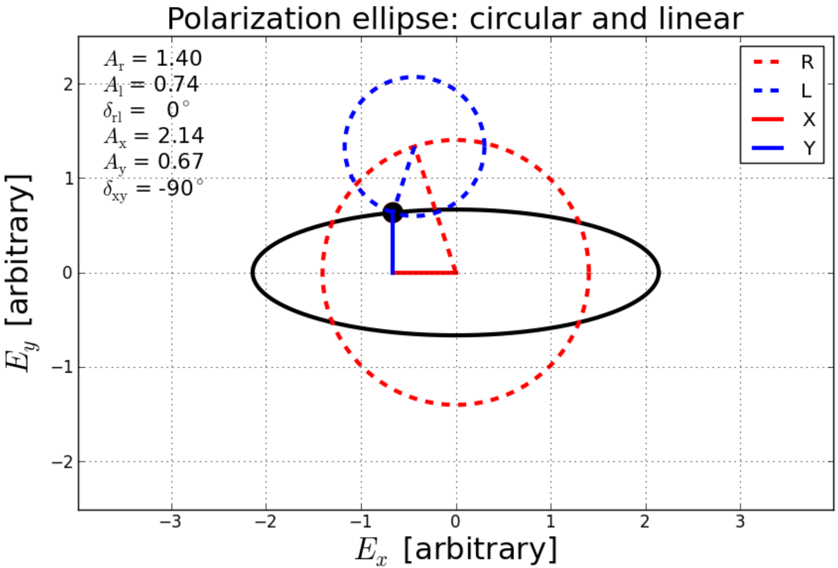

Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3

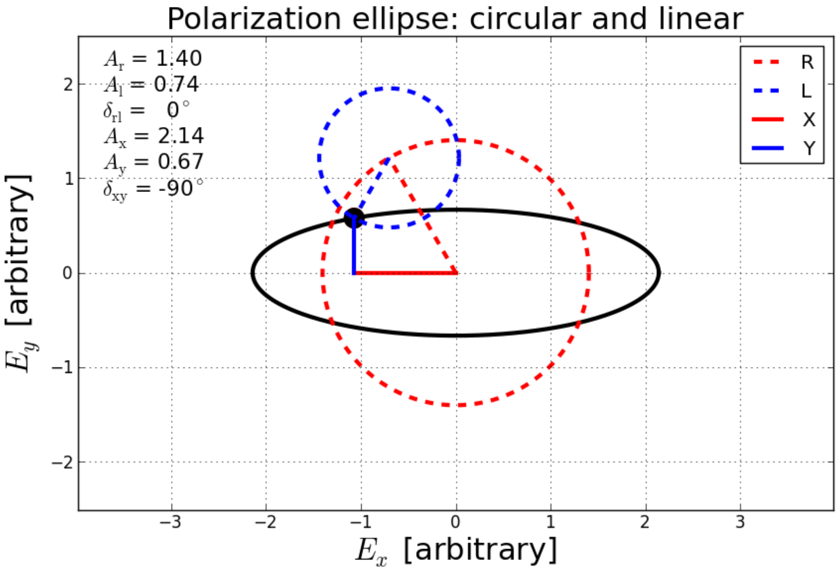
Polarization ellipse: circular and linear 006

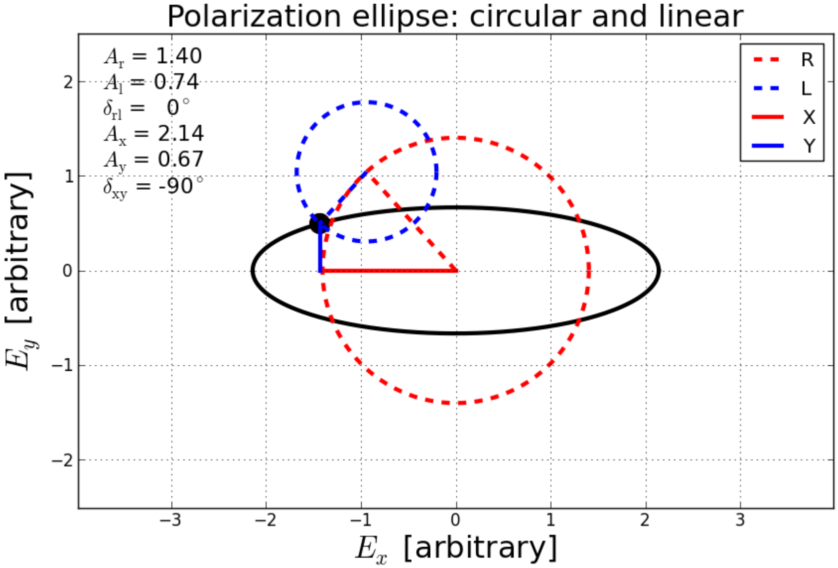



Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
−3
−2
−1
0
1
2
3
−2
−1
0
1
2


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
−3
−2
−1
0
1
2
3
−2
−1
0
1
2


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
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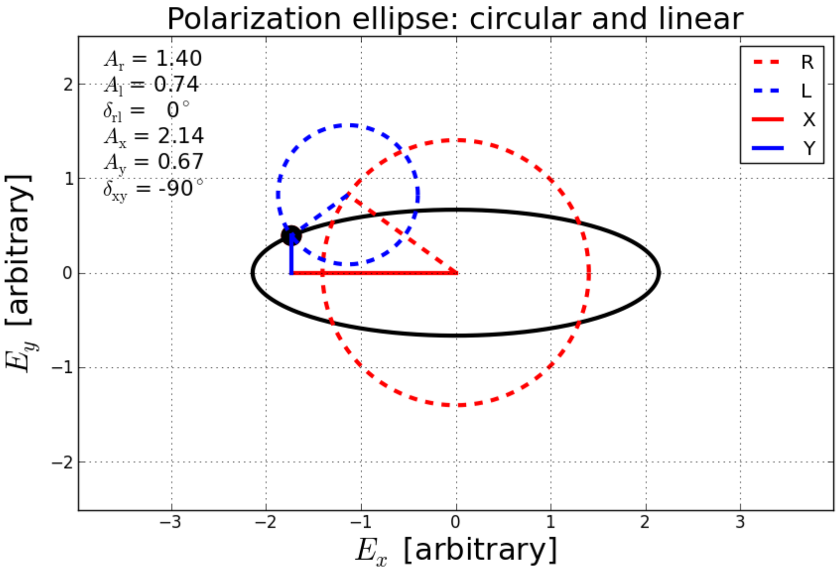
0
−1
−2
−3
−2
−1
0
1
2
3


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
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−1
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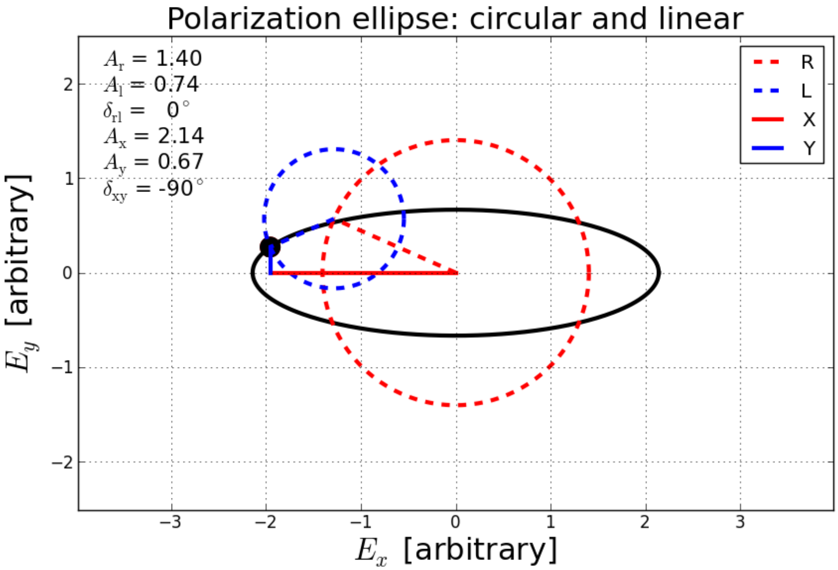
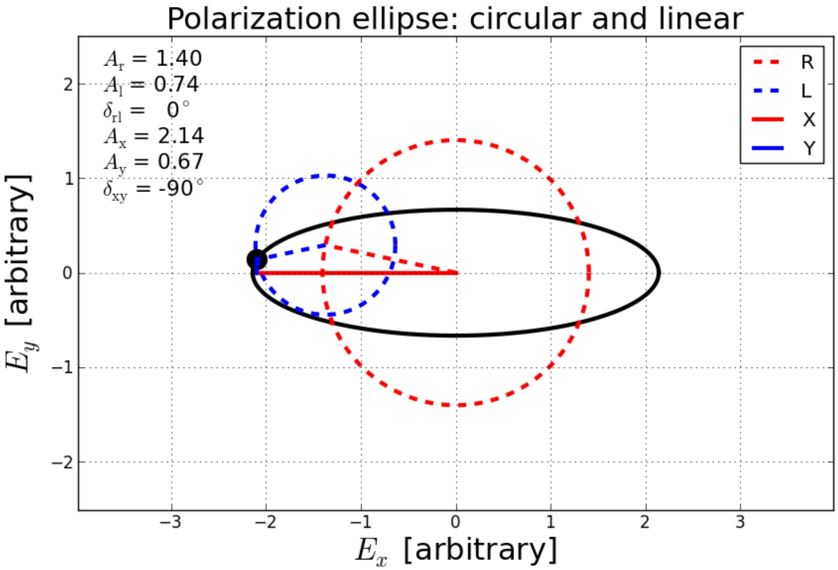
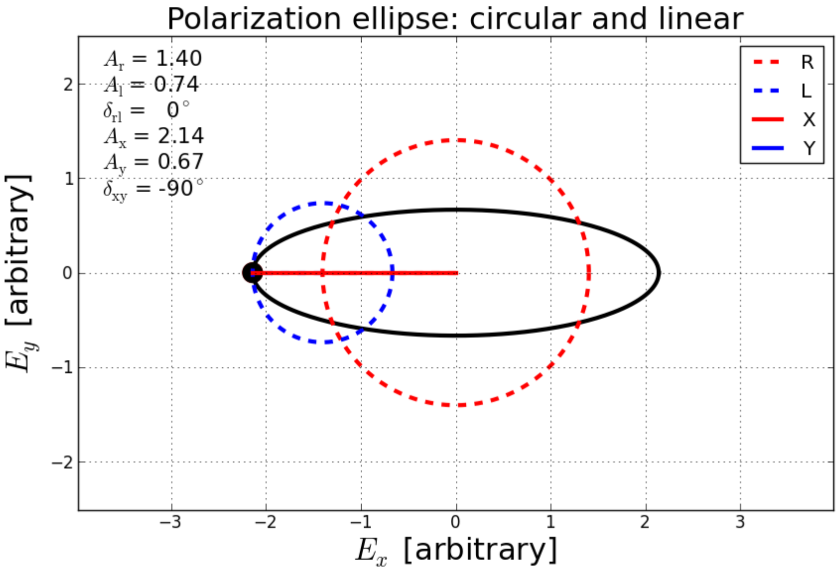
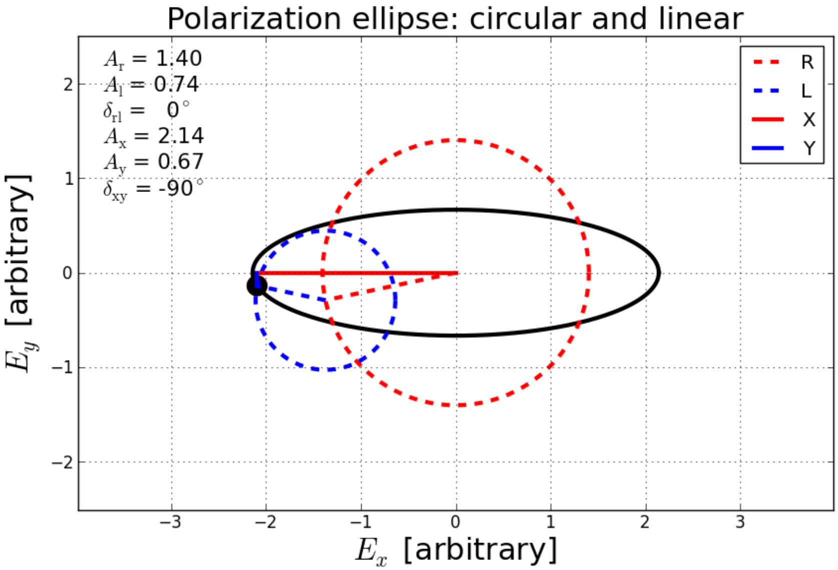
−2
−1
0
1
2
3


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
−3
−2
−1
0
1
2
3
2
1
0
−1
−2


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3

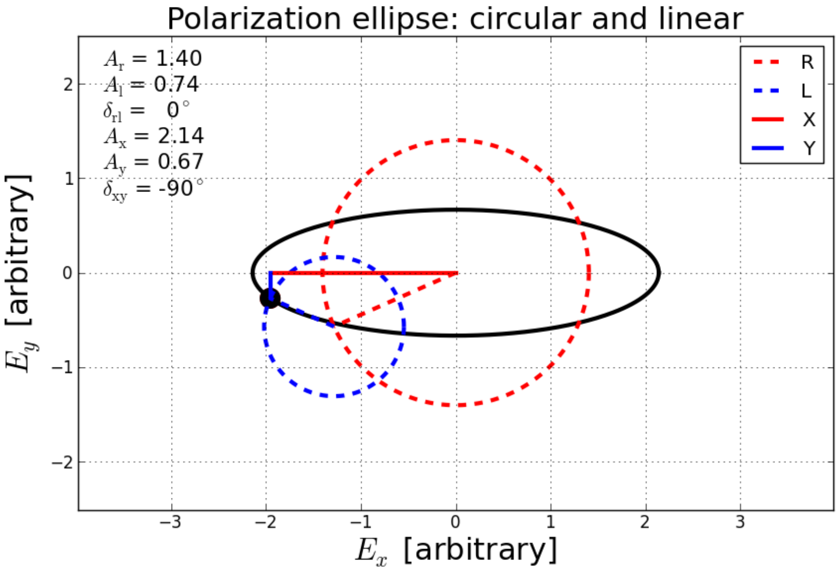
Polarization ellipse: circular and linear 015

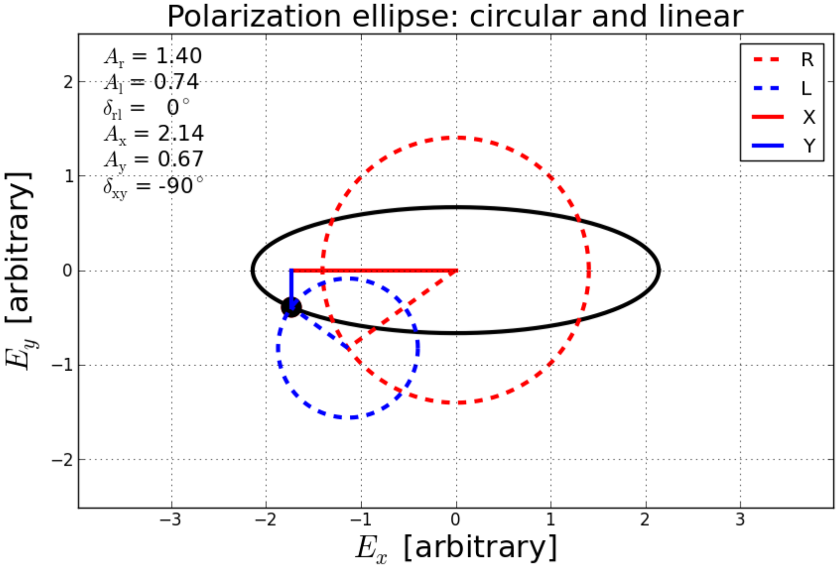




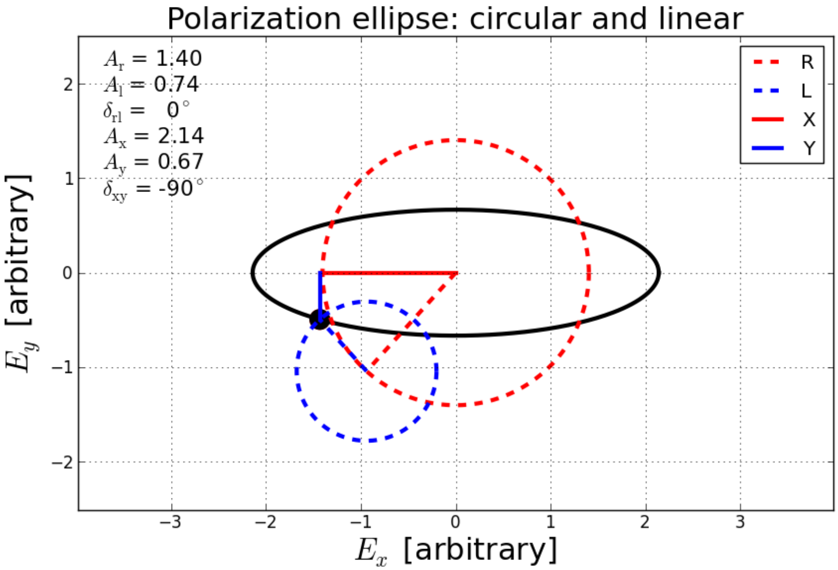
Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

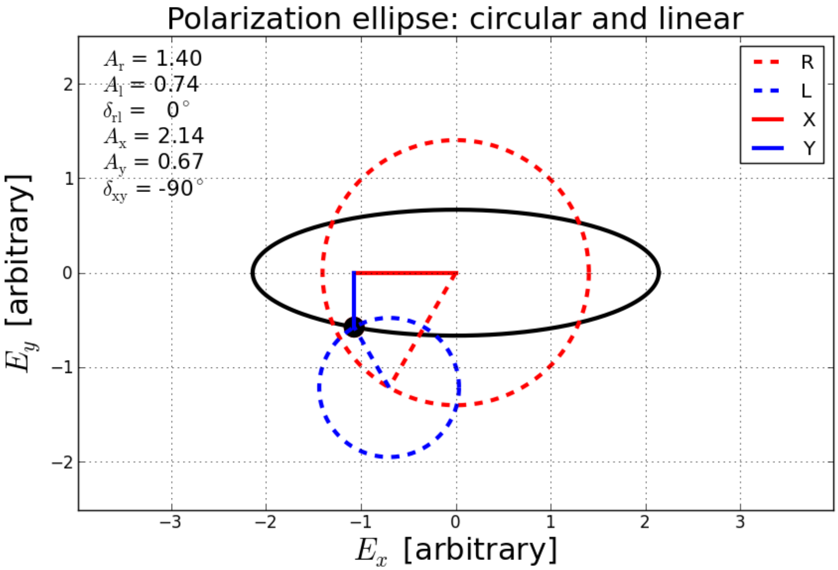
Polarization ellipse: circular and linear 017

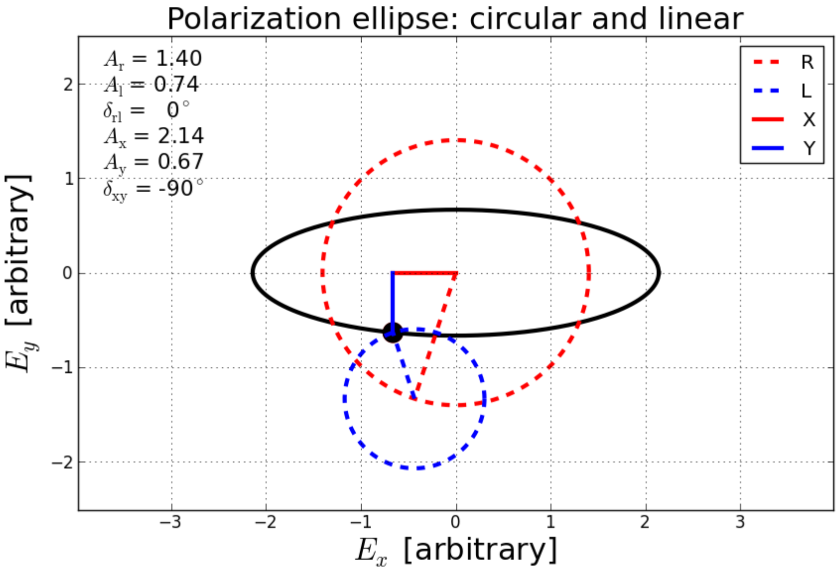


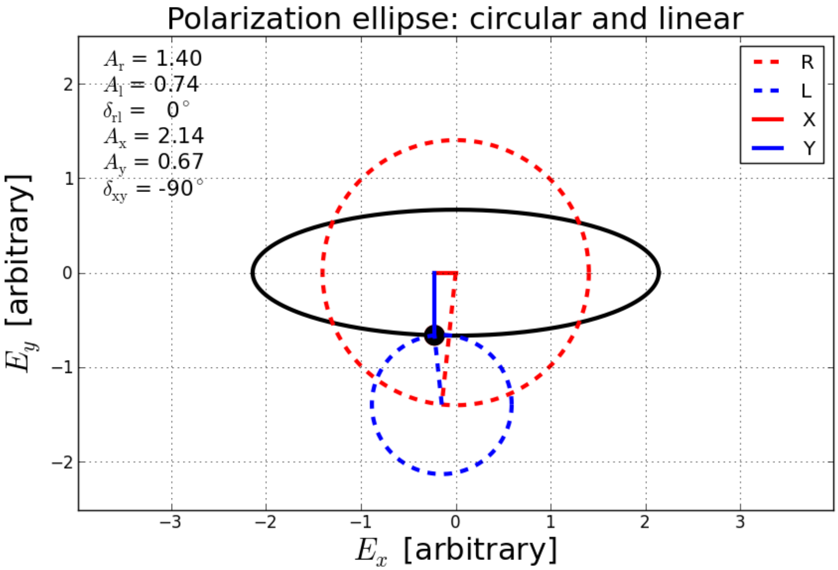

Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3

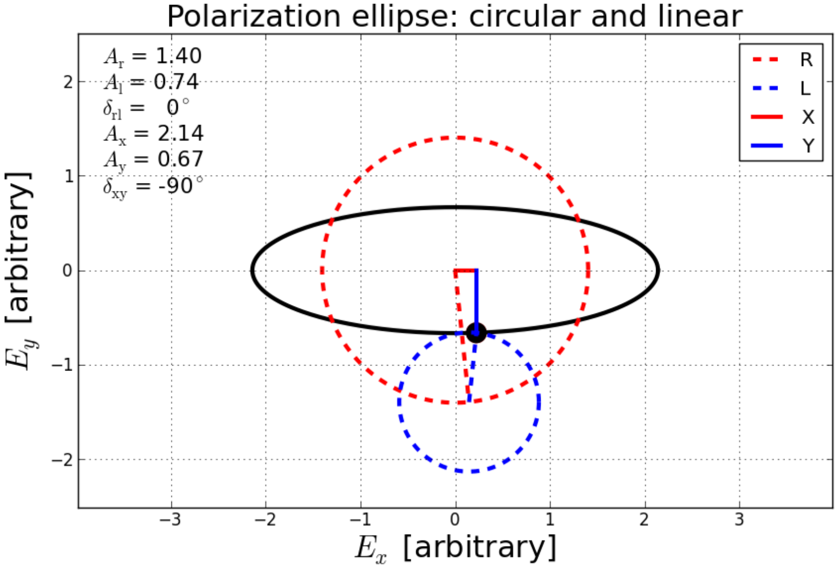
Polarization ellipse: circular and linear 019



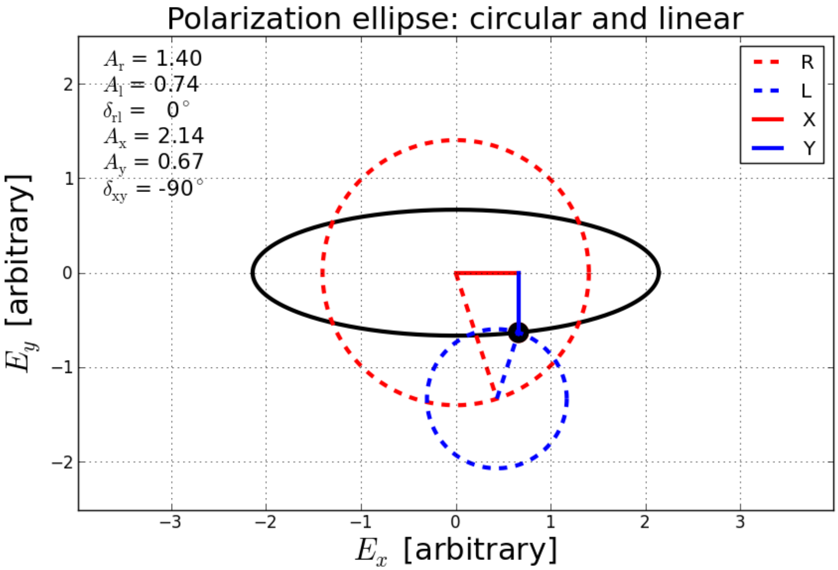

Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3

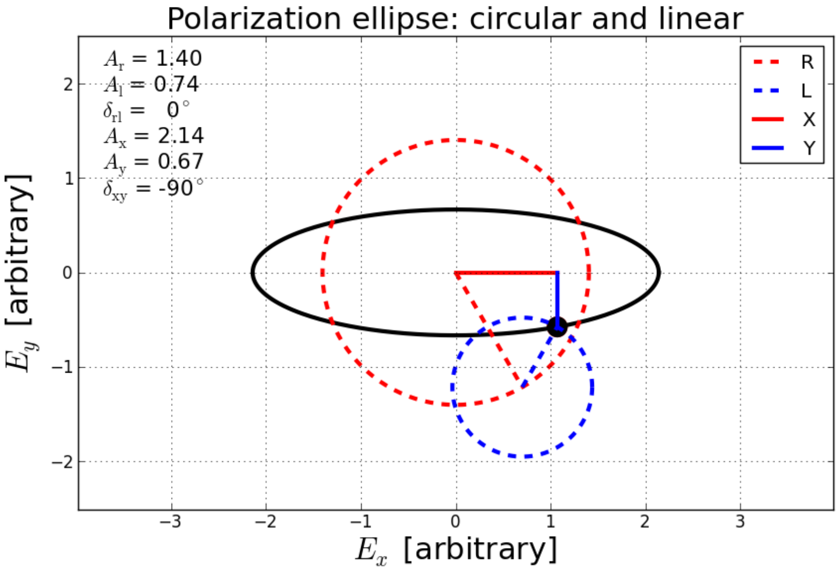

Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3

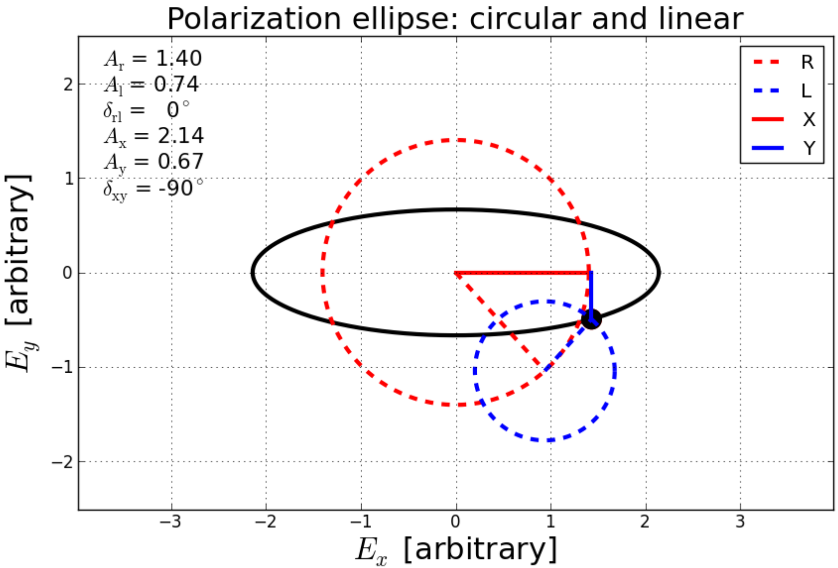
Polarization ellipse: circular and linear 024

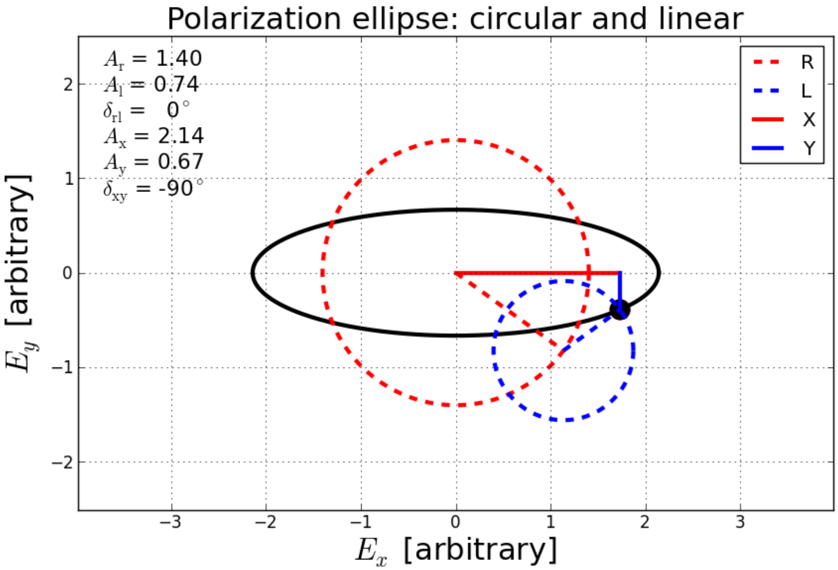


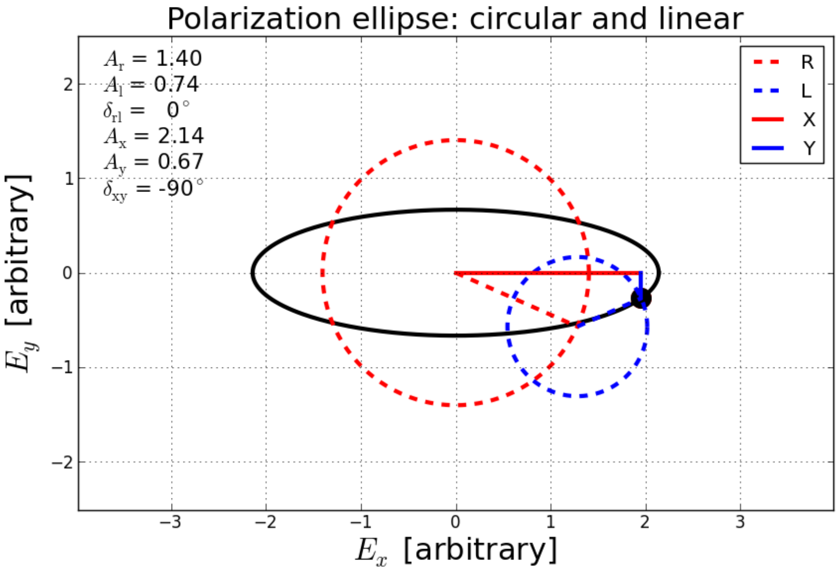
Polarization ellipse: circular and linear 025



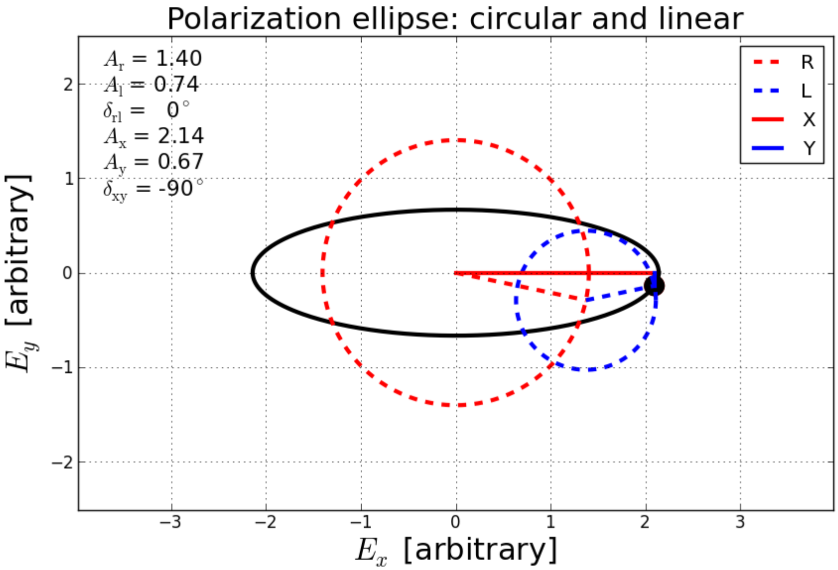
ASTRON


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

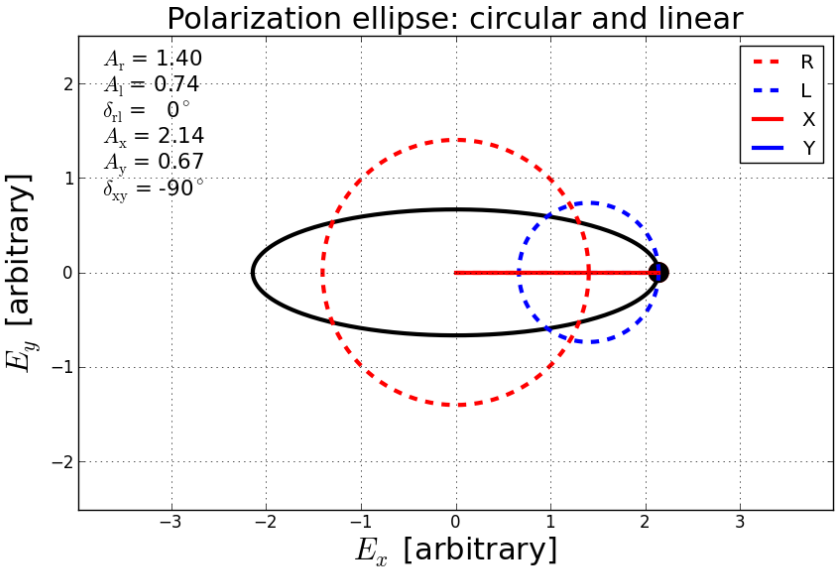

Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
3


Polarization ellipse: circular and linear
A_r = 1.40
A_l = 0.74
δ_{rl} = 0°
A_x = 2.14
A_y = 0.67
δ_{xy} = -90°
R
L
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3

Polarization ellipse: circular and linear 029



Polarization ellipse: circular and linear 030



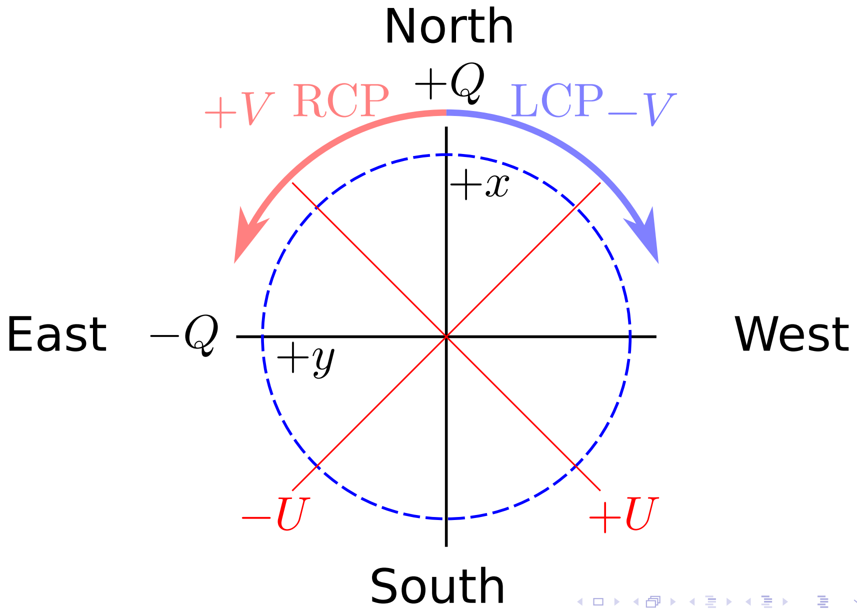
- Three parameters enough
- Same units is convenient
- George Stokes defined four parameters (1856)
- Chandrasekhar introduced them to astronomy (1946)

$$I = A_x^2 + A_y^2 \qquad I = A_r^2 + A_l^2$$
$$Q = A_x^2 - A_y^2 \qquad Q = 2A_r A_l \cos\delta_{rl}$$
$$U = 2A_x A_y \cos\delta_{xy} \qquad U = 2A_r A_l \sin\delta_{rl}$$
$$V = 2A_x A_y \sin\delta_{xy} \qquad V = A_r^2 - A_l^2$$

- Monochromatic wave 100% polarized:

$$I^2 = Q^2 + U^2 + V^2$$

IAU conventions



Stokes parameters revisited

Quasi-monochromatic approximation

- Monochromatic radiation does not exist
- Finite bandwidth $\Delta\nu$; averaging time $\tau \gg \Delta\nu^{-1}$

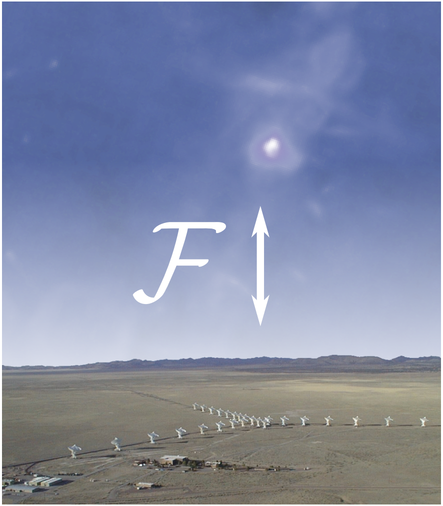
$$
\begin{aligned}
I &= \langle A_{\mathrm{x}}^2 \rangle + \langle A_{\mathrm{y}}^2 \rangle \\
Q &= \langle A_{\mathrm{x}}^2 \rangle - \langle A_{\mathrm{y}}^2 \rangle \\
U &= \langle 2A_{\mathrm{x}}A_{\mathrm{y}} \cos\delta_{\mathrm{xy}} \rangle \\
V &= \langle 2A_{\mathrm{x}}A_{\mathrm{y}} \sin\delta_{\mathrm{xy}} \rangle
\end{aligned}
\qquad
\begin{aligned}
I &= \langle A_{\mathrm{r}}^2 \rangle + \langle A_{\mathrm{l}}^2 \rangle \\
Q &= \langle 2A_{\mathrm{r}}A_{\mathrm{l}} \cos\delta_{\mathrm{rl}} \rangle \\
U &= \langle 2A_{\mathrm{r}}A_{\mathrm{l}} \sin\delta_{\mathrm{rl}} \rangle \\
V &= \langle A_{\mathrm{r}}^2 \rangle - \langle A_{\mathrm{l}}^2 \rangle
\end{aligned}
$$

$$I^2 \geq Q^2 + U^2 + V^2$$

- Fractional linear pol: $p = \sqrt{Q^2 + U^2}/I \leq 1$
- Fractional circular pol: $v = \|V\|/I \leq 1$

Outline

Introducing Stokes visibilities



$$
\begin{aligned}
\mathcal{I}(u,v) &= \mathcal{F}^{+}(I(l,m))\\
\mathcal{Q}(u,v) &= \mathcal{F}^{+}(Q(l,m))\\
\mathcal{U}(u,v) &= \mathcal{F}^{+}(U(l,m))\\
\mathcal{V}(u,v) &= \mathcal{F}^{+}(V(l,m)),
\end{aligned}
$$

where

$$
\mathcal{F}^{+}(f) = \int_{lm} f \mathrm{e}^{+2\pi \mathrm{i}\nu(ul+vm)/c} \mathrm{d}l \mathrm{d}m
$$

Cartesian

$$E_x = \Re\left\{A_x e^{2\pi i\nu t}\right\}$$
$$E_y = \Re\left\{A_y e^{i\delta_{xy}} e^{2\pi i\nu t}\right\}$$

$$
\begin{aligned}
I &= \langle A_x^2\rangle + \langle A_y^2\rangle &&= \langle E_x E_x^*\rangle + \langle E_y E_y^*\rangle \\
Q &= \langle A_x^2\rangle - \langle A_y^2\rangle &&= \langle E_x E_x^*\rangle - \langle E_y E_y^*\rangle \\
U &= \langle 2A_x A_y \cos\delta_{xy}\rangle &&= \langle E_x E_y^*\rangle + \langle E_y E_x^*\rangle \\
V &= \langle 2A_x A_y \sin\delta_{xy}\rangle &&= i\left(\langle E_x E_y^*\rangle - \langle E_y E_x^*\rangle\right)
\end{aligned}
$$

Circular

$$E_{\mathrm{r}} = \Re\left\{A_{\mathrm{r}} \mathrm{e}^{2\pi\mathrm{i}\nu t}\right\}$$

$$E_{\mathrm{l}} = \Re\left\{A_{\mathrm{l}} \mathrm{e}^{\mathrm{i}\delta_{\mathrm{rl}}} \mathrm{e}^{2\pi\mathrm{i}\nu t}\right\}$$

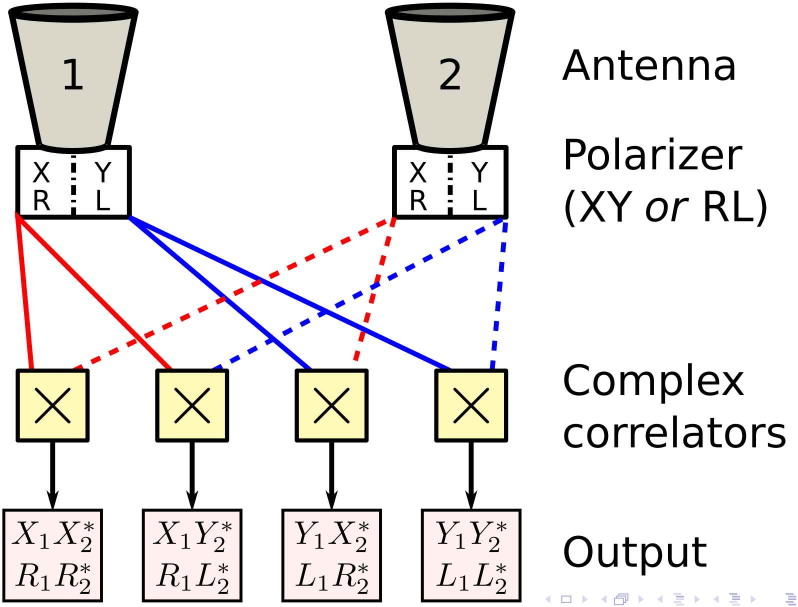
$$I = \langle A_{\mathrm{r}}^2 \rangle + \langle A_{\mathrm{l}}^2 \rangle = \langle E_{\mathrm{r}} E_{\mathrm{r}}^* \rangle + \langle E_{\mathrm{l}} E_{\mathrm{l}}^* \rangle$$

$$Q = \langle 2 A_{\mathrm{r}} A_{\mathrm{l}} \cos \delta_{\mathrm{rl}} \rangle = \langle E_{\mathrm{r}} E_{\mathrm{l}}^* \rangle + \langle E_{\mathrm{l}} E_{\mathrm{r}}^* \rangle$$

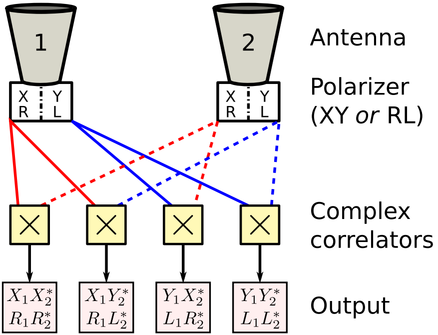
$$U = \langle 2 A_{\mathrm{r}} A_{\mathrm{l}} \sin \delta_{\mathrm{rl}} \rangle = \mathrm{i}\left(\langle E_{\mathrm{r}} E_{\mathrm{l}}^* \rangle - \langle E_{\mathrm{l}} E_{\mathrm{r}}^* \rangle\right)$$

$$V = \langle A_{\mathrm{r}}^2 \rangle - \langle A_{\mathrm{l}}^2 \rangle = \langle E_{\mathrm{r}} E_{\mathrm{r}}^* \rangle - \langle E_{\mathrm{l}} E_{\mathrm{l}}^* \rangle$$

Correlating interferometer



Stokes visibilities from correlator outputs



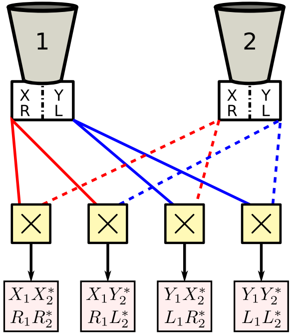
Cartesian

$$\mathcal{I} = x_1 x_2^* + y_1 y_2^*$$
$$\mathcal{Q} = x_1 x_2^* - y_1 y_2^*$$
$$\mathcal{U} = x_1 y_2^* + y_1 x_2^*$$
$$\mathcal{V} = \mathrm{i}\,(x_1 y_2^* - y_1 x_2^*)$$

Circular

$$\mathcal{I} = r_1 r_2^* + l_1 l_2^*$$
$$\mathcal{Q} = r_1 l_2^* + l_1 r_2^*$$
$$\mathcal{U} = \mathrm{i}\,(r_1 l_2^* - l_1 r_2^*)$$
$$\mathcal{V} = r_1 r_2^* - l_1 l_2^*$$

- From here on, $\langle\cdot\rangle$ is implied for correlator outputs.



From here on, *p* and *q* designate either *x* and *y*, or *r* and *l*.

- Polarizers produce vector:

$$\mathbf{e}_i = \begin{pmatrix} p_i \\ q_i \end{pmatrix}$$

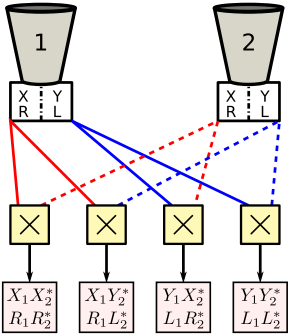
- Correlator multiplies:

$$\mathbf{E}_{ij} = \mathbf{e}_i \mathbf{e}_j^\dagger = \begin{pmatrix} p_i \\ q_i \end{pmatrix} \begin{pmatrix} p_j^* & q_j^* \end{pmatrix}$$

$$\mathbf{E}_{ij} = \begin{pmatrix} p_i p_j^* & p_i q_j^* \\ q_i p_j^* & q_i q_j^* \end{pmatrix}$$

- $\mathbf{E}_{ij}$ is the coherency matrix

ASTRON



Until now...

- Assumed all systems perfect

From now...

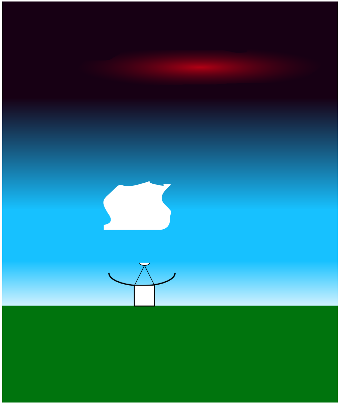
- Assume all systems linear:

$$\mathbf{e}'_i = \mathbf{J}_i \mathbf{e}_i$$

- $\mathbf{J}_i$ (2×2) is Jones matrix
- Cross correlation:

$$\begin{aligned}
\mathbf{E}'_{ij} &= \mathbf{e}'_i \mathbf{e}'^{\dagger}_j \\
\mathbf{E}'_{ij} &= \mathbf{J}_i \mathbf{e}_i \left(\mathbf{J}_j \mathbf{e}_j\right)^{\dagger} \\
\mathbf{E}'_{ij} &= \mathbf{J}_i \mathbf{e}_i \mathbf{e}_j^{\dagger} \mathbf{J}_j^{\dagger} \\
\mathbf{E}'_{ij} &= \mathbf{J}_i \mathbf{E}_{ij} \mathbf{J}_j^{\dagger}
\end{aligned}$$

Outline



- The measurement equation:

$$\mathbf{E}'_{ij} = \mathbf{J}_i \mathbf{E}_{ij} \mathbf{J}_j^{\dagger}$$

- Invertable!

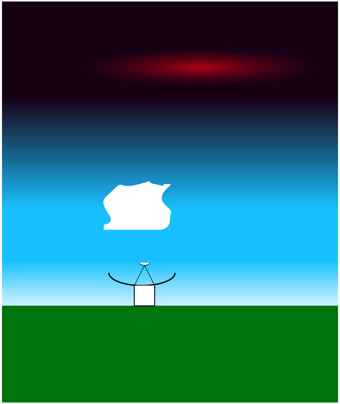
$$\mathbf{E}_{ij} = \mathbf{J}_i^{-1} \mathbf{E}_{ij} \mathbf{J}_j^{\dagger -1},$$

- where

$$\mathbf{J} = \mathbf{RPDOWTF} \ldots$$

- . . . riiiiight. . .

Measurement equation: examples



- Perfect instrument:

$$\mathbf{J} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

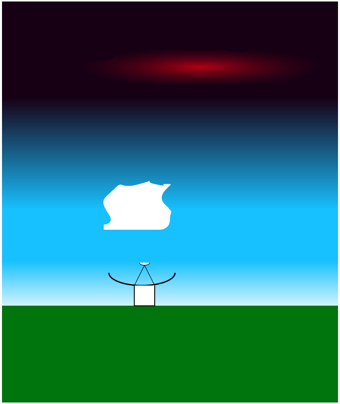
- Ionospheric time delay:

$$\mathbf{J} = \begin{pmatrix} \mathrm{e}^{2\pi\mathrm{i}\nu\tau} & 0 \\ 0 & \mathrm{e}^{2\pi\mathrm{i}\nu\tau} \end{pmatrix}$$

- Receiver gain:

$$\mathbf{J} = \begin{pmatrix} g_{\mathrm{p}} & 0 \\ 0 & g_{\mathrm{q}} \end{pmatrix}$$

Measurement equation: examples



- Polarization leakage:

$$\mathbf{J} = \begin{pmatrix} g_{\mathrm{p}} & d_{\mathrm{qp}} \\ d_{\mathrm{pq}} & g_{\mathrm{q}} \end{pmatrix}$$

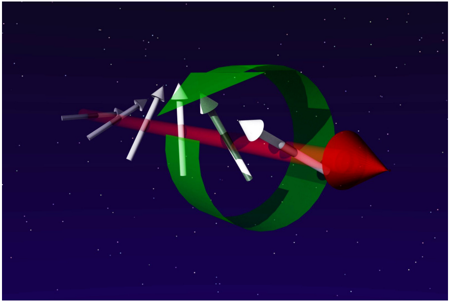
- Parallactic angle or feed rotation XY:

$$\mathbf{J} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

- Parallactic angle or feed rotation RL:

$$\mathbf{J} = \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\theta} \end{pmatrix}$$

ASTRON

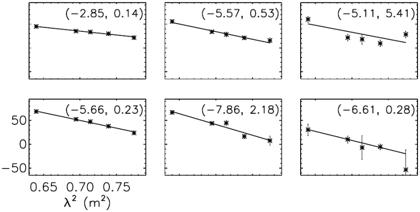


Process

- Modifies polarization state
- Delay between LCP and RCP
- Rotates linear pol angle
- $\Delta\chi = \chi_0 + \phi\lambda^2$

$$\phi = 0.812 \int_{\text{there}}^{\text{here}} n_e \mathbf{B} \cdot d\mathbf{l}$$

λ^2 law *Haverkorn et al. (2001)*



Polarimetry provides

- Source plasma properties
- Intervening plasma properties
- Rare cases: 3D tomography

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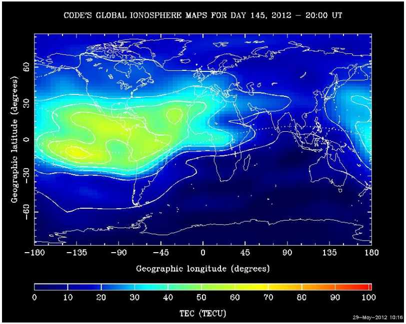
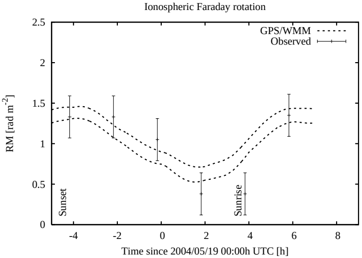
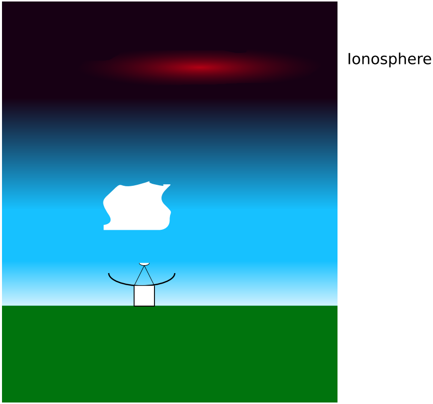




- Remember: $\Delta\chi = \chi_0 + \phi\lambda^2$
- Faraday depth

$$\phi = 0.812 \int_{\text{there}}^{\text{here}} n_e \mathbf{B} \cdot \mathrm{d}\mathbf{l}$$

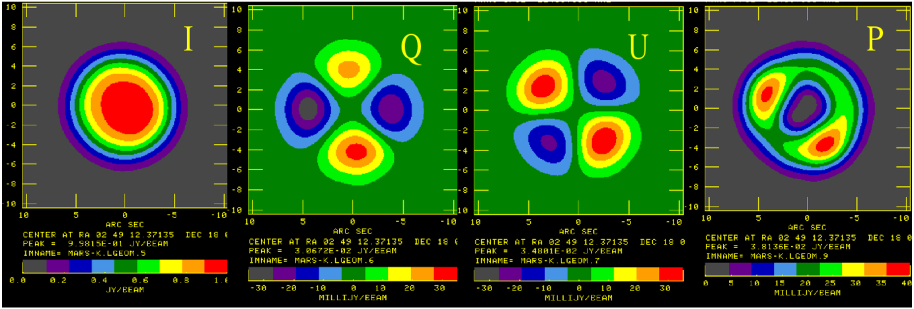
- ionosphere: plasma within Earth's magnetic field
- $\phi \approx$ -10 – +10 rad m^{-2}
- *Very* significant below 1 GHz
- Use TEC/IGRF models for correction, check with pulsar.



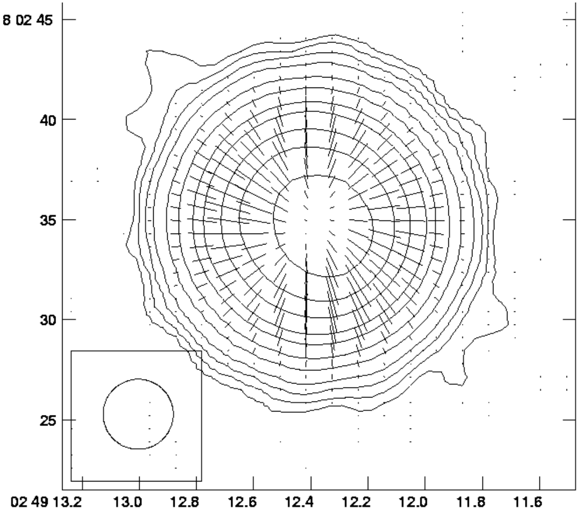
- $\Delta\chi = \chi_0 + \phi\lambda^2$
- Rotation of linear pol = delay between RCP and LCP
- Antennas see different ionosphere
- Leakage from LL to RR or v.v. after cross correlation
- Rotates $\begin{pmatrix} \mathcal{I} \\ \mathcal{V} \end{pmatrix}$ vector
- Important below 300 MHz at baselines $\geq$ 20 km

Outline

Mars with VLA at 23.4 GHz *Perley*



- Thermal emission
- What do pol vectors look like?
- Why is it even polarized?



Peak contour flux = 9.9738E-01 JY/BEAM
Levs = 9.974E-03 * (-0.250, 0.250, 0.500, 1, 2,
5, 10, 20, 30, 50, 70, 90)

Reading list

- Born & Wolf *Principles of optics*
- Thompson, Moran & Swenson *Interferometry and Synthesis in Radio Astronomy*
- Taylor, Carilli & Perley *Synthesis Imaging in Radio Astronomy II*
- Bracewell *The Fourier Transform & Its Applications*
- Hamaker, Bregman & Sault *Understanding radio polarimetry: paper I*(1996)
- Sault, Hamaker& Bregman *paper II*(1996)
- Hamaker & Bregman *paper III* (1996)
- Hamaker *paper IV* (2000)
- Hamaker *paper V* (2006)
- Brentjens & de Bruyn *Faraday rotation measure synthesis* (2005)