Noise Budget Analysis of the FPA

Rob Maaskant, June 21st 2005

Outline:

- Do we need to load the dummy (passive) array elements for achieving the highest A_{eff}/T_{sys}?
  - Positive effects of resistive loading
  - Negative effects of resistive loading

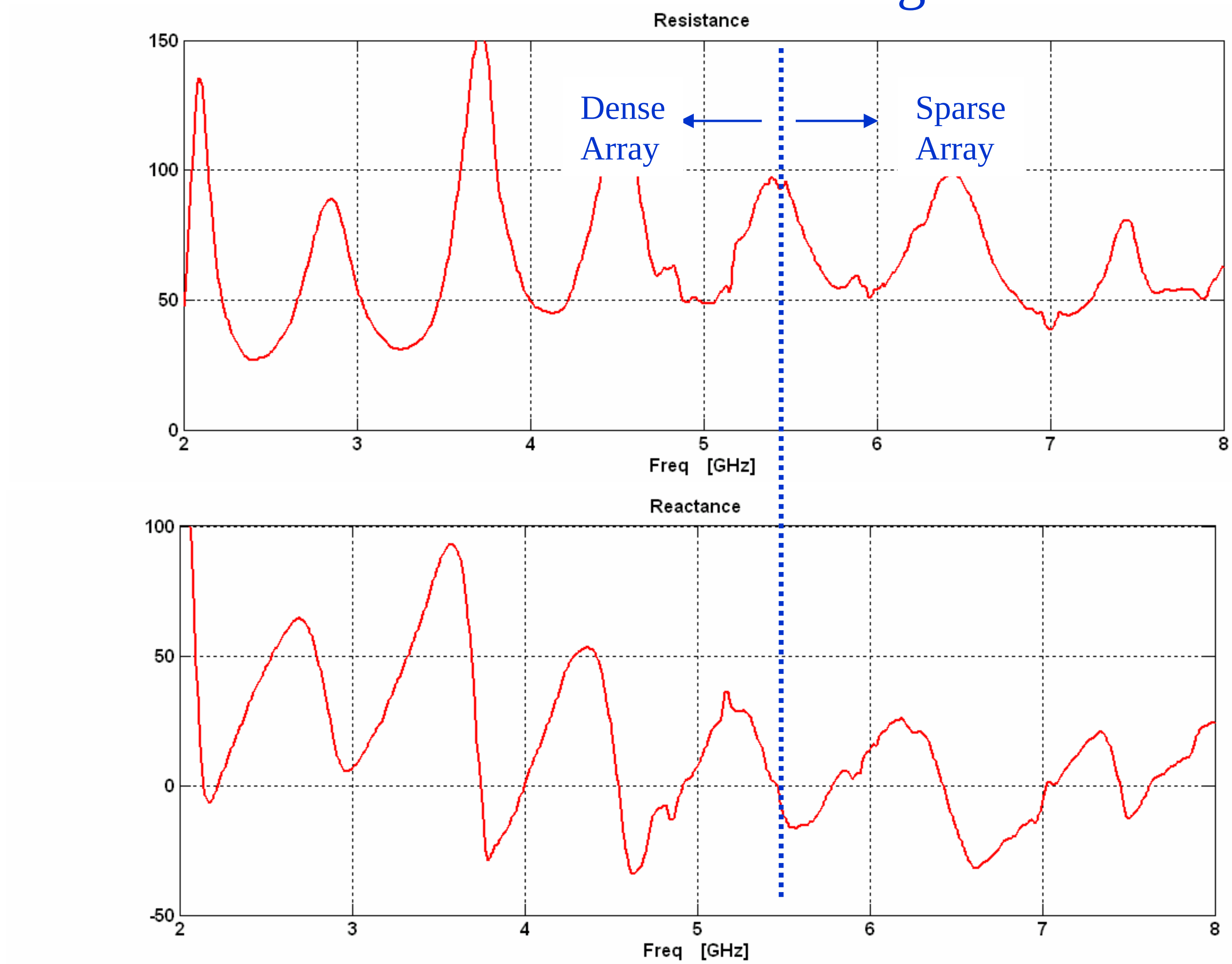
- A Noise Budget Analysis for the FPA
  - Predicted T_{sys} (theoretical model)
  - Measured T_{sys} (hot-cold measurement)

- Conclusions

R. Maaskant, E.E.M. Woestenburg and M.J. Arts, "A Generalized Method of Modeling the Sensitivity of Array Antennas at System Level", 34th EUMC Amsterdam 2004, pp. 1541-1544.

Resistive Loading

Advantages

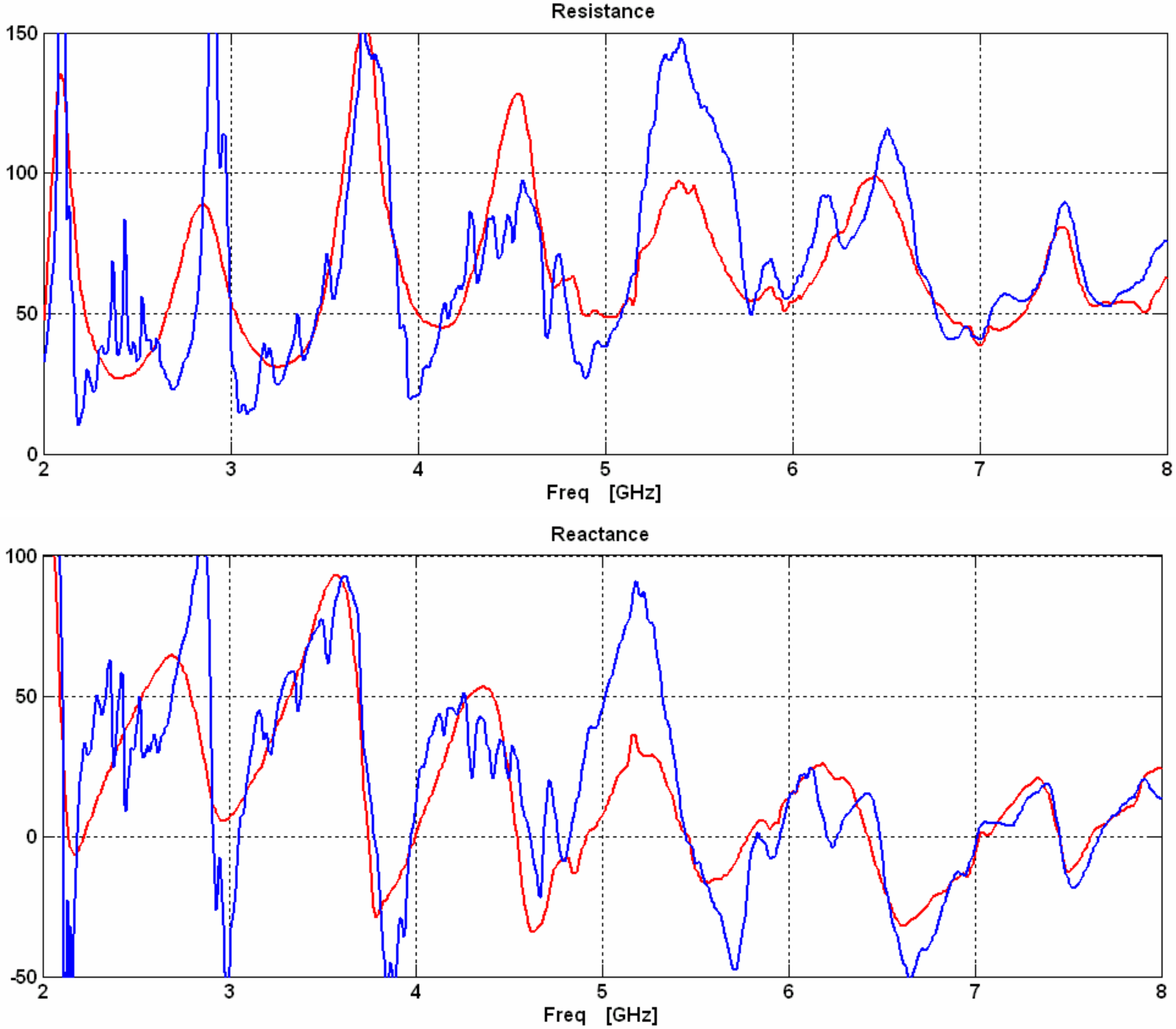


Shown in figures:
Measured passive antenna input impedance (resistance and reactance) of central element

Surrounding elements resistively loaded

Resistive Loading

Advantages



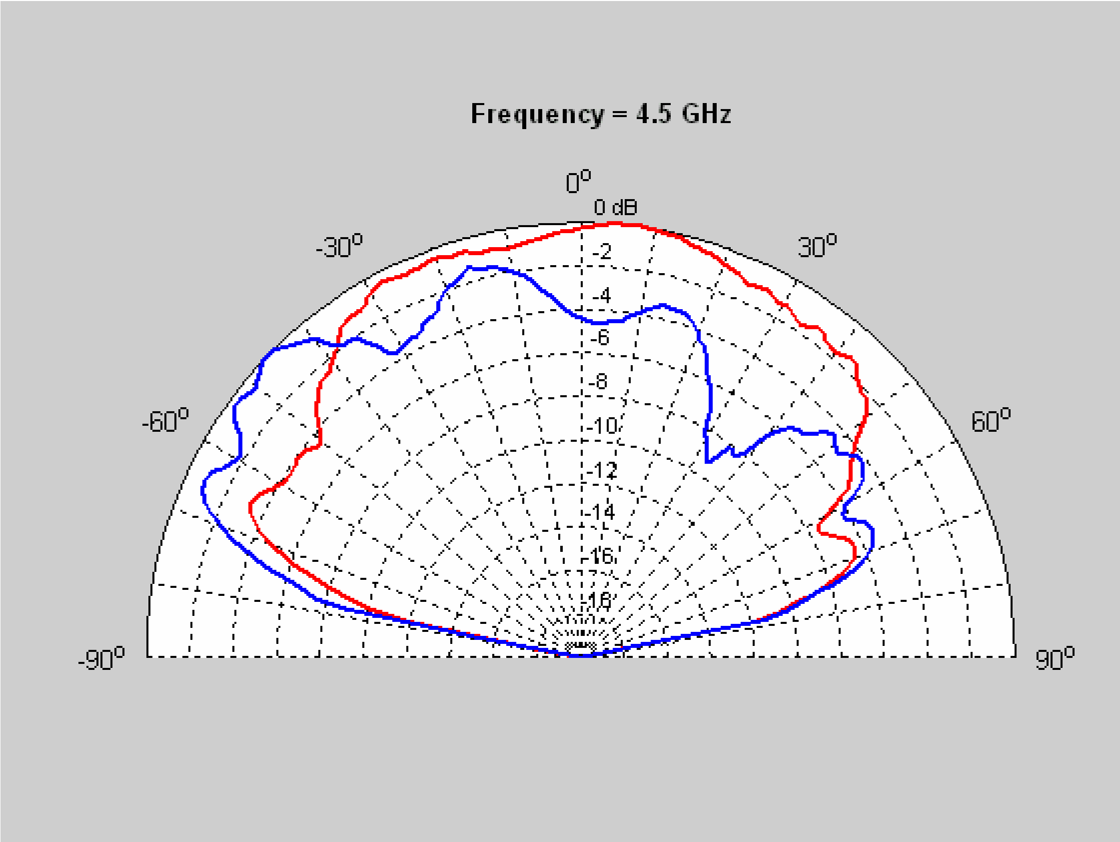
Shown in figures:
Measured passive antenna input impedance (resistance and reactance) of central element

Surrounding elements resistively loaded

Surrounding elements left open

Resistive Loading

Advantages



Surrounding elements resistively loaded (red line)

Surrounding elements left open (blue line)

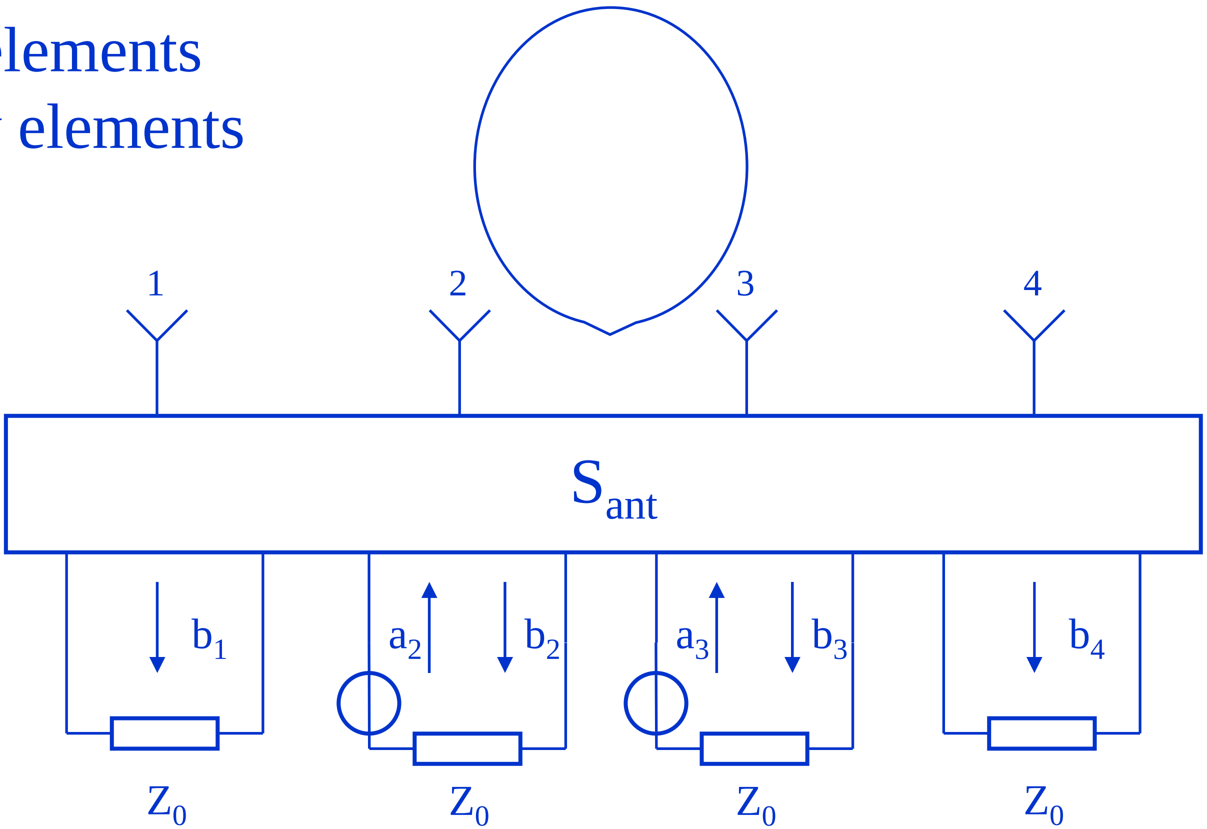
Advantages summarized:

- Impedance smoother over frequency band, especially in the dense regime where coupling is large
- Loading seems to improve symmetry of patterns, finiteness of array is less remarkable
- Ripples up to 5 dB are suppressed
- Frequency dependence of patterns decreases

Resistive Loading

Disadvantages

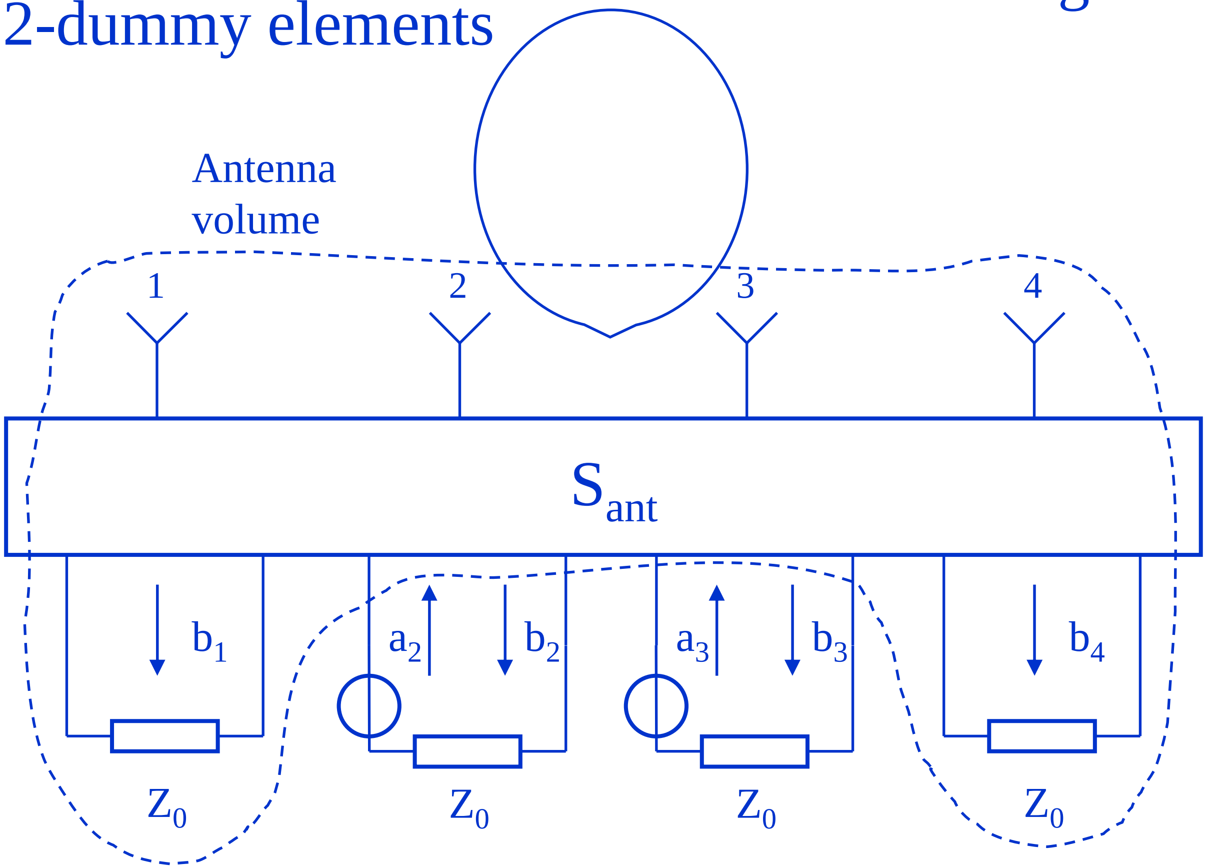
2-active elements
2-dummy elements



Resistive Loading

2-active elements
2-dummy elements

Disadvantages



$$\underline{b} = S^{ant}\,\underline{a}$$

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \begin{pmatrix} s_{11}^{ant} & s_{12}^{ant} & s_{13}^{ant} & s_{14}^{ant} \\ s_{21}^{ant} & s_{22}^{ant} & s_{23}^{ant} & s_{24}^{ant} \\ s_{31}^{ant} & s_{32}^{ant} & s_{33}^{ant} & s_{34}^{ant} \\ s_{41}^{ant} & s_{42}^{ant} & s_{43}^{ant} & s_{44}^{ant} \end{pmatrix} \begin{pmatrix} 0 \\ a_2 \\ a_3 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} = \begin{pmatrix} s_{12}^{ant} a_2 + s_{13}^{ant} a_3 \\ s_{22}^{ant} a_2 + s_{23}^{ant} a_3 \\ s_{32}^{ant} a_2 + s_{33}^{ant} a_3 \\ s_{42}^{ant} a_2 + s_{43}^{ant} a_3 \end{pmatrix}$$

$$P_{inc} = |a_2|^2 + |a_3|^2$$

$$P_{refl} = |b_2|^2 + |b_3|^2$$

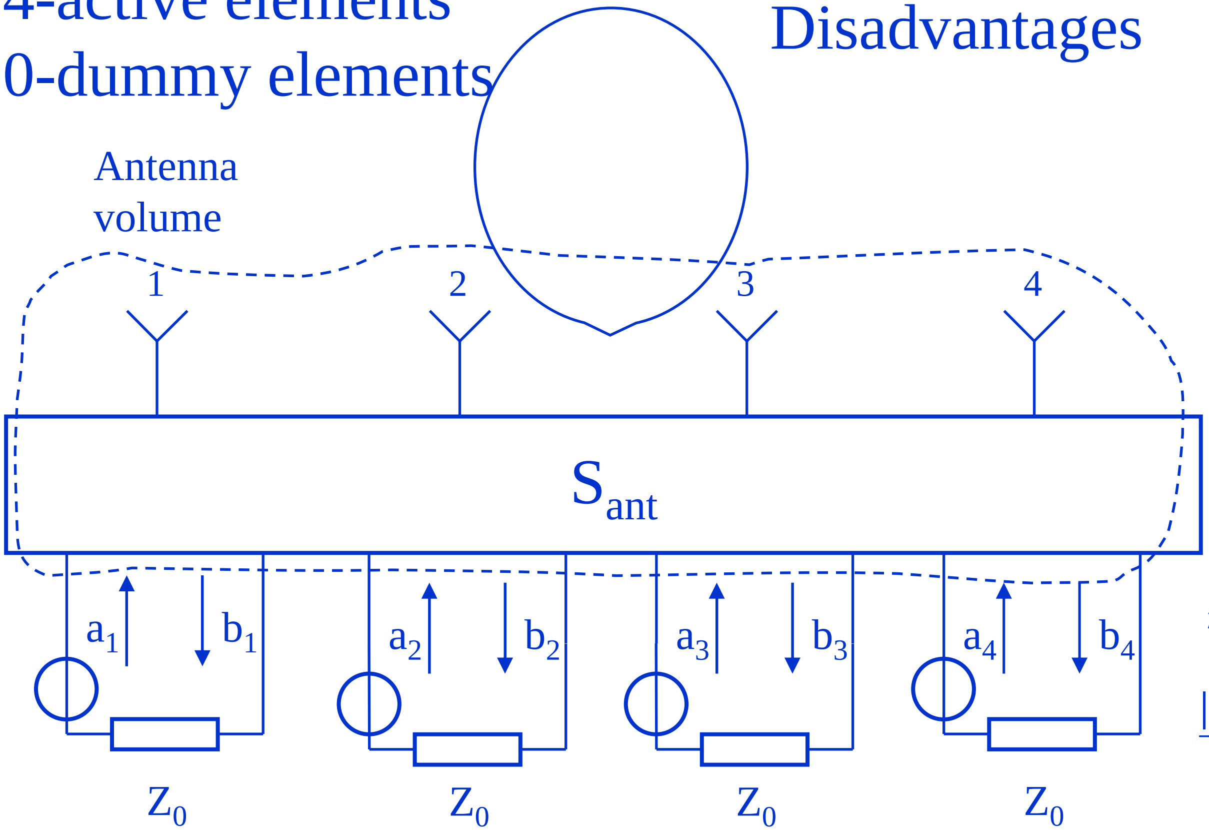
$$P_{loss}^{R} = |b_1|^2 + |b_4|^2$$

$$\eta^{ant} = \frac{P_{rad}}{P_{inc}} = \frac{P_{inc} - P_{refl} - P_{loss}}{P_{inc}} \approx \frac{P_{inc} - P_{refl} - P_{loss}^{R}}{P_{inc}} \Rightarrow \eta^{ant}(a_2, a_3) = \frac{|a_2|^2 + |a_3|^2 - |b_2|^2 - |b_3|^2 - |b_1|^2 - |b_4|^2}{|a_2|^2 + |a_3|^2}$$

Resistive Loading

4-active elements
0-dummy elements

Disadvantages



$$P_{inc} = |\cancel{a_1}|^2 + |a_2|^2 + |a_3|^2 + |\cancel{a_4}|^2$$

$$P_{refl} = |b_1|^2 + |b_2|^2 + |b_3|^2 + |b_4|^2$$

$$P_{loss}^R = 0 \, !$$

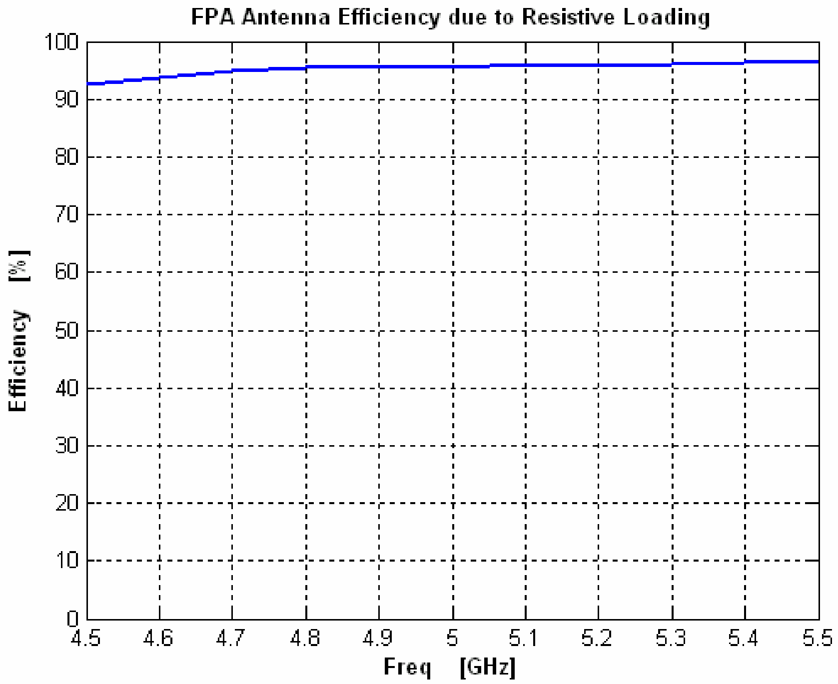
$$\eta^{ant} = \frac{P_{inc} - P_{loss}^R - P_{refl}}{P_{inc}} = \frac{|a_2|^2 + |a_3|^2 - |b_1|^2 - |b_4|^2 - |b_2|^2 - |b_3|^2}{|a_2|^2 + |a_3|^2}$$

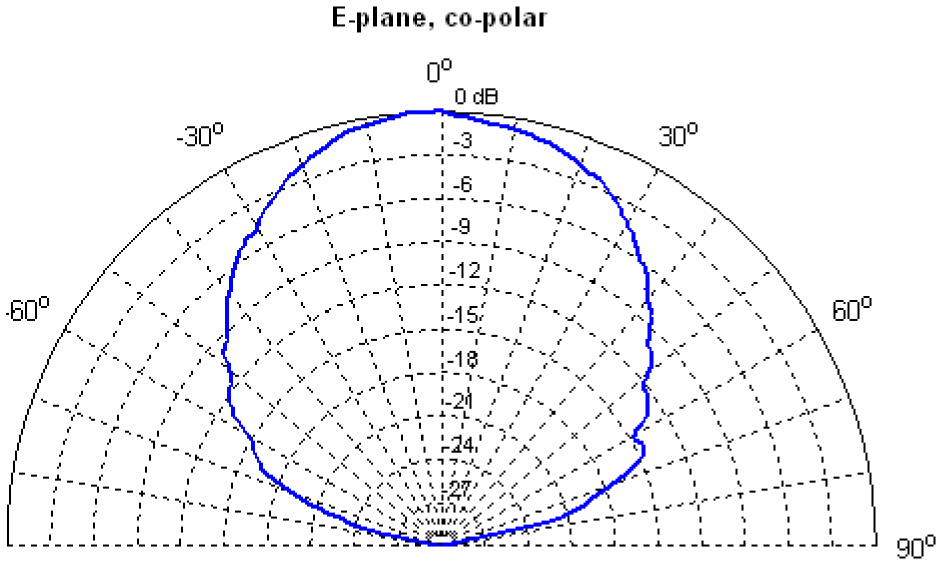
To obtain the same total illumination pattern, a_1=0 and a_4=0, because this total pattern is determined by a unique set of element patterns and weightings (element patterns are orthogonal)

The antenna efficiency remains the same, because the reflected power will be higher!

Resistive Loading

Disadvantages

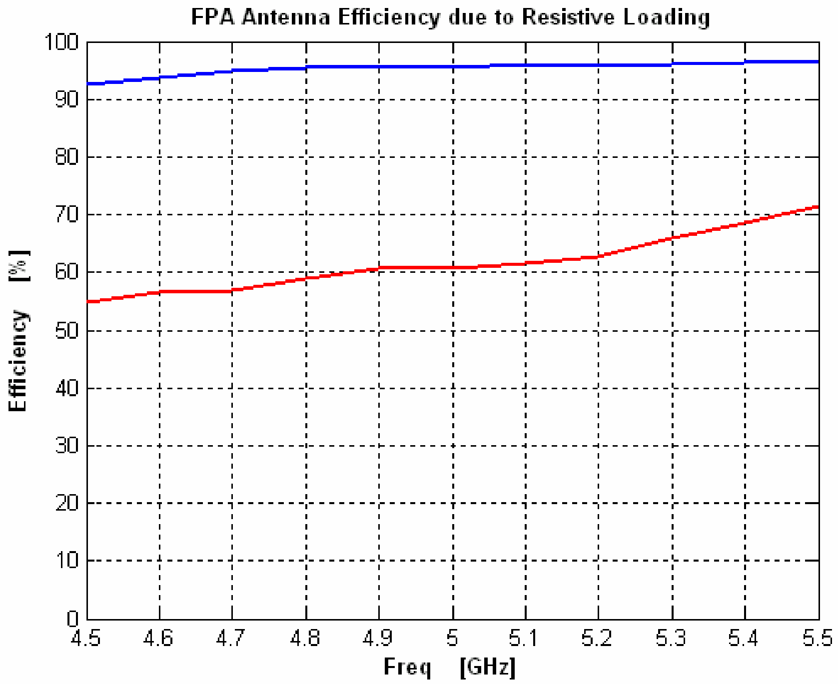


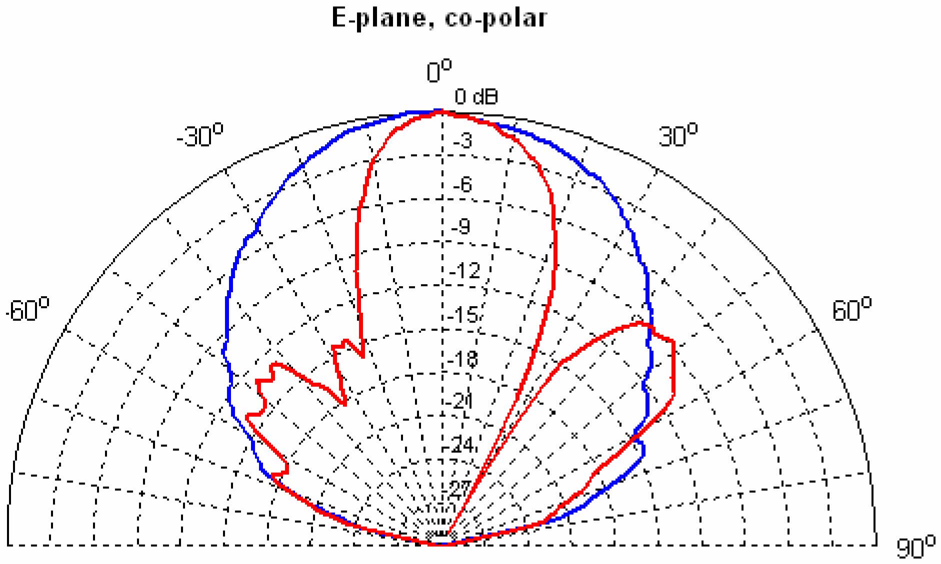


@ 5.0 GHz:

Resistive Loading

Disadvantages





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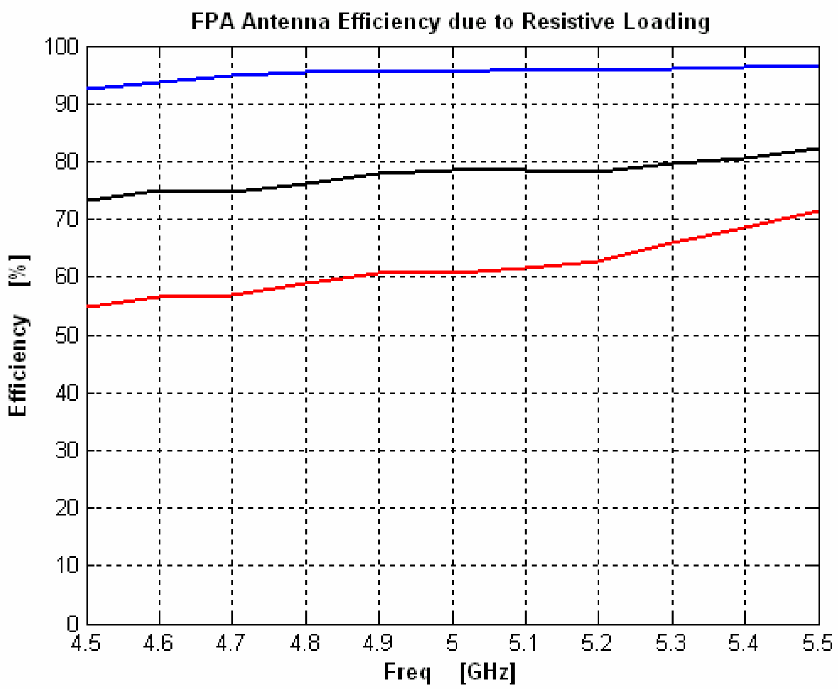
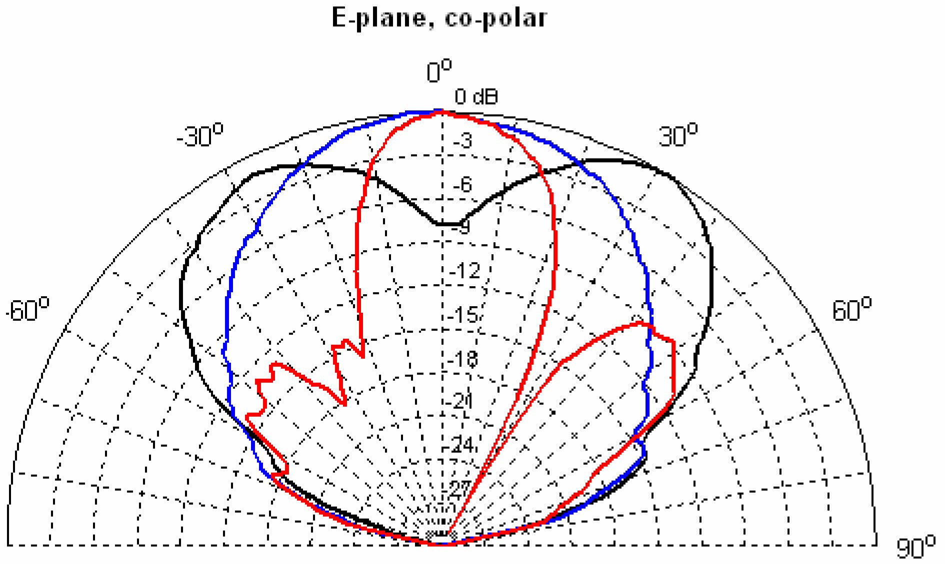
Resistive Loading

Disadvantages





@ 5.0 GHz:

Resistive Loading

Disadvantages

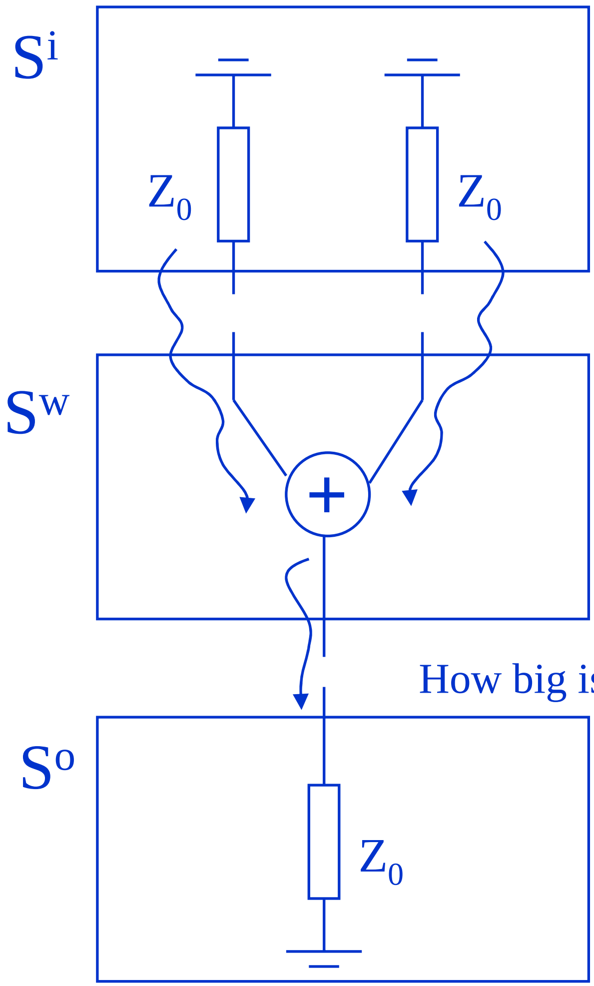
@ 5.0 GHz:

$$SENSITIVITY = \frac{A_{eff}}{T_{sys}} = \frac{A_{eff}^{MAX} \cdot 0.8}{T_{sys}^{MIN}}$$

?

Noise Wave Concept

Explained using an example

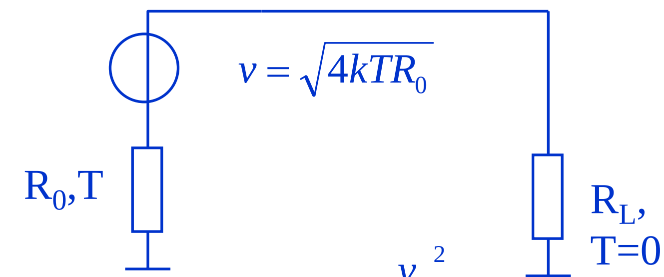


Two noisy resistors both at temp. T=300K

Wilkinson Power Combiner

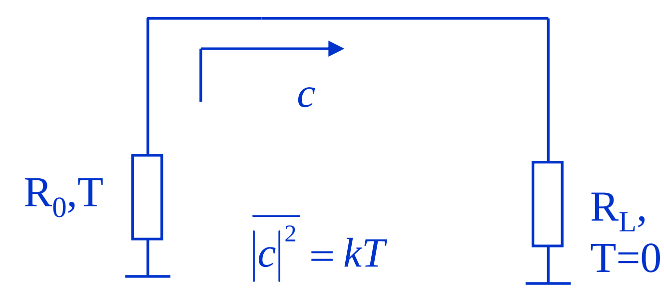
How big is the outgoing noise power ?

Characteristic termination at T=0K



$$P_L = \frac{v_L^{\,2}}{R_L}$$

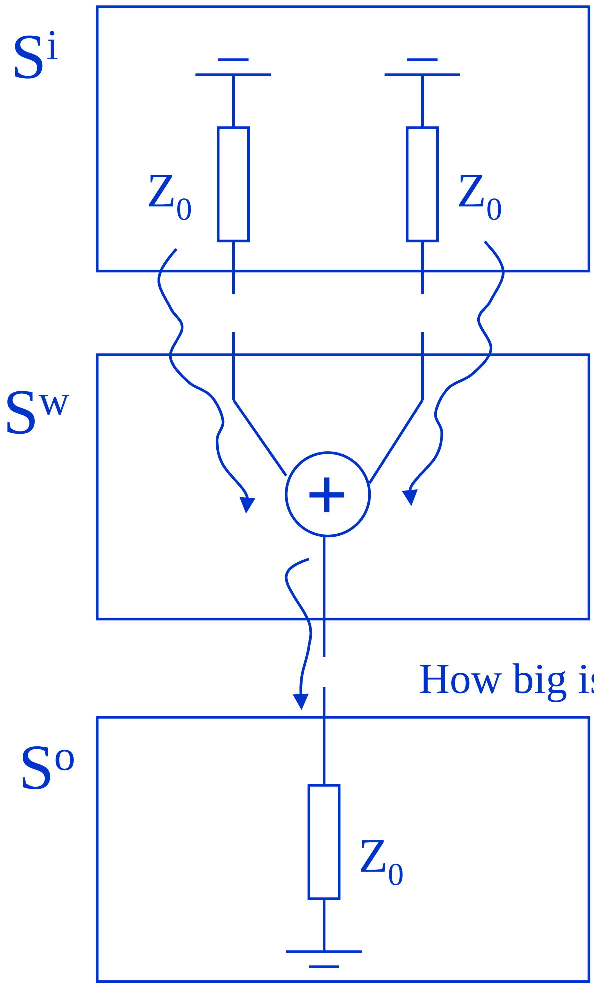
$$= \frac{\left(v\dfrac{R_L}{R_0+R_L}\right)^2}{R_L} = 4kT\frac{R_0R_L}{(R_0+R_L)^2}$$



$$P_L = \overline{|c|^2}\left(1-|\Gamma|^2\right) = kT\left(1-\left(\frac{R_L-R_0}{R_L+R_0}\right)^2\right)$$

Noise Wave Concept

Explained using an example



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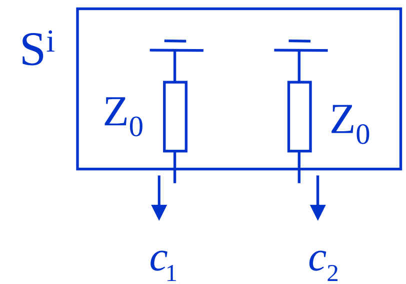
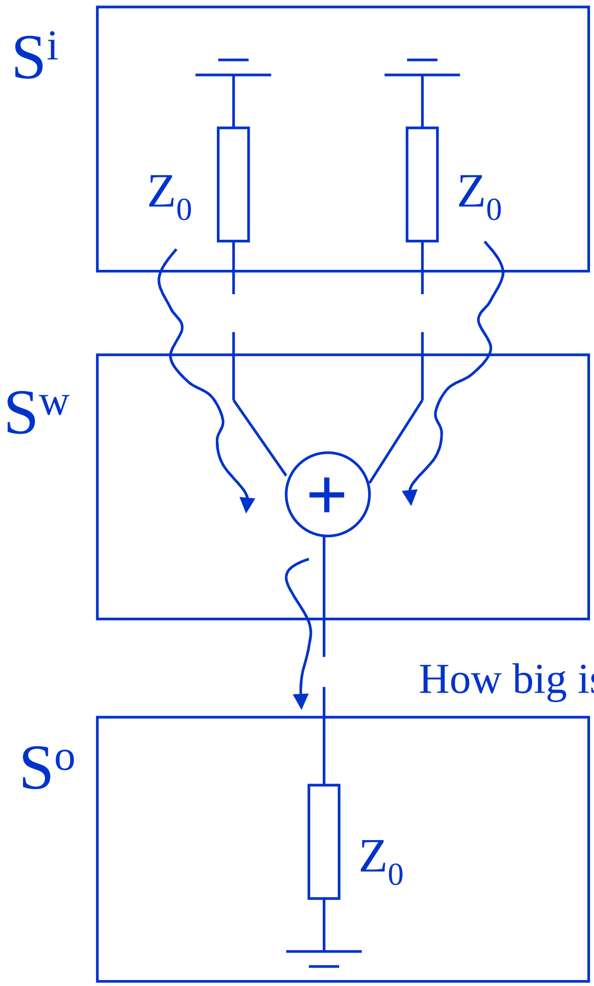


$$C = \begin{pmatrix} \overline{|c_1|^2} & \overline{c_1 c_2^*} \\ \overline{c_2^* c_2} & \overline{|c_2|^2} \end{pmatrix} = \begin{pmatrix} kT & 0 \\ 0 & kT \end{pmatrix}$$

Noise Wave Concept

Explained using an example



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Wilkinson Power
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How big is the outgoing noise power ?

Characteristic termination
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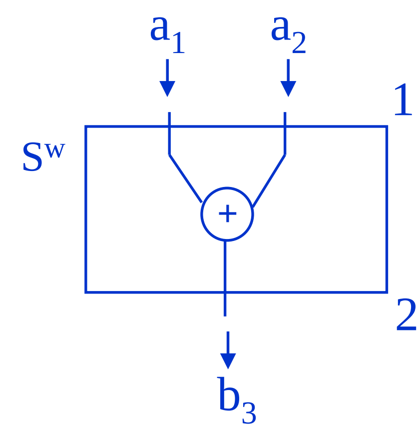


$$b_3 = \frac{1}{\sqrt{2}}(a_1 + a_2)$$

Why $\frac{1}{\sqrt{2}}$?

$$a_1 = a_2 = a$$

$$P_{in} = |a_1|^2 + |a_2|^2 = 2|a|^2$$

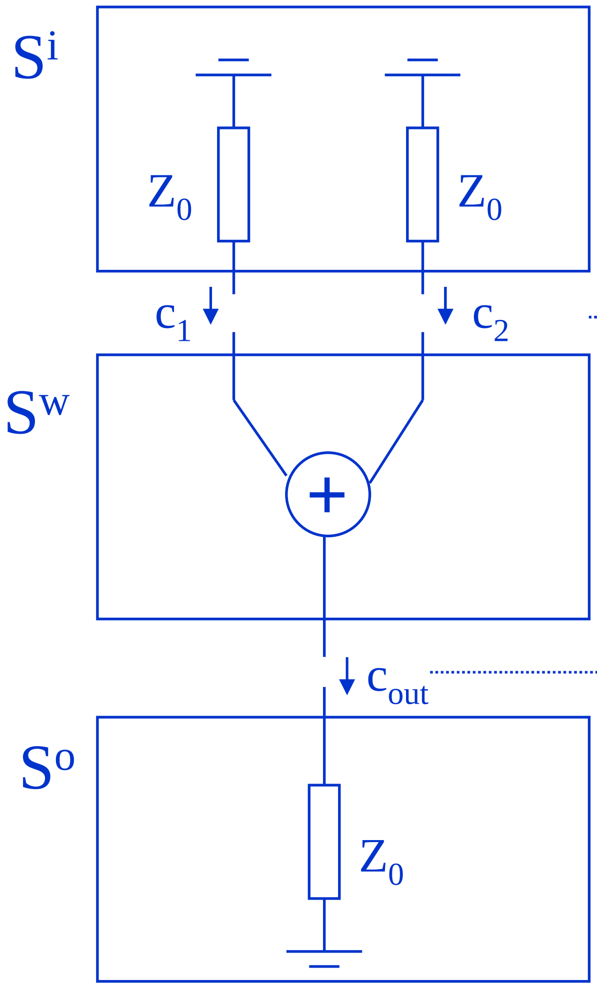
$$P_{out} = |b_3|^2 = \left|\frac{1}{\sqrt{2}}(a+a)\right|^2 = \frac{1}{2}|2a|^2 = 2|a|^2$$

Because of Power Normalization: $P_{in} = P_{out}$

$$b_3 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = S^w_{21}\underline{a}$$

Noise Wave Concept

Explained using an example



Two noisy resistors
both at temp. T=300K

Wilkinson Power
Combiner

?

Characteristic termination
at T=0K

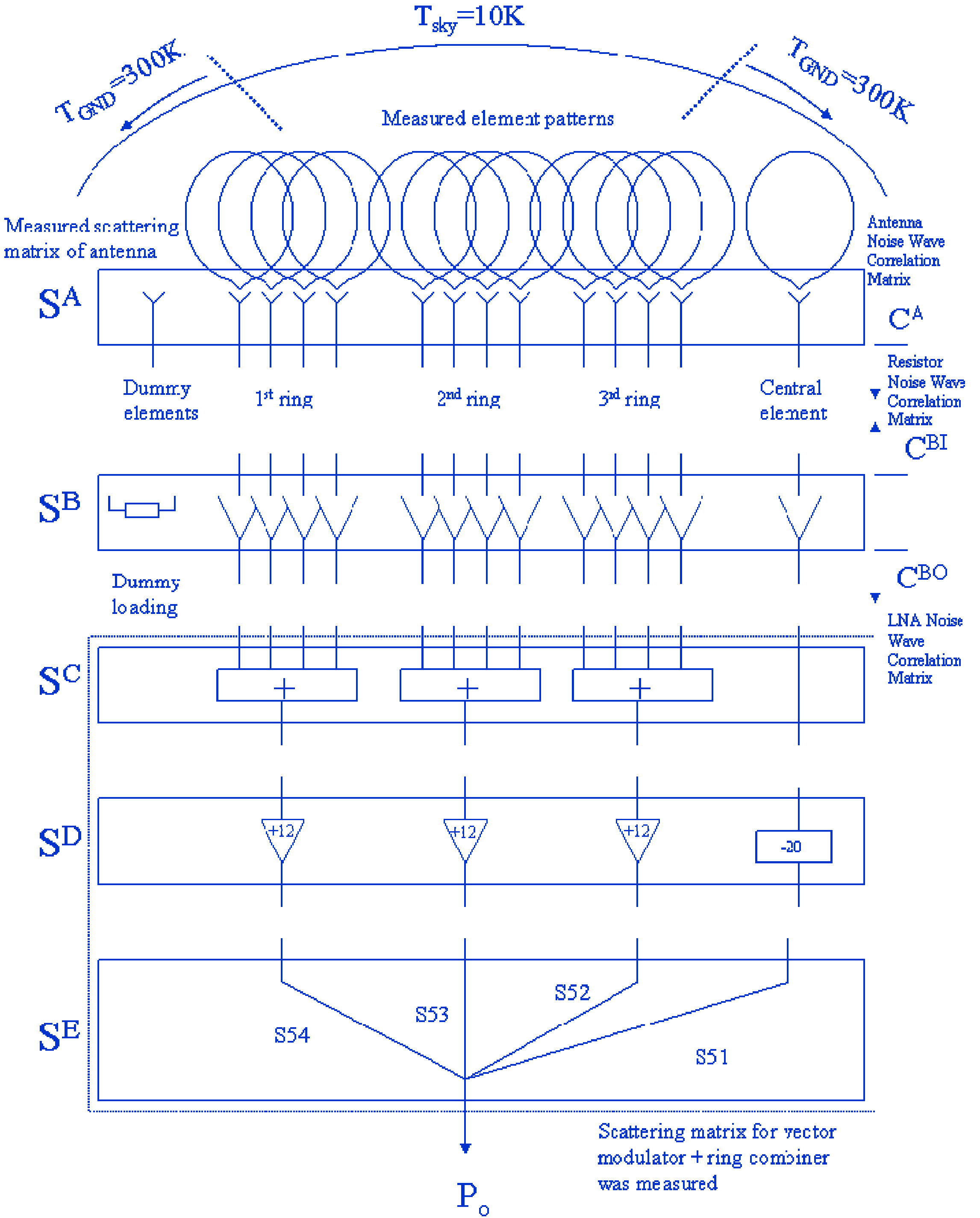
$$C_{in} = \begin{pmatrix} \overline{|c_1|^2} & \overline{c_1 c_2^*} \\ \overline{c_2^* c_2} & \overline{|c_2|^2} \end{pmatrix} = \begin{pmatrix} kT & 0 \\ 0 & kT \end{pmatrix}$$

$$S_{21}^{w} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$C_{out} = \overline{|c_{out}|^2} = \overline{c_{out} c_{out}^H} = \overline{\left(S_{21}^{w} \underline{c}_{in}\right)\left(S_{21}^{w} \underline{c}_{in}\right)^H}$$

$$= S_{21}^{w} \overline{\underline{c}_{in} \underline{c}_{in}^H} S_{21}^{w^H} = S_{21}^{w} \underline{C}_{in} S_{21}^{w^H}$$

$$C_{out} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} kT & 0 \\ 0 & kT \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = kT$$



FPA System Model

MODELED NOISE SOURCES:

- LNAs emit noise at its outputs (C^{BO}: 13x13 diagonal matrix)
- Dummy loads produce thermal noise (C^{BI}: 144x144 diag. matrix)
- Sky is treated as a black body at finite temperature, planck's law is used (C^A: 144x144 full matrix)

FPA
System Model

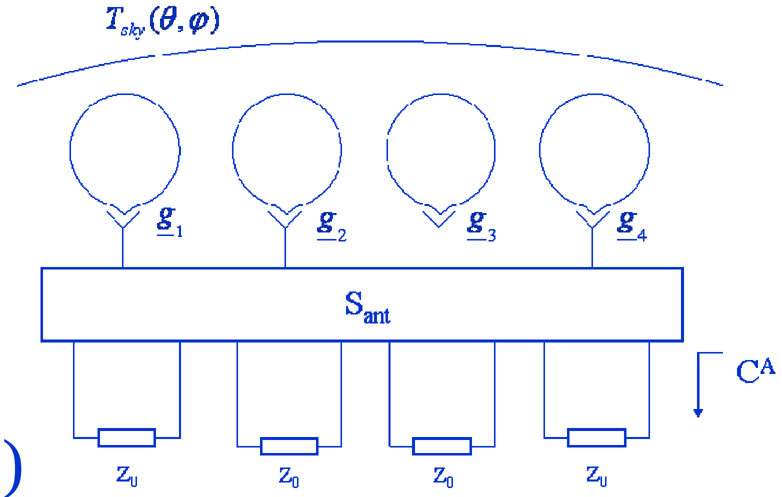
Noise Wave Correlation Matrix for Antenna (C^A):

$$C^A = LVL^H$$

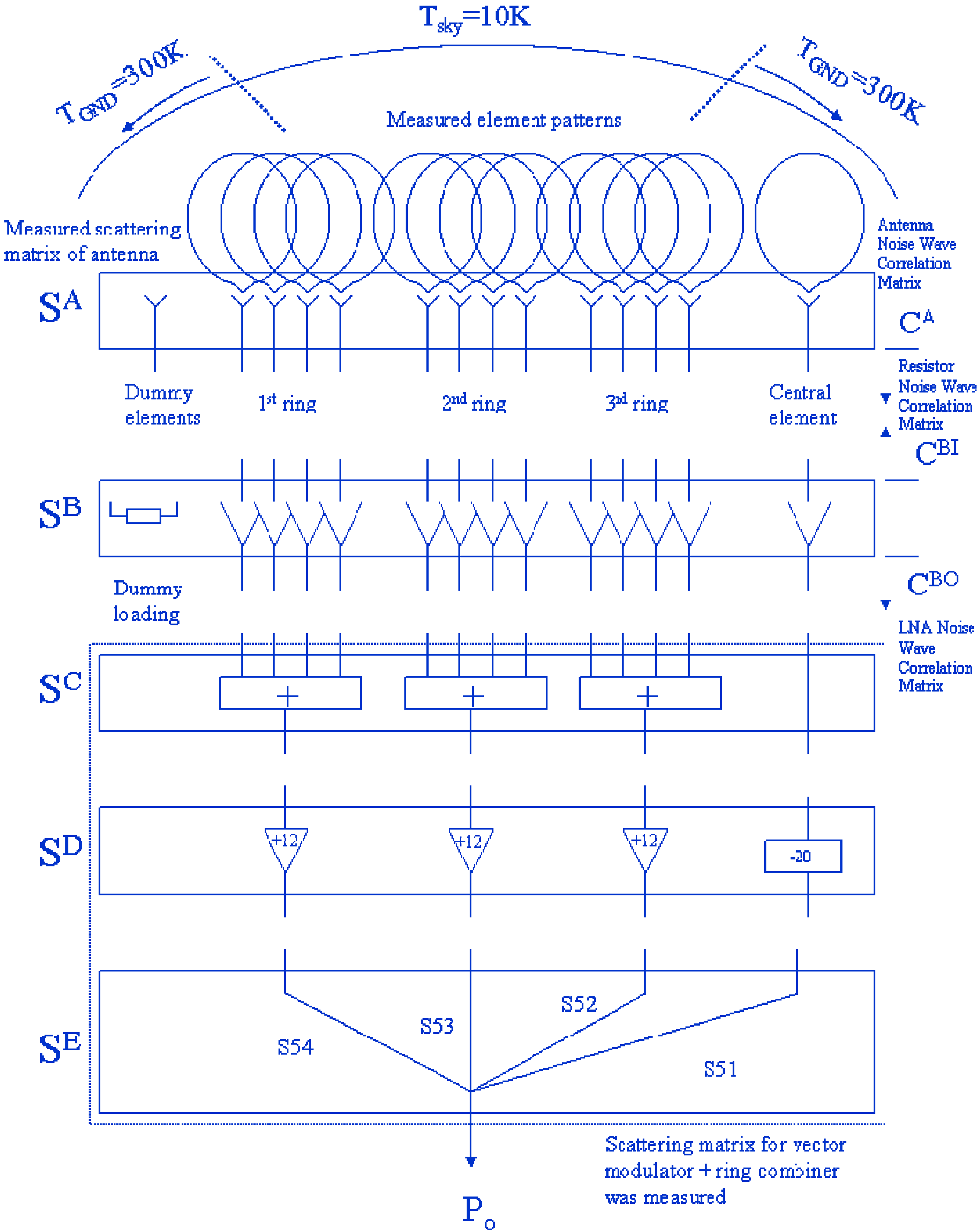
$$L = (Z^A + Iz_0)^{-1} I \sqrt{z_0}$$

Voltage correlation matrix derived using antenna reciprocity

$$Z^A = z_0 (I - S^A)^{-1} (I + S^A)$$



$$\overline{V_m V_n^*} = C \iint T_{sky}(\theta,\varphi) \left[\underline{g}_m(\theta,\varphi) \bullet \underline{g}_n^*(\theta,\varphi) \right] \sin\theta d\theta d\varphi$$



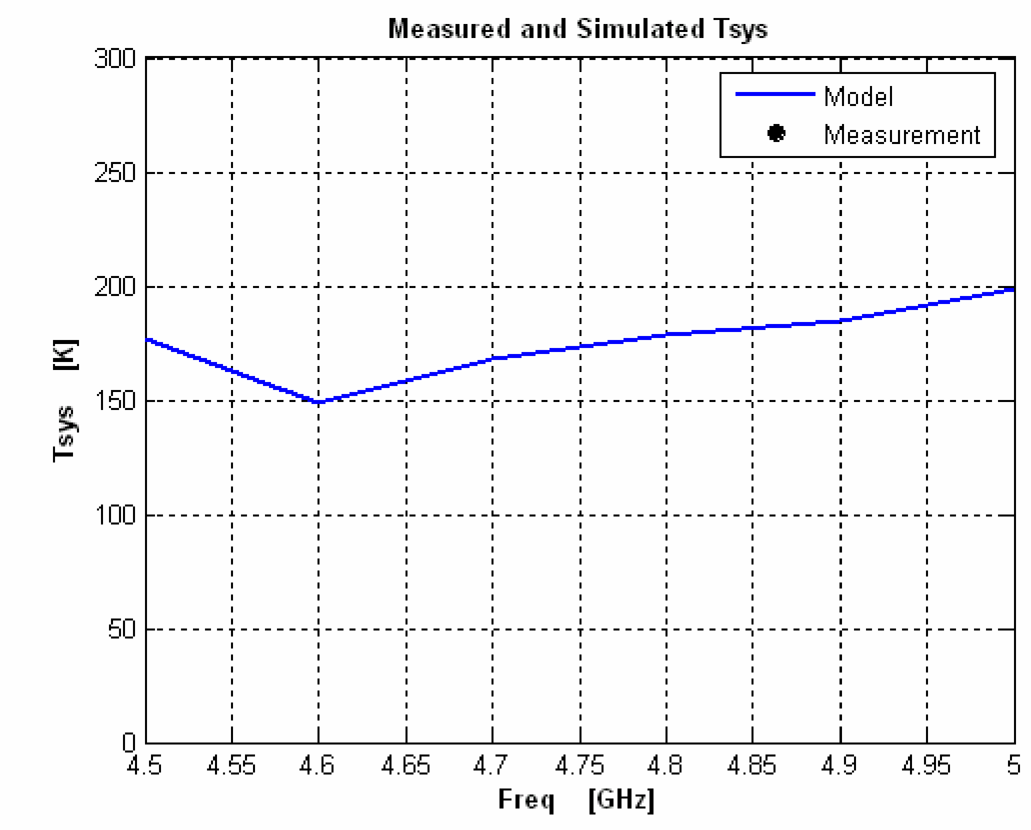
FPA System Model

Procedure hot-cold simulation to determine T_{sys}:

- Hot simulation: Tsky and Tgnd at T_h=300k
- Cold simulation: Tsky and Tgnd at T_c=10k
- Compute output power for hot and cold case, P_h and P_c resp.
- $Y=P_h/P_c$
- $T_{sys}=T_c+(T_h-T_c*Y)/(Y-1)$

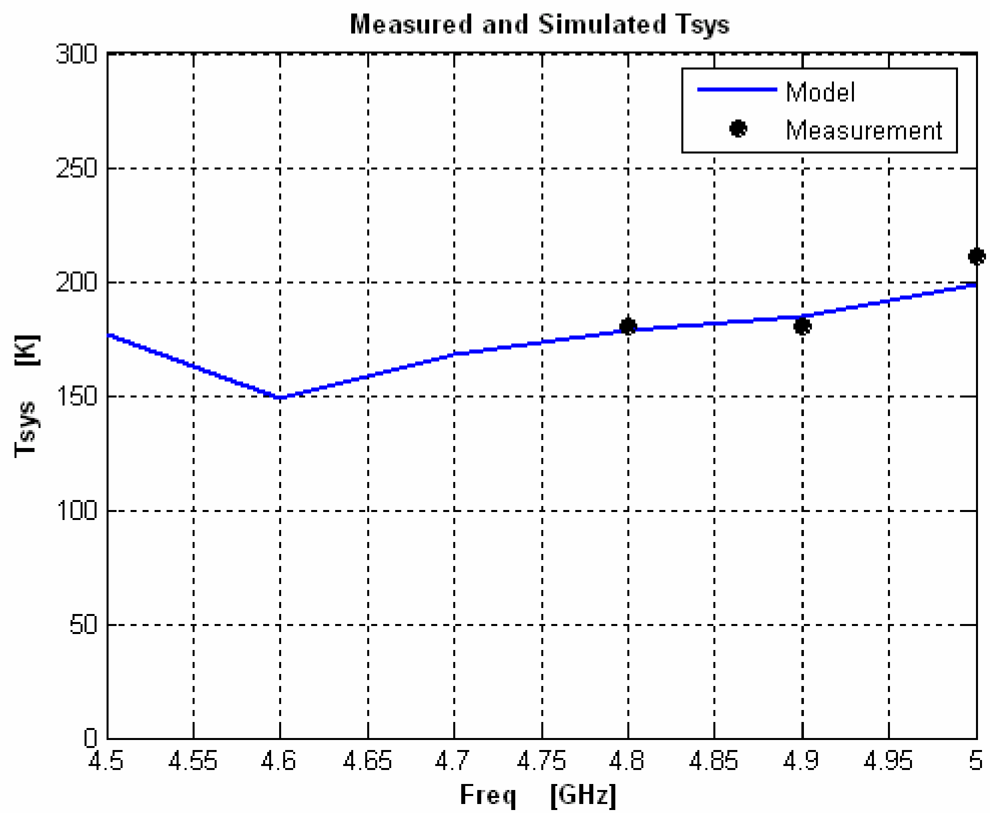
Resistive Loading

Disadvantages



Resistive Loading

Disadvantages



@ 5.0 GHz:

$$T_{sys} \approx T_{SKY}^{cold}(10K) + T_{LNA}(150K) + T_R(40K)$$

Resistive Loading

Disadvantages

@ 5.0 GHz:

$$SENSITIVITY = \frac{A_{eff}}{T_{sys}} = \frac{A_{eff}^{MAX} \cdot 0.8}{T_{sys}^{MIN} + 40}$$

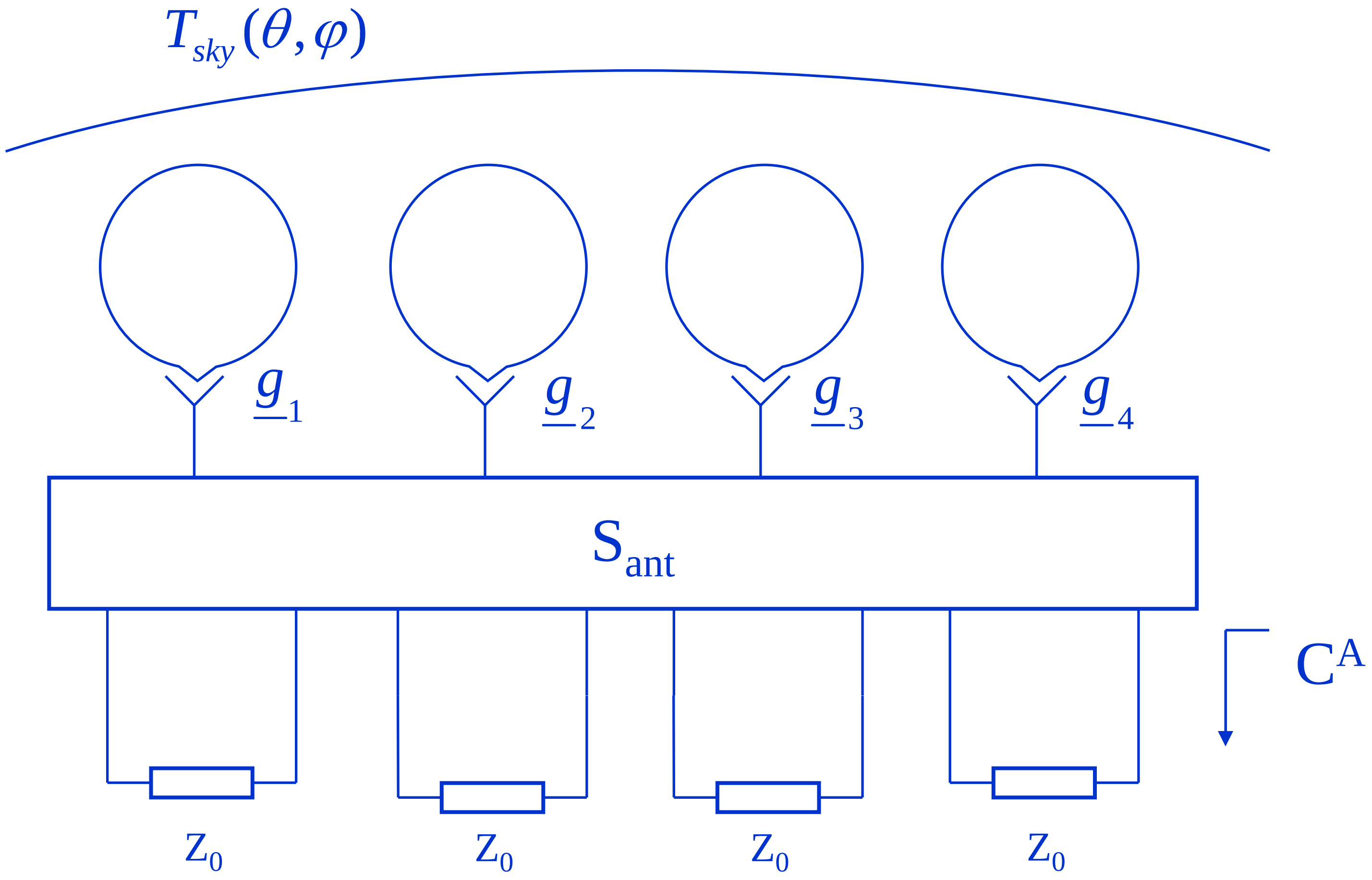
Remarks:

• From this analysis it turns out that we also need to consider/optimize the overall sensitivity of the receiving system to determine the best loading scheme. Although individual element patterns look bad in case dummy elements are not loaded, many elements used for beamforming could still yield a satisfactory illumination pattern (see presentation by Marianna).

• Considering active loading (e.g. LNAs) or cooled loads to reduce Tsys, etc.

Conclusions:

- Resistive loading of dummy elements in an array generally improve the antenna input impedance and element patterns over the frequency band, finite array effects become less severe
- Resistive loading does degrade the sensitivity Aeff/Tsys, i.e., Aeff will decrease while Tsys will increase
- A system model was implemented and used to predict Tsys sufficiently accurate w.r.t. the measured Tsys
- One of the parameters that needs to be accounted for during optimizing the overall sensitivity is the loading scheme of the dummy (passive) elements in the array



$T_{sky}(\theta,\varphi)$
$\underline{g}_1$
$\underline{g}_2$
$\underline{g}_3$
$\underline{g}_4$
S_{ant}
C^A
Z_0
Z_0
Z_0
Z_0