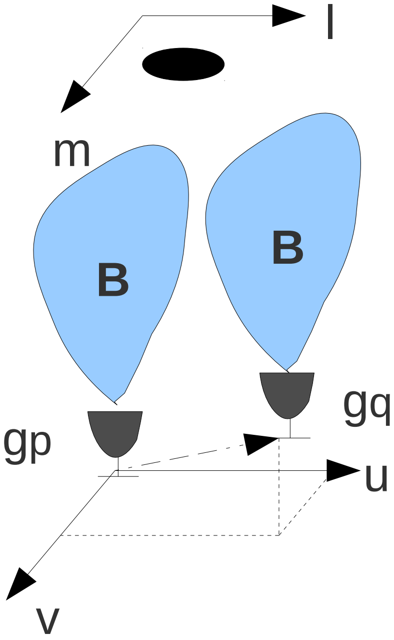
# LOFAR Imager: taking Direction Dependent Effects into account using A-Projection

Cyril Tasse, Ger van Diepen, Joris van Zwieten, Bas van der Tol

Sanjay Bhatnagar, Urvashi Rau, Kumar Golap

# Traditional Calibration and imaging (scalar)




$$V_{pq} = (g_p . g_q^*) \int_S \mathrm{B(l,m).I(l,m)}$$
$$. \exp\left(-2\pi i (u_{pq} l + v_{pq} m + w_{pq} . (\sqrt{1 - l^2 - m^2} - 1)\right)) dl.dm$$

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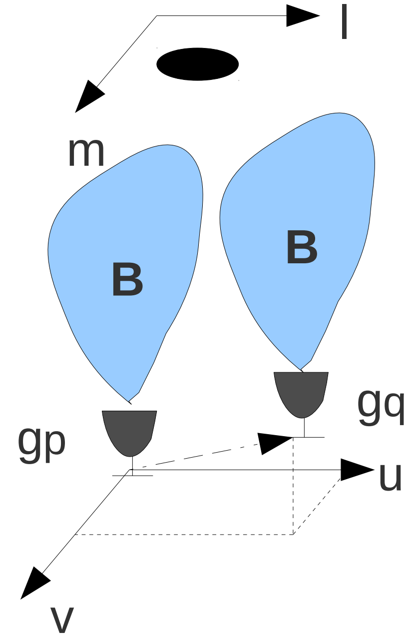


$$V_{pq} = (g_p.g_q^*) \int_S \mathrm{B(l,m).I(l,m)}$$
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**Small field of view**

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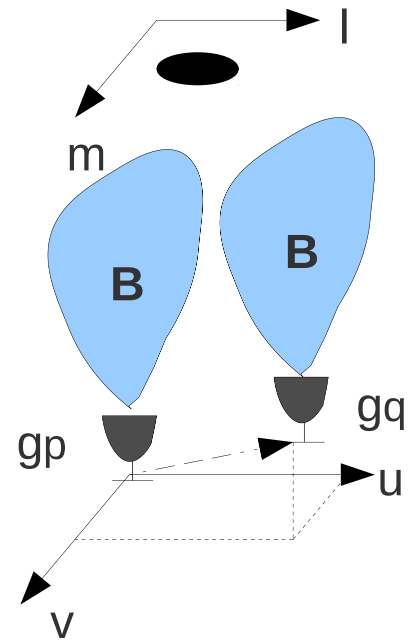


- Calibration

$$V_{pq} = (g_p . g_q^*) \int_S \mathrm{B(l,m).I(l,m)}$$
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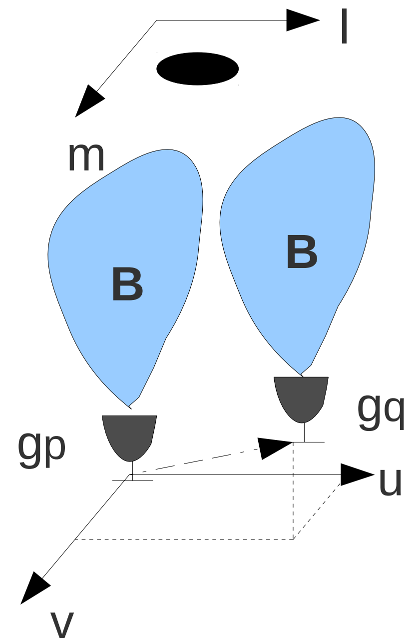


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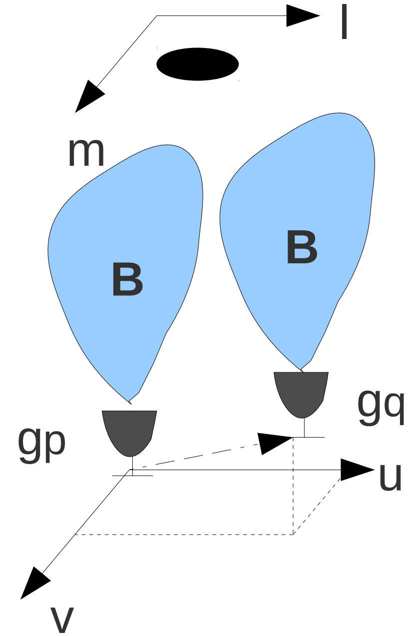
$$V_{pq} = (g_p . g_q^*) \int_S \mathrm{B(l,m).I(l,m)} \; . \exp\left(-2\pi i(u_{pq}l + v_{pq}m + w_{pq}.(\sqrt{1-l^2-m^2}-1))\right) dl.dm$$

**Small field of view**

- Imaging

$$\mathrm{I(l,m)} = \frac{1}{\mathrm{B(l,m)}} \mathrm{FT}\left(\frac{V(u,v)}{[g.g^*](u,v)}\right)$$

# Traditional Calibration and imaging (scalar)

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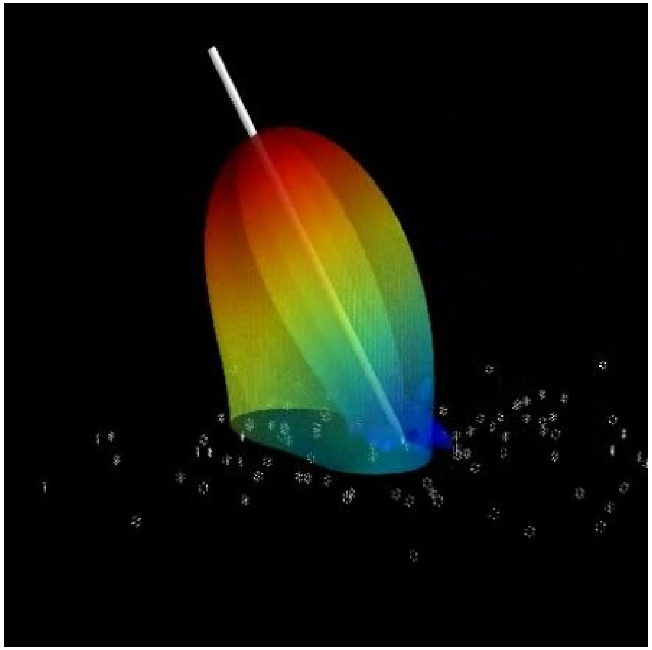
**Small field of view**

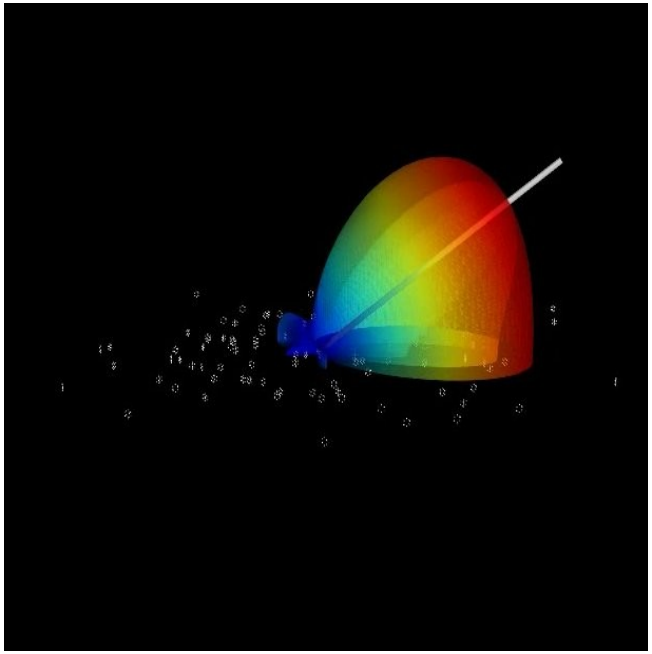
- Imaging

$$\mathrm{I(l,m)} = \frac{1}{\mathrm{B(l,m)}} \mathrm{FT}\left(\frac{V(u,v)}{[g.g^*](u,v)}\right)$$

Beam correction in the image plane

# … When Direction Dependent Effects (DDE) become a problem : Beam



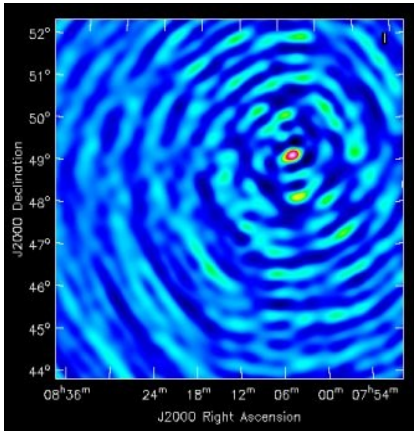


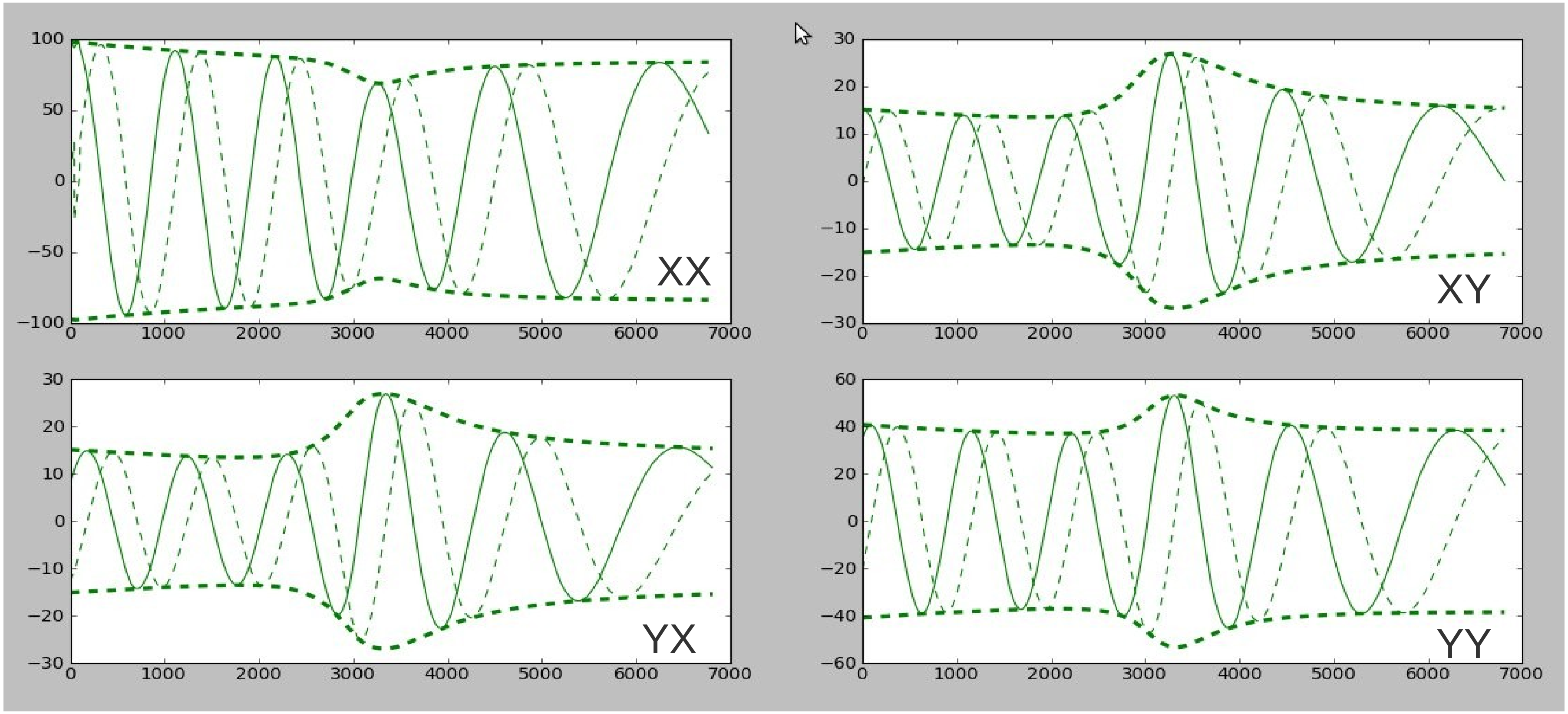
**LOFAR stations are phased arrays**

- Beam is variable in frequency and time
- Beam can be station-dependent

# … When Direction Dependent Effects (DDE) become a problem : Beam

**One off-axis source IQUV=(100, 40, 20 10)**



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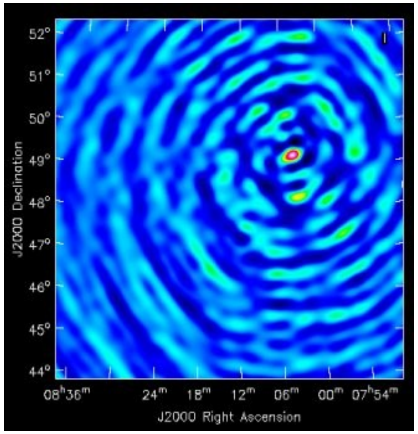
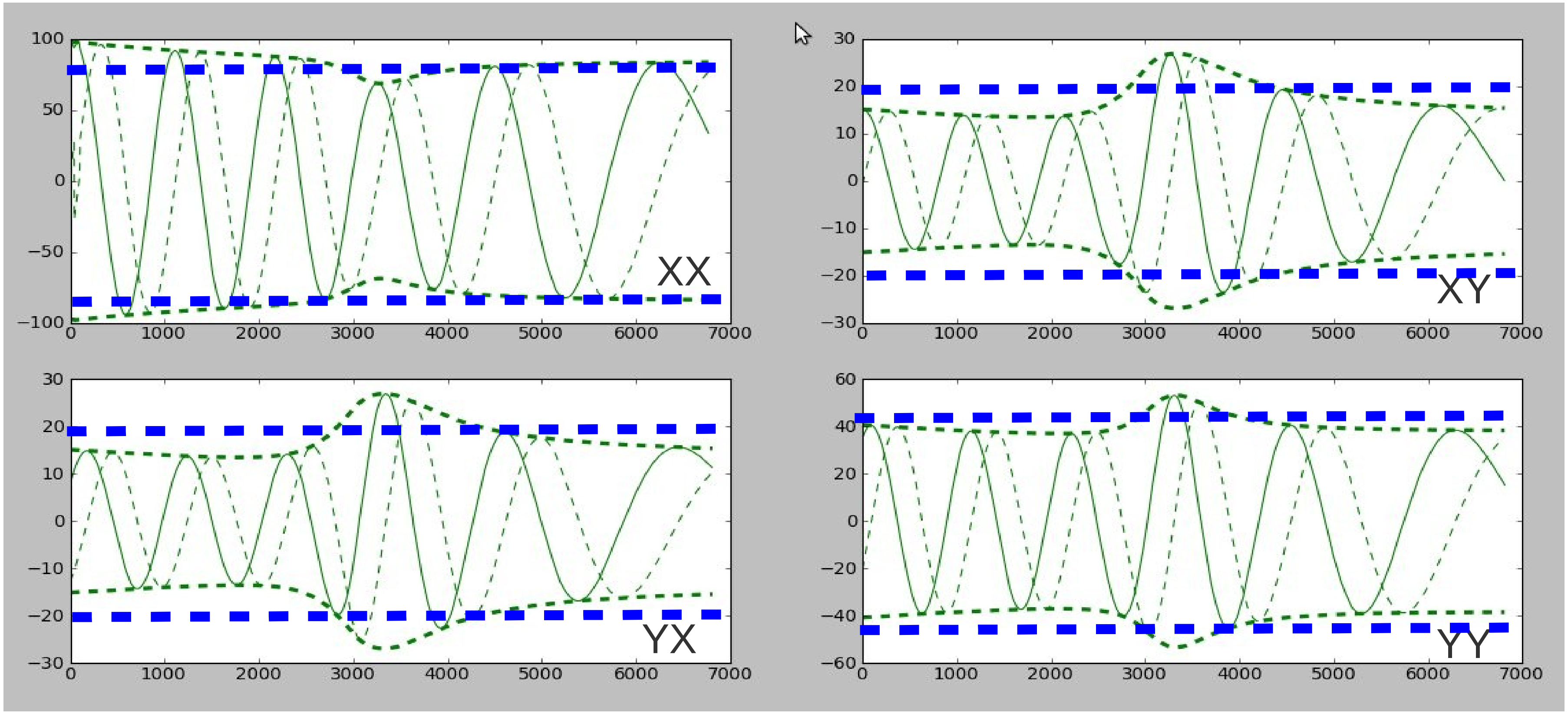
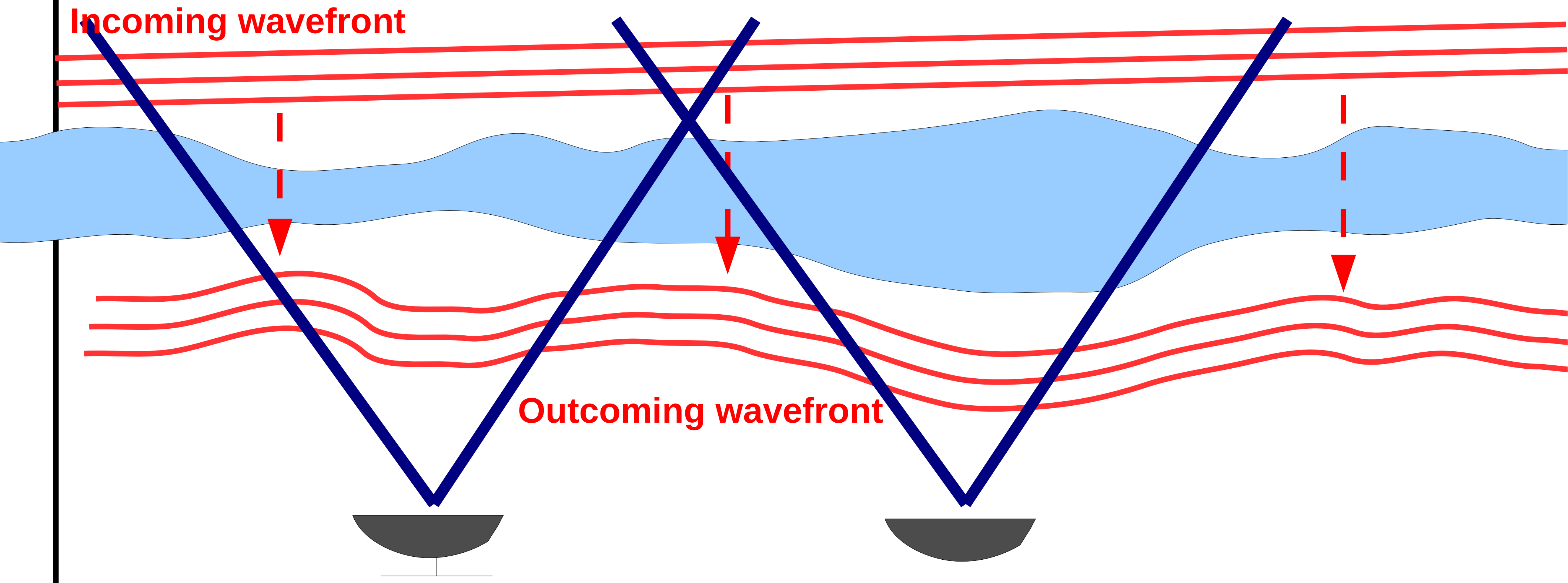



**One off-axis source IQUV=(100, 40, 20 10)**

**"Traditional" imager removes visibility with constant amplitude**



# … When Direction Dependent Effects (DDE) become a problem : Ionosphere




**Big field of view : station, direction, time and frequency dependent**

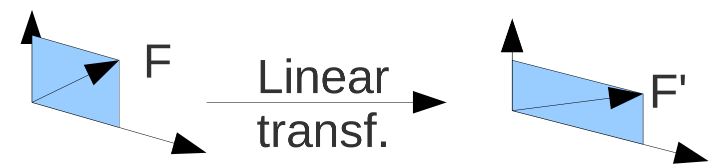
**Other direction dependent effects :**

- Projection of the dipoles on the sky
- Faraday rotation

**+ Effect on the polarisation**

# The Measurement Equation

Hamaker 1996




Direction independent | Direction dependent | Source coherency

$$V_{pq} = G_p \left( \sum_{i=1}^{N} \mathrm{B}_{pi}\mathrm{K}_{pi}\mathrm{I}_{pi}F_i \; . \; F_i^{+}\mathrm{I}_{qi}^{+}\mathrm{K}_{qi}^{+}\mathrm{B}_{qi}^{+} \right) G_q^{+}$$

**[Voltage antenna p] x [ Voltage antenna q]***

Beam | Geometrical delay +Correlator | Ionosphere | Electric field

$$\mathrm{K}_p\mathrm{K}_q^{+} = \exp(-2i\pi\phi_{pq}) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\phi_{pq} = u_{pq}l + v_{pq}m + w_{pq}(\sqrt{1 - l^2 - m^2} - 1)$$

# A-Projection

Bhatnagar 08

Convolution function (4*4)

Beam (4*4)

W term (scalar)

$$\mathrm{Vec}(V_{pq}) = (G_q^* \otimes G_p)\mathrm{FT}\left[\left(E^*_{q,\vec{s}} \otimes E_{p,\vec{s}}.\exp\left(-2\pi i w_{pq}.\left(\sqrt{1-l^2-m^2}-1\right)\right)\right)\right]$$

$$\star \int_S \mathrm{Vec}(X_{\vec{s}}).\exp\left(-2\pi i(u_{pq}l + v_{pq}m)\right)dl.dm$$

Convolution

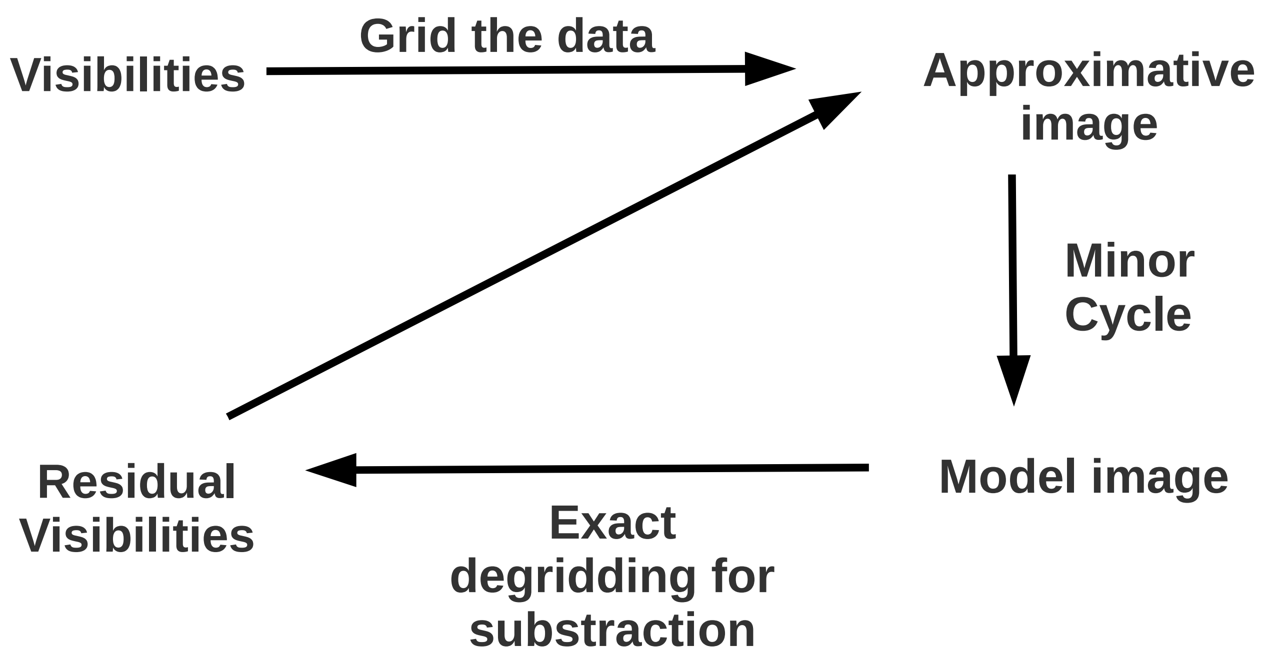
2D FFT

**This is an EXACT map from sky plane to the Visibilities in the UVW space!**

**BUT: The inverse map is approximative! (based on pseudo-inverse)**

# JAWS: the practice

- Plug in the casa architecture
- Full Polarization
- Convolution function is mapped by i,j,t, nu
- Ionosphere easy to plug in
- Will run in parallel

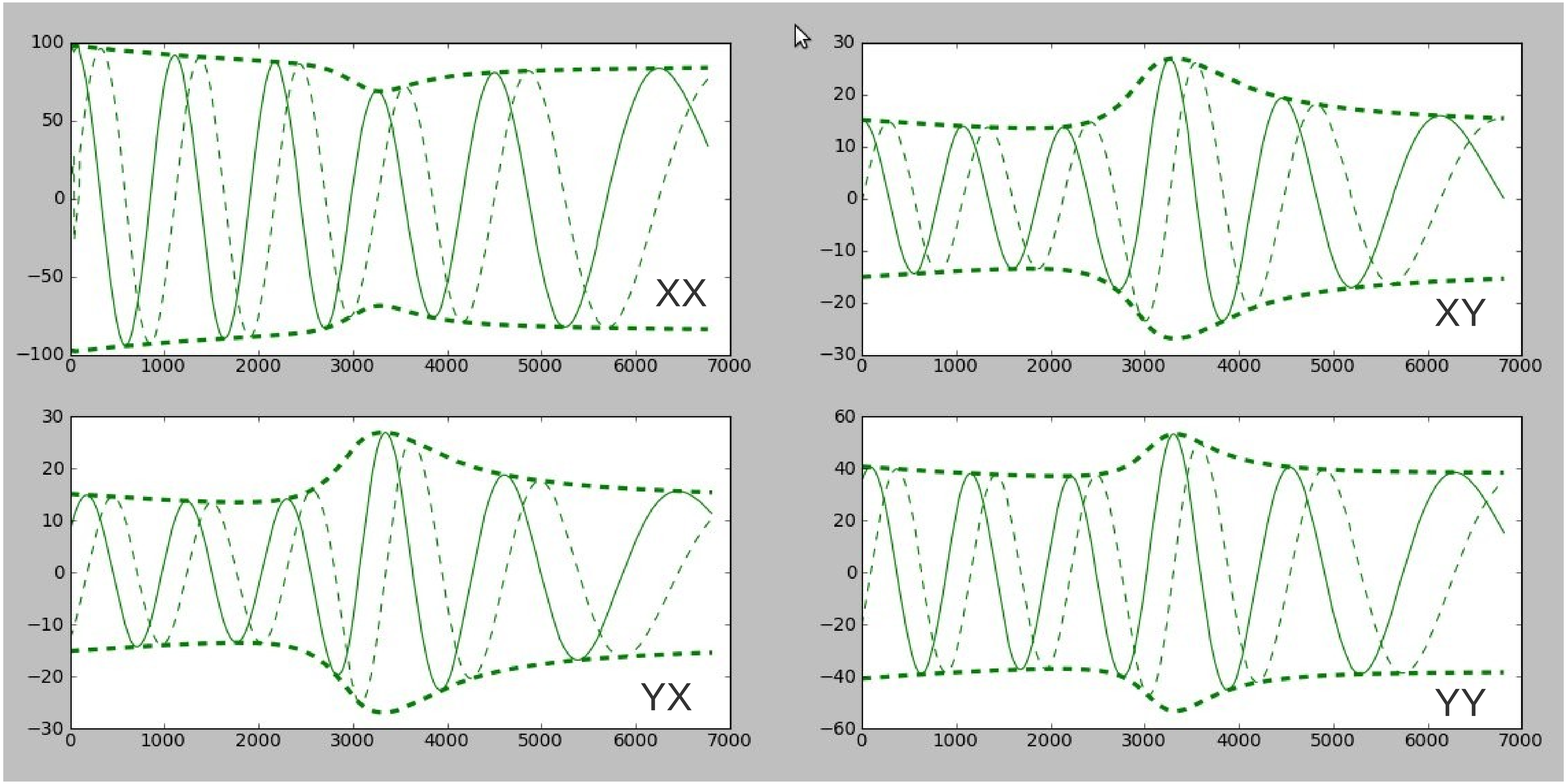



After a number of iteration, the flux in the clean component converges to the true values (to be studied)

# Mathematical framework-works

## One off-axis source IQUV=(100, 40, 20 10)

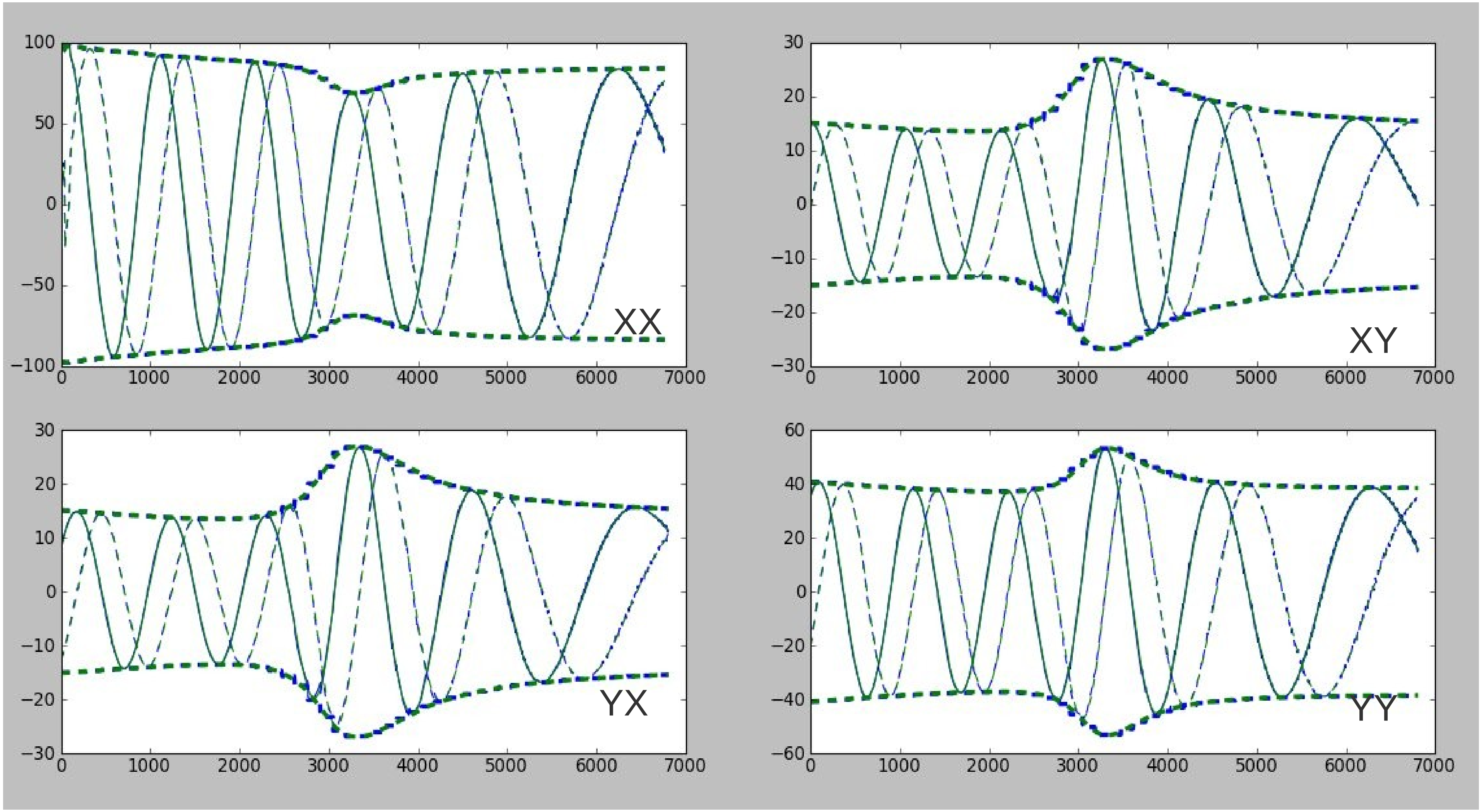
BBS predict (DFT)



# Mathematical framework-works

**BBS predict (DFT)**

**AW degridding (clean component put by hand)**



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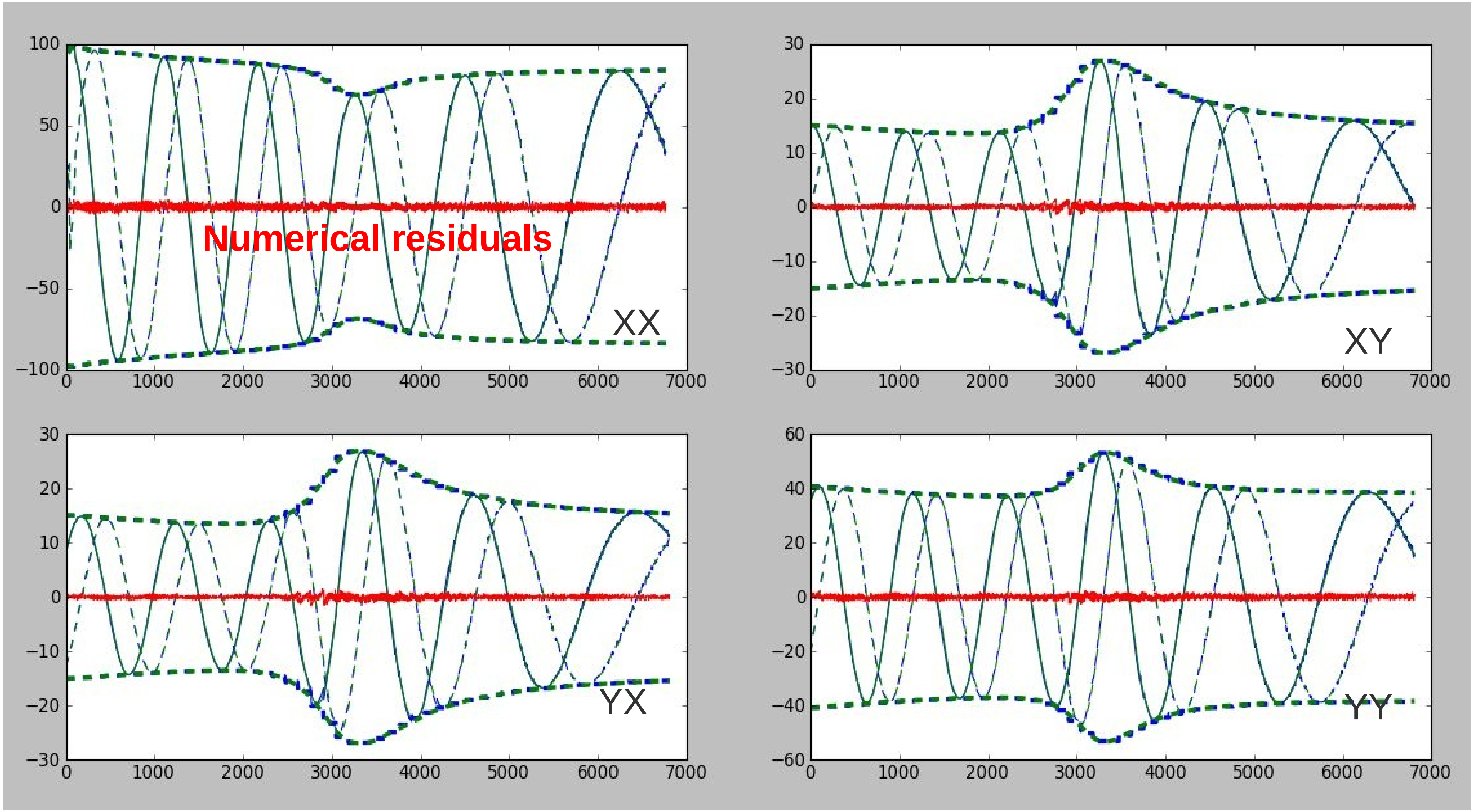
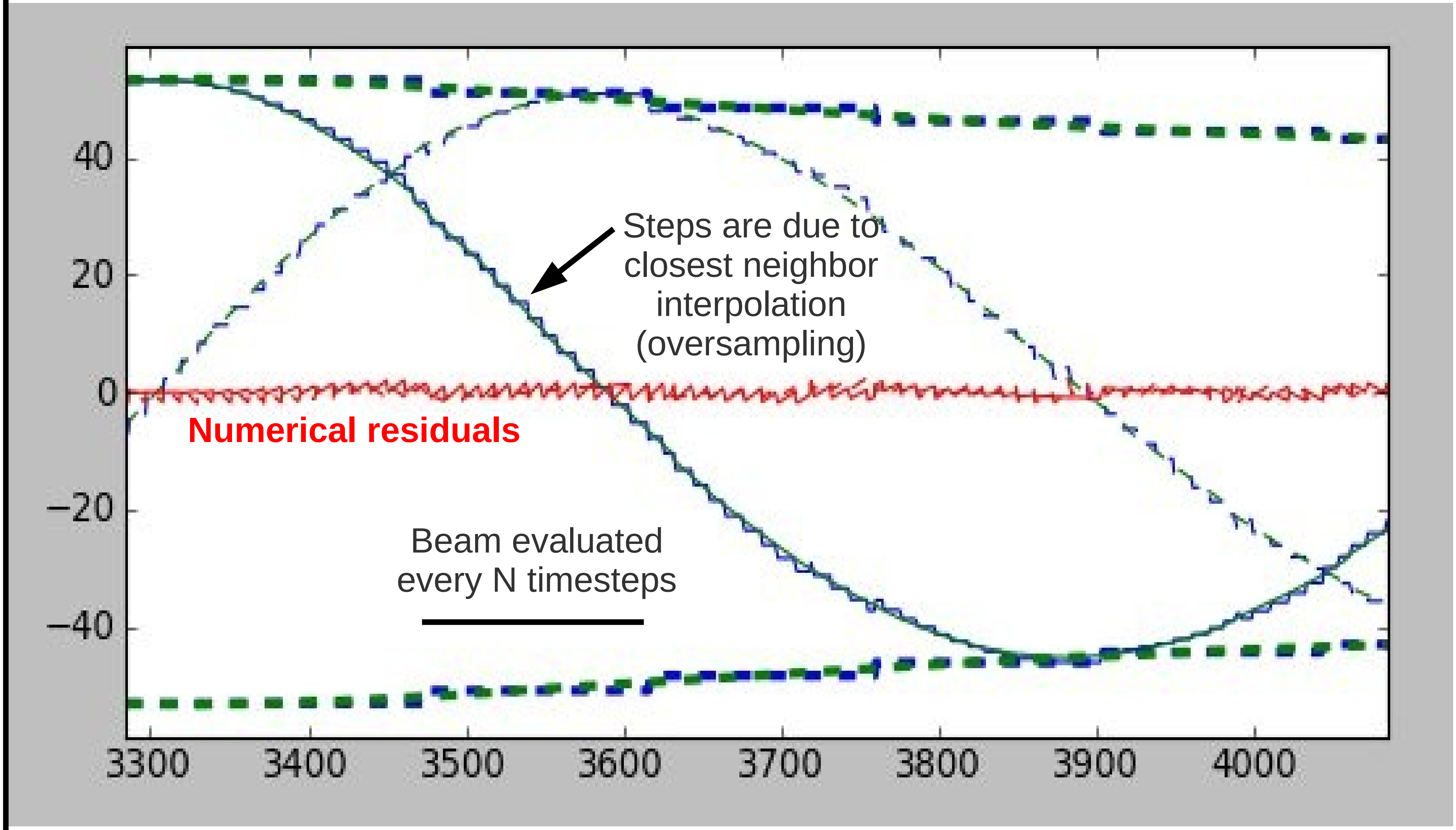
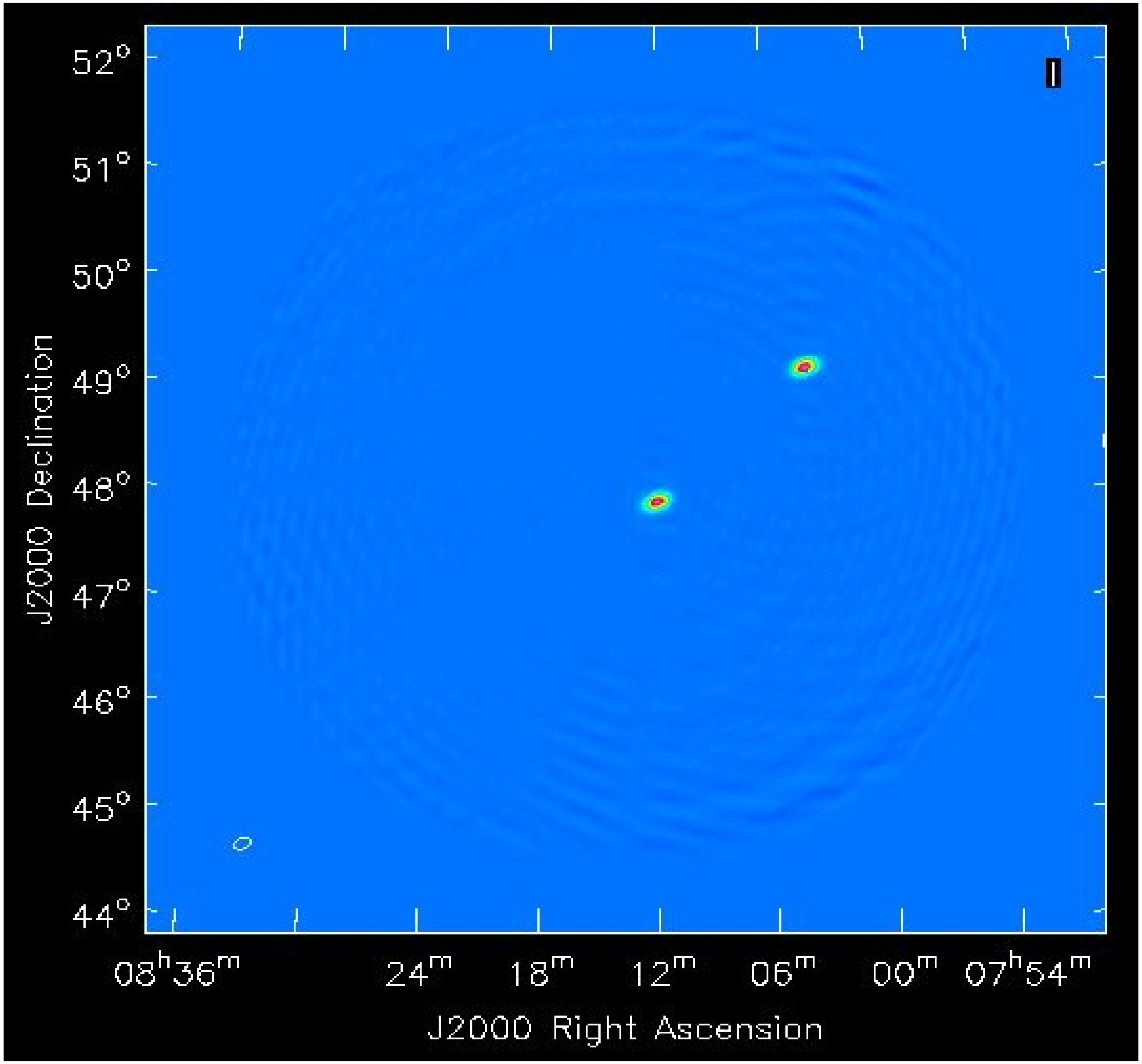


# Mathematical framework-works



# Mathematical framework-works

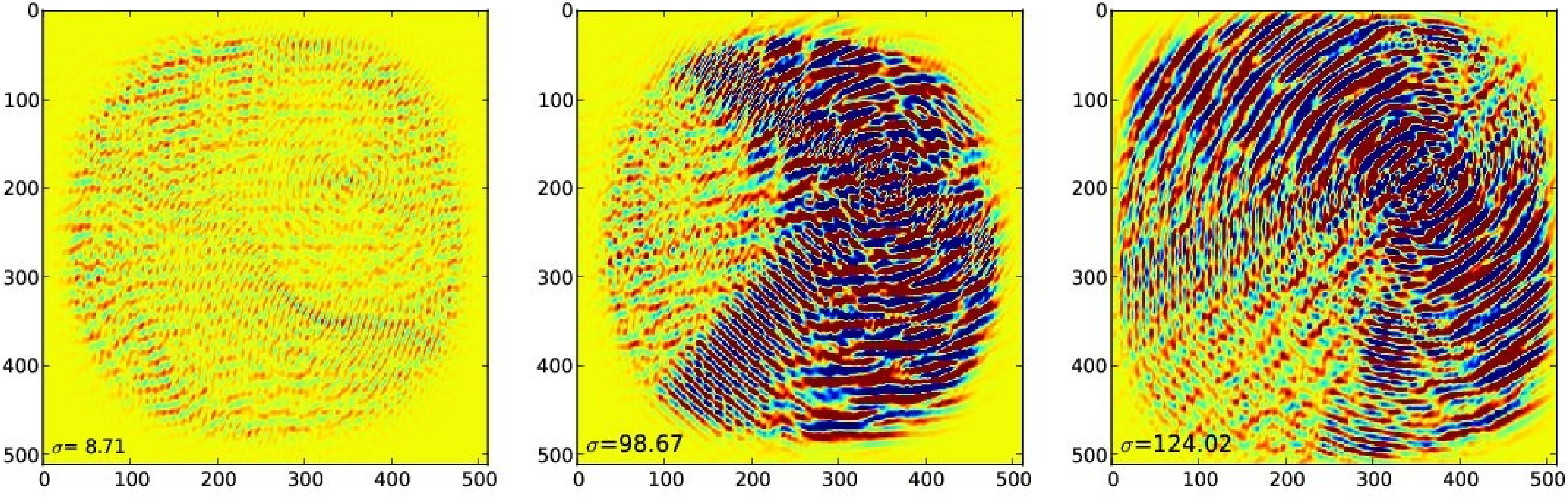



**Recovered IQUV=(100, 40, 20 10) fluxes to better than 1%**

# Mathematical framework-works

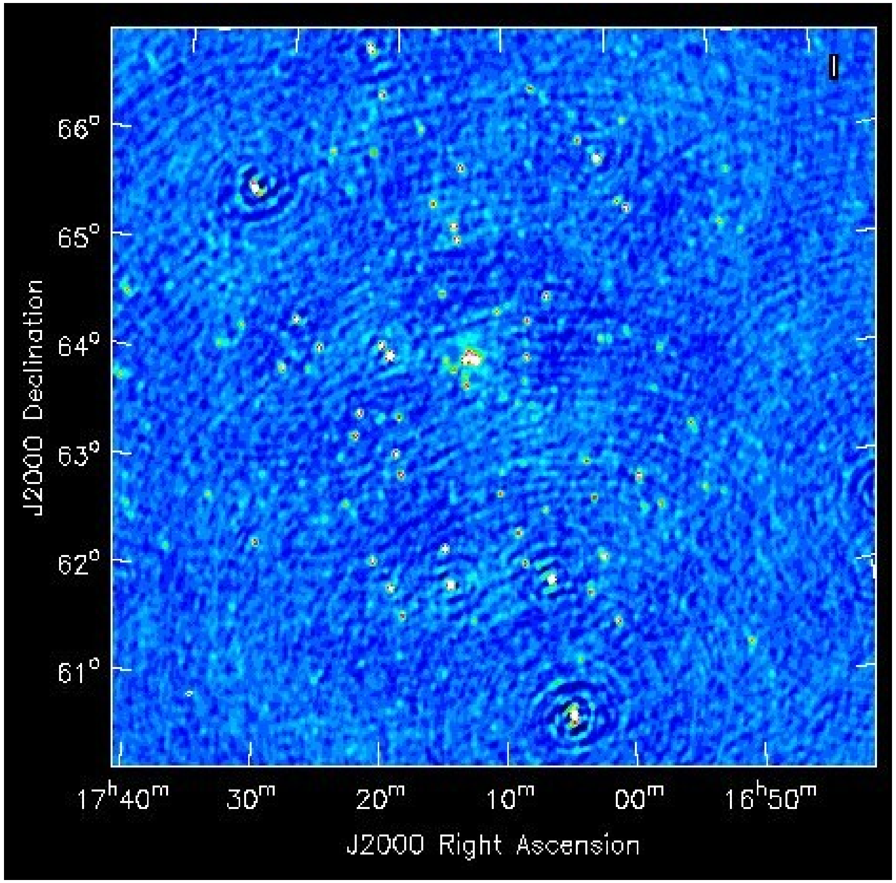
**Same simulated dataset with one off-axis source and the beam (IQUV=100,40,20,10)**

**Residual images, Stokes I**

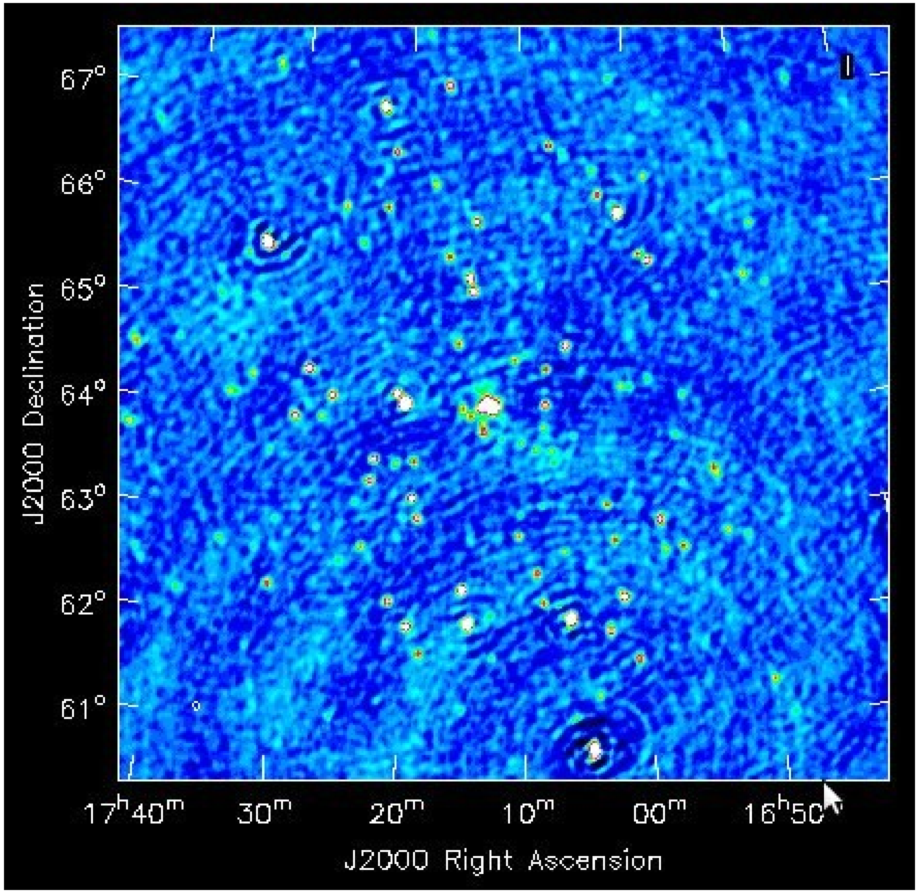



**AW projection** | **AW, only diag terms** | **W projection only**

# On real data (A2255)




**Casa**




**JAWS**

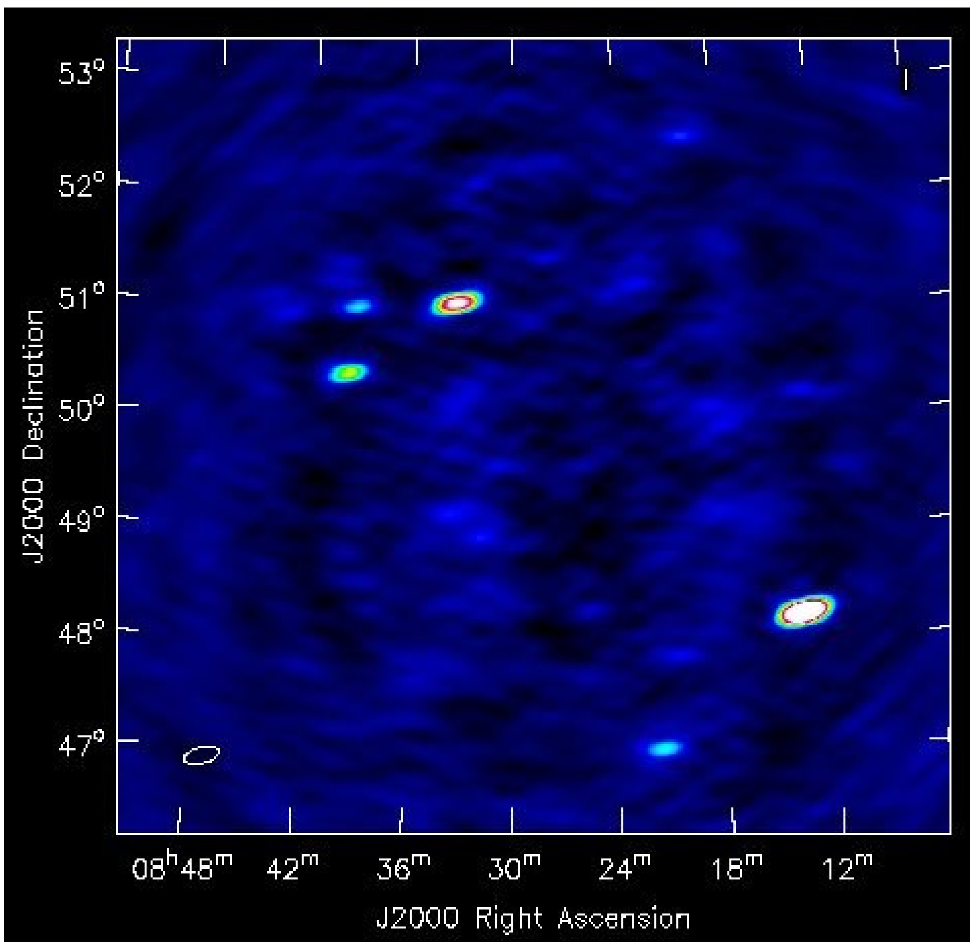
**See Roberto Pizzo talk**

# On real data (3C196)

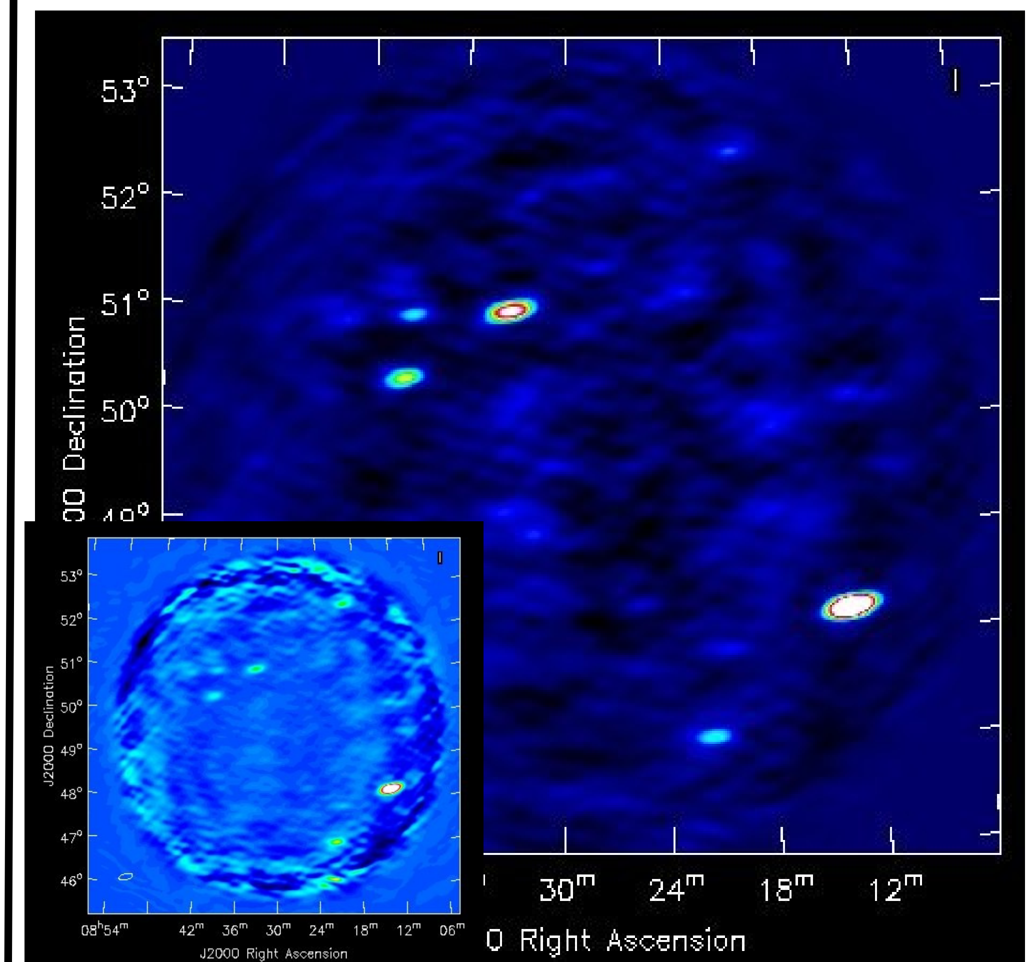
## 3C196 off axis ~150MHz

- Calibrated using 3C196+2 sources sources
- AW visibility estimates for those. Little difference?

**NOT Taking the beam into account**




**Taking the beam into account**



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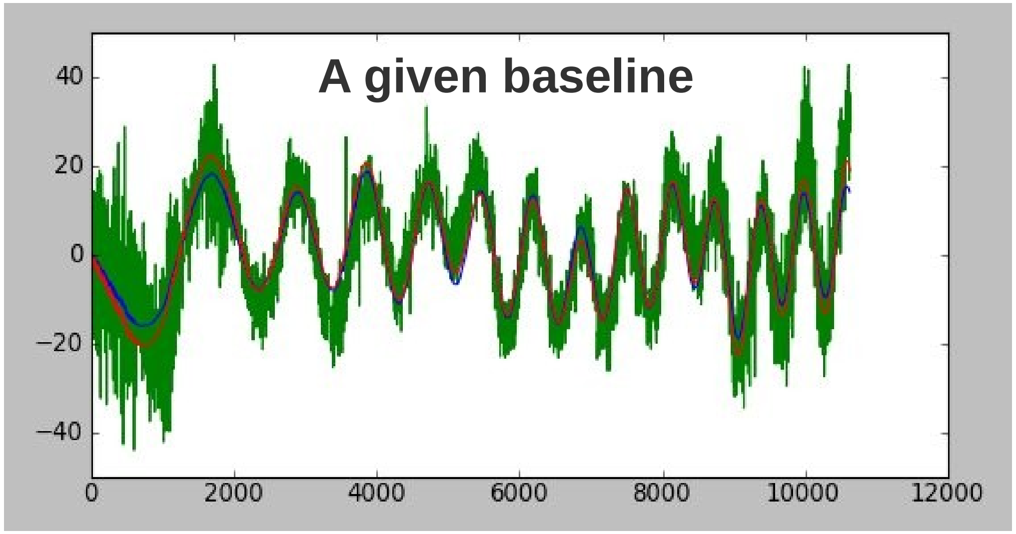
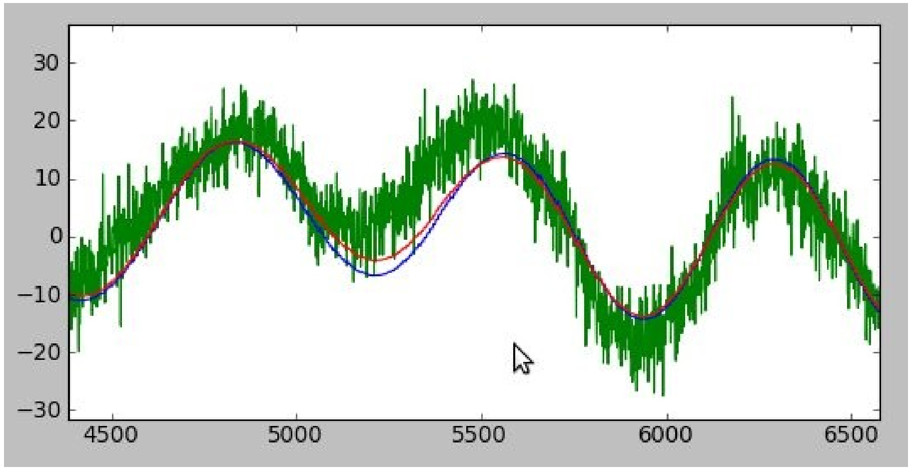
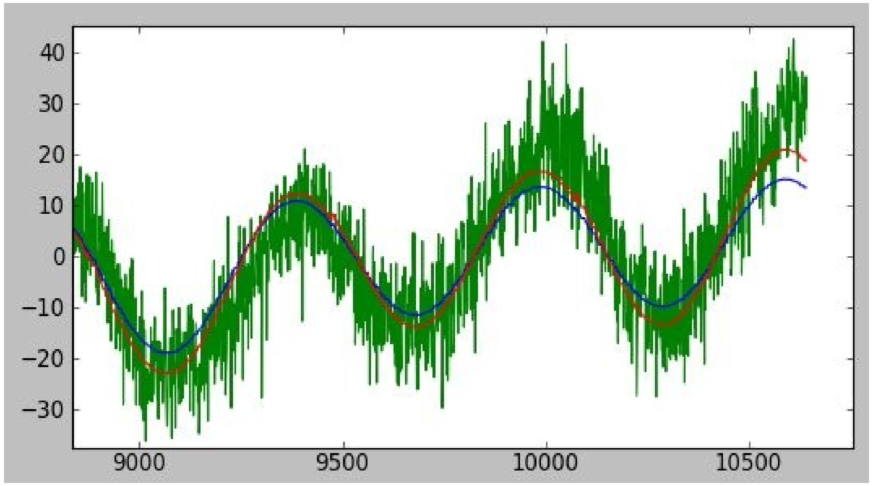


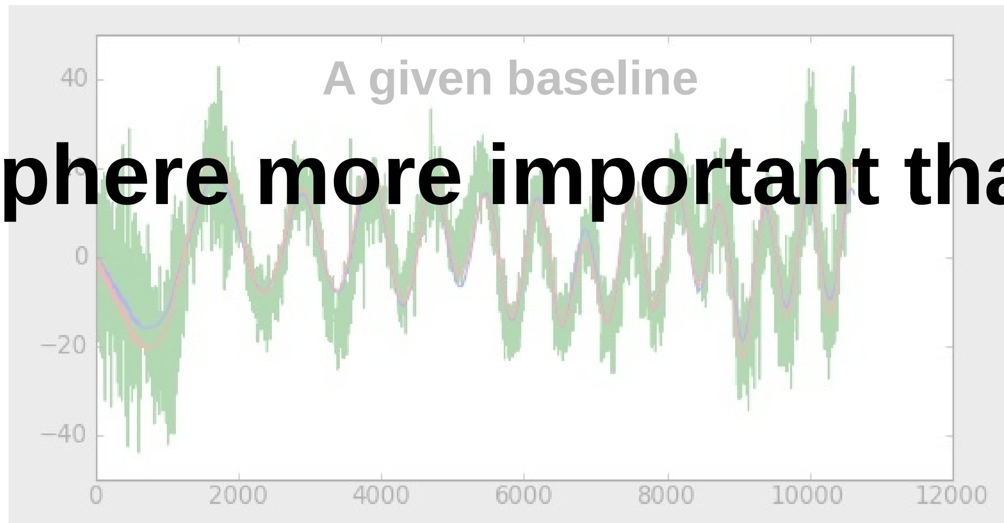

Beam taken into account





No Beam taken into account

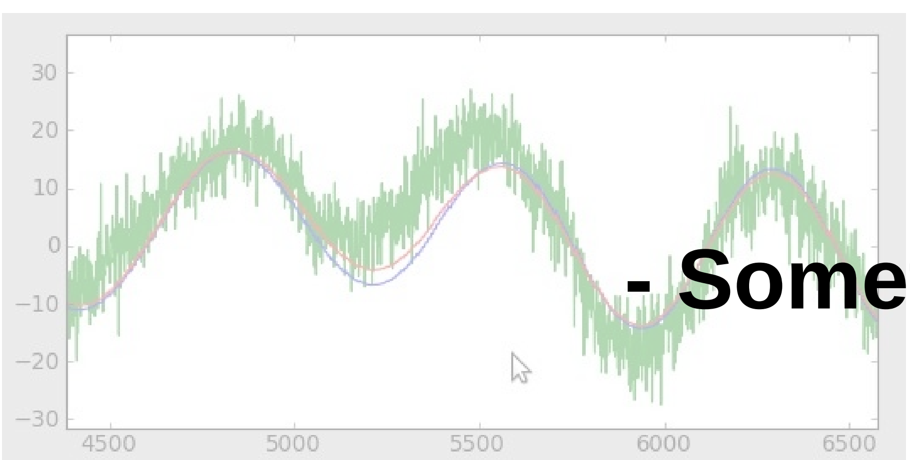
# On real data (3C196)

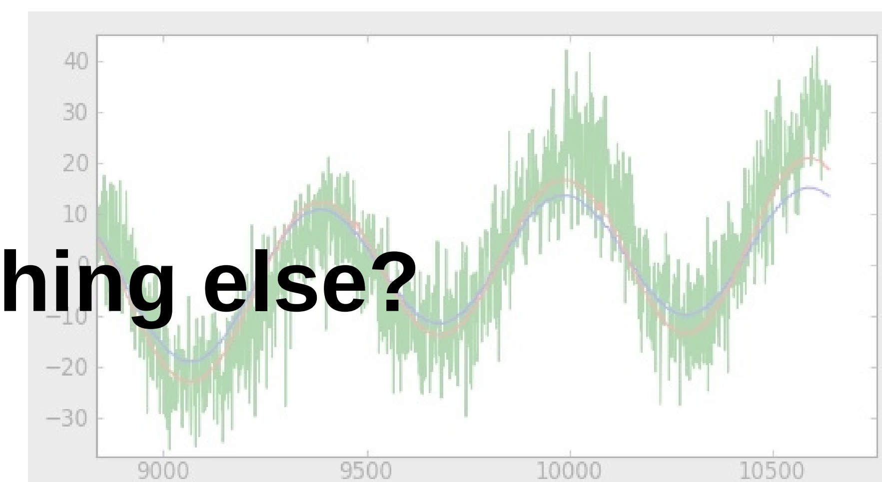



**- Ionosphere more important than beam ?**

**- Sky model too wrong? Do SelfCal?**

Beam taken into account




**- Something else?**

No Beam taken into account

# Conclusion and Next steps

**Conclusion:**

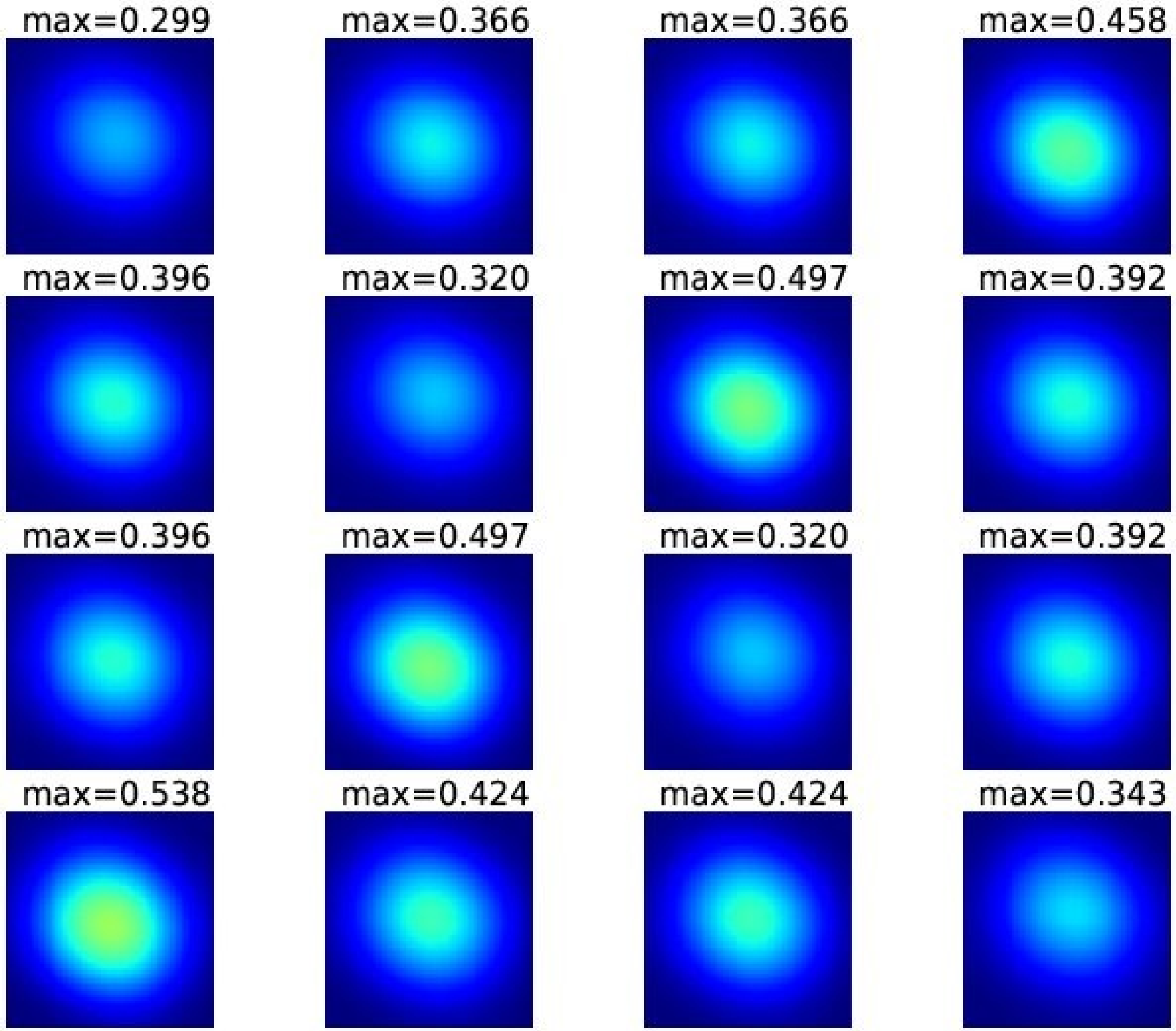
- Full Polarisation Framework based on Measurement Equation is working
- Doesn't do miracles
- Very flexible
- Effect will be seen at higher dynamical range?

**Next steps:**

- Optimise code
- Study convergence major cycle & SelfCal
- Ionosphere phase screen model
- Full Multi-Frequency cleaning
- Faraday Rotation?

… Start doing serious survey science

# LOFAR Beam: The Mueller Matrix varying over the image plane

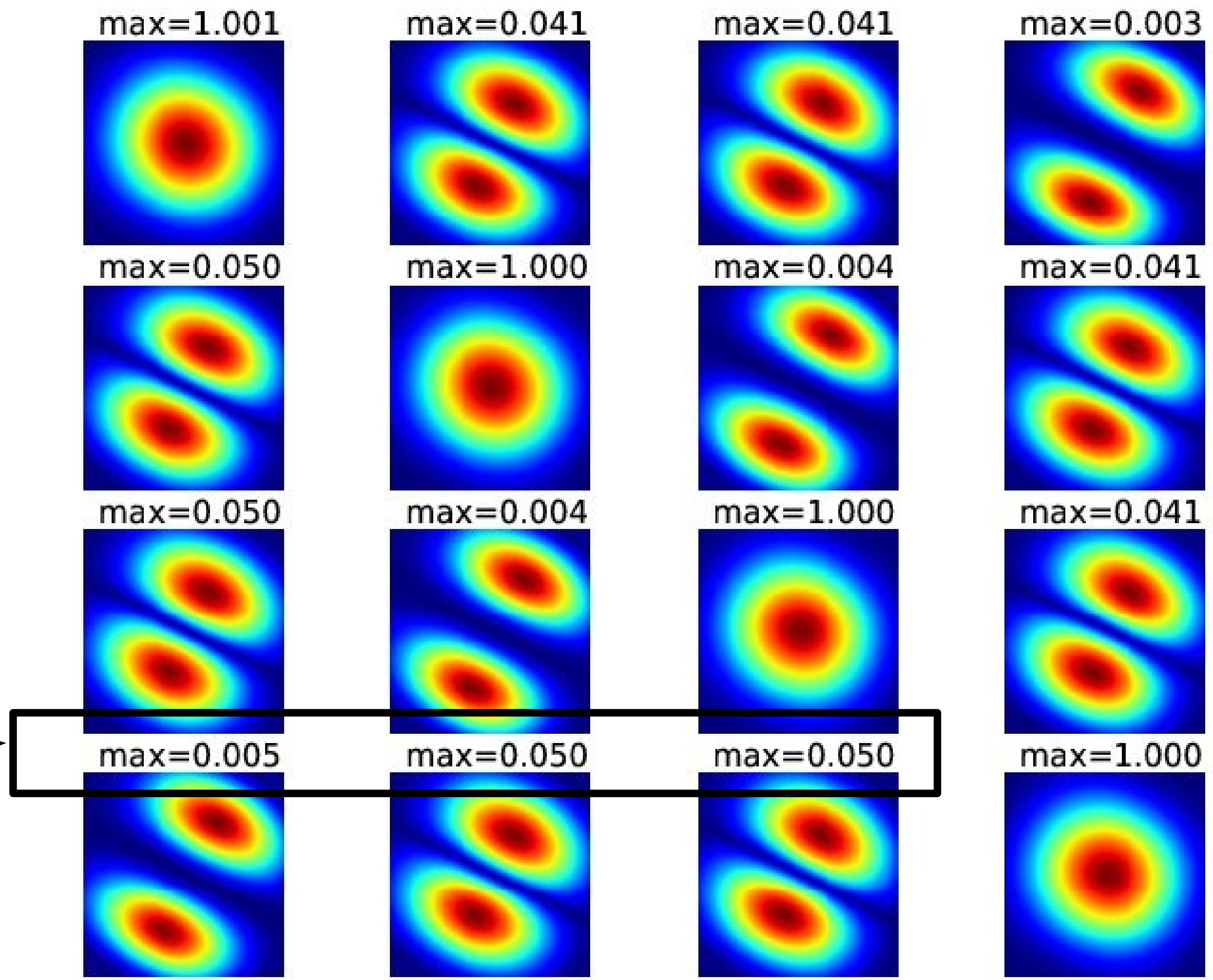



One pair of antennae, one time and frequency value

# LOFAR Beam: The Mueller Matrix varying over the image plane

**Beam bormalized by Beam Jones matrix at the center of the field (we correct the visibilities accordingly before the imaging)**

**Off axis small but perhaps not negligible for the degridding ?**




**!!! Color bar is adapted to the image here otherwise you don't see anything!!!**