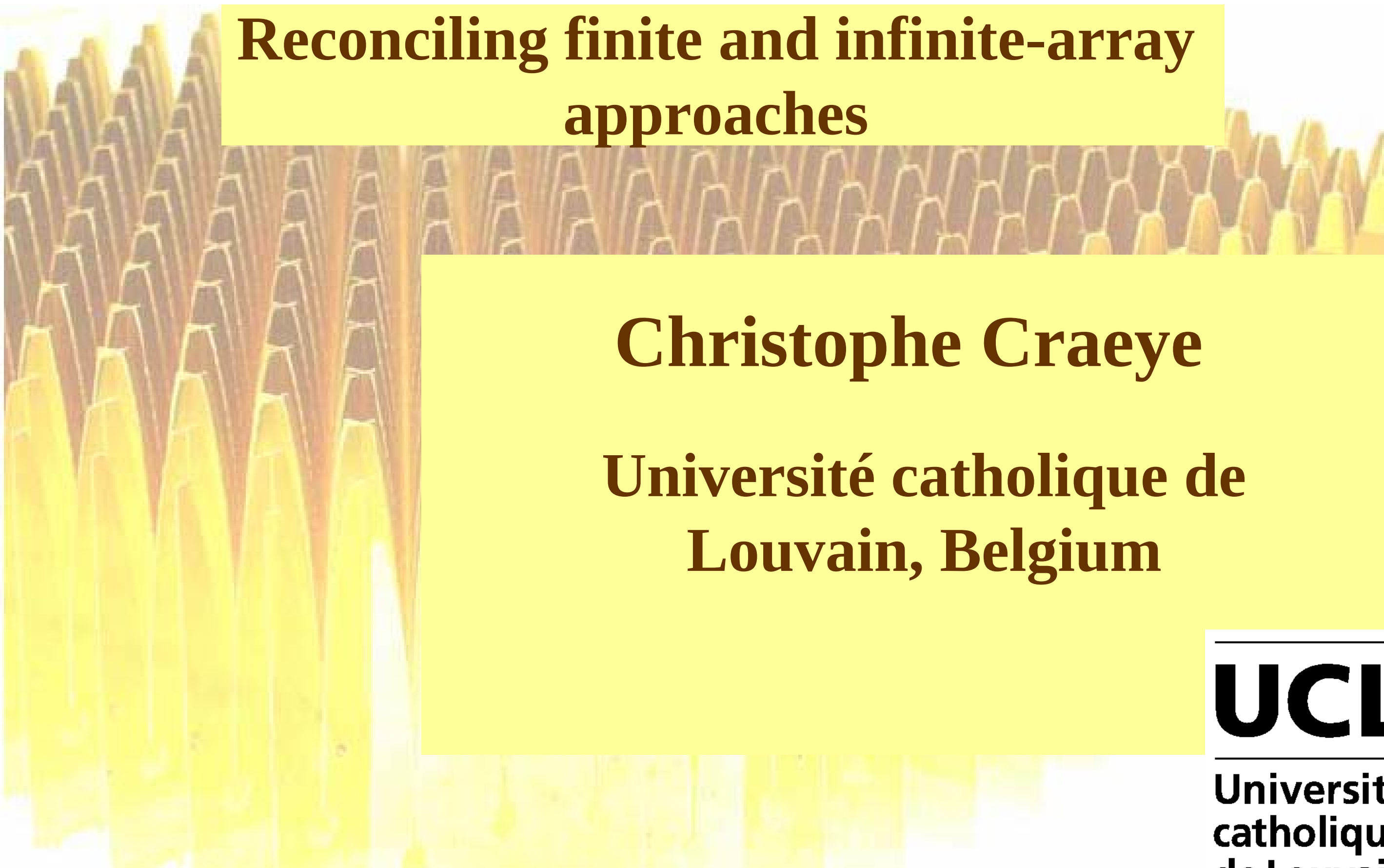
Efficient simulation of finite wideband arrays



Reconciling finite and infinite-array approaches

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Topics: metamaterials, array analysis, small wideband antennas.

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Courses taught: antennas, electromagnetics, transmission lines, numerical methods.

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Outline

- Tutorial on infinite-array versus finite-array solutions
- Macro basis functions and Array Scanning methods
- Fast solutions for finite arrays with finite dielectric parts

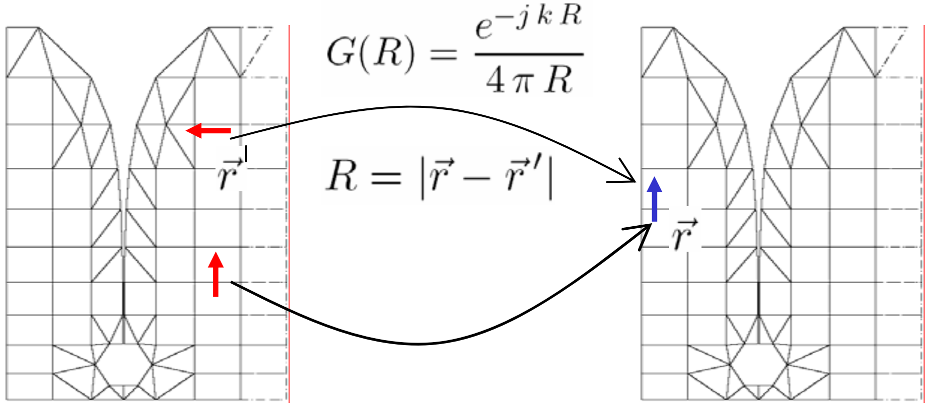
Efficient simulation of large finite arrays

Integral equation approach

$$\vec{J}(\vec{r}\,') = \sum_{i=0}^{N} x_i \, \vec{J}_{b,i}(\vec{r}\,')$$

→ Basis functions ($\vec{J}_{b,i}$)

→ Unknowns (x_i)



$$G(R) = \frac{e^{-j\,k\,R}}{4\,\pi\,R}$$

$$R = |\vec{r} - \vec{r}\,'|$$

Testing: $\iint_S \vec{J}_{t,j} \cdot \left(\vec{E}_s + \vec{E}_i\right) \, dS = 0$

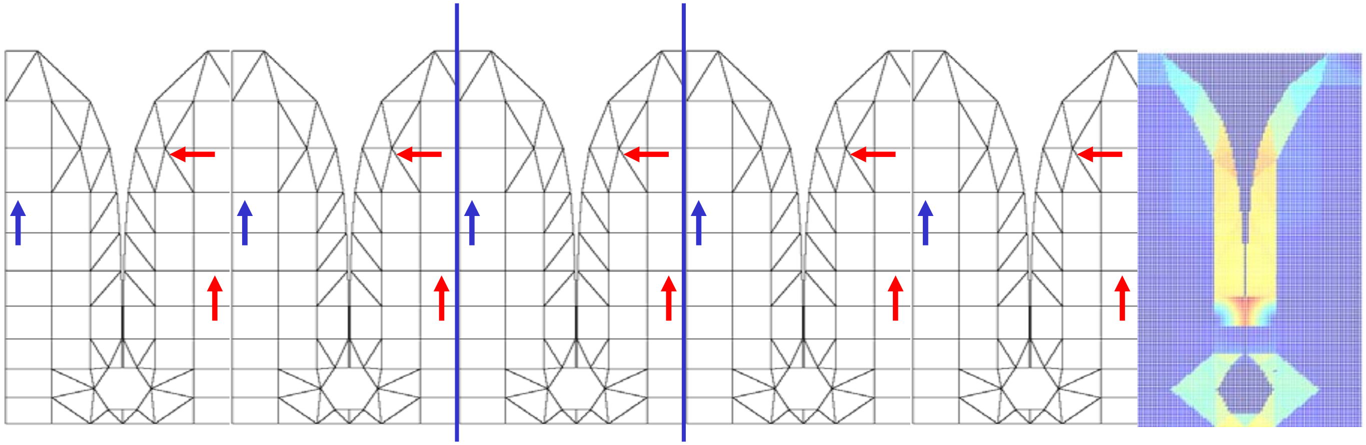
→ N equations with N unknowns

N^2 memory and N^3 CPU time

Efficient simulation of large finite arrays

Infinite array approach

Unit cell



Plane waves

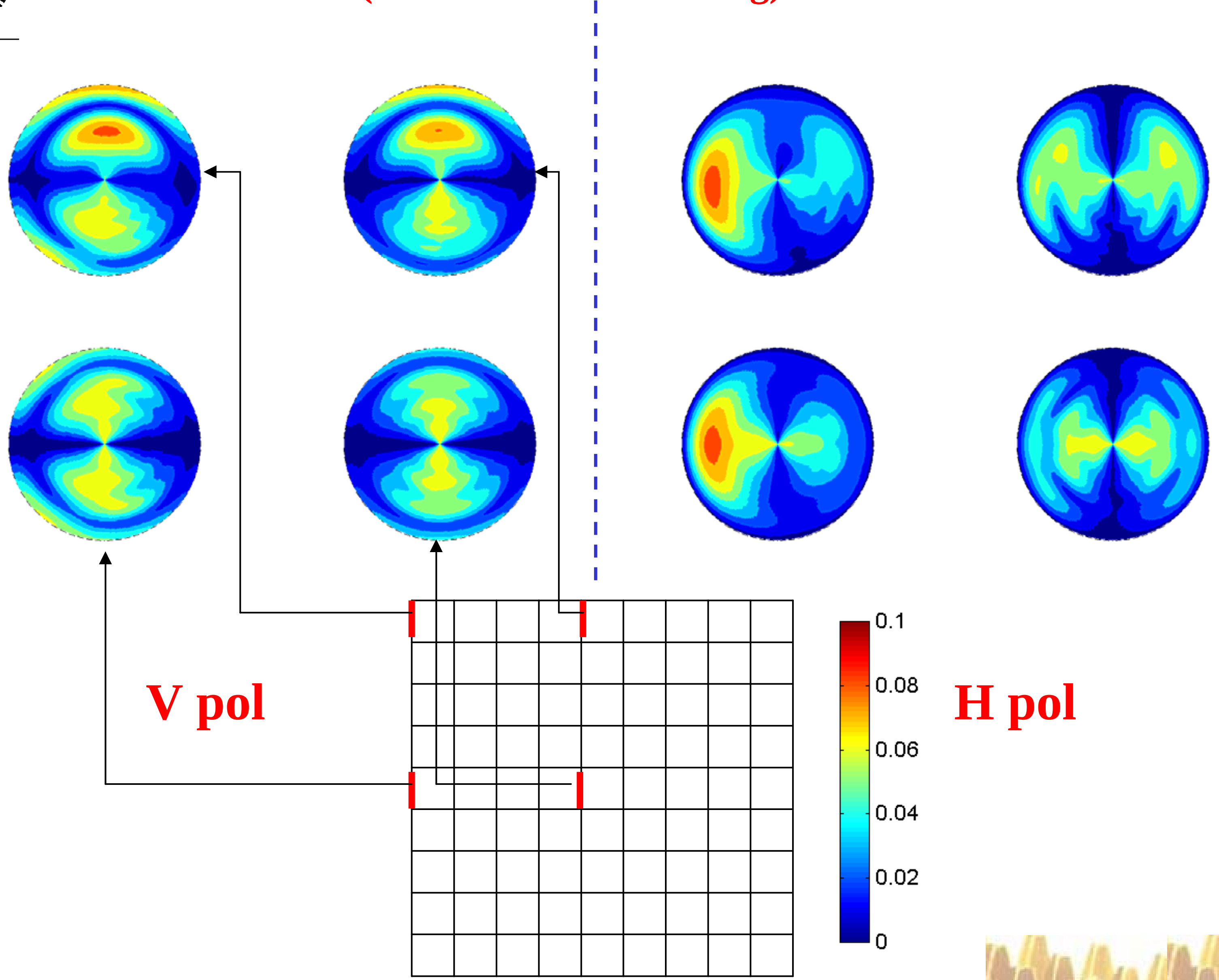
Green's function G

$$
G = \begin{cases} \displaystyle\sum_{q=-\infty}^{\infty} \sum_{p=0}^{\infty} \frac{e^{-j(k_{xp}x+k_{yq}y+k_{zpq}z)}}{2jd_xd_yk_{zpq}} & \rightarrow \text{Infinite-by-infinite} \\ \displaystyle\sum_{q=-\infty}^{\infty} \frac{e^{-j\,k_{yq}\,y}}{4\,j\,d_y} H_0^{(2)}\left(k_{\rho q}\,R_m\right)\ e^{-j\,m\,k_{x0}\,d_x} & \rightarrow \text{Finite-by-infinite} \end{cases}
$$

Cylindrical waves

Element patterns at 1 GHz : exact

(from 2005 FPA meeting)



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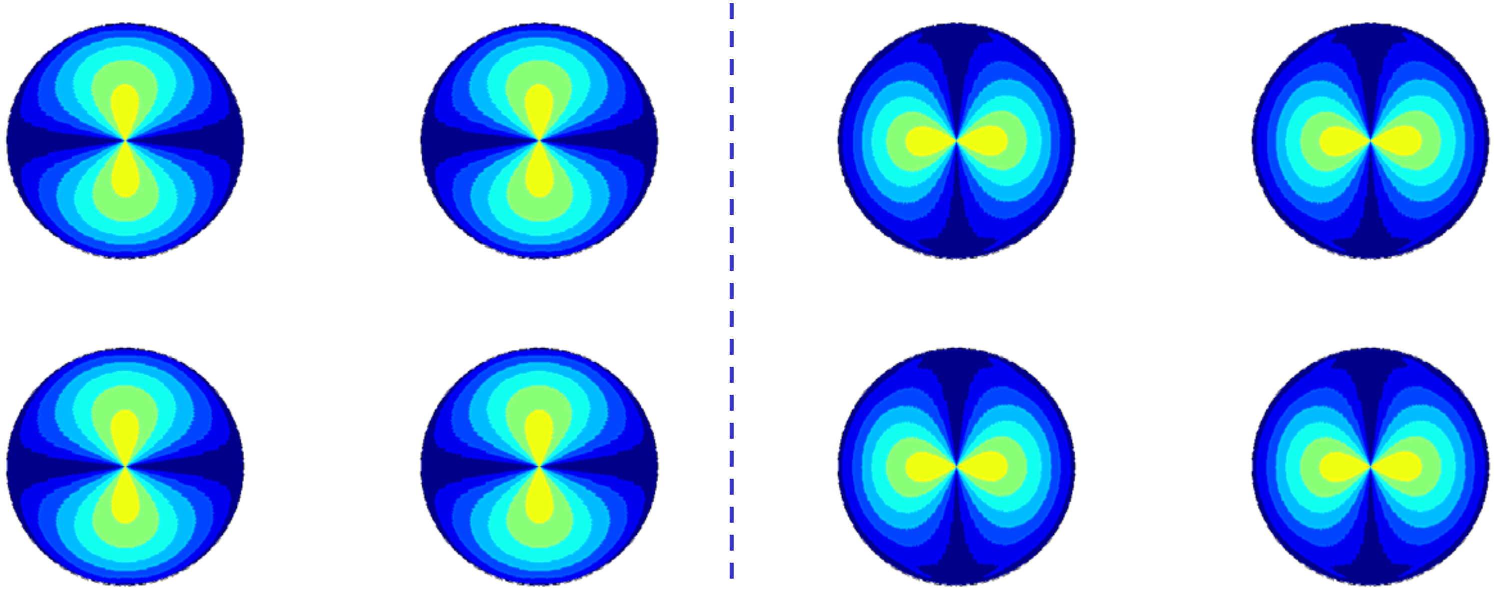
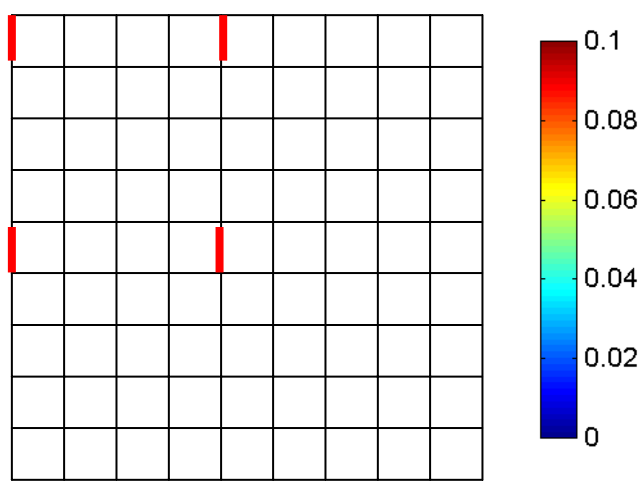
Element patterns at 1 GHz : infinite array (from FPA2005 meeting)



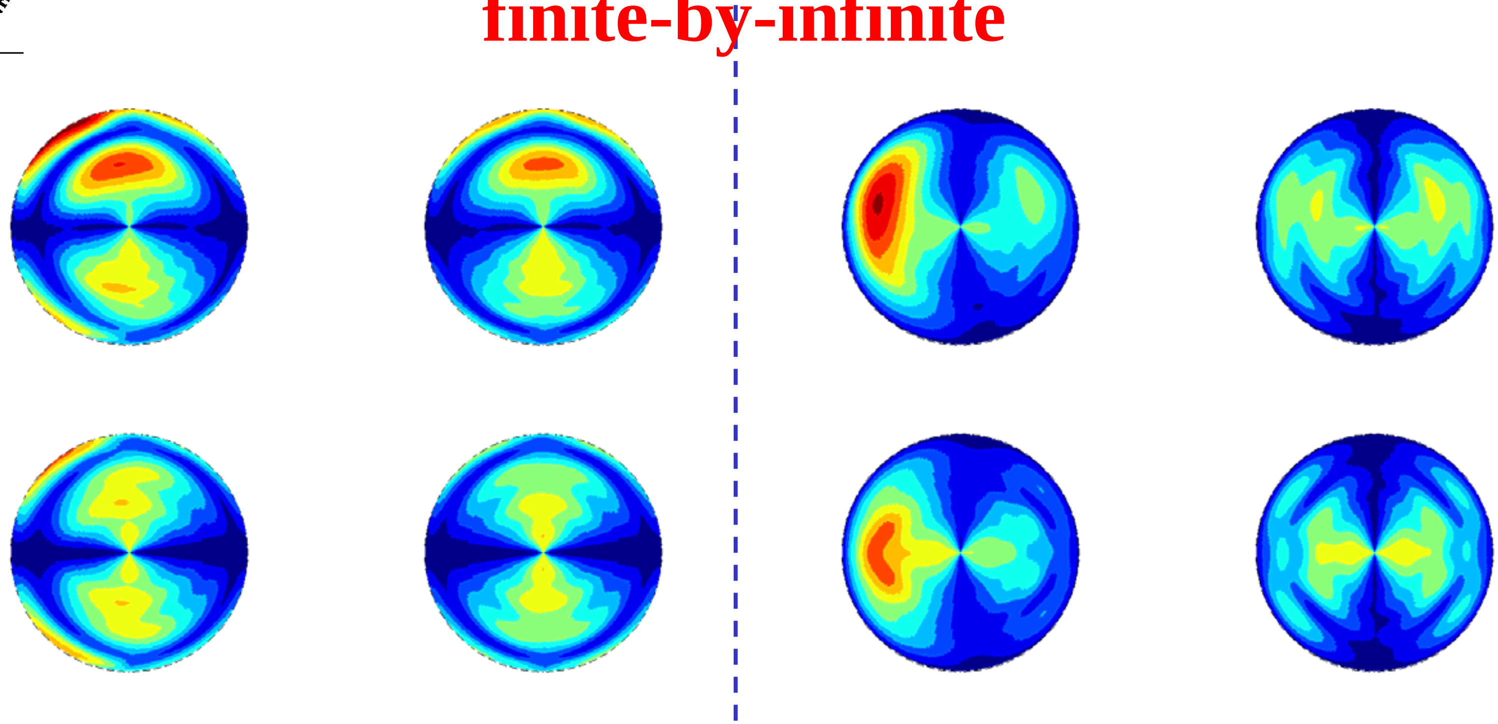
V pol

H pol

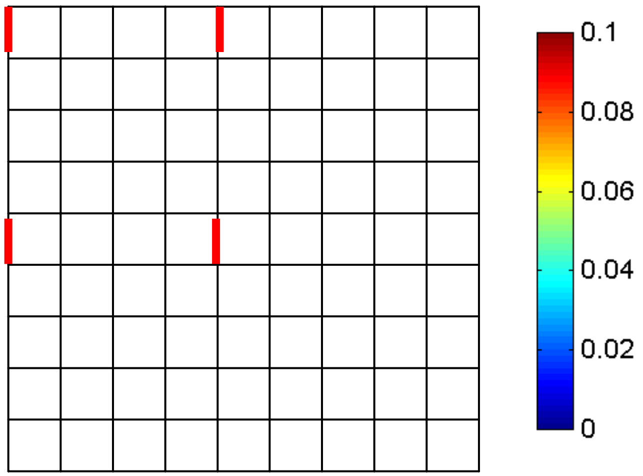


Array seen from top.
Elements are 12.7x20.8 cm

Element patterns at 1 GHz : finite-by-infinite

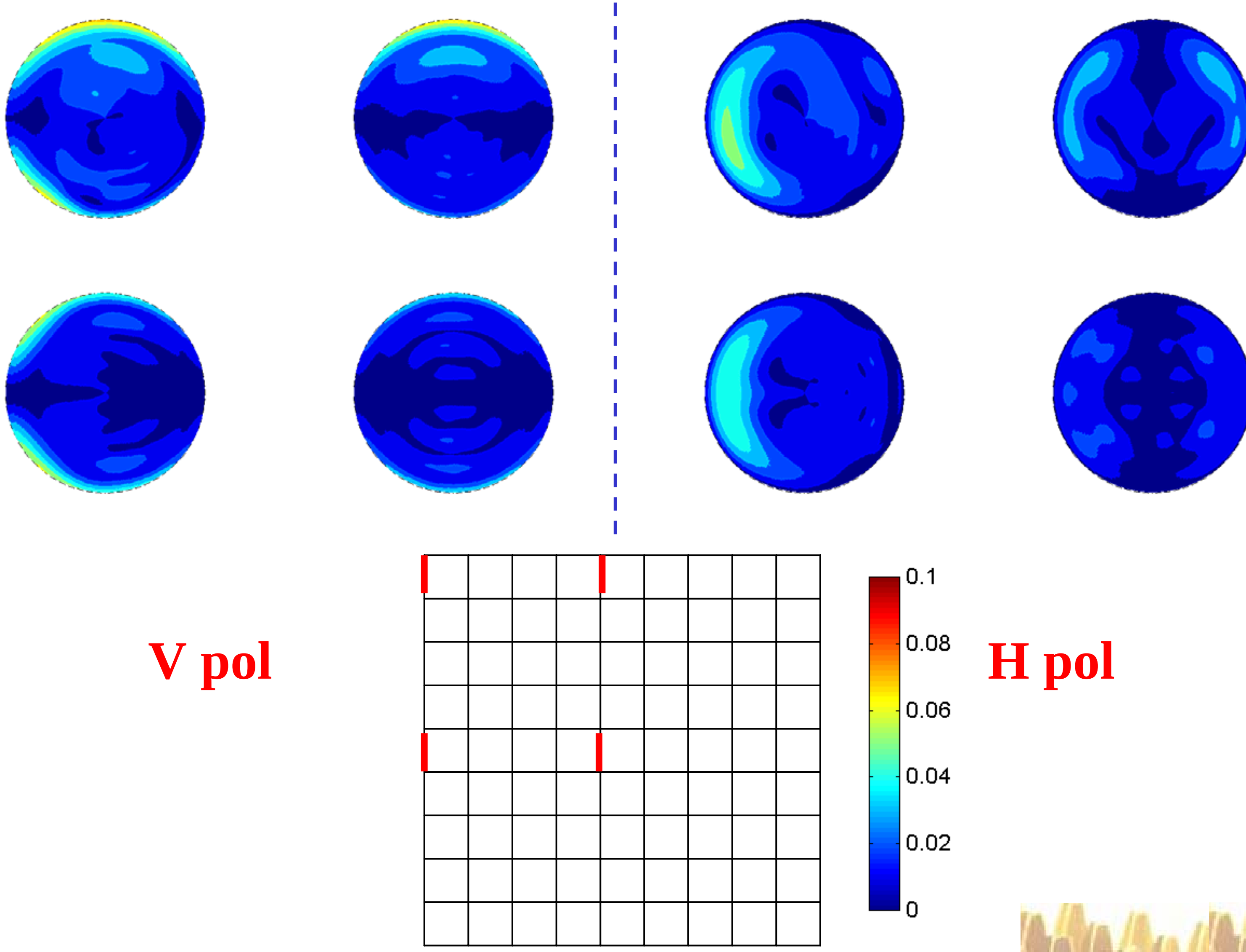


V pol

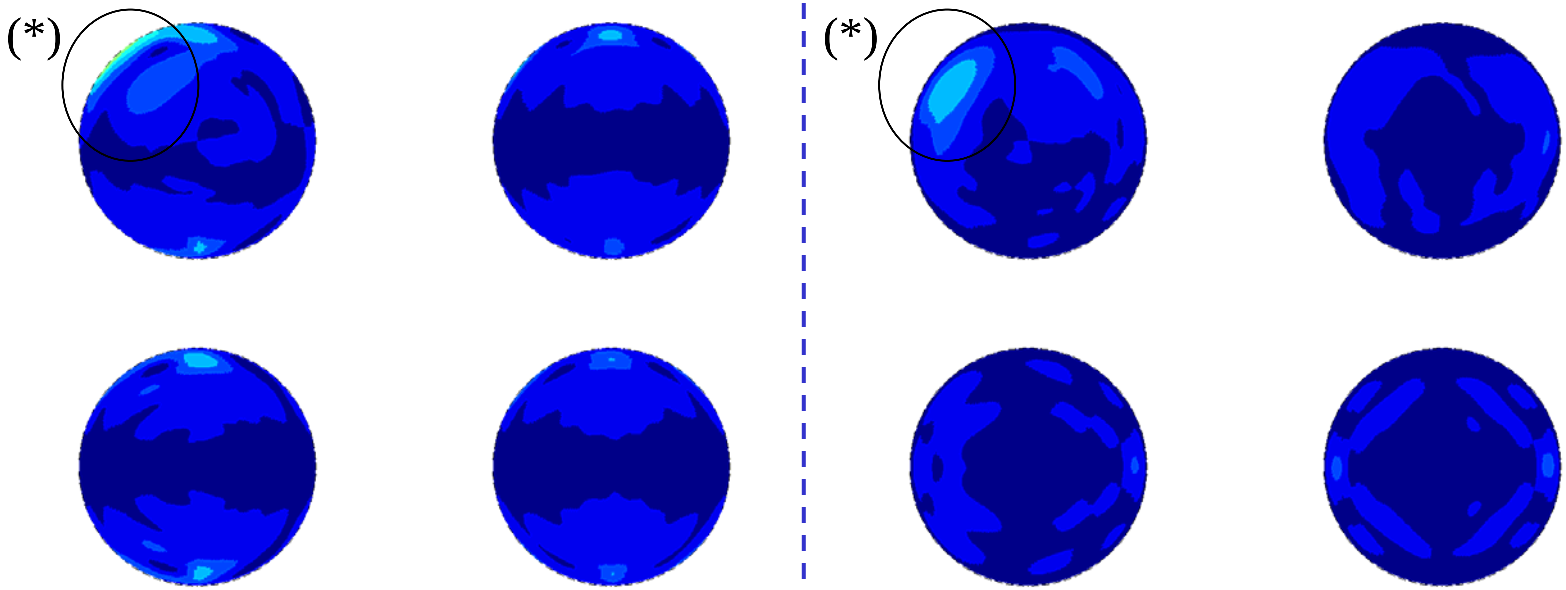


H pol

Element patterns at 1 GHz : error from infinite-array

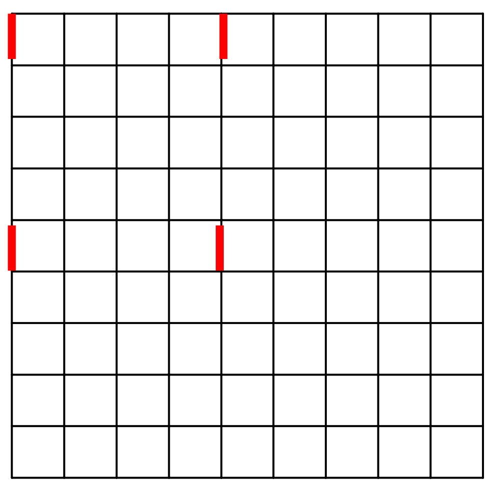


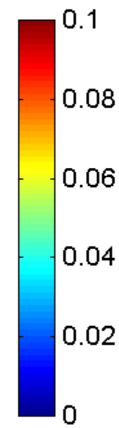
Element patterns at 1 GHz : Error from finite-by-infinite



V pol

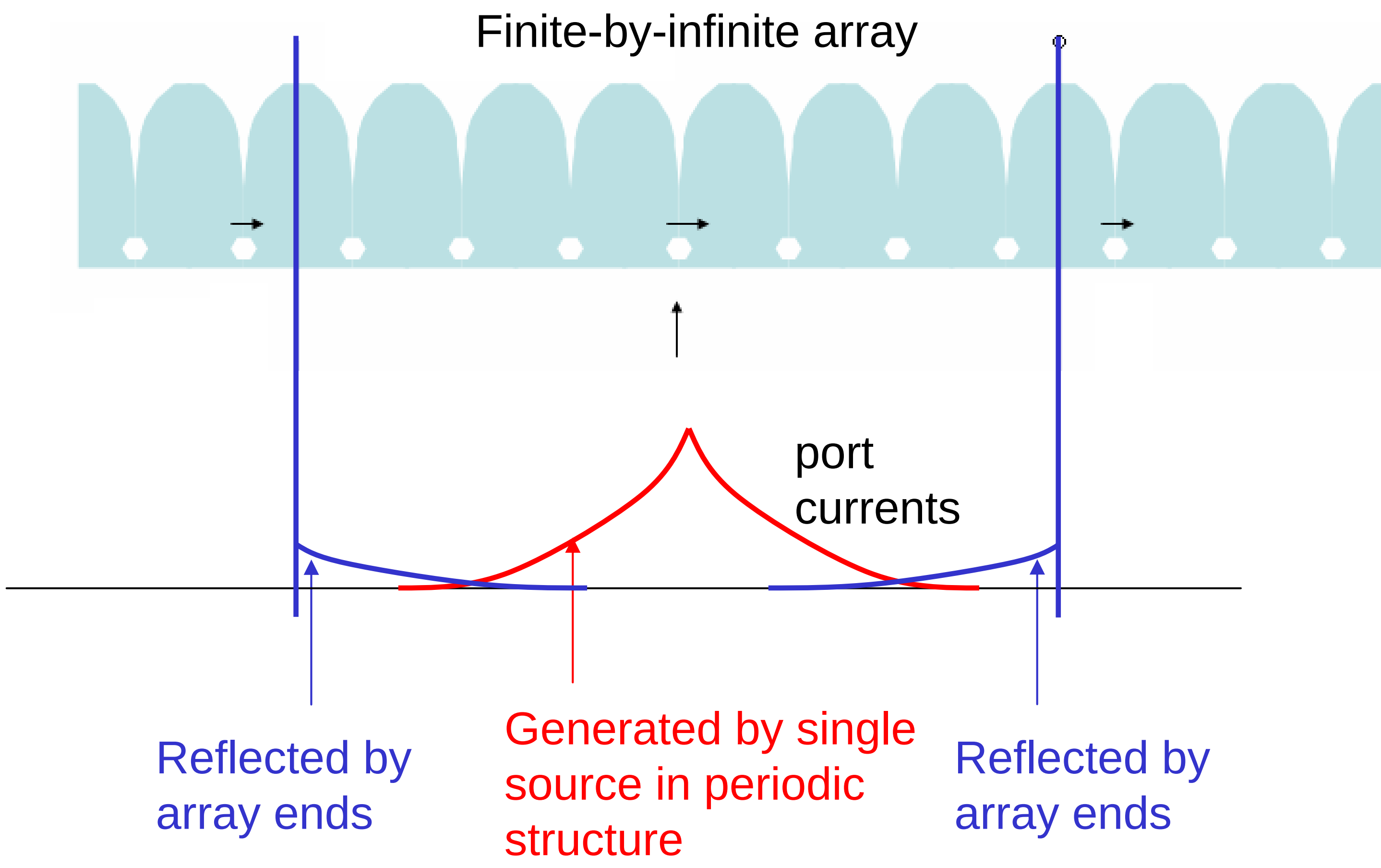
H pol





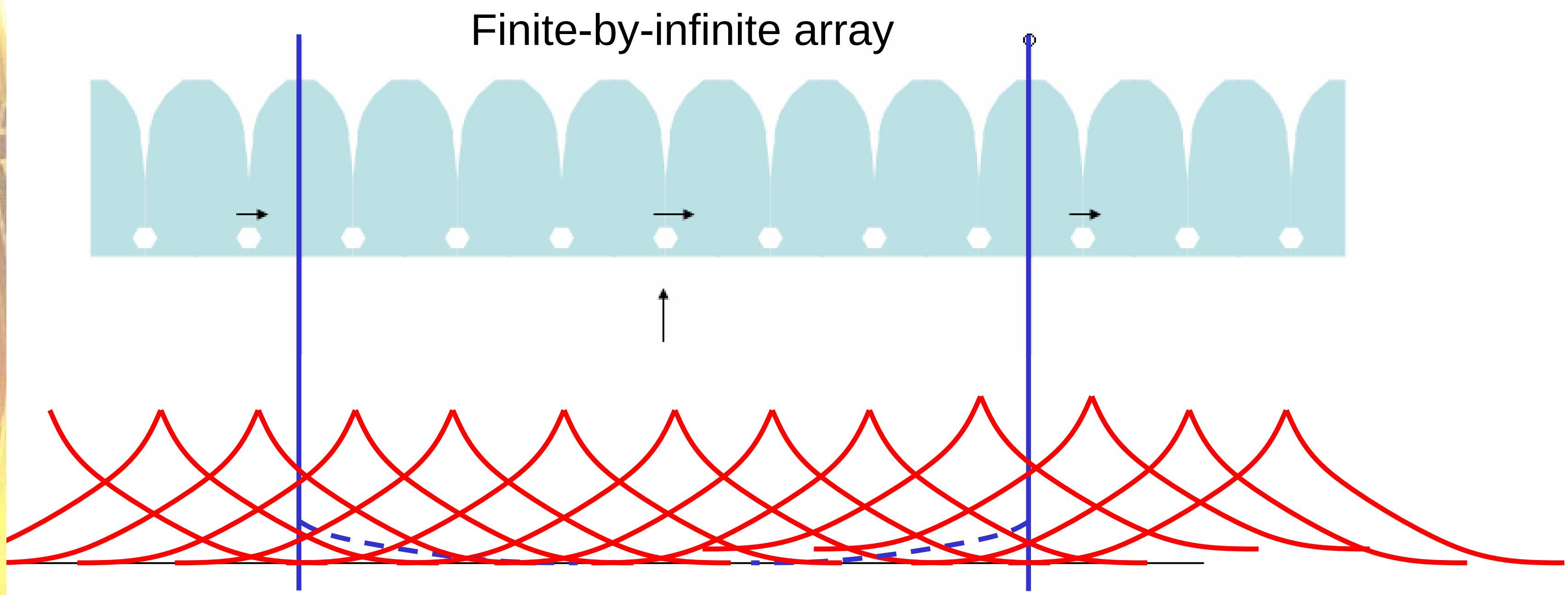
(*) larger errors in corners corrected with special procedure

Wave phenomenology in finite arrays



~ same distributions as forward wave, except for edge elements

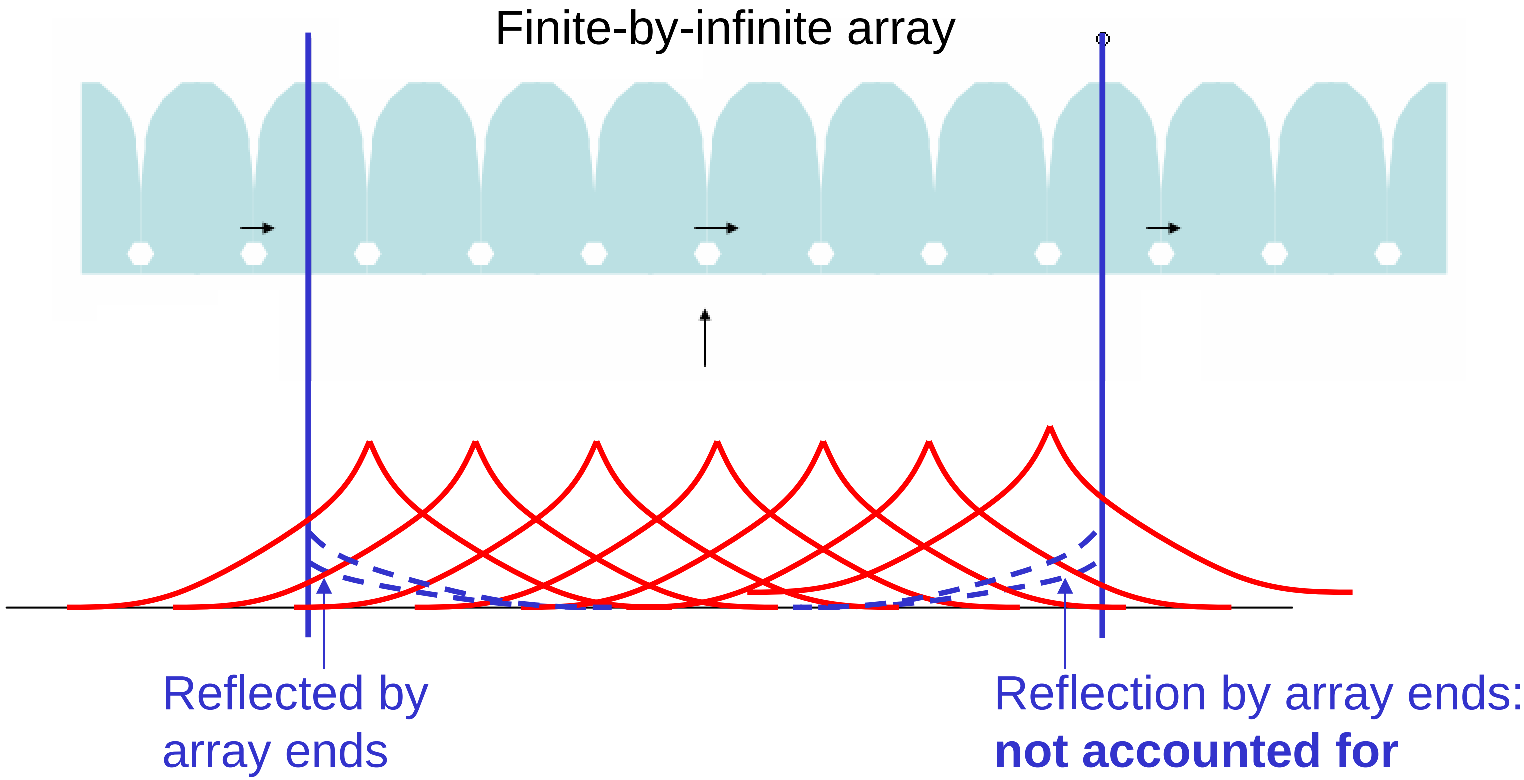
Brute infinite-array solution



Brute truncation of infinite-array current distributions:

- scattering by edges is omitted
- parasitic contributions from complementary arrays
- limited to periodic excitation

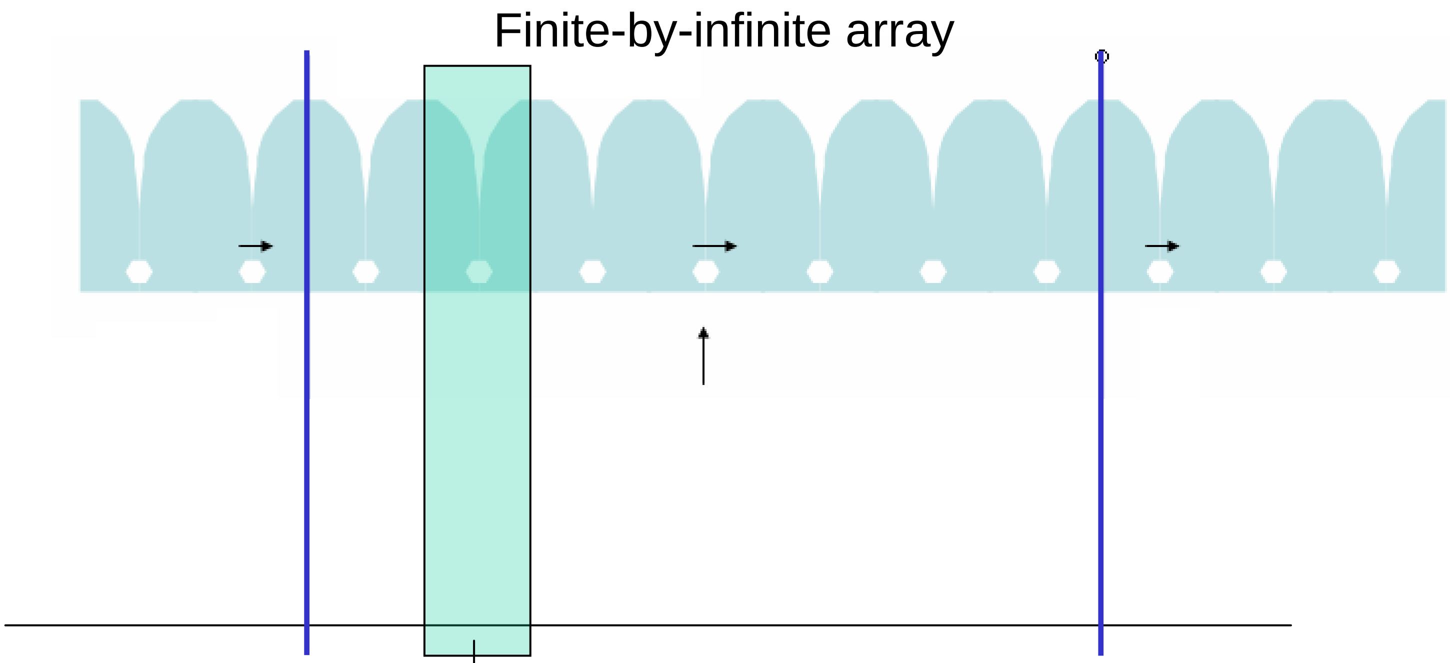
Windowing 1 method



Array pattern= embedded element pattern, sampled and convoluted by FT of window

A. Roederer, ``Etude des réseaux finis de guides rectangulaires à parois épaisses,'' *L'onde Electrique*, vol. 51, pp. 854-861, Nov. 1971.

Windowing 2 method

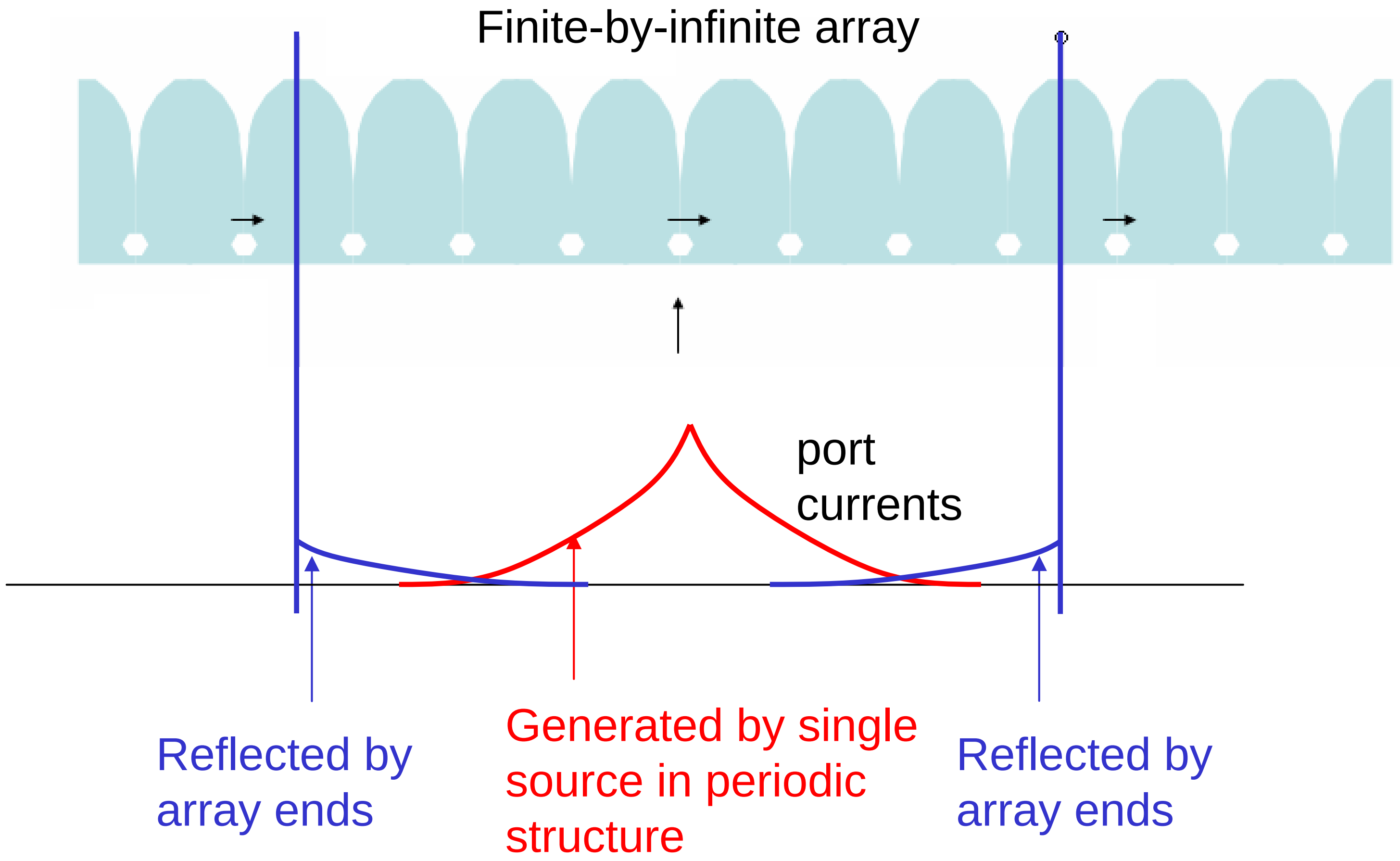


Solve for this element

Assuming that others have the same (phase-shifted) current distribution (periodic excitation)

A.K. Skrivervik and J.R. Mosig, Analysis of finite phased arrays of microstrip patches, *IEEE Trans. Antennas Propagat.*, vol. 41, pp. 1105-1114, Aug. 1993.

Wave phenomenology in finite arrays



~ same distributions as forward wave, except for edge elements

Array Scanning Method with finite resolution

$$\underbrace{I(m)}_{\text{Current at ant. m for ant. 0 excited}} = \frac{1}{2\pi} \int_0^{2\pi} \underbrace{I^{\infty}(\psi)}_{\text{Infinite-array solution for phase shift } \psi \text{ between elements}} e^{-j\, m\, \psi}\, d\psi$$

Current at ant. m
for ant. 0 excited

Infinite-array solution for phase shift ψ between elements

$$I(m) \simeq \frac{1}{N} \sum_{p=0}^{N-1} I^{\infty}(\psi_p)\, e^{-j\, m\, \psi_p}$$

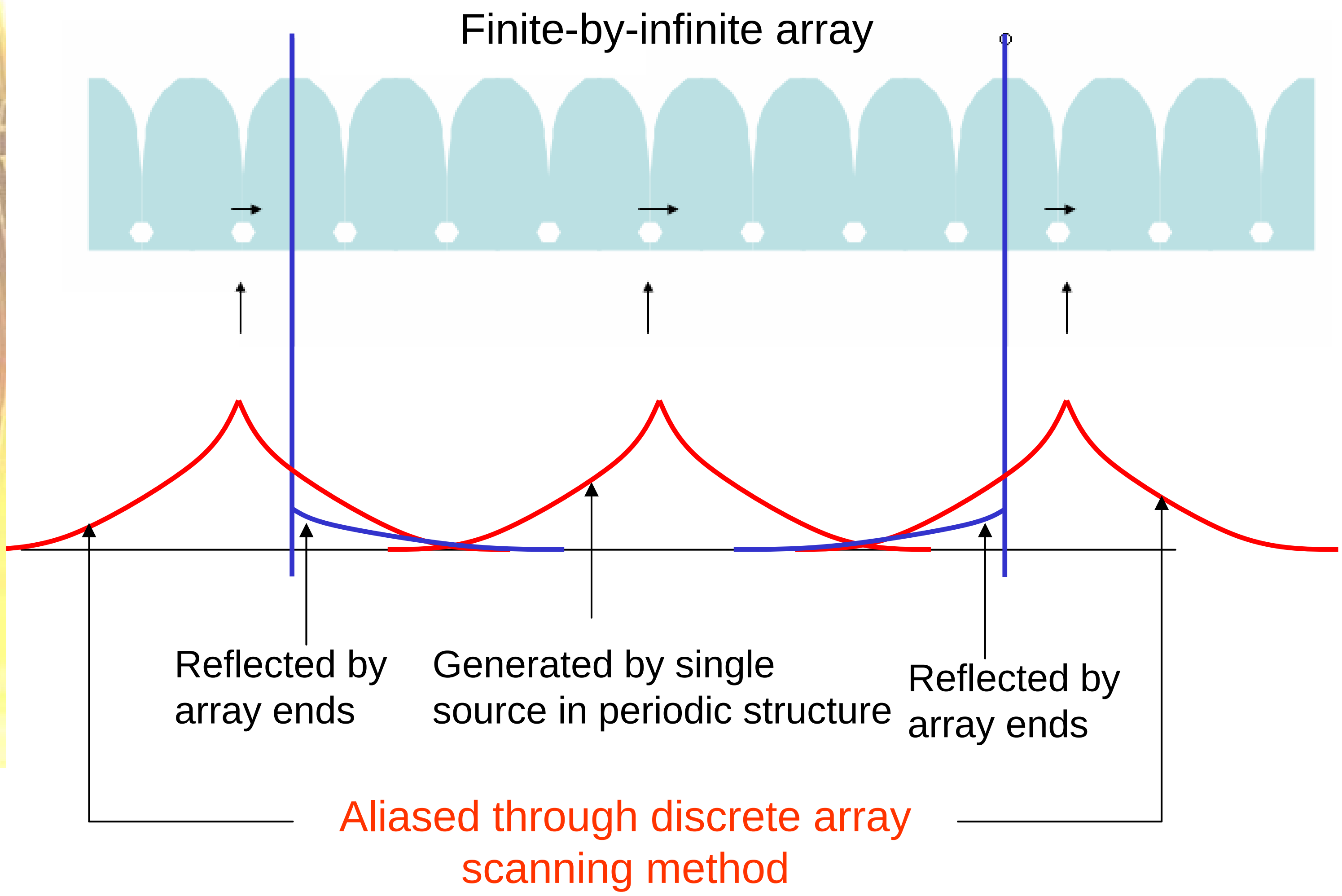
$$\psi_p = 2\,\pi\, p/N$$

Aliasing:

Repetition of source every N elements

15 min

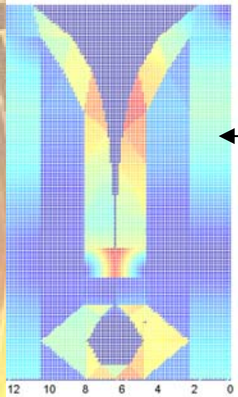
Array Scanning Method with finite resolution



C. Craeye and R. Sarkis, Finite array analysis through combination of Macro Basis Functions and Array Scanning methods, ACES conference 2007.

Macro Basis Function approach

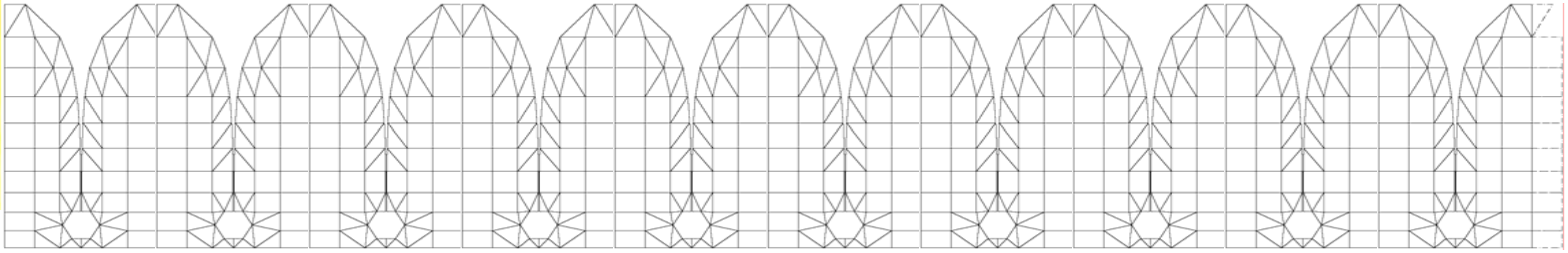
Current distribution = superposition of distributions obtained in small problems



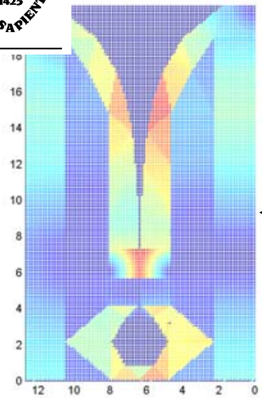
e.g. a single antenna with « a whole spectrum » of excitations

Problem with connected elements: edge currents

Possible solution: extend the element somewhat (cf. Maaskant et al.) and/or introduce some resistive tapering at the edges.

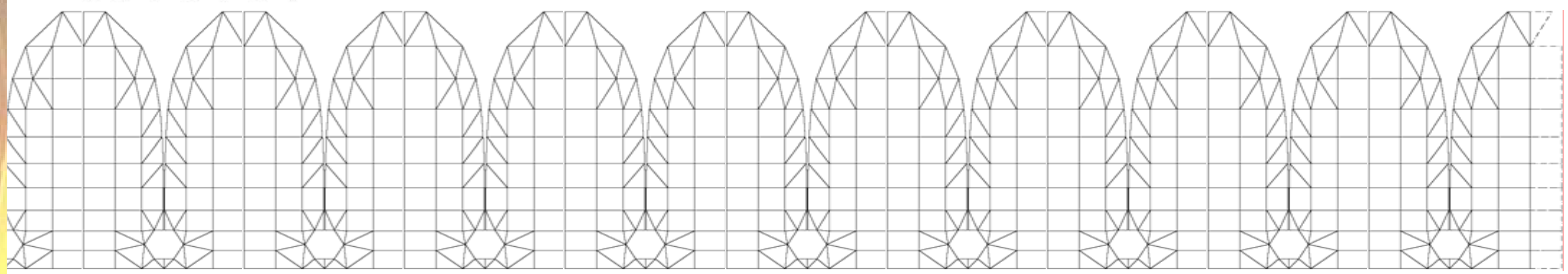


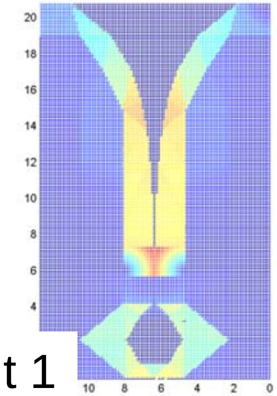
Macro Basis Function approach



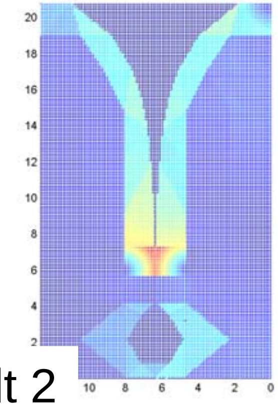
Isolated element

Use ASM solutions as MBF's ☺ !

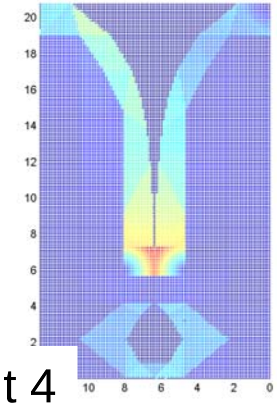




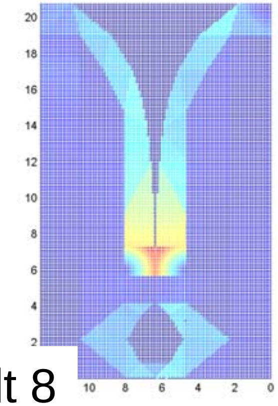
Elt 1



Elt 2



Elt 4



Elt 8

Macro Basis Function approach

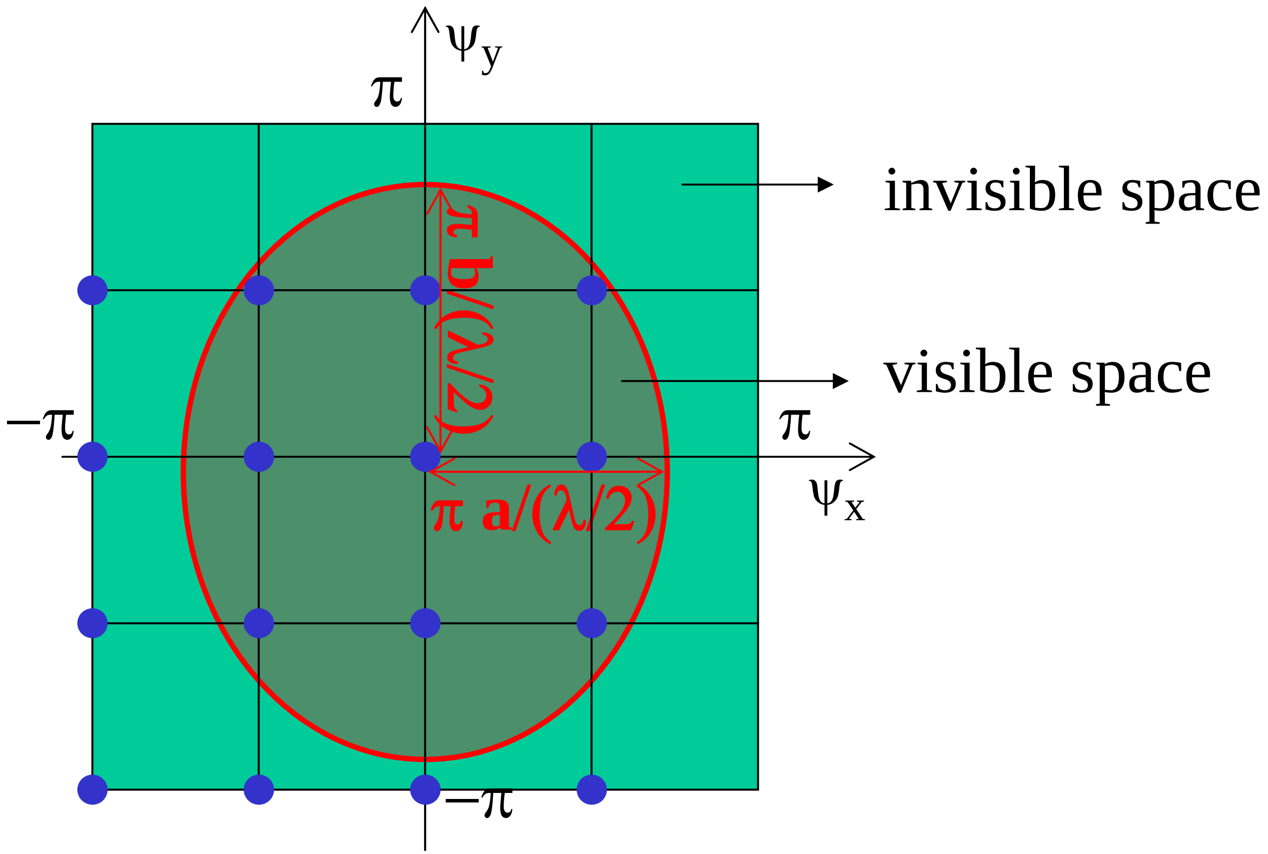
Use ASM solutions as MBF's ☺ !

ASM solutions: linear combinations of infinite-array solutions

Use infinite-array solutions !

Requirement: regularly distributed in reciprocal space (ψ space)

Macro Basis Function approach

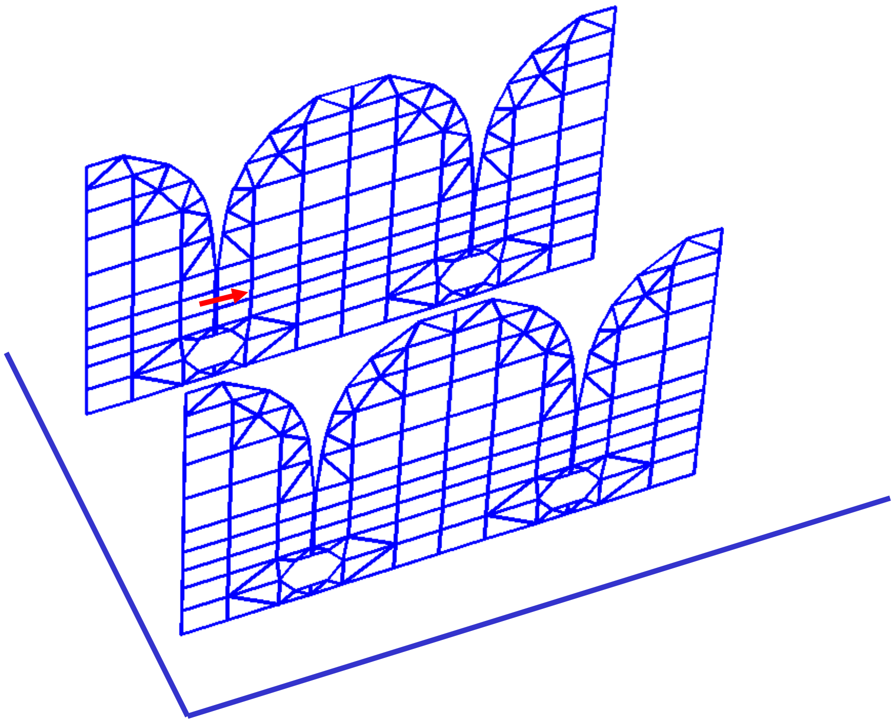


Requirement: regularly distributed in reciprocal space (ψ space)

NB: Can be relaxed somehow (under study)

Add edge current distributions

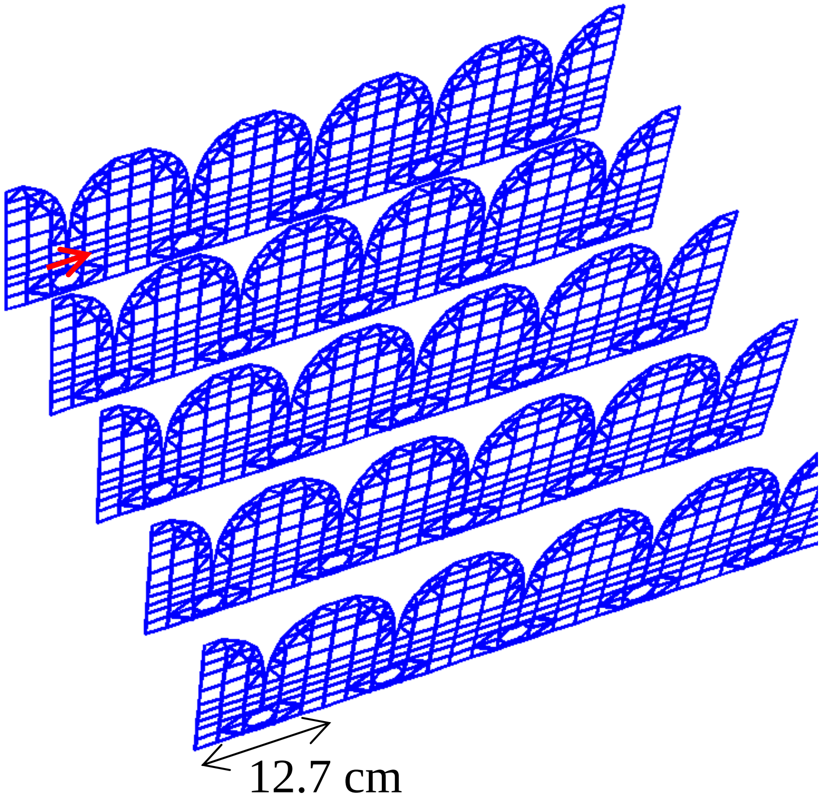
2X2 array simulations

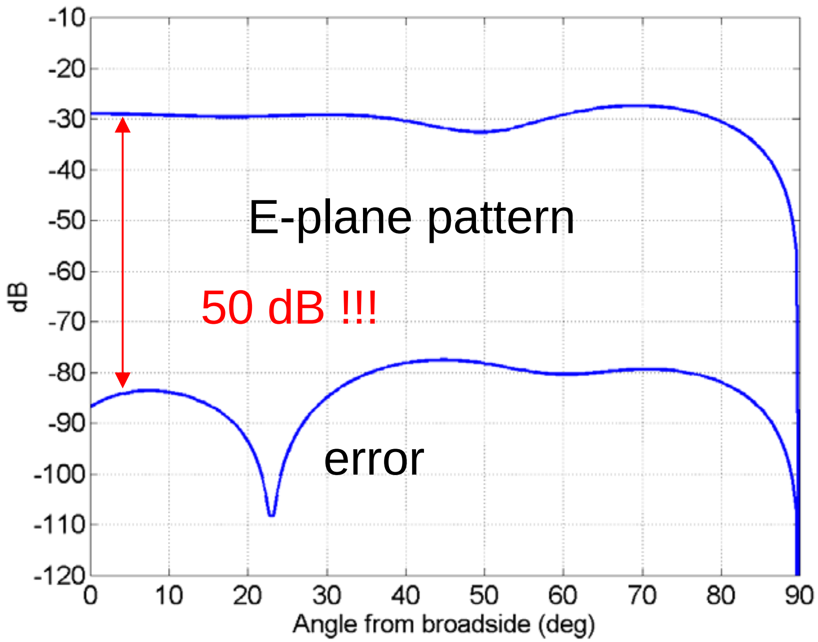


→ add 8 independent current distributions

NB: can be reduced to single cell problem through symmetry

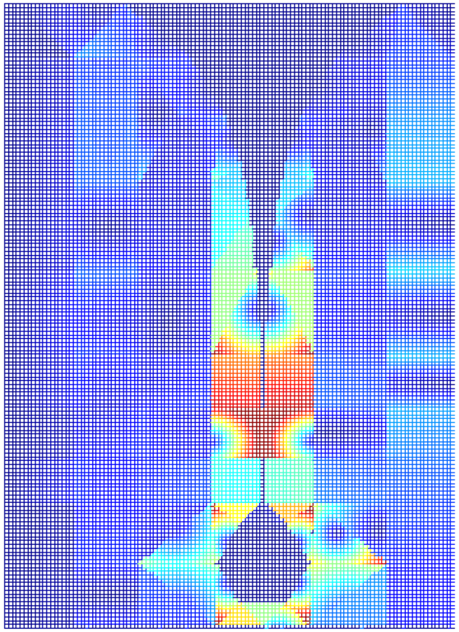
Numerical results for a 5X5 array



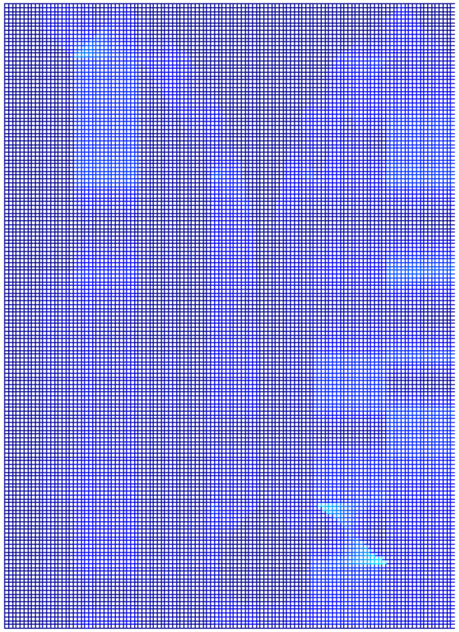


Embedded element pattern for corner element at 1 GHz

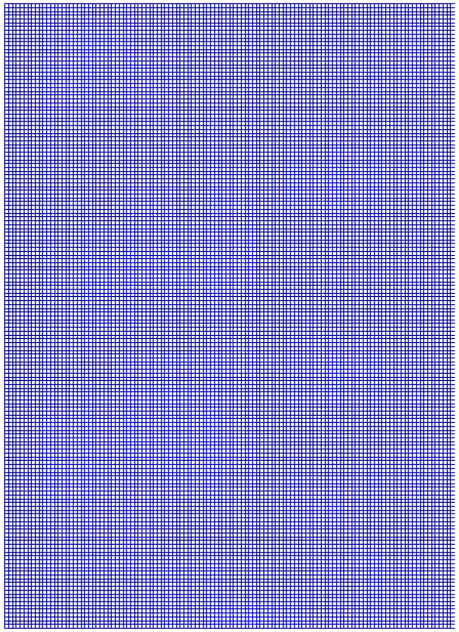
Current distribution in « tough » truncation case



λ=60, corner element, for center element excited

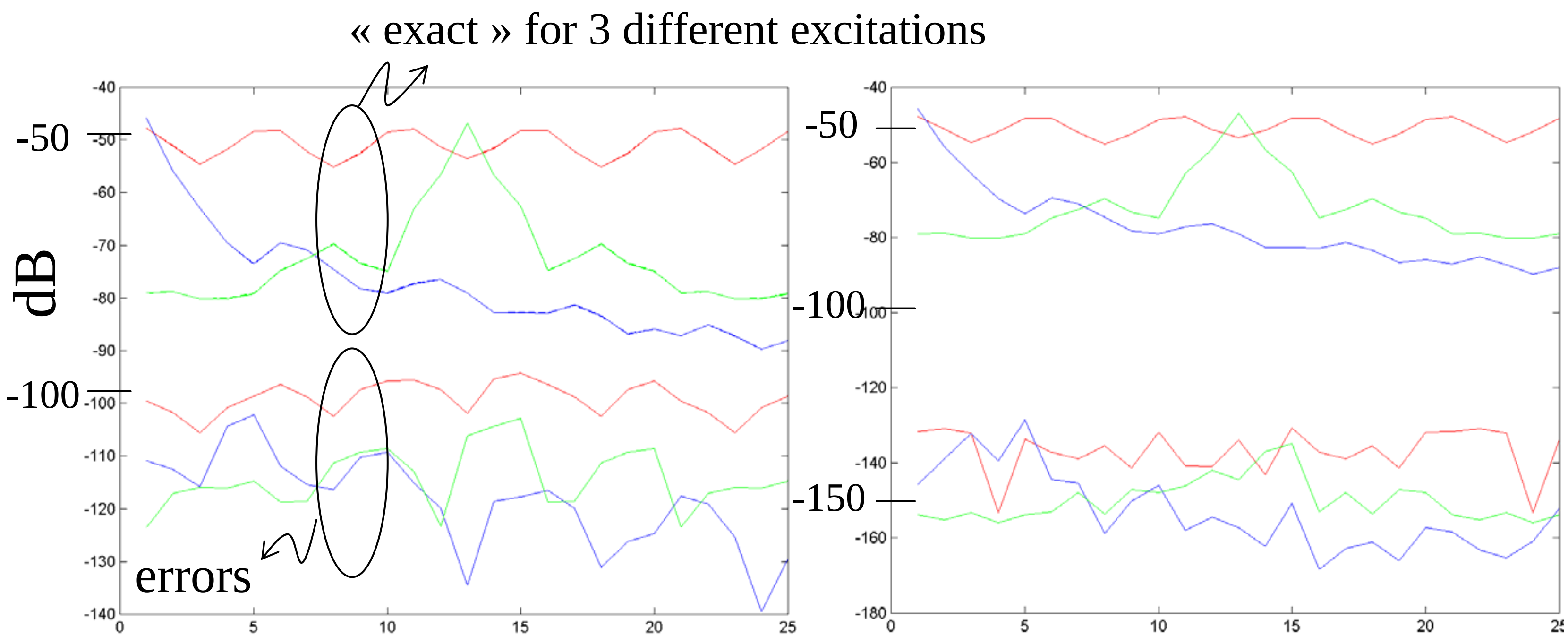


Error for 2X2 ASM



Error for 4X4 ASM

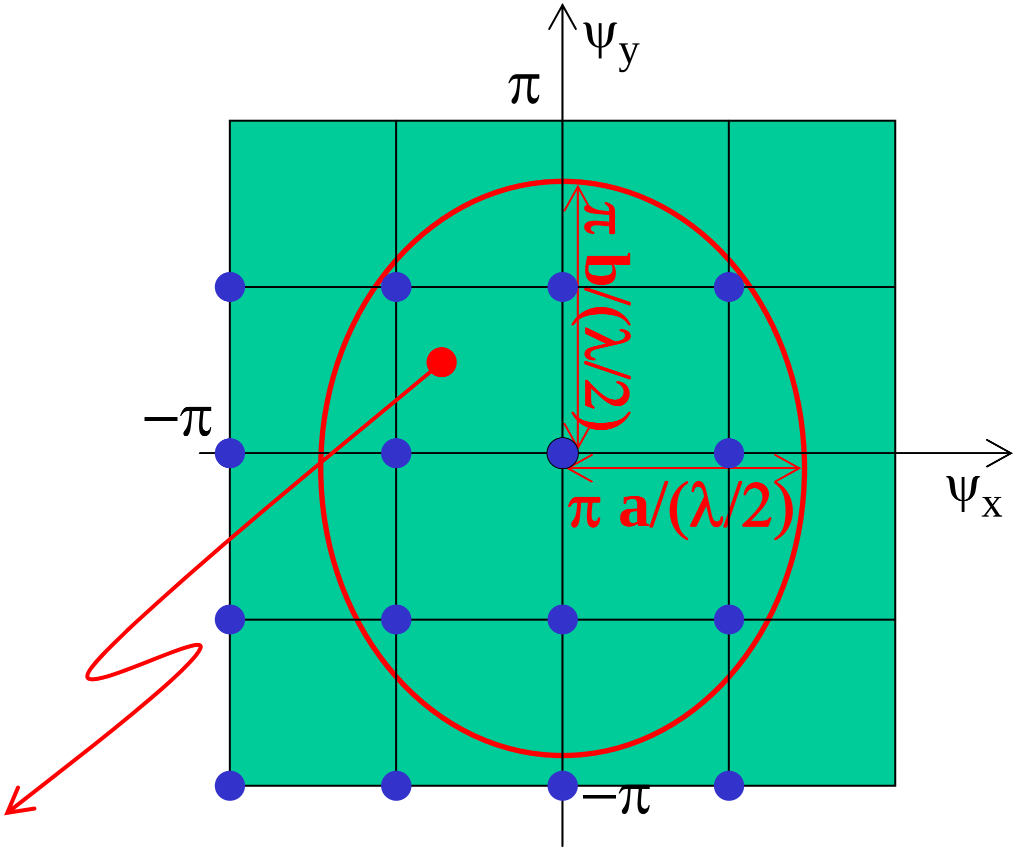
Port currents at 500 MHz



MBFs from 2X2 ASM

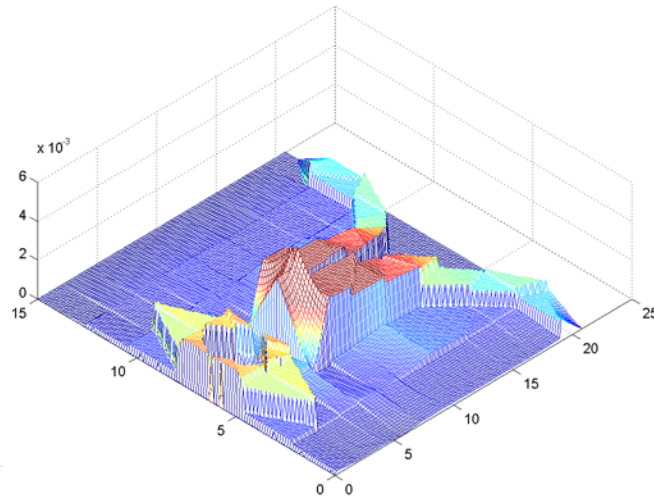
MBFs from 4X4 ASM

Analysis of anomalies

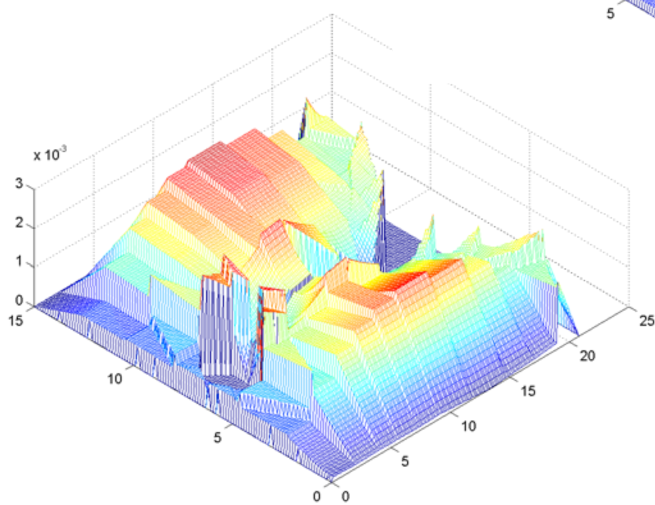


Singularities lead to slowly decaying current distributions.
Also captured in spectral (ψ_x, ψ_y) domain.

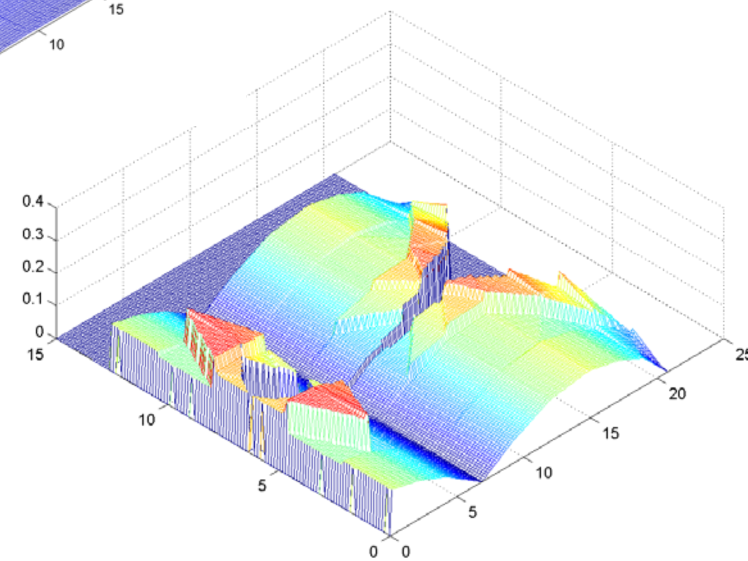
Analysis of anomalies



1 GHz



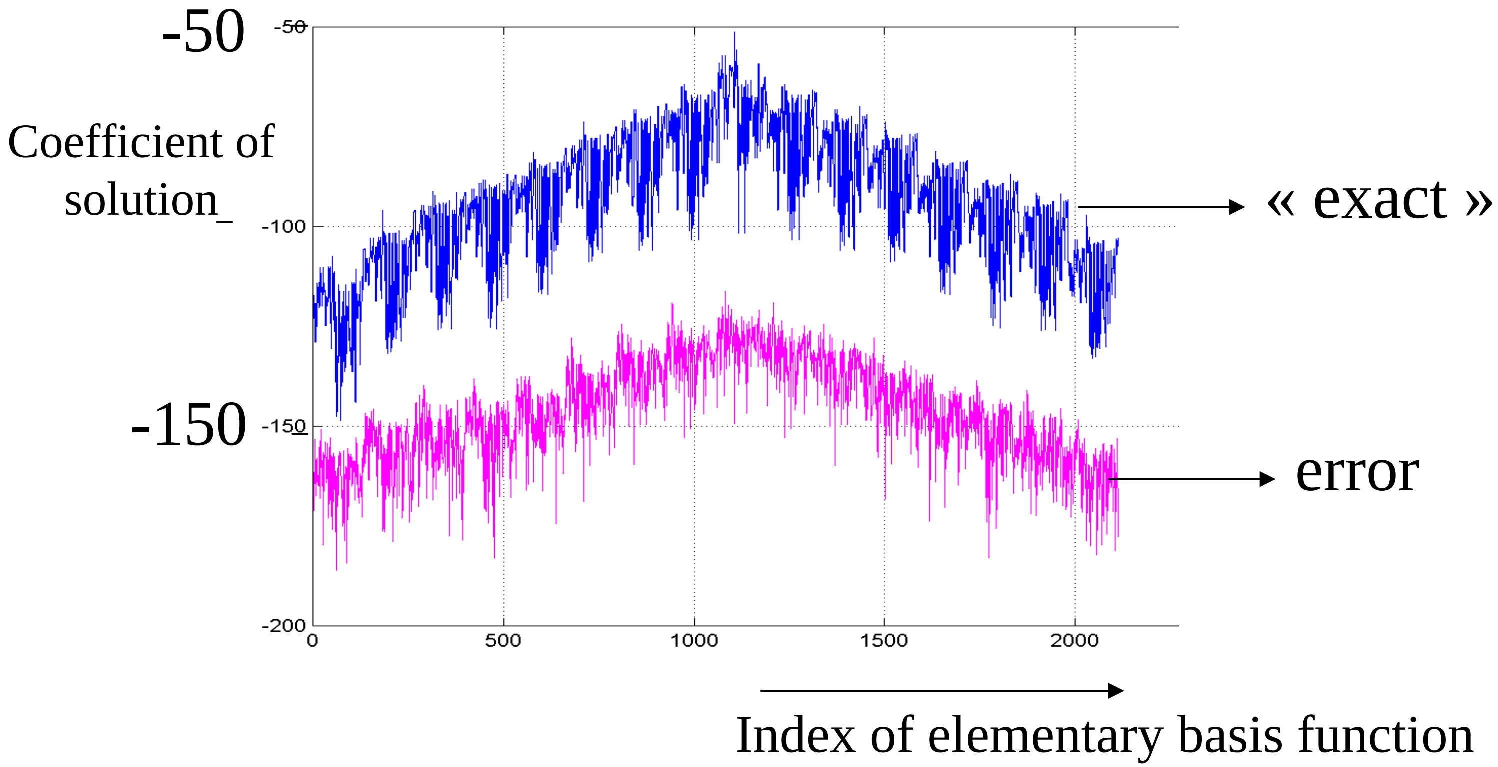
1.56 GHz anomalous currents at broadside



1.56 GHz, eigenvector with lowest eigenvalue

Analysis of anomalies

16x∞ array with element 9
excited at 1.56 GHz

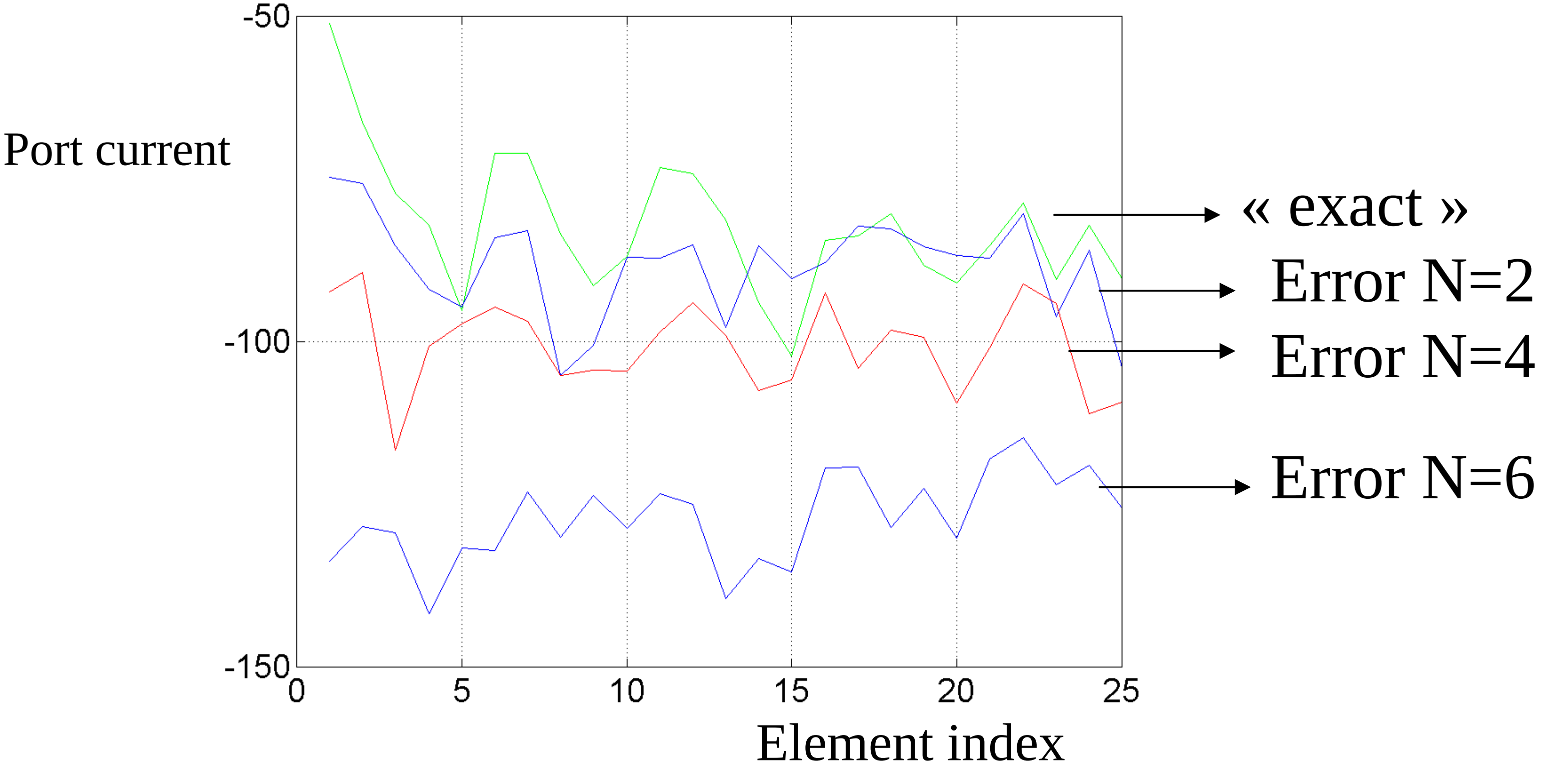


Only 8 MBFs are used

NB: anomaly outside regular grid can be treated in similar way

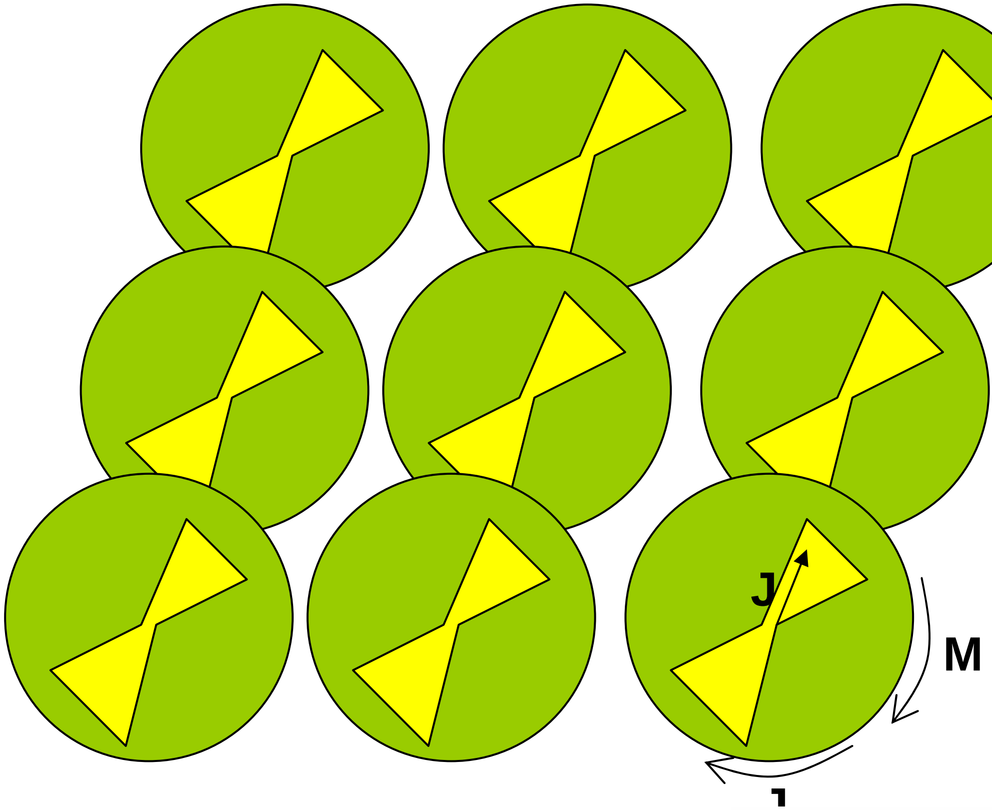
Analysis of anomalies

5x5 array with first element excited at 1.56 GHz



Fast treatment of elements with finite dielectrics

Method of Moments, with surface equivalence



Vector of unknowns on a given antenna: $\mathbf{x} = \{\mathbf{j}_m, \mathbf{j}_s, \mathbf{m}_s\}$

30 min

Structure of MoM impedance matrix

$$\begin{pmatrix} Z_{11} & Z_{12} & & Z_{1N} \\ Z_{21} & Z_{22} & & \\ & & \ddots & \\ Z_{N1} & & & Z_{NN} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix} = \begin{pmatrix} v_1 \\ \vdots \\ v_N \end{pmatrix}$$

Diagonal block

Z^{EJ}	Z^{EJ}	Z^{EM}	metal
Z^{EJ}	Z^{EJ}	Z^{EM}	dielectric
Z^{HM}	Z^{HJ}	Z^{HM}	
metal	dielectric		

Off-diagonal block

0	0	0
0	Z^{EJ}	Z^{EM}
0	Z^{HJ}	Z^{HM}

Large time saving for complex elements

Choice of Macro Basis Functions

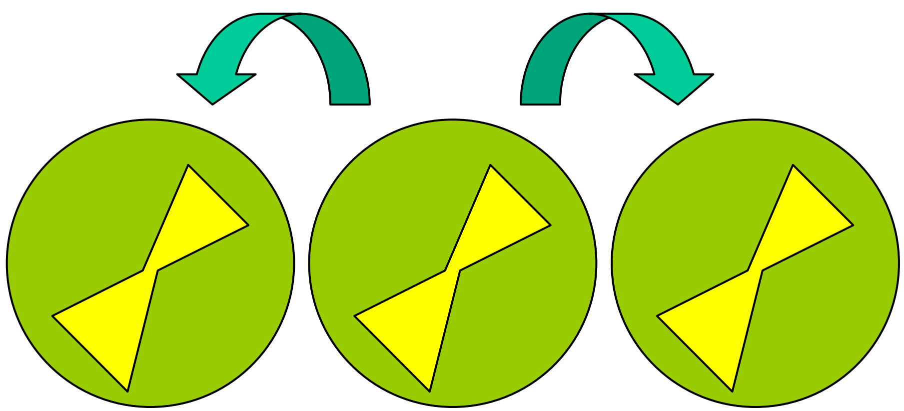
Unknowns

$$\{\mathbf{j_m}, \mathbf{j_s}, \mathbf{m_s}\} = \sum_{p=1}^{P} C_p \, \{\mathbf{j}^{\circ}_{\mathbf{m,p}}, \mathbf{j}^{\circ}_{\mathbf{s,p}}, \mathbf{m}^{\circ}_{\mathbf{s,p}}\}$$

Solution on any given unit cell

Solutions from small problems

« macro » or « characteristic » basis functions



secondary solution | primary solution | secondary solution

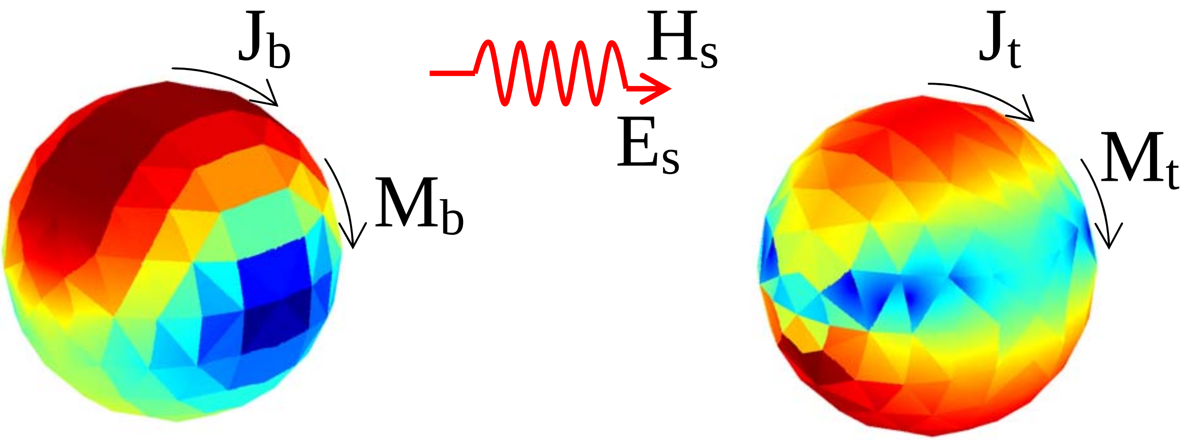
Will be replaced by ASM approach (see above)

J. Yeo, V. Prakash, and R. Mittra, *Efficient analysis of a class of microstrip antennas using the CBFM*, MOTL, vol. 39, pp. 456–464, Dec. 2003.

Testing procedure

Interaction between MBF's:

$$I = \iint \left(\vec{J}_t^\star \,.\, \vec{E}_s + \vec{M}_t^\star \,.\, \vec{H}_s \right) \; dS$$



Involves only equivalent surface currents

➡ **Independent from complexity of antennas inside !!!**

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Interaction between MBF's

Matrix form:

e.g., for non-self-block:

$$\begin{pmatrix} J^*_m & J^*_s & M^* \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & Z^{EJ} & Z^{EM} \\ 0 & Z^{HJ} & Z^{HM} \end{pmatrix} \begin{pmatrix} J_m \\ J_s \\ M \end{pmatrix}$$

Does not involve the metallic part inside the dielectric volume !!!

Slow !!! Nb^2 process, with Nb number of surface unknowns, complex elementary computations

Fast interaction using Multipoles:

Interaction over unit sphere

$$I = \frac{k}{(4\,\pi)^2} \iint (P_e + P_h) \underbrace{T(k, \vec{r}_{mn}, \hat{u})} \, dU$$

E and H interaction between MBF patterns

Multipole translation function

$$T(k, \vec{p}_{mn}, \hat{u}) \simeq \sum_{l=0}^{L} j^l \; (2l+1) \; h_l^{(2)}(k\, p_{mn}) \; P_l(\hat{u}\,.\,\hat{p}_{mn})$$

If Nb of the order of number of directions (a few hundred), Nb-process !!!

NB: no aggregation nor disaggregation

Fast interaction using Multipoles:

The special MBF patterns:

$$\vec{F}_i(\hat{u}) = \iint \vec{J}_i(\vec{r})\, e^{j\,k\,\hat{u}\,.\,\vec{r}}\, dS$$

Pattern of divergence

$$P_e = \vec{\mathcal{F}}_t \,.\, \left(-\omega\,\mu\,\vec{\mathcal{F}}_b^\star - k\,\hat{u}\times\vec{\mathcal{G}}_b^\star\right) + \frac{1}{\omega\,\epsilon}\,\mathcal{DF}_t\,\mathcal{DF}_b^\star$$

$$P_h = \vec{\mathcal{G}}_t \,.\, \left(-\omega\,\epsilon\,\vec{\mathcal{G}}_b^\star + k\,\hat{u}\times\vec{\mathcal{F}}_b^\star\right) + \frac{1}{\omega\,\mu}\,\mathcal{DG}_t\,\mathcal{DG}_b^\star$$

NB: 2 times « cheaper» formulation in preparation

$\mathcal{F}$: pattern from electric current

$\mathcal{G}$: pattern from magnetic current

Substript: *t* for testing MBF
b for basis MBF

Array impedance matrix

Reduced admittance matrix: $(\mathbf{Z}^{r})^{-1} = \mathbf{Y}^{\mathbf{r}}$

$\mathbf{Q} = [\mathbf{x}_1^{\circ} \ldots \mathbf{x}_P^{\circ}] \longrightarrow$ Matrix with list of MBFs

$\mathbf{x}_s$ line corresponding to feeding point (delta-gap source) with width *W*

Element of array admittance matrix $Y(i,j) = -W^2\, \mathbf{x}_s\, \mathbf{Y}^{\mathbf{r}}_{\mathbf{i},\mathbf{j}}\, \mathbf{x}_s^H$

Array pattern and/or embedded element pattern

1. Get solution for given excitation from MBF approach, e.g. excitation at a single port.
2. Compute far field pattern by brute-force from equivalent currents on dielectric-air interface:

polarisation

$$F_x(\hat{u}) = -j\,\omega\,\mu \iint_S \sum_{m.n} \left(\vec{J}_{mn}(\vec{r}_o)\,.\,\hat{e}_x + (\hat{u} \times \hat{e}_x)\,.\,\vec{M}_{mn}(\vec{r}_o)/\eta \right) e^{j\,k\,\vec{u}.\vec{r}_o}\;\; e^{j\,k\,\vec{u}.\vec{r}_{mn}}\;\; dS$$

Sum on array elements

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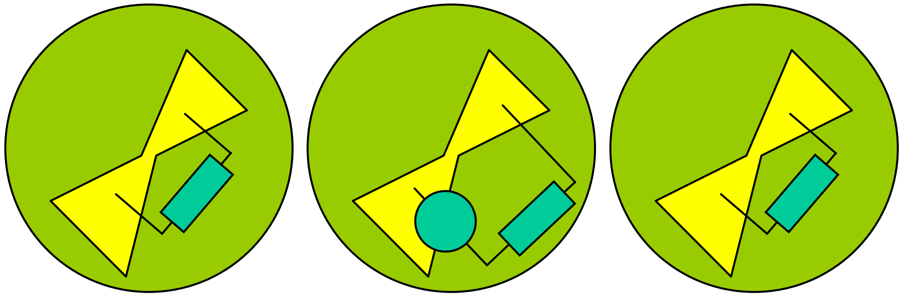
Array factorisation



MBF pattern can be factored out !!!

	Antennas		
MBF 1	c_{11}	c_{21}	c_{31}
MBF 2	c_{12}	c_{22}	c_{32}
⋮	⋮	⋮	⋮

Coefficients for **IDENTICAL** current distribution

Embedded element pattern

$$\mathbf{x}_m = \sum_{p=1}^{P} C_{m,p}\, \mathbf{x}_p^{\circ}$$

Finite series of pattern multiplication problems:

$$\vec{F}(\hat{u}) = \sum_{p=1}^{P} A_p(\hat{u})\, \vec{F}_p(\hat{u})$$

Pattern of a given MBF:

$$F_{p,x} = \iint_S \left(\vec{J}_p(\vec{r}_o) \,.\, \hat{e}_x + (\hat{u} \times \hat{e}_x) \,.\, \vec{M}_p(\vec{r}_o)/\eta \right) e^{j\,k\,\vec{u}.\vec{r}_o}\, dS$$

Have been computed in the course of MBF-Multipole computation !!!

Embedded element pattern

Array factor:

$$A_p(\hat{u}) = -j\,\omega\,\mu \sum_{m,n} C_{mnp}\, e^{j\,k\,\vec{u}.\vec{r}_{mn}}$$

$$= A_p(r,s) = -j\,\omega\,\mu\,\mathrm{FFT2}\left(C^{\star}_{mnp}\,(-1)^{mn}\right)^{\star}$$

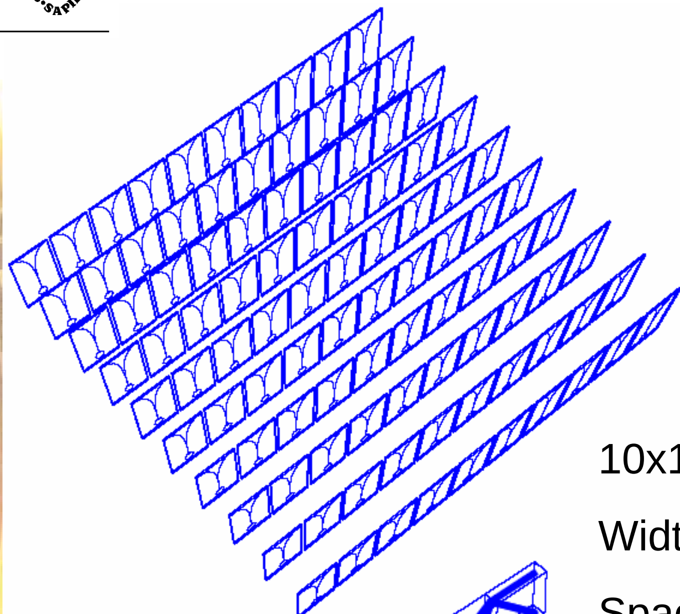
$$\begin{cases} u_x(r) = \lambda/a\,(-1/2 + r/N) \\ u_y(s) = \lambda/b\,(-1/2 + s/N) \end{cases}$$

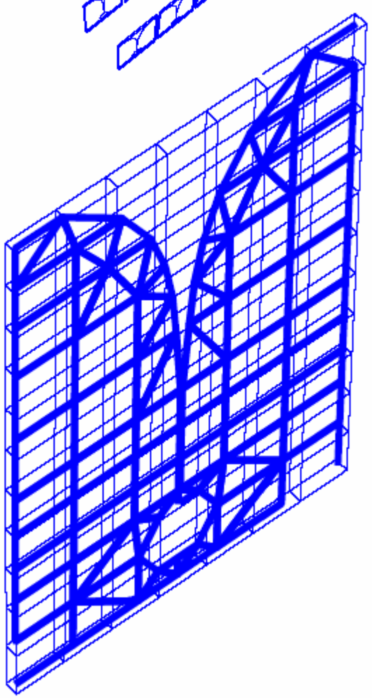
N log N complexity, with N = Max { M, D} !

↓ ↓

Antennas; directions

Array of Vivaldi antennas





10x10 array

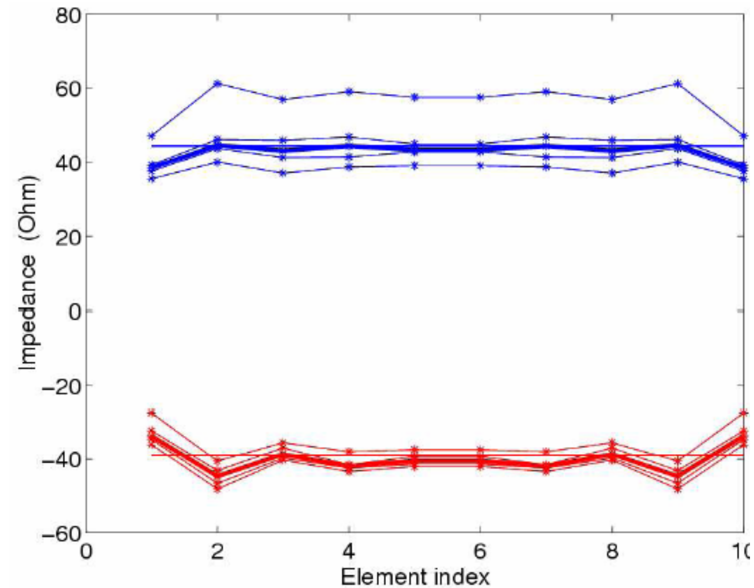
Width 13 cm

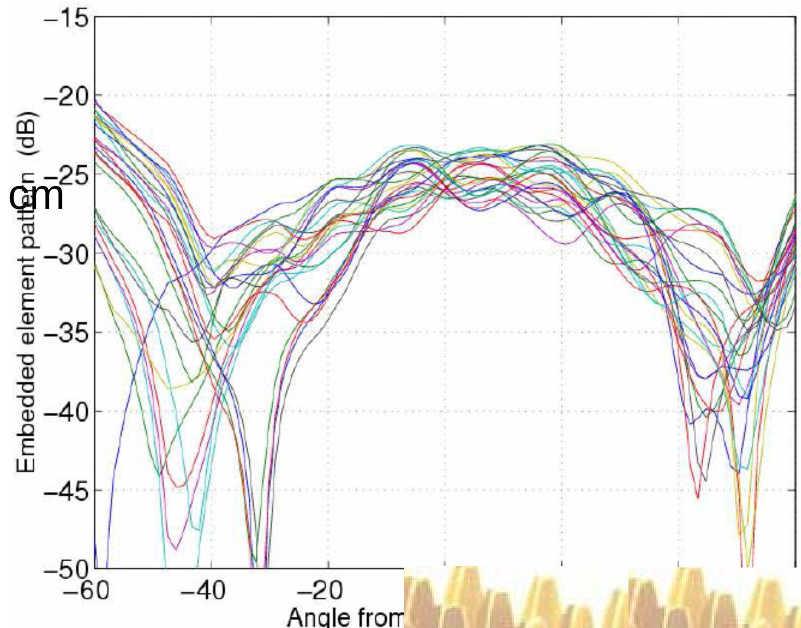
Spacing: 14 cm

Height: 21 cm

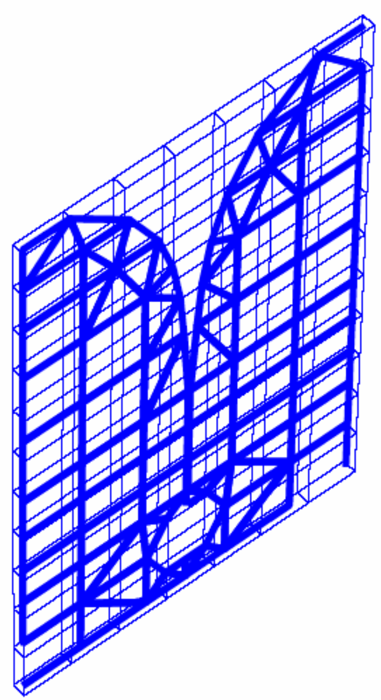
Wavelength: 30 cm

εr= 4





Validation for 4X4 array



4X4 array

Width 13 cm

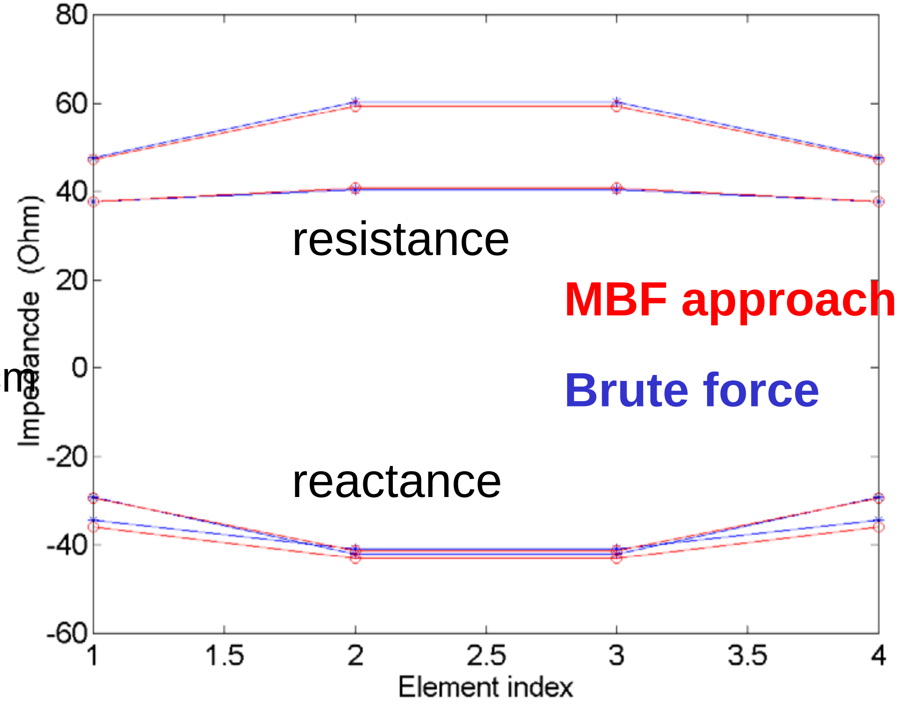
Spacing: 14 cm

Height: 21 cm

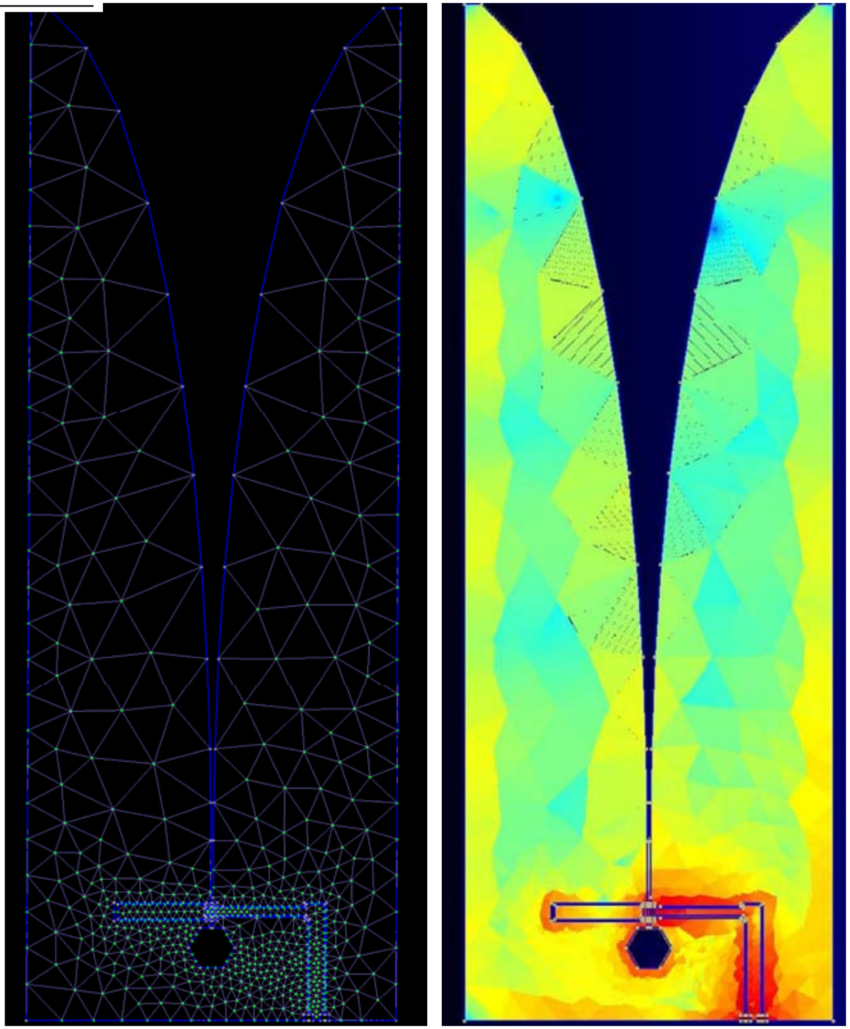
Wavelength: 30 cm

$\varepsilon_r = 4$

Active input impedances



More complex antenna



Balun structure from DRAO, Penticton, Canada. Meshing and runnig by C. Grevet, Toulouse, France.

Tapered-slot antenna with complex balun

- 1402 unknowns on the metallic antenna
- 2 x 380 unknowns on the dielectric box
- 16 X 16 array

553.472 unknowns

256 solutions in ~30 minutes on old laptop

Conclusions

- ASM is a key tool for infinite and finite array analysis.
- Link with « windowing » methods.
- MBFs from ASM + 2X2 array analysis.
- Highly accurate, allows to catch « anomalies ».
- MBFs extended to antennas with finite dielectrics
- Interactions between MBFs accelerated with Multipoles (less-than-N complexity for complex antennas)
- Array impedance matrix obtained from reduced MoM matrix
- Very fast pattern evaluation:

    Finite series of pattern multiplication problems

    With MBF patterns computed in multipoles process

    With array factors computed with FFT

Extensions in progress

- Treatment of cells connected through dielectrics
- Extension to multiple dielectric materials
- Further investigation of analysis of anomalies
- Analysis of complex eigenmodes (e.g. for leaky waves)
- Fast analysis of irregular arrays with similar method
- Analysis of complex feeding structures
- Joint analysis of antennas and supporting structures
- Experimental validation

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Journal papers in the field of array analysis

[1] Craeye C., Smolders A.B., Schaubert D.H., Tijhuis A.G., An efficient scheme for the computation of the Green's function of a two-dimensional semi-infinite array, *IEEE Trans. AP*, vol. 51, pp. 766-771, April 2003.

[2] Craeye C., Arts M., Modulated oscillations appearing in the scan impedance of a finite phased array, *IEEE Trans. AP*, vol.51, pp. 2504-2506, September 2003.

[3] Craeye C., Tijhuis A.G., Schaubert D.H., An efficient MoM formulation for finite-by-infinite arrays of two-dimensional antennas arranged in a three-dimensional structure, *IEEE Trans. AP*, January 2004.

[4] Craeye C., Arts M., On the receiving cross-section of an antenna in infinite linear and planar arrays, *Radio Science*, April 2004.

[5] Craeye C., Parvais B., Dardenne X., Signal-to-noise patterns in infinite and finite receiving antenna arrays, *IEEE Trans. AP*, December 2004.

[6] Craeye C., Including spatial correlation of thermal noise in the noise model of high-sensitivity arrays, *IEEE Transactions on Antennas and Propagation*, vol. 53, pp. 3845 – 3848, Nov. 2005.

[7] Craeye C., Dardenne X., Element pattern analysis of wideband arrays with the help of a finite-by-infinite array approach, *IEEE Transactions on Antennas and Propagation*, Vol. 54, pp. 519 – 526, Feb. 2006.

[8] Craeye C., Capolino F., Accelerated computation of the free-space Green's function of two-dimensional semi-infinite and infinite arrays, *IEEE Trans. AP*, vol. 54, 1037-1040, March 2006.

[9] Craeye C., A.O. Boryssenko, D.H. Schaubert, Wave propagation and coupling in linear arrays with application to the analysis of large arrays, *IEEE Trans. AP*, vol. 54, pp. 1971-1978, July 2006.

[10] Craeye C., Fast impedance and pattern computations in finite antenna arrays, IEEE Trans. AP, vol. 54, pp. 3030-3034, October 2006.

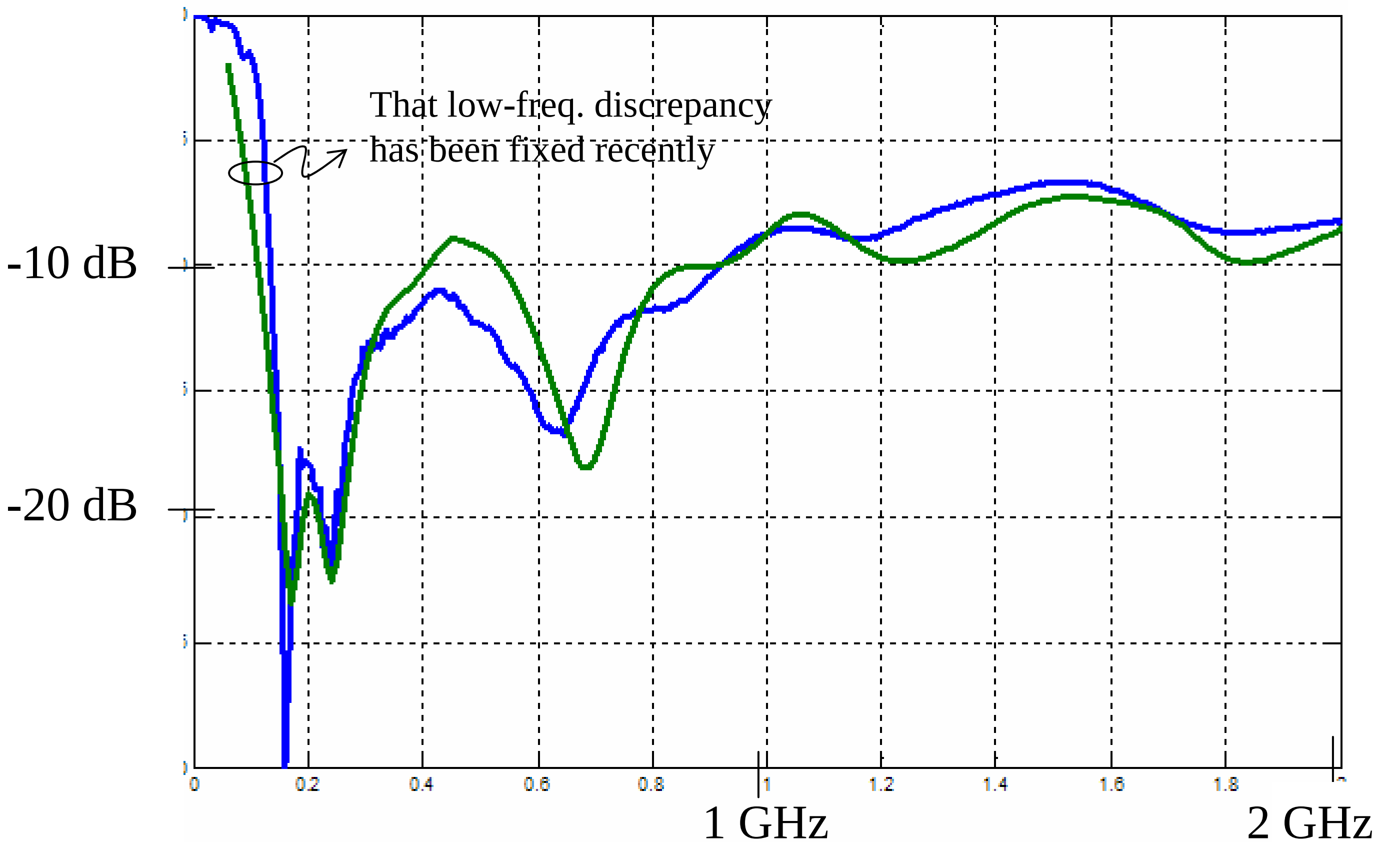
[11] Craeye C. and R. Sarkis, Finite array analysis through combination of array scanning and macro basis function approaches, Accepted for publication in *ACES* Journal.

[12] X. Dardenne and C. Craeye, Method of Moments simulation of infinitely periodic structures combining metal with connected dielectric objects, submitted to IEEE Transactions AP, July 2007.

[13] C. Craeye, Th. Gilles and C. Craeye, Fast analysis of finite arrays of antennas involving dielectric parts, submitted to Radio Science, November 2007.

Simulations versus measurements

S_{11} for Ultra-wideband monopole with complex shape



by Farshad Keshmiri and David Gonzalez, Nov. 2007

frequency

S12 validations in progress