Beam Sensitivity, Efficiency, and Receiver Noise Models for Focal Plane Arrays

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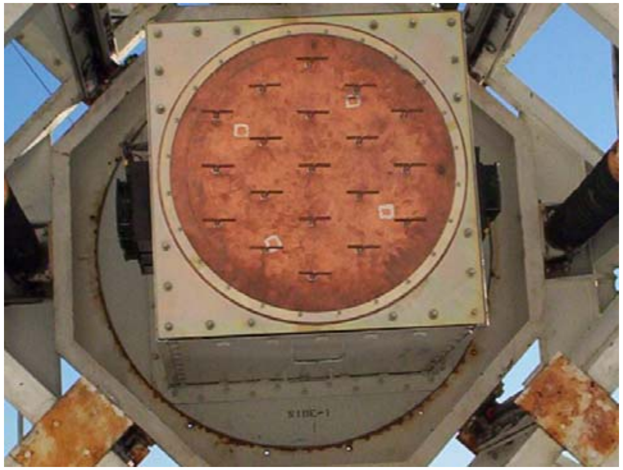
Overview

Goal: model beam sensitivity and efficiency in terms of antenna characteristics and amplifier network parameters

- Array and reflector model
- Amplifier noise model
- Beam efficiency and sensitivity
- Optimal noise matching
- Conclusions and open questions
- This presentation: theory
- Next presentation: applications



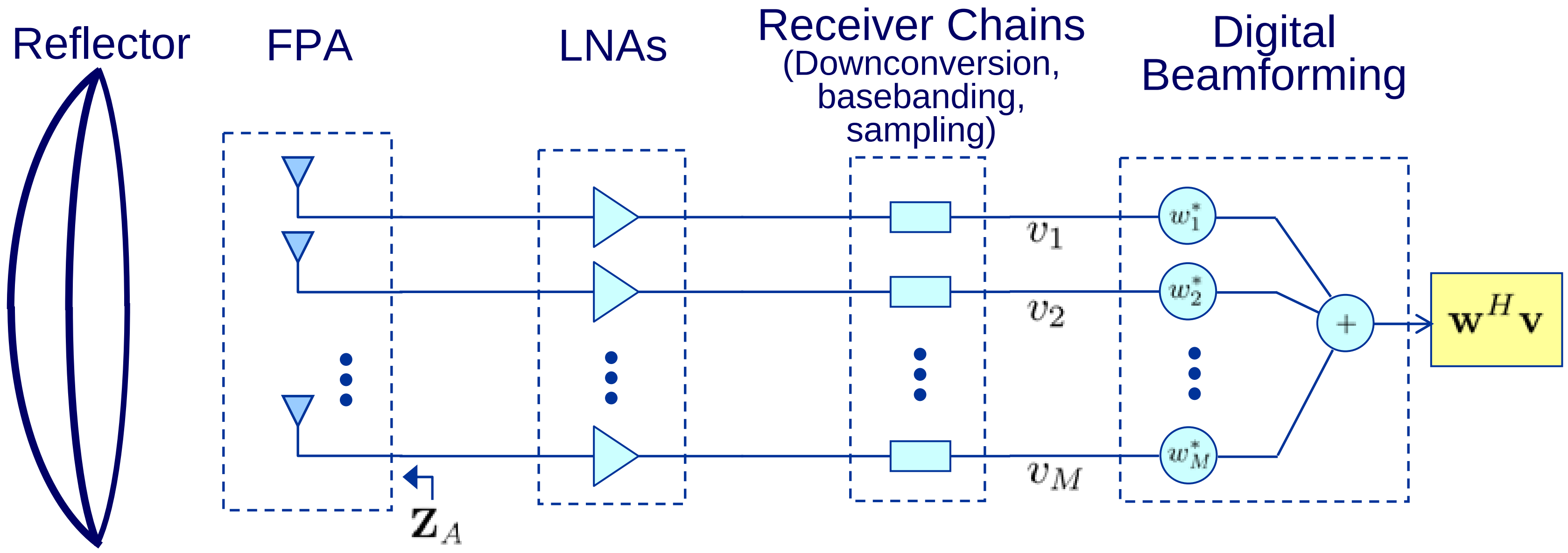
References

K. F. Warnick and B. D. Jeffs, “Gain and aperture efficiency for a reflector antenna with an array feed,” *IEEE Antennas and Wireless Propagation Letters*, Vol. 5, No. 1, pp. 499-502, 2006.

K. F. Warnick and M. A. Jensen, “Optimal noise matching for mutually-coupled arrays,” IEEE Transactions on Antennas and Propagation, Vol. 55, No. 6, pp. 1726-1731, June 2007.

K. F. Warnick and B. D. Jeffs, “Beam Efficiency and System Temperature for a Focal Plane Array,” Technical Report, Brigham Young University, http://hdl.handle.net/1877/588, 2007.

FPA System Model



Physical Optics

Full wave EM simulation

Network Theory

Signal, noise covariances

Array Signal Processing Theory

Beam sensitivity: $\mathrm{SNR} \sim \frac{A_e}{T_{\mathrm{sys}}}$

Aperture efficiency

Spillover efficiency

Receiver noise

Receiver Output Covariance Matrix

Array response correlation matrix: $$\hat{\mathbf{R}}_{\mathbf{v}} = \frac{1}{N}\sum_{n=1}^{N} \mathbf{v}[n]\mathbf{v}[n]^{H}$$

Wide sense stationary environment: $$\mathbf{R}_{\mathbf{v}} = \lim_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N} \mathbf{v}[n]\mathbf{v}[n]^{H}$$

Beam time average output power (1 Ohm Load): $P = \frac{1}{2}\mathbf{w}^{H}\mathbf{R}_{\mathbf{v}}\mathbf{w}$

Signal and Noise Covariance Matrices

$$\mathbf{R}_{\mathbf{v}} = \mathbf{R}_{\mathrm{sig}} + \mathbf{R}_{\mathrm{n}}$$

$\mathbf{R}_{\mathrm{sig}}$ Signal of interest

$$\left.\begin{array}{ll}\mathbf{R}_{\mathrm{sp}} & \text{Spillover noise}\\ \mathbf{R}_{\mathrm{loss}} & \text{Noise due to loss in antenna elements and feed structure}\\ \mathbf{R}_{\mathrm{rec}} & \text{Receiver noise}\\ \mathbf{R}_{\mathrm{int}} & \text{Interference}\end{array}\right\} \mathbf{R}_{\mathrm{n}}$$

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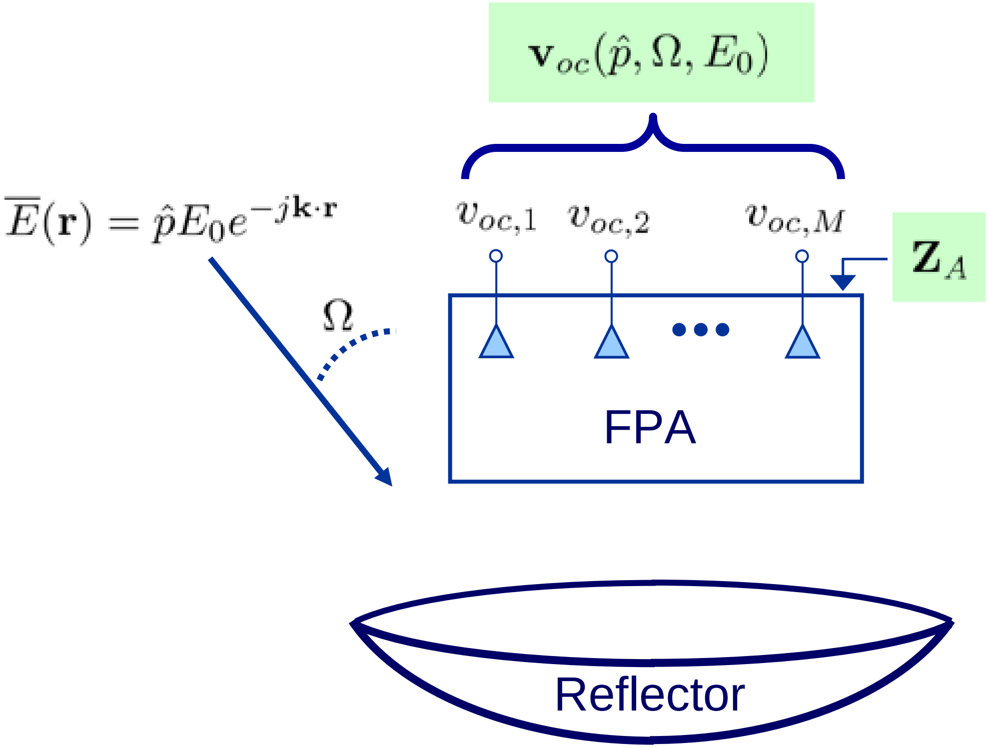
Receive Array Model

Thevenin equivalent source network:

Embedded open circuit loaded receiving voltage patterns: $\mathbf{v}_{oc}(\hat{p}, \Omega, E_0)$

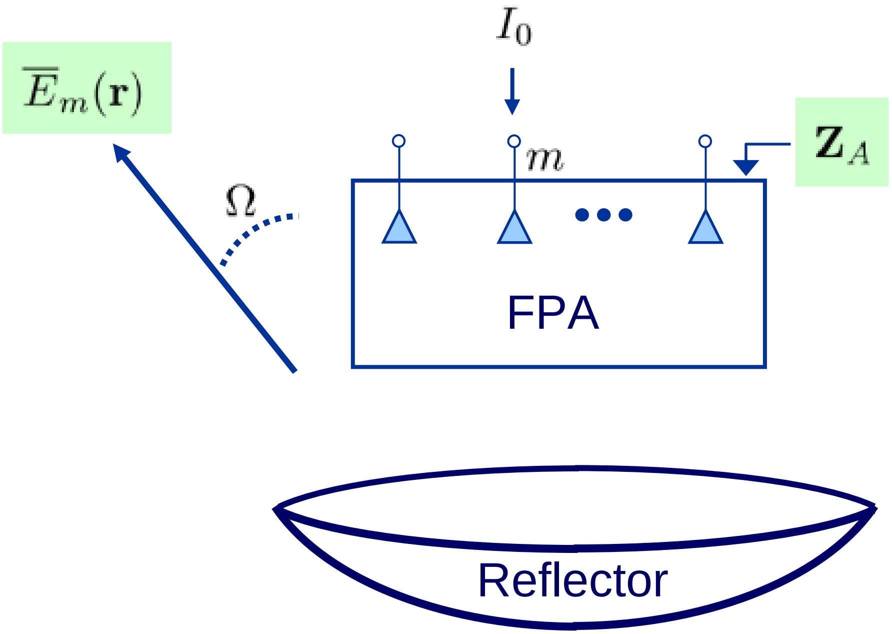
Mutual impedance matrix: $\mathbf{Z}_A$



Transmit Array Model

Embedded open circuit loaded radiation field patterns: $\overline{E}_m(\mathbf{r})$

Mutual impedance matrix: $\mathbf{Z}_A$



Reciprocity

Incident field

$$v_{oc,m}(\hat{p}, E_0, \Omega) = \frac{4\pi j r e^{jkr}}{\omega\mu I_0} E_0 \hat{p} \cdot \overline{E}_m(\mathbf{r})$$

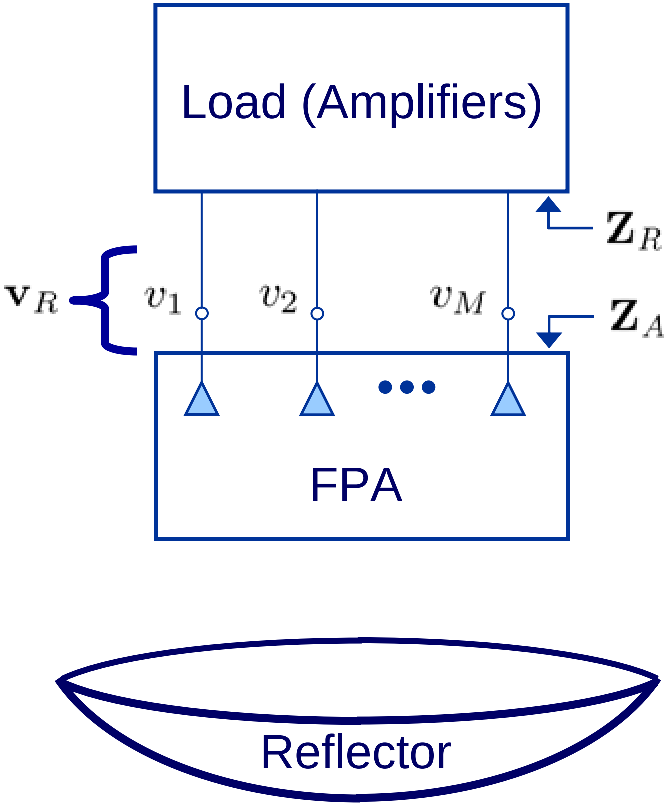
Receiving pattern

Radiation pattern

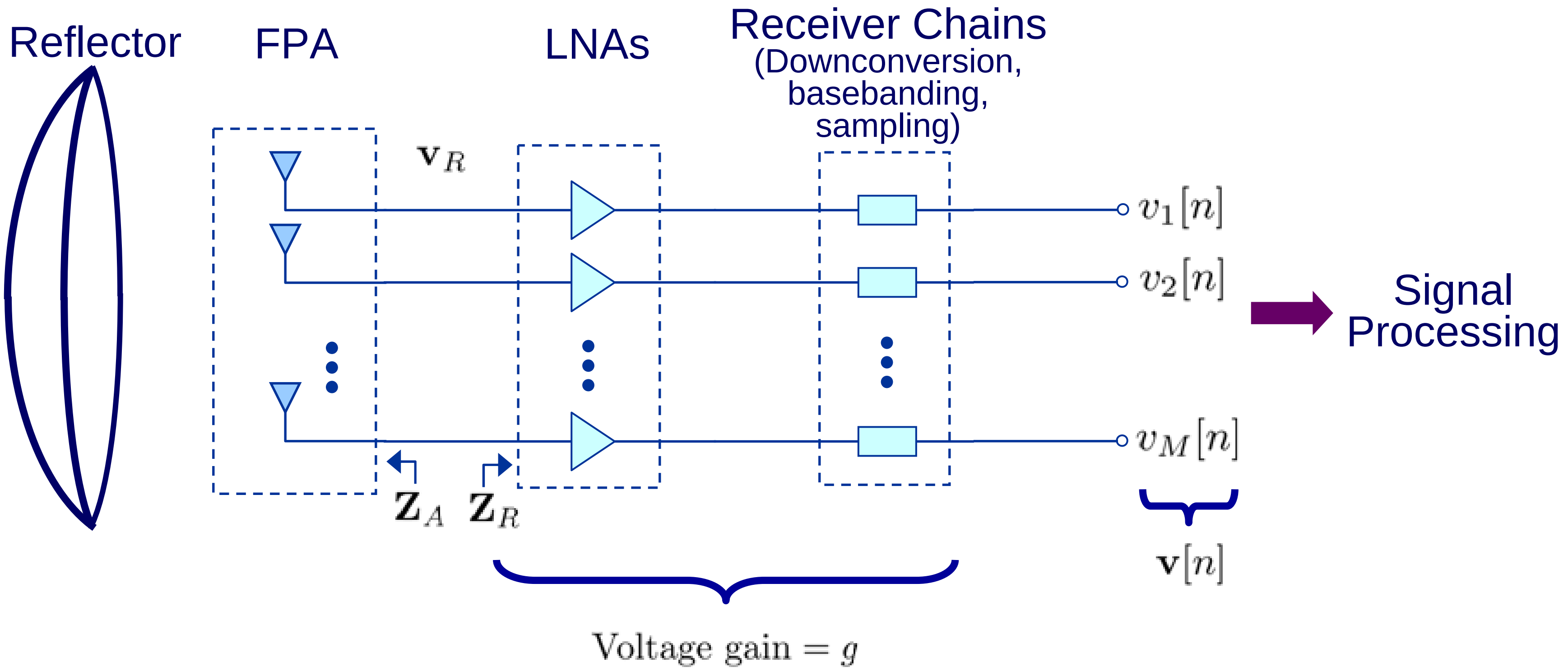
$\mathbf{r} = (r, \Omega)$

Loaded Receiving Pattern

$$\mathbf{v}_R = \mathbf{Z}_R(\mathbf{Z}_R + \mathbf{Z}_A)^{-1}\mathbf{v}_{oc}$$



Receiver Output Voltages

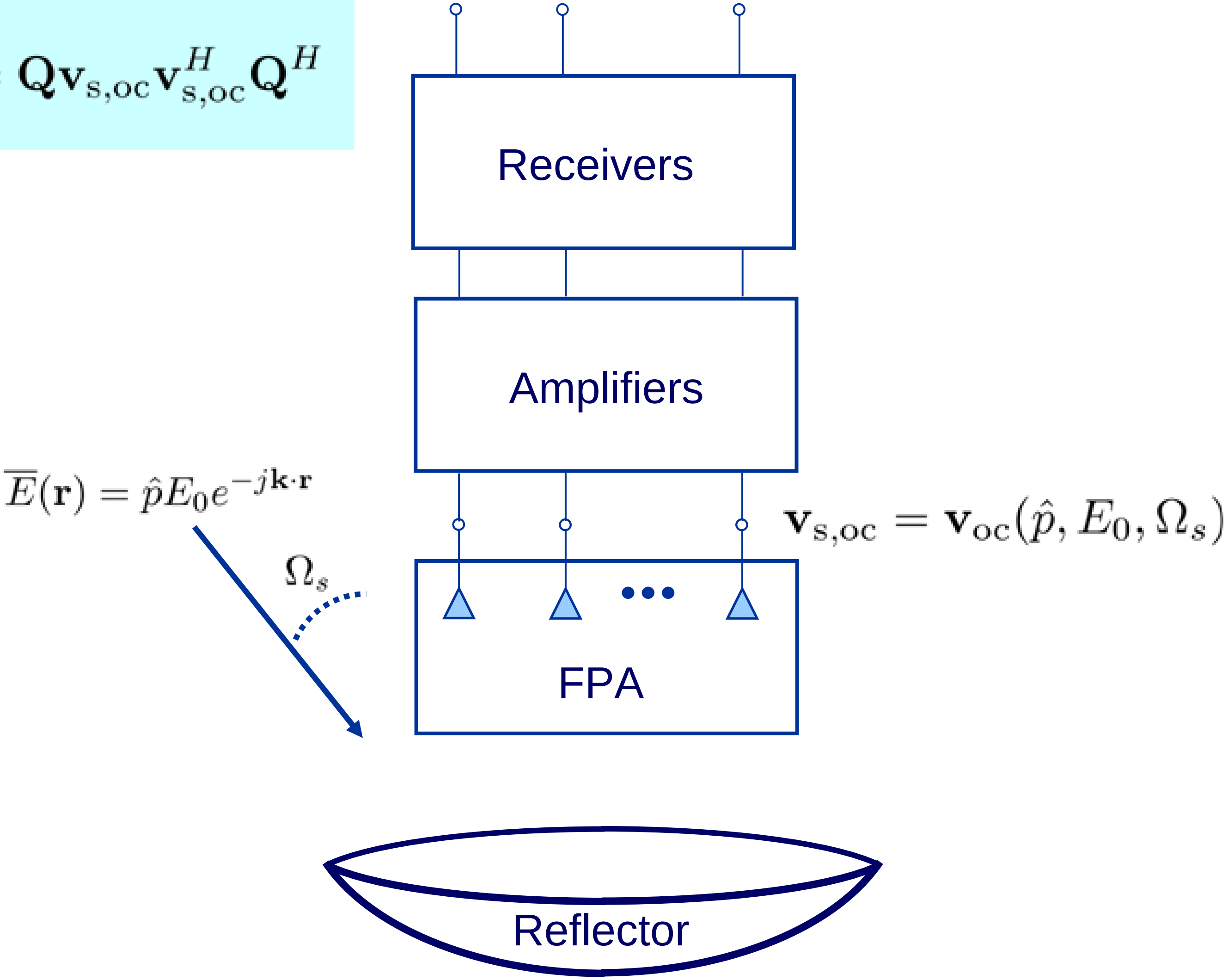


Uncoupled receiver chains:

$$\mathbf{v} = \underbrace{g\mathbf{Z}_R(\mathbf{Z}_R + \mathbf{Z}_A)^{-1}}_{\mathbf{Q}}\mathbf{v}_{oc}$$

Signal of Interest

$$\mathbf{R}_{\text{sig}} = \mathbf{Q}\mathbf{v}_{\text{s,oc}}\mathbf{v}_{\text{s,oc}}^{H}\mathbf{Q}^{H}$$



Arbitrary Thermal Noise Distribution

$$\mathbf{R}_{T(\Omega)} = \mathbf{Q}\mathbf{R}_{oc,T(\Omega)}\mathbf{Q}^H$$

$$R_{oc,T(\Omega),mn} = \frac{16k_bB}{|I_0|^2} \underbrace{\frac{1}{2\eta}\int T(\Omega)\overline{E}_m(\mathbf{r}) \cdot \overline{E}_n^*(\mathbf{r})r^2 d\Omega}_{\text{Weighted pattern overlap integral}}$$

k_b = Boltzman's constant

B = Noise equivalent bandwidth

η = Characteristic impedance of space

$T(\Omega)$ = Thermal noise brightness temperature distribution

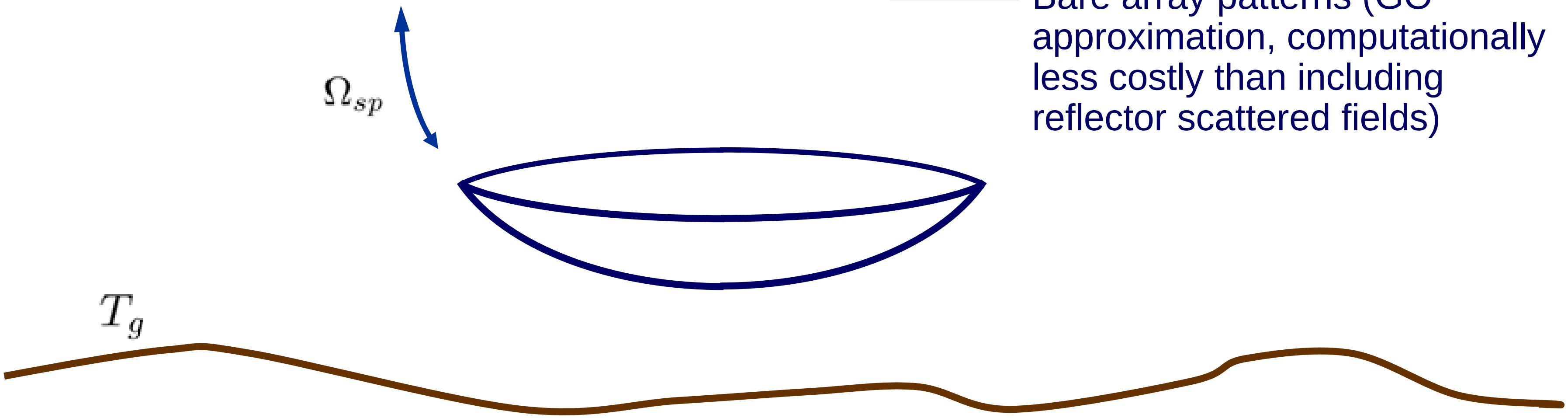
Spillover Noise

Spillover noise distribution: $T(\Omega) = \begin{cases} T_g & \Omega \in \Omega_{sp} \\ 0 & \text{otherwise} \end{cases}$

$$\mathbf{R}_{\text{sp}} = \frac{1}{|I_0|^2} 16 k_b T_g B \mathbf{Q} \mathbf{A}_{\text{sp}} \mathbf{Q}^H$$

$$A_{sp,mn} = \frac{1}{2\eta} \int_{\Omega_{sp}} \overline{E}_m(\mathbf{r}) \cdot \overline{E}_n^*(\mathbf{r}) r^2 d\Omega$$

Bare array patterns (GO approximation, computationally less costly than including reflector scattered fields)



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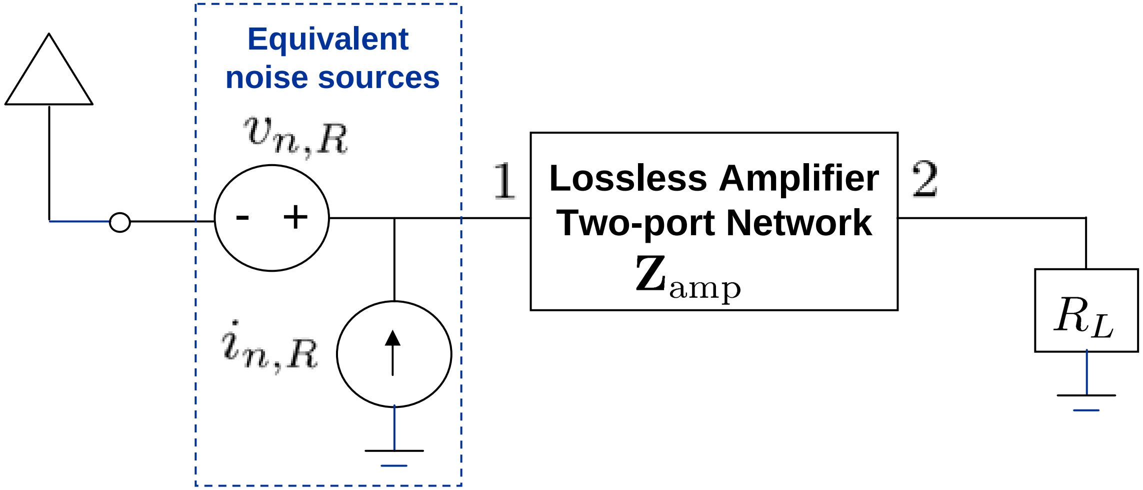
Amplifier Model



Network parameters:

$$\mathbf{Z}_{\text{amp}} = \begin{bmatrix} Z_r & 0 \\ 2g_r Z_r & Z_0 \end{bmatrix}$$

$g_r =$ voltage gain

Noise parameters:

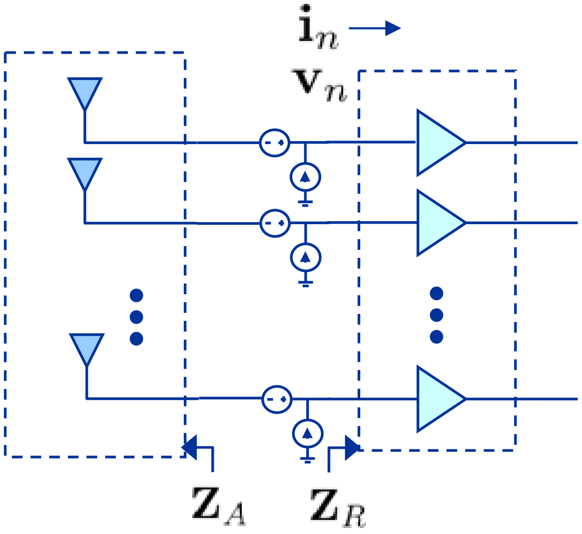
$$i_{n,R} = Y_c v_{n,R} + i_u$$

Y_c = correlation admittance, i_u and $v_{n,R}$ are uncorrelated

Network Relationships

$$\mathbf{v}_n = \mathbf{Z}_R \mathbf{i}_n$$

$$\mathbf{v}_{n,R} - \mathbf{v}_n = \mathbf{Z}_A(\mathbf{i}_{n,R} - \mathbf{i}_n)$$



Receiver noise voltage vector at receiver outputs:

$$\mathbf{v}_{\text{rec}} = \underbrace{g\mathbf{Z}_R(\mathbf{Z}_R + \mathbf{Z}_A)^{-1}\mathbf{Z}_A}_{\mathbf{P}} \mathbf{i}_{n,R} + \underbrace{g\mathbf{Z}_R(\mathbf{Z}_R + \mathbf{Z}_A)^{-1}}_{\mathbf{Q}} \mathbf{v}_{n,R}$$

Receiver Noise Covariance

$$\mathbf{R}_{\mathrm{rec}} = 2B\left[\bar{v}_{n,R}^2(\mathbf{Q}\mathbf{Q}^H + Y_c\mathbf{P}\mathbf{Q}^H + Y_c^*\mathbf{Q}\mathbf{P}^H) + \bar{i}_{n,R}^2\mathbf{P}\mathbf{P}^H\right]$$

$\bar{v}_{n,R} = \text{RMS noise voltage densitity } (\mathrm{V}/\sqrt{\mathrm{Hz}})$

$\bar{i}_{n,R} = \text{RMS noise current densitity } (\mathrm{A}/\sqrt{\mathrm{Hz}})$

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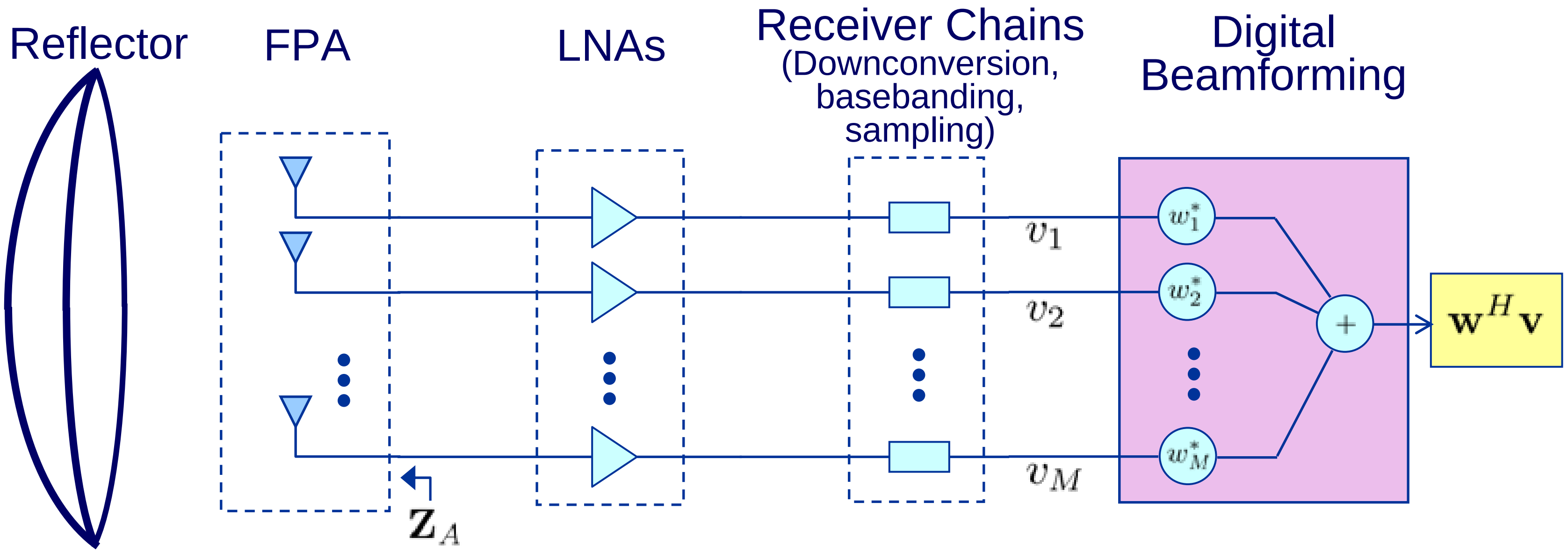
Beamforming



Reflector

FPA

LNAs

Receiver Chains
(Downconversion,
basebanding,
sampling)

Digital
Beamforming

Beam Sensitivity

Key figure of merit: $\mathrm{SNR} = \frac{P_{\mathrm{sig}}}{P_{\mathrm{n}}} = \frac{\mathbf{w}^H \mathbf{R}_{\mathrm{sig}} \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{\mathrm{n}} \mathbf{w}}$

Beam sensitivity: $\frac{A_e}{T_{\mathrm{sys}}} = \frac{k_b B}{S^{\mathrm{sig}}} \mathrm{SNR}\ (\mathrm{m}^2/\mathrm{K})$

where S^{sig} (W/m^2) is the signal flux density in one polarization
k_b is Boltzman's constant
B is the system noise equivalent bandwidth

Beam Efficiencies and System Temperature

- To characterize FPA performance and facilitate comparisons to single feeds, we would like to know the beam aperture efficiency, spillover efficiency, radiation efficiency, and system equivalent noise temperature

- Consider aperture efficiency:

Signal output power includes array loading, receiver gain, and beamforming coefficients:

$$P_{\text{sig}} = \mathbf{w}^H \mathbf{Q} \mathbf{v}_{s,oc} \mathbf{v}_{s,oc}^H \mathbf{Q}^H \mathbf{w}$$

The ratio of the beamformer output power to incident power density is essentially arbitrary

How can aperture efficiency be defined for an active array?

Aperture Efficiency for an Active Array

- **Embedded element efficiency**

  A transmit quantity, but can be defined for a receiver using reciprocity

  Problem for receiving arrays:

  - Effective receiving area is defined to be available signal power
  - Embedded element efficiency includes the active reflection coefficient
  - For a receiver, by convention reflection loss increases amplifier noise and should not reduce aperture efficiency

- **Receiving pattern directivity**

  Normalize output by integrated receiving pattern

- **Isotropic noise gain**

  Normalize output by isotropic noise response

  [Warnick and Jeffs, AWPL, 2005].

Isotropic Noise Response

$\mathbf{R}_{\mathrm{iso}}$ Array response covariance due to an isotropic noise distribution at temperature *T:*

For a passive antenna in an isotropic noise field, the output power into a matched load is

$$P = k_b T B \qquad (B \text{ is the system bandwidth})$$

For an active array, the natural absolute reference for output powers is

$$P = k_b T B \frac{\mathbf{w}^H \mathbf{R}_{\mathbf{v}} \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{\mathrm{iso}} \mathbf{w}}$$

Lossless Array Case

Isotropic noise covariance, mutual resistance matrix, and pattern overlap Integrals are essentially equivalent:

Pattern overlap integrals:

$$A_{mn} = \frac{1}{2\eta_0} \int \overline{E}_m(\mathbf{r}) \cdot \overline{E}_n^*(\mathbf{r}) r^2 d\Omega, \quad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$\mathbf{R}_{\text{iso}} = \frac{1}{|I_0|^2} 16 k_b T B \mathbf{Q} \mathbf{A} \mathbf{Q}^H$$

Mutual resistance matrix [Twiss, 1955]:

$$\mathbf{R}_{\text{iso}} = 8 k_b T B \mathbf{Q} \text{Re}[\mathbf{Z}_A] \mathbf{Q}^H$$

Isotropic noise covariance may be directly measurable using Y factor approach with cold sky and warm absorber cover

Sensitivity and Efficiencies – Lossless Case

$$\frac{A_e}{T_{\mathrm{sys}}} = \frac{k_b B}{S^{\mathrm{sig}}} \frac{\mathbf{w}^H \mathbf{R}_{\mathrm{sig}} \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{\mathrm{sp}} \mathbf{w} + \mathbf{w}^H \mathbf{R}_{\mathrm{rec}} \mathbf{w}} \frac{\frac{T}{\mathbf{w}^H \mathbf{R}_{\mathrm{iso}} \mathbf{w}}}{\frac{T}{\mathbf{w}^H \mathbf{R}_{\mathrm{iso}} \mathbf{w}}}$$

A_e

$$= \frac{\frac{k_b T B}{S^{\mathrm{sig}}} \frac{\mathbf{w}^H \mathbf{R}_{\mathrm{sig}} \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{\mathrm{iso}} \mathbf{w}}}{\frac{T \mathbf{w}^H \mathbf{R}_{\mathrm{sp}} \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{\mathrm{iso}} \mathbf{w}} + \frac{T \mathbf{w}^H \mathbf{R}_{\mathrm{rec}} \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{\mathrm{iso}} \mathbf{w}}} = \frac{\eta_{\mathrm{ap}} A_p}{(1 - \eta_{\mathrm{sp}})T + T_{\mathrm{min}}/\eta_{\mathrm{n}}}$$

T_{spill}

T_{rec}

T_{min} is minimum noise temperature of one LNA

Sensitivity and Efficiencies – Lossy Case

$$\frac{A_\mathrm{e}}{T_\mathrm{sys}} = \frac{\eta_\mathrm{rad}\eta_\mathrm{ap}A_p}{\eta_\mathrm{rad}(1-\eta_\mathrm{sp})T_\mathrm{g} + (1-\eta_\mathrm{rad})T_a + \eta_\mathrm{rad}T_\mathrm{min}/\eta_\mathrm{n}}$$

Spillover efficiency: $$\eta_\mathrm{sp} = 1 - \frac{\mathbf{w}^H\mathbf{R}_\mathrm{sp}\mathbf{w}}{\mathbf{w}^H\mathbf{R}_\mathrm{iso}\mathbf{w}}$$

Aperture efficiency: $$\eta_\mathrm{ap} = \frac{k_bTB}{A_pS^\mathrm{sig}}\frac{\mathbf{w}^H\mathbf{R}_\mathrm{sig}\mathbf{w}}{\mathbf{w}^H\mathbf{R}_\mathrm{iso}\mathbf{w}}$$

Radiation efficiency: $$\eta_\mathrm{rad} = \frac{\mathbf{w}^H\mathbf{R}_\mathrm{iso}\mathbf{w}}{\mathbf{w}^H(\mathbf{R}_\mathrm{iso}+\mathbf{R}_\mathrm{loss})\mathbf{w}}$$

Noise matching efficiency: $$\eta_\mathrm{n} = \frac{T_\mathrm{min}}{T}\frac{\mathbf{w}^H\mathbf{R}_\mathrm{iso}\mathbf{w}}{\mathbf{w}^H\mathbf{R}_\mathrm{rec}\mathbf{w}}$$

T_min is minimum noise temperature of one LNA

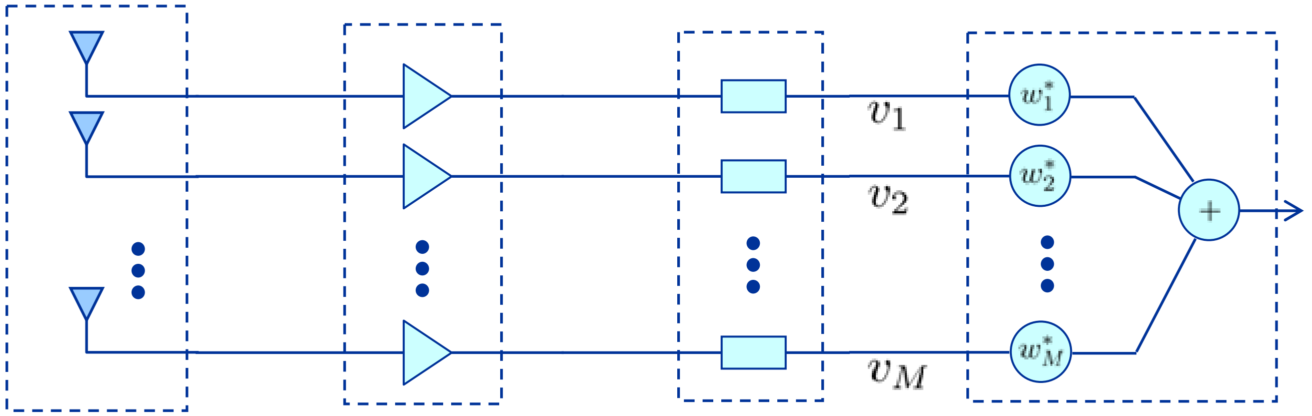
Antenna Efficiency

$$\eta_{ant} = \eta_{\mathrm{rad}} \frac{k_b T B}{A_p S^{\mathrm{sig}}} \frac{\mathbf{w}^H \mathbf{R}_{\mathrm{sig}} \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{\mathrm{iso}} \mathbf{w}}$$

$$= \frac{1}{8 A_p S^{\mathrm{sig}}} \frac{\mathbf{w}^H \mathbf{R}_{\mathrm{sig}} \mathbf{w}}{\mathbf{w}^H \mathbf{Q} \mathrm{Re}[\mathbf{Z}_A] \mathbf{Q}^H \mathbf{w}}$$

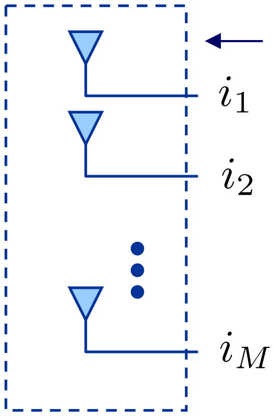
Transmit/Receive Equivalence

RX



$$\mathbf{w}^H\mathbf{v} = \mathbf{w}^H\mathbf{Q}\mathbf{v}_{oc}$$

TX



$$\mathbf{i} = \mathbf{Q}^H\mathbf{w}$$

$$\boxed{\frac{\lambda^2}{4\pi}G} = \eta_{\mathrm{rad}}\eta_{\mathrm{ap}}A_p = \boxed{\eta_{\mathrm{rad}}\frac{k_bTB}{S^{\mathrm{sig}}}\frac{\mathbf{w}^H\mathbf{R}_{\mathrm{sig}}\mathbf{w}}{\mathbf{w}^H\mathbf{R}_{\mathrm{iso}}\mathbf{w}}}$$

Coupling Efficiency

Noise matching efficiency:

$$\eta_{\mathrm{n}} = \frac{T_{\mathrm{min}}}{T}\frac{\mathbf{w}^H\mathbf{R}_{\mathrm{iso}}\mathbf{w}}{\mathbf{w}^H\mathbf{R}_{\mathrm{rec}}\mathbf{w}}$$

$$= 8k_bT_{\mathrm{min}}BZ_0\frac{\mathbf{w}_{\mathrm{f}}^H(\mathbf{I}-\mathbf{S}_A\mathbf{S}_A^H)\mathbf{w}_{\mathrm{f}}^H}{\mathbf{w}_{\mathrm{f}}^H\mathbf{R}_{\mathrm{rec,f}}\mathbf{w}_{\mathrm{f}}} \quad \text{(Bosma's theorem)}$$

RX forward wave effective beamformer weights

Coupling efficiency:

$$\eta_c = 1 - \frac{\mathbf{a}^H\mathbf{S}_A^H\mathbf{S}_A\mathbf{a}}{\mathbf{a}^H\mathbf{a}}$$

$$= \frac{\mathbf{a}^H(\mathbf{I}-\mathbf{S}_A^H\mathbf{S}_A)\mathbf{a}}{\mathbf{a}^H\mathbf{a}}$$

TX forward wave amplitudes

Equal if: $\mathbf{R}_{\mathrm{rec}} = 8k_bT_{\mathrm{min}}BZ_0\mathbf{I}$ (uncorrelated receiver noise forward waves)

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Noise Matching

Equivalent receiver noise temperature: $T_{\mathrm{rec}} = \eta_{\mathrm{rad}} T_{\mathrm{min}} / \eta_{\mathrm{n}}$

Noise matching efficiency: $$\eta_{\mathrm{n}} = \frac{T_{\mathrm{min}}}{T} \frac{\mathbf{w}^H \mathbf{R}_{\mathrm{iso}} \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{\mathrm{rec}} \mathbf{w}}$$

T_{min} is minimum noise temperature of one LNA

$\eta_{\mathrm{n}} < 1$

Amplifier input noise mismatch

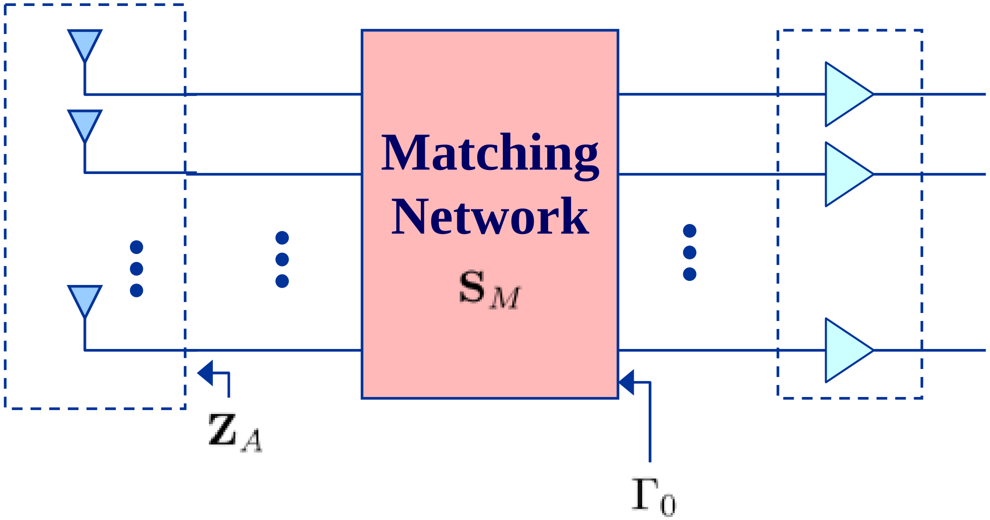
In the single channel case, receiver noise increases if amplifier is not optimally noise matched

Mutual coupling

Front end amplifier (LNA) noise couples through the array

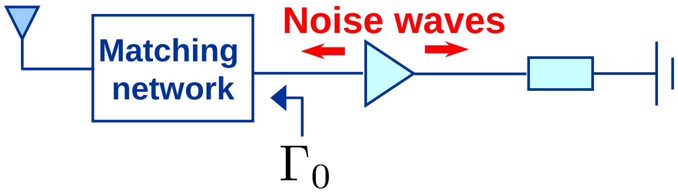
Coupled noise causes an increase in receiver temperature

Multiport Optimal Noise Matching



How should the multiport matching network be designed to maximize the beam SNR?

Classical Two-port Noise Matching



Exchangeable amplifier noise temperature in terms of noise wave parameters (amplifier noise referred to input):

Minimize

$$\frac{\partial T_{\mathrm{ee}}}{\partial \Gamma_0} = 0$$

$$T_{\mathrm{ee}} = \frac{T_\alpha + |\Gamma_0|^2 T_\beta - T_\gamma \Gamma_0 - T_\gamma^* \Gamma_0^*}{1 - |\Gamma_0|^2}$$

$T_\alpha,\ T_\beta,\ T_\gamma$ are transistor noise wave parameters

$$\Gamma_0 = \Gamma_{\mathrm{opt}}(T_\alpha, T_\beta, T_\gamma), \quad T_{\mathrm{LNA}} = T_{\mathrm{min}}$$

Optimal source reflection coefficient maximizes cancellation between reverse amplifier noise wave and partially correlated forward noise wave.

Exchangeable Amplifier Noise Covariance

How can one characterize multichannel noise performance?

Scalar exchangeable noise temperature generalizes to exchangeable noise correlation matrix (covariance of amplifier noise voltages referred to source network):

$$\mathbf{R}_{\mathrm{ee}} = (\mathbf{I} - \mathbf{\Gamma}_0\mathbf{\Gamma}_0^\dagger)^{-1/2}\mathbf{R}_\eta(\mathbf{I} - \mathbf{\Gamma}_0\mathbf{\Gamma}_0^\dagger)^{-1/2}$$

$$\mathbf{R}_\eta = k_b B\left(T_\alpha\mathbf{I} + T_\beta\mathbf{\Gamma}_0\mathbf{\Gamma}_0^\dagger - T_\gamma\mathbf{\Gamma}_0 - T_\gamma^*\mathbf{\Gamma}_0^\dagger\right)$$

(Scalar case) $T_{\mathrm{ee}} = \frac{T_\alpha+|\Gamma_0|^2T_\beta-T_\gamma\Gamma_0-T_\gamma^*\Gamma_0^*}{1-|\Gamma_0|^2}$

Multiport Optimality Criterion

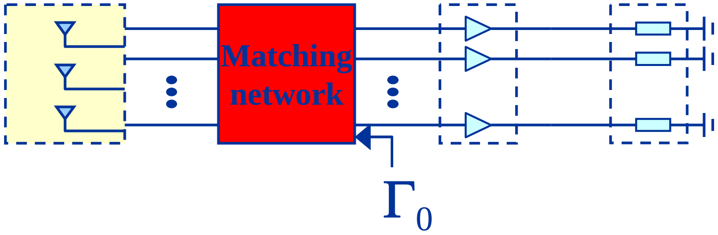
Optimal S-matrix to be presented by matching network to the amplifier input ports is

$$\boldsymbol{\Gamma}_0 = \Gamma_{\mathrm{opt}} \mathbf{I}$$

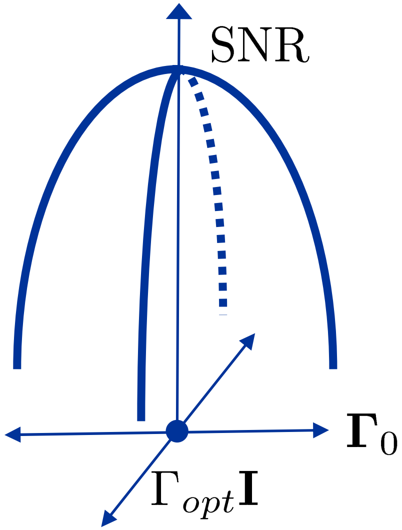
This matching condition simultaneously minimizes all singular values of $\mathbf{R}_{ee}$

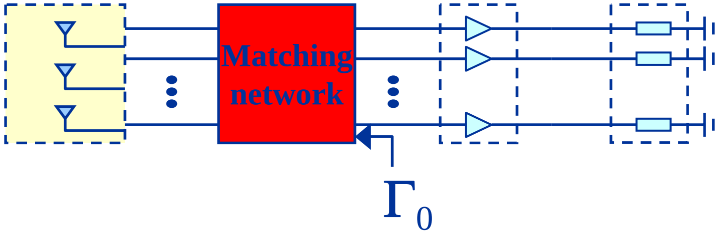
Since covariance matrices are Hermitian and positive definite, this is equivalent to minimum trace and determinant (but not to minimum 2-norm).

Noise matching efficiency is maximized: $\eta_{\mathrm{n}} = \dfrac{T_{\mathrm{min}}}{T} \dfrac{\mathbf{w}^H \mathbf{G} \mathbf{R}_{\mathrm{ee,iso}} \mathbf{G}^H \mathbf{w}}{\mathbf{w}^H \mathbf{G} \mathbf{R}_{\mathrm{ee}} \mathbf{G}^H \mathbf{w}}$



Beam Sensitivity/SNR





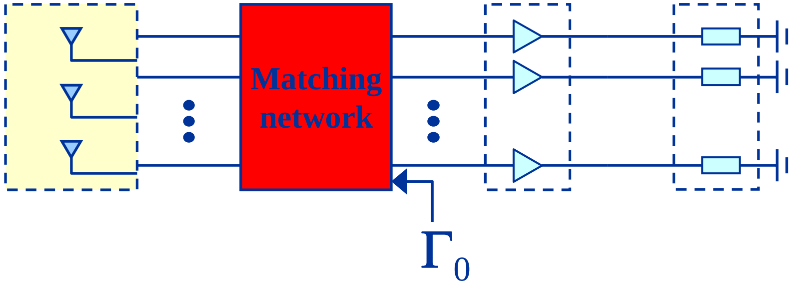
Physical Interpretation

Optimal S-matrix is diagonal

For best noise performance, the array must be decoupled

Noise from one amplifier is uncorrelated with noise from all other amplifiers, so if reverse noise from one amplifier scatters through the array to other amplifiers, it can only hurt the overall noise performance.

Real part of mutual impedance matrix is proportional to element pattern overlap integral matrix, so decoupled array implies orthogonal effective receiving patterns.



Qualifications

- For a given signal arrival angle (beam), other matching networks may achieve the maximal output SNR, but the exchangeable noise covariance has at least one larger eigenvalue.
- Optimal matching requires a complex, fully coupled network.
- *Loss*: Analysis assumes lossless matching network. If the matching network loss is large, a suboptimal network may be better.
- Possible suboptimal networks: self-impedance match, near-neighbor decoupling.

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Conclusions

- Array is characterized using embedded element patterns and mutual impedance matrix
- Network model provides signal response, spillover noise, receiver noise, and antenna thermal noise
- Beam aperture efficiency, spillover efficiency, radiation efficiency, and system temperature can be defined using either the array isotropic noise response or element pattern overlap integrals
- Minimum noise temperature over all possible beams requires decoupled array ports

Open Questions

- Can the isotropic noise response be measured directly?
- What is the optimal diagonal matching network for an FPA?
- How large is the penalty in the noise matching efficiency for suboptimal matching?
- Can the noise matching efficiency be improved by changing the array design to reduce coupling?