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(NETHERLANDS FOUNDATION FOR RESEARCH IN ASTRONOMY)

Calculation of phased array noise temperature

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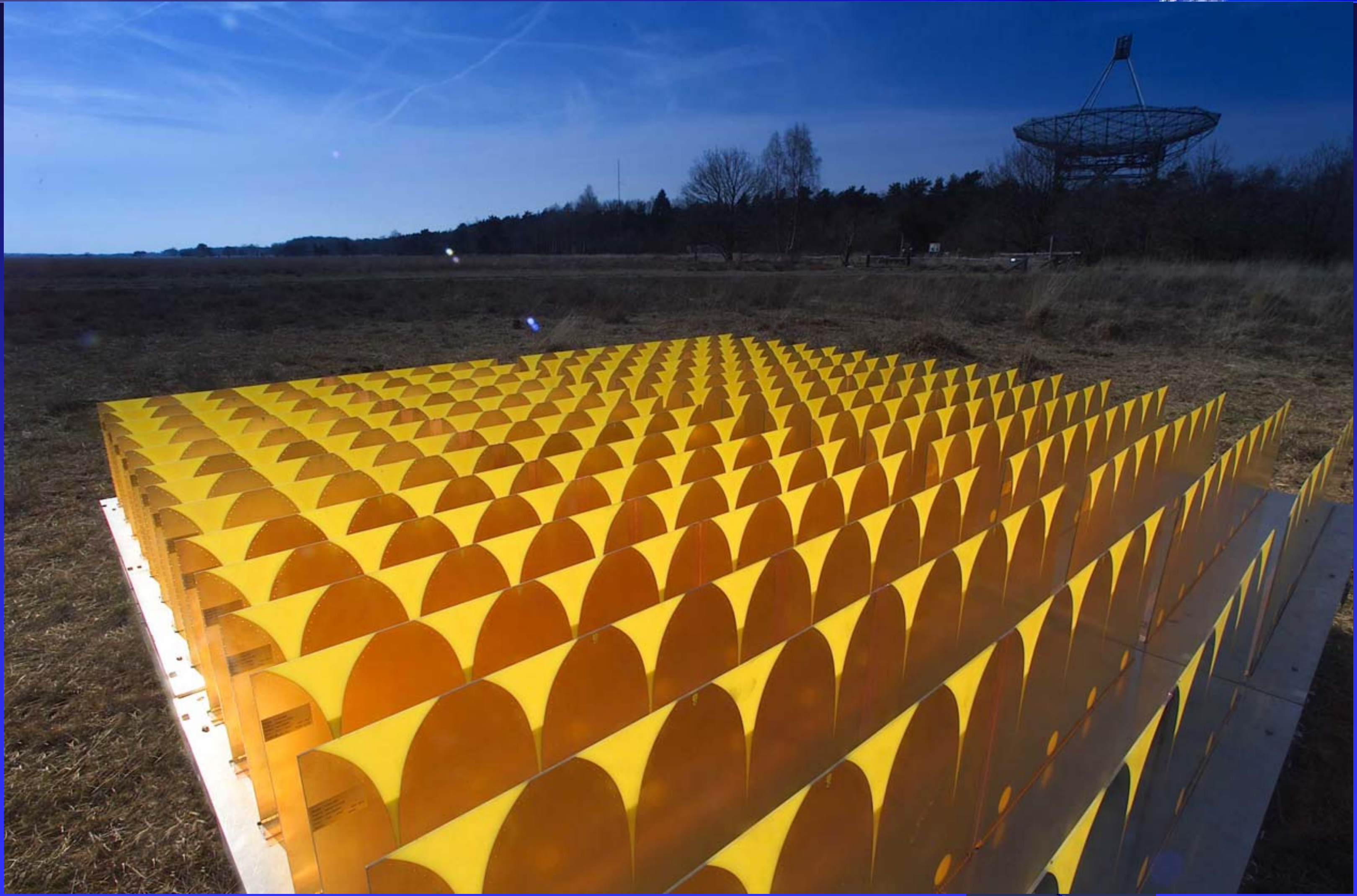
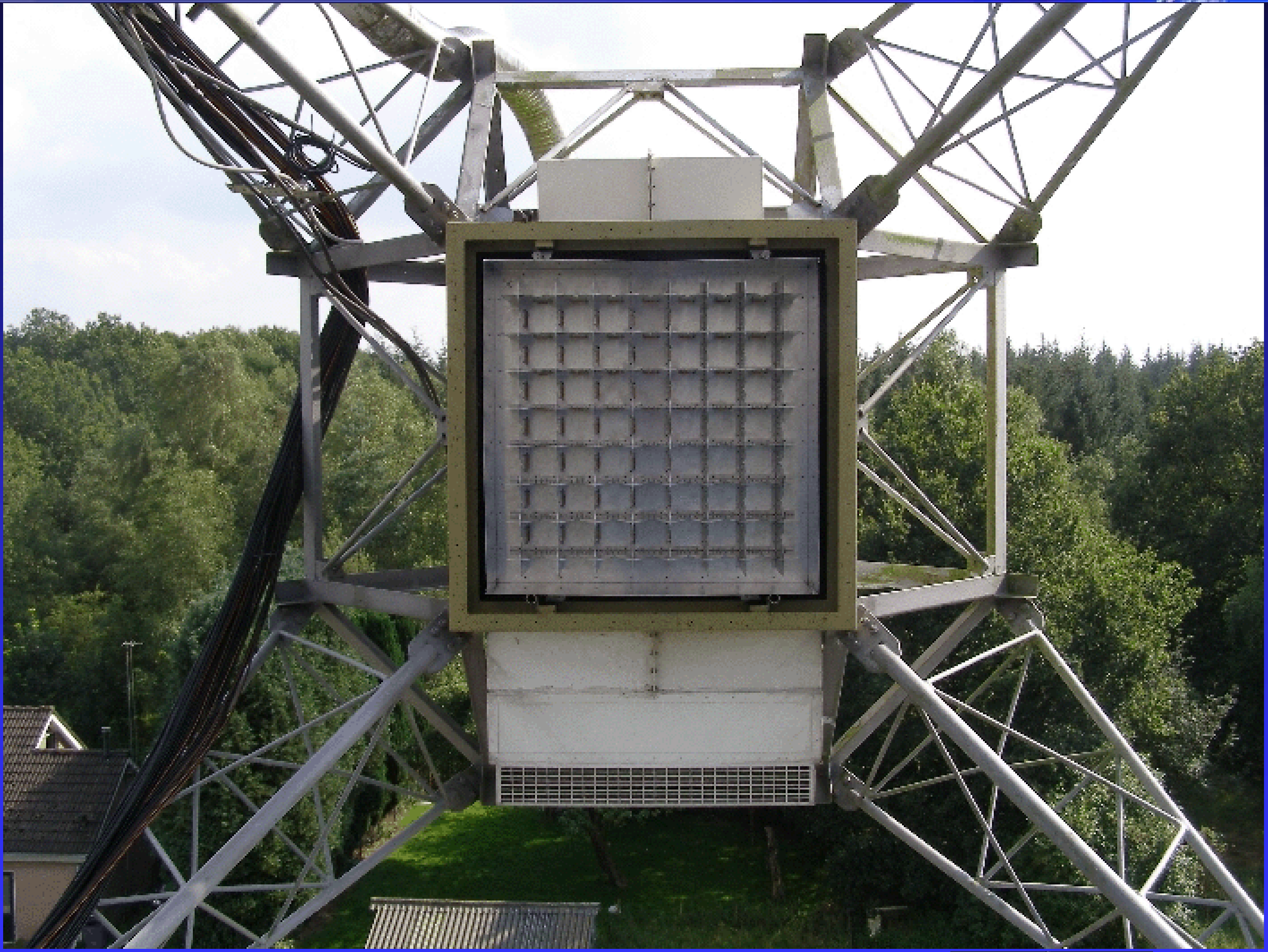
THEA

Aperture array



Digestif

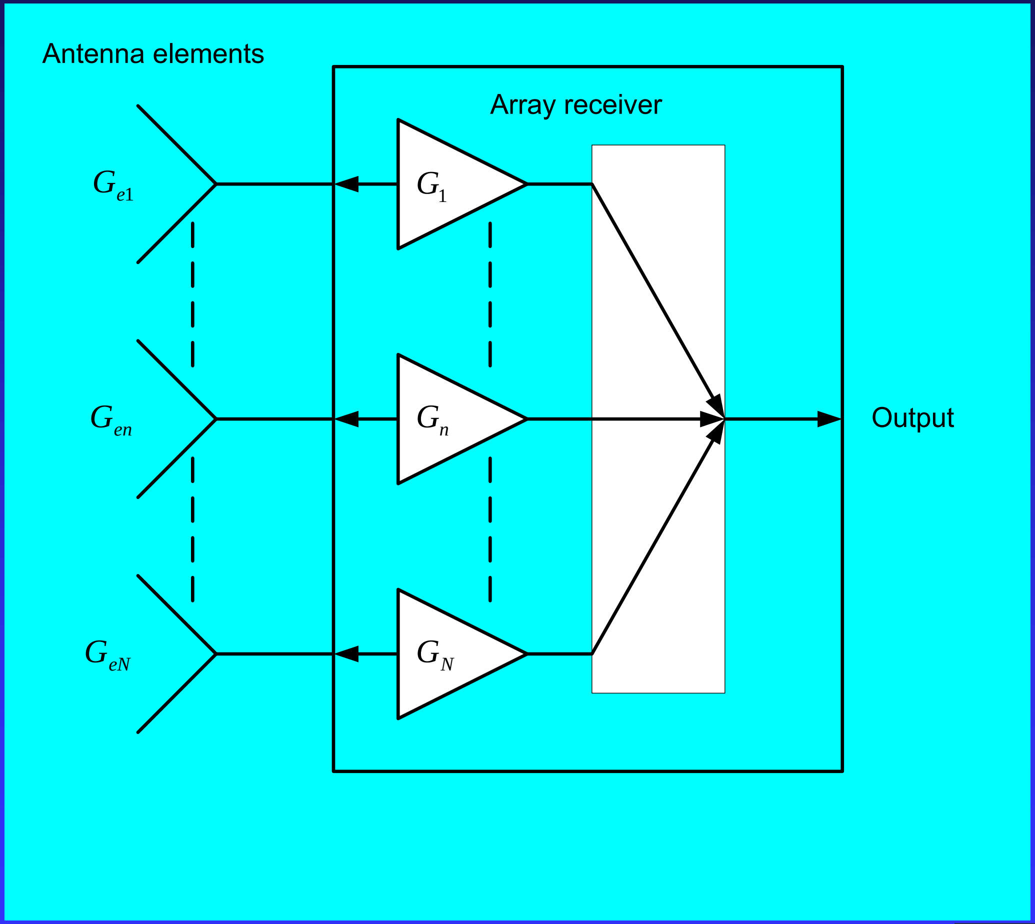
Focal Plane Array



Outline

- Modeling of an array system with wave parameters
- Modeling an array system as a combination of two-ports
- Example of WW-procedure for FARADAY
- Calculation methods (4) for array noise temperature
- Examples of calculations for some simple arrays
- Conclusion

A practical array receiver system



Array modeling with wave parameters

- Noise output due to internal noise sources

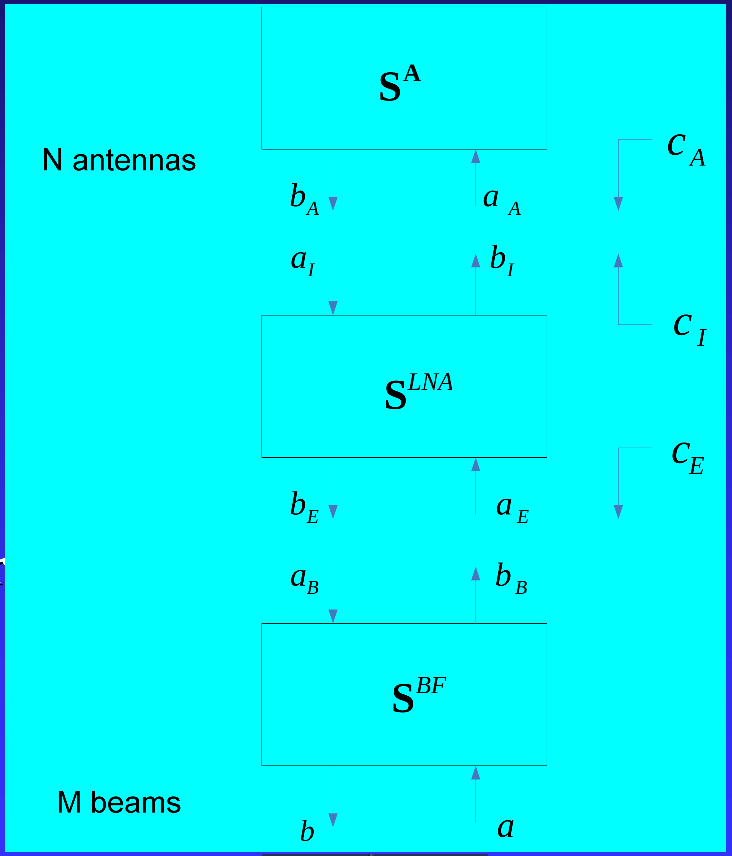
$$\underline{b}_E = \underline{c}_E + \mathbf{S}_{21}^{LNA}\mathbf{S}^{\mathbf{A}}(\mathbf{I}-\mathbf{S}_{11}^{LNA}\mathbf{S}^{A})^{-1}\underline{c}_I$$

$$\underline{b} = \mathbf{S}_{21}^{BF}\,\underline{b}_E$$

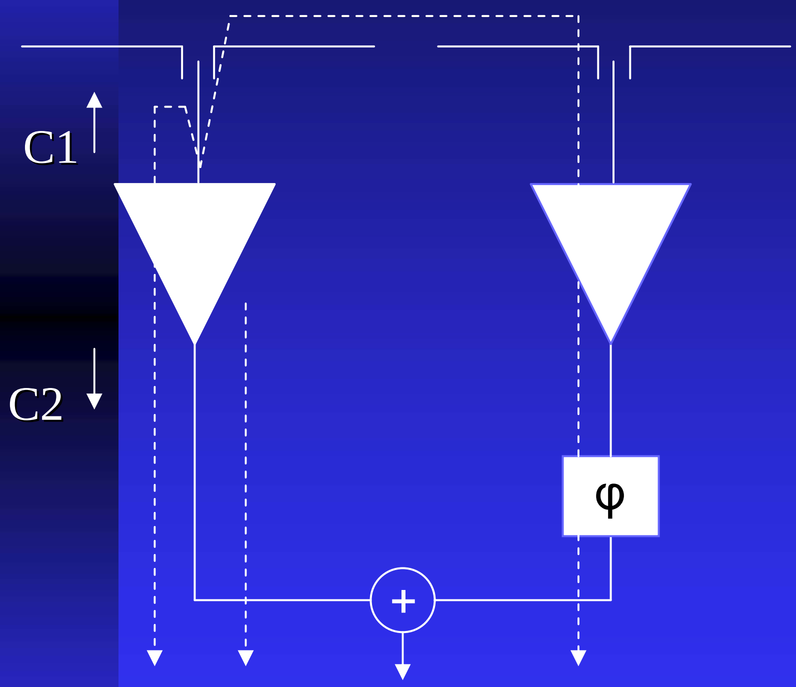
fully describes array noise behavior; basis for array noise calculations with CAESAR

- Matrix description using active reflection coefficient does not easily show criteria for optimization

$$\underline{b} = \underline{w}\mathbf{C}_{\mathbf{E}} + S_{21}\,\underline{w}\mathbf{R}_{\mathbf{act}}(\mathbf{I}-\mathbf{S}_{11}^{LNA}\mathbf{S}^{\mathbf{A}})^{-1}\mathbf{C}_{\mathbf{I}}$$

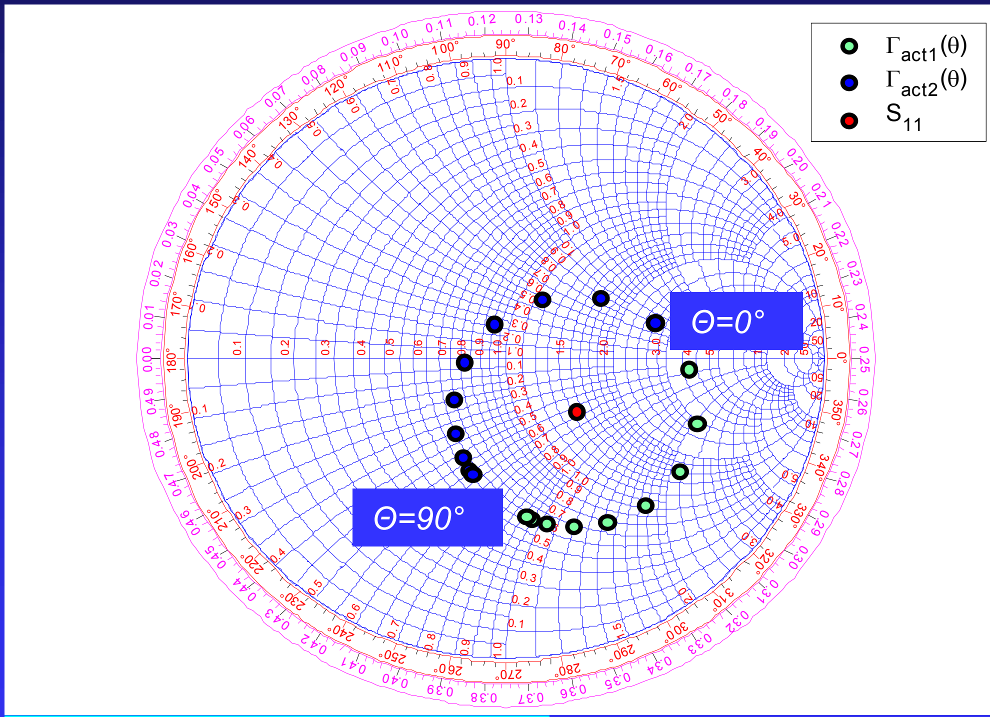


A two-element dipole array



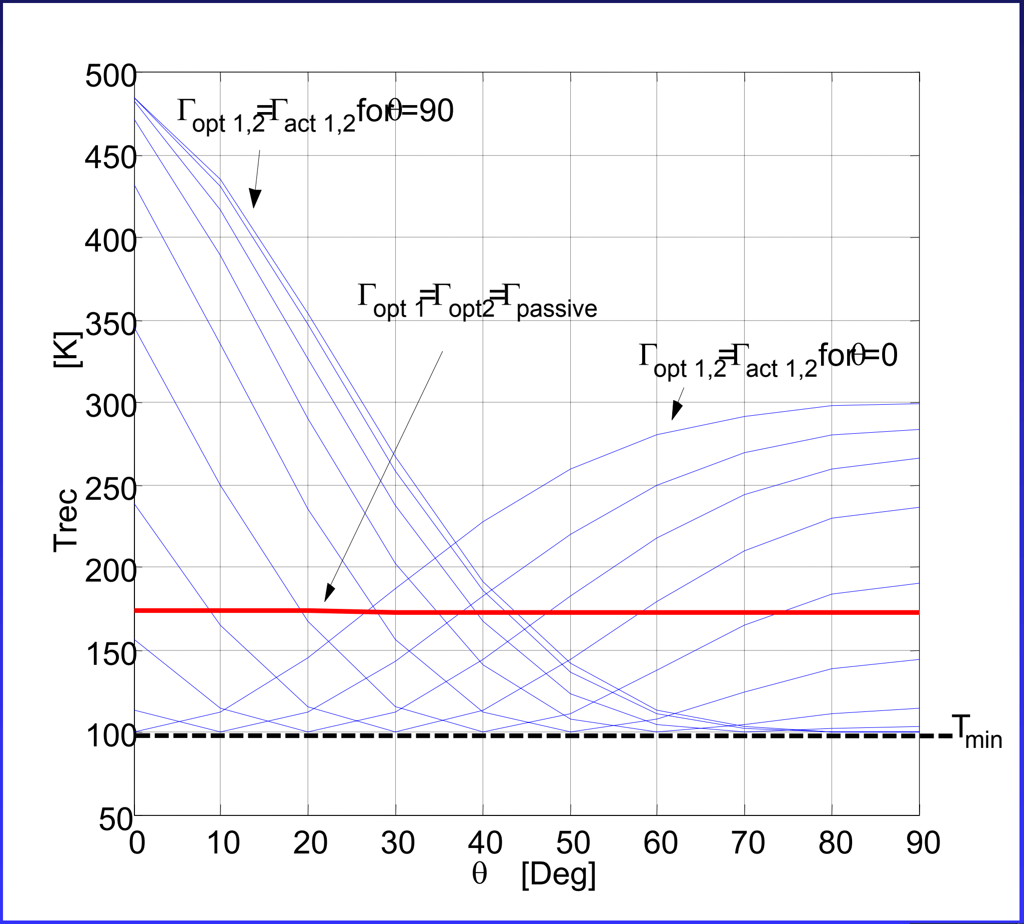
Ctot=Direct+Refl+Coupl

Reflection coefficients in a 2x1 dipole array



$$\Gamma_{act_i} = \sum_j S_{ji} e^{j(\Phi_j - \Phi_i)}$$

Trec as a function of scan angle (results with CAESAR)



Modeling the array as a combination of two-ports

- Replace the array system by an equivalent single-channel system
- The single antenna represents the array beam properties
- The single receiver represents the array gain and noise temperature
- The receiver gain and noise temperature are expressed in the properties of the individual two-port receivers, embedded in the array environment
- Thus the effects of mutual coupling in the array on the two-port noise and gains are included, effectively defining independent two-port receiver channels

Embedded two-port noise temperature and available gain formulas

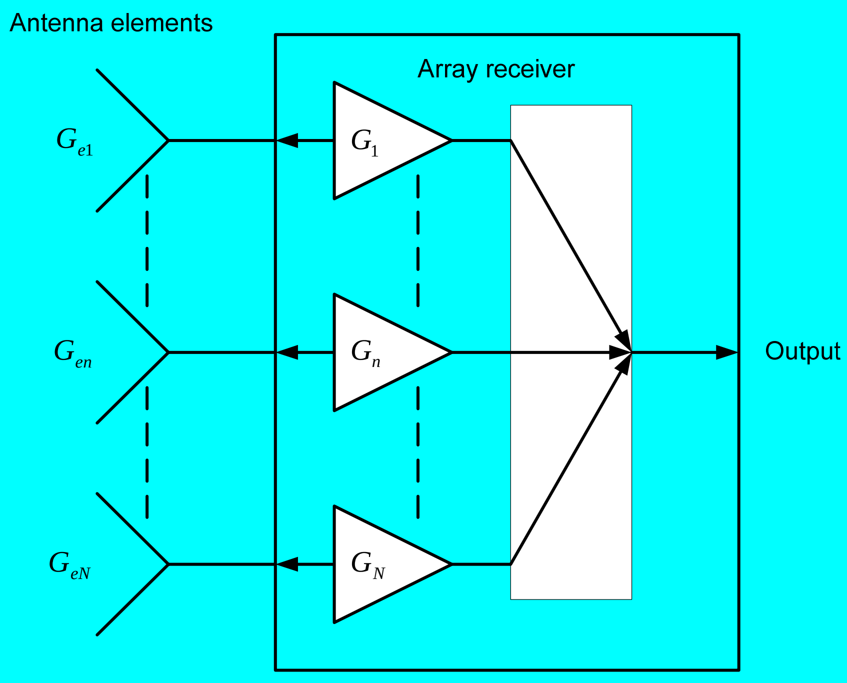
$$T_n = T_{\min} + \frac{4R_n T_0}{Z_0}\frac{\left|\Gamma_{act} - \Gamma_{opt}\right|^2}{\left|1+\Gamma_{opt}\right|^2\left(1-\left|\Gamma_{act}\right|^2\right)}$$

$$\left|\Gamma_{act}\right| \neq 1$$

Available gain: $$G_{av} = \frac{\left|S_{21}\right|^2\left(1-\left|\Gamma_{act}\right|^2\right)}{\left|1-\Gamma_{act}S_{11}\right|^2\left(1-\left|S_{22}'\right|^2\right)},\quad S_{22}' = S_{22} + \frac{S_{12}\Gamma_{act}S_{21}}{1-\Gamma_{act}S_{11}}$$

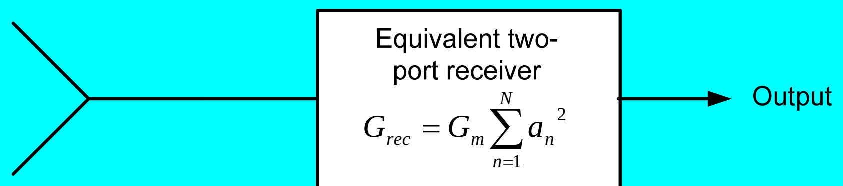
Definition of array receiver gain

Replace the array system by an equivalent single-channel system



$$S_o = P_o G_m \left| \sum_{n=1}^{N} \left(\sqrt{G_{en}}\right) a_n \exp(j\theta_n) \right|^2$$

$$a_n = \sqrt{\frac{G_n}{G_m}}$$



Equivalent single-output antenna

$$G_a = \left| \sum_{n=1}^{N} \left(\sqrt{G_{en}}\right) a_n \exp(j\theta_n) \right|^2 \Big/ \sum_{n=1}^{N} a_n^2$$

$$S_o = P_o G_a G_m \sum_{n=1}^{N} a_n^2$$

$$G_{rec} = G_m \sum_{n=1}^{N} a_n^2 = \sum_{n=1}^{N} G_n$$

Definition of array receiver noise temperature

Calculate from excess output noise density N_o

of receiver: $T_{rec} = \dfrac{N_o}{kG_{rec}}$

Array receiver noise temperature is weighted average of the individual embedded receiver channel noise temperatures

$$T_{rec} = \frac{\sum_{n=1}^{N} G_n T_n}{\sum_{n=1}^{N} G_n}$$

Calculation of equivalent receiver gain and noise, including combiner

- Consider simple N-channel array of identical two-ports, formed by a cascade of two-ports, and one combiner with amplitude taper. Gain of two-port:

$$G_{rec} = \sum_{n=1}^{N} G_n = G \sum_{n=1}^{N} G_{cn} = G G_c$$

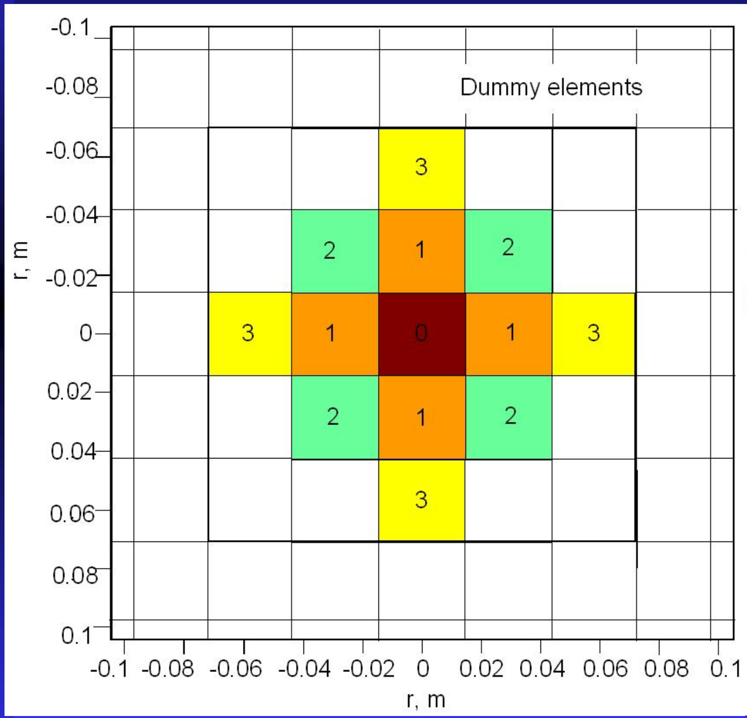
- Excess output noise density $N_o = kTG_{rec} + kT_0(1 - G_c)$ gives noise temperature, according to cascade noise formula for one channel two-port with a second stage passive lossy two-port, representing the combiner

$$T_{rec} = T + \frac{T_0}{G}\left(\frac{1}{G_c} - 1\right)$$, with $$T = \frac{\sum_{n=1}^{N} G_n T_n}{\sum_{n=1}^{N} G_n}$$

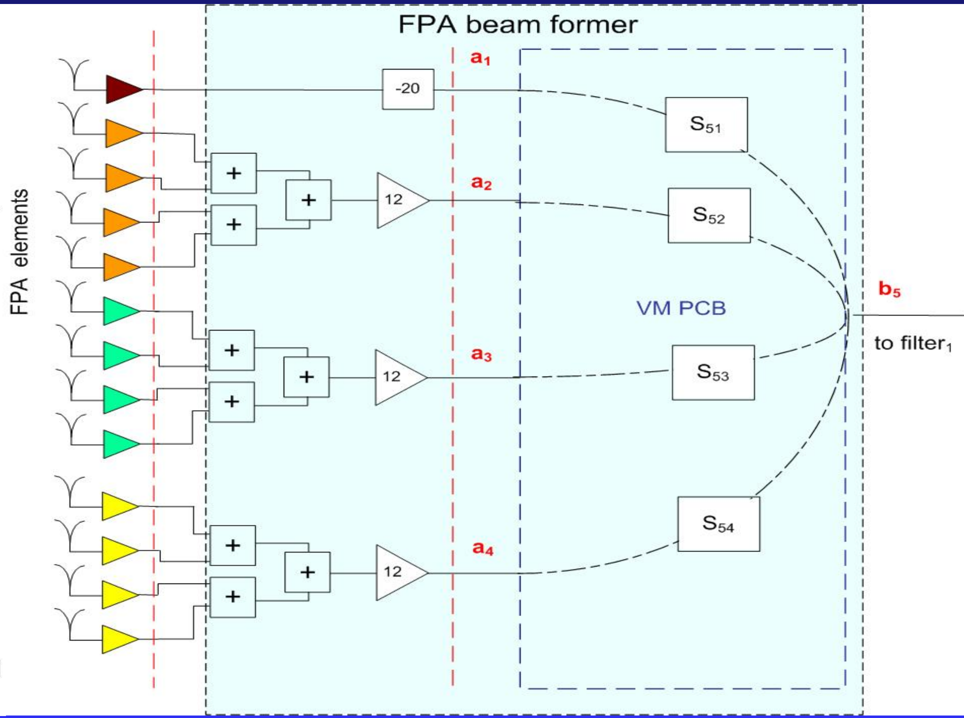
References

- Waldman, A., Wooley, G.J., “Noise Temperature of a Phased Array Receiver”, The Microwave Journal, pp. 89-96, September 1966
- Lee, J.J., “G/T and Noise Figure of Active Arrays”, IEEE Transactions on Antennas and Propagation, Vol.41, No. 2, pp. 241-244, February 1993
- Woestenburg, E.E.M., “Definition of Array Receiver System Gain and Noise Temperature”, Poster at Paris SKA Workshop, September 2006, ASTRON Doc.nr. RP-123, August 2006
- Maaskant, R., Woestenburg, E.E.M., “Applying the Active Antenna Impedance to Noise Match in Receiving Array Antennas”, Proc. APS, June 2007, Hawaii
- Maaskant, R., Yang, B., “A Combined Electromagnetic and Microwave Antenna System Simulator for Radio Astronomy”, Proc. EuCAP, November 2006, Nice

Example of analysis of the FARADAY array with the WW-procedure

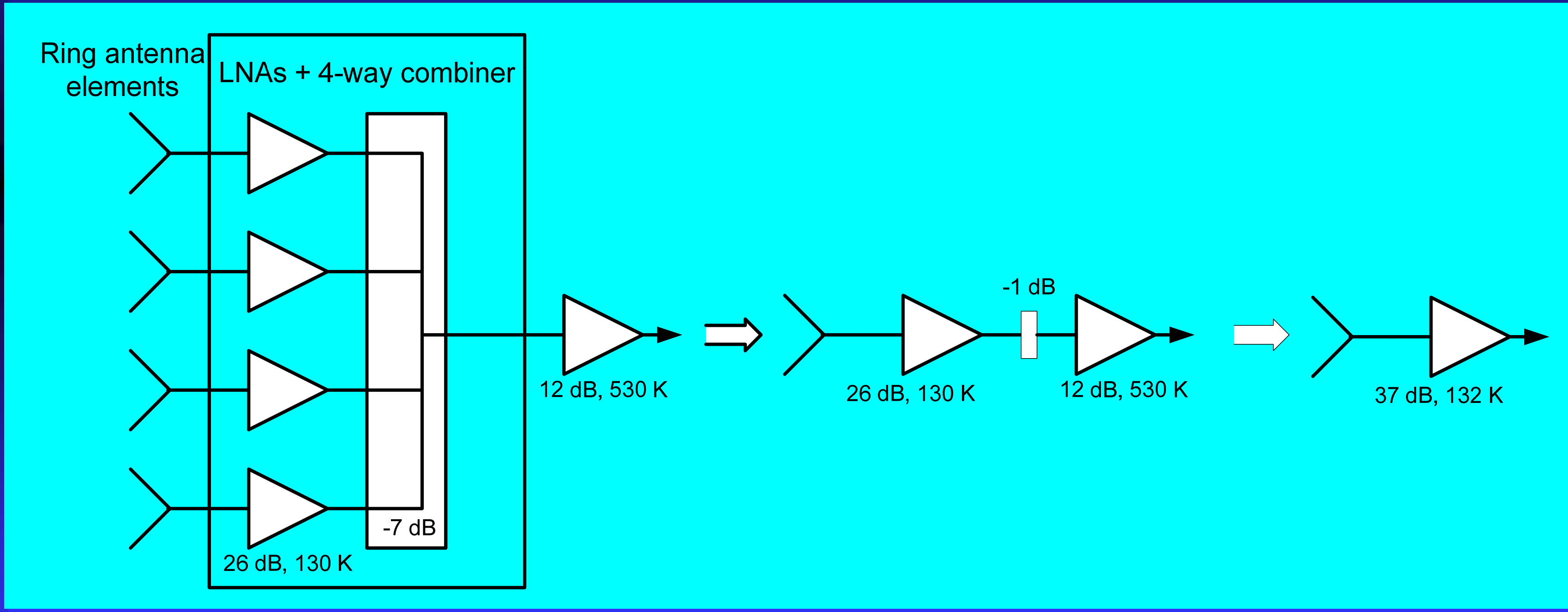


Top view of the array

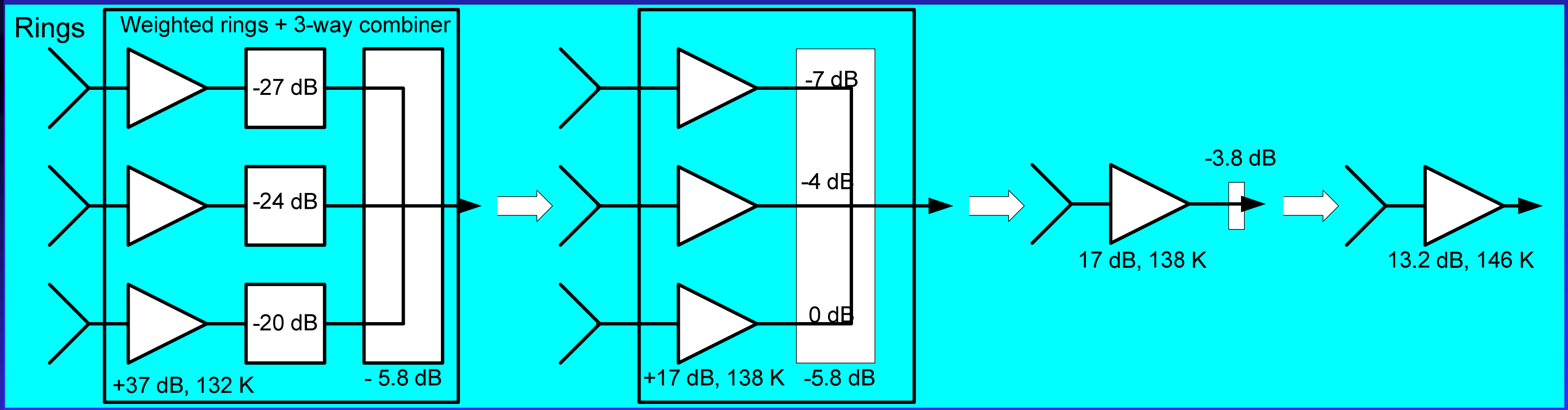


Beam former configuration 2005

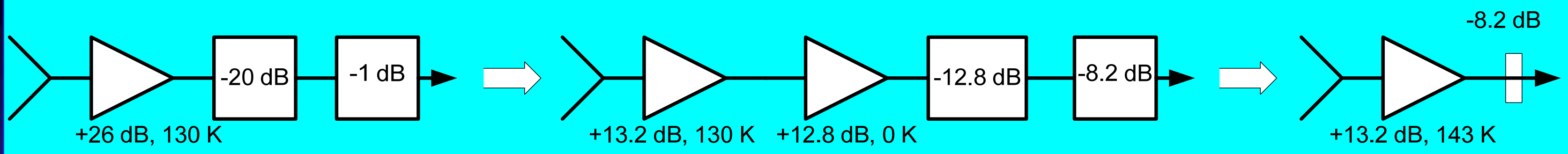
Transforming a ring system into an equivalent two-port receiver



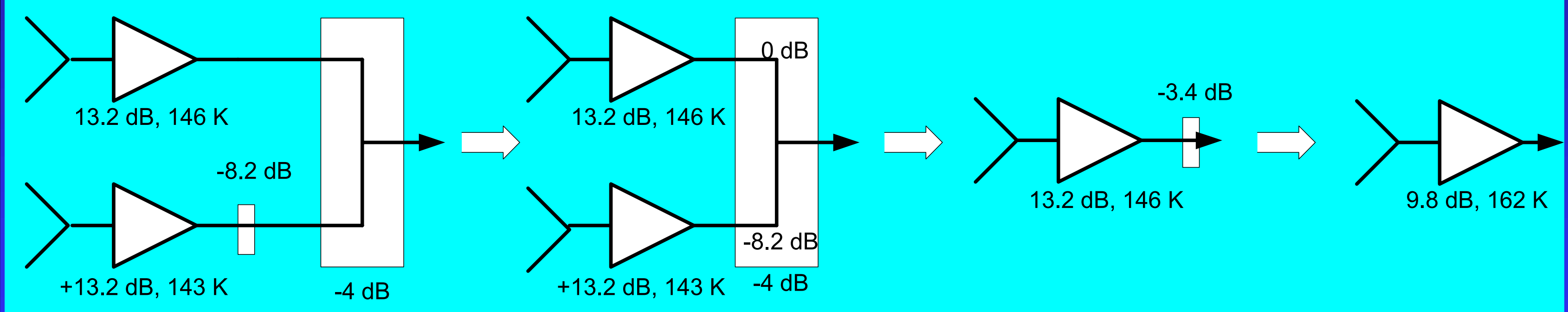
Combination of the weighted rings



The equivalent central element



The central element added, to give the equivalent FPA receiver



Calculation of array noise temperature (Method 1)

- Weighted average of embedded two-port noise temperatures, using active reflection coefficients

$$T_{array} = \frac{\sum_{n=1}^{N} G_{av_n} T_{rec_n}}{\sum_{n=1}^{N} G_{av_n}}$$

$$T_{rec_n} = T_{\min_n} + \frac{4 R_{n_n} T_0}{Z_0} \frac{\left|\Gamma_{act_n} - \Gamma_{opt_n}\right|^2}{\left|1 + \Gamma_{opt_n}\right|^2 \left(1 - \left|\Gamma_{act_n}\right|^2\right)}$$

$$\Gamma_{act_n} = \sum_{m=1}^{N} \frac{G_m}{G_n} S_{nm} e^{j(\phi_m - \phi_n)}$$

$$G_{av_n} = G_{rec_n} (1 - \left|\Gamma_{act_n}\right|^2)$$

- Represents noise temperature at array input, as would result from a measurement with a hot and cold load at the array input
- Calculation method implemented in Excel spread sheet

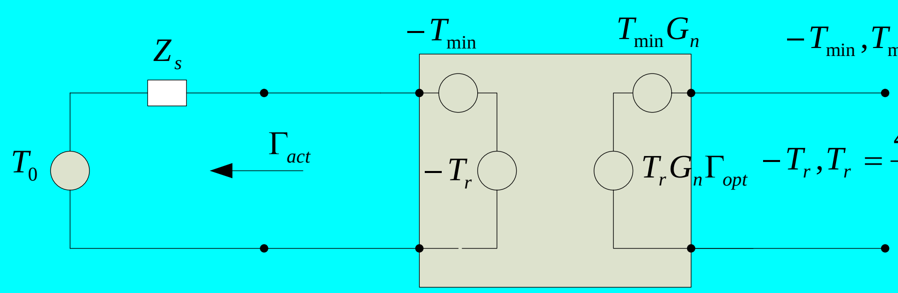
Calculation of array noise temperature (Method 2)

- CAESAR, numerical tool based on MM for modeling antenna structures, combined with microwave system simulator, using noise wave correlation and S-matrices to calculate array output noise for a certain sky brightness temperature. Program input includes LNA parameters
- Two calculations are performed to determine the array noise temperature, as the array response to a sky signal, thus by definition the noise temperature at the input of the antenna array

$$T_{array} = \frac{P_{out}(T_{sky} = 0)}{P_{out}(T_{sky} = 1, T_{rec} = 0)}$$

Calculation of array noise temperature (Method 3)

Calculate array output noise from individual channel two-port contributions, using 4 noise sources derived from noise wave formulas given by Wedge



$-T_{\min}, T_{\min}$, uncorrelated sources

$-T_r, T_r = \frac{4 R_n T_0}{Z_0} \frac{1}{\left|1+\Gamma_{opt}\right|^2}$, correlated sources

Correlated output noise density $kT_r G_n \left|\Gamma_{opt} - \Gamma_{act}\right|^2$

Uncorrelated output noise density $(1-\left|\Gamma_{act_n}\right|^2) kT_{\min} G_n$

Use array available gain $G_{av_{array}} = \sum_{n=1}^{N} G_{av_n} = \sum_{n=1}^{N} G_n (1-\left|\Gamma_{act_n}\right|^2)$ to calculate array input noise temperature:

$$T_{array} = \frac{\sum_{n=1}^{N} \left[(1-\left|\Gamma_{act_n}\right|^2) T_{\min_n} G_n + T_{r_n} \left|\Gamma_{opt_n} - \Gamma_{act_n}\right|^2 G_n \right]}{\sum_{n=1}^{N} G_{av_n}}$$

Calculation of array noise temperature (Method 4)

- Use of Y-parameter models with associated noise sources (O'Sullivan – CSIRO)
- Different from other three methods, but gives the same numerical results in comparing calculations for a 7-element array with other methods

Available array gain and coupling efficiency

- Array coupling and excitation are incorporated in the active reflection coefficient, which may be >1 in FPAs

$$\Gamma_{act_n} = \sum_{m=1}^{N} \frac{G_m}{G_n} S_{nm} e^{j(\phi_m - \phi_n)}$$

- Coupling efficiency as defined by Kehn et al is also a measure of array coupling and excitation

$$\varepsilon_c = \frac{P_{rad}}{P_{inc}} = 1 - \frac{P_c}{P_{inc}} = 1 - \frac{\sum_{n=1}^{N} |b_n|^2}{\sum_{m=1}^{M} |a_m|^2}, b_n = \sum_{m=1}^{M} S_{mn} a_m$$

- Relation $\varepsilon_c = 1 - \frac{\sum_{n=1}^{N} G_n |\Gamma_{act_n}|^2}{\sum_{m=1}^{M} G_m}$ → $\varepsilon_c \sum_{n=1}^{N} G_n = \sum_{n=1}^{N} G_n (1 - |\Gamma_{act_n}|^2) = G_{av}$

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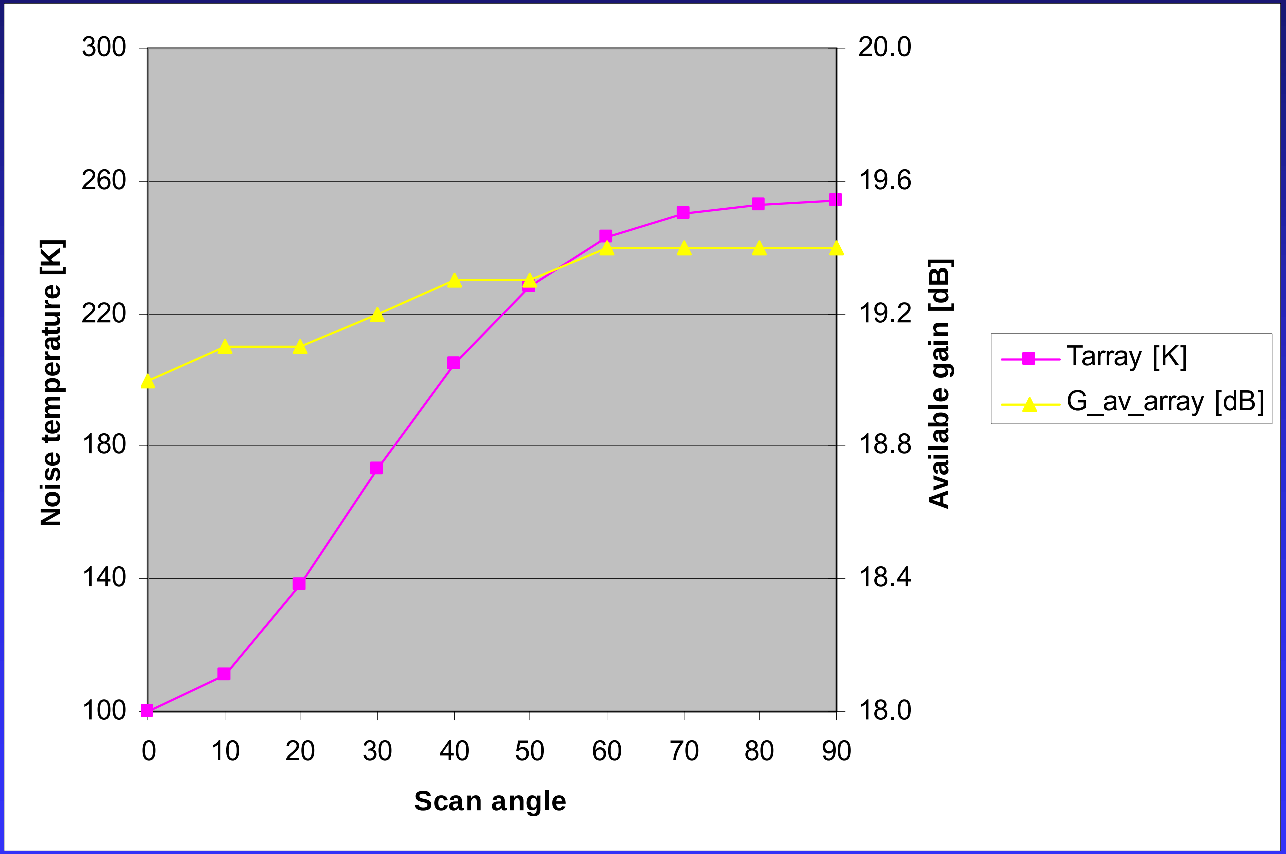
References

- Wedge, S.W., Rutledge, D.B., “Wave Techniques for Noise Modeling and Measurement”, IEEE MTT, Vol.40. No.11, November 1992, pp. 2004-2012
- Kehn, M.N.M., Ivashina, M.V., Kildal, P.-S., Maaskant, R.,”Coupling Efficiency of Wideband Dense Focal Plane Array Feeds for Reflector Antennas – Part I: Definitions and Theoretical Study on a Hypothetical Hard Waveguide Array”, submitted to IEEE Trans.Antennas Propagat., July 2007
- Woestenburg, E.E.M., “Calculation of the Noise Temperature of Focal Plane Array Receiver Systems”, ASTRON Doc. No. RP 269, September 2007

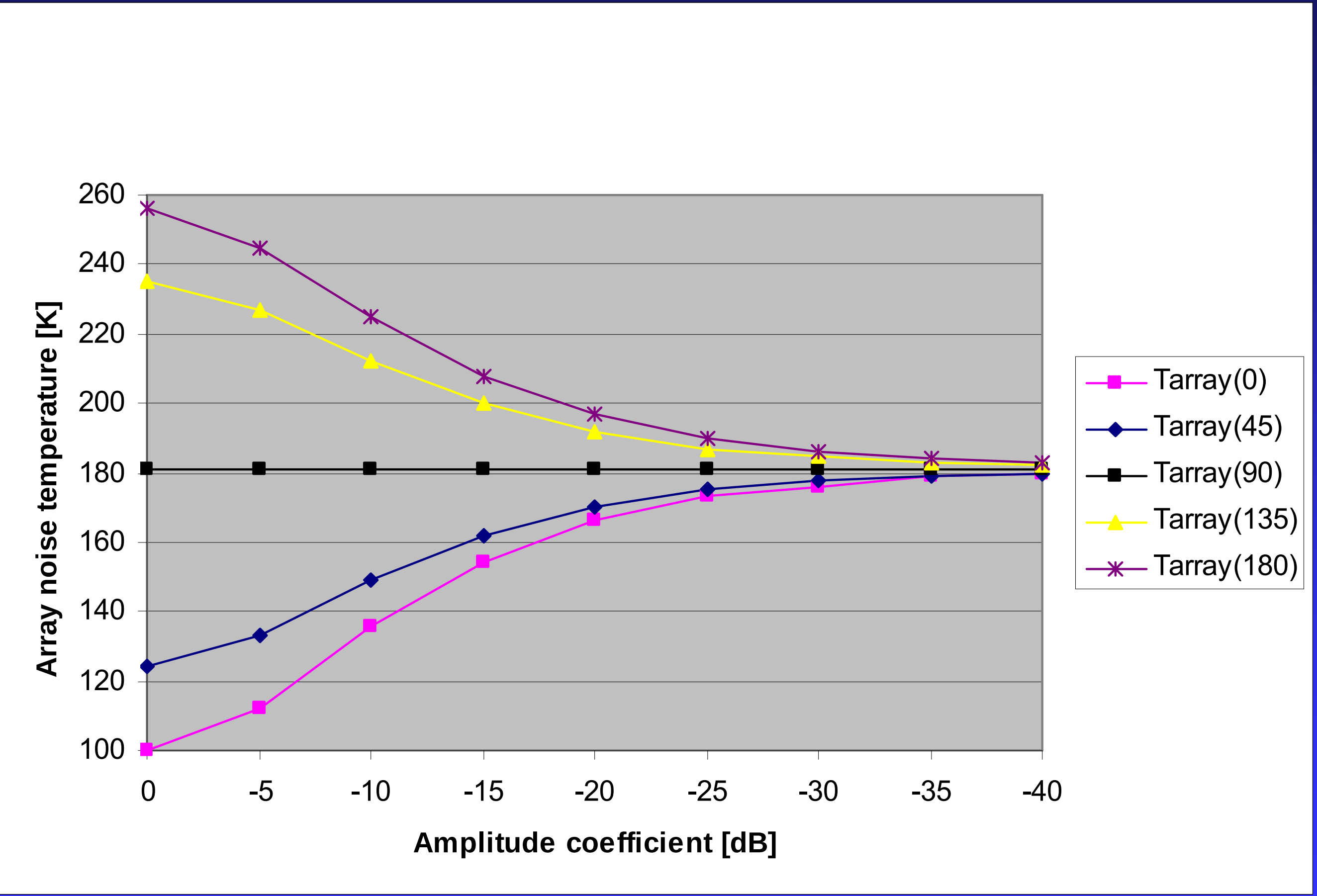
Calculation examples for simple FPAs

- Two-element dipole array with equal weighting gives the same results with simulator and noise formula calculations
- 3-element array with equal weighting of outer elements
- 5-element FPA at 2.3 GHz also gives consistent results (not presented here)
- 7-element array (Y-parameters O'Sullivan)

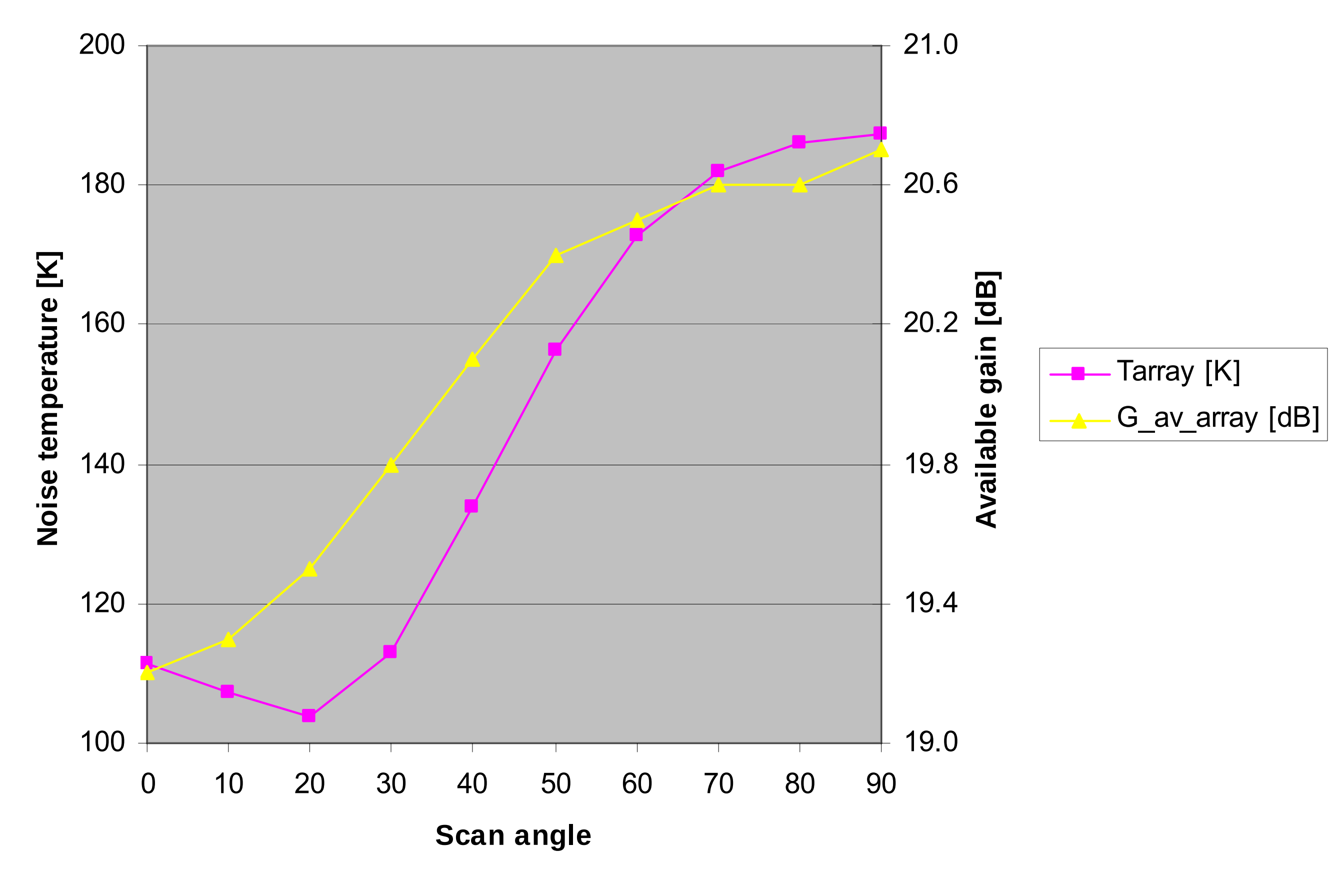
Noise temperatures and available gains as a function of scan angle for the two-element array (method 2) for methods 1 and 3, showing overlapping results



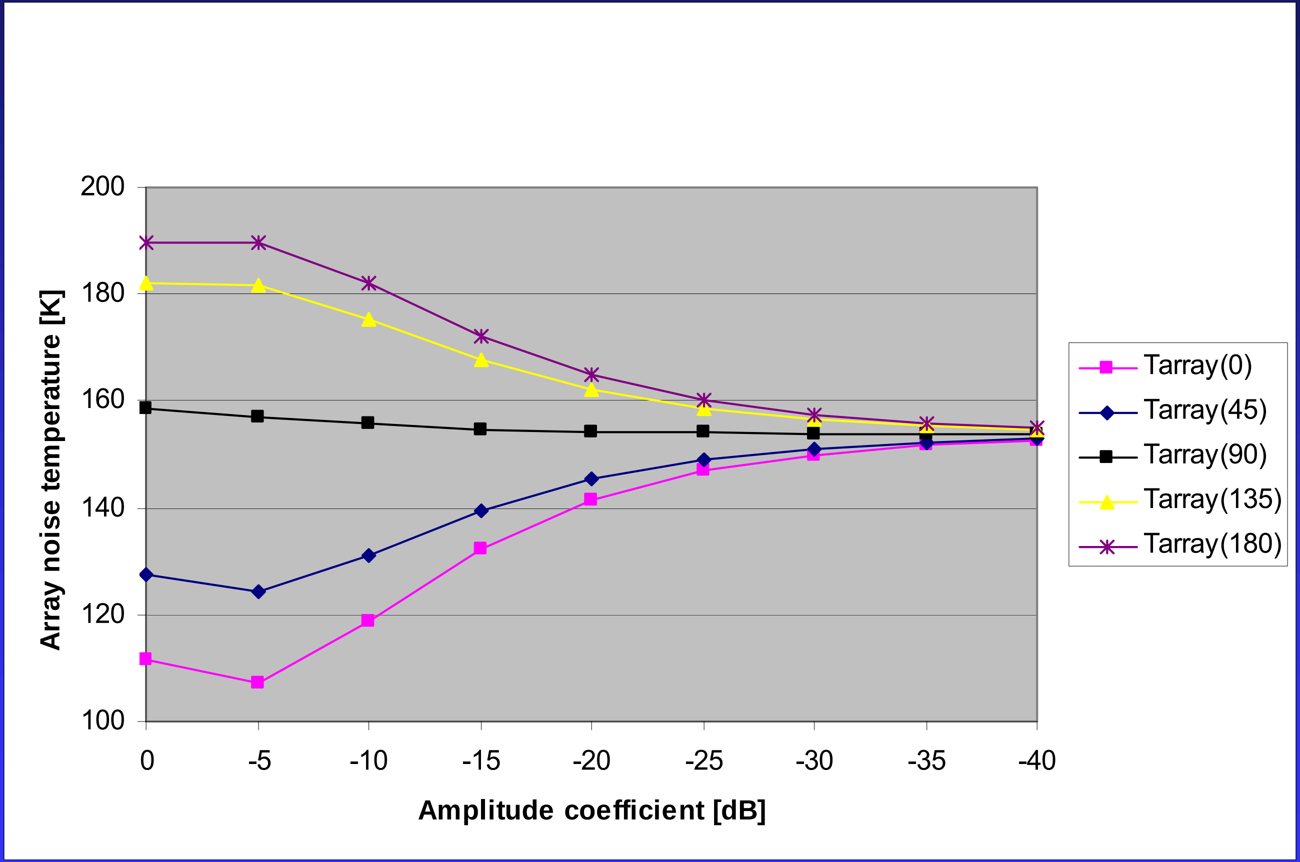
Two-element array noise temperature as a function of amplitude coefficients for five different phases (Methods 1 and 3)



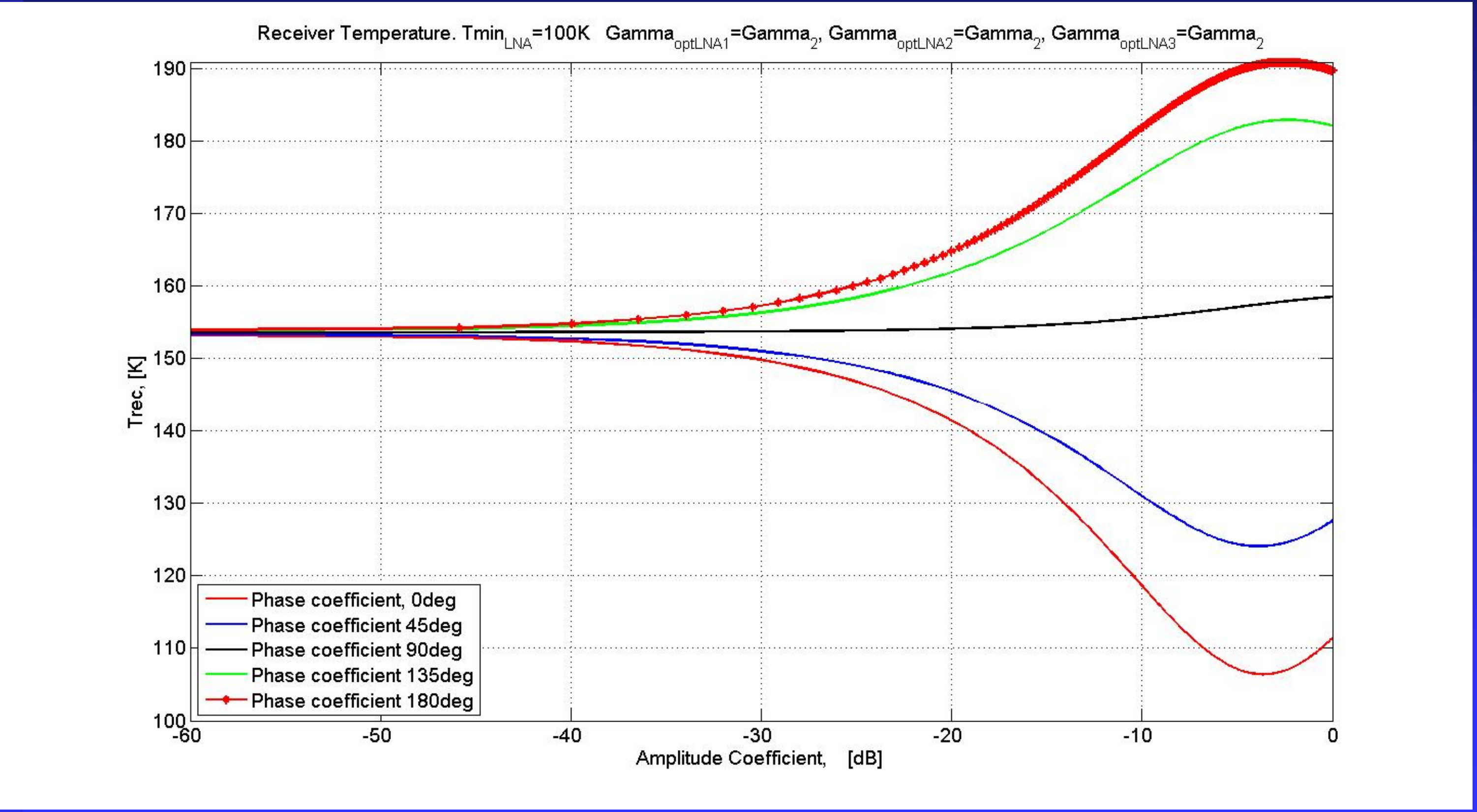
Noise temperature and available gain as a function of scan angle for a three-element array (Methods 1 and 3)



Three-element array noise temperature as a function of amplitude coefficient for the outer elements, at five different phases (Method 1 and 3)

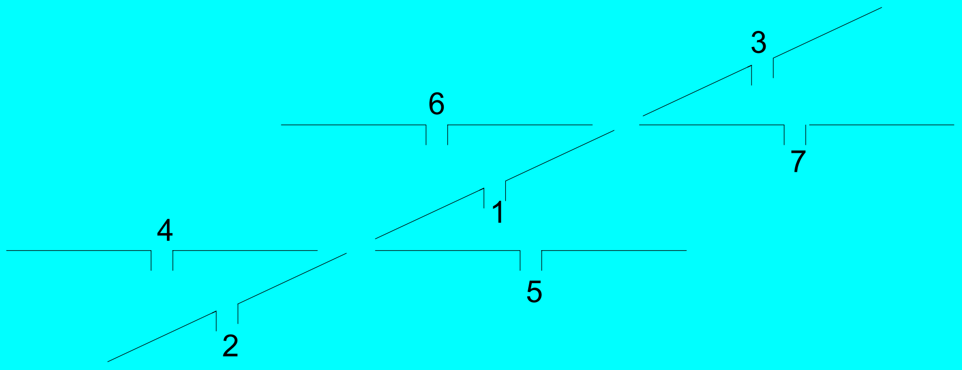


Results obtained with CAESAR for the three-element array for comparison with previous picture. Results are exactly the same for Methods 1, 2 and 3



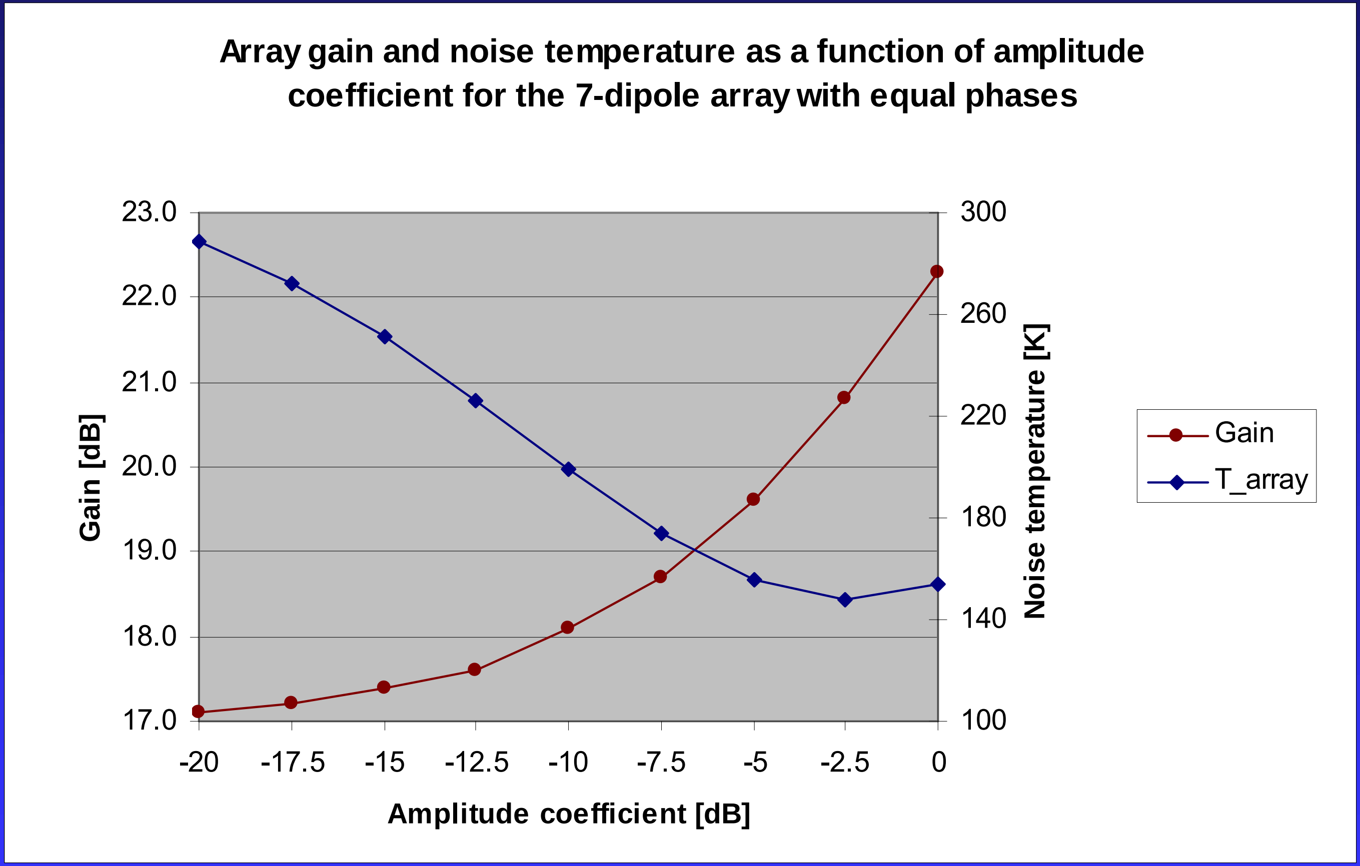
Geometry and configuration of a 7-element array

Dipole length	0.150 m
Dipole width	0.001 m
Distance between dipoles	0.152 m
Distance to ground plane	0.075 m
Frequency	1 GHz

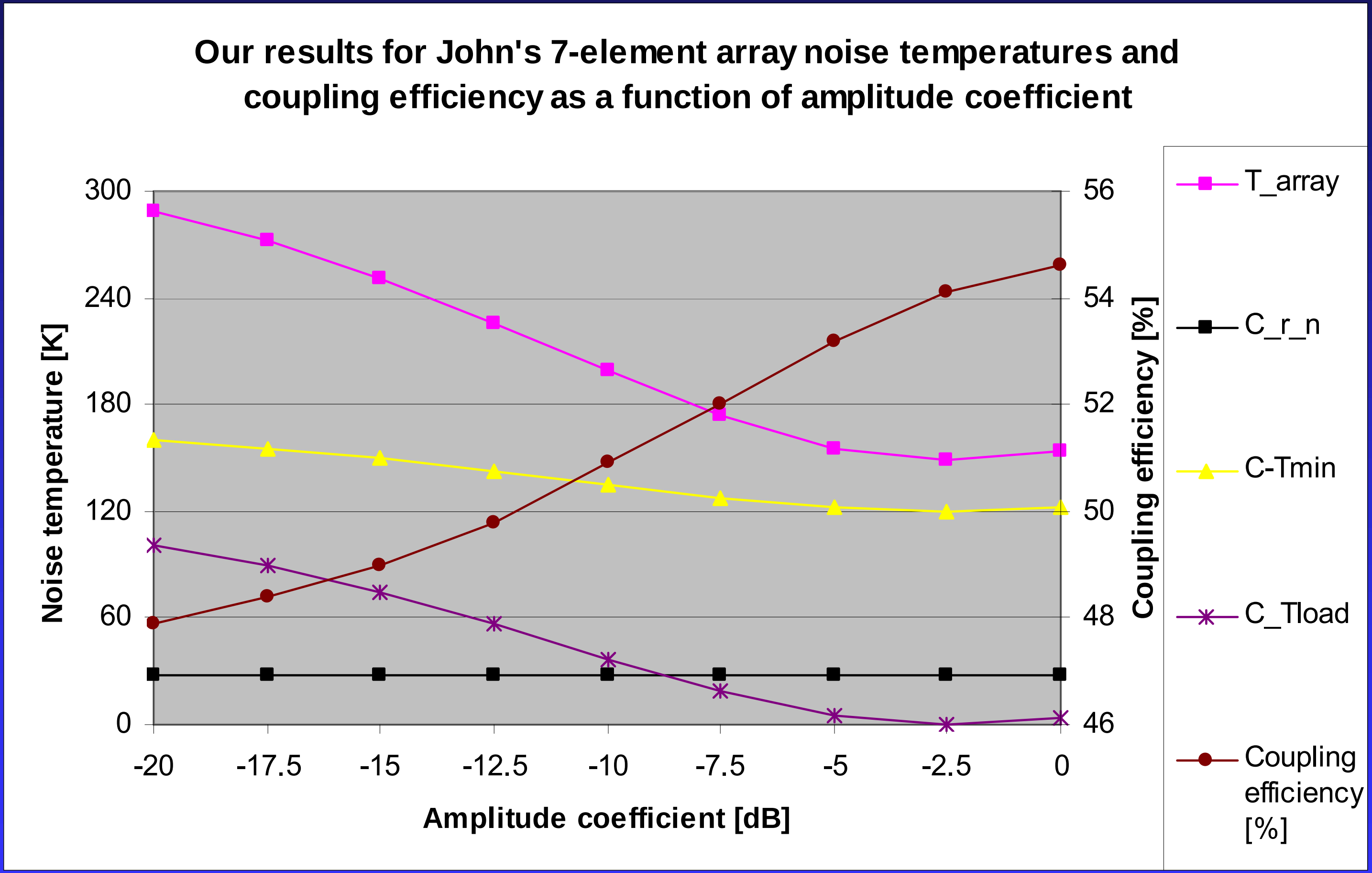


Elements 1, 2 and 3 have LNAs with $T_{\min} = 120K, \Gamma_{opt} = S_{11}, R_n = 25$
Equal weights for elements 2 and 3
Elements 4, 5, 6 and 7 have loads at 300 K

Results with Methods 1, 2 and 3 for the 7-element array, compared to noise temperature calculations with Method 4 (blue dots)



Detailed comparison of noise contributions for ASTRON and CSIRO calculations



Conclusions

- Arrays may be described in terms of a single antenna and receiver chain. (Uncorrelated) gain and noise of the array receiver are found by replacing the array receiver with a single receiver chain with weighted properties of the array receiver elements. The antenna gain accounts for the phasor summation of signals from the individual antenna elements
- Receiver element noise and available gain should include contributions from array coupling, which may be described by the active reflection coefficient
- Calculation of noise properties of complex FPAs may be done with a simulation tool (CAESAR), but also easy to use two-port noise formulas
- Examples of verification with 4 methods for a number of relatively simple FPAs, with excellent agreement