# Low Noise Amplifier fundamentals

Bert Woestenburg

# WSRT

Synthesis Array

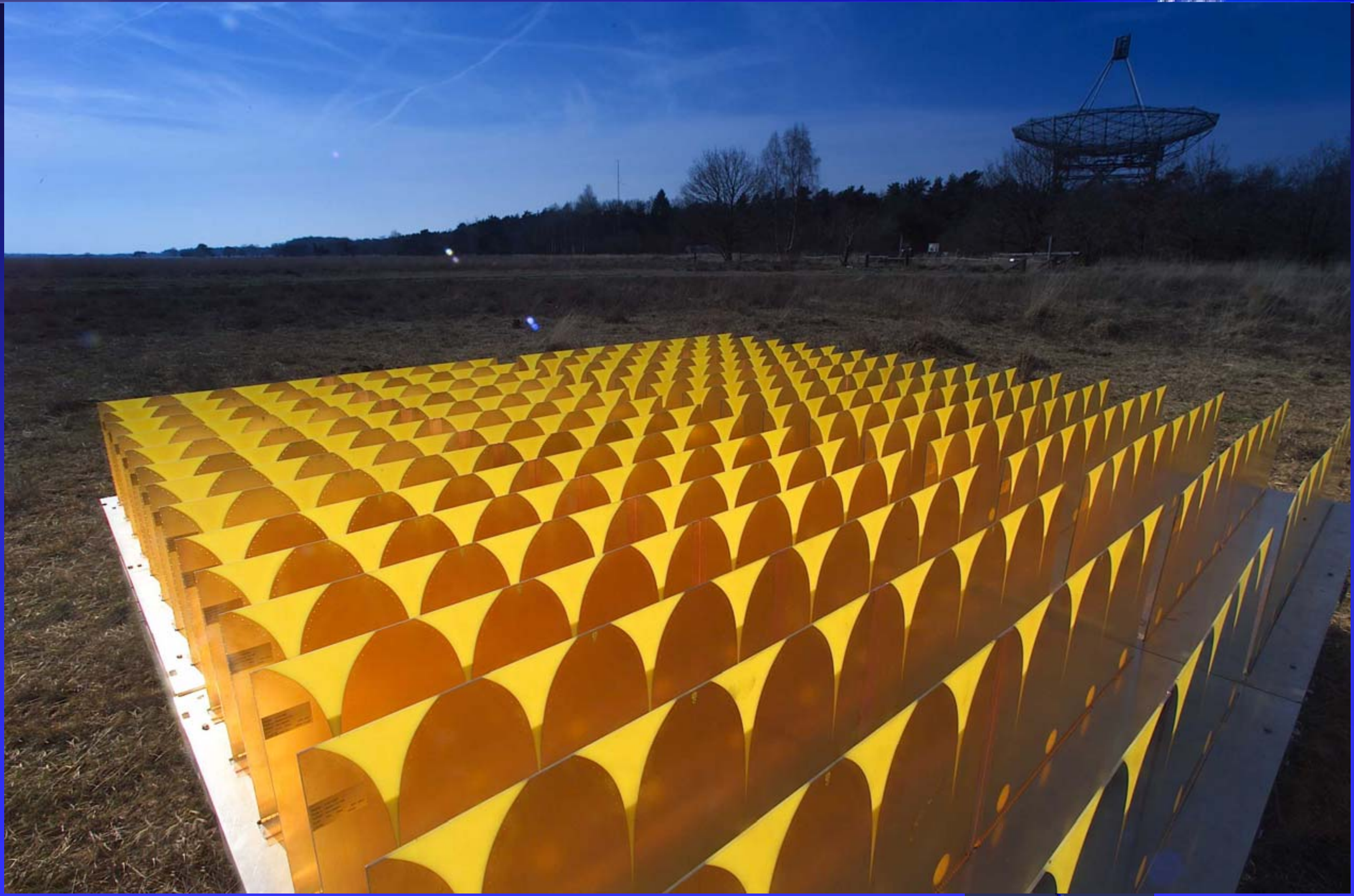
# MFFE

WSRT Multi Frequency Front End (MFFE), showing two single pixel feeds and a two-element dual-polarization array

The WSRT covers a frequency range from 110 MHz to 8.7 GHz in 9 different bands

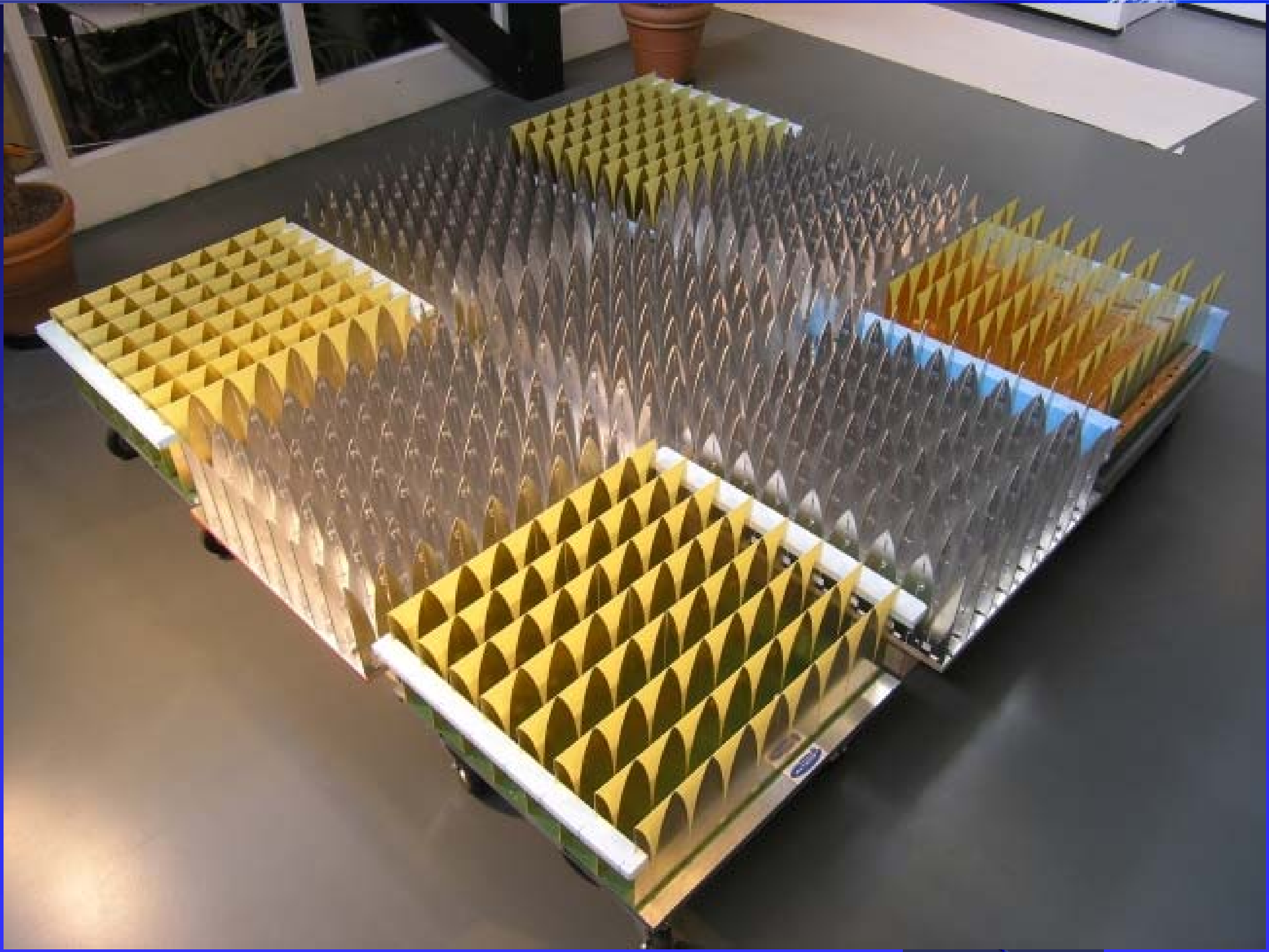
# THEA

## Aperture array



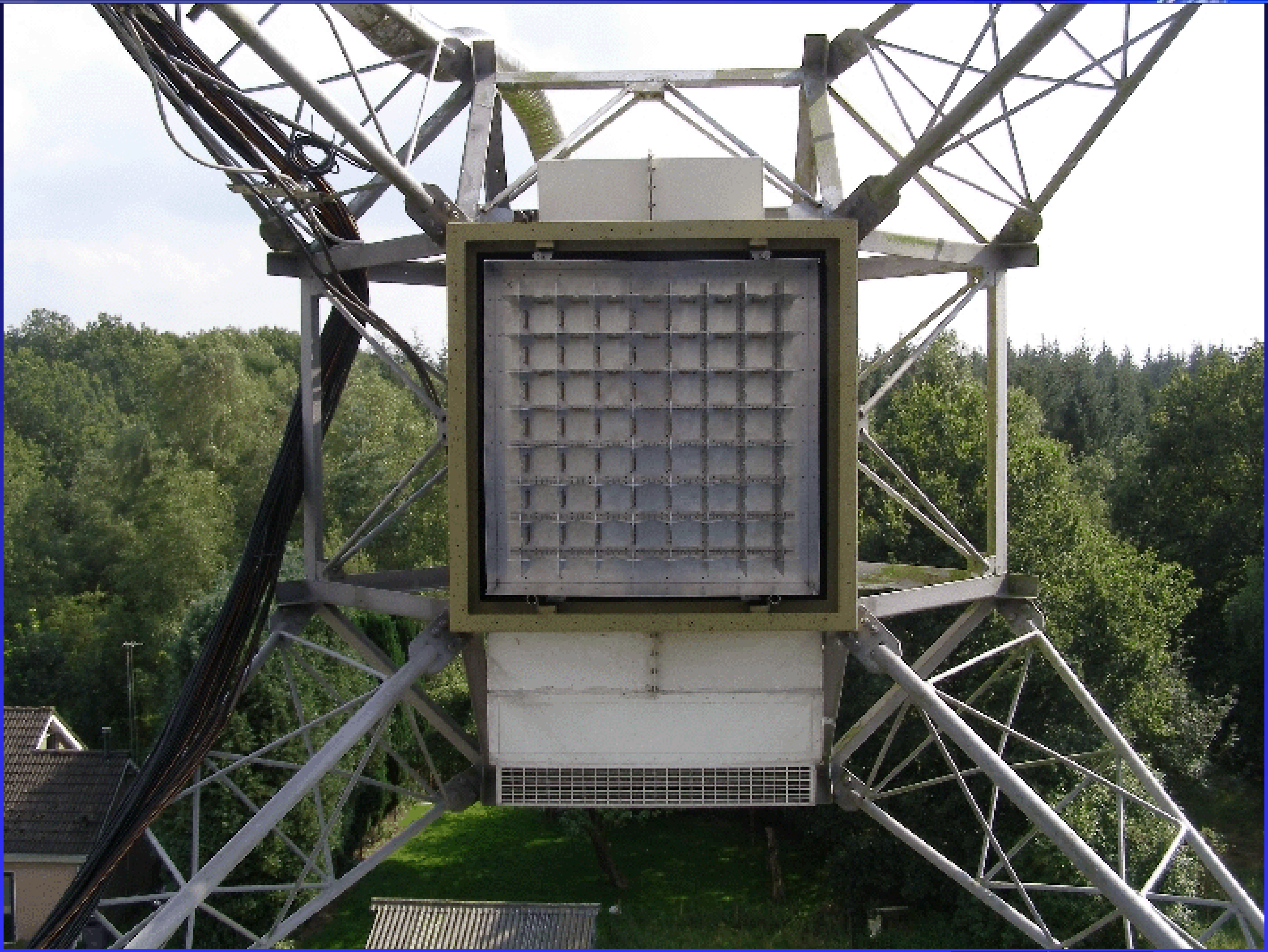
# EMBRACE

Aperture array



# Digestif

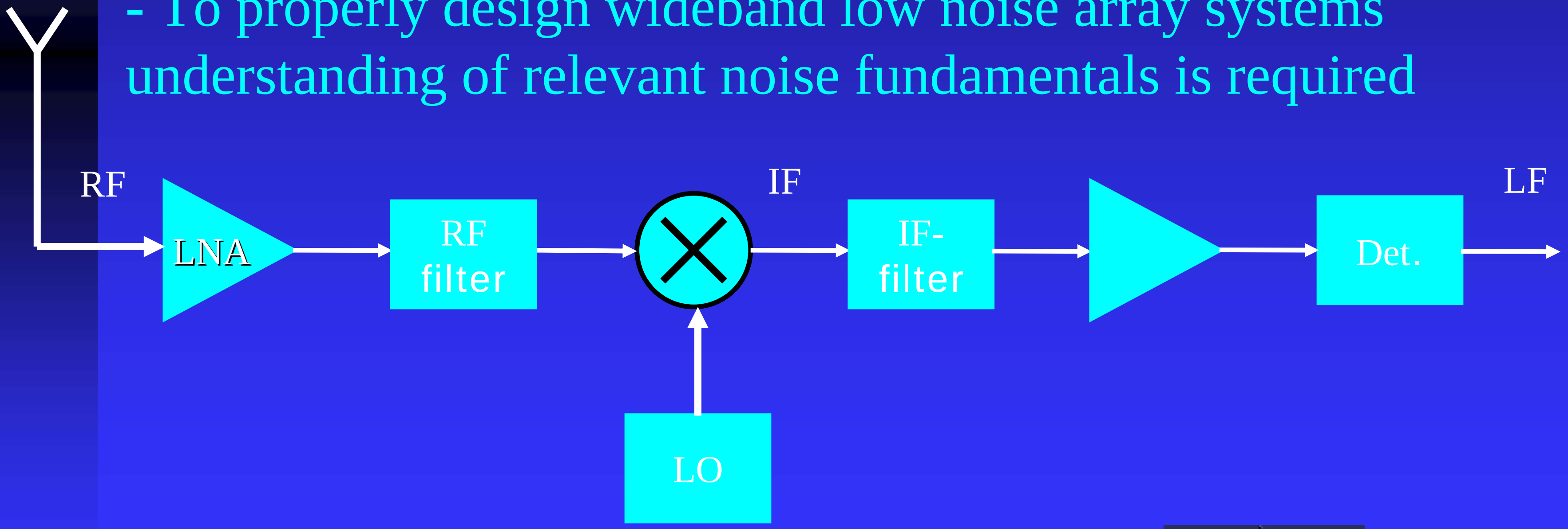
## Focal Plane Array



- The Receiving Array Systems should be sensitive to process weak signals, obscured by noise added by the receiver system components

- Common to all array systems is a number of single receiver chains, with an LNA as a first stage

- To properly design wideband low noise array systems understanding of relevant noise fundamentals is required



# Outline

- Introduction to noise quantities and quality factors
- Description of LNA signal and noise properties
- Noise matching
- Noise temperature measurement
- Modeling of array receivers
- Noise coupling in dense arrays

# Introduction to noise quantities (1)

- Noise is added by receiver system components, which produce current (shot) noise or noise of thermal nature with available power $N_{av} = kTB$
- S/N-ratios at LNA in- and output determine LNA quality, giving a noise quality factor, based on the concept of noise figure, defined by Harold Friss ('44):

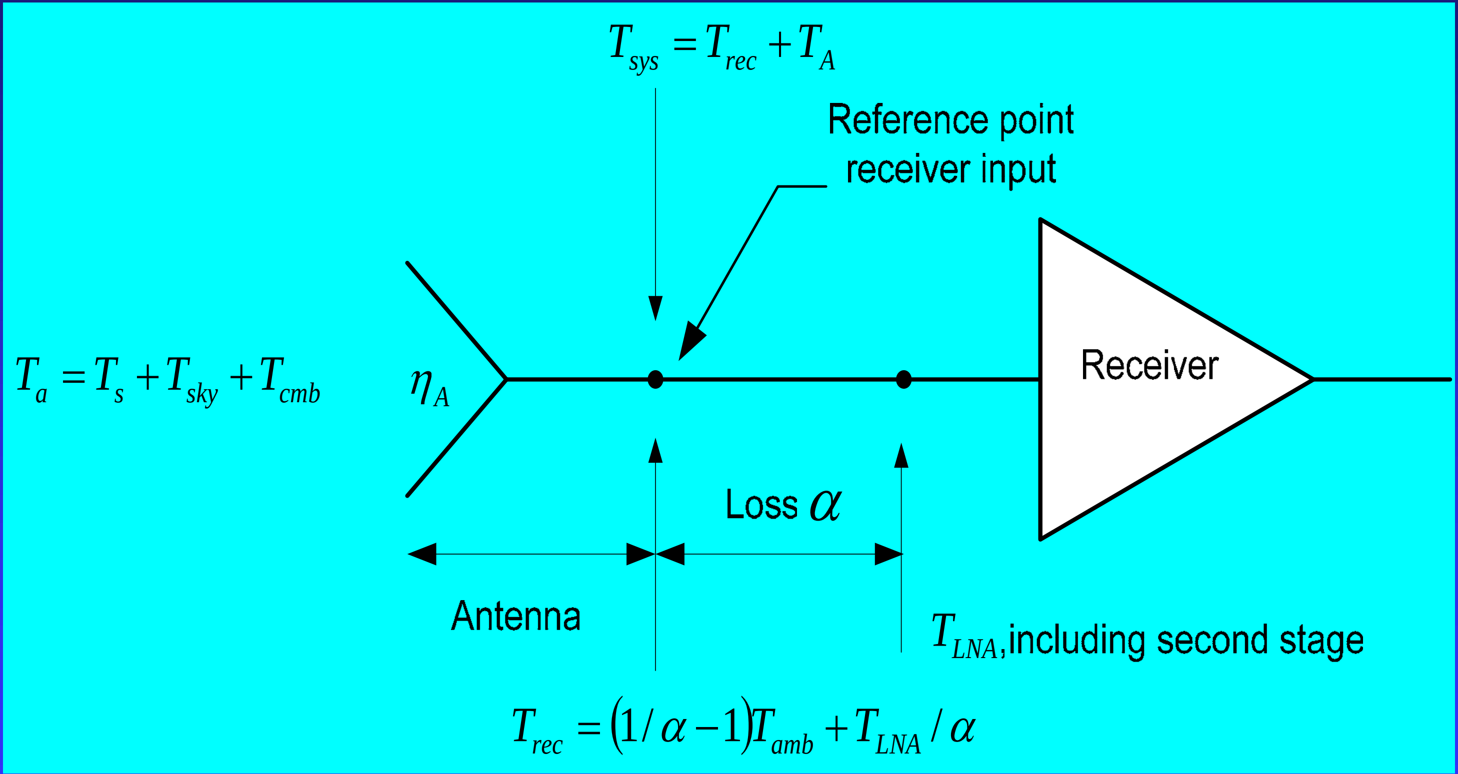
$$F = \frac{S_i / N_i}{S_o / N_o} => \frac{S_i / N_i}{G_{av} S_i / (N_a + G_{av} N_i)} = \frac{N_a + G_{av} N_i}{G_{av} N_i} = 1 + \frac{N_a}{G_{av} N_i}$$

- Relation between noise figure and noise temperature:

$$F = 1 + \frac{k T_a B G_{av}}{k T_0 B G_{av}} = 1 + \frac{T_a}{T_0}, T_0 = 290K$$

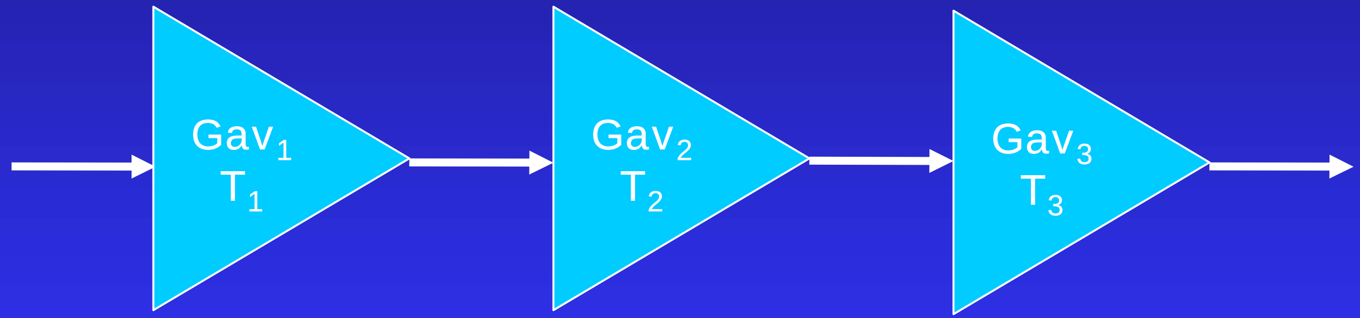
# Introduction to noise quantities (2)

## Noise temperatures in a single-channel antenna/receiver system



# Introduction to noise quantities (3)

- Noise Figure (dB): NF = 10 log F (0.1 dB ~ 7 K)
- Friis' formula for noise temperature of cascaded two-ports

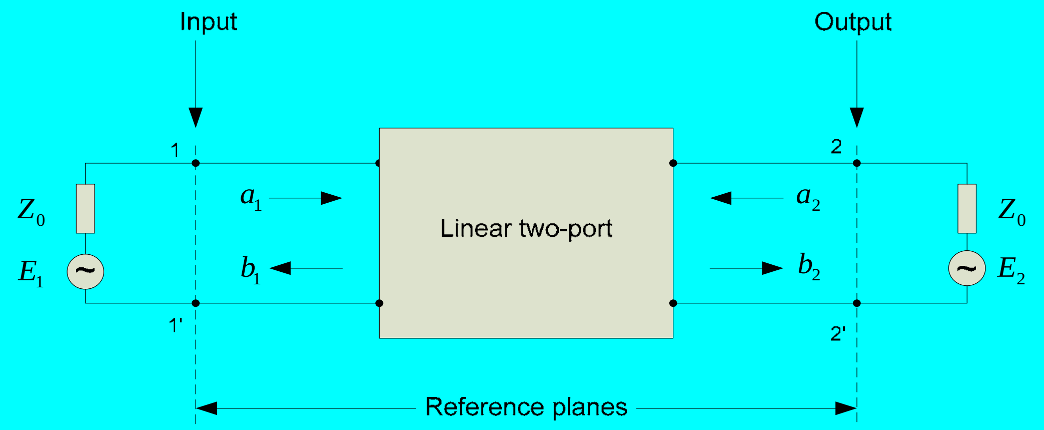



$$T_T = T_1 + \frac{T_2}{Gav_1} + \frac{T_3}{Gav_1 Gav_2} + \ldots + \frac{T_n}{Gav_1 Gav_2 \ldots Gav_{n-1}}$$

# Description of LNA signal and noise properties as a two-port device

- Signal behavior may be described with various sets of parameters: Z-, Y- or H-parameters, related to controlled voltage and/or current sources, or S(cattering)-parameters, related to waves. We will use S-parameters, which describe the two-port with a set of four complex parameters. The S-parameters can be converted into any of the other sets of parameters
- Similarly the noise behavior may be fully modeled with four noise parameters, related to noise current and noise voltage sources or noise waves, characterized by a noise temperature

# Two-port S-parameters

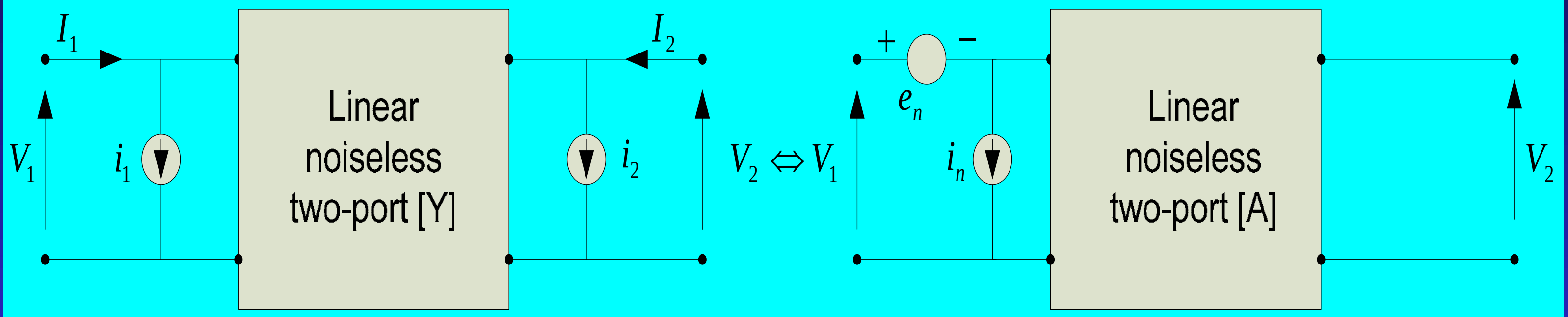



Relations between in- and outgoing waves
$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

Source reflection coefficient
$$\Gamma_s = \frac{Z_s - Z_0}{Z_s - Z_0}$$

Two-port available gain
$$G_{av} = \frac{|S_{21}|^2 \left(1 - |\Gamma_s|^2\right)}{|1 - \Gamma_s S_{11}|^2 \left(1 - |S'_{22}|^2\right)}, \text{ with } S'_{22} = S_{22} + \frac{S_{12} \Gamma_s S_{21}}{1 - \Gamma_s S_{11}}$$

# Two noise representations in linear two-ports

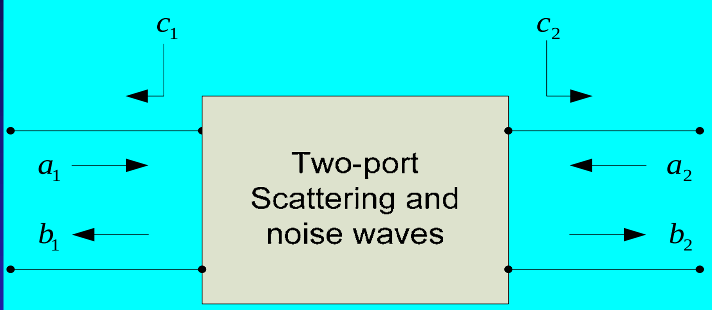



$$G_1 = \frac{\overline{|i_1|^2}}{4kT_0\Delta f} \qquad G_2 = \frac{\overline{|i_2|^2}}{4kT_0\Delta f} \qquad \rho_c = \frac{\overline{i_1^* i_2}}{\sqrt{\overline{|i_1|^2}\,\overline{|i_2|^2}}}$$

$$R_n = \frac{\overline{|e_n|^2}}{4kT_0\Delta f} \qquad g_n = \frac{\overline{|i_n|^2}}{4kT_0\Delta f} \qquad \rho = \frac{\overline{e_n^* i_n}}{\sqrt{\overline{|e_n|^2}\,\overline{|i_n|^2}}}$$

Two equivalent noise parameter sets, natural for admittance and ABCD-matrix representation, respectively (Rothe and Dahlke, 1956)

# Noise parameters based on noise waves

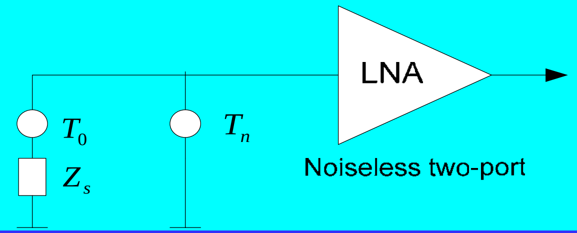



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

Noise correlation matrix $C_s = \begin{bmatrix} \overline{|c_1|^2} & \overline{c_1 c_2^*} \\ \overline{c_2 c_1^*} & \overline{|c_2|^2} \end{bmatrix}$

Relation between noise waves and temperature $kT_n = \dfrac{\overline{|c_2|^2}}{|s_{21}|^2}$ for $\Gamma_s = 0$

Particularly useful description as noiseless two-port with input noise source with equivalent noise temperature $T_n$, described with noise parameters $T_{\min}, R_n, \Gamma_{opt}$



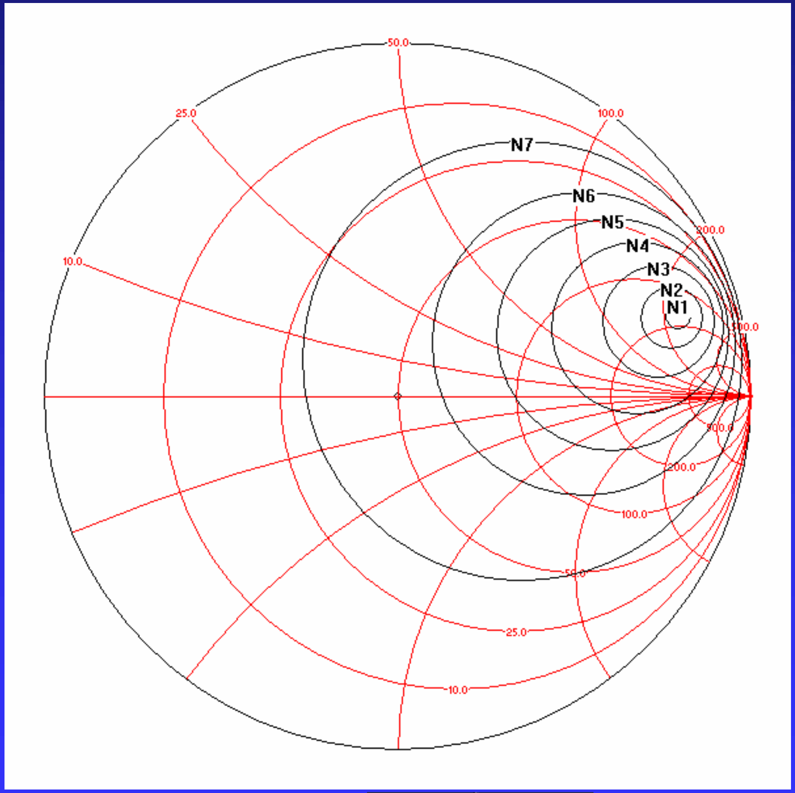

$$T_n = T_{\min} + \frac{4R_n T_0}{Z_0} \frac{|\Gamma_s - \Gamma_{opt}|^2}{|1 + \Gamma_{opt}|^2 \left(1 - |\Gamma_s|^2\right)}$$

# Noise matching

- Noise circles in Smith-chart: constant noise temperature as a function of impedance
- Distance between $\Gamma_s$ and $\Gamma_{opt}$ determines noise increase with respect to $T_{\min}$
- Noise matching (N1)

$$\Gamma_s = \Gamma_{opt} \Rightarrow T_n = T_{\min}$$

- Minimize Rn by transistor design/technology and LNA circuit design

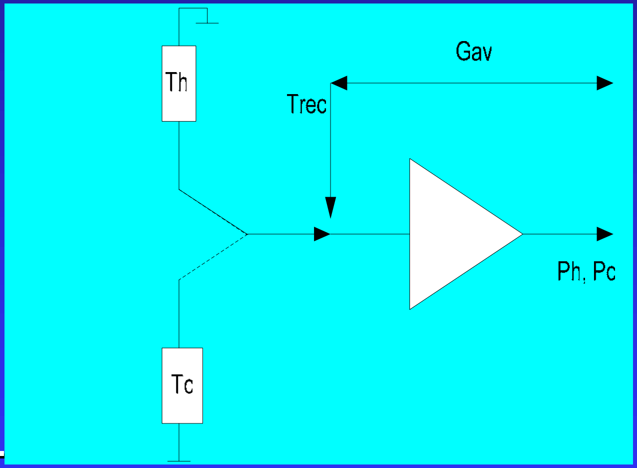


# Noise measurement (1)

- Absolute measurement of available output power, requires accurate gain calibration $P_{av} = kTBG_{av}$

- Y-factor method, using 'hot' and 'cold' reference noise sources

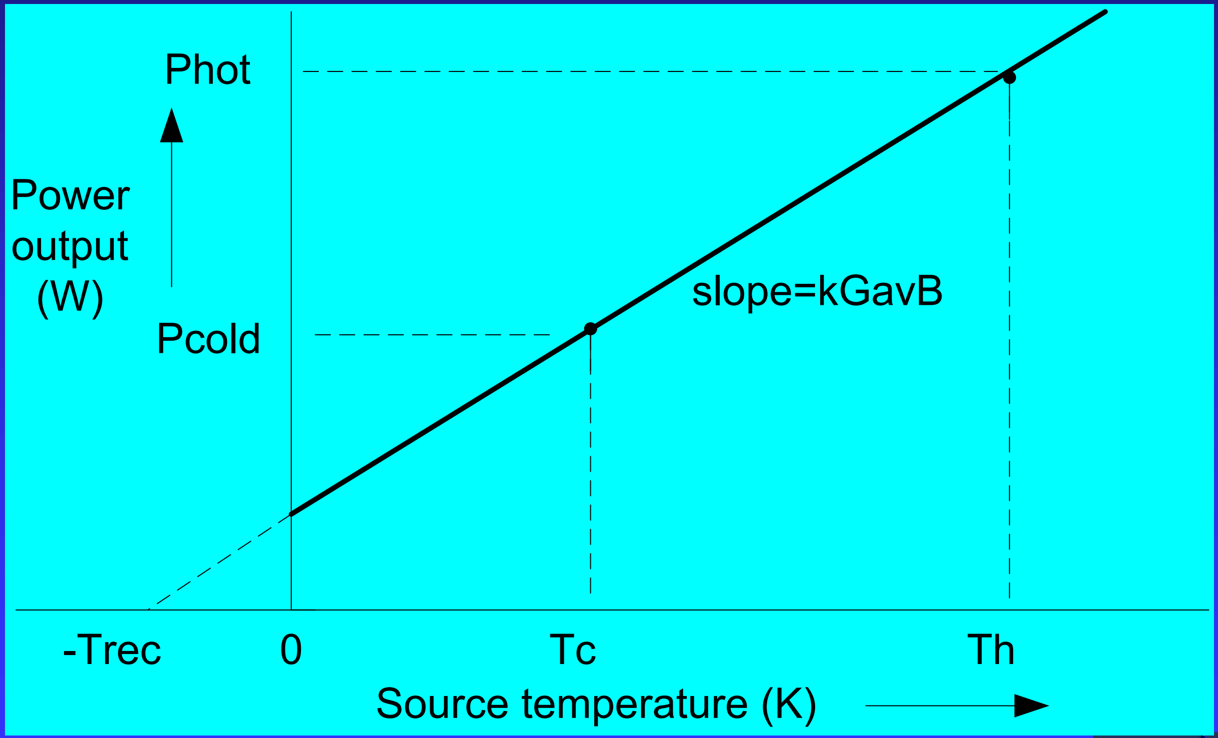
$$Y = \frac{P_{av_h}}{P_{av_c}} = \frac{k(T_h + T_{rec})BG_{av}}{k(T_c + T_{rec})BG_{av}} = \frac{T_h + T_{rec}}{T_c + T_{rec}} \Rightarrow T_{rec} = \frac{T_h - YT_c}{Y - 1}$$




Use of diode noise source; microwave resistors at 300 K and 78 K; absorber in front of array and cold sky

# Noise measurement (2)

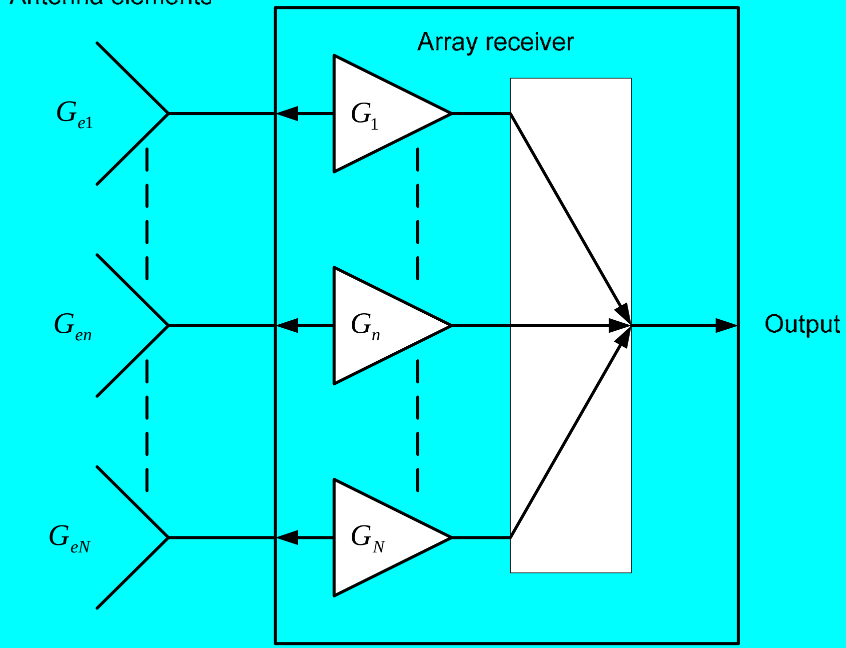
- Regression line method: larger number of points increases accuracy



# Modeling of array receivers (1)

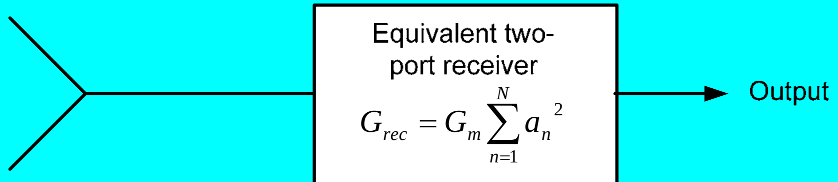
## Definition of array gain

## Replace the array system by an equivalent single-channel system




$$S_o = P_o G_m \left|\sum_{n=1}^{N}\left(\sqrt{G_{en}}\right)a_n \exp\left(j\theta_n\right)\right|^2$$

$$a_n = \sqrt{\frac{G_n}{G_m}}$$




Equivalent single-output antenna

$$G_a = \left|\sum_{n=1}^{N}\left(\sqrt{G_{en}}\right)a_n \exp\left(j\theta_n\right)\right|^2 \Big/ \sum_{n=1}^{N} a_n^2$$

$$S_o = P_o G_a G_m \sum_{n=1}^{N} a_n^2$$

$$G_{rec} = G_m \sum_{n=1}^{N} a_n^2 = \sum_{n=1}^{N} G_n$$

# Modeling of array receivers (2)

## Definition of array noise temperature

Excess output noise density of receiver: $N_o$, $T_{rec} = \frac{N_o}{kG_{rec}}$

$$\Rightarrow T_{rec} = \frac{\sum_{n=1}^{N} G_n T_n}{\sum_{n=1}^{N} G_n}$$

Antenna related noise: $T_A = \frac{1}{2k} \int G_a(\theta,\varphi) w(\theta,\varphi) d\Omega$

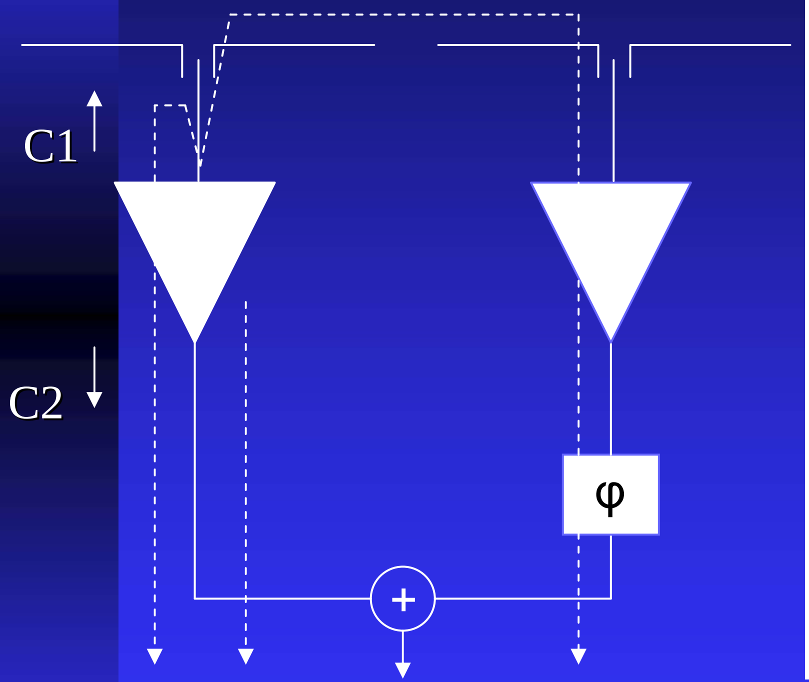
Total output noise density: $k(T_A + T_{rec})G_{rec} = kT_{sys}G_{rec}$

$T_{sys}$ defined at the input of equivalent conventional receiver

# Noise coupling in dense arrays

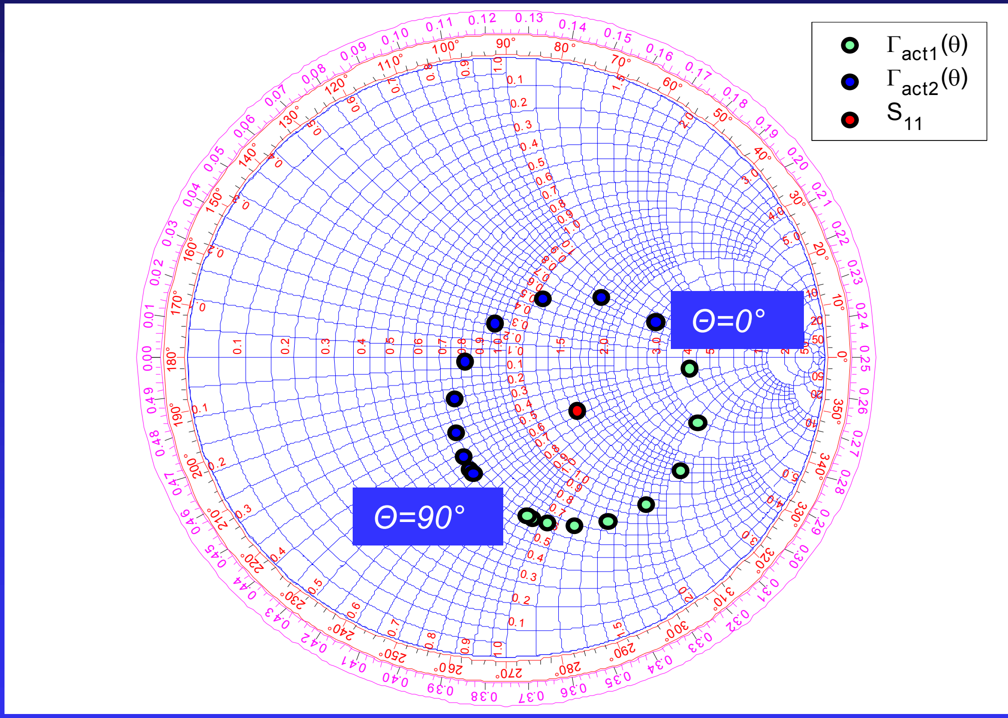
- Description
- Active reflection coefficient
- Effect on noise and available gain
- Use of the modified two-port noise formula

# Noise coupling and active reflection coefficient




Ctot=Direct+Refl+Coupl

Reflection coefficients in a 2x1 dipole array




$$\Gamma_{act_i} = \sum_j S_{ji} e^{j(\Phi_j - \Phi_i)}$$

# Noise formula for array two-ports, including noise coupling

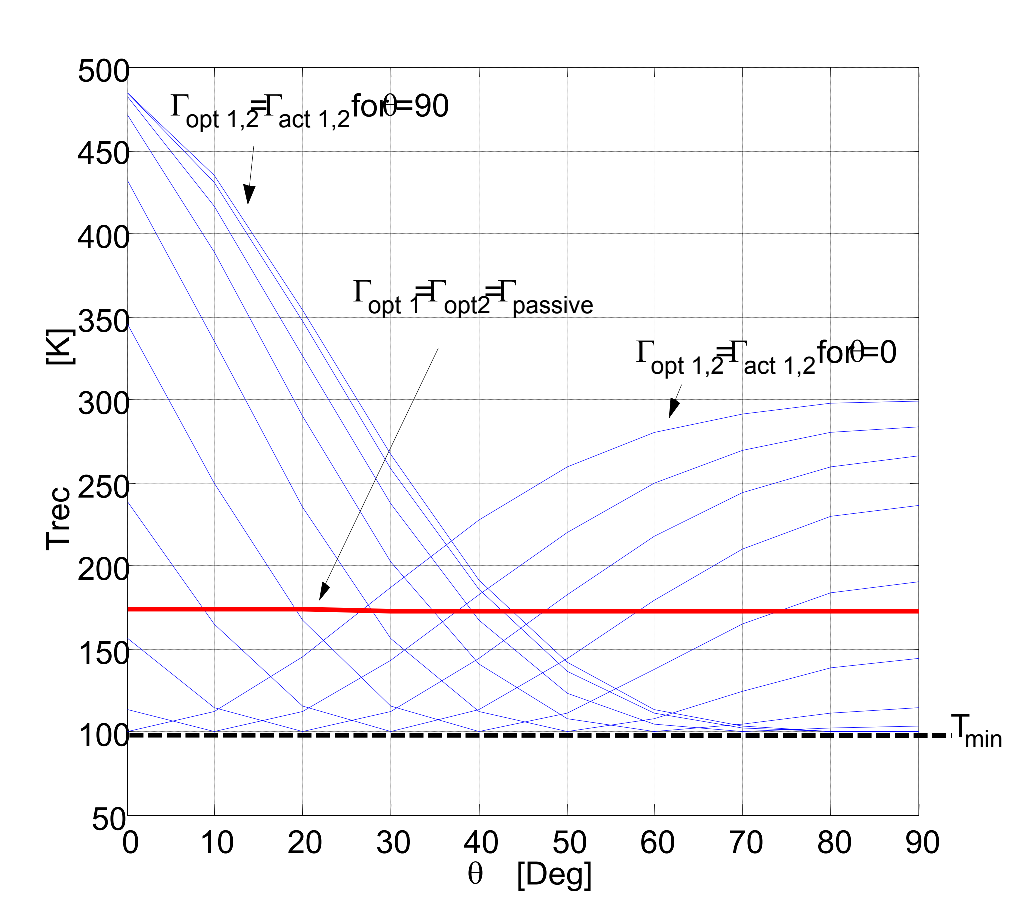
$$T_n = T_{\min} + \frac{4R_n T_0}{Z_0} \frac{\left|\Gamma_{act} - \Gamma_{opt}\right|^2}{\left|1+\Gamma_{opt}\right|^2 \left(1 - \left|\Gamma_{act}\right|^2\right)}$$

Valid for $\left|\Gamma_{act}\right| \neq 1$

Available gain: $G_{av} = \dfrac{\left|S_{21}\right|^2 (1 - \left|\Gamma_{act}\right|^2)}{\left|1 - \Gamma_{act} S_{11}\right|^2 (1 - \left|S_{22}'\right|^2)}$, $S_{22}' = S_{22} + \dfrac{S_{12}\Gamma_{act} S_{21}}{1 - \Gamma_{act} S_{11}}$

With these formulas the array may be considered to consist of independent two-ports, resulting in an equivalent single channel two-port

# Trec as a function of scan angle



# The effect of noise coupling

- Noise coupling generally increases the effective input noise temperature of an LNA, which becomes dependant on array weighting factors.
- For an infinite array with equal elements this will lead to a similar increase in system temperature, that can only be compensated by noise matching to the active reflection coefficient of individual array elements.
- For finite arrays or infinite arrays with unequal elements, the weighting factors may be corrected to maximize S/N-ratio.

# In summary relevant noise related items for the 'Design of Wideband Receiving Array Systems'

- Basics for two-ports: Noise figure/temperature, S- and noise waves, Noise parameters, Two-port noise formula, Noise matching, Noise (Y-factor) measurements
- Gain and noise modeling of array receiver by replacement with an equivalent single channel system, as a combination of independent two-ports
- Noise coupling in arrays, Active reflection coefficient, modification of two-port noise and available gain formulas for individual two-ports
- This shows that effect of coupling may be incorporated in independent two-ports, which are combined to find the equivalent array two-port