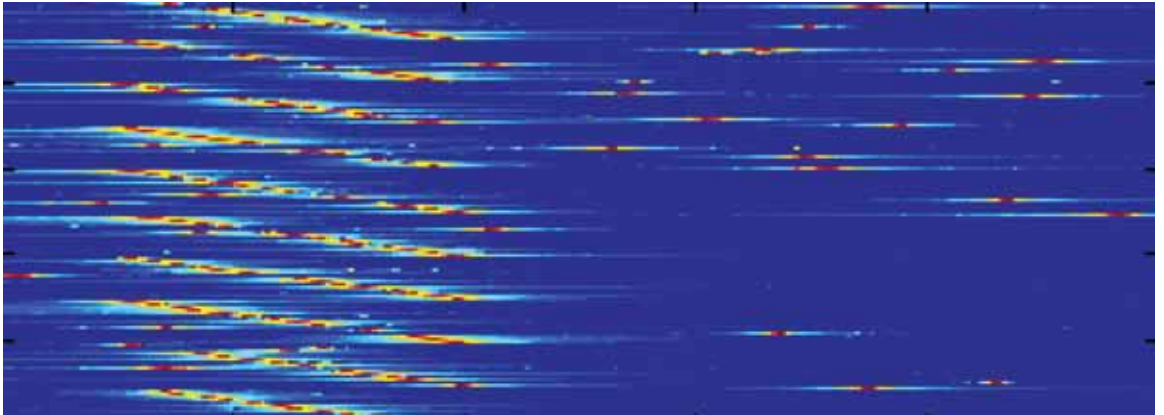


# Finding the Interference

## Karhunen-Loève Transform as an Instrument to Detect Weak RF signals



**GNSS signals, like other RF signals in general, are exposed to interference and jamming. In GNSS however, the problem is far more challenging and difficult to solve. The low signal power on the ground resulting from the large distance the satellite signals have travelled make them particularly sensitive to interference, even if the power levels of the interfering signals are modest. This column discusses a mathematical tool – the Karhunen-Loève Transform – that can help detect interfering signals with power levels similar to or even lower than those usually received on earth from GNSS satellites today.**

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**T**he signal-processing methods currently used to detect interfering signals have reached their practical limits and have little more to offer in dealing with problem of very weak signal detection. The most popular of these include:

- the Fast Fourier Transform (FFT), which directly transforms a signal into its spectral representation
- the Short Time Fourier Transform (STFT), based on the Fourier Transform (FT), which gives information about frequency and time of the signal
- the Wigner-Ville probability distribution, which also provides frequency and time information.

This column presents a very competitive alternative to FT, namely the Karhunen-Loève Transform (KLT).

In contrast to the FT, the KLT offers a solution to problems that are today still intractable.

The KLT has indeed been proposed for applications to process the signals collected by astronomers worldwide in the framework of the SETI program (Search for Extra Terrestrial Intelligence). In a 2010 paper (cited in the Additional Resources section at the end of this column), Dr. Claudio Maccone presented his most recent discoveries concerning the KLT theory and its application to the detection of very weak signals hidden in noise. He also presented an example of the successful detection of an unknown sinusoidal signal with a signal-to-noise ratio (SNR) of as low as -23 decibels. He concluded that an even lower SNR could be mastered by the KLT.

Following the ideas of the SETI scientists, this column will first discuss the advantages and limitations of the KLT,

especially with respect to its application to detect typical interferences of GNSS signals. After providing some theoretical insight into the KLT approach, we will present examples of a successful detection of a very weak wideband signal, including a performance comparison of the KLT to the FFT, the STFT, and the Wigner-Ville methods.

### **KLT: Worth Our Attention**

The KLT was not chosen for weak signal detection by accident. Several factors explain why KLT could be an appropriate mathematical tool for that purpose. As the Fourier Transform is of paramount importance in signal processing nowadays, this article uses it as the reference transform against which the KLT properties are compared.

The main advantages of the KLT compared to the FT are the following:

1. The KLT works equally well for narrowband and wideband signals,

while the FT is optimized for narrowband signals only. This feature makes the KLT more adjustable to bandwidths characteristic for GNSS signals.

2. Both transforms decompose the signal using a set of base functions. When comparing these functions, one notices that the KLT is a more flexible transform, because its basis functions can be of any form. This results in a better decomposition of the signal. The FT is very limited here because its basis functions are strictly limited to sines and cosines.
3. The KLT merges deterministic and stochastic analyses of the signal, which is a very powerful attribute not found in other methods. Although the set of KLT basis functions is deterministic, it does also provide information on the stochastic nature of the signal, rather characterizing an expectation of the power of the basis functions than their exact value. Consequently the KLT grades the basis functions with respect to their probable power contribution thus allowing one to efficiently distinguish the signal from the noise. This implies that, when using the KLT, the processed signal can be filtered by keeping only the most interesting, non-stochastic part and omitting the rest, which is then defined as background noise. On the other hand, when using FT an exact power value can be determined to each sine and cosine, this being the only parameter defining the signal.
4. Finally, according to experiments performed by Maccone and presented in the previously cited paper, the KLT is able to detect much weaker signals than the FT. Although this capability still must be confirmed by practical applications, it could have an enormous influence in the future.

In spite of its significant advantages, the KLT has not replaced FTs yet. In particular, associated complexity and, thus, the computational burden still speak against KLT and in fact favor classical FFT. The Fourier Transform has its fast,

	Fast Fourier Transform	Karhunen-Loève Transform
Decomposition functions	Sines and cosines	Any orthonormal functions
Type of signal analysis	Deterministic	Deterministic and stochastic
Optimal frequency range	Narrowband signals	Narrowband and wideband signals
Potential to detect feeble signal	Small	High
Complexity	$O(n \cdot \log(n))$	$O(n^2)$

TABLE 1. Comparison of selected characteristics of the Fast Fourier Transform and Karhunen-Loève Transform

numerical implementation called Fast Fourier Transform with a complexity of  $O(n \cdot \log(n))$  (i.e.  $n \cdot \log(n)$  addition/multiplication operations on data of length  $n$ ). The complexity of the numerical implementation of the KLT is much higher —  $O(n^2)$ .

The underlying reason for this difference is that the FFT uses a predefined set of orthogonal functions (sines and cosines), whereas the KLT looks for the best representation of the orthogonal function for each individual signal. A comparative summary of the characterizations is presented in Table 1.

### KLT and FFT – Analogies and Differences

The Karhunen-Loève Transform and the Fourier Transform have many analogies but also important differences. To gain further insight into the mathematical characteristics of each of them, let us compare the equations for the Fourier series of a deterministic periodic signal  $x(t)$  and its analogous KL expansion of a stochastic process of the signal  $X(t)$  are compared. (Stochastic variables are denoted by capitals).

As is well known, any periodic signal can be expressed in terms of a Fourier series as follows:

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(\omega_n t) + b_n \sin(\omega_n t)] \quad (1)$$

where the angular frequencies are defined by  $\omega_n = n(2\pi / T)$  with the period of the signal being  $T = t_2 - t_1$ .

This equation defines a deterministic signal  $x(t)$  as a sum of sines and cosines that have an amplitude equal to the corresponding coefficients  $a_n$  and  $b_n$ . The norm of the square of these coefficients gives information about the power at the correspondent to the coefficients frequency. By definition, the FFT basis

functions are extended between minus infinity and infinity, which allows the calculation of the exact amplitudes of the functions for each frequency.

The coefficients  $a_n$  and  $b_n$  of the Fourier series are defined by the following formulas:

$$a_n = \frac{2}{T} \int_{t_1}^{t_2} x(t) \cos(\omega_n t) dt \quad (2)$$

and

$$b_n = \frac{2}{T} \int_{t_1}^{t_2} x(t) \sin(\omega_n t) dt \quad (3)$$

These are integrated projections of the sine and cosine on the signal.

Applying the KLT methodology to a stochastic process  $X(t)$  over the finite time interval  $0 \leq t \leq T$  can be represented by equation (4) which shows some analogies to the Fourier series:

$$X(t) = \sum_{n=1}^{\infty} Z_n \Phi_n(t) \quad (4)$$

The deterministic functions  $\Phi_n(t)$  are called *eigenvectors* or *eigenfunctions*, while  $Z_n$  are random scalar variables. This is a very short equation but the consequences and possibilities standing behind it are enormous. As we can see, this is a generic decomposition in vectorial space using eigenfunctions, which can have in principle any form. We should mention that, in both the Fourier series and the KL expansion, the eigenfunctions have to be orthonormal, i.e., orthogonal and normalized to one. In effect, these functions are uncorrelated and can be a base of any signal in infinite dimensional Euclidean space spanned by these functions.

Another important note: the eigenfunctions  $\Phi_n(t)$  are to the KL expansion what the set of sines and cosines is to the Fourier series. However, the eigenfunctions' shape is in principle unknown and

adapts to the signal being processed. Also, unlike the Fourier series, the KLT eigenfunctions have a finite support, making the requirement for a periodic signal unnecessary. This is an important advantage of the KL expansion compared to the FT, because the data to be processed usually has a finite duration.

But the most revolutionary aspect of the KL expansion is still to come.

Unlike the FT, the coefficients  $Z_n$  of the KL expansion of a stochastic process  $X(t)$  are pure random variables. In contrast to  $a_n$  and  $b_n$  of the Fourier series, the KL coefficients reflect the stochastic nature of the data under analysis. In his work, Maccone proposed computation of the random variable  $Z_n$  by using a very similar equation as the formulas (2) and (3):

$$Z_n = \int_0^T X(t)\Phi_n(t)dt \tag{5}$$

Similar to the coefficients of the Fourier series, equation (5) describes the projection of the processed signal onto eigenfunctions. It is worth highlighting the fact that the integral boundaries, which are finite, cover the entire signal duration  $0 \leq t \leq T$ ; thus, KLT also applies well for non-periodic signals.

We must emphasize that the *variance* of the KL expansion coefficients  $Z_n$  is more important than the coefficients themselves. This variance is called eigenvalue and is denoted  $\lambda_n$ .

The eigenvalue  $\lambda_n$  corresponds to  $\Phi_n(t)$  and represents the expected power of the corresponding eigenfunction and is significant for the filtering capabilities of the KLT, which can be used for detection of feeble signals. Theoretically, what Maccone demonstrates in his article is that a signal containing only pure noise is characterized by KLT eigenvalues, which are uniformly distributed and equal to one. This is true for a continuous signal only. Consequently, we can already state that eigenvalues larger than one can identify that a corresponding eigenfunction is correlating better with the hidden signal than with the remaining noise.

So, how does one compute the eigenvalues and eigenfunctions of a signal? The

KLT computes the covariance of a processed signal which is then used to find its eigenvalues and eigenvectors. The following equations are derived in Maccone's article, with (6) providing a fundamental equation to compute the unknown eigenvalues and eigenfunctions:

$$\int_0^T E\{X(t_1)X(t_2)\}\Phi_n(t_1)dt_1 = \lambda_n\Phi_n(t_2) \tag{6}$$

Equation (6) is analytically derived from the KL expansion equation (4). In this equation  $E\{X(t_1)X(t_2)\}$  represents the autocorrelation of a stochastic process  $X(t)$  at instants  $t_1$  and  $t_2$ . This is the only known variable of this equation. The linear autocorrelation function is calculated directly from the processed signal.

Thanks to this, the eigenfunction characterize the structure of a processed signal at the same time that it becomes sensitive to the signal's harmonic behavior. This is the main advantage of the KLT over the FT. Because of this sensitivity it is now possible to identify eigenfunctions related to the non-stochastic content of the signal. As mentioned earlier, a parameter that allows this identification is the eigenvalue.

During simulations for which results are presented later in this article, discrete signals were processed and the length of the autocorrelation was finite. Under these conditions, the eigenvalues of a noise-only signal converge to one and are equal to one on average.

For digital applications we work on sampled signals. So, as usual, replacing the integral by summation brings us from continuous to discrete data, thus:

$$\sum_{k=1}^N E\{X_k X_l\}\Phi_{nk}\Delta t = \lambda_n\Phi_{nl} \tag{7}$$

which is a set of  $N$  linear equations with  $N$  unknowns;  $E\{X_k X_l\}$  is a Toeplitz autocorrelation matrix of the size  $N \times N$ , and, finally,  $\Delta t$  denotes time duration of the sample.

Equation (7) is solved with the rules of classical linear algebra. Eigenvalues and eigenvectors can be determined using common methods. This equation can always be solved; however, the only drawback of the KLT resides here. This set of equations carries with it a signifi-

cant computational burden on the order of  $O(N^2)$ . Maccone demonstrated the first successful approach to the problem of computation time, which will be discussed later on in this article.

This theoretical introduction leads us to the practical part. Our purpose here was to investigate the capabilities of the KLT as applied to weak signal detection, especially those that can be harmful to GNSS signals on the receiver side and are to be detected from far distance.

In order to gain better insight into the detection capabilities offered by the various well-known methodologies, we tested them using the same input data. We structured the tests in three stages. The first example demonstrates how the KLT decomposes a signal and identifies the eigenvalues and eigenfunctions of the data. Next, a narrowband, single sinusoidal signal was selected to check the performance of the KLT.

The final example addresses the wideband signal detection capabilities of the various approaches. We carried out the last exercise (or stage) for two kinds of signals: a BPSK (binary phase shift keying) modulated signal, representing a stationary signal to reveal the KLT's limitations on wideband signals (Example 3) and a wideband chirp, which represents Example 4 and enabled the testing of the KLT's behavior in the presence of a non-stationary signal.

### Example 1: Decomposition of the Signal

This simple example shows how the decomposition of a signal  $X(t)$  is performed using the KLT. The decomposition starts with the computation of the linear autocorrelation function of the received signal. The next step is to build an autocorrelation Toeplitz matrix and use equation (7) to solve a set of linear equations. The result of this equation is the set of eigenvalues and their corresponding eigenfunctions.

The parameters presented in **Table 2** were selected for this example.

**Figure 1** shows the time representation of the example signal. Because the sine signal component shows up at the identical power level as the noise com-



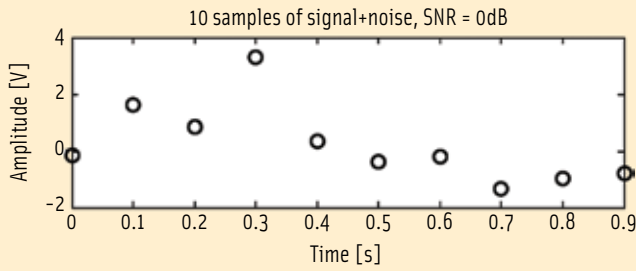


FIGURE 1  $X(t)$ , sine signal at 1 hertz plus noise ; SNR = 0 decibel, 10 samples

ponent, the desired signal can barely be identified in the plot.

Figure 2 presents the computed eigenvalues and the corresponding eigenfunctions of the signal. The plot clearly shows that the first two eigenvalues are significantly higher than all others. The first two eigenfunctions corresponding to these eigenvalues are also characteristic as they show sine-like behavior of a frequency of one hertz, which relates to the hidden sine in the processed signal.

Moreover, the eigenfunctions allow us to distinguish the non-stochastic content of the signal from the noise content, which is represented by the remaining eigenfunctions.

of the KLT technique to detect signals in very strong noise environment. In the following test the KLT performance is compared also against that of an FFT. Note that only one eigenfunction is used to estimate the signal.

However, any number of computed eigenfunctions may be chosen, depending on the level of filtering required. The more eigenfunctions you consider, the more information about the signal (including noise) becomes part of the estimation. Table 3 presents the parameters selected for this example.

From Table 3 we can see that the frequency resolution of the estimated spectrum is different for both methods. The FFT resolution is inversely propor-

tional to the signal length while the KLT is inversely proportional to the length of the autocorrelation function (ACF). Because the ACF length is shorter than the signal length, the FFT provides better spectral resolution.

The comparative results of signal detection for FFT and KLT are presented in Figure 3, which clearly shows that the KLT detects a hidden non-stochastic signal with much higher sensitivity than the FFT does. The peak of the hidden signal frequency as detected by the KLT is very high and leaves no doubt whether this is the desired signal or noise. The power spectrum estimates resulting from use of an FFT technique are much more difficult to interpret.

To complement the overview of this simulation scenario, Figure 4 shows the first 10 eigenvalues (out of a total of 500) of a signal derived using KLT estimation. Eigenvalues of the signal + noise are represented in black; eigenvalues of only the noise component are indicated in red. As in Figure 2, the first two eigenvalues clearly stick out. Additional simulations could verify that this behavior is typical for a single sine signal hidden in noise.

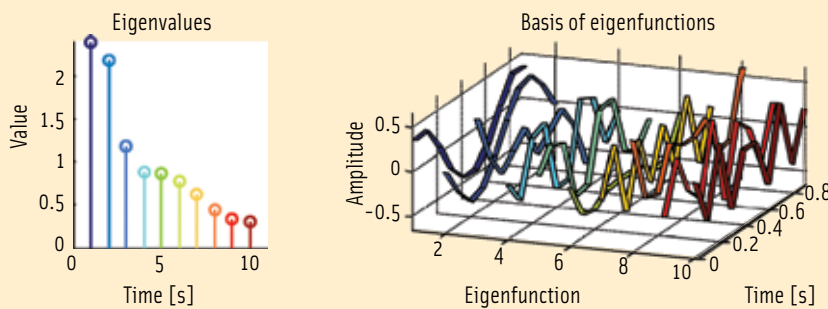


FIGURE 2 Eigenfunctions and eigenvalues of the example signal

Decomposition method	KLT
Hidden signal type	Sine at 1 Hz
Signal length	10 samples
Sampling frequency	10 Hz
autocorrelation function (ACF) length	10 samples
SNR	0 dB

TABLE 2. Simulation parameters; decomposition of the sine signal

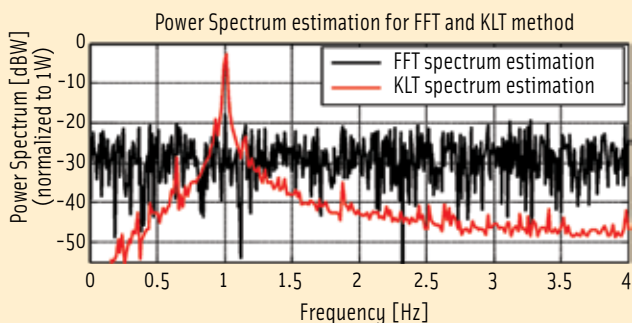


FIGURE 3 Power spectrum estimates applying FFT and KLT techniques

Power Spectrum estimation method	FFT	KLT
Hidden signal type	Sine at 1 MHz	
Signal length	1,000 samples	
Sampling frequency	8 MHz	
Autocorrelation function (ACF) length	N/A	500 samples
Number of eigenfunctions taken for the estimation	N/A	1
Resolution	8,000Hz	16,000Hz
SNR	-16 dB	

TABLE 3. Simulation parameters, narrowband signal detection

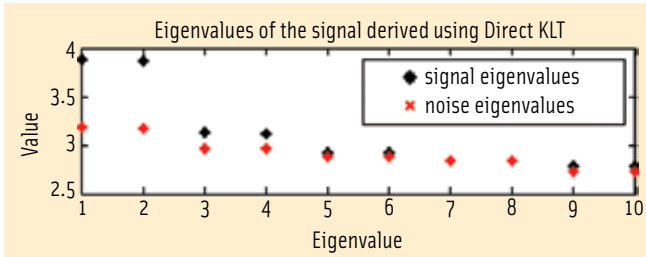


FIGURE 4 Eigenvalues of the signal. Black asterisks are the eigenvalues of the received signal + noise, and red ones are the eigenvalues of the noise component in this signal.

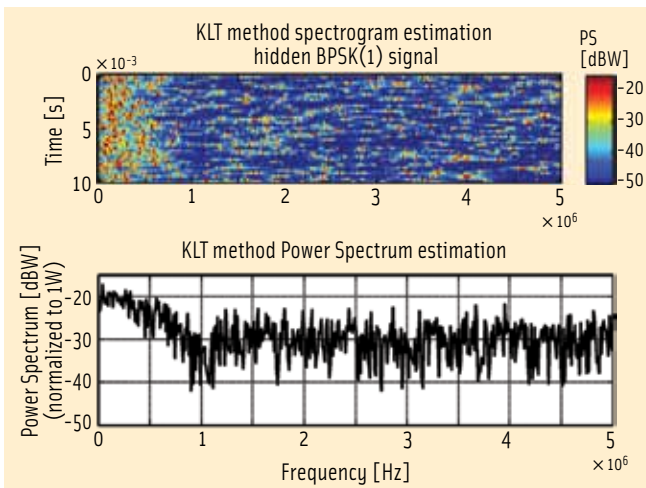


FIGURE 5 The KLT spectrogram and power spectrum estimation; BPSK(1) signal, SNR = -12 decibels

Power Spectrum estimation method	STFT	Wigner-Ville Method	KLT
Hidden signal type	BPSK(1) modulated with fundamental frequency equal to 1.023MHz		
Signal length	100,000 samples		
Sampling frequency	10MHz		
Window length	10,000 samples		
Window type	HFT90D		
Window overlapping	76%		
Autocorrelation function (ACF) length	N/A	N/A	1,000 samples
Number of eigenfunctions taken to the estimation	N/A	N/A	50
Frequency resolution	1,000Hz	1,000Hz	10,000Hz
Time resolution	263 microseconds		
SNR	-12 dB		

TABLE 4. Simulation parameters; wideband signal detection - BPSK(1) modulated signal

We also want to highlight the fact that a significantly higher eigenvalue is not the only requirement for the KLT being able to detect a non-stochastic signal correctly. The eigenvalue corresponding to the desired signal also has to be higher than the corresponding eigenvalue of the noise. Although each noise

eigenvalue in theory is equal to one in numerical calculations, only the average of all eigenvalues is equal to one. Therefore, individual noise eigenvalues are different from one; consequently, we cannot rely on a threshold equal to one to distinguish between noise and signal.

In Figure 4 the first two noise eigenvalues are clearly not close to one, but higher. By using a longer autocorrelation function, the noise eigenvalues converge to one, but they are still not uniformly distributed.

### Example 3: Wideband Signal Detection

The previous example showed that the KLT performs better than the FFT in narrowband signal detection. Next, the KLT is applied to wideband signals. A BPSK-modulated signal serves as the sample signal for detection.

We present the results in the time-frequency domain (see, for example Figure 5). This allows us to see changes in the estimated spectrums for different instants of a signal observation.

A fundamental requirement for achieving a proper estimation of the spectrum is the selection of a windowing function that preserves the original spectral characteristics of the signal without introducing significant distortions due to the estimation process.

In the article by G. Heinzel *et alia* cited in Additional Resources, the HFT90D window type is proposed as optimum for narrowband signals because it determines the exact amplitude of a sinusoidal component. Further analyses are recommended to determine a most appropriate window for wideband signals. Nevertheless and for demonstration purposes the HFT90D window is made use of in the following wideband example.

Compared to other flat-top windows — those window functions that are as flat as possible in the frequency domain — the HFT90D has a relatively narrow three-decibel bandwidth and very strong side-lobe attenuation. According to G. Heinzel *et alia*, the recommended overlapping of window functions applied to the datastream is 76 percent. Table 4 lists the parameters selected for the wideband signal detection example.

In order to achieve a sufficiently high-frequency resolution of the power spectrum estimation using the KLT technique, the ACF takes 1,000 samples. This, however, comes at the cost of a longer computation time.

Consequently, the KLT needs a high number of eigenfunctions to detect a wideband signal without losing part of the information that describes the non-stochastic signal. In this particular case 50 out of the 1,000 eigenfunctions were chosen. We must keep in mind, however, that the number of eigenfunctions taken to estimate the signals spectrum is indeed the variable KLT parameter that determines the level of filtering.

In the following sections, we present the results of applying KLT, STFT, and Wigner-Ville methods to detect a hidden BPSK signal with an SNR level of -12 decibels. For each method two plots are presented. The first shows the spectrogram of the processed signal, and the second plot shows the power spectrum estimation obtained by averaging the spectrogram.

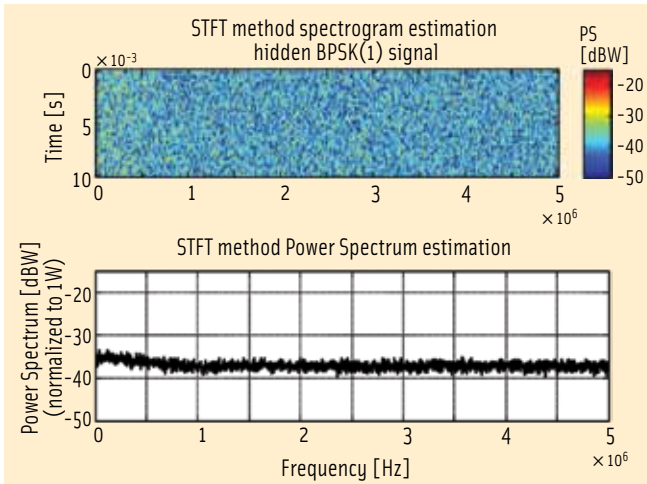


FIGURE 6 The STFT spectrogram and power spectrum estimation; BPSK(1); SNR = -12dB

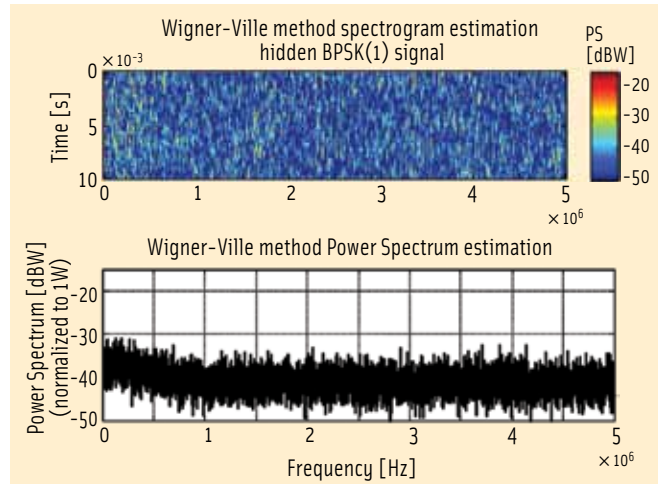


FIGURE 7 The Wigner-Ville spectrogram and power spectrum estimation; BPSK(1); SNR = -12dB

## KLT

The top plot in Figure 5 shows the spectrogram of the signal computed by the Karhunen-Loève Transform. Most of the power clearly appears to be located in the lower band of the spectrum, with much less power at higher frequencies. However, this spectrogram may not be precise enough to correctly identify the BPSK signal. Therefore, the data is averaged over time, with the obtained power spectrum estimation of the averaged spectrogram (lower plot) resembling quite clearly the BPSK(1) signal.

## STFT

In similar fashion as Figure 5, Figure 6 provides the results of the spectrum estimation using a Short Time Fourier

Transform. The spectrogram as well as the power spectrum show that the STFT method fails to detect the wideband signal.

Still, the lower frequencies have slightly more power compared to the average. However, the noise dominates the signal; so, we cannot recognize the BPSK(1) signal.

## Wigner-Ville

Figure 7 shows the results using the Wigner-Ville method. As with STFT, Wigner-Ville fails to detect the wideband signal.

To summarize the preceding simulation results, one can observe that the KLT technique indeed is able to detect wideband signals even in the presence of very strong noise, whereas the STFT

and Wigner-Ville methods clearly fail. Moreover, one can see that, even though the SNR level is slightly higher compared that used in Example 2 (narrowband signal detection), more samples of the signal were required for estimating the power spectrum. A wideband signal carries a lot of information that has to be observed for a longer period of time compared to a narrowband signal in order to recognize its non-stochastic structure.

## Example 4: Chirp Signal Detection

We also analyzed a chirp signal with a wide frequency boundary. This type of signal was chosen to enable us to evaluate the performance of KLT in detecting a dynamic, non-stationary signal. Table 5 lists the simulation parameters for this test.

In order to be able to achieve a spectrogram estimation with good time resolution, the window length was shortened to 1,000 samples. Also the ACF length was reduced to 500 samples. This setting speeds up the simulation, but reduce the frequency resolution of the KLT spectrum estimation.

Figure 8 shows the KLT spectrogram and the power spectrum. For a signal with SNR = -12dB the spectrogram is a little frayed but still readable. The KLT is able to detect the chirp signal in the noise.

The challenge here is to define a proper observation window length. It has to be long enough to detect the sig-

Power Spectrum Estimation Method	STFT	Wigner-Ville method	KLT
Hidden signal type	Linear chirp, frequency range: (0.5-2MHz), chirp time: 1millisecond		
Signal length	100,000 samples		
Sampling frequency	10MHz		
Window length	1000 samples		
Window type	HFT90D		
Window overlapping	76%		
Autocorrelation function (ACF) length	N/A	N/A	500 samples
Number of eigenfunctions taken for the estimation	N/A	N/A	2
Frequency resolution	10,000Hz	10,000Hz	20,000Hz
Time resolution	24.2 microseconds		
SNR	-12 dB		

TABLE 5. Simulation parameters; wideband signal detection - chirp modulated signal



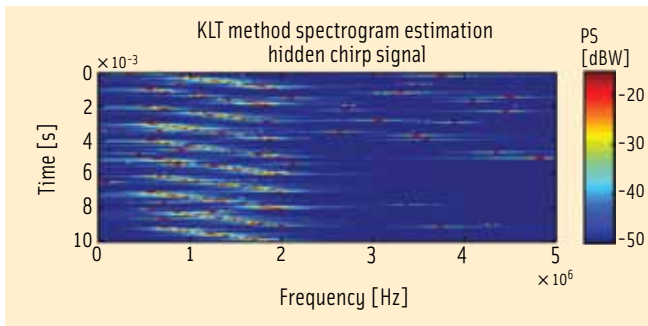


FIGURE 8 Spectrogram KLT; chirp; SNR = -12 dB

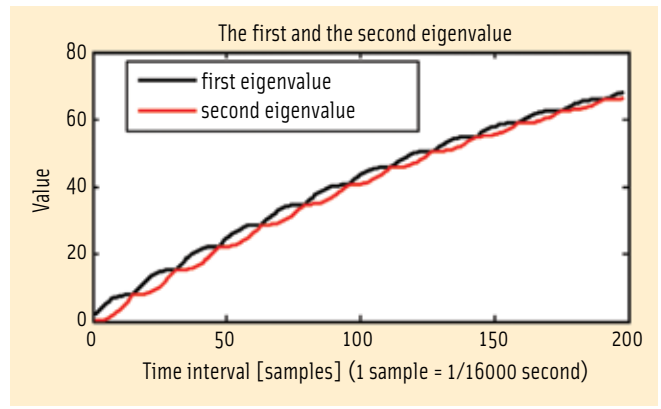


FIGURE 9 The first and the second eigenvalue of a sine signal computed using BAM-KLT

nal but also short enough to reach the required time resolution. If only a short time period of the signal is considered, the detection process for a chirp signal is similar to the detection of a narrowband signal and, therefore, allows us to rely on only a few initial eigenfunctions.

Although results are not shown here, as in the previous example the STFT and Wigner-Ville methods both failed to detect the chirp signal under the given conditions.

### BAM-KLT: A Step Closer to Fast KLT

Again, the biggest drawback of the KLT is its complexity and the resulting high computational burden. As with the Fourier Transform, however, which became popular when its fast implementation (the FFT) became available, the KLT has the potential to experience a similar boost if a fast KLT implementation is discovered.

Maccone has already presented an innovative way of using the KLT, which paves the way towards a faster algorithm. He called this method the

BAM-KLT (Bordered Autocorrelation Method KLT), and his paper provides an example of successful sine signal detection using the BAM-KLT to correctly detect a signal hidden in noise (SNR = -23 decibels), while the FFT failed.

This work uncovered the fact that eigenvalues are functions of a final instant  $T$  (the same  $T$  that appears in the definition of the KL expansion of the final instant of the signal). Using this discovery, Maccone defined “the Final Variance Theorem” expressed by an equation:

$$\sigma_{X(T)}^2 = \sum_{n=1}^{\infty} \frac{\partial \lambda_n(T)}{\partial T} \tag{8}$$

This theorem states that the variance of a stochastic process  $X(t)$  is equal to the sum of the series of partial derivatives of eigenvalues  $\lambda_n(T)$  with respect to the final instant  $T$ . Computing a spectrum of such a partial derivative was found to obtain the partial spectrum of the signal. Similar to the KL expansion, the first terms — i.e., the partial derivatives of the eigenvalues — contain the most useful information about the non-stochastic content of the signal.

In theory, using this method in signal processing, one could reduce the computational burden. Instead of calculating both eigenfunctions and eigenvalues, just determining the set of eigenvalues is now sufficient. Additionally, the KLT filtering feature can also be used in this theorem. If only that information kept in the derivative of the first eigenvalue is used, equation (8) can be simplified to:

$$\sigma_{X(T)}^2 \approx \frac{\partial \lambda_1(T)}{\partial T} \tag{9}$$

We must point out that BAM-KLT is still a technique that needs to be explored and further tested before it can be used in a predictable manner.

As an example, BAM-KLT shows unwanted behavior at one half of the Nyquist frequency: When using normal frequency representation for the signal spectrum, the signal is displayed at twice its original frequency. This problem was discussed in the paper by Maccone, but not solved.

Based on the parameters in Table 6, the following simulation shows another, very interesting behavior of the eigenvalues beyond those discussed by Maccone, which requires further explanation.

Let’s consider only the first two eigenvalues of the signal as a function of the time interval and their derivatives. The eigenvalues are plotted in Figure 9. By definition the first eigenvalue always has to be bigger than the second one. But here it seems that the first two eigenvalues become equal at some points. Those represent points of discontinuity of the eigenvalue functions of the time interval. These eigenvalues might also be related to each other, which was not expected.

One possible explanation could lie in the eigenfunctions. They are orthogonal, but they can also have the same spectrum representation (like, for example, sine and cosine). As a consequence, depending on the time span of the signal, one eigenfunction or another has the biggest chance to appear in the signal.

The situation becomes even more complicated when one is processing more complex signals. In that case, one can observe that the eigenvalues inter-

Power Spectrum estimation method	BAM-KLT
Signal type	Sine at 500Hz without noise
Signal length	200 samples
Sampling frequency	16kHz
autocorrelation function (ACF) length	200 samples
number of eigenfunctions taken to the estimation	2

TABLE 6. Simulation parameters for the BAM-KLT example

weave not only in pairs but also with further “neighbors.”

This has an effect on the derivatives as well. When computing the FFT of the derivatives of the first and the second eigenvalue, the result is a spectrum sine with harmonics, because of the discontinuity of the function. Moreover, the derivatives contain a direct current (DC) component because the mean value of the function is not equal to zero.

This issue is presented here to make users aware of the unexpected behaviour of the BAM-KLT and also to show areas for improvement and future investigation.

### BAM-KLT and Wideband Signals

Because very promising results of single-tone detection in a strong noise were presented in Maccone’s paper, we thought it worthwhile to test the detection capabilities of BAM-KLT for wideband signals. The following results are based on identical test conditions as in the previous tests for estimation of a wideband signal spectrum. First the results of hidden BPSK(1) signal detection are presented, followed by the detection of a chirp.

As in the previous examples of BPSK signal detection, **Figure 10** presents the result as a spectrogram and the power spectrum estimation. Unfortunately, the BPSK(1) characteristic is not visible and, therefore, the signal detection fails.

A typical behavior of the BAM-KLT is the dominating DC component in

both plots. The signal detection capability of BAM-KLT is much worse than the one of the original KLT examples (Figures 5 and 6) and even worse than what was obtained using the STFT and Wigner-Ville methods. Obviously, the BPSK signal cannot be detected by the BAM-KLT for the given conditions.

### BAM-KLT and Chirp Signal Detection

BAM-KLT shows a much better performance when it comes to chirp signals. But it is, of course, still not as good as the original KLT. From the spectrogram (**Figure 11**) it can be seen that the chirp was detected successfully. Nonetheless, a very strong DC component appears in the spectrum, which could make the detection of a low-frequency signal impossible. One can say that the BAM-KLT is already powerful enough to detect narrowband signals, excluding low-frequency signals, which are buried by strong DC component observed in the plots.

### Conclusions

This article shows that the KLT technique has very high potential and can outperform the classical signal detection methods that are used today. The KLT is equally capable of detecting both narrowband and wideband signals, which deserves close attention if we think of future applications in the field of GNSS.

The simulations presented here indicate that this transform allows us to detect more feeble

signals than FFT, STFT, and Wigner-Ville methods can. The main strength of the KLT is its natural feature of adapting to the characteristics of the processed signal. As a result, it allows the filtering of a non-stochastic signal from noise.

The KLT is also more flexible and configurable than other known methodologies. This permits the user to directly control the level of filtering. The remarkable detection capabilities of KLT, however, must be paid for by a high computational burden.

BAM-KLT, which shows excellent detection performance when applied to pure sinusoidal functions, is capable of significantly reducing the computational load. Therefore, some analogies between the BAM-KLT and the FFT can be drawn. Indeed, the introduction of the FFT made it possible to exploit the Fourier Transform in a numerically efficient way.

A similar future could be expected for the BAM-KLT. However, before BAM-KLT or a closely related methodology can be efficiently applied, the spectrum of detectable signals needs to be significantly enlarged. This article shows that the BAM-KLT is not yet capable of unambiguously detecting wideband signals such as BPSKs appearing as noise or interference in a GNSS-like signal. So, further research work needs to be invested in this area. One potential starting point could be to identify relations between the signals to be detected, the windowing function, and the number of characteristic eigenvalues.

In summary, we can say that the KLT is a very high-performing mathematical tool and could prove to be of great value

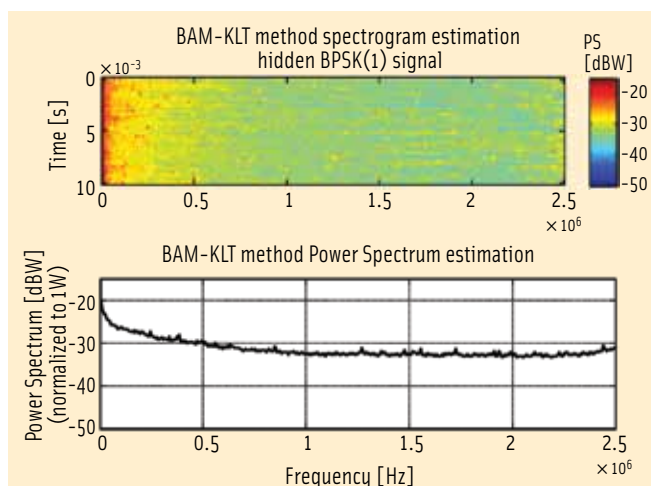


FIGURE 10 Spectrogram BAM-KLT; BPSK(1); SNR = -12dB

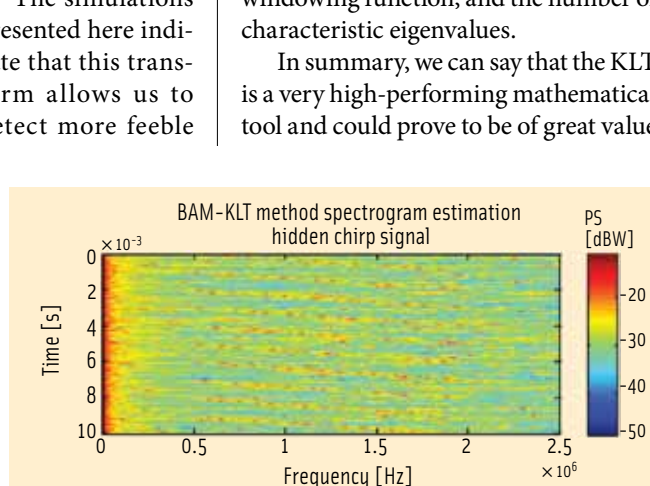


FIGURE 11 Spectrogram BAM-KLT; chirp; SNR = -12dB



for the detection of feeble interfering signals. However, the method still needs further development in order to become faster, more efficient, and easier to use.

Future work on this topic could be to explore the limitations of the KLT in wideband signal detection, in order to get a better idea of the full capabilities of KLT in regards to wideband signals in general. Another promising work would be to reduce the complexity of a KLT implementation. Here the BAM-KLT approach already offers a promising step towards a competitive signal analysis method that meets the needs of the scientific community.

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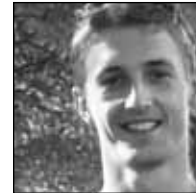
**Additional Resources**

[1] Heinzel, G., and A. Rüdiger and R. Schilling, "Spectrum and spectral density estimation by the Discrete Fourier transform (DFT), including a comprehensive list of window functions and some new at-top windows," Max-Planck-Institut für Gravitationsphysik (Albert-Einstein-Institut) Teilinstitut Hannover, February 15, 2002

[2] Maccone, C., "The KLT (Karhunen-Loeve Transform) to extend SETI searches to broad-band and extremely feeble signals," *Acta Astronautica* (2010), doi:10.1016/j.actaastro.2010.05.002


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