

Evolution of Life on Exoplanets and SETI

Evo-SETI

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ABSTRACT

SETI and ASTROBIOLOGY have long been regarded as two separate research fields. But the growing number of discovered exoplanets of various sizes and characteristics requires a **CLASSIFICATION OF EXOPLANETS** based on the question:

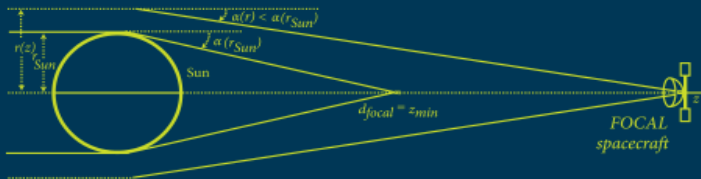
WHERE DOES AN EXOPLANET STAND ON ITS WAY TO DEVELOP LIFE ?

This author has tried to answer this question in a mathematical fashion by virtue of his "Evo-SETI" ("Evolution and SETI") mathematical model described in this seminar.

700-pages BOOK about “Mathematical SETI”

This book introduces the Statistical Drake Equation where, from a simple product of seven positive numbers, the Drake Equation is turned into the product of seven positive random variables. The mathematical consequences of this transformation are demonstrated and it is proven that the new random variable N for the number of communicating civilizations in the Galaxy must follow the lognormal probability distribution when the number of factors in the Drake equation is allowed to increase at will.

Mathematical SETI also studies the proposed FOCAL (Fast Outgoing Cyclopean Astronomical Lens) space mission to the nearest Sun Focal Sphere at 550 AU and describes its consequences for future interstellar precursor missions and truly interstellar missions. In addition the author shows how SETI signal processing may be dramatically improved by use of the Karhunen-Loève Transform (KLT) rather than Fast Fourier Transform (FFT). Finally, he describes the efforts made to persuade the United Nations to make the central part of the Moon Far Side a UN-protected zone, in order to preserve the unique radio-noise-free environment for future scientific use.



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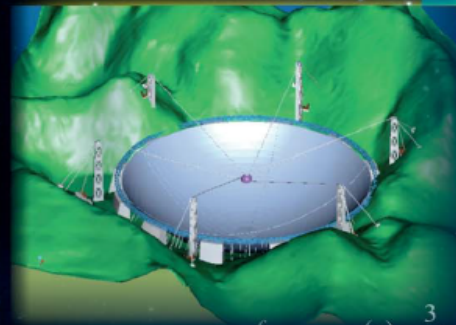
Maccone



Mathematical SETI

Mathematical SETI

Statistics,
Signal Processing,
Space Missions



$$f_{ET_Distance}(r) = \frac{3}{r} \cdot \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(\ln(n)-\mu)^2}{2\sigma^2}}$$

Claudio Maccone

$$X(t) = \sum_{n=1}^{\infty} Z_n \phi_n(t) \quad \text{with } 0 \leq t \leq T.$$

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$$N = N_s \cdot f_p \cdot n_e \cdot f_l \cdot f_i \cdot f_c \cdot f_L$$

$$f_N(n) = \frac{1}{n} \cdot \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(\ln(n)-\mu)^2}{2\sigma^2}} \quad (n \geq 0)$$

$$\left(\ln \left[\frac{6 R_{Galaxy}^2 h_{Galaxy}}{r^3} \right] - \mu \right)^2$$

TALK' s SCHEME 1/2

Part 1: The STATISTICAL DRAKE EQUATION

Part 2: LIFE as a b-LOGNORMAL in time

Part 3: Darwinian EXPONENTIAL GROWTH

Part 4: Geometric Brownian Motion (GBM)

Part 5: Darwinian EVOLUTION as a GBM

Part 6: ENTROPY as EVOLUTION MEASURE

TALK' s SCHEME 2/2

Part 7: Aztecs vs. Spaniards

Part 8: Future up to 10 million years

Part 9: BIG HISTORY

Part 10: MASS EXTINCTIONS

Part 1:

**THE STATISTICAL
DRAKE EQUATION**

The Classical Drake Equation /1

- ▶ In 1961 Frank Drake introduced his famous “Drake equation” described at the web site http://en.wikipedia.org/wiki/Drake_equation. It yields the number N of communicating civilizations in the Galaxy:

$$N = N_s \cdot f_p \cdot n_e \cdot f_l \cdot f_i \cdot f_c \cdot f_L$$

- ▶ Frank Donald Drake (b. 1930)



The Classical Drake Equation /2

- ▶ The meaning of the seven factors in the Drake equation is well-known.
- ▶ The middle factor f_l is Darwinian Evolution.
- ▶ In the classical Drake equation the seven factors are just **POSITIVE NUMBERS**. And the equation simply is the **PRODUCT** of these seven positive numbers.
- ▶ It is claimed here that Drake's approach is too "simple-minded", since it does NOT yield the **ERROR BAR** associated to each factor!

The STATISTICAL Drake Equation /1

- ▶ If we want to associate an **ERROR BAR** to each factor of the Drake equation then...
- ▶ ... we must regard each factor in the Drake equation as a **RANDOM VARIABLE**.
- ▶ Then the number N of communicating civilizations also becomes a random variable.
- ▶ This we call the **STATISTICAL DRAKE EQUATION** and studied in our mentioned reference paper of 2010 (Acta Astronautica, Vol. 67 (2010), pages 1366-1383)

The STATISTICAL Drake Equation /2

- ▶ Denoting each random variable by capitals, the **STATISTICAL DRAKE EQUATION** reads

$$N = \prod_{i=1}^7 D_i$$

- ▶ Where the $D_{sub\ i}$ ("D from Drake") are the 7 random variables, and N is a random variable too ("to be determined").

Generalizing the STATISTICAL Drake Equation to ANY NUMBER OF FACTORS /1

- ▶ Consider the statistical equation

$$N = \overset{\text{any number}}{\prod_{i=1}} D_i$$

- ▶ This is the generalization of our Statistical Drake Equation to the product of ANY finite NUMBER of positive random variables.
- ▶ Is it possible to determine the statistics of N ?
- ▶ Rather surprisingly, the answer is "yes" !

Generalizing the *STATISTICAL* *Drake Equation* to *ANY NUMBER OF FACTORS /2*

- ▶ First, you obviously take the natural log of both sides to change the finite product into a finite sum

$$\ln(N) = \sum_{i=1}^{\text{any number}} \ln(D_i)$$

- ▶ Second, to this finite sum one can apply the **CENTRAL LIMIT THEOREM OF STATISTICS**. It states that, in the limit for an infinite sum, the distribution of the left-hand-side is **NORMAL**.
- ▶ This is true **WHATEVER** the distributions of the random variables in the sum **MAY BE**.

Generalizing the STATISTICAL Drake Equation to ANY NUMBER OF FACTORS /3

- ▶ So, the random variable on the left is **NORMAL**, i.e.

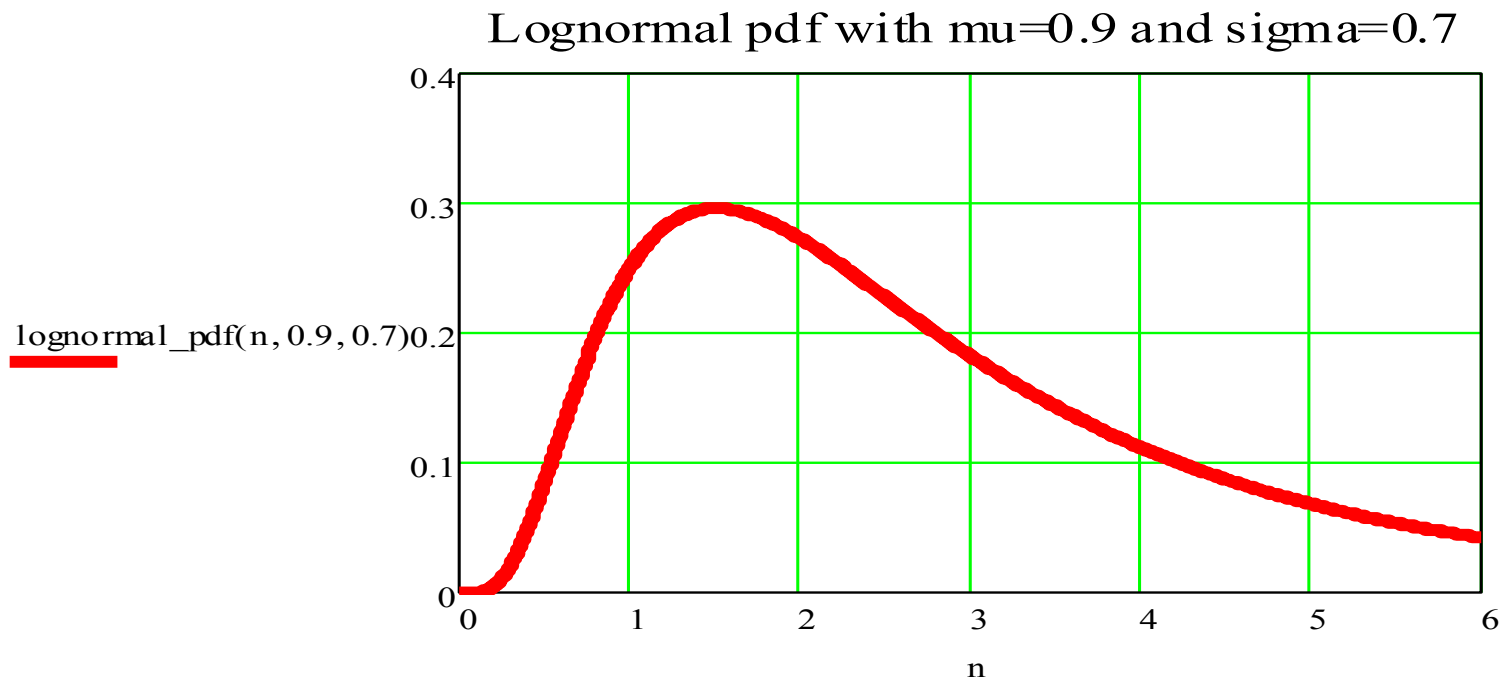
$$\ln(N)$$

- ▶ Thus, the random variable N under the log must be **LOG-NORMAL** and its distribution is determined!
- ▶ One must, however, determine the mean value and variance of this log-normal distribution in terms of the mean values and variances of the factor random variables. This is **DIFFICULT**. But it can be done, for example, by a suitable numeric code that this author wrote in MathCad language.

LOGNORMAL pdf

$$\text{lognormal_pdf}(n, \mu, \sigma) = \frac{e^{-\frac{(\log(n)-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi} \sigma n}.$$

This pdf starts at $n = 0$, that is: $0 \leq n \leq \infty$.



Conclusion

The number of Signaling Civilizations is LOGNORMALLY distributed

- ▶ Our Statistical Drake Equation, now Generalized to any number of factors, embodies as a special case the Statistical Drake Equation with just 7 factors.
- ▶ The conclusion is that the random variable N (the number of communicating ET Civilizations in the Galaxy) is LOG-NORMALLY distributed.
- ▶ The classical “old pure-number Drake value” of N is now replaced by the **MEAN VALUE** of such a log-normal distribution.
- ▶ But we now also have an **ERROR BAR** around it !

REFERENCE PAPER :

- ▶ The Statistical Drake Equation
- ▶ Acta Astronautica, V. 67 (2010), p. 1366-1383.

Acta Astronautica 67 (2010) 1366–1383

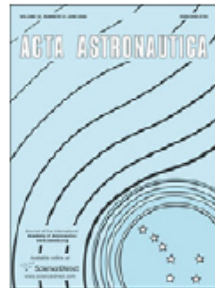


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The Statistical Drake Equation

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Technical Director of the International Academy of Astronautics (IAA) and Co-Chair, SETI Permanent Study Group of the IAA

Part 2:

b-LOGNORMALS

as the LIFE

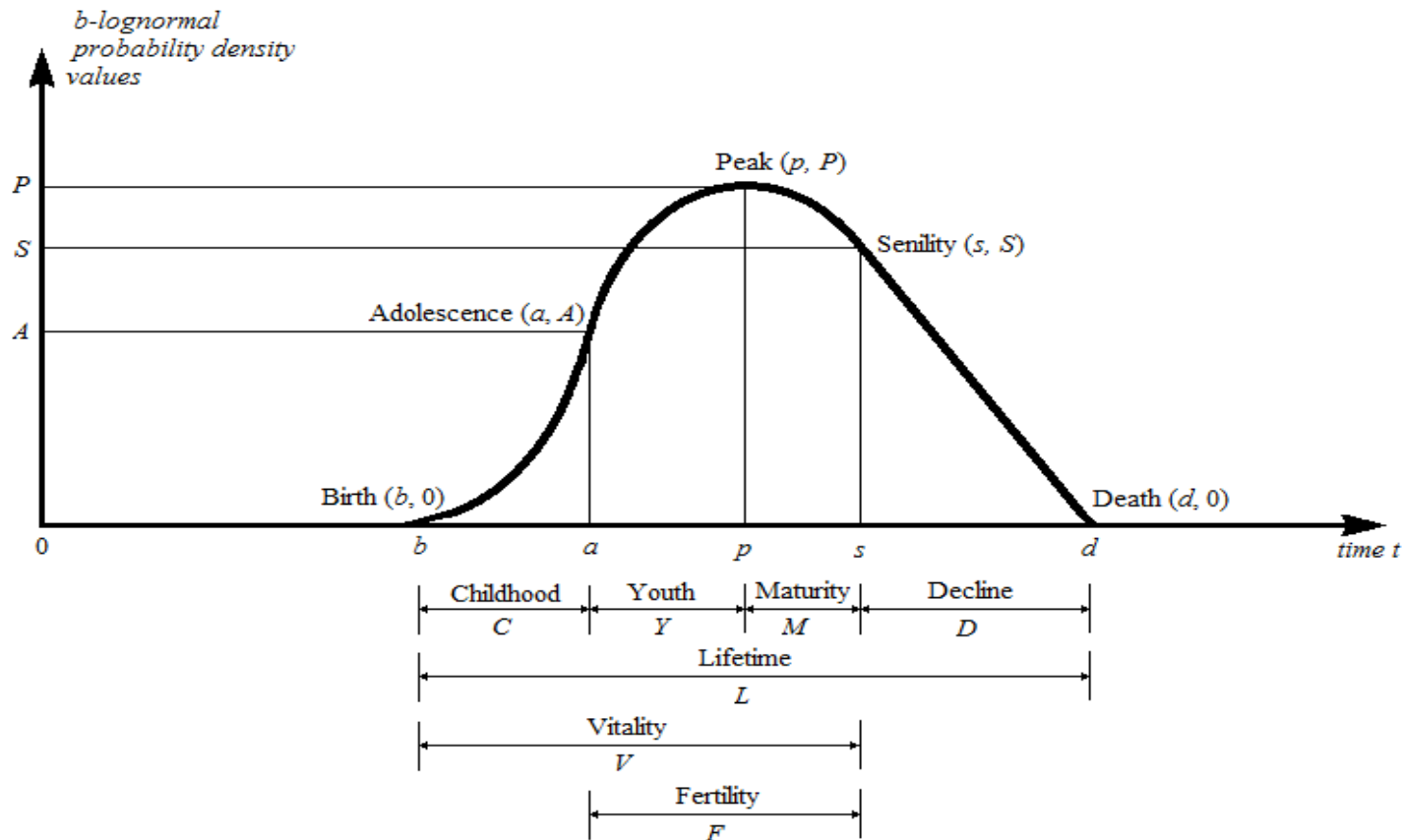
of a cell, of an animal,

of a human, a civilization

(f sub i) even ET (f sub L)

LIFE as a FINITE b-LOGNORMAL

- ▶ The lifetime of a cell, an animal, a human, a civilization can be modeled as a b-lognormal with tail REPLACED at **senility** by the descending TANGENT. The interception at time axis is **DEATH=d**.



LIFE as a FINITE b-LOGNORMAL

- ▶ The equation of a **INFINITE b-lognormal** is :

$$\text{b-lognormal_pdf}(t, \mu, \sigma, b) = \frac{e^{-\frac{(\log(t-b)-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi} \cdot \sigma \cdot (t-b)}.$$

This pdf only starts at time $b = \textit{birth}$, that is: $b \leq t \leq \infty$.

- ▶ The lifetime of a cell, an animal, a human, a civilization can be modeled as a **FINITE b-lognormal**: namely an infinite b-lognormal whose TAIL has been REPLACED at **senility** by the descending TANGENT STRAIGHT LINE. The interception of this straight line at time axis is **DEATH=d**.

LIFE as a FINITE b-LOGNORMAL

b = birth time is supposed to be known.

$$\text{adolescence} \equiv a = b + e^{-\frac{\sigma\sqrt{\sigma^2+4}}{2} - \frac{3\sigma^2}{2} + \mu}$$

$$\text{b-lognormal}_{\text{peak}} \equiv p = b + e^{\mu} e^{-\sigma^2} = b + e^{\mu - \sigma^2}$$

$$\text{senility} \equiv s = b + e^{\frac{\sigma\sqrt{\sigma^2+4}}{2} - \frac{3\sigma^2}{2} + \mu}$$

$$d = b + \frac{(\sqrt{\sigma^2+4} + \sigma)^2 e^{\frac{\sigma\sqrt{\sigma^2+4}}{2} - \frac{3\sigma^2}{2} + \mu}}{4}$$

$$\text{Childhood} \equiv C = a - b = e^{-\frac{\sigma\sqrt{\sigma^2+4}}{2} - \frac{3\sigma^2}{2} + \mu}$$

$$\text{Youth} \equiv Y = p - a = e^{\mu - \sigma^2} - e^{-\frac{\sigma\sqrt{\sigma^2+4}}{2} - \frac{3\sigma^2}{2} + \mu}$$

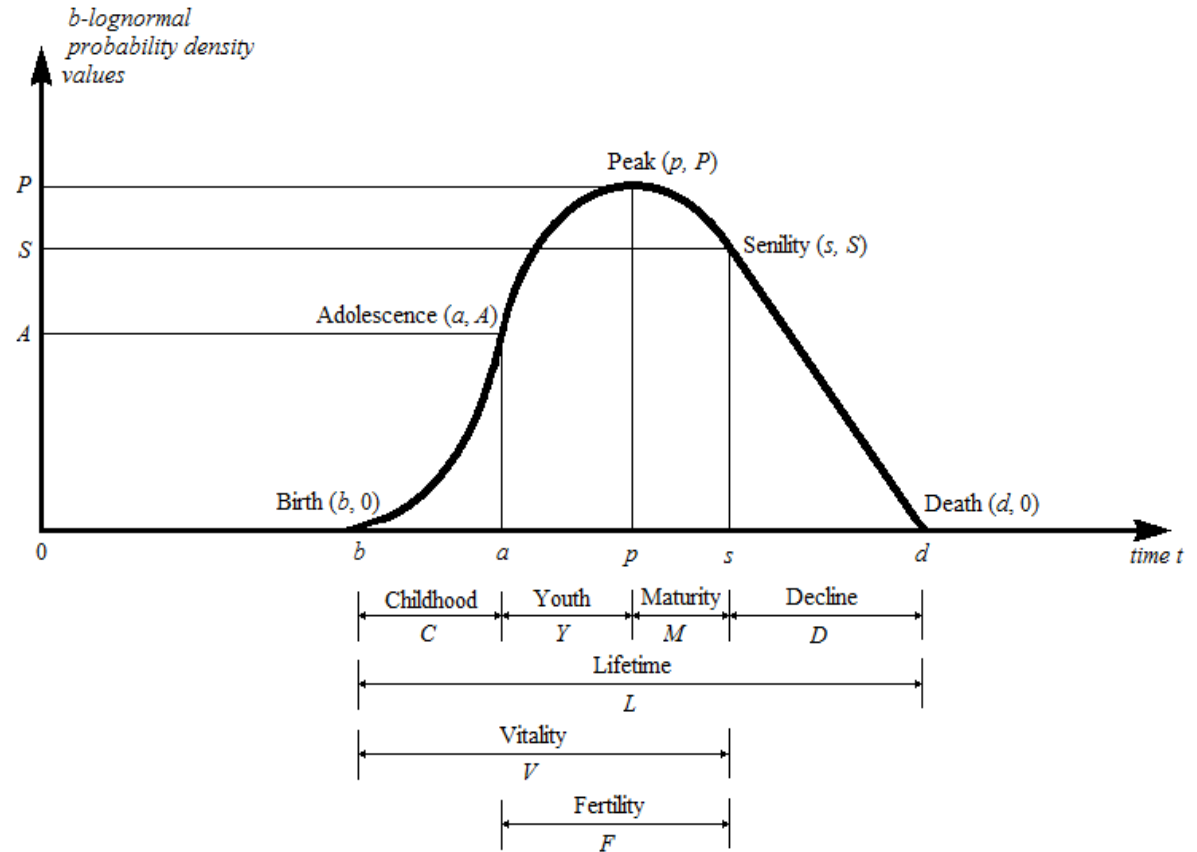
$$\text{Maturity} \equiv M = s - p = e^{\frac{\sigma\sqrt{\sigma^2+4}}{2} - \frac{3\sigma^2}{2} + \mu} - e^{\mu - \sigma^2}$$

$$\text{Decline} \equiv D = d - s = \frac{(\sqrt{\sigma^2+4} + \sigma)^2 e^{\frac{\sigma\sqrt{\sigma^2+4}}{2} - \frac{3\sigma^2}{2} + \mu}}{4} - e^{\frac{\sigma\sqrt{\sigma^2+4}}{2} - \frac{3\sigma^2}{2} + \mu} = e^{\frac{\sigma\sqrt{\sigma^2+4}}{2} - \frac{3\sigma^2}{2} + \mu} \cdot \frac{\sigma(\sqrt{\sigma^2+4} + \sigma)}{2}$$

$$\text{Lifetime} \equiv L = d - b = \frac{(\sqrt{\sigma^2+4} + \sigma)^2 e^{\frac{\sigma\sqrt{\sigma^2+4}}{2} - \frac{3\sigma^2}{2} + \mu}}{4}$$

$$\text{Fertility} \equiv F = s - a = e^{\frac{\sigma\sqrt{\sigma^2+4}}{2} - \frac{3\sigma^2}{2} + \mu} - e^{-\frac{\sigma\sqrt{\sigma^2+4}}{2} - \frac{3\sigma^2}{2} + \mu} = 2 \cdot e^{-\frac{3\sigma^2}{2} + \mu} \cdot \sinh\left(\frac{\sigma\sqrt{\sigma^2+4}}{2}\right)$$

$$\text{Vitality} \equiv V = s - b = e^{\frac{\sigma\sqrt{\sigma^2+4}}{2} - \frac{3\sigma^2}{2} + \mu}$$



LIFE as FINITE b-LOGNORMAL

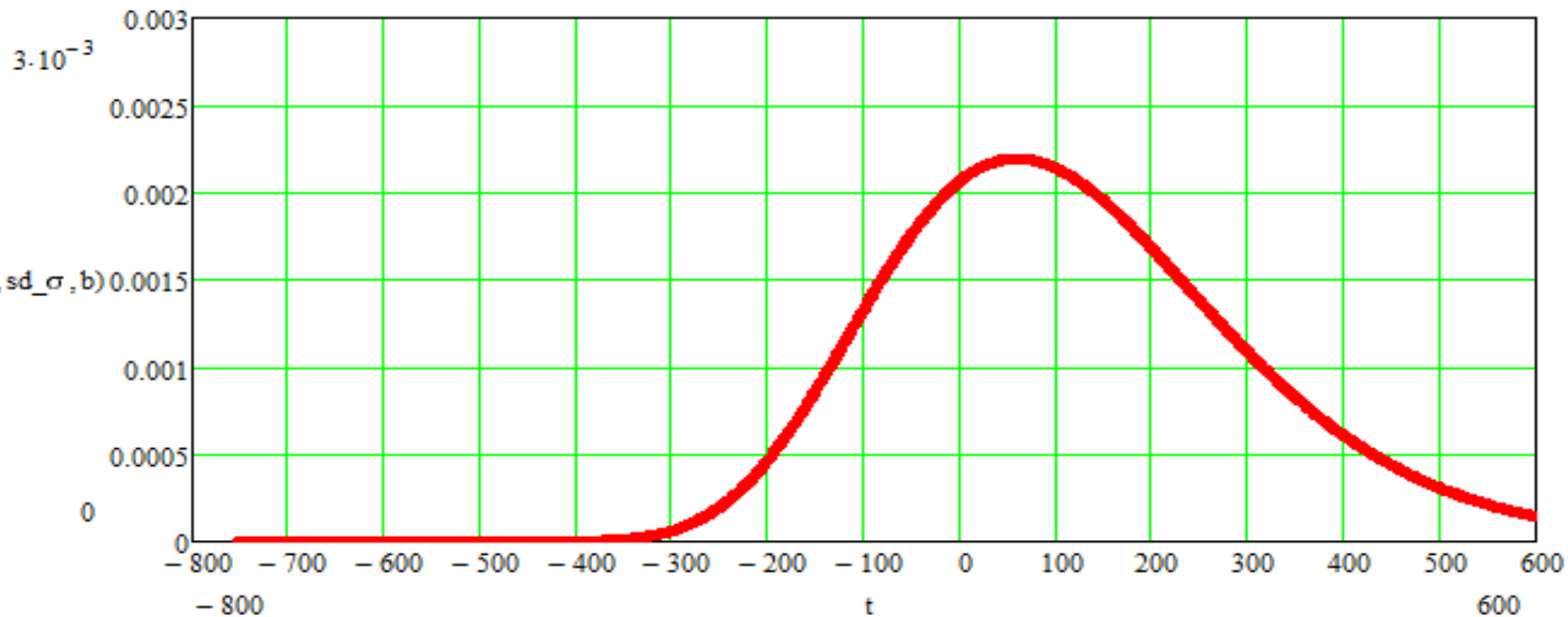
- ▶ Let a = increasing inflexion, s = decreasing inflexion.
- ▶ Then any b-lognormal has birth time (b), adolescence time (a), peak time (p) and senility time (s).
- ▶ Rome's civilization: $b=-753$, $a=-146$, $p=59$, $s=235$.
- ▶ HISTORY FORMULAE : GIVEN (b , s , d) it is always possible to compute the corresponding b-lognormal by virtue of the HISTORY FORMULAE :

$$\left\{ \begin{array}{l} \sigma = \frac{d - s}{\sqrt{d - b} \sqrt{s - b}} \\ \mu = \ln(s - b) + \frac{(d - s)(b + d - 2s)}{(d - b)(s - b)}. \end{array} \right.$$

LIFE as INFINITE b-LOGNORMAL

- ▶ Let a = increasing inflexion, s = decreasing inflexion.
- ▶ Then any b-lognormal has birth time (b), adolescence time (a), peak time (p) and senility time (s).
- ▶ Rome's civilization: $b=-753$, $a=-146$, $p=59$, $s=235$.

b-lognormal of Rome's CIVILIZATION, 753 B.C. thru 476 A.D.

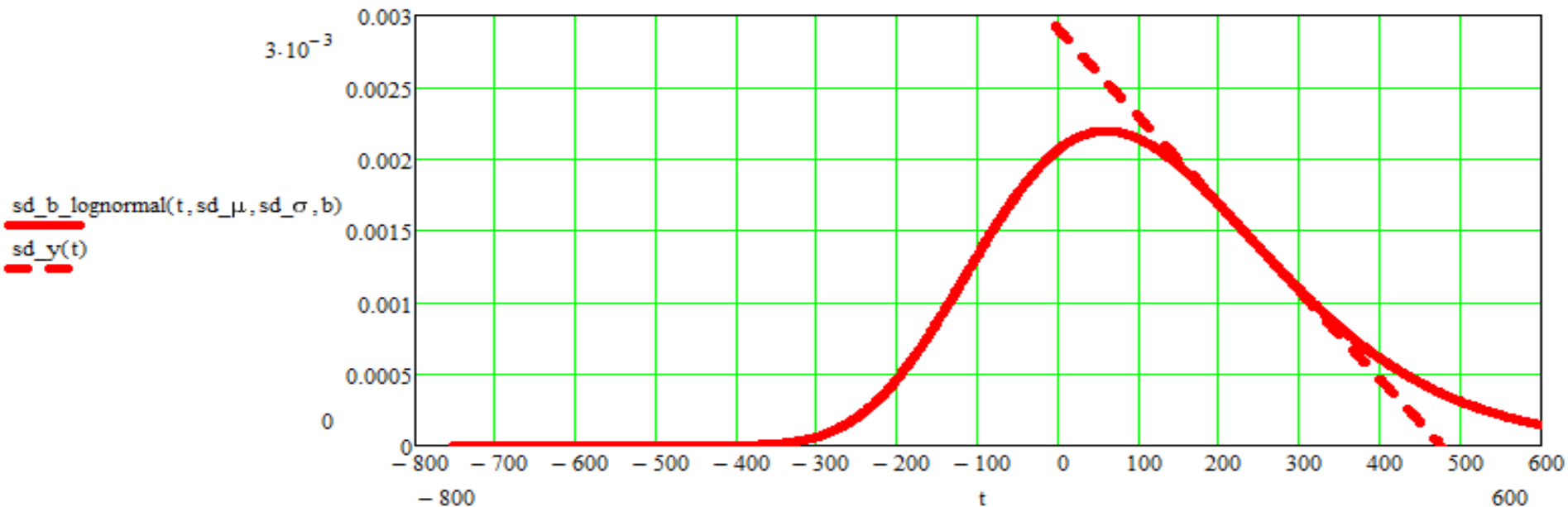


Years (B.C. = negative years, and A.D. = positive years)

LIFE as a FINITE b-LOGNORMAL

- ▶ ANY **FINITE LIFE** may be modeled as a b-lognormal with tail REPLACED at **senility** by the descending TANGENT. The interception at time axis is **DEATH=d**.
- ▶ (e.g. for Rome civilization one has **b=-753**, **d=476**)

b-lognormal of Rome's CIVILIZATION, 753 B.C. thru 476 A.D.



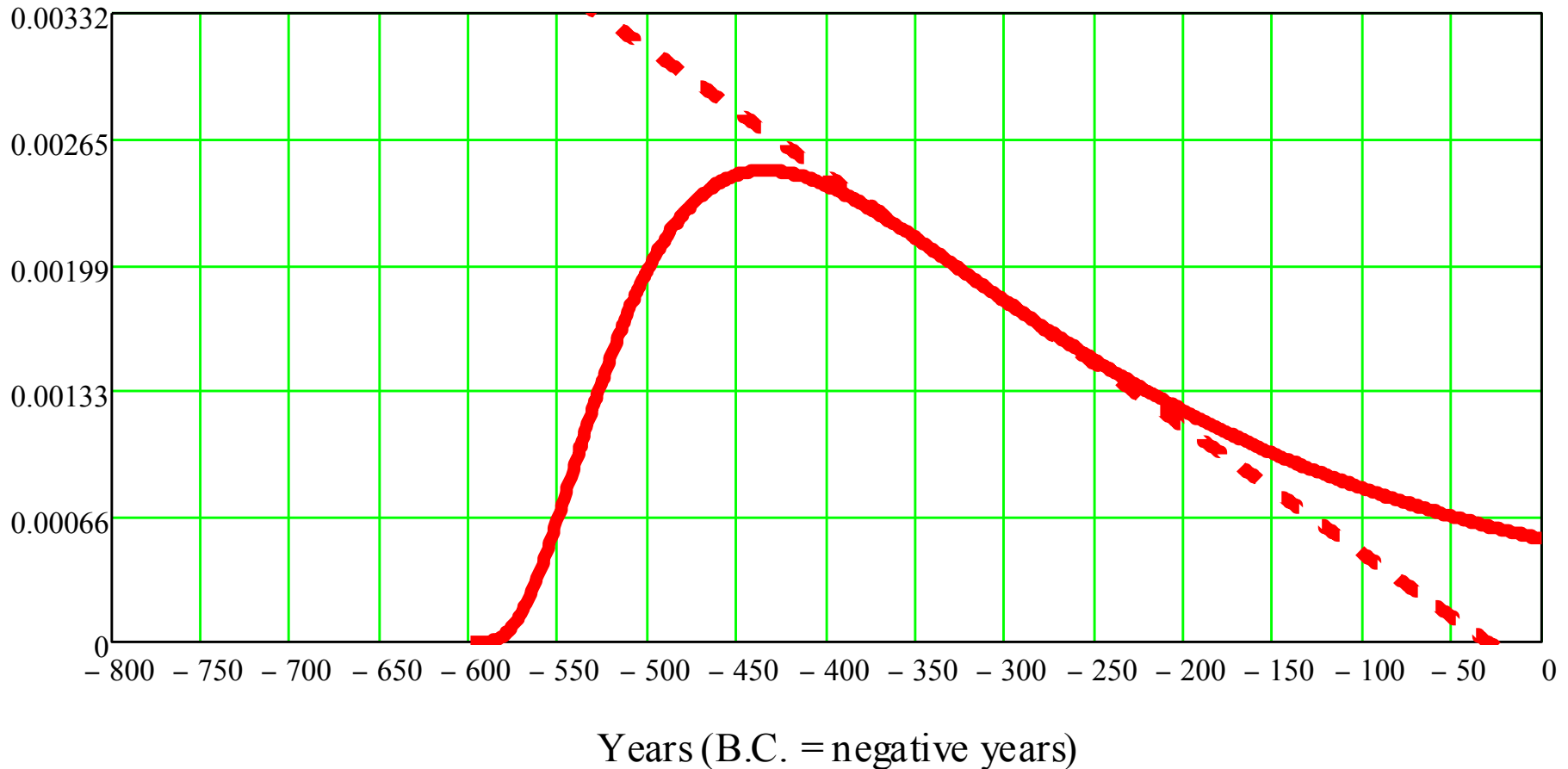
Years (B.C. = negative years, and A.D. = positive years)

FINITE b -LOGNORMAL Civilizations

	b = birth time	s = senility time	d = death time	p = peak time
Ancient Greece	600 BC Mediterranean Greek coastal expansion.	323 BC Alexander the Great's death. Hellenism starts.	30 BC Cleopatra's death: last Hellenistic queen	434 BC Pericles' Age. Democracy peak. Arts and science peak.
Ancient Rome	753 BC Rome founded. Italy seized by Romans by 270 BC.	235 AD Military Anarchy starts. Rome not capital.	476 AD Western Roman Empire ends. Dark Ages start.	59 AD Christianity preached in Rome by Saints Peter and Paul against slavery
Renaissance Italy	1250 Frederick II dies Middle Ages end. Free Italian towns.	1564 Council of Trent. Tough Catholic and Spanish Rule.	1660 1600 G. Bruno executed, 1642 Galileo dies, 1667 Cimento Academy shut.	1450 Renaissance art and architecture. Science. Copernican revolution.
Portuguese Empire	1419 Madeira island discovered	1822 Brazil independent, colonies retained.	1999 Last colony Macau lost.	1716 Black slave trade to Brazil at its peak.
Spanish Empire	1492 Columbus discovers America.	1805 Spanish fleet lost at Trafalgar.	1898 Last colonies lost to the USA.	1741 California to be settled by Spain, 1759-76.
French Empire	1524 Verrazano first in New York bay.	1815 Napoleon defeated at Waterloo.	1962 Algeria lost, as most colonies.	1732 French Canada and India conquest tried.
British Empire	1588 Spanish Armada Defeated.	1914 World War One won at a high cost.	1973 The UK joins European EEC.	1868 Victorian Age. Science: Faraday and Maxwell.
USA Empire	1898 Philippines, Cuba, Puerto Rico seized.	2001 9/11 terrorist attacks.	2050 ? Will the USA yield to China ?	1973 Moon landings, 1969-72

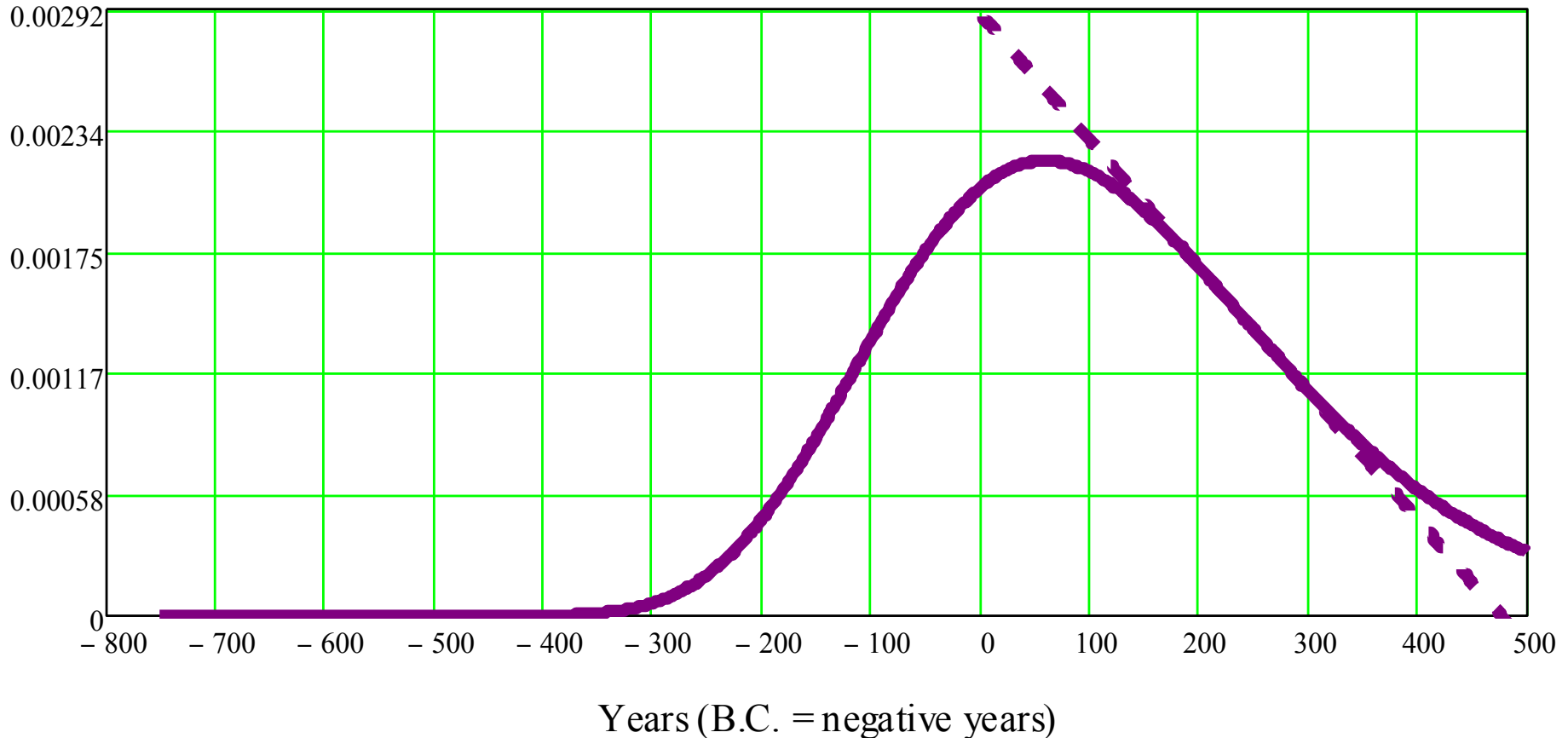
ANCIENT GREECE (600 BC - 323 BC - 30 BC)

FINITE b-lognormal of the GREEK CIVILIZATION (600 B.C. - 30 B.C.).



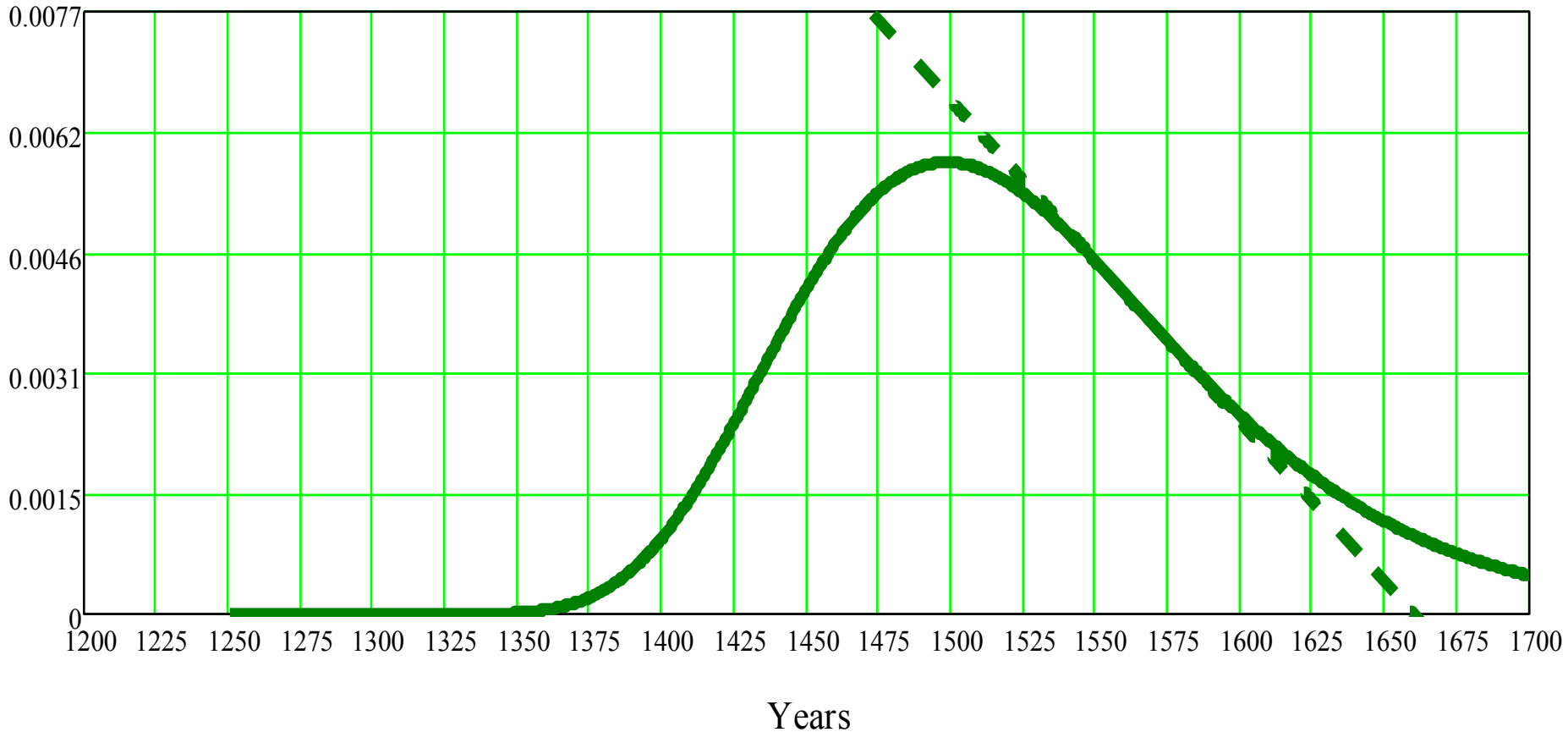
ANCIENT ROME (753 BC – 235 AD – 476 AD)

FINITE b-lognormal of the CIVILIZATION OF ROME (753 B.C. - 476 A.D).



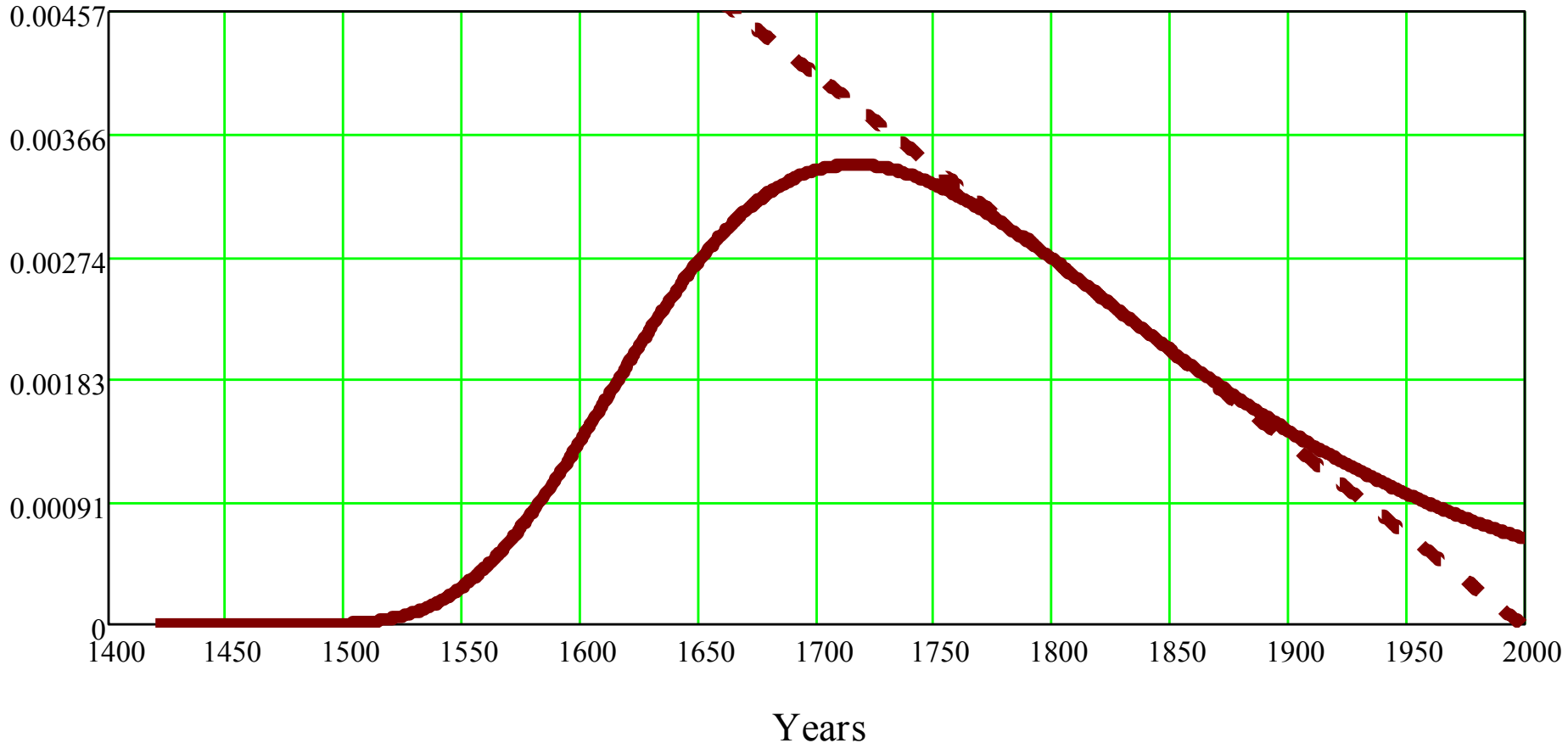
ITALIAN RENAISSANCE (1250 - 1564 - 1660)

FINITE b-lognormal of the ITALIAN RENAISSANCE CIVILIZATION (1250 - 1660).



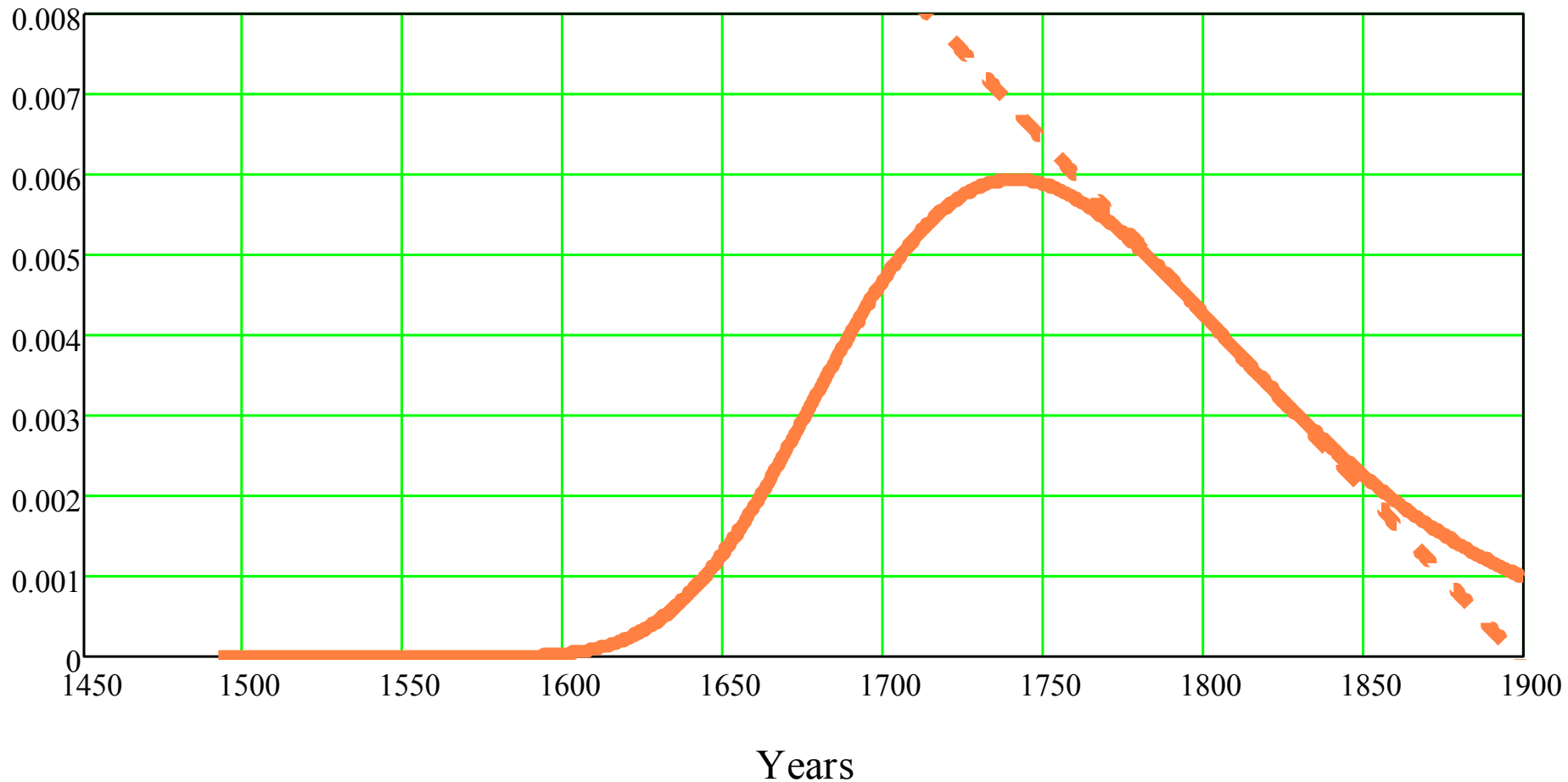
PORTUGAL (1419 - 1822 - 1999)

FINITE b-lognormal of the PORTUGUESE CIVILIZATION (1419 - 1999).



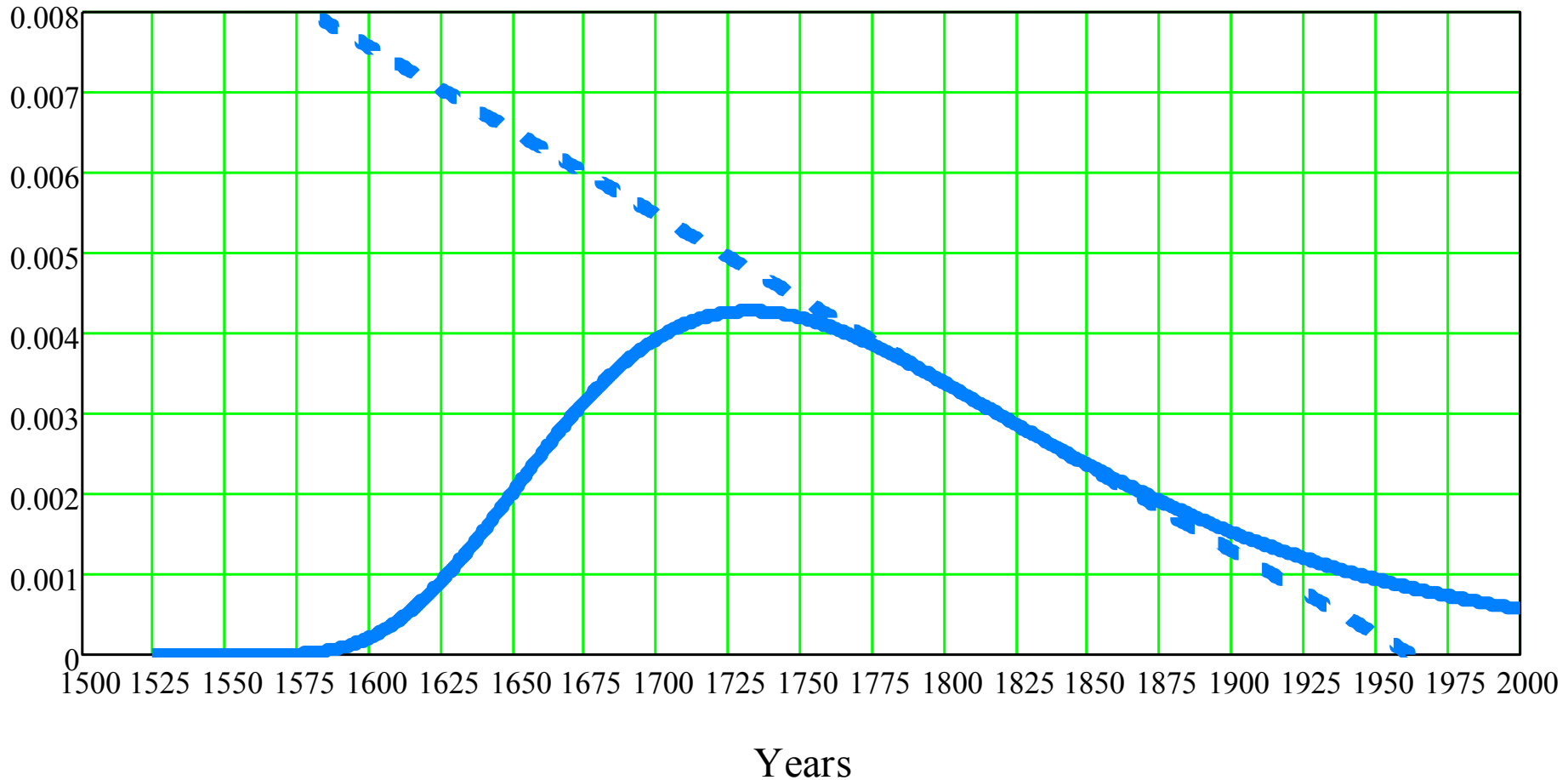
SPAIN (1492 - 1805 - 1898)

FINITE b-lognormal of the SPANISH EMPIRE, 1492-1898.



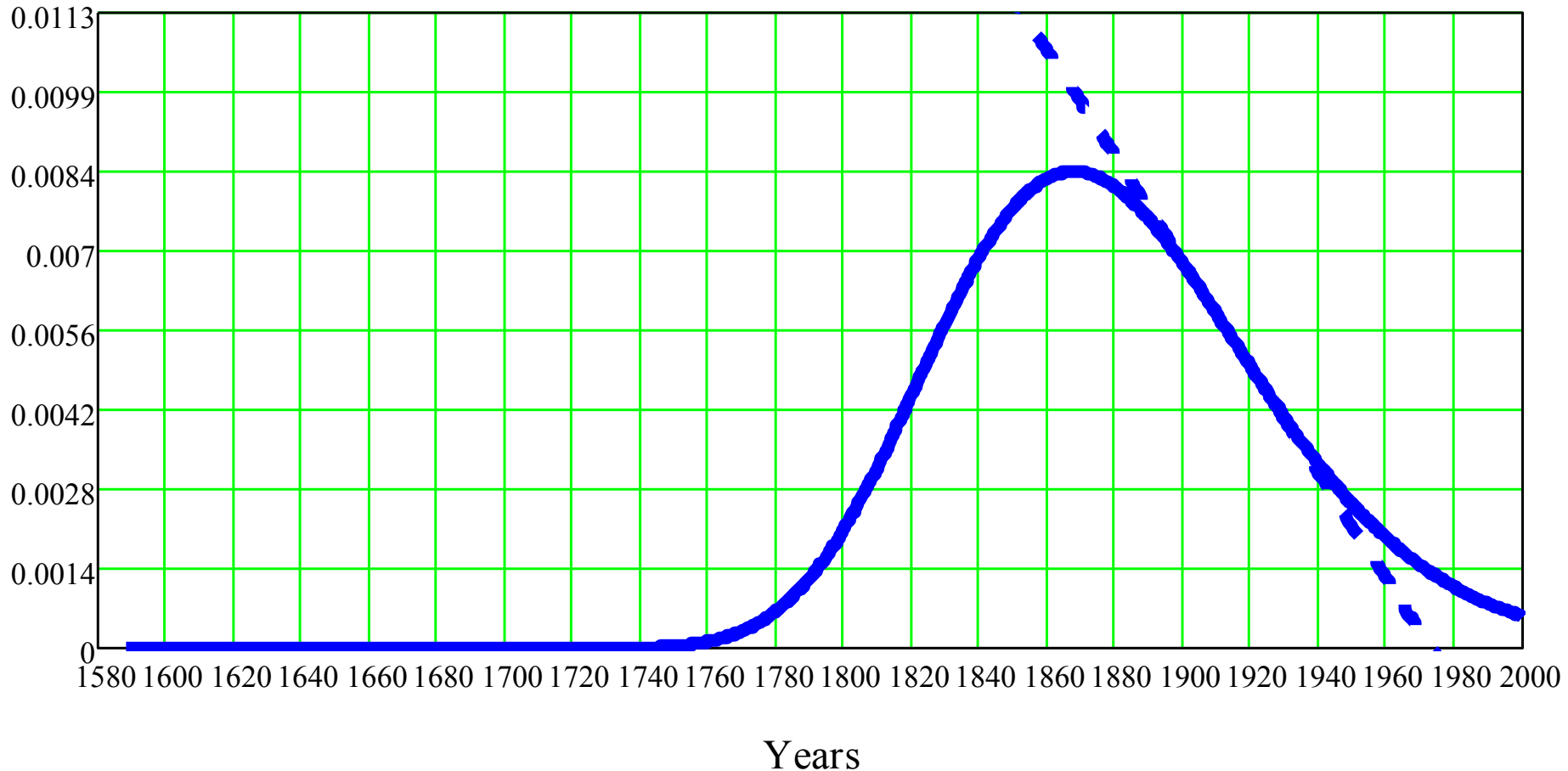
FRANCE (1524 - 1870 - 1962)

FINITE b-lognormal of the FRENCH COLONIAL EMPIRE, 1525-1962.



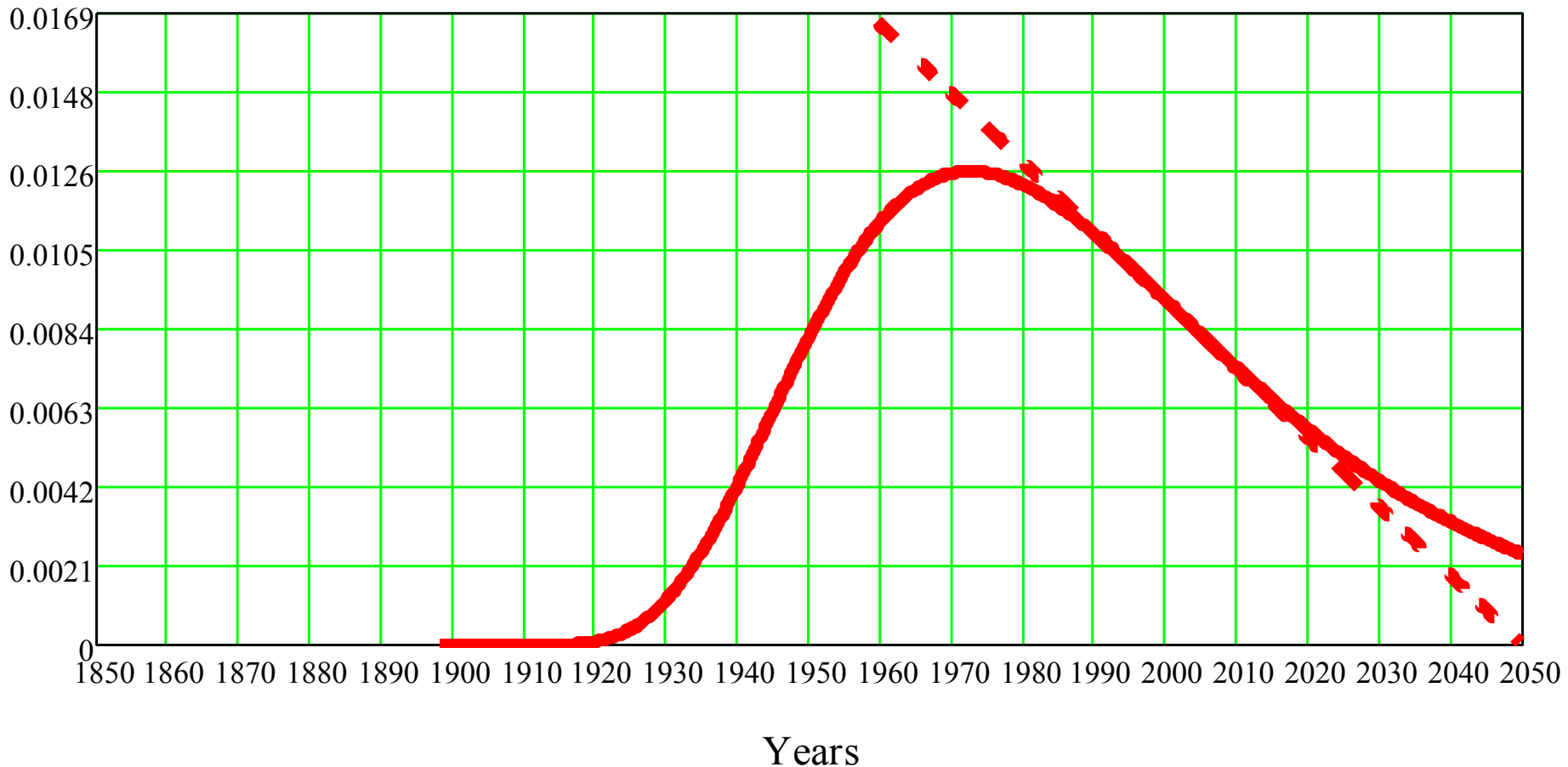
BRITAIN (1588 - 1914 - 1974)

FINITE b-lognormal of the BRITISH EMPIRE, 1588-1973.



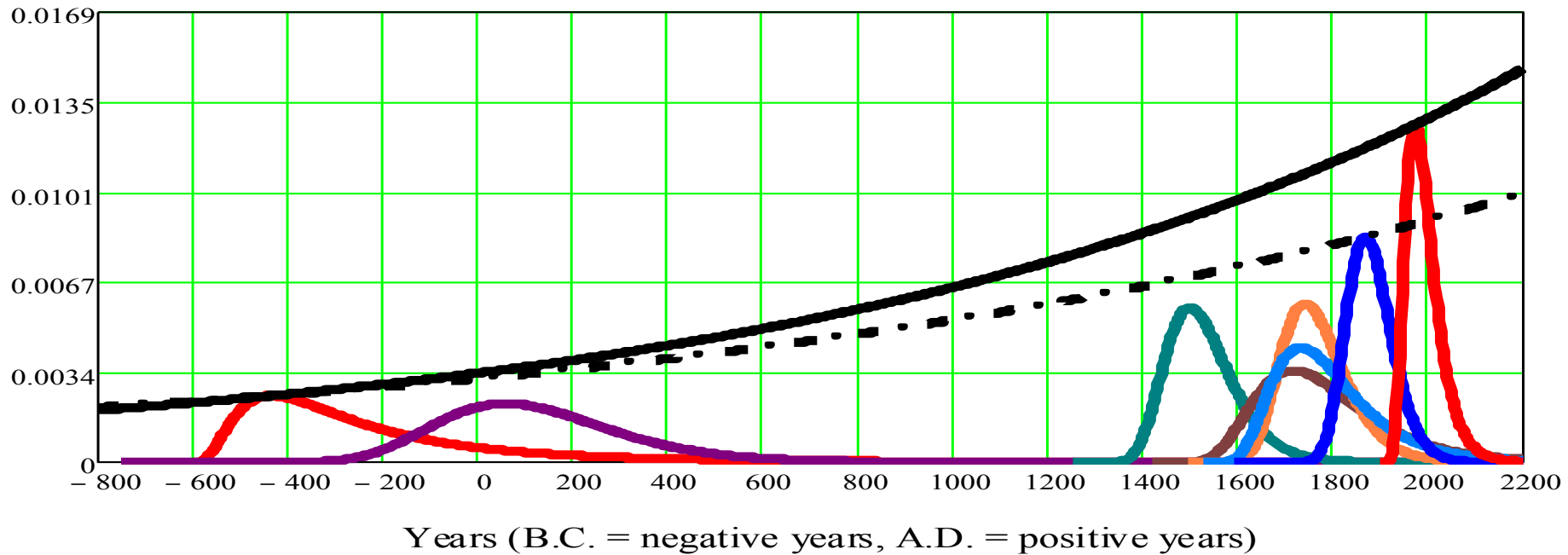
USA (1898 - 2001 - ? 2050 ?)

FINITE b-lognormal of the AMERICAN EMPIRE, 1898 - 2050.



FINITE b-LOGNORMAL Civilizations

Two ENVELOPES for ALL CIVILIZATIONS (800 B.C. - 2200 A.D.).



- Greece 600 BC - 30 BC
- Rome 753 BC - 476 AD
- Renaissance Italy 1250-1660
- Portugal 1419-1974
- Spain 1492-1898
- France 1524-1962
- Britain 1588-1974
- USA 1898-2050
- · - Greece-to-Britain EXPONENTIAL ENVELOPE
- Greece-to-USA EXPONENTIAL ENVELOPE

EXPONENTIAL through two points

- ▶ Given TWO POINTS with coordinates :

$$(p_1, P_1) \text{ and } (p_2, P_2)$$

- ▶ The EXPONENTIAL $A e^{Bt}$ must have:

$$\left\{ \begin{array}{l} A = \frac{\frac{p_2}{P_1^{p_2-p_1}}}{\frac{p_1}{P_2^{p_2-p_1}}} \\ B = \frac{\ln\left(\frac{P_2}{P_1}\right)}{p_2 - p_1} \end{array} \right.$$

Part 3:

Darwinian

EXPONENTIAL GROWTH

as LOCUS of

b-LOGNORMAL PEAKS

REFERENCE PAPER :

- ▶ A Mathematical Model for Evolution and SETI
- ▶ Origins of Life and Evolution of Biospheres (OLEB), Vol. 41 (2011), pages 609-619.

Orig Life Evol Biosph (2011) 41:609–619
DOI 10.1007/s11084-011-9260-3

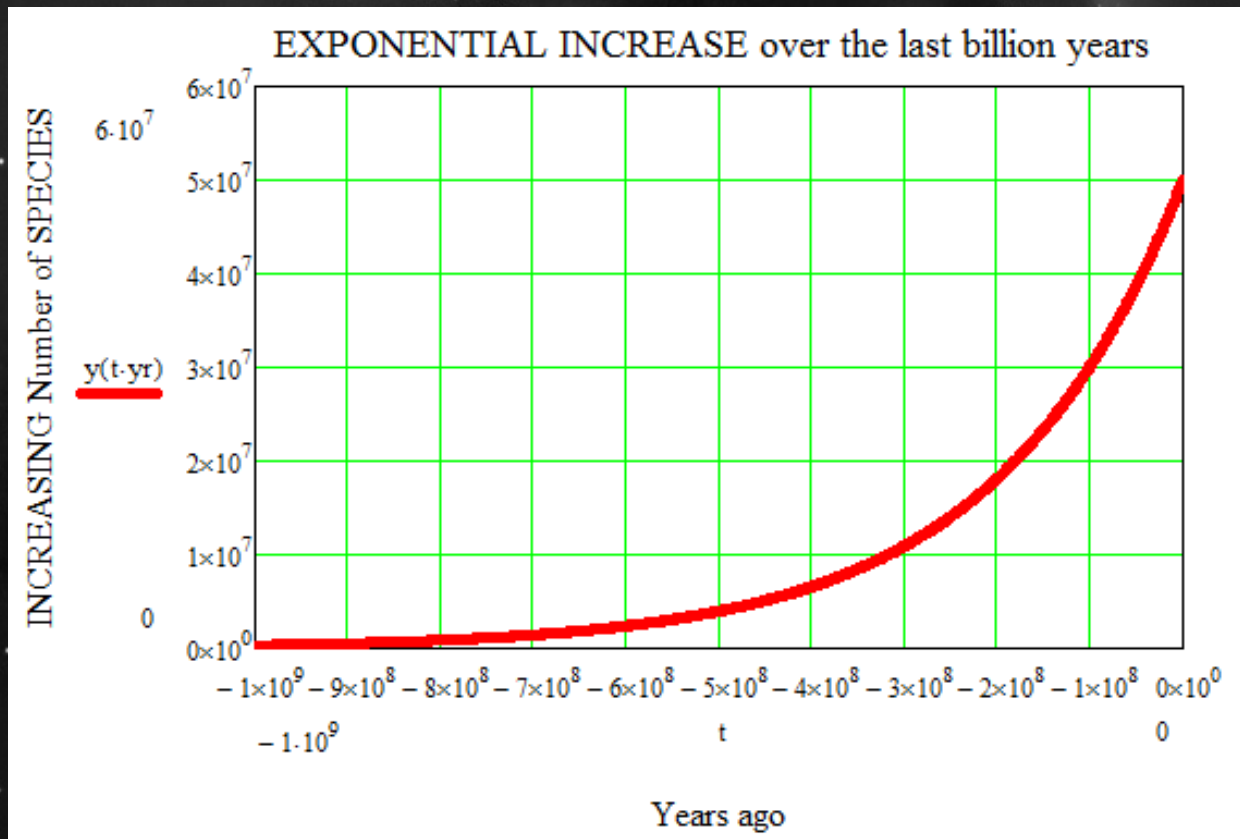
EVOLUTIONARY PERSPECTIVES

A Mathematical Model for Evolution and SETI

Claudio Maccone

Darwinian EXPONENTIAL GROWTH

- ▶ Life on Earth evolved since 3.5 billion years ago.
- ▶ The number of Species GROWS EXPONENTIALLY: assume that today 50 million species live on Earth
- ▶ Then:



Darwinian EXPONENTIAL GROWTH

- ▶ Life on Earth evolved since 3.5 billion years ago.
- ▶ The number of Species GROWS EXPONENTIALLY: assume that today 50 million species live on Earth
- ▶ Then:

exponential curve in time : $E(t) = A e^{Bt}$

- ▶ with:

$$\left\{ \begin{array}{l} A = 50 \text{ million species} = 5 \cdot 10^7 \text{ species} \\ B = -\frac{\ln(E(t_2))}{t_1} = -\frac{\ln(5 \cdot 10^7)}{-3.5 \cdot 10^9 \text{ year}} = \frac{1.605 \cdot 10^{-16}}{\text{sec}} \end{array} \right.$$

b-LOGNORMALS i.e. LOGNORMALS starting at $b = \text{birth}$

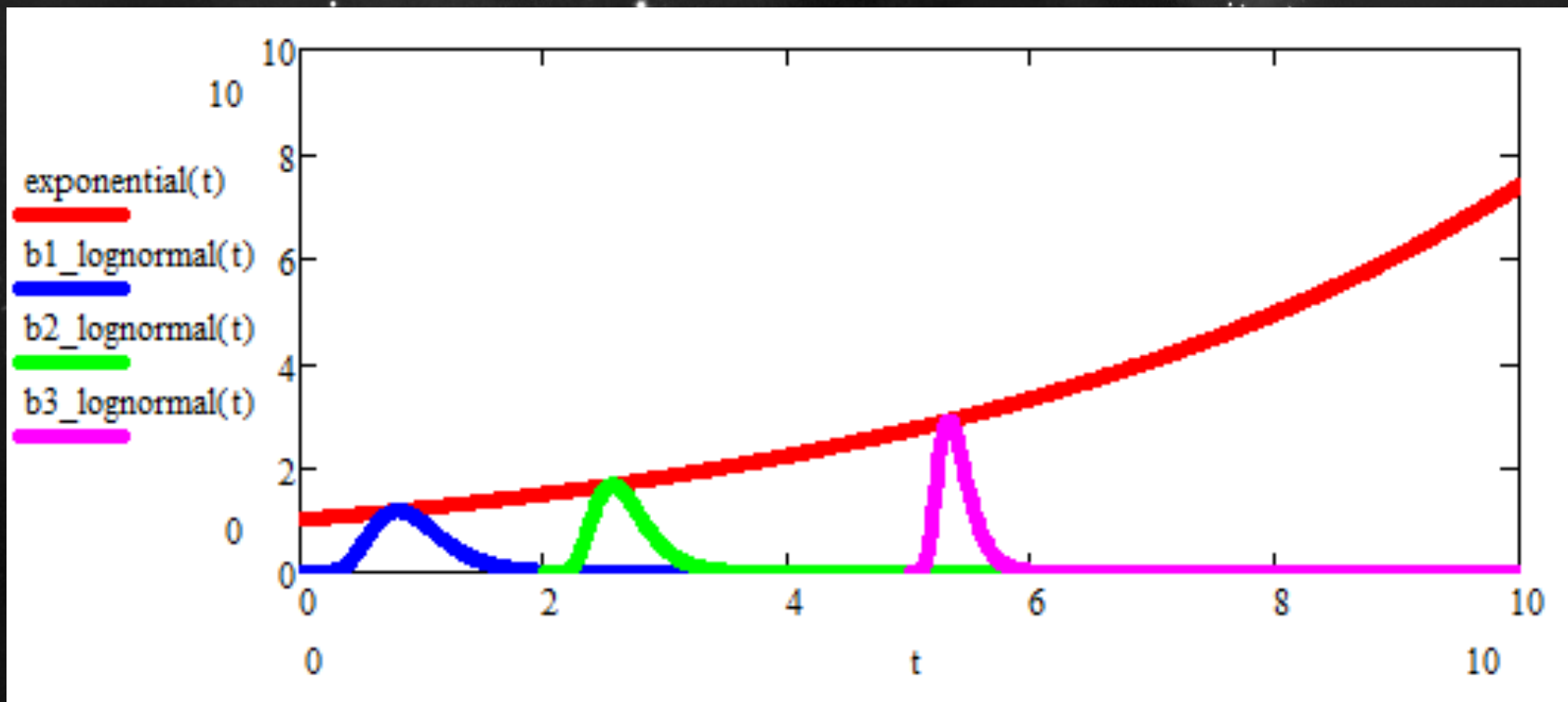
$$\text{b-lognormal}(t, \mu, \sigma, b) = \frac{e^{-\frac{(\log(t-b) - \mu)^2}{2\sigma^2}}}{\sqrt{2\pi} \cdot \sigma \cdot (t-b)}.$$

This pdf only starts at time $b = \text{birth}$, that is: $b \leq t \leq \infty$.

- ▶ b-lognormals are just lognormals starting at any instant b , supposed to be known.
- ▶ b-lognormals are thus a family of probability density functions with three real and positive parameters: μ , σ , and b .

EXPONENTIAL as “ENVELOPE” of b-LOGNORMALS

- ▶ Each b-lognormal has its peak on the exponential.
- ▶ PRACTICALLY an “Envelope”, though not so formally.



b-LOGNORMAL PEAK /1

$$\left\{ \begin{array}{l} \text{b-lognormal peak abscissa} \equiv p = b + e^{\mu - \sigma^2} \\ \text{b-lognormal peak ordinate} \equiv P = \frac{e^{\frac{\sigma^2}{2} - \mu}}{\sqrt{2\pi\sigma}} \end{array} \right.$$

- It is POSSIBLE to match the second equation (peak ordinate) with the EXPONENTIAL curve of the increasing number of Species upon setting:

b-LOGNORMAL PEAK /2

$$\left\{ \begin{array}{l} \text{exponential ordinate at } t = p \text{ reads: } E(p) = A e^{B p} \\ \text{b-lognormal peak ordinate at } t = p : P = \frac{e^{\frac{\sigma^2}{2} - \mu}}{\sqrt{2\pi\sigma}} \end{array} \right.$$

- We noticed that it is POSSIBLE to MATCH these two equations EXACTLY just upon setting:

$$\left\{ \begin{array}{l} A = \frac{1}{\sqrt{2\pi\sigma}} \\ B p = \frac{\sigma^2}{2} - \mu. \end{array} \right.$$

b-LOGNORMAL PEAK /3

► Moreover, the last two equations can be INVERTED, i.e. solved for μ and σ EXACTLY, thus yielding:

$$\begin{cases} \sigma = \frac{1}{\sqrt{2\pi A}} \\ \mu = \frac{\sigma^2}{2} - Bp = \frac{1}{4\pi A^2} - Bp. \end{cases}$$

► These two equations prove that, knowing the exponential (i.e. A and B) and peak time p , the b-lognormal HAVING ITS PEAK EXACTLY ON THE EXPONENTIAL is perfectly determined (i.e. its μ and σ are perfectly determined given A , B and p). This is the **BASIC RESULT** to make further progress.

Part 4:

**GEOMETRIC
BROWNIAN MOTION
(GBM)**

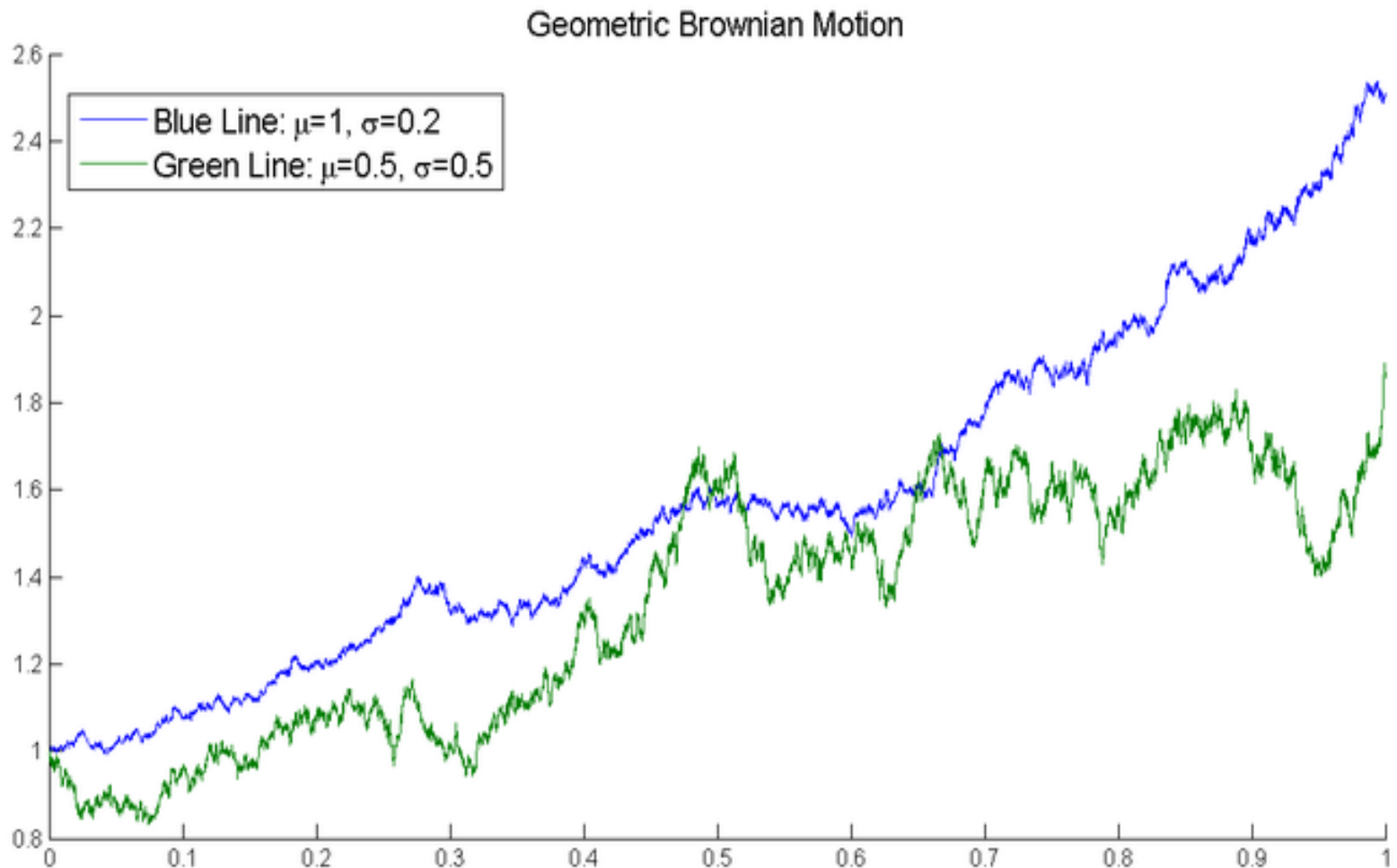
WARNING !!!

GEOMETRIC BROWNIAN MOTION
is a WRONG DENOMINATION:

This process is NOT a Brownian Motion since its probability density function is a LOGNORMAL, and NOT A GAUSSIAN!

So, the pdf ranges between ZERO and INFINITY, and NOT between minus infinity and plus infinity!! *Period.*

GEOMETRIC BROWNIAN MOTION (GBM): exponential mean value



GEOMETRIC BROWNIAN MOTION
(GBM): exponential mean value :

$$\langle N(t) \rangle = N_0 e^{\mu t}.$$

GEOMETRIC BROWNIAN MOTION
lognormal probability density :

$$N(t) \text{ pdf}(n, N_0, \mu, \sigma, t) = \frac{e^{-\frac{\left[\ln(n) - \left(\ln N_0 + \mu t - \frac{\sigma^2 t}{2} \right) \right]^2}{2 \sigma^2 t}}}{\sqrt{2\pi} \sigma \sqrt{t} n}.$$

GEOMETRIC BROWNIAN MOTION
is the extension in time of the
STATISTICAL DRAKE EQUATION:

$$\left\{ \begin{array}{l} t = 1 \\ \sigma_{GBM} = \sigma_{Drake} = \sigma \\ \mu_{GBM} = \mu_{Drake} = \mu \\ N_0 = e^{\frac{\sigma^2}{2}} \end{array} \right.$$

The two lognormals (of movie & picture) then COINCIDE.

In other words still:

1) The CLASSICAL DRAKE EQ. is STATIC, and is a SUBSET of the STATISTICAL DRAKE EQUATION.

2) In turn, the STATISTICAL DRAKE EQUATION is the STATIC VERSION (i.e. the STILL PICTURE) of the GEOMETRIC BROWNIAN MOTION (the MOVIE).

Part 5:

**Darwinian EXPONENTIAL
GROWTH**

**as GBM in the number
of LIVING SPECIES**

THREE REFERENCE PAPERS

- ▶ A Mathematical Model for Evolution and SETI
- ▶ Origins of Life and Evolution of Biospheres (OLEB), Vol. 41 (2011), pages 609-619.

Orig Life Evol Biosph (2011) 41:609–619
DOI 10.1007/s11084-011-9260-3

EVOLUTIONARY PERSPECTIVES

A Mathematical Model for Evolution and SETI

Claudio Maccone

- ▶ SETI, Evolution and Human History Merged into a Mathematical Model.
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International Journal of Astrobiology 12 (3): 218–245 (2013) doi:10.1017/S1473550413000086

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SETI, Evolution and Human History Merged into a Mathematical Model

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- ▶ Evolution and History in a new
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Acta Astronautica ■ (■■■) ■■-■■

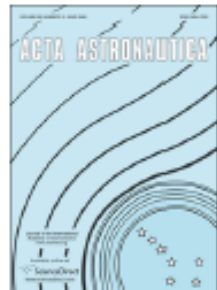


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Academy Transactions Note

Evolution and History in a new “Mathematical SETI” model

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Part 6:

ENTROPY

as the

EVOLUTION MEASURE

b-LOGNORMAL ENTROPY

- ▶ **Shannon ENTROPY** for a probability density (in bits) :

$$\begin{aligned} H &= -\int_{-\infty}^{\infty} f_X(x) \cdot \log_2 [f_X(x)] dx = \\ &= -\frac{1}{\ln 2} \int_{-\infty}^{\infty} f_X(x) \cdot \ln [f_X(x)] dx. \end{aligned}$$

- ▶ **Shannon ENTROPY** for b-lognormals (in bits) :

$$H_{\text{b_lognormal_in_bits}}(\mu, \sigma) = \frac{1}{\ln 2} \left[\ln(\sqrt{2\pi} \sigma) + \mu + \frac{1}{2} \right].$$

b-LOGNORMAL ENTROPY

- ▶ But μ ONLY is a function of the peak abscissa p :

$$\begin{cases} \sigma = \frac{1}{\sqrt{2\pi A}} \\ \mu(p) = -B p + \frac{1}{4\pi A^2} \end{cases}$$

- ▶ Shannon ENTROPY for the b-lognormal in bits :

$$\begin{aligned} H_{\text{b_lognormal_in_bits}}(p) &= \frac{1}{\ln 2} \cdot [\mu(p) + \text{part_not_depending_on_p}] = \\ &= -\frac{B}{\ln 2} \cdot p + \text{another_part_not_depending_on_p}. \end{aligned}$$

CIVILIZATION LEVEL DIFFERENCE

- ▶ The ENTROPY DIFFERENCE among any two Civilizations having their two peak abscissae at $p_{sub 1}$ and $p_{sub 2}$ is given by

$$\Delta_{H_in_bits} = -\frac{B}{\ln 2} \cdot (p_2 - p_1) \quad B = \frac{4.0376473009027558 \cdot 10^{-10}}{\text{sec}}$$

- ▶ ENTROPY IS THUS A MEASURE OF THE LEVEL OF PROGRESS REACHED BY EACH CIVILIZATION.
- ▶ ENTROPY DIFFERENCE measures the DIFFERENCE in civilization level among any two Civilizations.
- ▶ If it is known WHEN the two Civilizations reached their two peaks, the above formula yields their **CIVILIZATION LEVEL DIFFERENCE.**

EXAMPLES

CIVILIZATION DIFFERENCE

- ▶ The DIFFERENCE in Civilization Level between the Spaniard and Aztecs in 1519 was about 3.84 bits per individual.
- ▶ The DIFFERENCE in Civilization Level between a Victorian Briton and a Pericles Greek was about 1.76 bits per individual.
- ▶ The DIFFERENCE in Civilization Level between Humanity and the first Alien Civilization is UNKNOWN, of course, but...
- ▶ ... but now we have a Mathematical Theory to ESTIMATE IT on the basis of the messages we get.

EXAMPLE

EVOLUTION DIFFERENCE

► The DIFFERENCE in Darwinian Evolution between two species on Earth is given by the same equation

$$\Delta_{H_in_bits} = -\frac{B}{\ln 2} \cdot (p_2 - p_1)$$

► As for the DIFFERENCE in Civilization Level, except we must now use the different numerical value of B the enveloping Darwinian exponential, found earlier.

► The result is that the DIFFERENCE IN EVOLUTION LEVEL between the first living being 3.5 billion years ago and Humans living now is about 25.57 bits per individual.

Part 7:

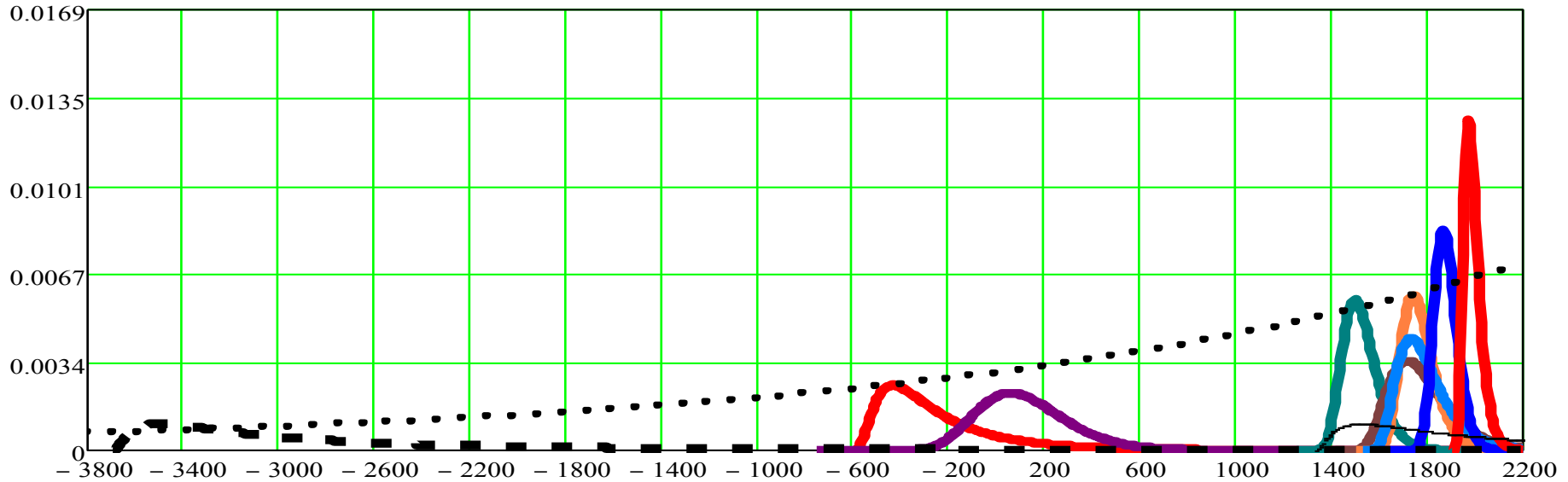
Example in the PAST

namely

AZTECS vs. SPANIARDS

VIRTUAL AZTEC b-lognormal

Greece-to-Spain ENVELOPE and ALL CIVILIZATIONS (3800 B.C. - 2200 A.D.).



Years (B.C. = negative years, A.D. = positive years)

- Greece 600 B.C. - 30 B.C.
- Rome 753 B.C. - 476 A.D.
- Renaissance Italy 1250-1660
- Portuguese Empire 1419-1974
- Spanish Empire 1492-1898
- French Empire 1524-1962
- British Empire 1588-1974
- USA Empire 1898-2050 (?)
- · · · Greece-to-Spain EXPONENTIAL ENVELOPE
- - - Virtual-Aztec Empire 3694 BC - 2627 BC
- · - · True-Aztec Empire 1325-1519

VIRTUAL AZTEC b-lognormal

$$\left\{ \begin{array}{l} -3500 = p_{VA} = b_{VA} + e^{\mu_{VA} - \sigma_{VA}^2} \\ \text{Greece_to_Spain_EXPONENTIAL} = \\ = 7.305 \cdot 10^{-4} = P_{VA} = \frac{e^{\frac{\sigma_{VA}^2}{2} - \mu_{VA}}}{\sqrt{2\pi} \sigma_{VA}}. \end{array} \right.$$

$$\text{Numerically_known} = (p_{VA} - b_{VA}) P_{VA} = \frac{e^{-\frac{\sigma_{VA}^2}{2}}}{\sqrt{2\pi} \sigma_{VA}}.$$

$$\text{Numerically_known} = (p_{VA} - b_{VA}) P_{VA} \approx \frac{1 - \frac{\sigma_{VA}^2}{2}}{\sqrt{2\pi} \sigma_{VA}}.$$

$$\sigma_{VA}^2 + 2\sqrt{2\pi} (p_{VA} - b_{VA}) P_{VA} \sigma_{VA} - 2 = 0.$$

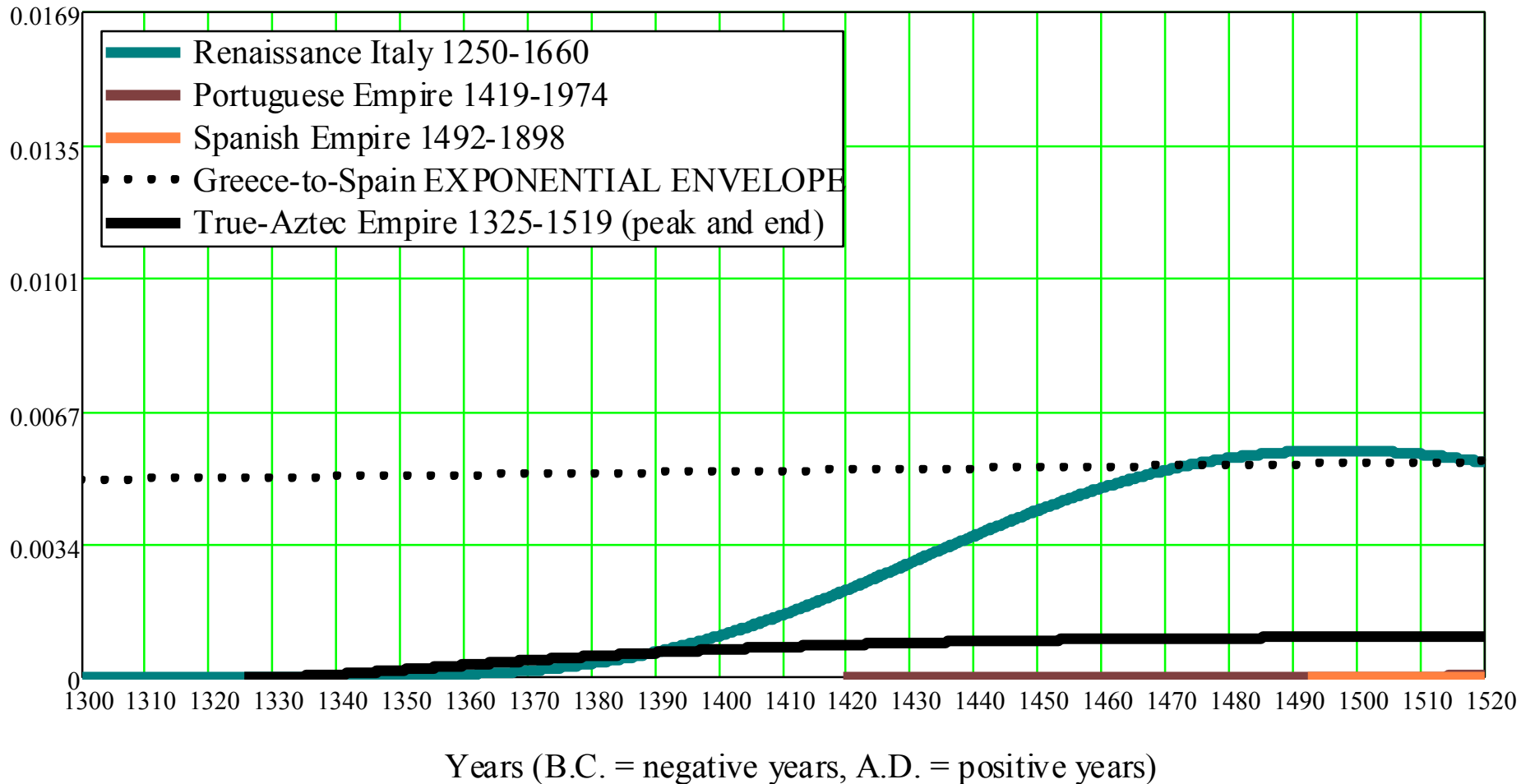
$$\sigma_{VA} = -\sqrt{2\pi} (p_{VA} - b_{VA}) P_{VA} \pm \sqrt{2} \sqrt{\pi (p_{VA} - b_{VA})^2 P_{VA}^2 + 1}.$$

$$\mu_{VA} = \sigma_{VA}^2 + \ln(p_{VA} - b_{VA}).$$

AZTECS vs. SPANIARDS

1300 - 1520

Renaissance Italy, Portuguese and Spanish Empires and TRUE-AZTECS (1300 - 1520).



Part 8:

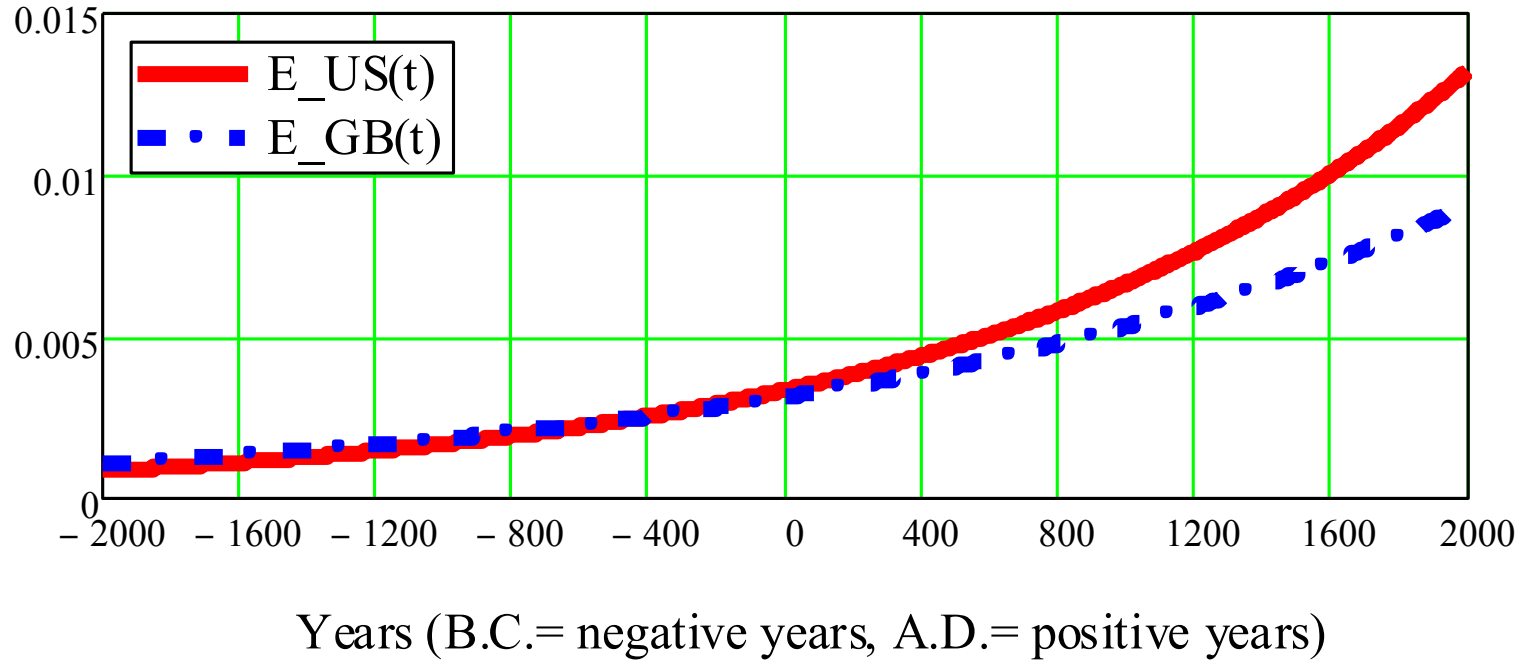
Example in the FUTURE

up to

10 MILLION YEARS

TRENDS: TWO EXPONENTIALS

Greece-to-USA and Greece-to-Britain EXPONENTIALS



$$A_GB := 0.0031332506554731$$

$$A_US := 0.0033528127662929$$

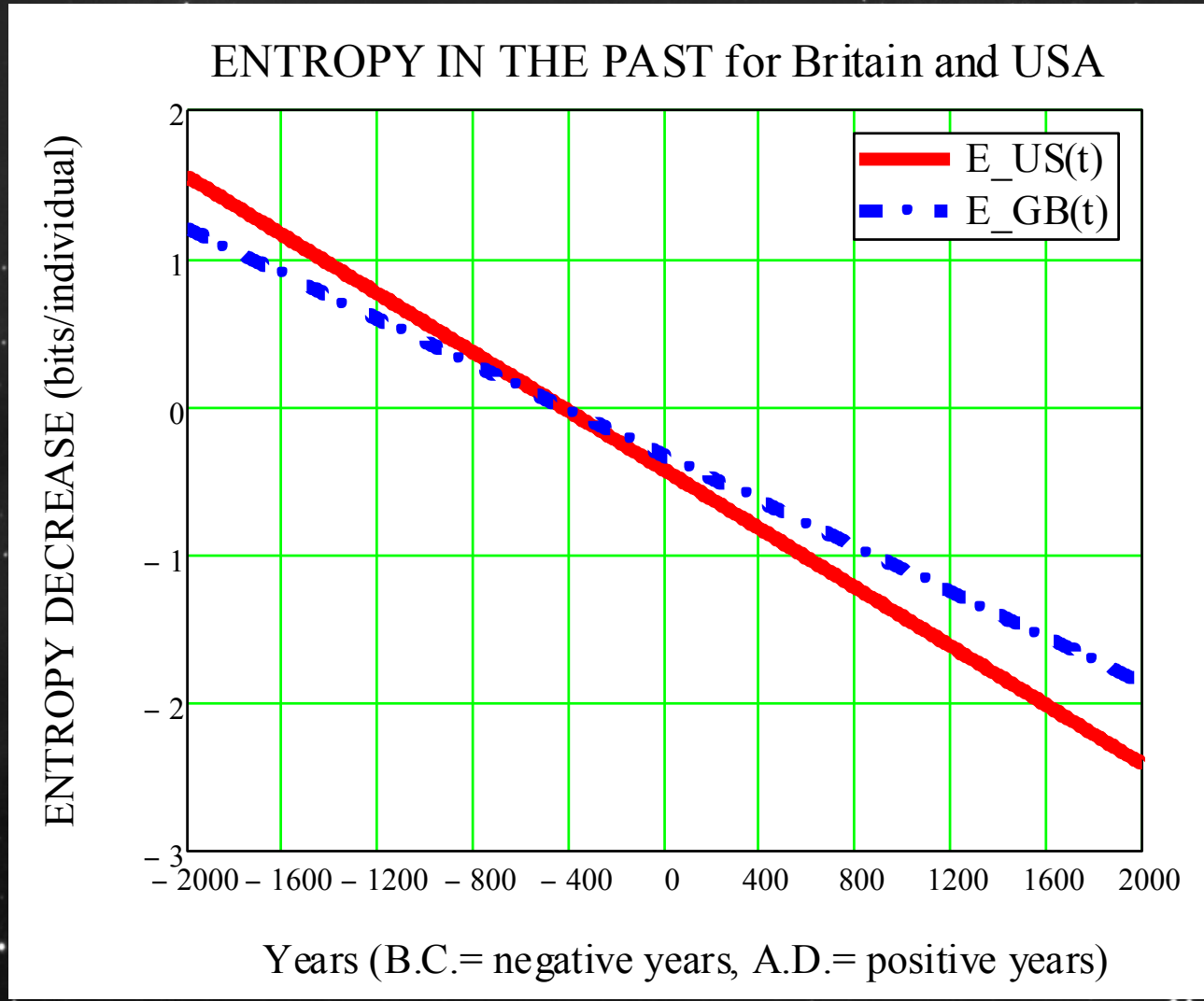
$$B_GB := \frac{5.3091024359570339 \cdot 10^{-4}}{\text{yr}}$$

$$B_US := \frac{6.8684730470799167 \cdot 10^{-4}}{\text{yr}}$$

$$E_GB(t) := A_GB \cdot e^{B_GB \cdot t \cdot \text{yr}}$$

$$E_US(t) := A_US \cdot e^{B_US \cdot t \cdot \text{yr}}$$

ENTROPY in the PAST for US & GB

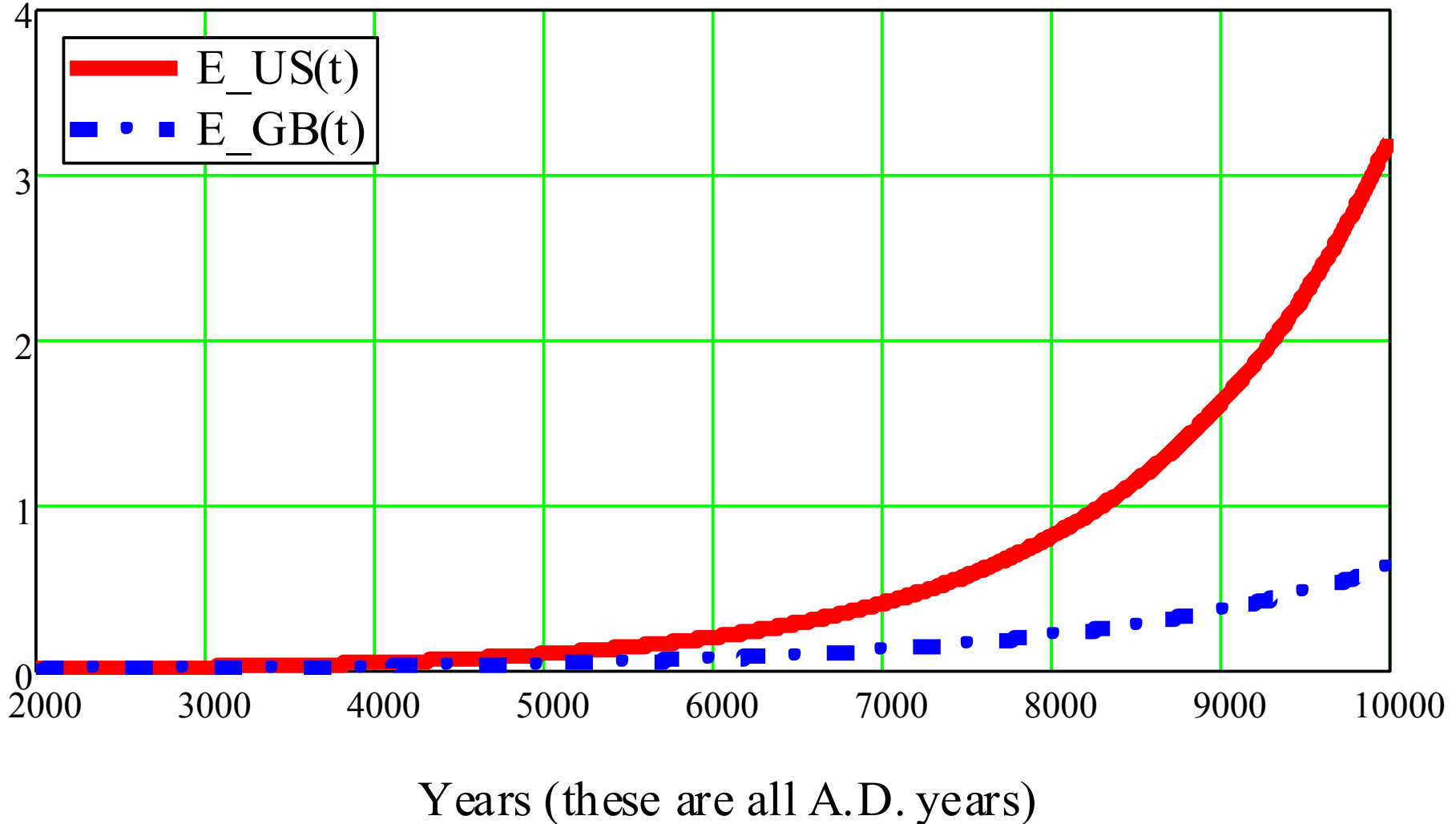


$$\Delta H_{\text{in_bits_GB}}(p) := -\frac{B_{\text{GB}}}{\ln(2)} \cdot (p + 434) \cdot \text{yr}$$

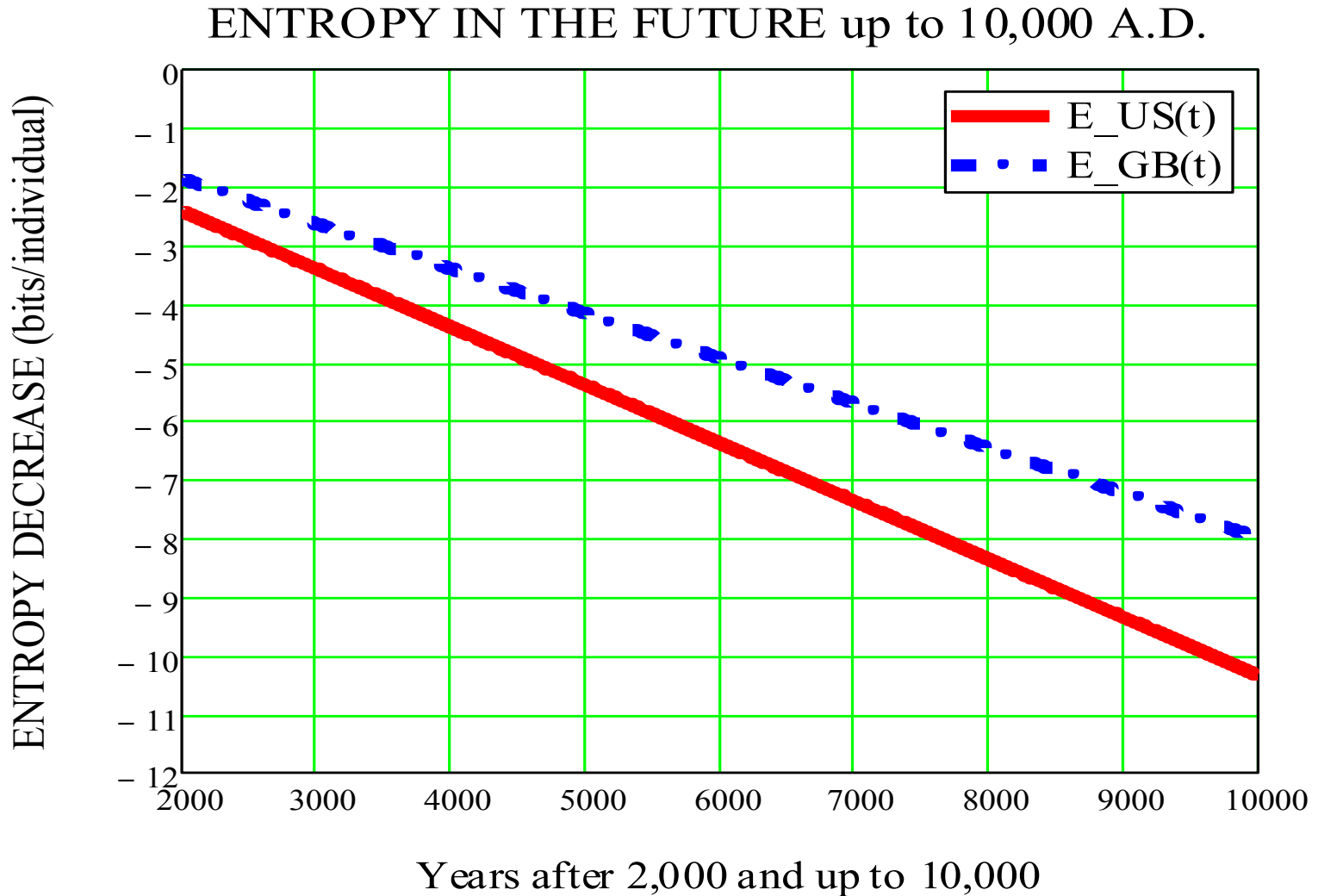
$$\Delta H_{\text{in_bits_US}}(p) := -\frac{B_{\text{US}}}{\ln(2)} \cdot (p + 434) \cdot \text{yr}$$

Exps EXTRAPOLATED to 10,000 AD

EXPONENTIALS EXTRAPOLATED up to 10,000 A.D.



FUTURE ENTROPY to 10,000 AD



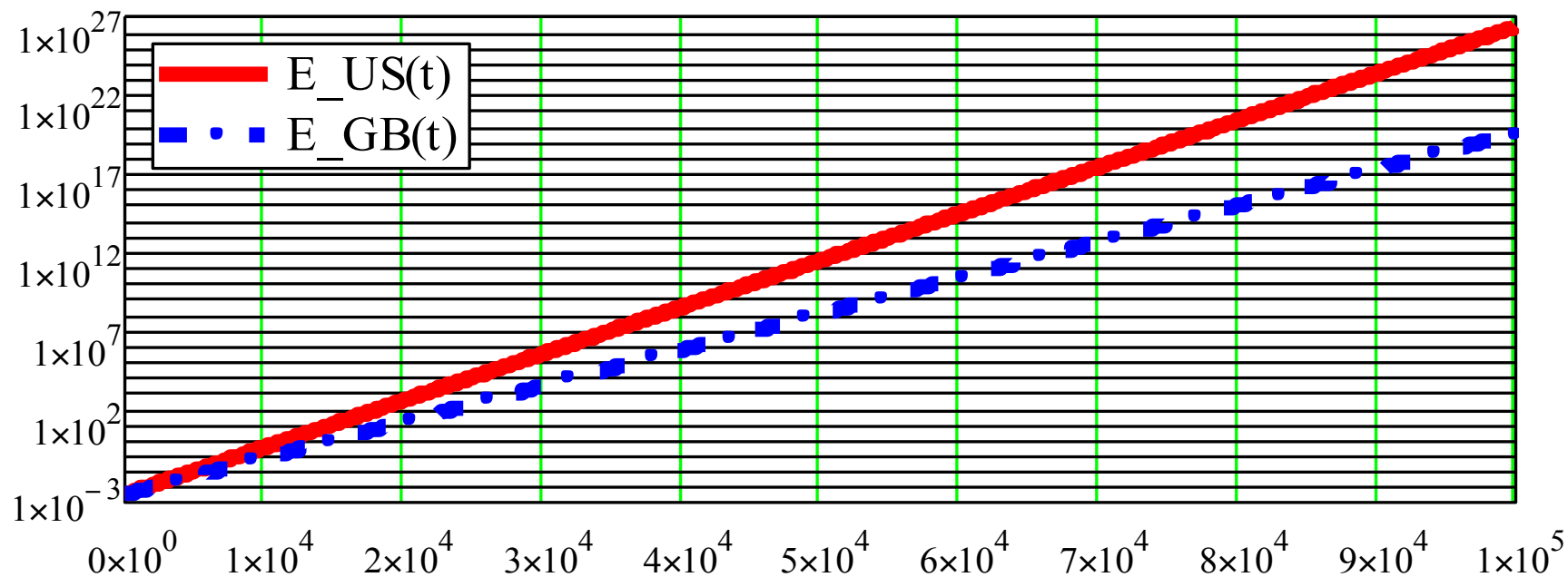
$\Delta H_{in_bits_GB}(10000) = -7.992$

$\Delta H_{in_bits_US}(10000) = -10.339$

Exps EXTRAPOLATED to 100,000 AD

$8.447 \cdot 10^{-3}$

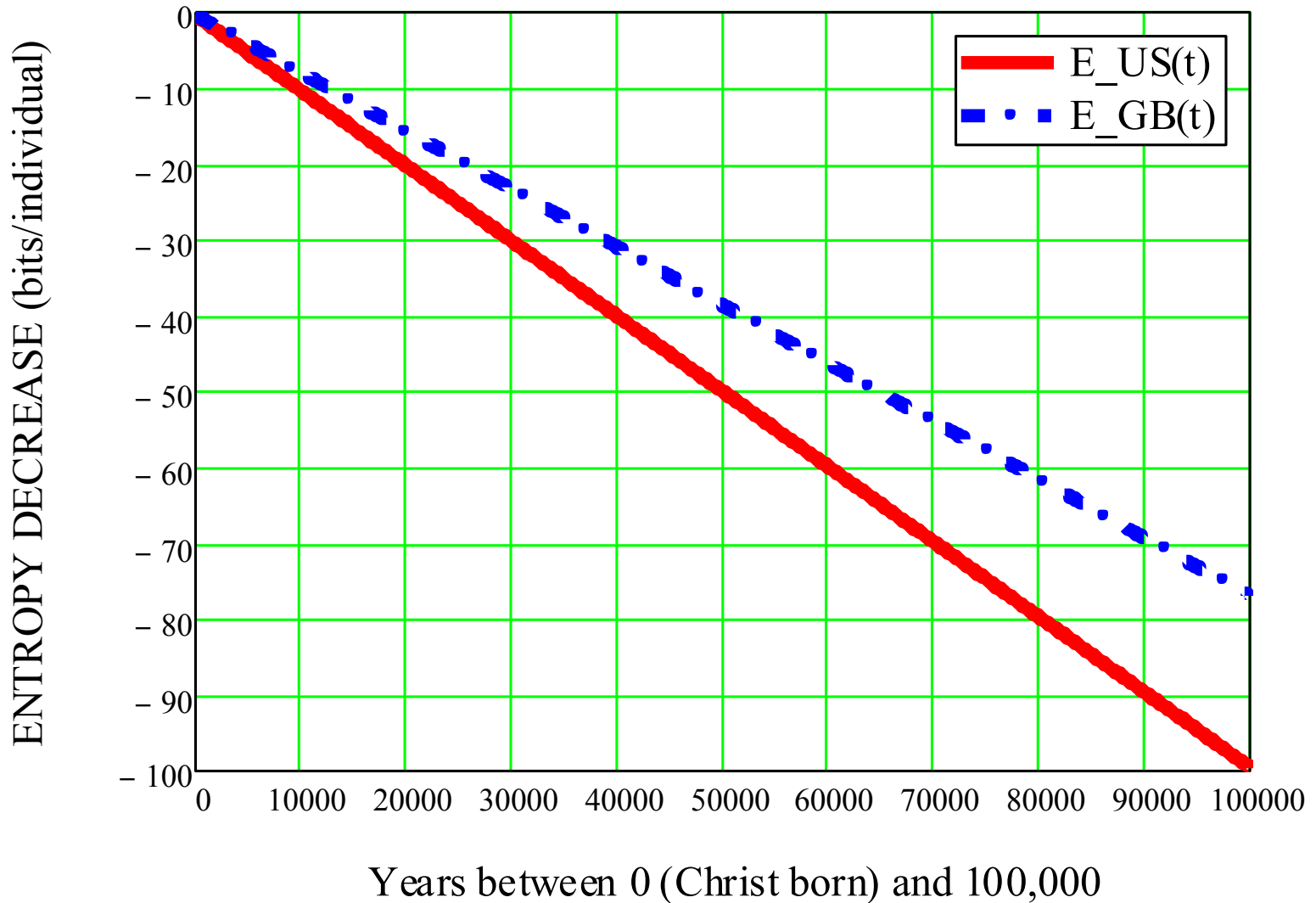
EXPONENTIALS EXTRAPOLATED up to 100,000 A.D.



Years between 0 (Christ born) and 100,000

FUTURE ENTROPY to 100,000 AD

ENTROPY EXTRAPOLATED up to 100,000 A.D.

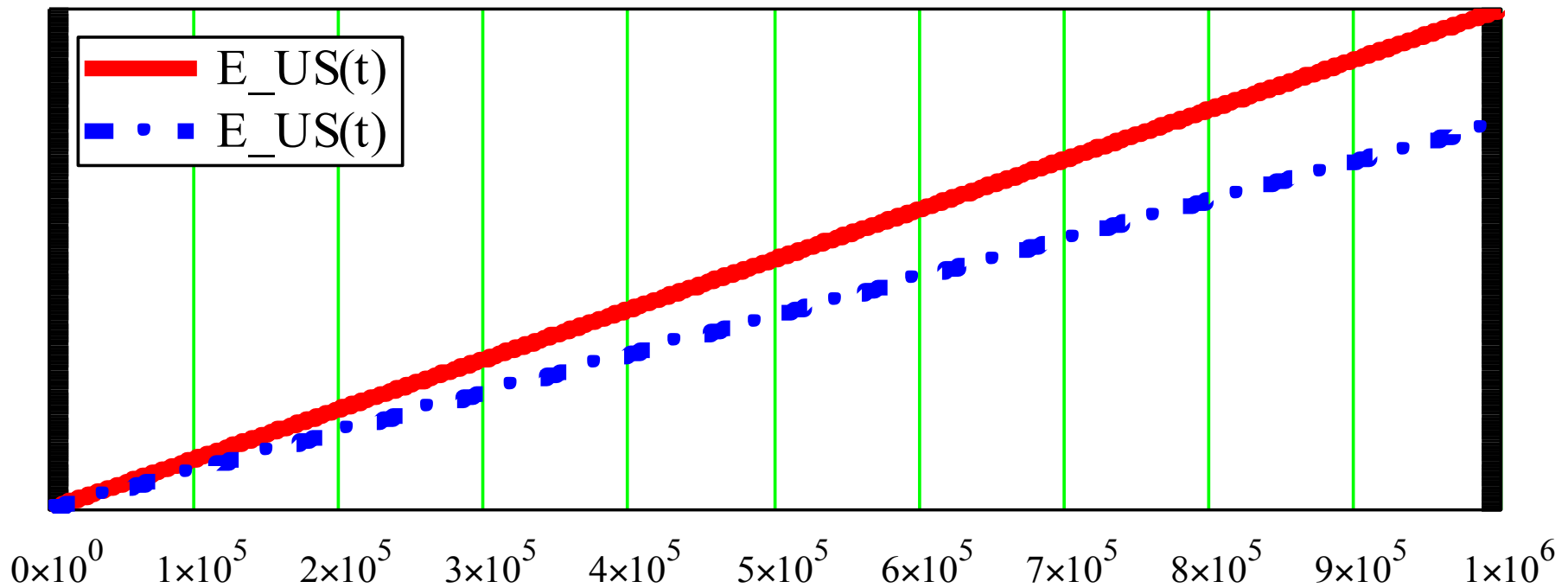


Exps EXTRAPOL. to 1 MILLION AD

$$E_{\text{GB}}(10^6) = 1.168 \times 10^{228}$$

$$E_{\text{US}}(10^6) = 6.598 \times 10^{295}$$

EXPONENTIALS EXTRAPOLATED up to 1 million A.D.



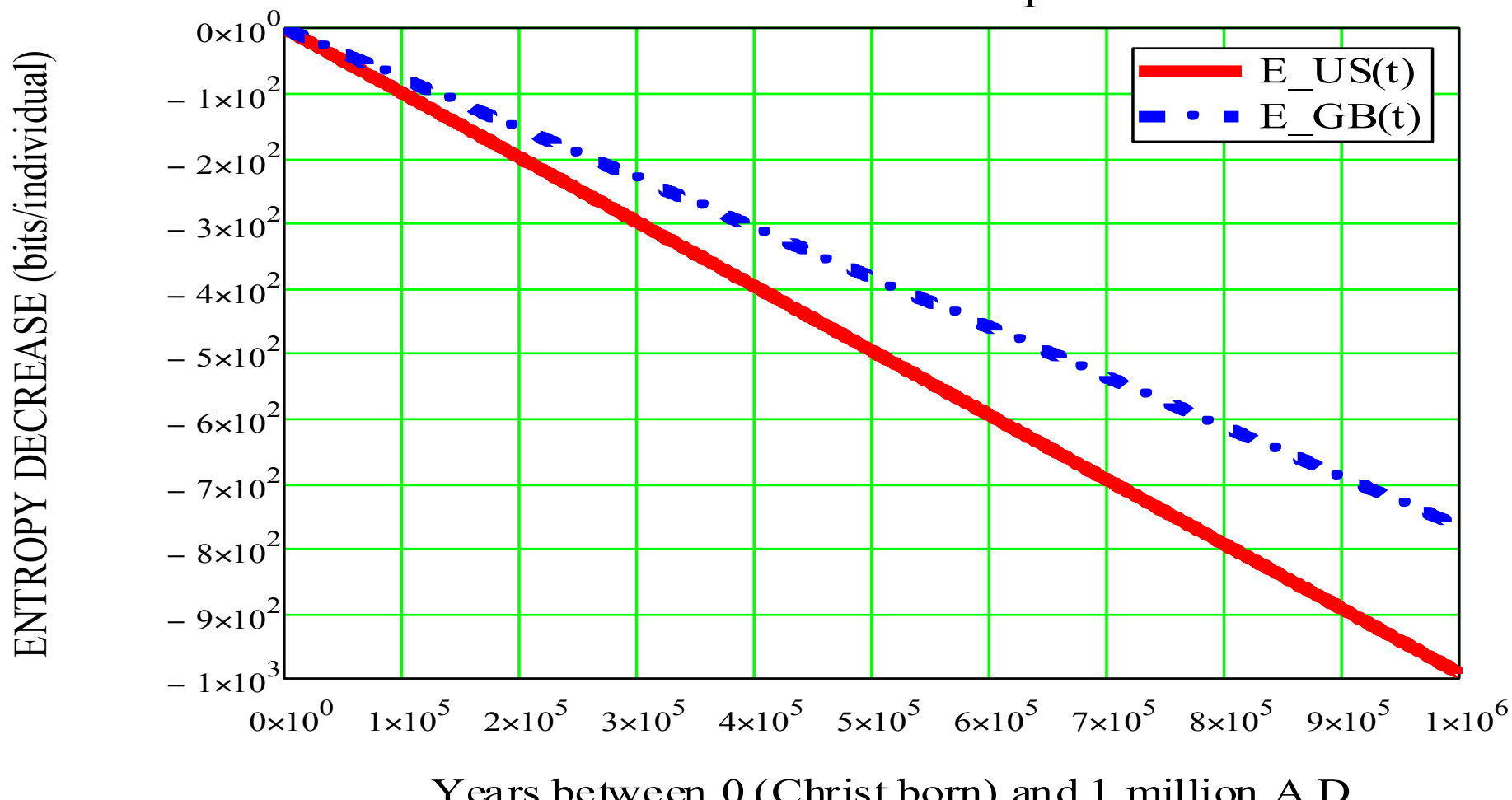
Years between 0 (Christ born) and 1 million A.D.

FUTURE ENTROPY to 1 MILLION AD

$$\Delta H_{\text{in_bits_GB}} \left(10^6 \right) = -766.274$$

$$\Delta H_{\text{in_bits_US}} \left(10^6 \right) = -991.341$$

ENTROPY EXTRAPOLATED up to 1 million A.D.

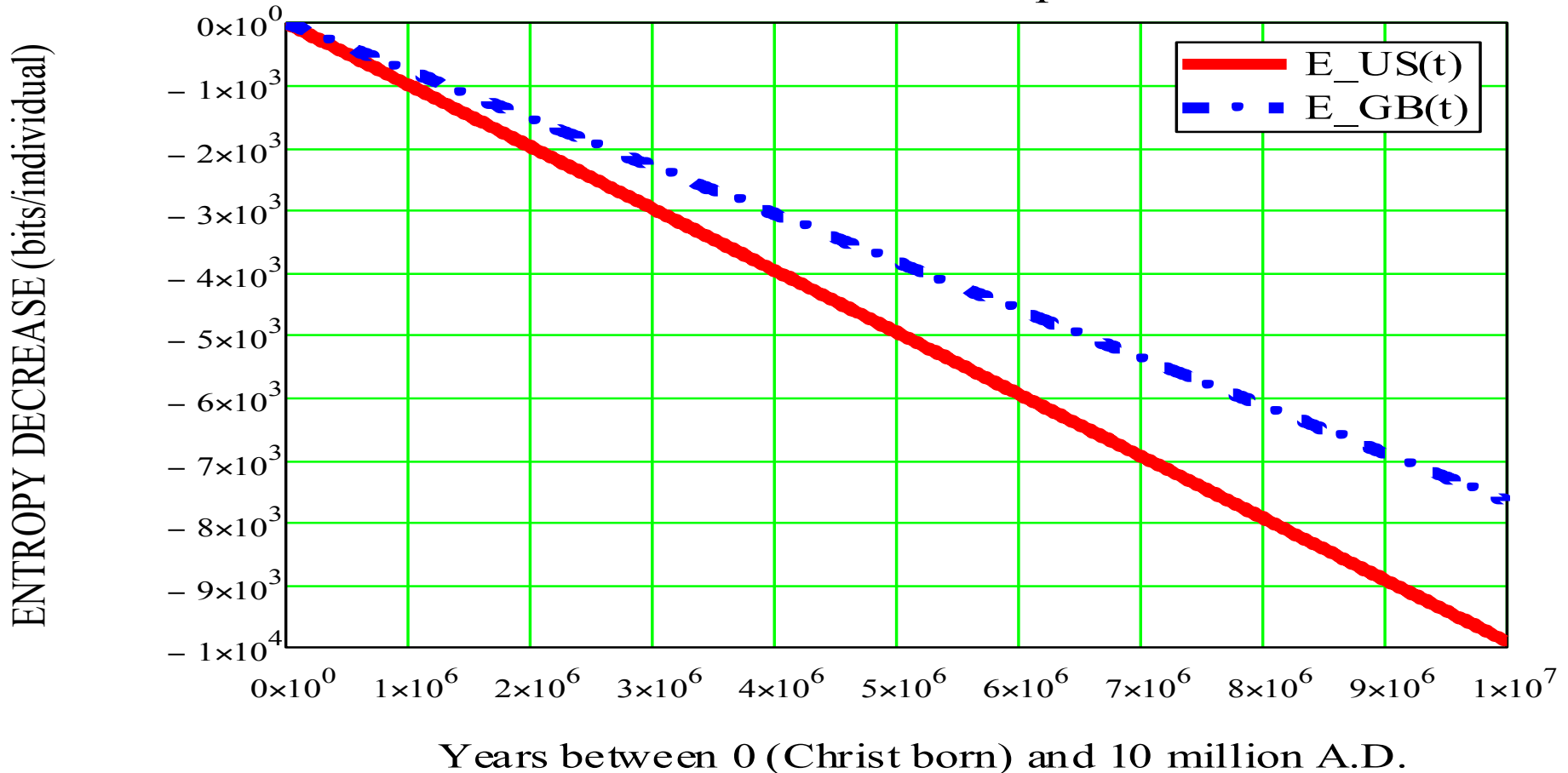


FUTURE ENTROPY to 10 MILLION AD

$$\Delta H_{\text{in_bits_GB}} (10^7) = -7.66 \times 10^3$$

$$\Delta H_{\text{in_bits_US}} (10^7) = -9.91 \times 10^3$$

ENTROPY EXTRAPOLATED up to 10 million A.D.



FERMI PARADOX (22 M-years): An estimate of how many bits/ individual of EVOLUTION are needed to settle the Galaxy.

Humans would be able to colonize the whole Galaxy only if they could improve themselves by some 10,000 bits/individual. This is about 400 times the 25 bits/individual improvement that Nature took on Earth to evolve over 3.5 billion years. Also, no other Alien Civilization would have to interfere, which is highly unlikely! Thus, our mathematical theory is crucial to estimate how much Aliens will be more advanced than Humans, when SETI succeeds.

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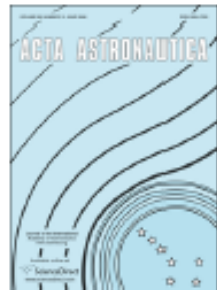
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Part 9:

BIG HISTORY :

A GBM

**IN THE NUMBER
OF CIVILIZATIONS
IN THE WHOLE
UNIVERSE**

BIG HISTORY is a GBM in the increasing number of CIVILIZATIONS

- ▶ Life in the WHOLE UNIVERSE (and NOT just in the Milky Way) evolved since, say, 10 billion years ago.

- ▶ $t_{START} = -10 \times 10^9$ years. $\langle N_{INCREASING}(t_{START}) \rangle = 1$

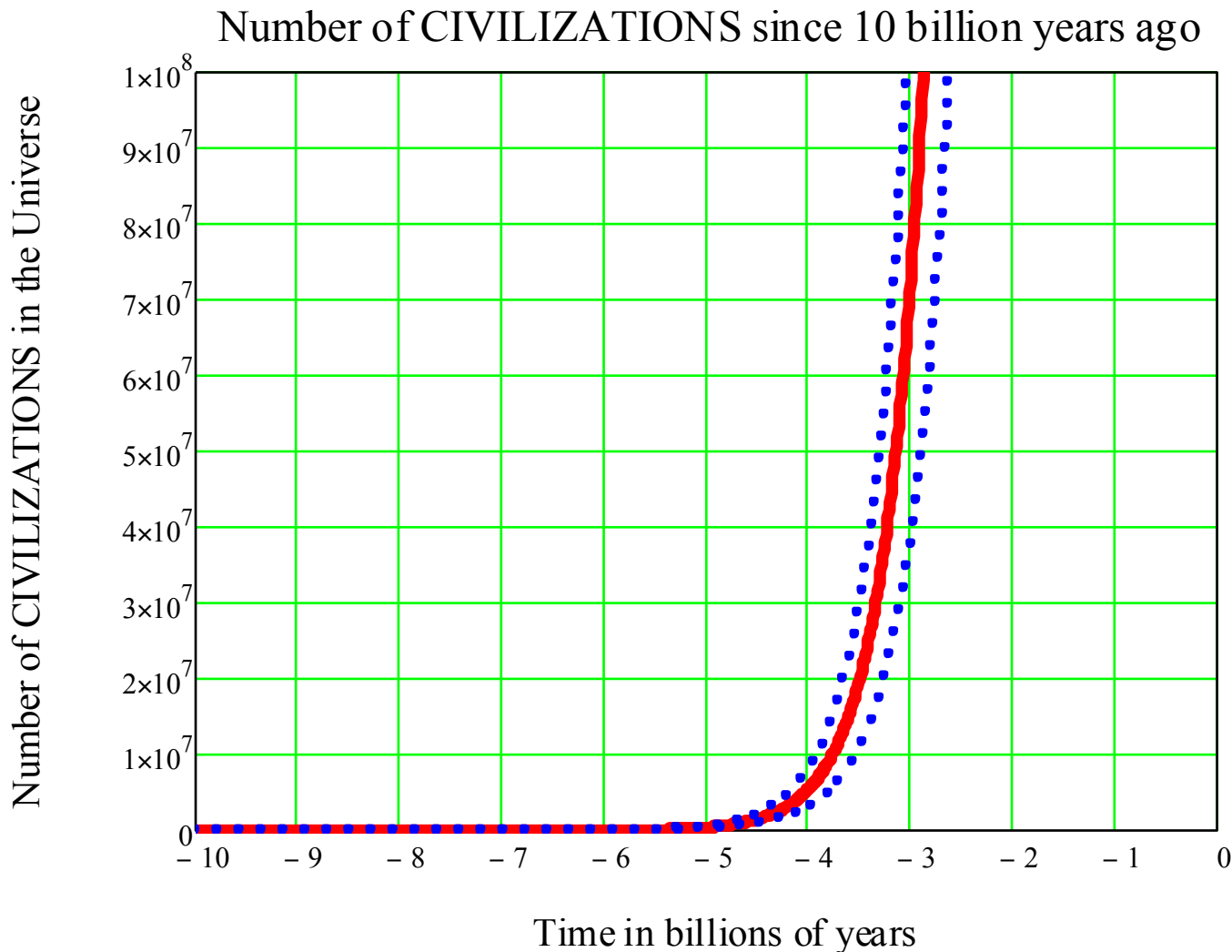
- ▶ Mean value $\langle N_{INCREASING}(t) \rangle = e^{\mu(t-t_{START})}$.

- ▶ GBM two parameters: μ and σ

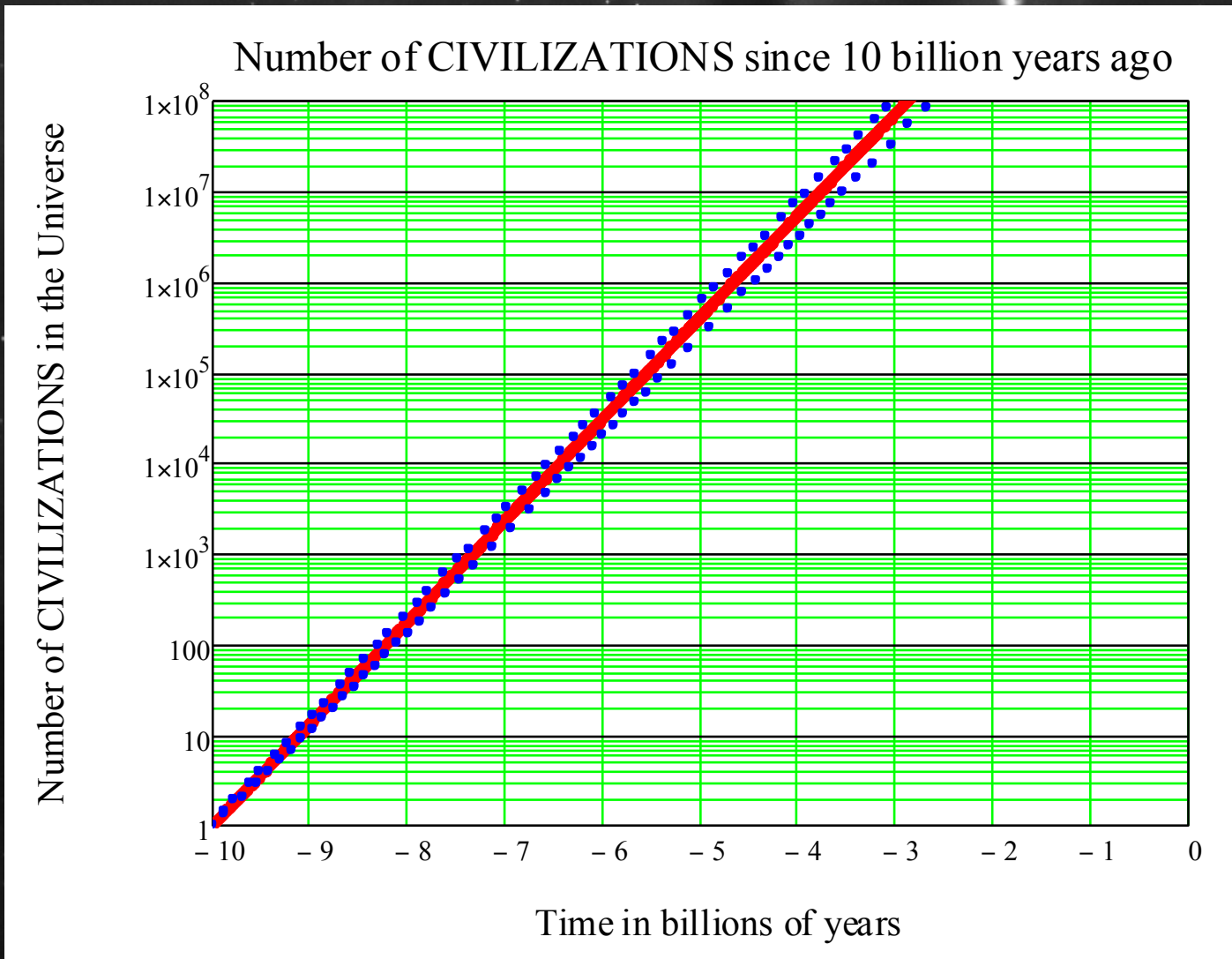
$$\mu = \frac{\ln(N_0)}{-t_{START}}$$

$$\sigma = \sqrt{\frac{\ln\left(1 + \frac{\delta N_0^2}{N_0^2}\right)}{-t_{START}}}$$

BIG HISTORY: is a GBM in the increasing number of Civilizations



BIG HISTORY: a GBM in the increasing number of Civilizations



Part 10:

MASS EXTINCTIONS :

GBMs

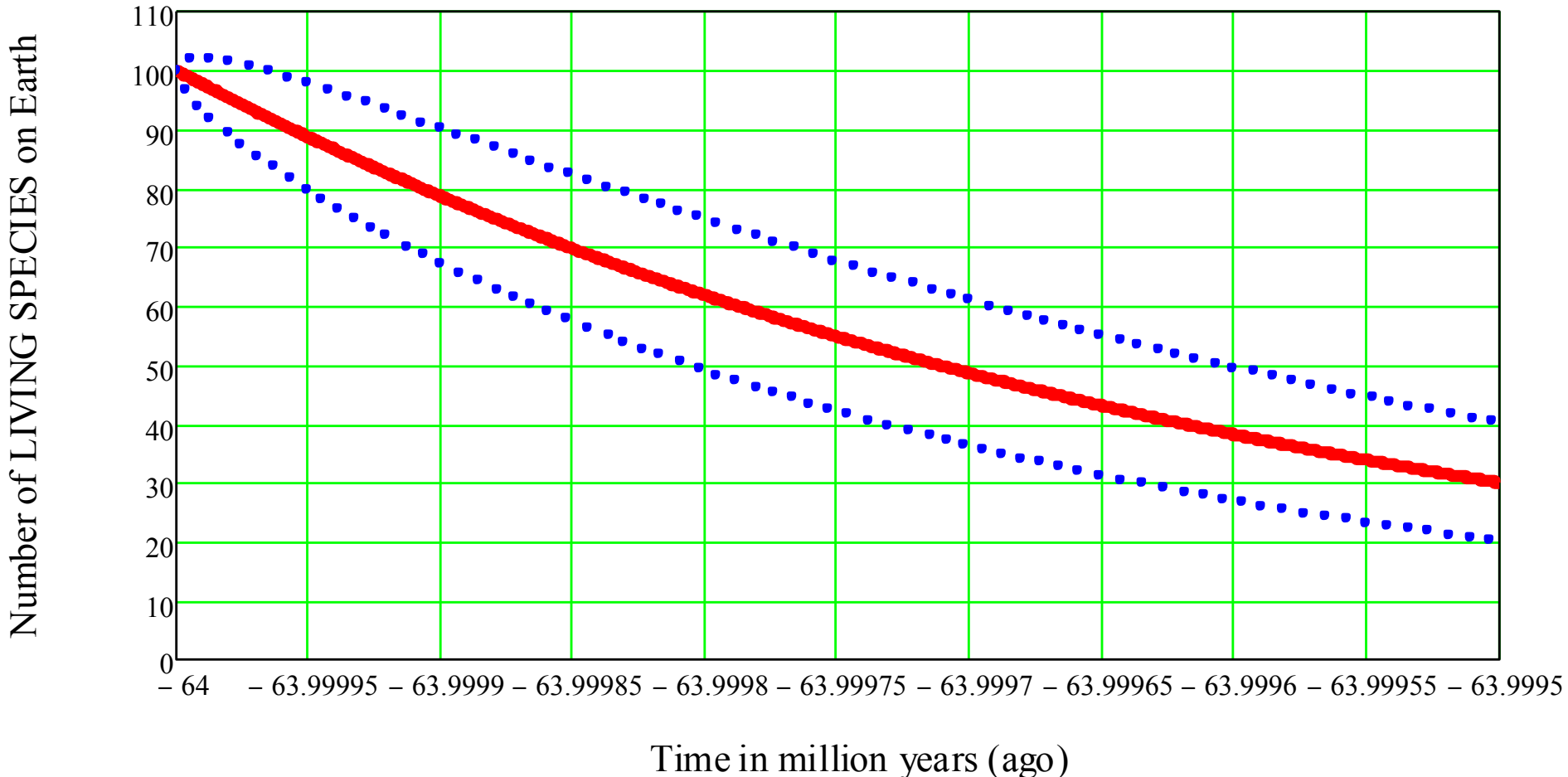
IN THE DECREASING

NUMBER OF

LIVING SPECIES

MASS EXTINCTION: a GBM in the decreasing number of living species

DECREASING number of species during the K-Pg MASS EXTINCTION



MASS EXXTINCTION: a GBM in the decreasing number of SPECIES

- ▶ The K-Pg IMPACT was 64 billion years ago. Suppose that its NUCLEAR WINTER lasted 1000 years:

$$t_{\text{Impact}} = -64 \times 10^6 \text{ years} .$$

$$t_{\text{End}} = -63.999 \times 10^6 \text{ year} .$$

$$t_{\text{end}} - t_{\text{Impact}} = 1000 \text{ year}$$

- ▶ In addition to the above two inputs, we must assign the following three more inputs:

$$N_{\text{Impact}} = 100 .$$

$$N_{\text{End}} = 30 .$$

$$\delta N_{\text{End}} = 10 .$$

MASS EXXTINCTION: a GBM in the decreasing number of SPECIES

- ▶ Then, the GBM mean value is given by:

$$\text{mean_value}(t) = N_{\text{Impact}} e^{\mu(t-t_{\text{Impact}})}$$

- ▶ While the GBM lognormal's μ and σ are given by:

$$\mu = - \frac{\ln \left(\frac{N_{\text{Impact}}}{N_{\text{End}}} \right)}{t_{\text{End}} - t_{\text{Impact}}}.$$

$$\sigma = \sqrt{\frac{\ln \left[1 + \left(\frac{\delta N_{\text{End}}}{N_{\text{End}}} \right)^2 \right]}{t_{\text{End}} - t_{\text{Impact}}}}.$$

MASS EXXTINCTION: a GBM in the decreasing number of SPECIES

- ▶ Finally the GBM upper and lower STANDARD DEVIATION CURVES are given by, respectively:

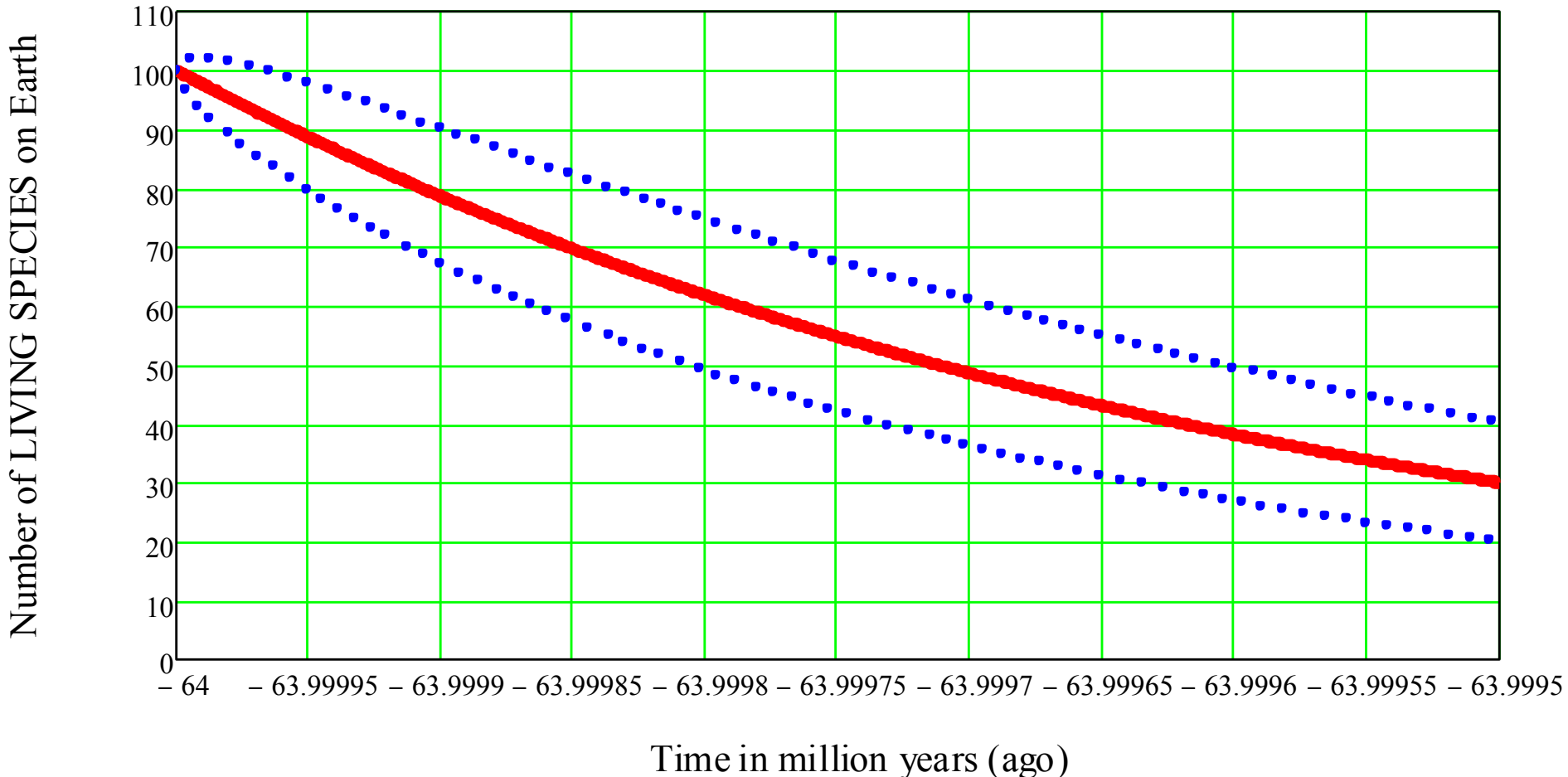
$$\text{st_dev_curves}(t) = N_{\text{Impact}} e^{\mu(t-t_{\text{Impact}})} \left[1 \pm \sqrt{e^{\sigma^2(t-t_{\text{Impact}})} - 1} \right]$$

- ▶ While the upper standard deviation curve has its maximum at the time:

$$t_{\text{Impact}} + \frac{1}{\sigma^2} \cdot \ln \left[\frac{2\mu \left(\sqrt{\mu^2 - 2\mu\sigma^2 - \sigma^4} \right) + \sigma^2 + 3\mu}{(\sigma^2 + 2\mu)^2} \right] \cdot$$

MASS EXTINCTION: a GBM in the decreasing number of living species

DECREASING number of species during the K-Pg MASS EXTINCTION



CONCLUSIONS

- 1) We developed here a new mathematical model embracing all of Big History, including Darwinian Evolution (RNA to Humans), Human History (Aztecs to USA) and then we extrapolated even that into the future up to 10 million years, the minimum time requested for a civilization to expand to the whole Milky Way (Fermi paradox).
- 2) Our mathematical model is based on the properties of lognormal probability distributions. It also is fully compatible with the Statistical Drake Equations, i.e. the foundational equation of SETI, the Search for Extra-Terrestrial Intelligence.
- 3) Merging all these apparently different topics into the larger but single topic called Big History is the achievement of this paper. As such, our statistical theory would be crucial to estimate how much more advanced than Humans the Aliens would be when SETI scientists will succeed in finding the first ET Civilization.



Thank you very much !