UNIVERSITY OF AMSTERDAM Master Astronomy and Astrophysics - 5214RAAS6Y

# Radio Astronomy

## Lecture 6

## The Techniques of Radio Interferometry I: Basics

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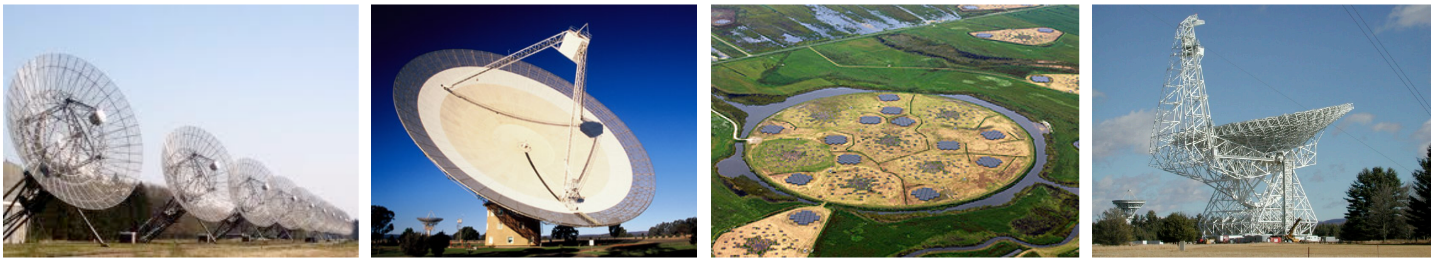
A1.20 - April 20th, 2015



- Welcome to Lecture 6 of Radio Astronomy, in which we'll be discussing the basics of radio interferometry.
- As you will see, aperture synthesis radio interferometry is a fascinating, powerful and extremely clever technique.
- In fact, the development of aperture synthesis led to a Nobel Prize for Martin Ryle (shared with Anthony Hewish for his discovery of pulsars).
- The technique is complex and multi-faceted. This is only the first in a series of 3 lectures in which we'll investigate the technique.
- We'll also investigate the technique as part of the practica: first a practicum (starting today!) in which you'll simulate your own radio interferometer and then one on calibration and imaging of real interferometric data.

# Lecture outline

- Quest for resolution (motivation for interferometry)
- Terminology
- Basic radio interferometer and correlator
- Visibilities and the uv-plane (core concepts we will investigate in the first practicum)
- Basics of making an image (to be explored more in the following two lectures)
- Other considerations

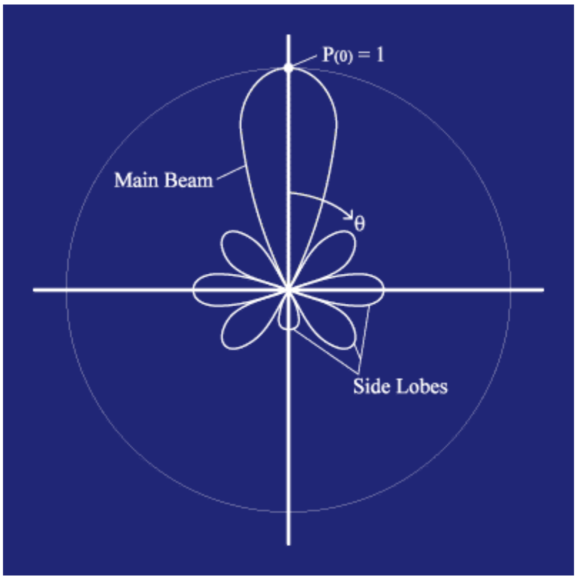
- This first lecture will cover the terminology and some of the fundamentals.
- The next two lectures will cover calibration and imaging, respectively.
- After we have covered imaging with radio interferometers, we will also have a lecture on how we can use interferometers not for imaging but for high-time-resolution "beam-formed" observations.

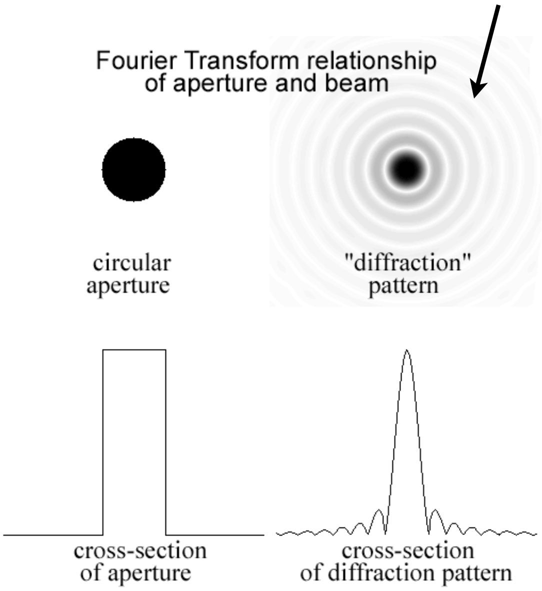
# Quest for resolution

- Why not just use single-dish radio telescopes? Why do we go through the much more complicated trouble of linking them together?
- The answer is we need widely separated dishes in order to achieve scientifically interesting angular resolution (e.g. arcsecond or better in order to match with optical observations).
- Also, linking multiple telescopes together simply gives more collecting area, and hence sensitivity.
- So even if you just want to use the telescope as a giant "light bucket", linking multiple telescopes is a good way to go (especially since we've reached the engineering limits of how big one can make a single dish!).

# Radio telescope FoV

Bessel Function





Sinc Function

- Remember that we'll use the terms "beam" and "field-of-view" (FoV) basically interchangeably.
- The beam/FoV of a radio telescope is a diffraction pattern.
- There is a "main beam", which we can loosely consider the FoV. This is the direction in which the telescope is most sensitive.
- However, there are also "side-lobes", i.e. other diffraction maxima that give sensitivity in other directions as well.
- **QUESTION: WHY DO WE CARE ABOUT THE SIDE-LOBES IF THEY GIVE LOWER SENSITIVITY?**
- To properly calibrate a radio telescope image, and to get an accurate radio image requires properly considering the effects of both the main and side-lobes (the telescope simply receives everything it is sensitivity to, and making a reliable image requires understanding the telescope's full sensitivity pattern on the sky).
- There is a Fourier transform relationship between the aperture and the beam. **DOES EVERYONE KNOW WHAT A FT IS? IF NOT, READ UP ON IT A BIT**
- A Fourier transform relates the time and frequency domains of a signal. It can also relate shapes and their "spatial frequencies".
- Above is an example for a single circular aperture, but this concept also holds for a collection of dishes/antennas distributed on the ground.
- This is a concept of fundamental importance for the whole course!

# The quest for resolution

We want sub-arcsecond resolution (cf. optical, X-ray)

$$\Theta_{\mathrm{rad}} \propto \frac{\lambda}{D}$$

Unlike large, ground-based optical telescopes (atm. limits!), radio telescopes are always diffraction limited.

$$\Theta_{\mathrm{arcsec}} \sim 2\frac{\lambda_{\mathrm{cm}}}{D_{\mathrm{km}}}$$

So to get 1 arcsec resolution at 21cm requires a 42km diameter!

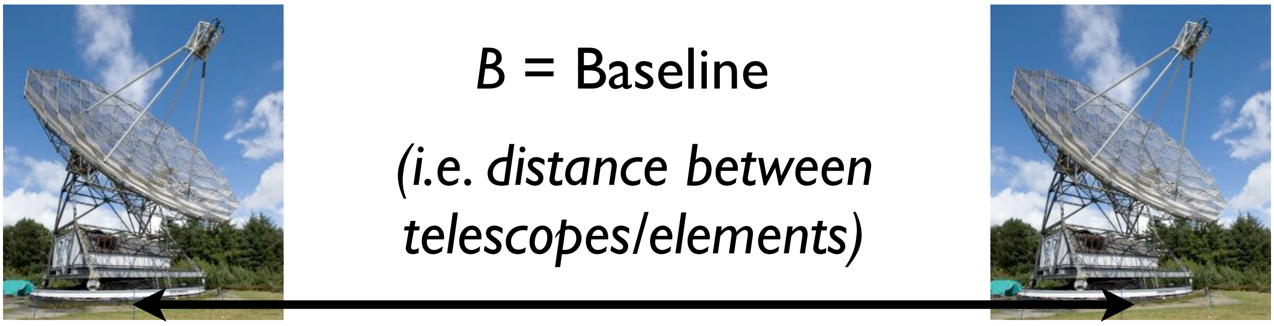
- Simply put, our achievable resolution is a function of the observing wavelength (lambda) and the size of the telescope.
- In the case of a single dish, the size of the telescope is basically the diameter of the dish.
- **QUESTION: HOW DO WE GET THE RESOLUTION WE WANT? WE CAN'T MAKE A 42-KM DISH!**
- In the case of an interferometer, the size of the telescope is the maximum "baseline" (i.e. distance) between the dishes/antennas.
- Note that to get 1 arsec resolution at a typical frequency of 1.4GHz (21cm) would require a telescope 42km across. We can't build such a dish, but we can coherently link the radio waves received by dishes separated by such a distance.

# The quest for resolution

Fortunately, we don’t have to build a single 42-km-wide radio dish

Aperture synthesis

$$\Theta \propto \frac{\lambda}{D} \propto \frac{\lambda}{B}$$



*B* = Baseline

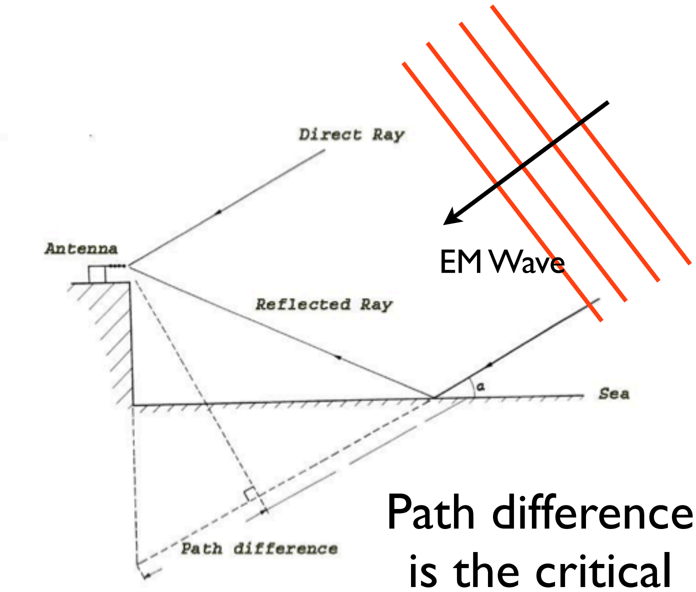
*(i.e. distance between telescopes/elements)*

- Again, sampling the astronomical radio wave at multiple geographically separated points allow use to “synthesize and aperture” (aperture synthesis technique) that is as large as you need.
- As we’ll see, this can mean linking radio dishes spread across the globe or even orbiting in space.

# “Sea” interferometry

(mid 1940s)

Recall from Lecture #1



Path difference is the critical ingredient

Dover Heights near Sydney, Australia

- Just to recall from something shown in the first lecture, one of the first demonstrations of this technique used the ocean to receive reflected radio waves.
- This creates an interference pattern at the antenna because there is a path length difference between the direct ray and reflected ray.
- The path difference and the observing wavelength quantify the angular resolution that can be achieved in this case.
- **QUESTION: WHY IS THE EM-WAVE A PARALLEL WAVE? ANSWER: DISTANCE TO SOURCE IS WAY LARGER THAN SEPARATION BETWEEN TELESCOPES**
- This is a cute experiment, but we don’t use such rudimentary techniques anymore...

# Modern interferometers

(see Lecture 1 for more details)

ATCA

JVLA

LOFAR

AMI

WSRT

GMRT

- Modern radio interferometers are much more sophisticated and provide angular resolution in two dimensions.
- **QUESTION: WHY IS 2-D RESOLUTION IMPORTANT? ANSWER: SO WE CAN ACCURATELY MAP RADIO SOURCES IN RA & DEC.**
- The multiple telescopes are linked via analog and digital signal paths that lead to a central "correlator", which is the machine (now in some cases a computer cluster) that "cross-correlates" the signals between antennas.
- The cross-correlation procedure is what allows us to compare the "amplitude" and "phase" of the radio signal at each antenna.
- This is the basic information we need to create a radio image.
- **QUESTION: WHY SO MANY RADIO INTERFEROMETERS? ARE THEY ALL DOING THE SAME THING?**
- The interferometers shown here have complementary scientific goals. They cover different ranges of the radio window (10MHz - 1000GHz) and they also have different spacings between antennas.
- The longest baselines give the highest angular resolution (i.e. for "compact emission") but the short spacing are necessary to image structure on broader angular scales (i.e. "diffuse emission").

# Modern interferometers

$$N_{\text{baselines}} = N_{\text{elements}}(N_{\text{elements}} - 1)/2$$

Avoid counting each telescope pair twice

JVLA

27 antennas
351 baselines

WSRT

14 antennas
91 baselines

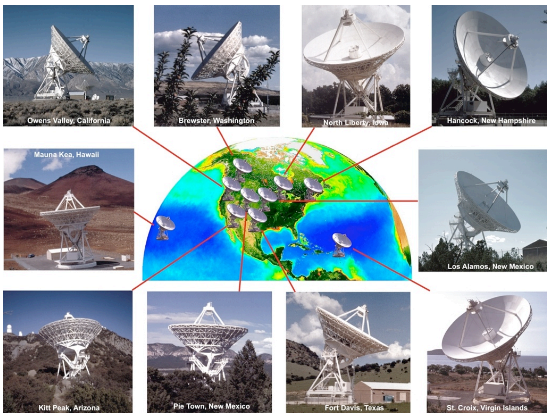
Want to compare signal **amplitude** and **phase** between these telescopes.

- The concept of a "baseline" is really important.
- Baselines have different "lengths" (distance between a pair of dishes/antennas) and they have different orientation with respect to the sky (RA,DEC).
- Note that the orientation of these baselines with respect to the sky changes during an observation (the Earth is rotating after all).
- This change in orientation complicates the picture but also provides a powerful way to accurately image the sky.
- Note that the number of baselines (telescope pairs) is related to the number of elements in the array - as shown by the equation above.

# Very Long Baseline Interferometry

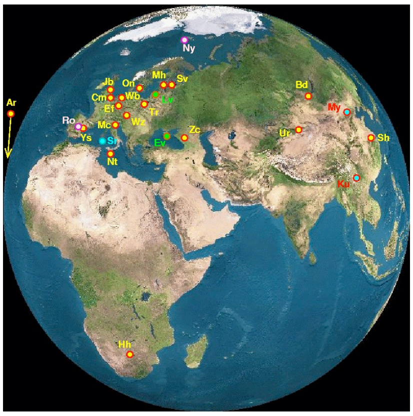
## 1000-km baselines

Data traditionally recorded locally and shipped to correlator (though moving more and more towards real-time)



VLBA, USA

EVN, Europe

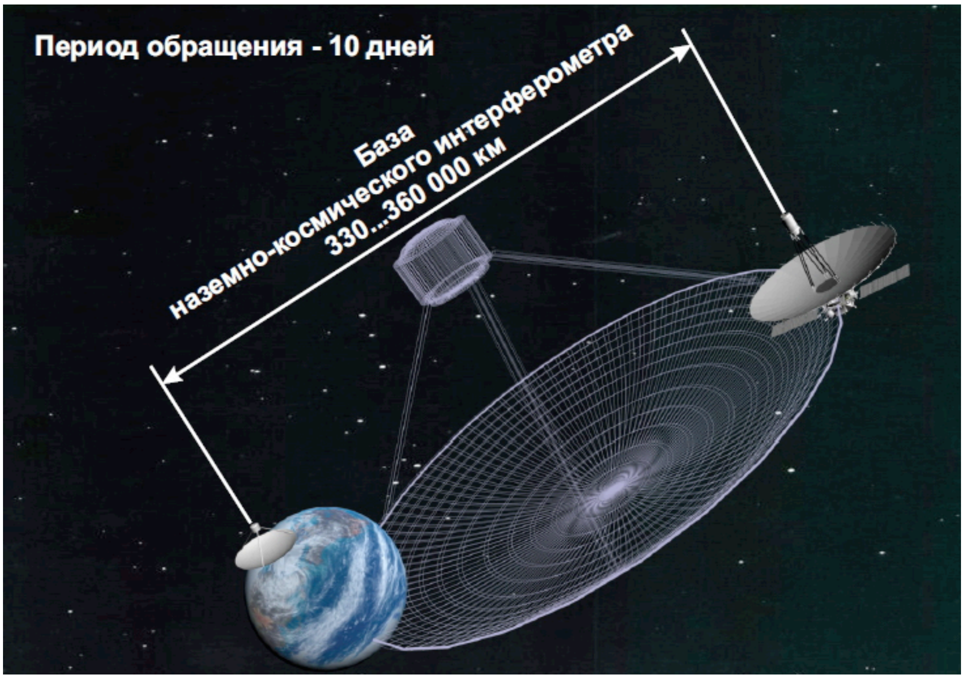


Global VLBI

- In the case of very-long-baseline interferometry (baselines of hundreds or thousands of kilometers) the data for each telescope has traditionally been recorded separately on site.
- What is recorded? The "raw voltage" (i.e. "baseband") signal from the telescope. This needs to be Nyquist-sampled data that allows us to measure both the amplitude and phase of the signal (the later is crucial for comparing signal phases between telescopes).
- The data from each telescope is then shipped to a central correlator where it is combined.
- This is what JIVE (Joint Institute for VLBI in Europe) does. There is a large correlator in the basement of the ASTRON/JIVE building.
- More and more the data transport is being done by fiber-optic cable, which now routinely can provide > 10GB/s capacity (the data capacity of these lines is what limits the total observing bandwidth that can be shipped).

# Very Long Baseline Interferometry

## 100,000-km baselines



RadioAstron

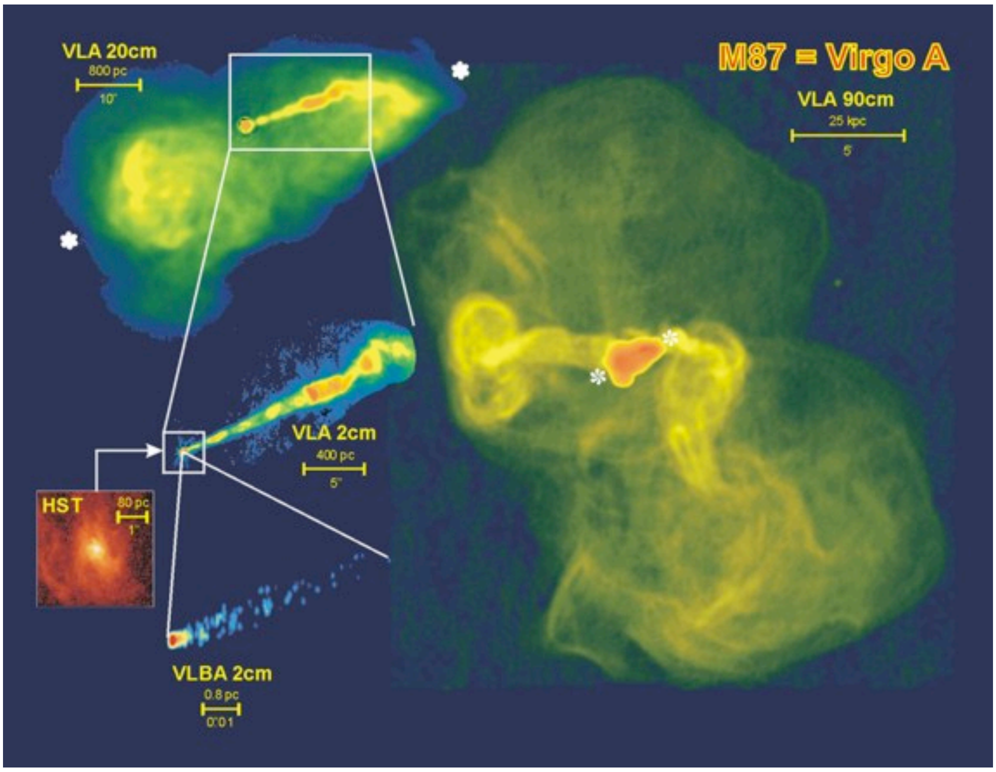
• The RadioAstron satellite (built and operated in Russia) takes VLBI even further: i.e. to baselines that are many Earth diameters.

• Crucial to making this work is knowing exactly where the RadioAstron satellite is positioned with respect to the telescopes on Earth.

• **QUESTION: WHY IS IT CRITICAL TO KNOWN THE PRECISE POSITION OF THE SATELLITE? HOW PRECISE IS NECESSARY?**

• If one wants to compare the phases of the radio signal at RadioAstron and the Earth-based telescopes, then ultimately we need to know their relative positions to a fraction of a wavelength.

# The quest for resolution



Probe milli-arcsecond scales

- It's important to appreciate that long-baseline interferometry allows resolutions that are beyond the reach of the resolutions achieved in other observing wavebands (optical, X-ray, etc.).
- You've seen this image of the M87 (i.e. Virgo A) radio galaxy in previous lectures.
- It shows how interferometry on a variety of different scales probes different parts of the ejecta from the central super-massive black hole.
- At the highest observing frequencies and longest baselines (using the Very Long Baseline Array [VLBA] at 2cm) it is possibly to probe sub-parsec length scales even though this source is at a distance of several mega-parsecs!!!
- **QUESTION: WHY DON'T WE ALWAYS JUST USE VLBI AND GET THE BEST POSSIBLE RESOLUTION? ANSWER: THE RESOLUTION WE USE SHOULD BE WELL MATCHED TO THE SIZE OF THE FEATURES WE WANT TO STUDY (IT'S POSSIBLE TO OVER OR UNDER RESOLVE A SOURCE)**

# Terminology

- Let’s now discuss a bit more the terminology that we will need.

# Terminology

## “Specific Intensity” or “Brightness”

Energy per unit time, area, frequency, and solid angle

$$[I(\vec{s},\nu,t)] = \mathrm{erg\ s^{-1}\ cm^{-2}\ Hz^{-1}\ ster.^{-1}}$$

## “Specific Flux Density”

Integrate over all angles

$$S = \int I(\vec{s},\nu,t)d\Omega$$

$$[S(\nu,t)] = \mathrm{erg\ s^{-1}\ cm^{-2}\ Hz^{-1}}$$

Another reason why radio telescopes are so big (not just resolution).

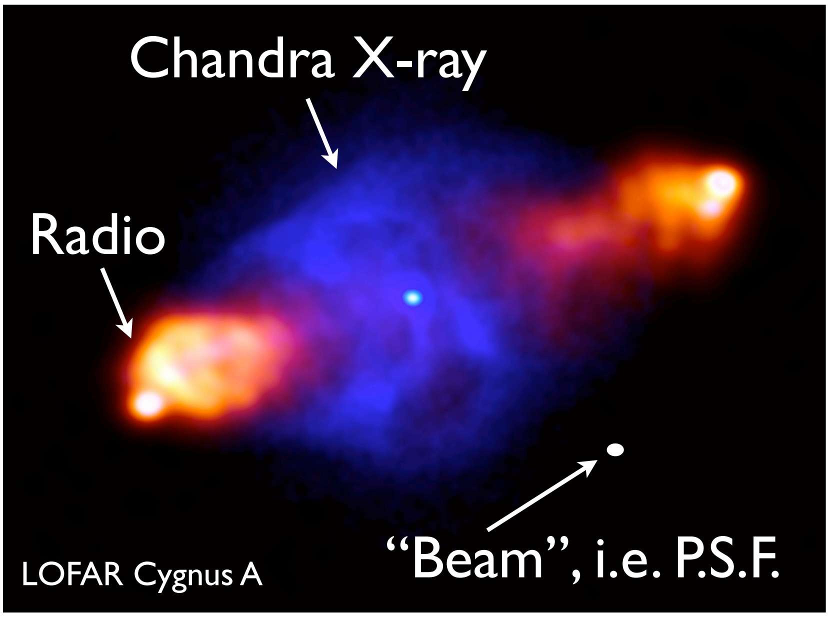
Measured in Janskys $1\mathrm{Jy} = 10^{-23}\mathrm{erg\ s^{-1}\ cm^{-2}\ Hz^{-1}}$

- First: “specific intensity” or “brightness”.
- These quantify the amount of energy received per unit time, area, frequency an solid angle.
- If we integrate over all observed angles, then this leads to the “specific flux density”, which in radio astronomy we measure in units of Janskys (after Karl Jansky, whose early experiments for Bell Labs made the first detections of cosmic radio emission [see Lecture 1]).
- Note that the Jansky corresponds to a tiny amount of energy. Most astronomical radio sources have flux densities of only milli- or even micro-Janskys! That’s another reason why radio telescopes are big.

# Terminology

## Brightness in "Jy/beam"

## (i.e. flux density per resolution element)

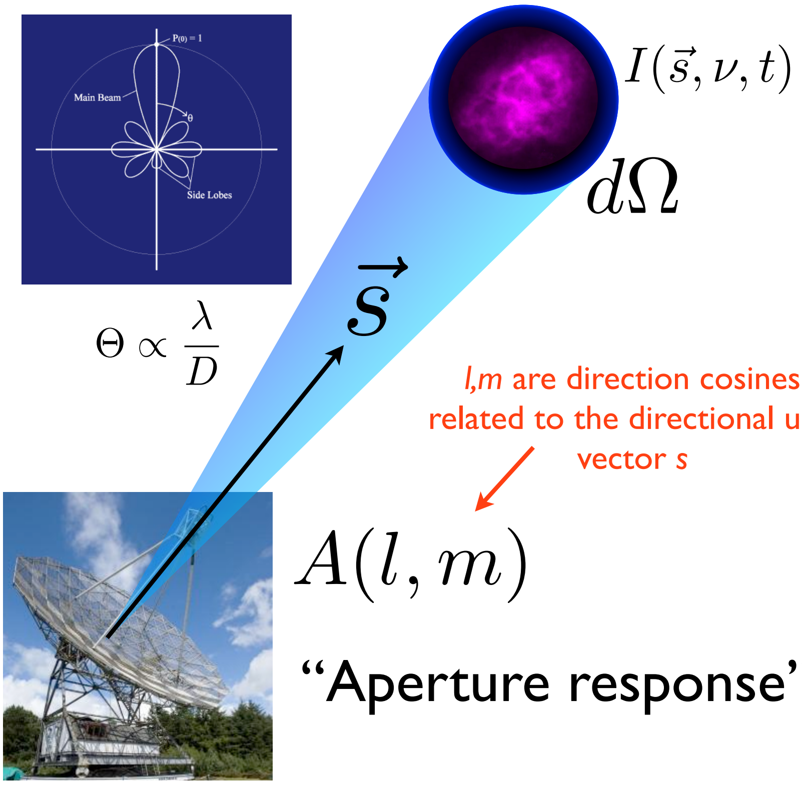


- 1 beam ~ 100 sq. arcseconds.
- Peak is ~10Jy/beam
- Total Source flux density ~1000 Jy

- There is also the concept of brightness in "Jy/beam", i.e. Jy per resolution element.
- The beam that is synthesized for an array has a particular "point spread function" (response to a perfect point source).
- The brightness of an extended source won't necessarily be uniform, so we can talk about "peak brightness" etc.

# Terminology

## “Primary beam” (main beam of individual elements)



$I(\vec{s}, \nu, t)$

$d\Omega$

$\vec{s}$

$\Theta \propto \frac{\lambda}{D}$

*l*,*m* are direction cosines related to the directional unit vector *s*

$A(l, m)$

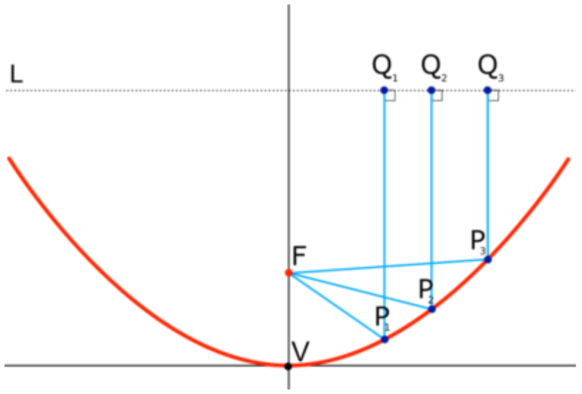
“Aperture response”

- This determines the total FoV, whereas the synthesized beam determines the resolution within that FoV.
- The aperture has a response as a function of direction and frequency.

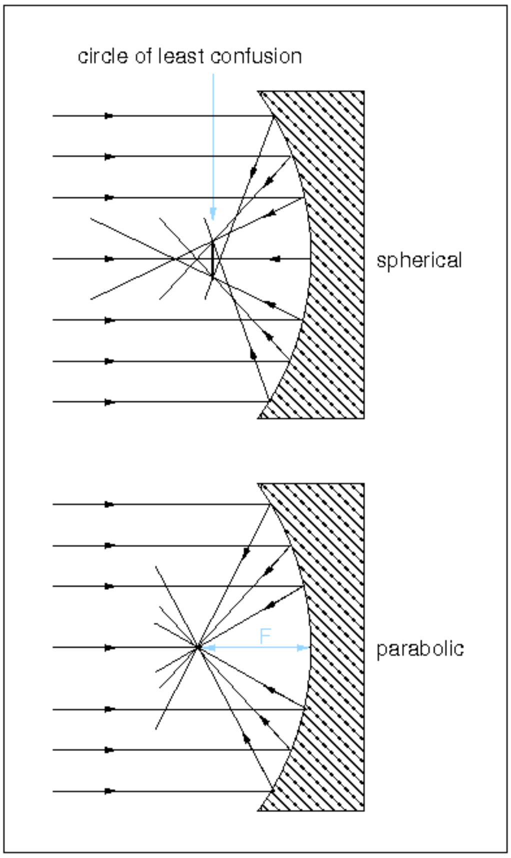
- For an interferometer, the “primary beam” is the main beam of the individual array elements.
- This determines the total FoV, whereas the synthesized beam determines the resolution within that FoV.
- The aperture has a response as a function of direction and frequency.

# Terminology

“Parabolic reflector”

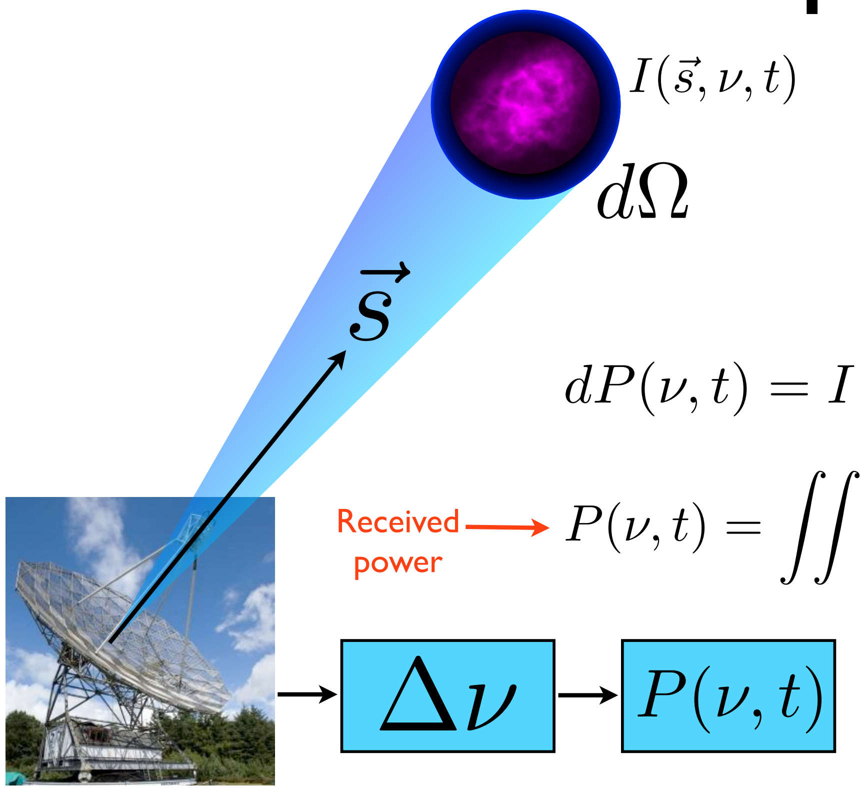


- Electric field is in phase at the focus because all rays from a parallel wavefront travel the same distance.
- NB: a spherical reflector (like Arecibo) will focus to a line.



• A parabolic reflector will focus the radio waves to a single point (where the receiver is placed).
• In the case of a spherical reflector (e.g. Arecibo) or something non-axisymmetric (e.g. GBT) secondary and even tertiary optics will be necessary before the receiver.
• Also note that a focal-plane-array (like the APERTIF system being installed on Westerbork) allows one to sample more of the focal plane instead of just a single focal point along the “bore axis” of the telescope. This increases the field-of-view substantially.

# From emitted brightness to received power



$$dP(\nu, t) = I(\vec{s}, \nu)\ A(\vec{s}, \nu)\ d\nu d\Omega$$

Received power →

$$P(\nu, t) = \iint I(\vec{s}, \nu)\ A(\vec{s}, \nu)\ d\nu d\Omega$$

Emitted brightness

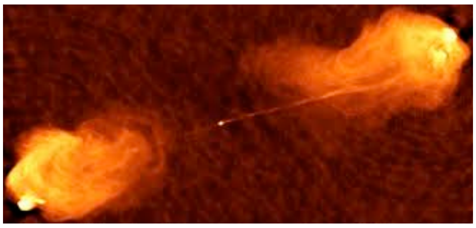
Aperture response

- The received power (energy per unit time) is simply a 2D integral - over frequency and solid angle - of the source brightness times the aperture response.

# Basic radio interferometer and correlator

- Now that we have the necessary terminology, let's consider radio interferometers and correlators in more detail.

# Radio interferometric imaging



$I(\vec{s}, \nu, t)$

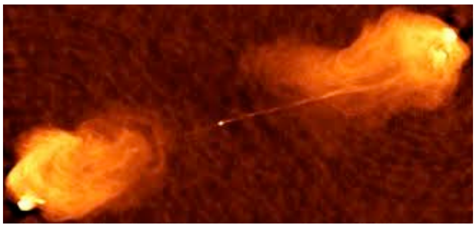
$P_0(\nu, t)$ $P_2(\nu, t)$ $P_1(\nu, t)$

Key questions for this lecture...

- How do we relate the brightness of the radio sky to the power received by the antennas?
- How do we turn this into a radio image?

• Very important to understand is how we can relate the quantities we measure at the telescope to the intrinsic brightness of the astronomical sources on the sky.

# Radio interferometric imaging



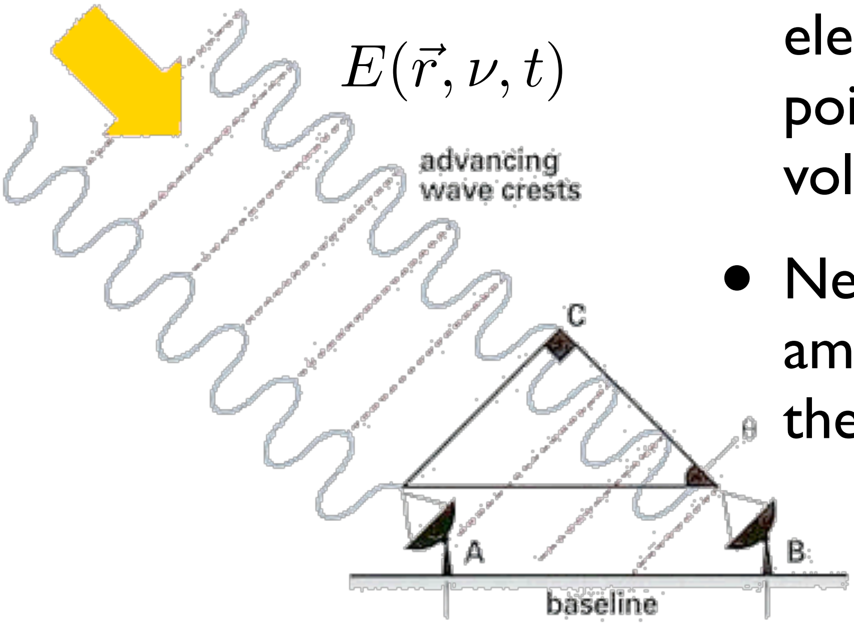
$I(\vec{s}, \nu, t)$

$P_0(\nu, t)$ $P_1(\nu, t)$ $P_2(\nu, t)$

- Need to correlate the received electric field (signal) at various geographically separate locations.
- Each element gives a signal **amplitude** and **phase**.

• As stated earlier, we need to correlate (compare) the received electric field (signal) at the different locations of the telescopes.

# Simple interferometer



$E(\vec{r}, \nu, t)$

$V_0(\nu, t)$ $V_1(\nu, t)$

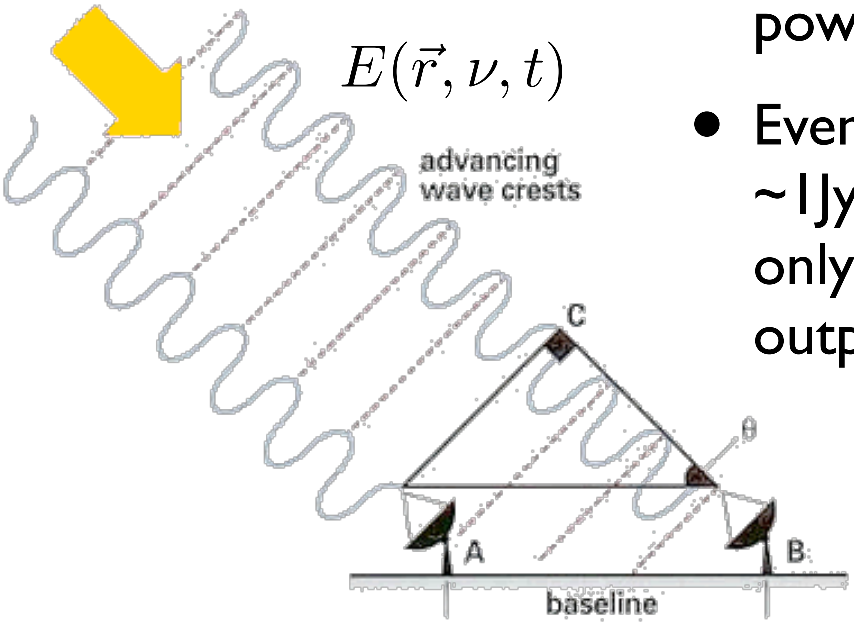
- Sample an incoming electric field at various points and convert to voltages.
- Need to register both amplitude and phase of the wave.

$$P(\nu, t) \propto V^2(\nu, t)$$

Power proportional to voltage squared

- For ease of discussion, let's start by considering just two dishes (a single baseline).
- The parallel electromagnetic waves at the two dishes will induce a time and frequency variably voltage at each receiver.
- This voltage has an amplitude and phase.
- The total power received is the square of the voltage.
- By "detecting" the voltage you are squaring the signal to get total intensity and thus losing phase information.

# Simple interferometer



$$E(\vec{r}, \nu, t)$$

$$V_0(\nu, t) \qquad V_1(\nu, t)$$

- Antenna adds additional power to the signal.
- Even a bright source of ~1Jy will still constitute only ~0.5% of the outputted power.

(Signal is buried in noise but signal is correlated between antennas and noise is not)

$$P(\nu, t) \propto V^2(\nu, t)$$

• Crucially the antenna itself also produces power, which adds noise to the signal we want to measure (see Lecture 5).
• So, even a bright astronomical source (say 1Jy) will still only contribute a percent of the total measured power.

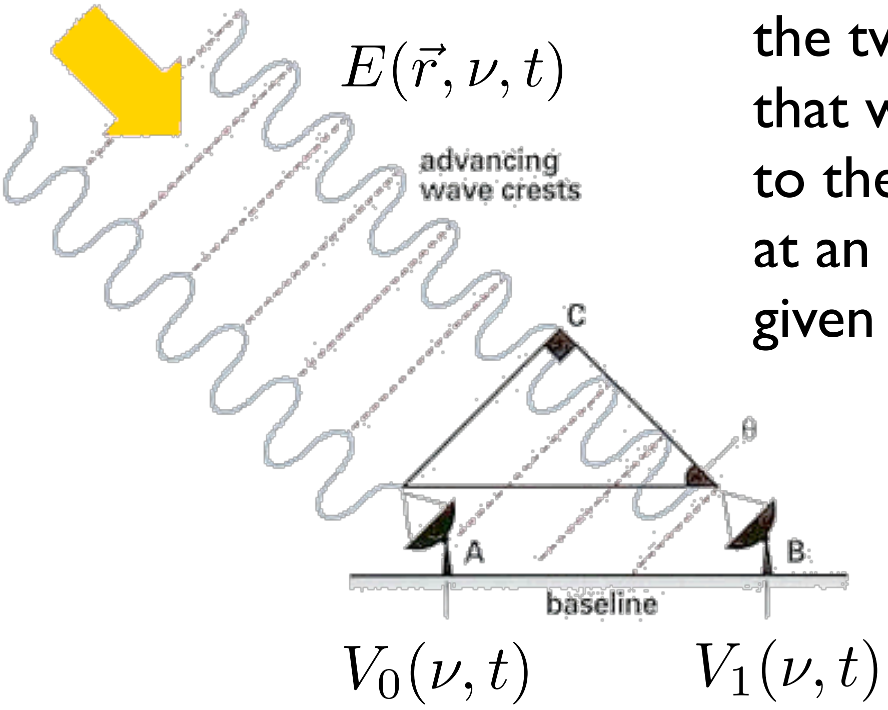
# Simple interferometer

Start with an overly simplistic example (also assumptions we'll make for the practicum)

- Interferometer is fixed w.r.t. the sky (an instant in time).
- Quasi-monochromatic waves. (single frequency)
- Interferometer directly measures the sky frequency ("RF interferometer").
- Single polarization.
- No distortions from ionosphere.
- Identical elements and perfect electronics.

- Let's restrict the problem by making even more assumptions.
- A real interferometer cannot make all these assumptions, but fortunately we know how to deal with each of these problems (that will come in later).

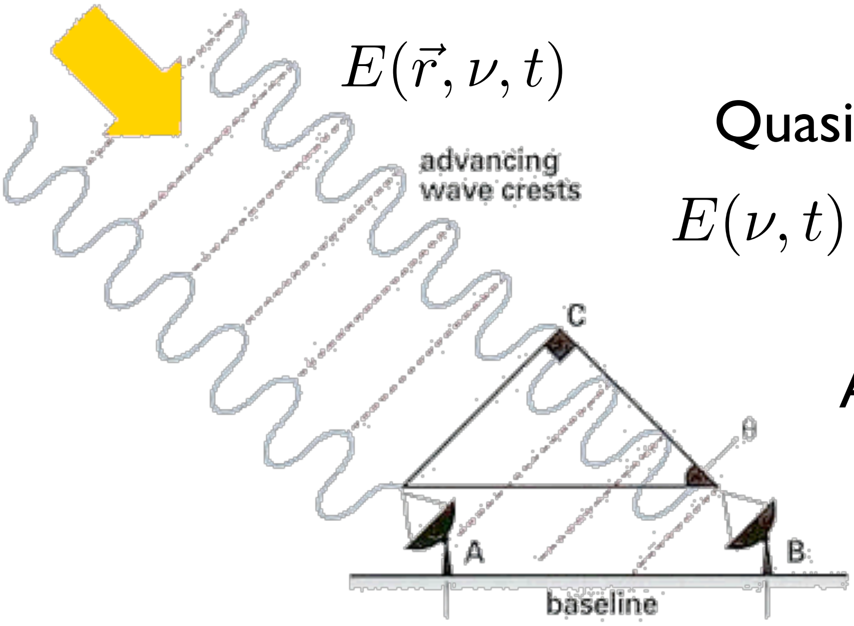
# Simple interferometer



- How can we combine the two **voltages** such that we can relate them to the **sky brightness** at an angular resolution given by the baseline?

- What method do we use to compare/relate the separate voltage signals at each antenna?

# Simple interferometer



$$E(\vec{r},\nu,t)$$

$$V_0(\nu,t) \qquad V_1(\nu,t)$$

Quasi-monotonic waves

$$E(\nu,t) = E\cos(2\pi\nu t + \phi)$$

Amplitude

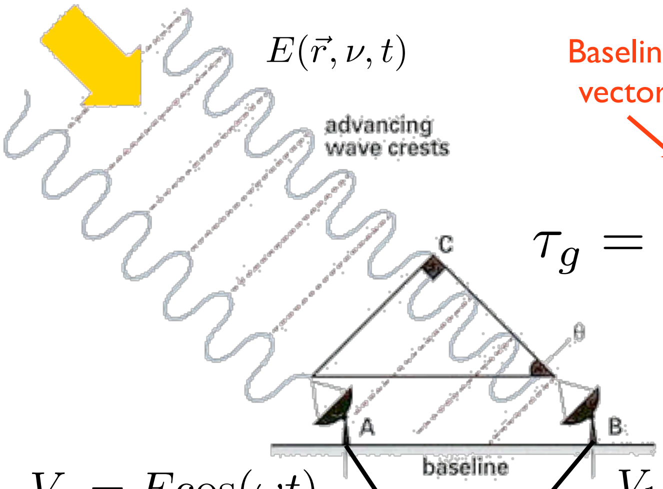
Phase

Amplitude and phase constant for

$$dt \sim 1/d\nu$$

- Let’s consider just a short instant of time over a narrow fraction of the radio band.

# Simple interferometer



$E(\vec{r}, \nu, t)$

Baseline vector

Source direction

$$\tau_g = \frac{\vec{b} \cdot \vec{s}}{c}$$

Time delay between antennas

$V_0 = E\cos(\omega t)$

$V_1 = E\cos[\omega(t - \tau_g)]$

Multiply $P[\cos(\omega\tau_g) + \cos(2\omega t - \omega\tau_g)]$

Average $R_C = P\cos(\omega\tau_g)$

Varies rapidly, averages to zero

- What do we get if we multiply the voltages at the two antennas?

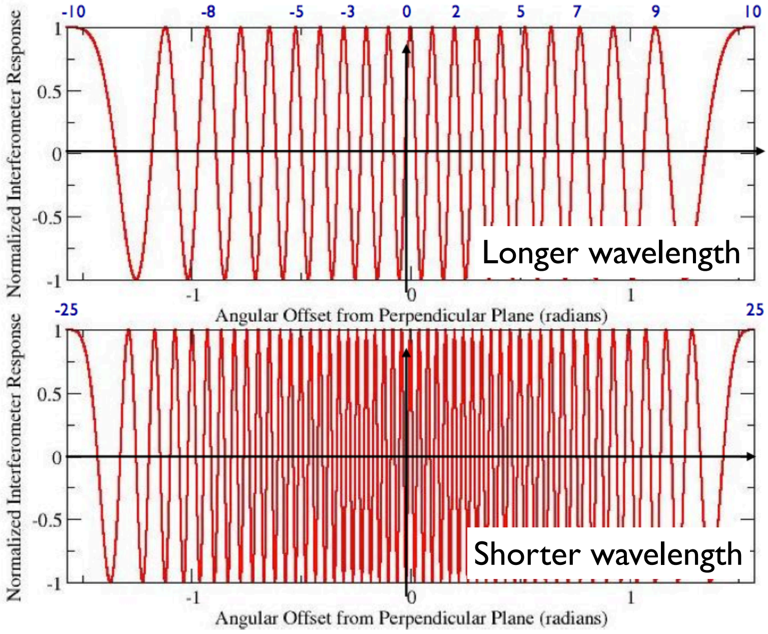
# Simple interferometer

$$R_C = P\cos(\omega\tau_g) = \underset{\substack{\uparrow \\ \text{Electric field}}}{P}\cos\left(2\pi\underset{\substack{\uparrow \\ \text{Geometric delay}}}{\frac{\vec{b}\cdot\vec{s}}{\lambda}}\right)$$

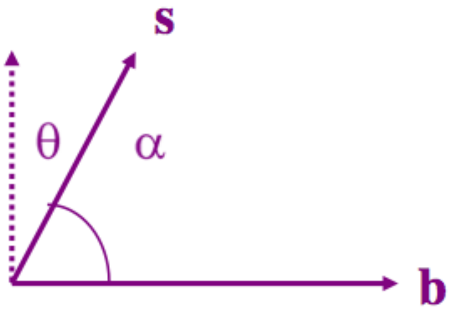
- In other words, the response *Rc* depends on the received field strength (amplitude squared) and orientation of the baseline w.r.t. the source.
- Doesn't depend on observation epoch, location of baseline (distance to source), or incoming signal phase (source is in the far field).

# Simple interferometer

$$\frac{\vec{b}\cdot\vec{s}}{\lambda} = u\mathrm{cos}\alpha = u\mathrm{sin}\Theta = ul$$

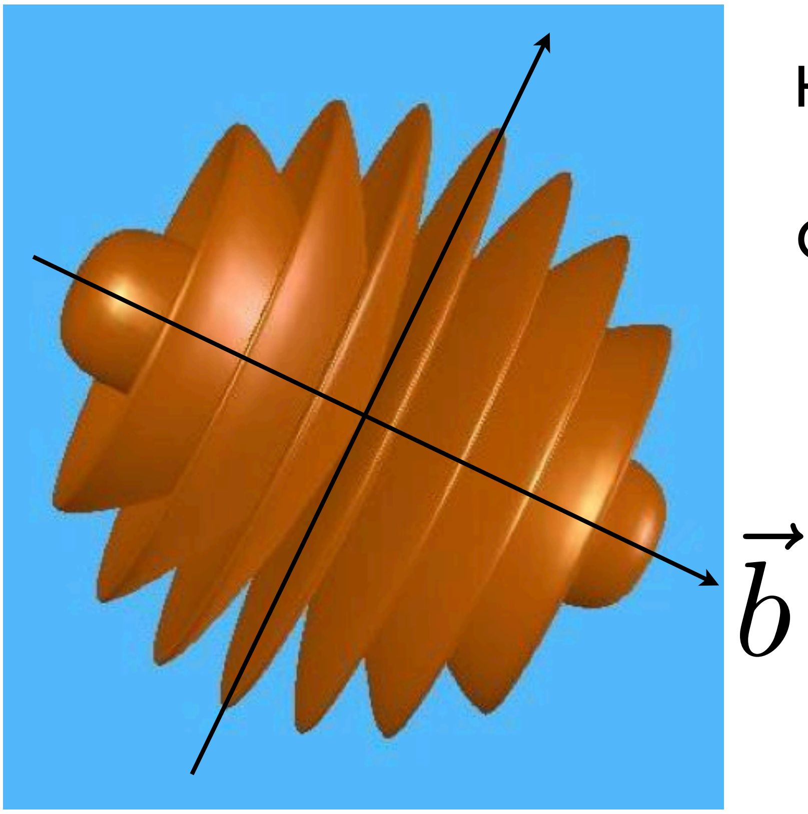


Now consider the baseline in terms of the wavelength



- Let’s now consider the projected baseline length (dot product of the baseline vector and unit vector towards source) in terms of the number of wavelengths it represents.
- Doing a 1-D cut, we see that the simple interferometer response oscillates on the sky. As an analogy, think of the Young double-slit experiment.

# Simple interferometer



How things actually look in 2-D

Over the whole sky, u=4 (i.e. baseline is 4 wavelengths long).

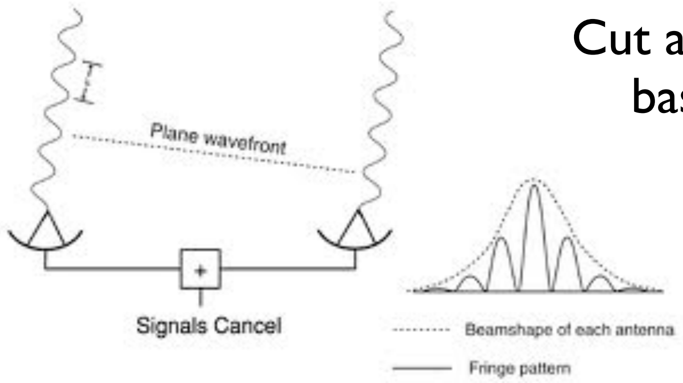
$\vec{b}$

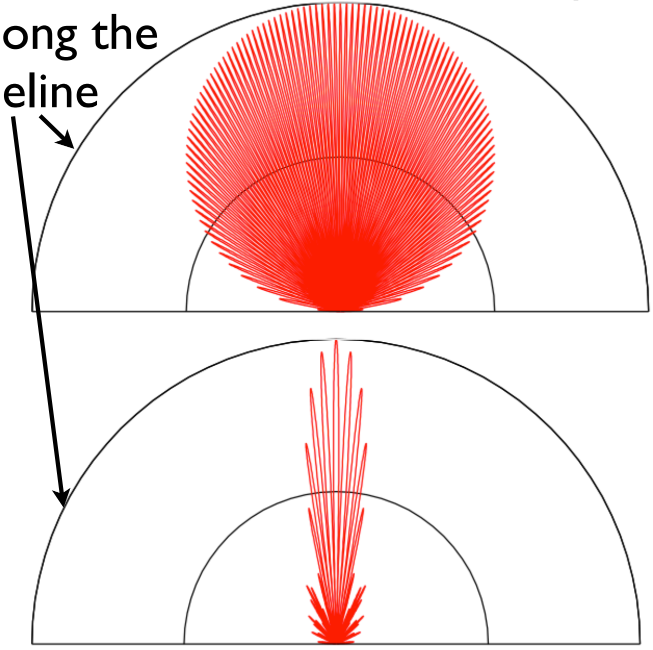
Think of a 2-element interferometer as creating fan beams

- A 2-element (or any 1-D configuration of baselines) creates a set of fan beams on the sky.
- These only provide resolution in one direction.
- **QUESTION: WILL THIS ALLOW US TO PROPERLY IMAGE SOURCES ON THE SKY? ANSWER: NOT REALLY, WE'RE ONLY GETTING RESOLUTION IN A SINGLE DIRECTION.**

# Simple interferometer

Don’t forget the effect of the aperture, which modulates the amplitude and phase of the received signal





LOFAR-like response

Cut along the baseline

This adds an envelope to the response

More directional response

- The preceding beam patterns didn’t account for the response of the individual antennas (i.w. the primary beams). This places an envelope on the beam pattern of the array response.

# Extended source response

For each baseline, sum over the sky and average:

$$R_C = \left\langle \int V_1 d\Omega_1 \int V_2 d\Omega_2 \right\rangle$$

Switch order of integral/average (assume the emission is spatially incoherent - i.e. no phase relation):

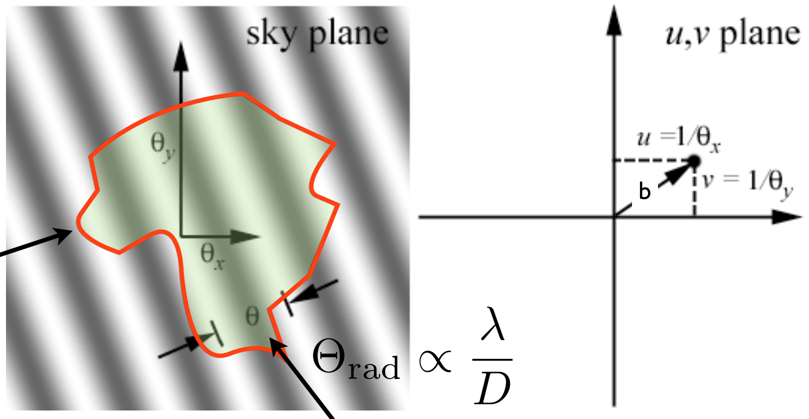
$$R_C = \iint I(\vec{s},\nu)\cos(2\pi\nu\vec{b}\cdot\vec{s}/c)d\Omega$$

We have now linked the interferometer response, *Rc*, and the sky intensity. But we'll need to invert to get *I*.

- Yey, here's our first equation linking a quantity measured at the telescope and the sky intensity.
- Still, what we're interested in is the sky intensity as a function of direction and frequency, so we'll have to invert this equation to get at it.

# Extended source response

Schematically



$$\Theta_{\mathrm{rad}} \propto \frac{\lambda}{D}$$

We'll be talking a lot about the "uv-plane" in the second half of this lecture

Extended Source

Orientation given by baseline geometry

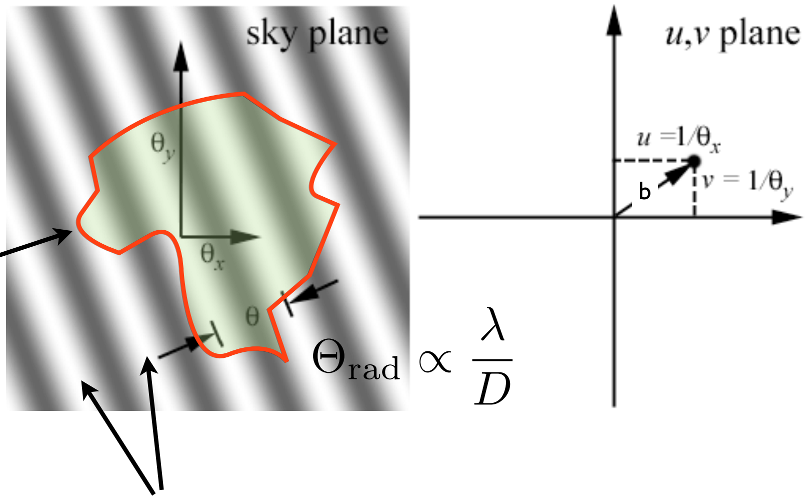
Spacing given by (projected) baseline length and wavelength.

- Here the greyscale (left) shows the response of a single baseline, which will have some orientation with respect to the sky.
- The orientation and length of the baseline can be represented in the "uv-plane" (right). In the uv-plane, the baseline creates a point at u = 1/(theta_x) and v = 1/(theta_y).

# Extended source response

Schematically

Extended Source



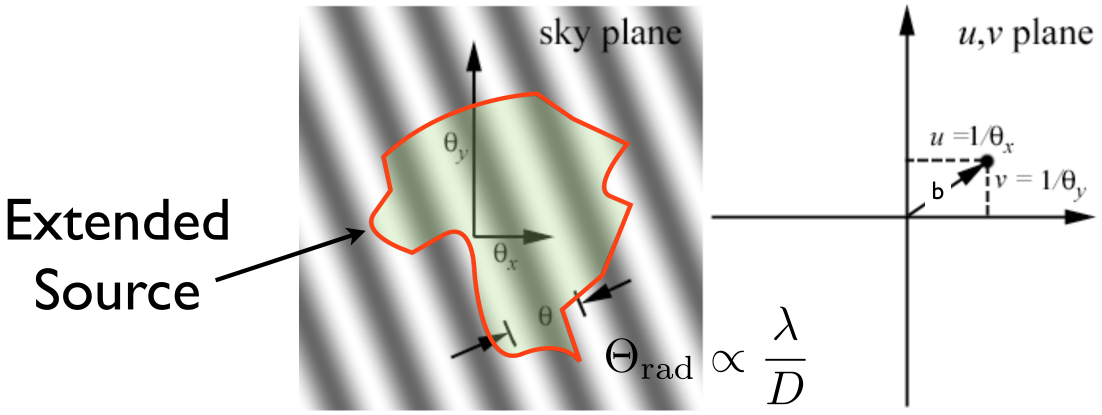
$$\Theta_{\text{rad}} \propto \frac{\lambda}{D}$$

"Fringes" (diffraction pattern): tightly packed for long baselines (and/or short wavelengths, widely separated for short baselines (and/or long wavelengths).

- The "fringe" pattern response on the sky has a spacing theta = lamda / D.

# Extended source response

## Schematically



$$\Theta_{\mathrm{rad}} \propto \frac{\lambda}{D}$$

**WAIT! Are we really sampling all the source flux in this way?**

- So the interferometer effectively places a cosine coherence pattern on the sky.
- **The interferometer response is then the source brightness multiplied by this pattern and integrated over the whole sky.**

• **QUESTION: ARE WE REALLY SAMPLING ALL THE SOURCE FLUX IN THIS WAY?**

# Odd/even functions

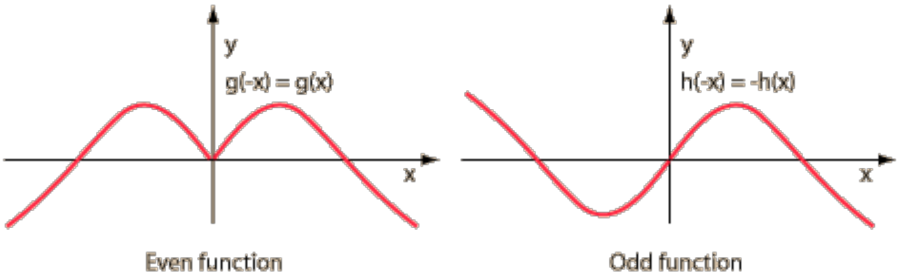
Any real function can be decomposed into an even and odd part:

$$I(x,y) = I_E(x,y) + I_O(x,y)$$

Such that:

$$I_E(x,y) = \frac{I(x,y) + I(-x,-y)}{2} = I_E(-x,-y)$$

$$I_O(x,y) = \frac{I(x,y) - I(-x,-y)}{2} = -I_O(-x,-y)$$



- The cosine response we just showed seems problematic though: we don't want to have a ripple pattern of sensitivity. Uniform response over the _whole_ source would be way better.
- Quick reminder of a basic piece of math: any real function can be decomposed into an even and an odd part.

# Correlator response

$I(\vec{s},\nu)$ is a real function with (potentially) both even and odd parts

Problem: *Rc* only samples the even part of $I(\vec{s},\nu)$

**Remember: cosine is an even function, while sine is an odd function**

$$R_C = \iint I(\vec{s},\nu)\cos(2\pi\nu\vec{b}\cdot\vec{s}/c)d\Omega$$

$$R_C = \iint I_O(\vec{s},\nu)\cos(2\pi\nu\vec{b}\cdot\vec{s}/c)d\Omega = 0 \qquad I_O(\vec{s}) = -I_O(-\vec{s})$$

We're missing some information about the source brightness!

- Obvious the sky brightness must be a real function of direction and frequency.
- So R_c is a quantity that doesn't tell the whole picture: it only gives the even part of the sky brightness.
- cos() is an even function. Multiplying by an odd function and integrating will give zero. In other words, R_c provides zero information about the odd part of the source brightness function.

# Cosine/sine terms

To recover the full source brightness, we need:

$$R_C = \iint I(\vec{s},\nu)\cos(2\pi\nu\vec{b}\cdot\vec{s}/c)d\Omega = \iint I_E(\vec{s},\nu)\cos(2\pi\nu\vec{b}\cdot\vec{s}/c)d\Omega$$

$$R_S = \iint I(\vec{s},\nu)\sin(2\pi\nu\vec{b}\cdot\vec{s}/c)d\Omega = \iint I_O(\vec{s},\nu)\sin(2\pi\nu\vec{b}\cdot\vec{s}/c)d\Omega$$

*Rc* samples the even part of *I(s)* and *Rs* samples the odd part of *I(s)*.

To get Rs, we simply add a 90 deg phase shift into one of the signal paths before multiplying.

- So let's introduce another quantity, R_s which does sample the odd part of the source brightness.
- To create this R_s quantity, we just need to rotate the phase of one of the antenna signals before multiplying.

# Intermission!

- Take a break, stretched your legs...

# Visibilities and the uv-plane

- We’ve slowly built up to two very important concepts: “visibilities” (the signal recorded for each baseline - i.e. telescope pair) and the uv-plane, which represents the distribution of the telescope on the ground and hence it’s response to sky brightness.

# Visibilities

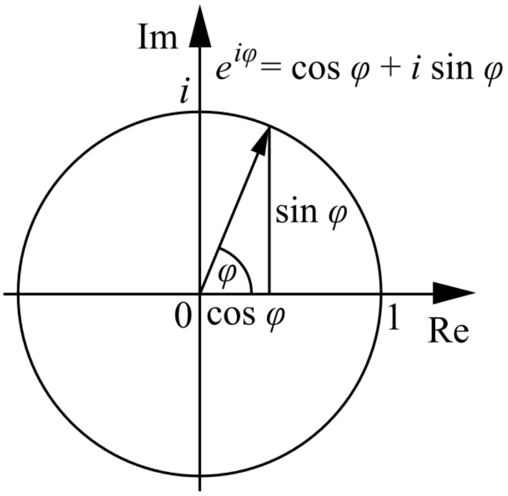
*Define* a complex function called the "visibility", which contains all the information for that baseline:

$$V = R_C - iR_S = Ae^{-i\phi} \qquad A = \sqrt{R_C^2 - R_S^2} \qquad \phi = \tan^{-1}\left(\frac{R_S}{R_C}\right)$$

Phase

Amplitude

Recall Euler's Formula



- We can encapsulate all the information available from one baseline using a complex function called the "visibility".
- The visibility is constructed from R_c and R_s, as previously defined, and gives us an amplitude and phase.

# Visibilities

*Define* a complex function called the "visibility", which contains all the information for that baseline:

$$V = R_C - iR_S = Ae^{-i\phi} \qquad A = \sqrt{R_C^2 - R_S^2} \qquad \phi = \tan^{-1}\left(\frac{R_S}{R_C}\right)$$

$$V_\nu(\vec{b}) = R_C - iR_S = \iint I_\nu(\vec{s})e^{-2\pi i\nu\vec{b}\cdot\vec{s}/c}d\Omega$$

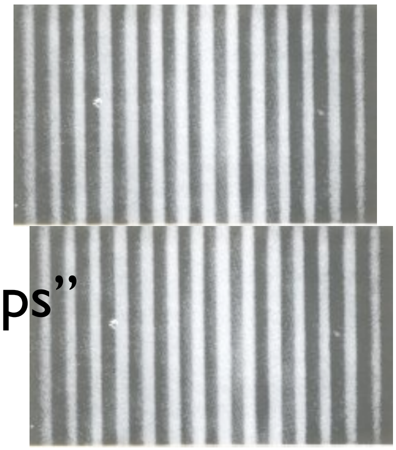
Hey look a 2D Fourier transform! (of spatial frequencies)

- *Complete* relation between the interferometer response and the source brightness.
- This is a 2-D Fourier relation under certain circumstances.

- This is now a complete relation between the interferometers response and the source brightness.
- This equation looks like a Fourier transform, and it is under certain circumstances (the simple approximations).
- Idea is to relate (projected) distances on the ground to angular scales on the sky.

# Complex correlator

- **Correlator**: machine to produce both the real and imaginary part of the visibilities (i.e. *Rc* and *Rs*).
- Effectively casts two sets of sinusoids on the sky, offset by 90deg.
- Again, both are needed if this pattern remains stationary w.r.t. the source.



"Fill-in the information gaps"

- A machine called a correlator creates the visibilities from the raw telescope voltage signals.
- Pictured here is the (former) correlator of the Joint Institute for VLBI in Europe (JIVE).

# Complex correlator

To makes the math easier, we use complex numbers (instead of cosine/sine). Re-write our antenna voltages as:

$$V_1 = A\cos(\omega(t)) = \mathrm{Re}\{Ae^{-i\omega(t)}\}$$

Delay time

$$V_2 = A\cos(\omega(t - \vec{b}\cdot\vec{s}/c)) = \mathrm{Re}\{Ae^{-i\omega(t-\vec{b}\cdot\vec{s}/c)}\}$$

Correlated power becomes:

$$P_{corr} = \langle V_1 V_2^* \rangle = A^2 e^{-i\omega\vec{b}\cdot\vec{s}/c}$$

Just a different version of equation on Slide 27

Time average

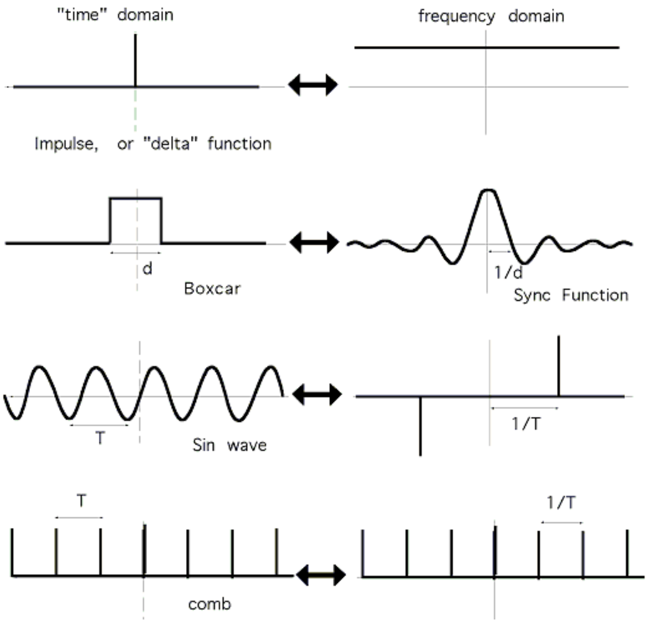
(term with *t* disappears when averaging over a much longer timescale)

Complex conjugate

# Visibility

$$V_\nu(u,v) \Leftrightarrow I_\nu(l,m)$$

## Fourier Transform



1-D

Again, you may want to remind yourself about FTs and their properties.

It's very useful to get an intuitive feel for FTs.

- Simplistically, the visibilities (which are functions of the u,v coordinates) are related to the sky brightness (which is a function of position on the sky) by a Fourier transform.

# Visibility

$$V_\nu(u,v) \Leftrightarrow I_\nu(l,m)$$

- *V* is a unique function of *I*.
- So far we've talked about a single baseline measuring a single frequency at a single time at a single (u,v) coordinate.
- This single visibility gives us limited information about the relevant spatial scales and morphology of the source we're observing.

- A single baseline/visibility gives only limited information about the sky brightness distribution. It's tuned to just one specific angular scale, with one specific orientation with respect to the sky.

# Visibility

$$V_\nu(u,v) \Leftrightarrow I_\nu(l,m)$$

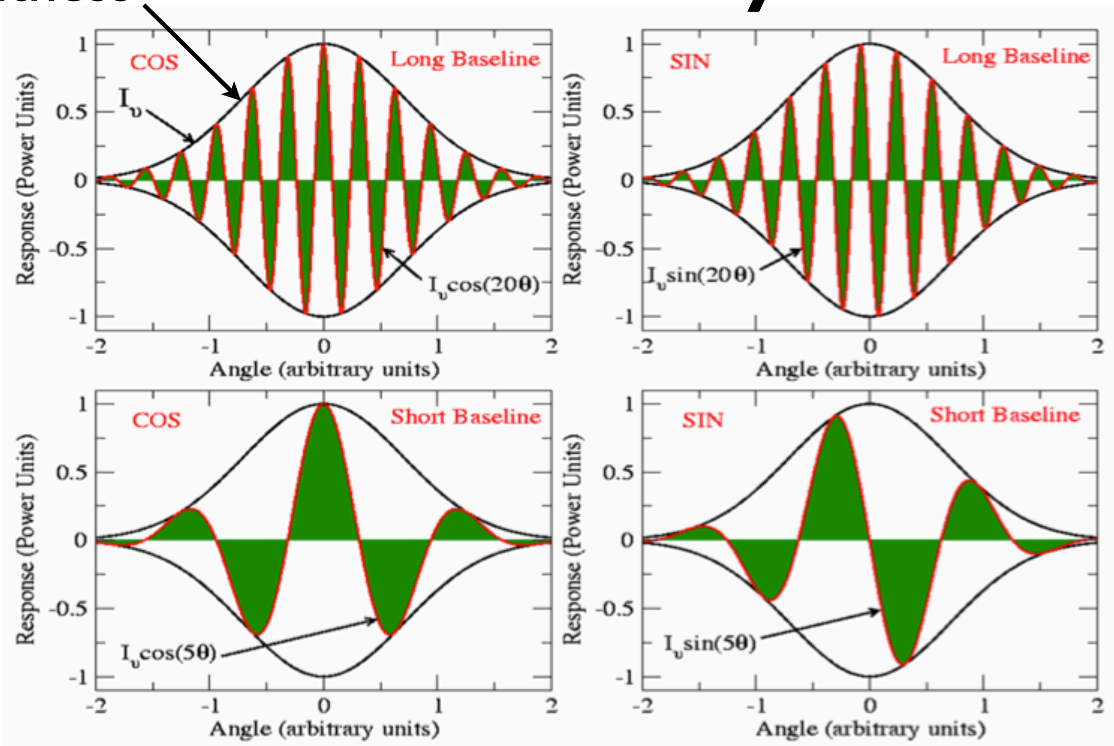
- Amplitude of visibility generally gets lower for increasingly long baseline.
- A source is "resolved out" when the visibility amplitude approaches zero.
- Visibility of "reversed baseline" is the complex conjugate of the original.

# Visibility

Source brightness

$R_C$

$R_S$

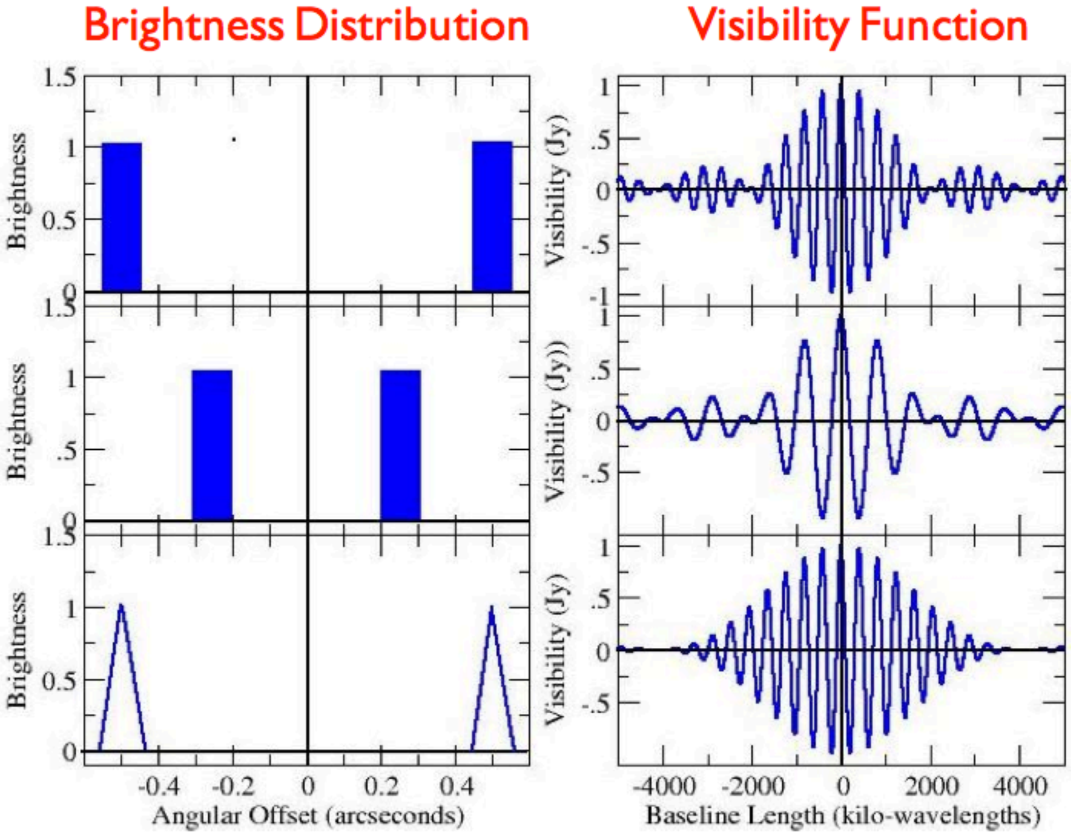


Long Baseline

Short Baseline

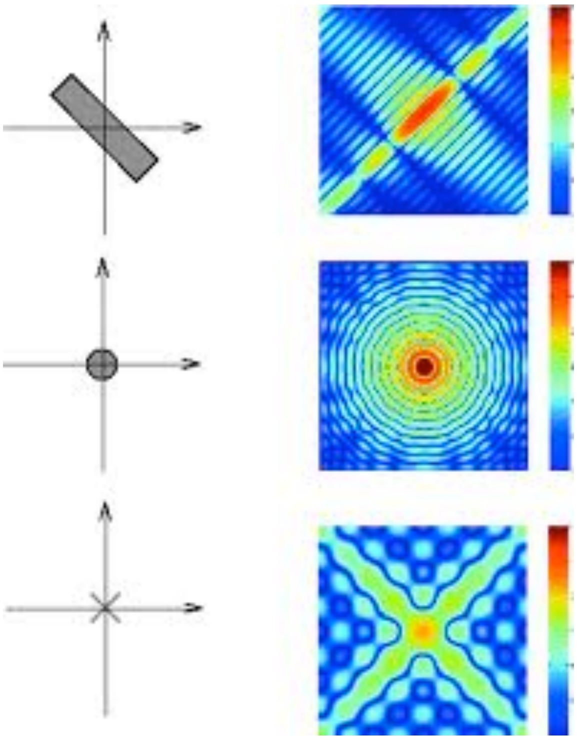
Red: fringes; green: response (visibility)

# Visibility


Brightness Distribution
Visibility Function
Brightness
Visibility (Jy)
Visibility (Jy))
1.5
1
0.5
.5
0
-.5
-1
-0.4
-0.2
0.2
0.4
Angular Offset (arcseconds)
-4000
-2000
2000
4000
Baseline Length (kilo-wavelengths)

# Moving to 2-D

- Of course, a more useful interferometer includes more than just two antennas and hence more than just 1 baseline.
- Also, ideally these will be distributed in 2-D.
- Recall: for an N-element interferometric array, there are N(N-1)/2 independent baselines.



2-D Fourier Transform

- Look at the 2-D Fourier transform examples on the right. These patterns are the same as the diffraction pattern you'd get by putting coherent light through a slit of that shape.

# uv-Plane (2D version)

Assume all interferometric elements are in a plane.

$$\vec{b} = (\lambda u, \lambda v, \lambda w) = (\lambda u, \lambda v, 0)$$

w is perpendicular to the observing plane. (u,v,w) in units of wavelengths.

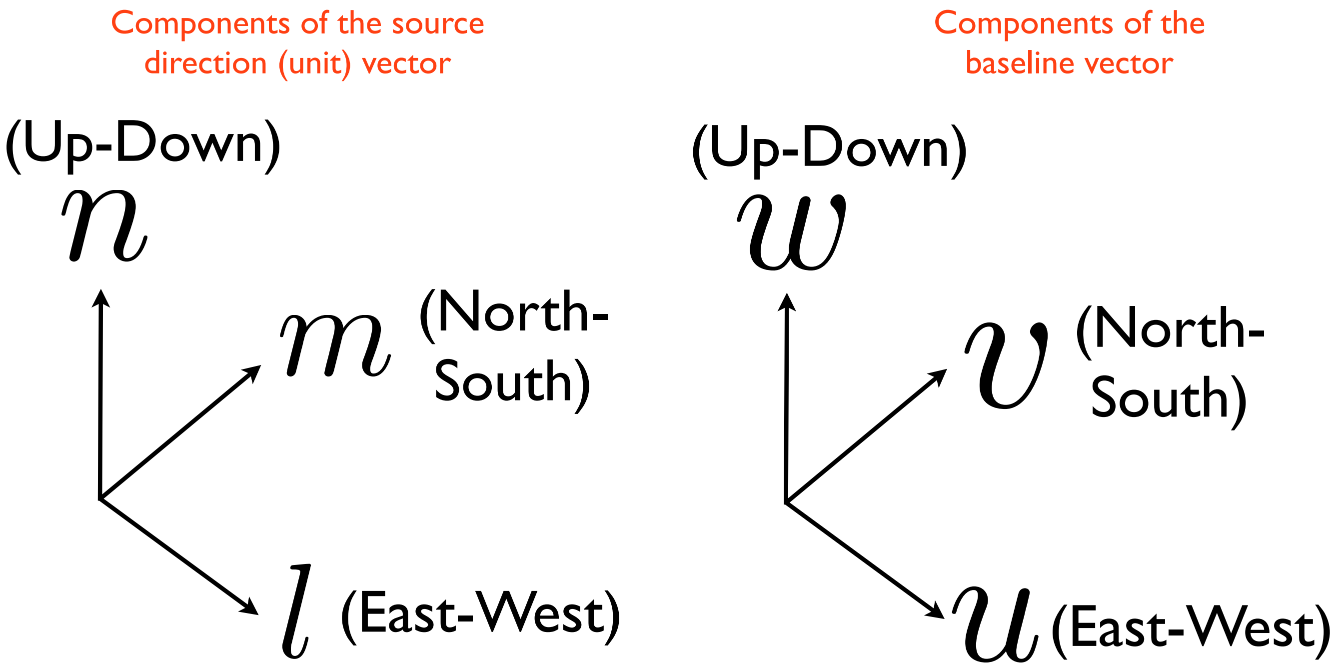
$$\vec{s} = (l, m, n) = (l, m, \sqrt{1 - l^2 - m^2})$$

(components of the unit direction vector)

- The "uv-plane" describes the distribution of the telescopes on the ground as a function of the baseline orientations and lengths in units of the observing wavelength.
- u is the E-W direction and v is the N-S direction. w (Up-Down) can be used to describe an array that doesn't lie in a plane, but that's a complication that we won't deal with here.

# uv-Plane (2D version)

“($l$,$m$,$n$) and ($u$,$v$,$w$) coordinates”



- Left: vector components describing the source direction.
- Right: vector components describing the baseline orientation.

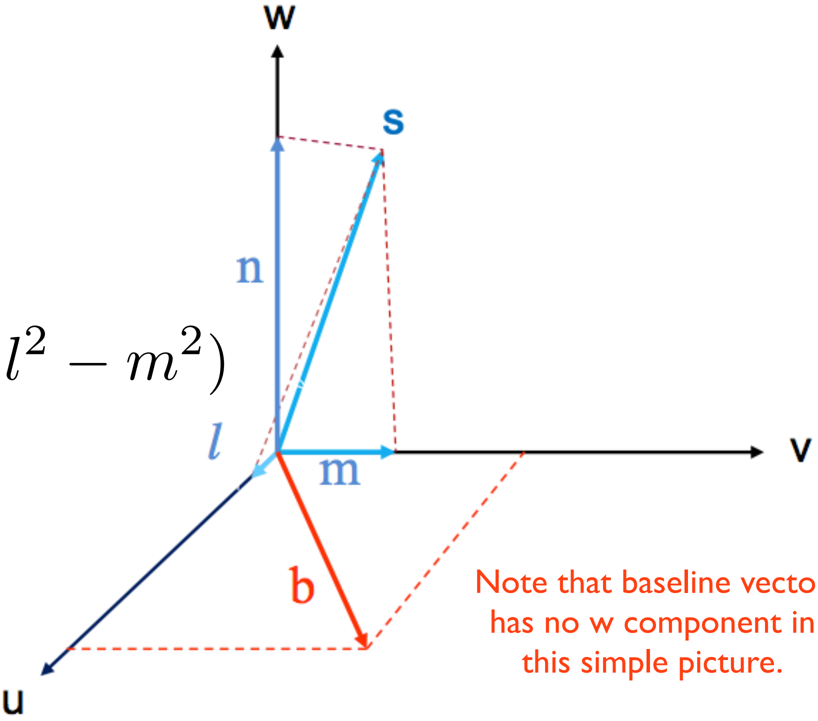
# uv-Plane (2D version)

Unit vector s (direction) is defined by its projections (l,m,n) onto the (u,v,w) axes. We call (l,m,n) the **direction cosines**.

$$l = \cos(\alpha)$$

$$m = \cos(\beta)$$

$$n = \cos(\theta) = \sqrt(1 - l^2 - m^2)$$



Note that baseline vector has no w component in this simple picture.

# 2-D Fourier relation

Infer the source brightness from the visibilities, as before

(see Slide 42)

$$V_\nu(u,v) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-i2\pi(ul+vm)} dl dm$$

Projection of array

$$I_\nu(l,m)/\cos(\Theta) \Leftrightarrow V_\nu(u,v)$$

$$\boxed{I_\nu(l,m) = \cos(\Theta) \iint V_\nu(u,v) e^{i2\pi(ul+vm)} du dv}$$

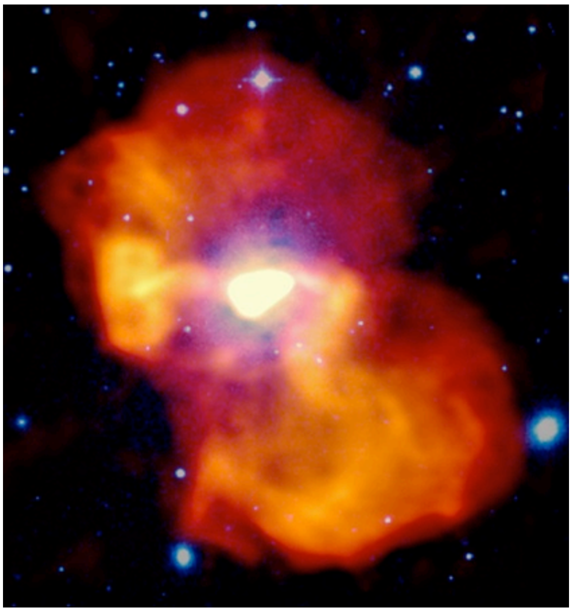
Now a 2-D Fourier transform

Situation becomes more complicated if w not zero (not co-planar array w.r.t. the source)

- Aha, a 2-D Fourier transform relation.
- We're assuming that all the telescopes lie perfectly in a plane. If they don't (e.g. different altitudes and/or Earth curvature is relevant) then the relation becomes more complicated in order to deal with this effect.

# uv-Coverage

- Spatial sampling of the source brightness distribution is dictated by what baselines are recorded and their orientation w.r.t. the sky.
- More baselines means more spatial information on a variety of scales.
- Ideally the interferometric elements will be randomly distribution in order to disperse small errors.

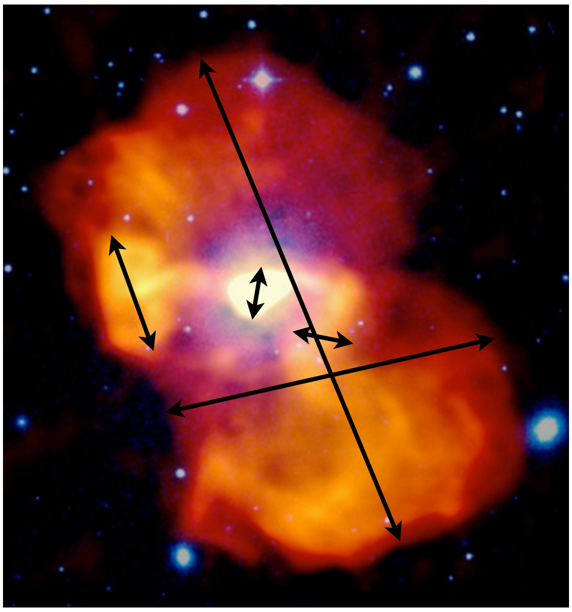


Virgo A - LOFAR

- Typically it’s better to have baselines with a nice distribution of orientations and lengths. This gives good sampling of different source features.
- That said: some arrays have incorporated redundant baselines (same orientation and length) in order to help with calibration (they should all give the same answer, so that can be checked by comparing them).

# uv-Coverage

- Coverage is never 100% complete, but want to sample the relevant angular scales and orientations.



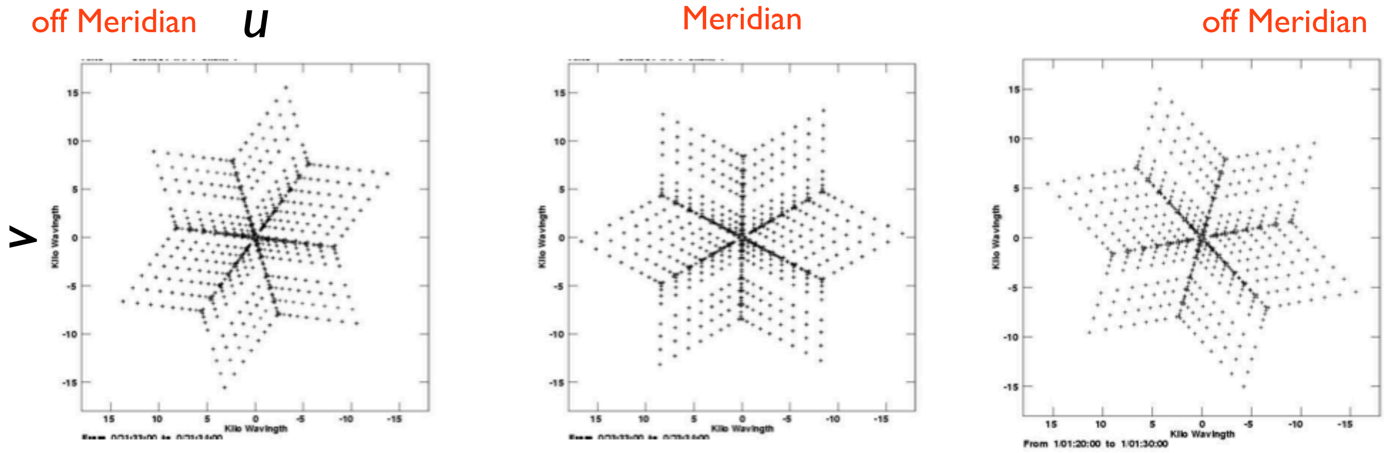
Virgo A - LOFAR

- This is just a simple demonstration of the angular scales that are relevant for this source, the radio galaxy Virgo A.

# Snapshot uv-Coverage

## Example for VLA (at dec = 50deg)

Projection off Meridian | Meridian | Projection off Meridian



HA = -2h | HA = 0h | HA = 2h

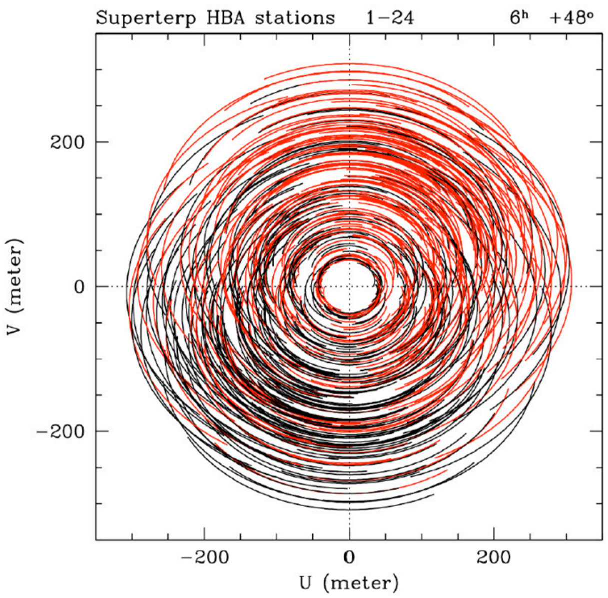
North-South baselines (none purely E-W) mean that *no* uv-tracks are centered on (u,v) = (0,0).
Helps a lot with imaging around the celestial equator.

# uv-Coverage and Beam

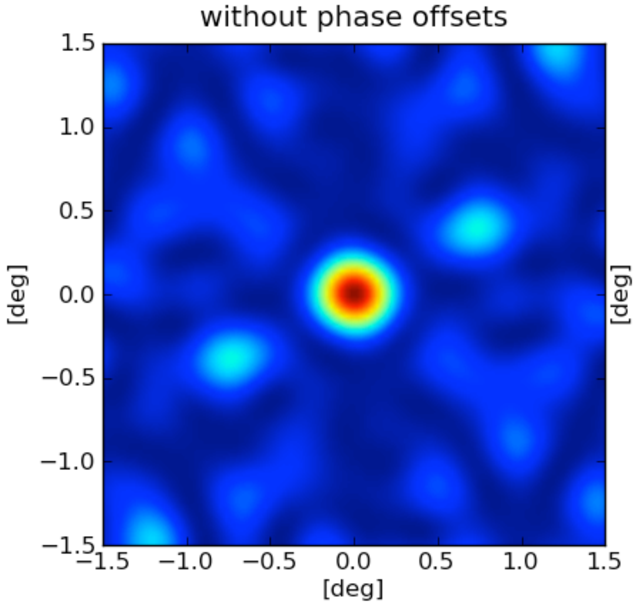
Here we see **tracks** over 6 hrs (fills in the uv coverage)

## LOFAR Superterp (Inner core)

Here we see the **instantaneous** beam shape



uv-coverage in 6 hours

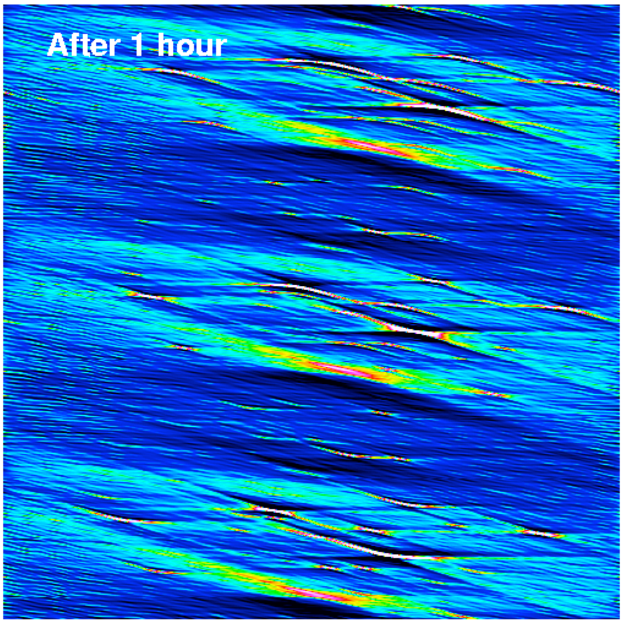


beam

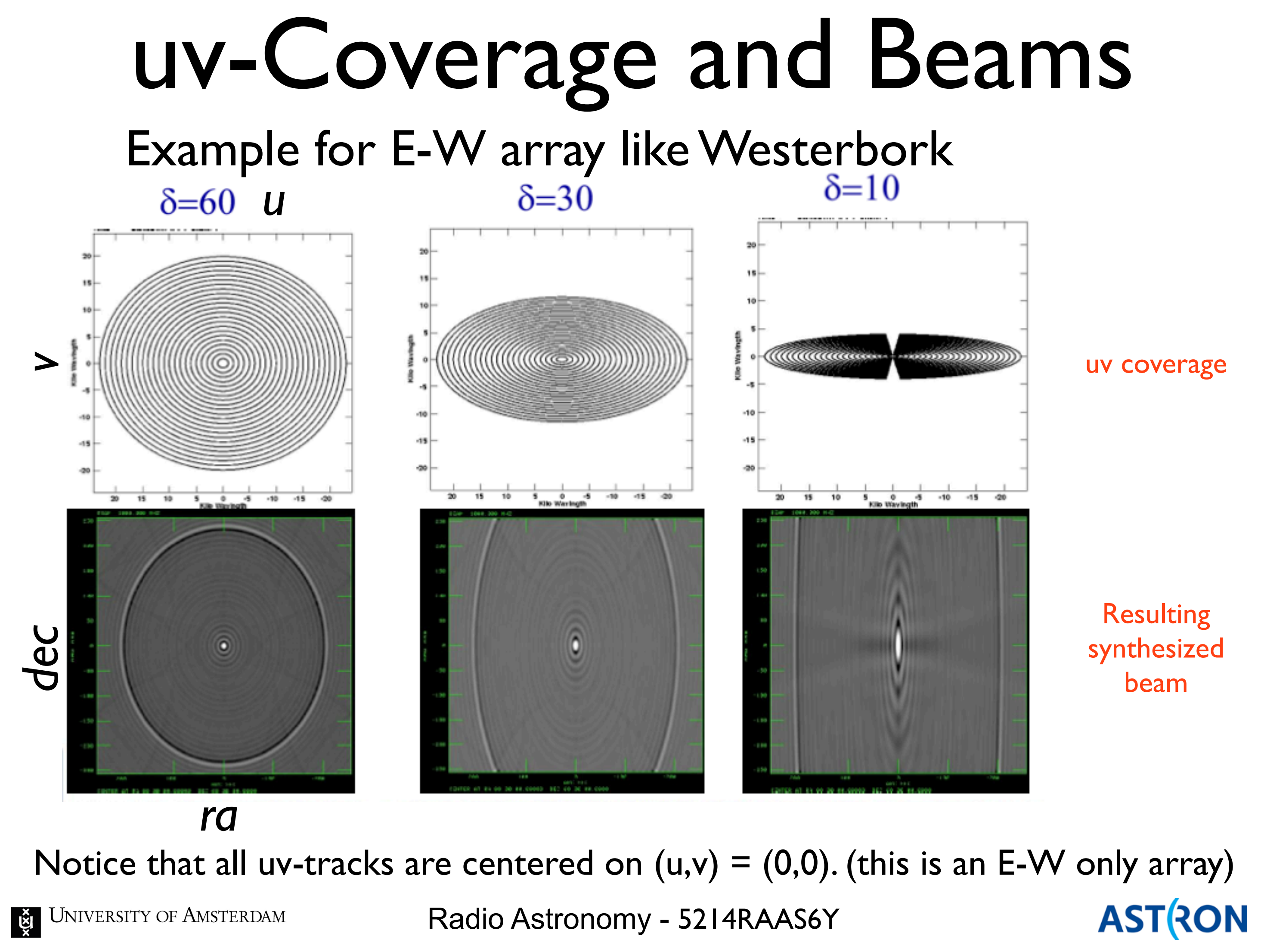
- On the left: the uv-coverage of the LOFAR "Superterp" (inner-most 12 sub-stations) is shown over the course of a 6-hr integration.
- The lines show how the point associated with an individual baseline follows a track in the uv-plane.
- The red/black curves show the baseline symmetry (reverse baseline is complex conjugate of the visibility).
- On the right: this is how the _instantaneous_ Superterp beam looks. There is a main lobe and two very prominent side-lobes. The side-lobes rotate with respect to the center and and wash out with time, while the main lobe does not.
- **QUESTION: WHAT CAUSES THE UV POINTS TO FOLLOW TRACKS WITH TIME? ANSWER: EARTH ROTATION. THE BASELINES HAVE A DIFFERENT ORIENTATION W.R.T. RA/DEC AND A DIFFERENT PROJECTION WITH TIME.**

# Earth rotation aperture synthesis

Baselines will rotate w.r.t. the sky as a function of time. Earth rotation "fills-in" the image.



- Luckily the "uv coverage" is not necessarily just dictated by the instantaneous distribution of the dishes with respect to the sky.
- The Earth is of course rotating with respect to the sidereal frame and hence the baselines will rotate with respect to the observe field and fill in the information about the sky brightness.
- For example, Westerbork is a purely linearly distributed array of 14 dishes but can still make nice 2-D images _if_ one records data for 12hrs.
- This building up of "uv coverage" with Westerbork is illustrated in the left-hand movie. After only one hour high resolution has only been achieved in one direction.
- **QUESTION: WHAT ARE ALL THOSE RINGS AND OTHER STRANGE FEATURES? ANSWER: THESE ARE RELATED TO THE SIDE-LOBES OF THE SYNTHESIZED BEAM AND NEED TO BE DECONVOLVED TO MAKE A CLEAN IMAGE.**



# uv-Coverage and Beams

Example for E-W array like Westerbork

uv coverage

Resulting synthesized beam

Notice that all uv-tracks are centered on (u,v) = (0,0). (this is an E-W only array)

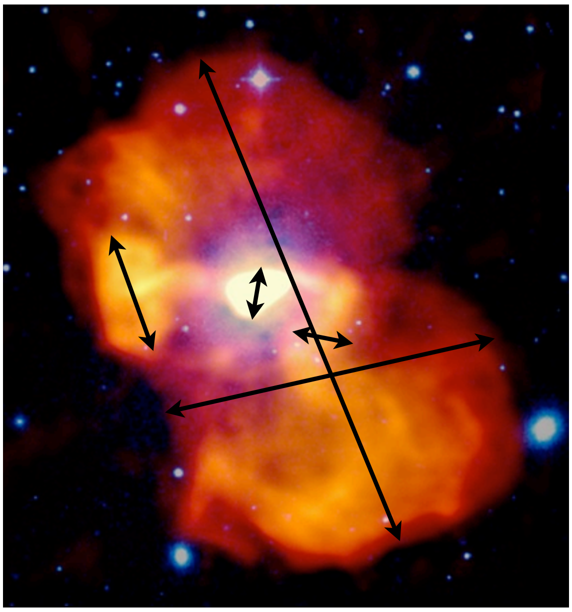
UNIVERSITY OF AMSTERDAM

Radio Astronomy - 5214RAAS6Y

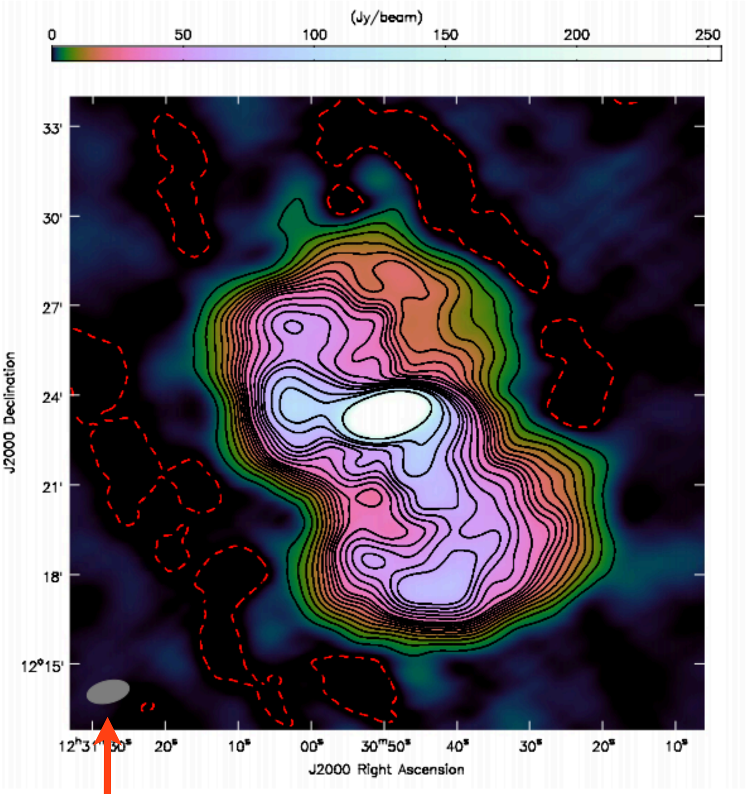
ASTRON

- **QUESTION: WHY DOES THE WESTERBORK UV COVERAGE AND SYNTHESIZED BEAM BECOME SQUISHED TOWARDS LOWER DECLINATIONS?**
- uv coverage will depend on the source declination. (the array will get "squashed" in size because of projection.
- Notice how the pattern and resulting beam get squished as one goes to low declinations?

# Synthesized beam



Virgo A - LOFAR



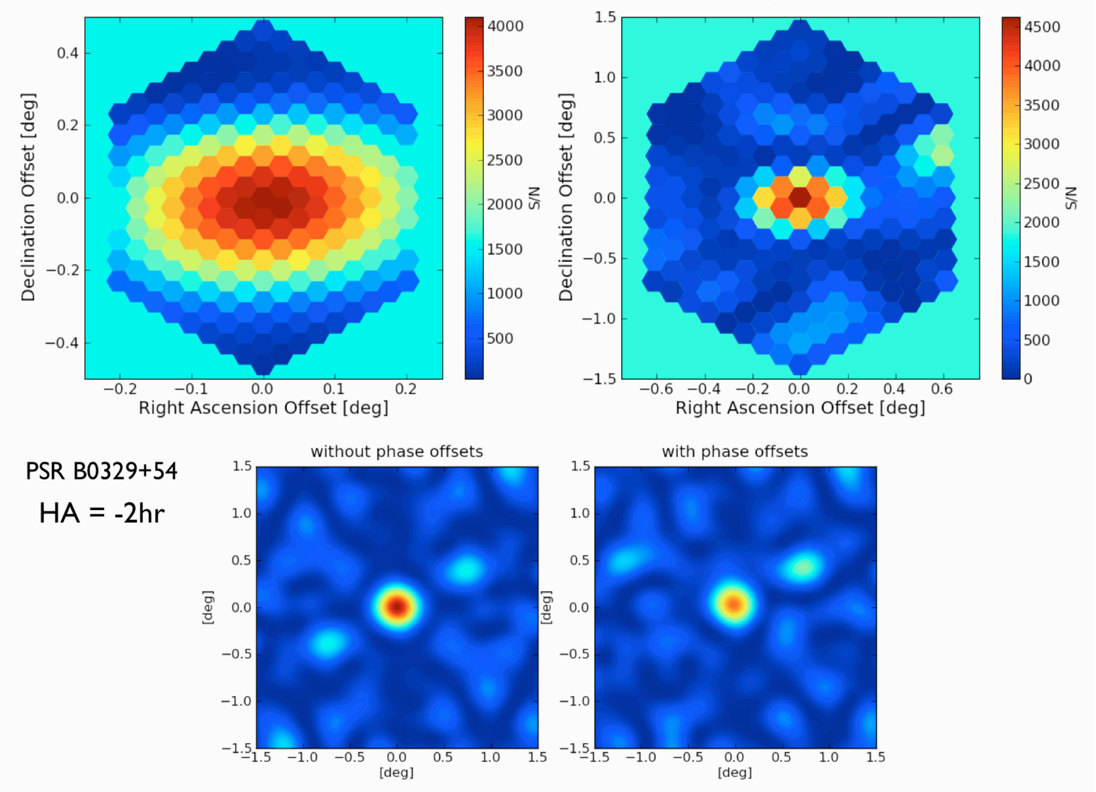
Synthesized beam

- Given a certain uv-coverage over the course of an observation, one achieves a “synthesized beam”, which is effectively the point-spread-function (basic resolution element).
- Radio interferometric images often shown the synthesized beam shape in the bottom left-hand corner of the image because one needs to take this into account when interpreting whether features in the image are intrinsic or related to the telescope response.

# uv-Coverage and Beam

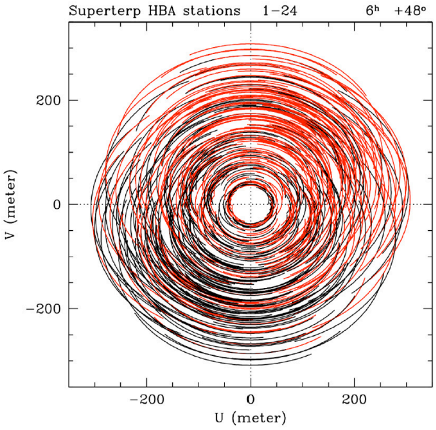
## LOFAR Superterp (Inner core)

Also relevant for understanding "tied-array" beams.



- When we later (Lecture 10) discuss high-time-resolution observations with a radio interferometer we'll also talk about "tied-array" beams.
- Tied-array beams are basically the instantaneous synthesized beam of the interferometer, which you can get by coherently adding the station signals together.
- The top panels compare empirical measurements of the LOFAR tied-array beam with theoretical expectations for what it should look like. This demonstrates that there is a small phasing error. This can be seen by the asymmetric shapes of the side-lobes.

# (u,v,w) Coordinates



Define antenna positions in an Earth-based coordinate system

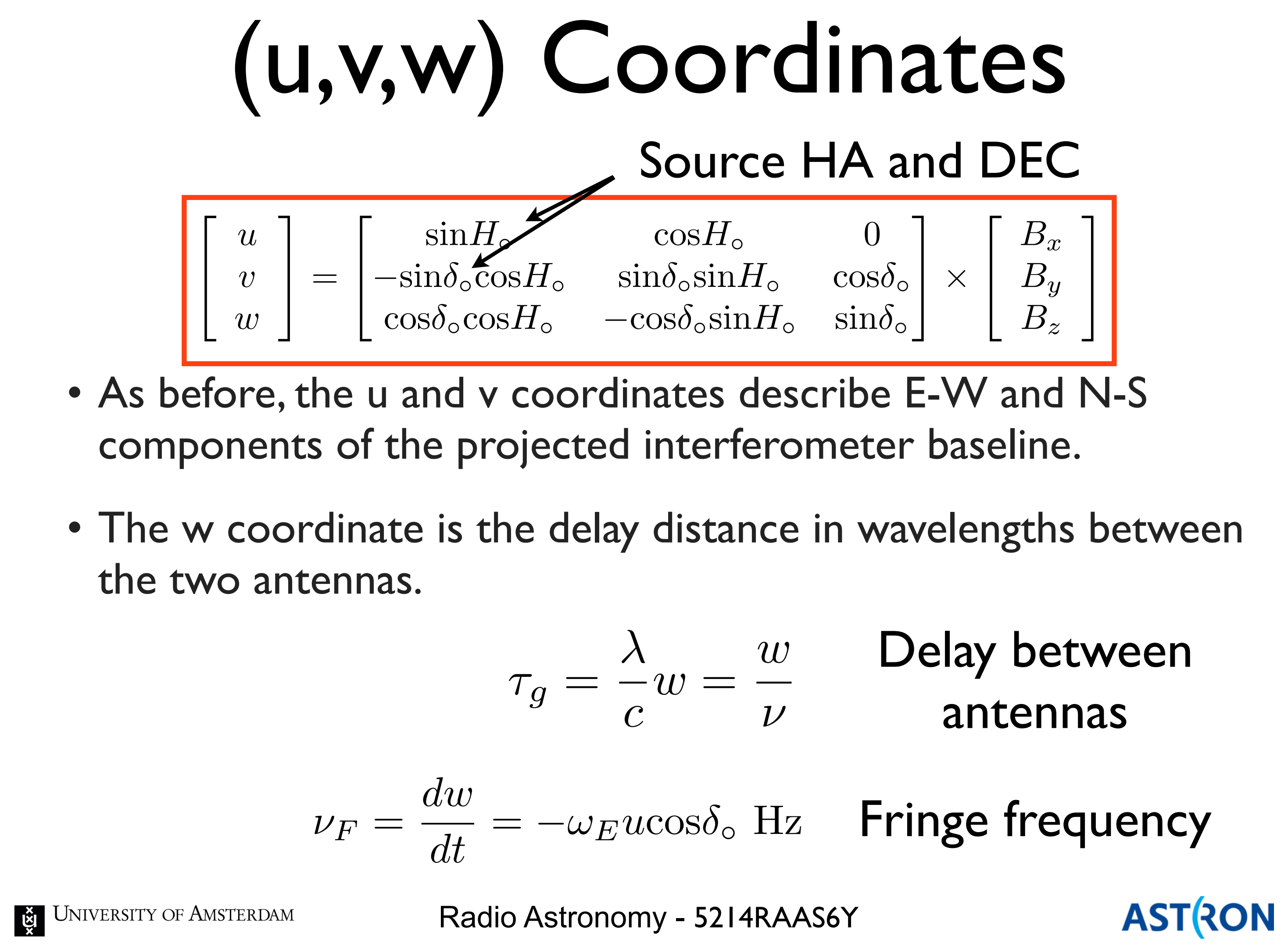
$X \equiv H = 0, \delta = 0$ (where Meridian intersects Celestial Equator)

$Y \equiv H = -6, \delta = 0$ (due East, on Equator)

$Z \equiv \delta = 90 (\mathrm{NCP})$ (points to North Celestial Pole)

(Bx,By,Bz) define - in number of wavelengths - the baseline in this coordinate system

- We will use an Earth-based coordinate system to describe the telescope positions (origin at the center of the Earth).
- The baseline lengths should be in units of number of wavelengths.
- NB: (X,Y,Z) are the individual antenna _positions_, while (Bx,By,Bz) are the baseline vectors between them.
- There should be N_tel sets of (X,Y,Z), while there should be N_tel (N_tel - 1) / 2 _independent_ sets of (Bx,By,Bz).
- (Bx,By) are the projected coordinates of the baseline (in wavelengths) onto the equatorial plane of the Earth.
- By is the E-W component.
- Bz follows the Earth's rotational axis.
- delta and H denote the declination and the hour angle of the source being observed.



# (u,v,w) Coordinates

Source HA and DEC

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \sin H_\circ & \cos H_\circ & 0 \\ -\sin\delta_\circ \cos H_\circ & \sin\delta_\circ \sin H_\circ & \cos\delta_\circ \\ \cos\delta_\circ \cos H_\circ & -\cos\delta_\circ \sin H_\circ & \sin\delta_\circ \end{bmatrix} \times \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$$

- As before, the u and v coordinates describe E-W and N-S components of the projected interferometer baseline.
- The w coordinate is the delay distance in wavelengths between the two antennas.

$$\tau_g = \frac{\lambda}{c} w = \frac{w}{\nu}$$

Delay between antennas

$$\nu_F = \frac{dw}{dt} = -\omega_E u \cos\delta_\circ \ \mathrm{Hz}$$

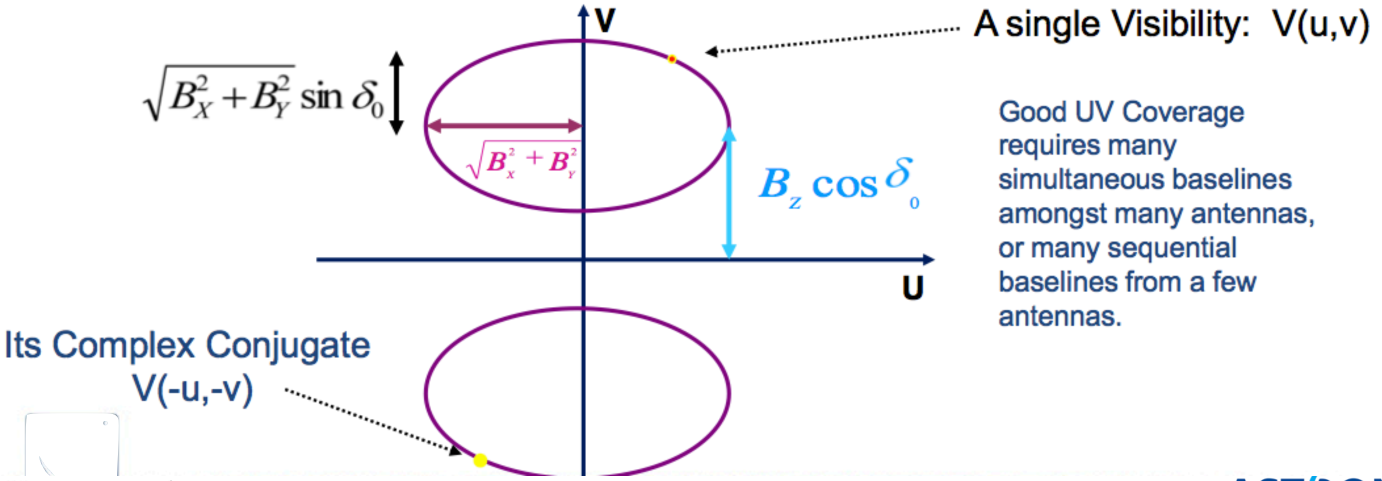
Fringe frequency

- This matrix equation allows us to calculate the uvw-coordinates of the various baseline vectors. (it can be derived using straightforward spherical trigonometry)
- u gives the E-W component of the projected baseline.
- v gives the N-S component of the projected baseline.
- w gives us the delay distance (in wavelengths) between the antennas.
- The “fringe frequency” is the time derivative of the w coordinate.

# Baseline locus

$$u^2 + \left(\frac{v - B_z \cos\delta_\circ}{\sin\delta_\circ}\right)^2$$

- Traces out ellipse in 24hrs.
- Brightness is real so: $V(-u,-v) = V^*(u,v)$
- E-W baselines have no v offset.



Good UV Coverage requires many simultaneous baselines amongst many antennas, or many sequential baselines from a few antennas.

- Over the course of 24 hrs, the u,v points of the various baselines will describe ellipses in the uv-plane.

# Fringe Frequency

$$\nu_F = \frac{dw}{dt} = -\omega_E u \cos\delta_\circ \ \mathrm{Hz}$$ Fringe frequency

- Fringe frequency is the rate at which a source passes through the fringe pattern of the interferometer.
- It is zero at the NCP and maximum at the Equator.
- Delay Rate tells us what extra delay is needed to keep the source at the same position w.r.t. the fringe pattern.

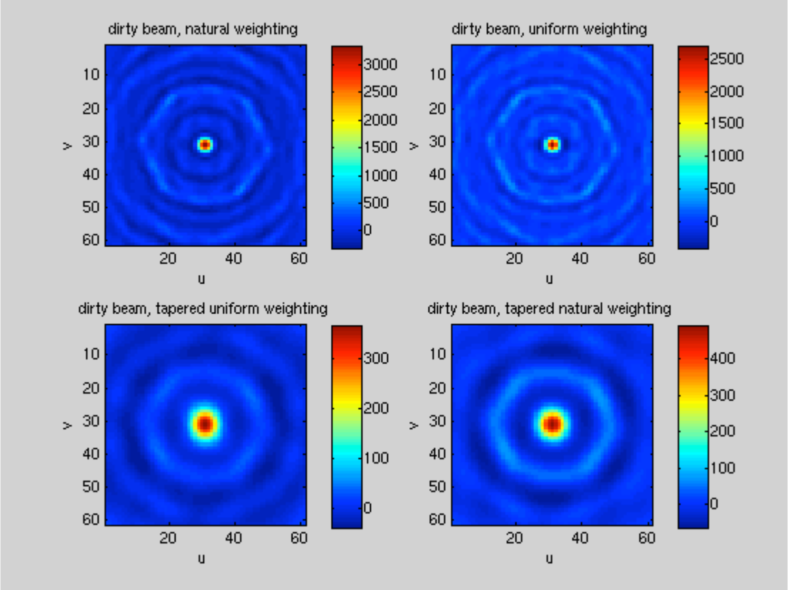
$$\frac{d\tau_g}{dt} = \frac{\lambda}{c}\frac{dw}{dt} = -\omega_E B_X \cos\delta_\circ / c$$

For Bx in km, delay rate is 0.24 Bx cos(dec) nsec/sec

- Want to keep the fringe pattern stationary w.r.t. the sky.

# Basics of making an image

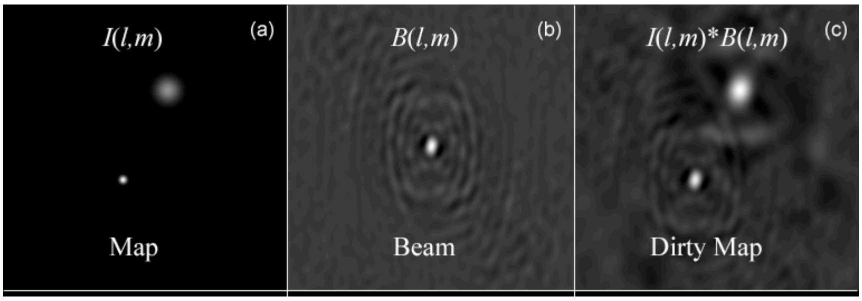
# Dirty Beam



- Essentially the synthesized beam we have considered so far.
- Not unlike the "point spread function" of the interferometer.
- Weighting the baselines differently will give different dirty beams.

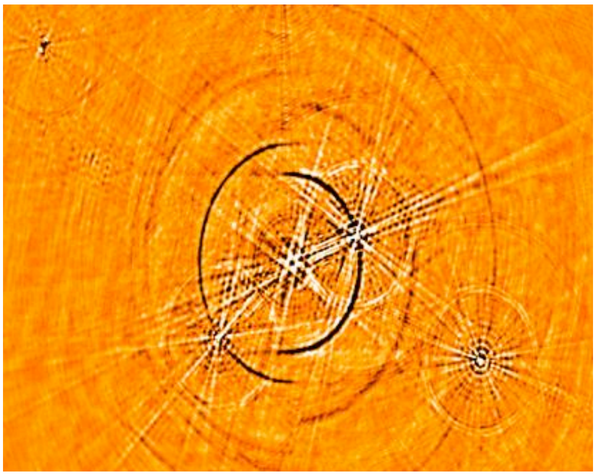
- Can make a 2-D Fourier transform of the uv-coverage in order to get the synthesized beam.

# Dirty Image



- The sky brightness convolved with the dirty beam (instrument response).
- Need to deconvolve the dirty beam in order to get the "clean image".
- More in the coming two lectures.

# Dirty Image



- Highly symmetric arrays create very distinct patterns in the dirty beam. If these are not de-convolved from the image perfectly, then they lead to artifacts.
- Randomized array (like LOFAR) is better in this regard.

# Other considerations

# A more realistic interferometer

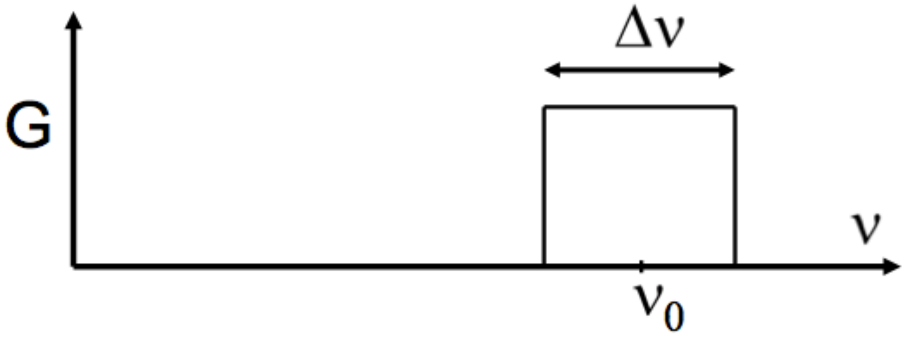
- Things aren't static. Need "delay tracking" to steer the interference pattern towards the region of interest. The sine and cosine fringe patterns move with the source.
- Frequency down-conversion. Electronics do not necessarily work at the (higher) observed sky frequency (RF).
- Non-monochromatic waves. Finite bandwidth.
- Losses from time averaging. Delay tracking only works perfectly at the phase center. All other sources are moving (slightly) w.r.t. the fringe patterns.

# Bandwidth response

- Need a finite bandwidth for good sensitivity etc.

$G(\nu)$ characterizes the amplitude and phase variation imparted on the signal by the instrument.

For example...



# Bandwidth response

Plug in to get finite bandwidth visibilities....

$$V = \int \left( \frac{1}{\Delta\nu} \int_{\nu_0 - \Delta\nu/2}^{\nu_0 + \Delta\nu/2} I(\vec{s},\nu) G_1(\nu) G_2^*(\nu) e^{-i2\pi\nu\tau_g} d\nu \right) d\Omega$$

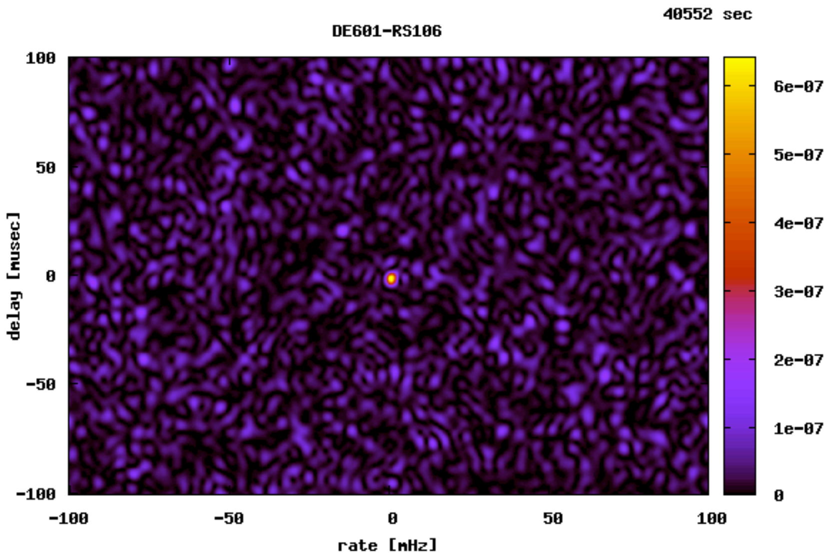
$$V = \iint I(\vec{s},\nu) \frac{\sin(\pi\tau_g\Delta\nu)}{\pi\tau_g\Delta\nu} e^{-i2\pi\nu_0\tau_g} d\Omega = \iint I(\vec{s},\nu) \text{sinc}(\tau_g\Delta\nu) e^{-i2\pi\nu_0\tau_g} d\Omega$$

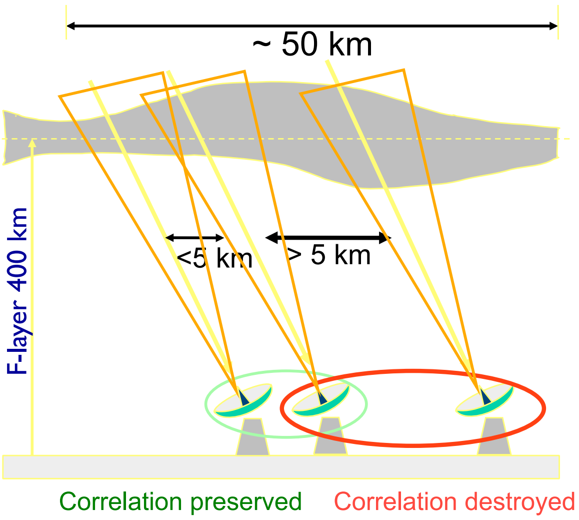
Assume source does not vary over this bandwidth and that the antennas provide the same response.

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \sim 1 - \frac{(\pi x)^2}{6} \quad (\text{for} x << 1)$$

Fringe attenuation function (limits FoV off meridian)

# Ionosphere





Greenhill

Introduces extra, dynamic phase delays between the antennas

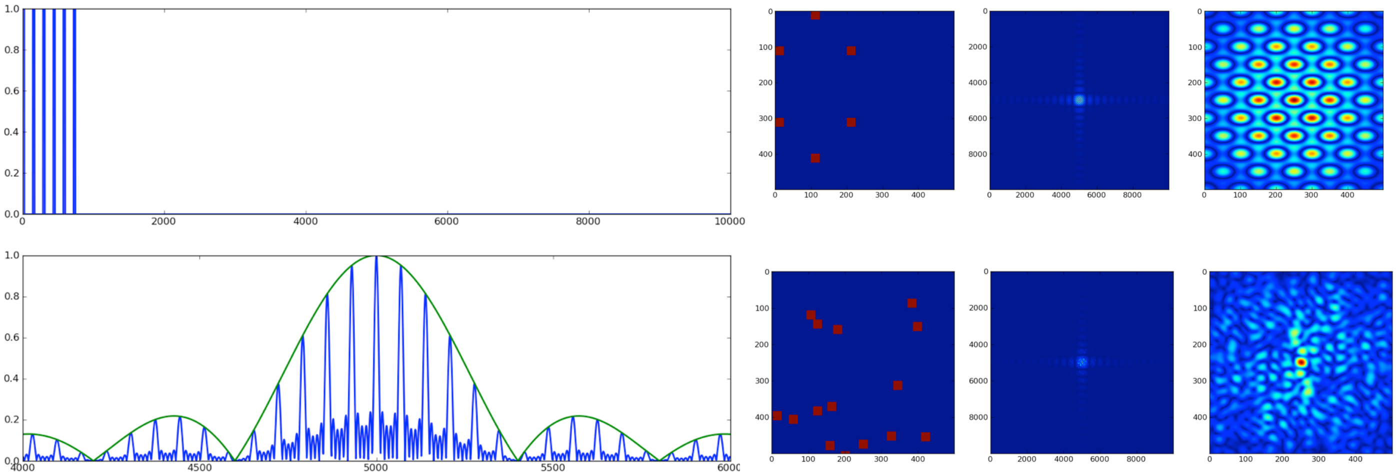
# Next Two Lectures

Lots more on calibration and imaging

# Questions?

# Today’s Practicum

## Simulate your own radio interferometer!



# Today's Practicum Suggestions

- Use Python (unless you're an expert with something else).
- Use the numpy and scipy Python packages.
- Start with 1-D case then expand to 2-D if.
- Try both random and symmetric configurations.
- Get the units right on the axes.

# Sources

**NRAO Synthesis Summer School Lectures (Perley)**

New Jersey IT Lectures

http://web.njit.edu/~gary/728

Other course slides (see links on course wiki page):

http://www.astron.nl/astrowiki/doku.php?id=uva_msc_radioastronomy_2013

# 3-D Fourier relation

Infer the source brightness from the visibilities

$$V_\nu(u,v,w) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-i2\pi(ul+vm+wn)} dldm$$

$$V_\nu(u,v,w) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-i2\pi[ul+vm+w(\sqrt{1-l^2-m^2}-1)]} dldm$$