UNIVERSITY OF AMSTERDAM

Master Astronomy and Astrophysics - 5214RAAS6Y

Radio Astronomy

Lecture 6

The Techniques of Radio Interferometry I: Basics

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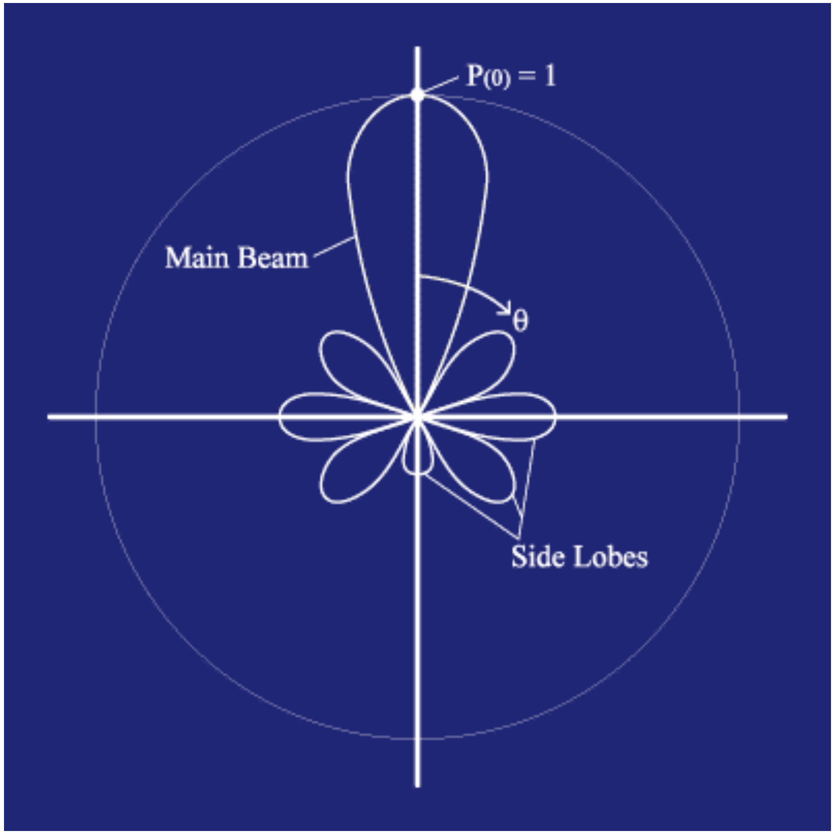
G2.213 - April 18th, 2013

Lecture outline

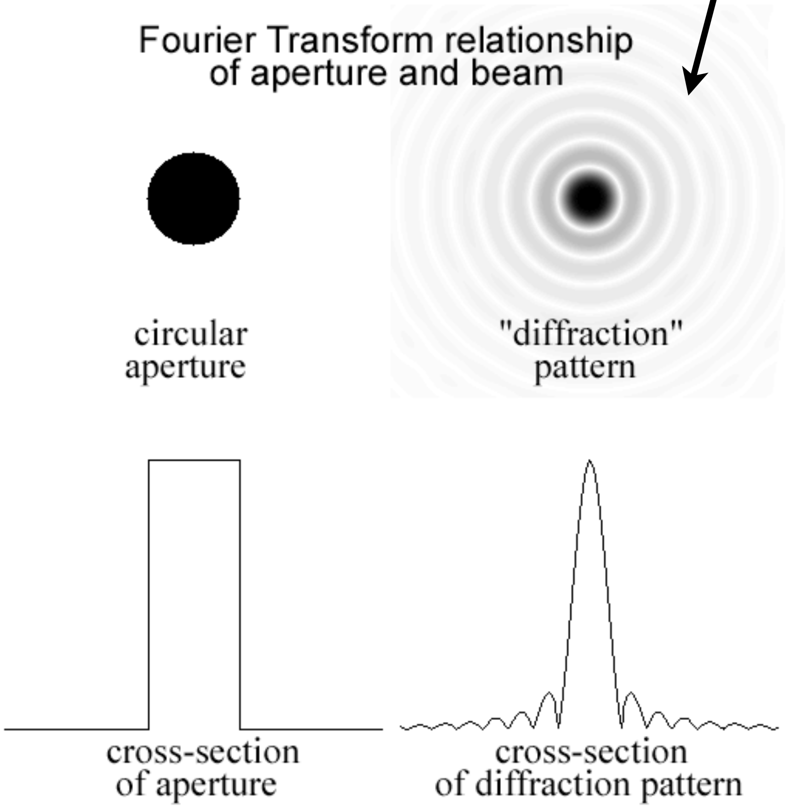
- Quest for resolution
- Terminology
- Basic radio interferometer and correlator
- Visibilities and the uv-plane
- Basics of making an image
- Other considerations

Quest for resolution

Radio telescope FoV



Bessel Function



Sinc Function

The quest for resolution

We want sub-arcsecond resolution (cf. optical, X-ray)

$$\Theta_{\mathrm{rad}} \propto \frac{\lambda}{D}$$

Unlike large, ground-based optical telescopes, radio telescopes are always diffraction limited.

$$\Theta_{\mathrm{arcsec}} \sim 2\frac{\lambda_{\mathrm{cm}}}{D_{\mathrm{km}}}$$

So to get 1 arcsec resolution at 21cm requires a 42km diameter!

The quest for resolution

Fortunately, we don’t have to build a single 42-km-wide radio dish

Aperture synthesis

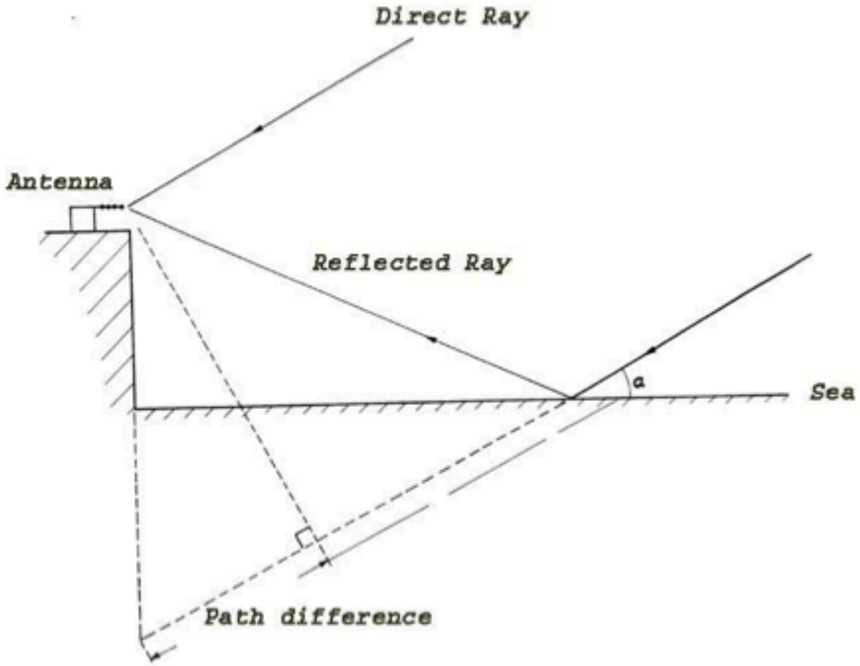
$$\Theta \propto \frac{\lambda}{D} \propto \frac{\lambda}{B}$$

B = Baseline

"Sea" interferometry

(mid 1940s)

Recall from Lecture #1



Dover Heights near Sydney, Australia

Modern interferometers

(see Lecture 1 for more details)

ATCA

JVLA

LOFAR

AMI

WSRT

GMRT

Modern interferometers

$$N_{\mathrm{baselines}} = N_{\mathrm{elements}}(N_{\mathrm{elements}} - 1)/2$$

27 antennas
351 baselines

JVLA

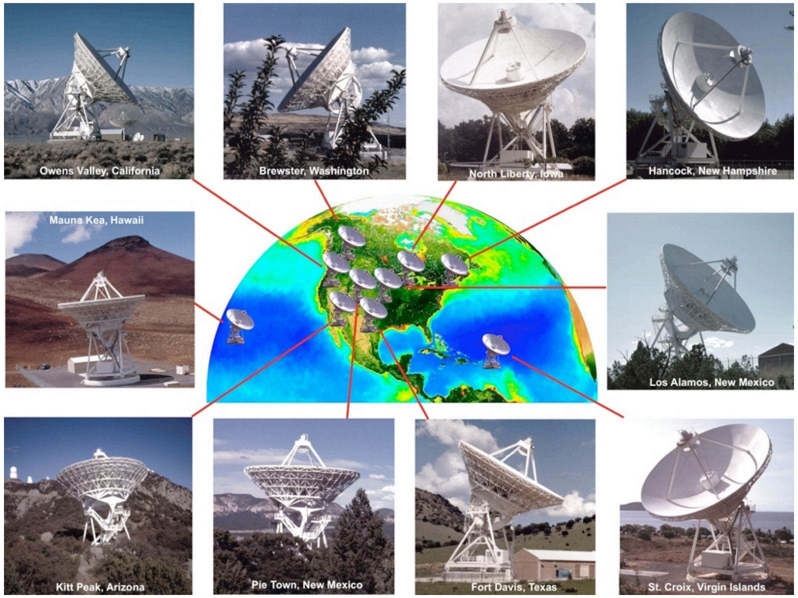
14 antennas
91 baselines

WSRT

Very Long Baseline Interferometry

1000-km baselines

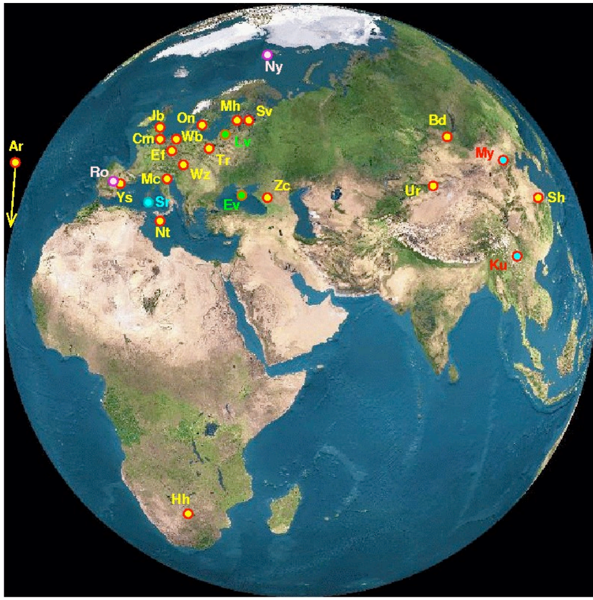
Data recorded locally and shipped to correlator (though moving towards more real-time)



VLBA, USA

Image by Paul Boven (boven@jive.nl), satellite image by NASA Visible Earth (visibleearth.nasa.gov)

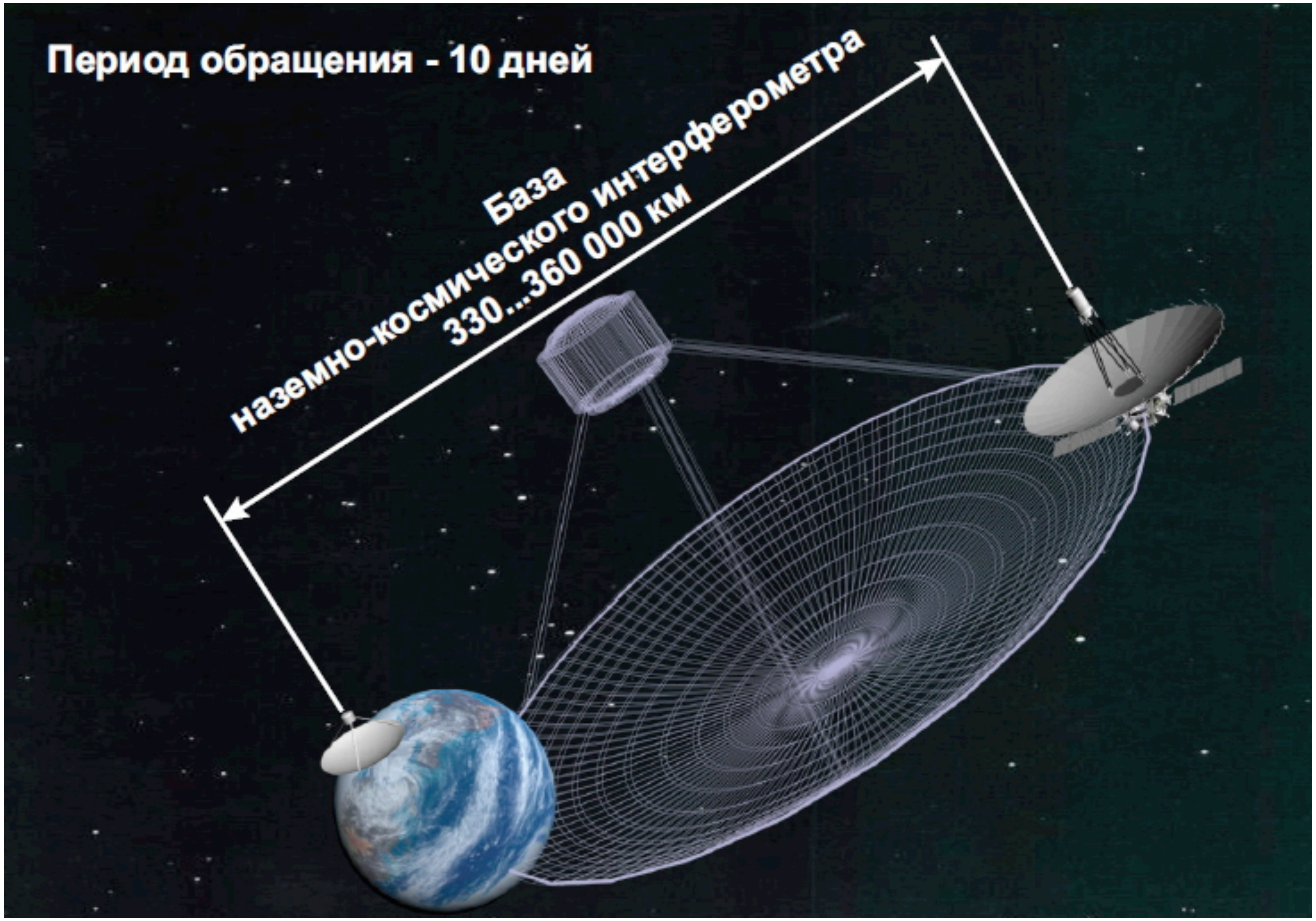
EVN, Europe



Global VLBI

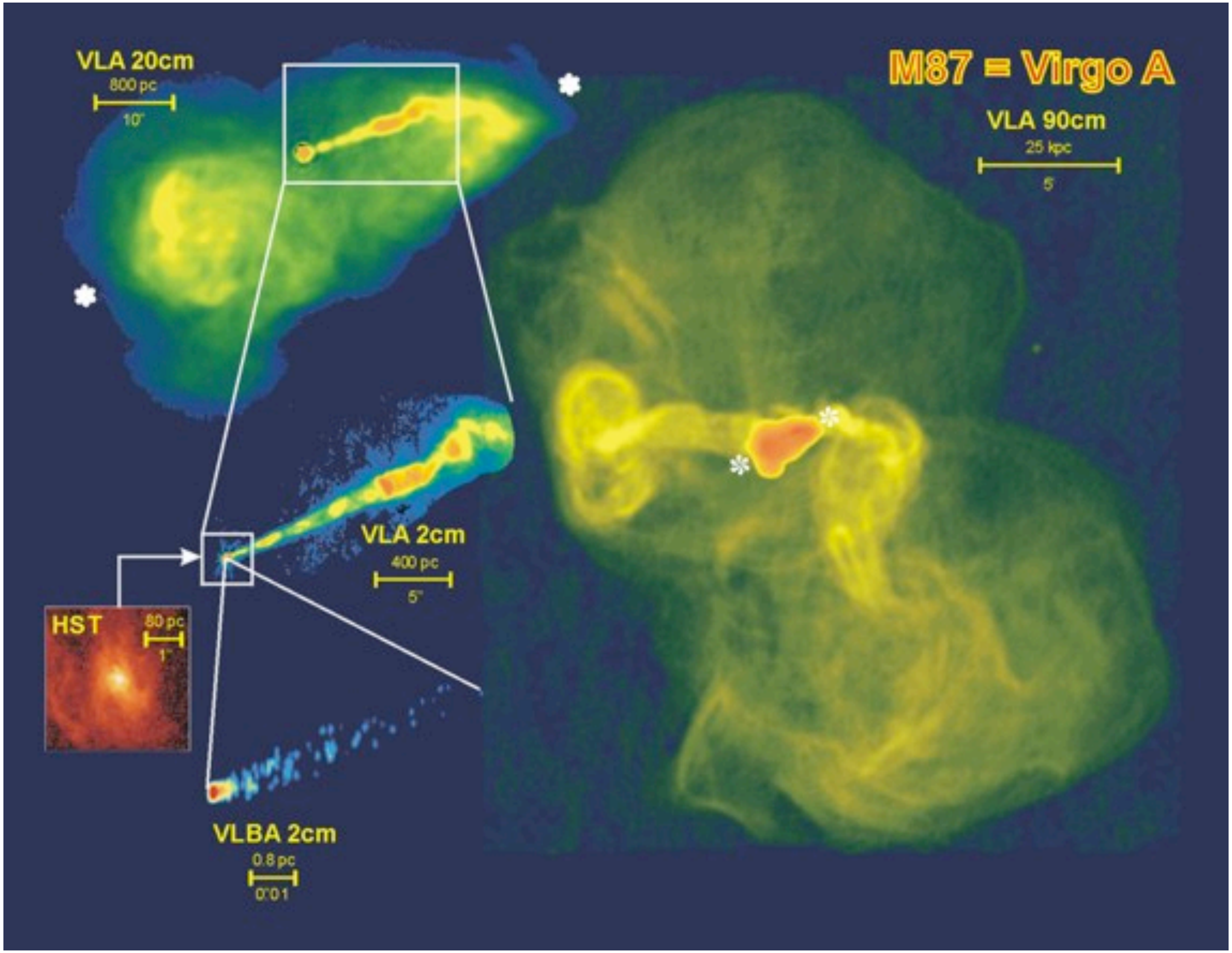
Very Long Baseline Interferometry

100,000-km baselines



RadioAstron

The quest for resolution



Probe milli-arcsecond scales

Terminology

Terminology

“Specific Intensity” or “Brightness”

Energy per unit time, area, frequency, and solid angle

$$[I(\vec{s}, \nu, t)] = \mathrm{erg\ s^{-1}\ cm^{-2}\ Hz^{-1}\ ster.^{-1}}$$

“Specific Flux Density”

Integrate over all angles

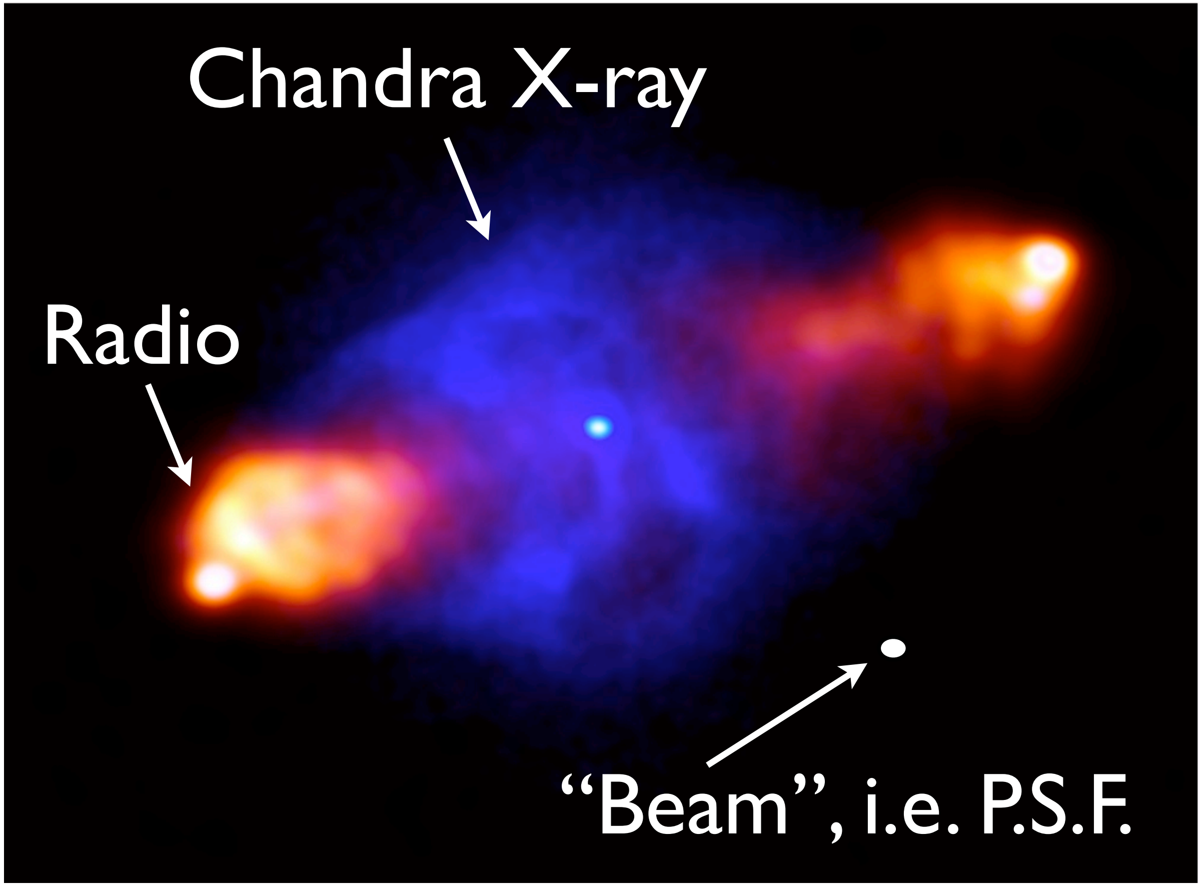
$$S = \int I(\vec{s}, \nu, t) d\Omega$$

$$[S(\nu, t)] = \mathrm{erg\ s^{-1}\ cm^{-2}\ Hz^{-1}}$$

Measured in Janskys $1\mathrm{Jy} = 10^{-23}\mathrm{erg\ s^{-1}\ cm^{-2}\ Hz^{-1}}$

Terminology

Brightness in “Jy/beam”

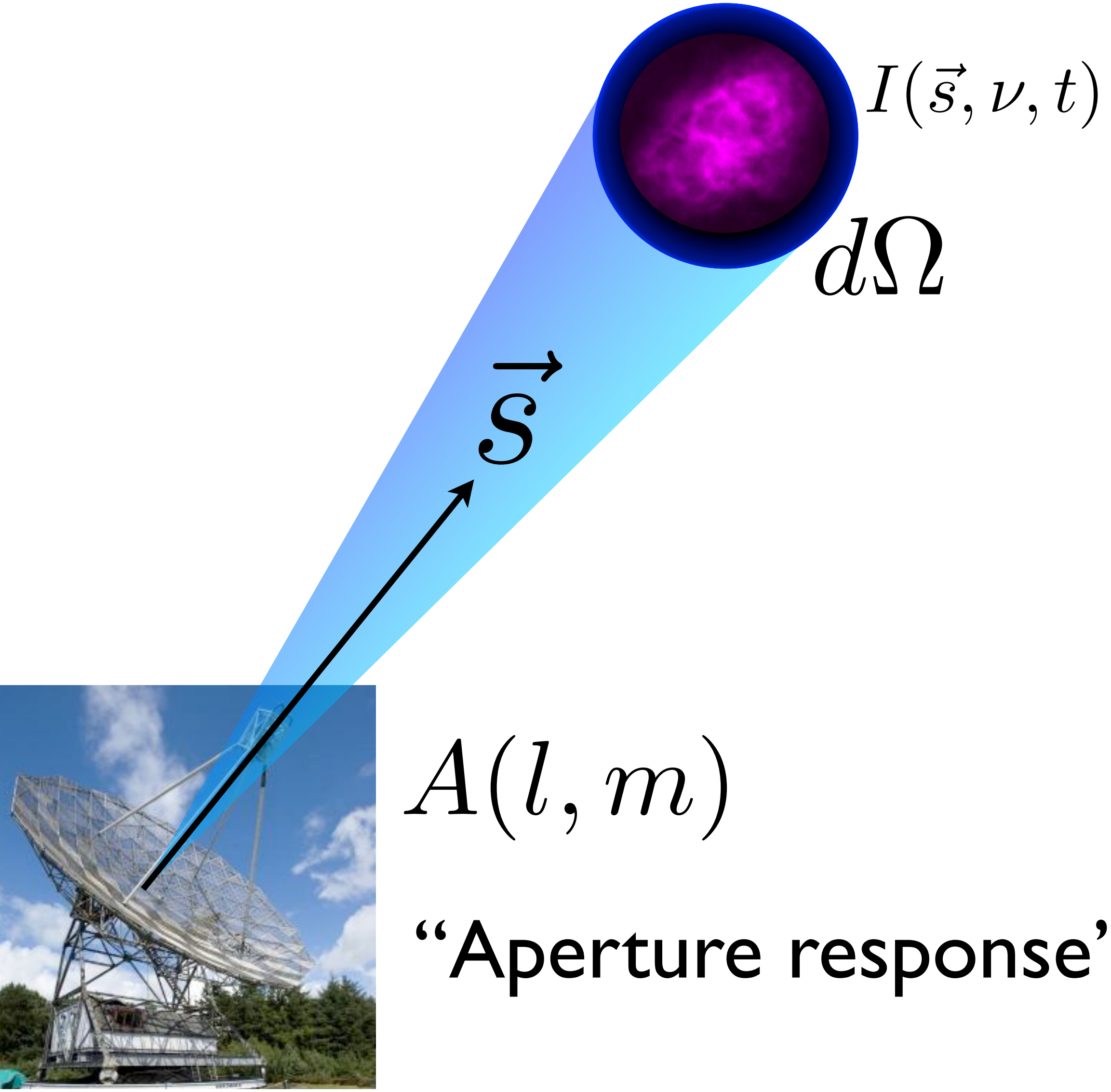


LOFAR Cygnus A

- 1 beam ~ 100 sq. arcseconds.
- Peak is ~10Jy/beam
- Source flux density ~1000 Jy

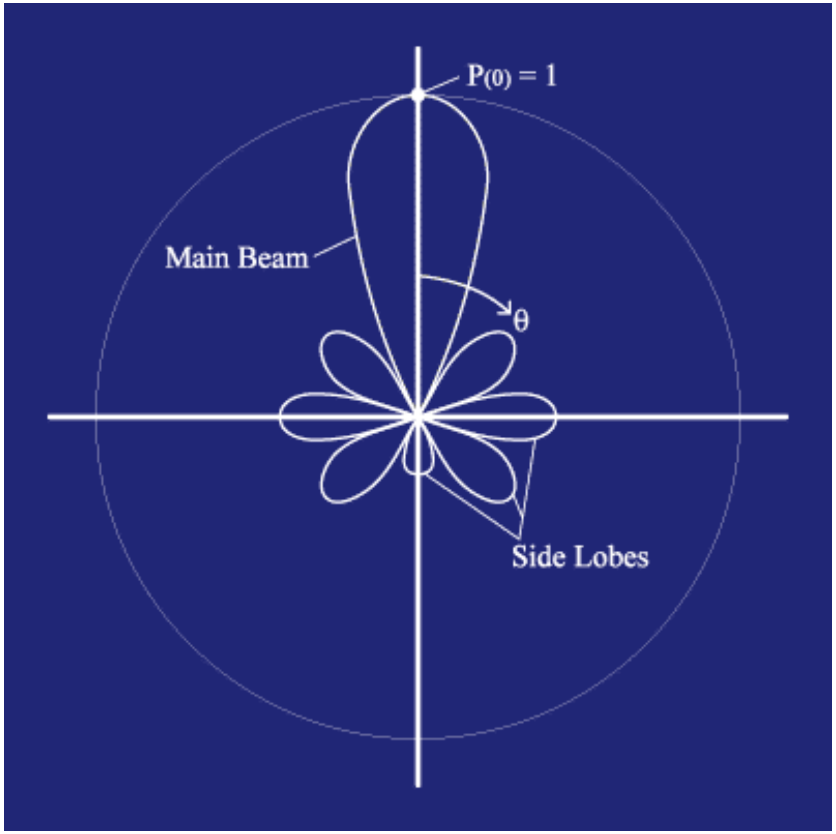
Terminology

"Primary beam"



$A(l, m)$

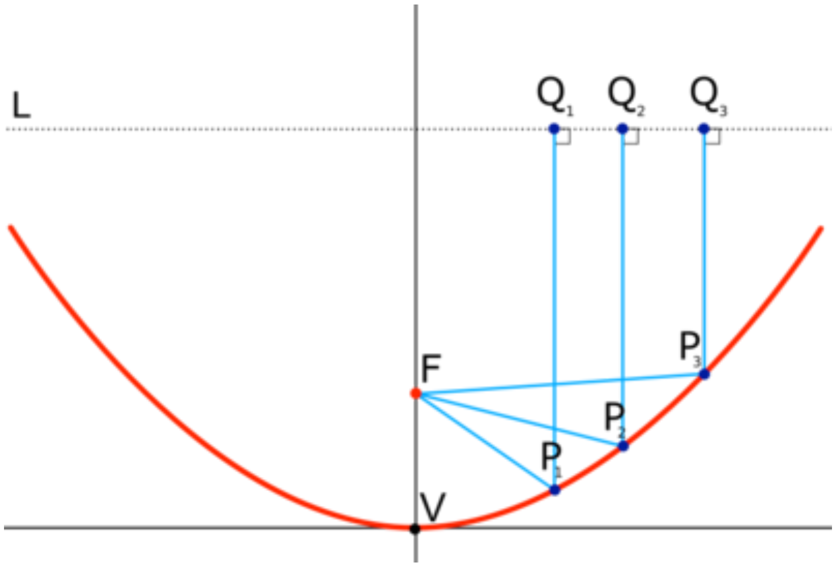
"Aperture response"



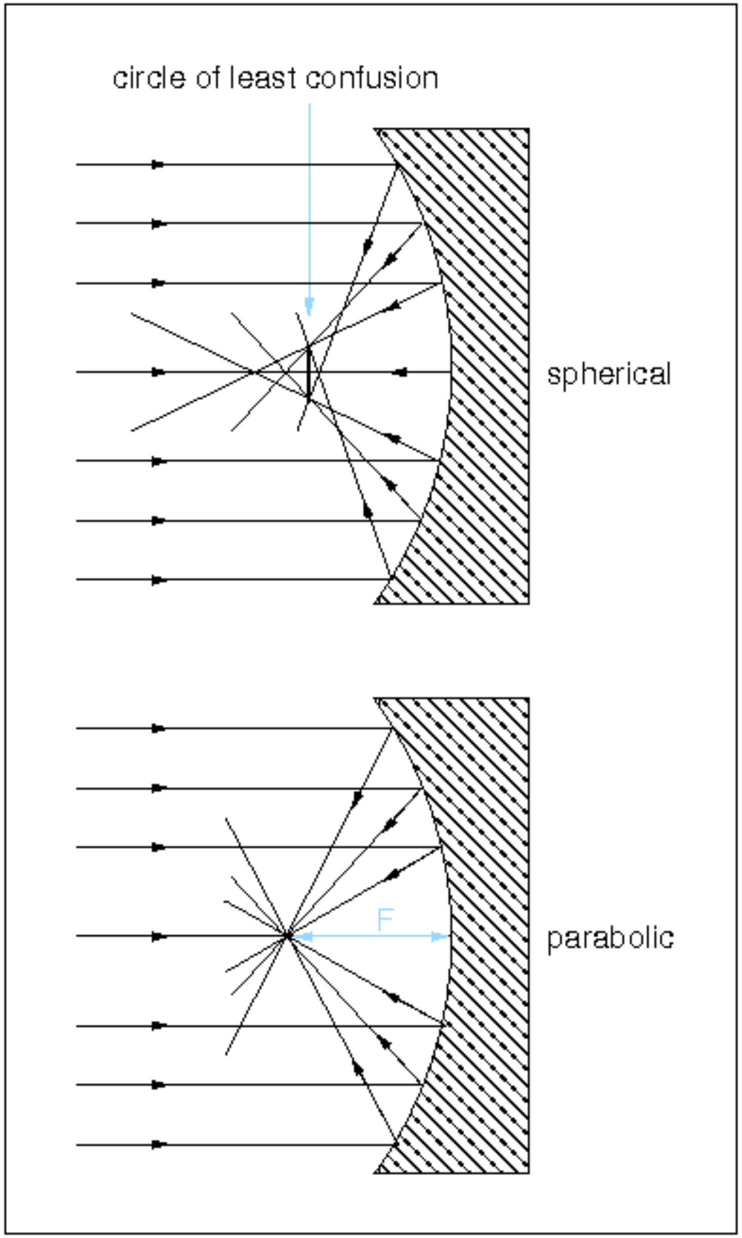
$$\Theta \propto \frac{\lambda}{D}$$

Terminology

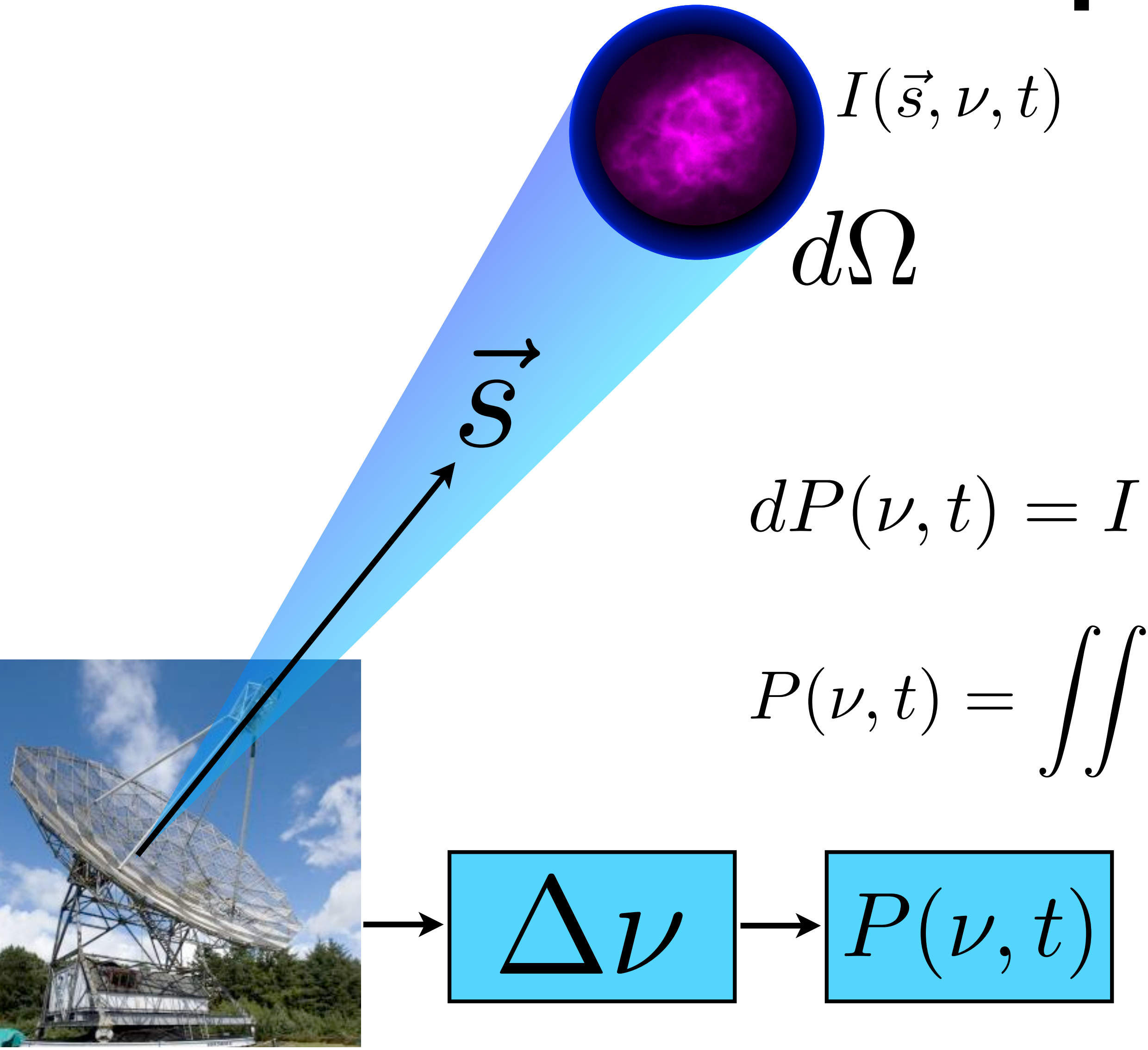
“Parabolic reflector”



- Electric field is in phase at the focus because all rays from a parallel wavefront travel the same distance.
- NB: a spherical reflector (like Arecibo) will focus to a line.



From emitted brightness to received power

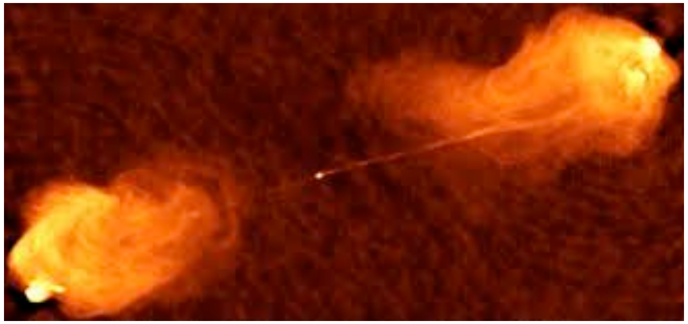


$$dP(\nu,t) = I(\vec{s},\nu)\ A(\vec{s},\nu)\ d\nu d\Omega$$

$$P(\nu,t) = \iint I(\vec{s},\nu)\ A(\vec{s},\nu)\ d\nu d\Omega$$

Basic radio interferometer and correlator

Radio interferometric imaging



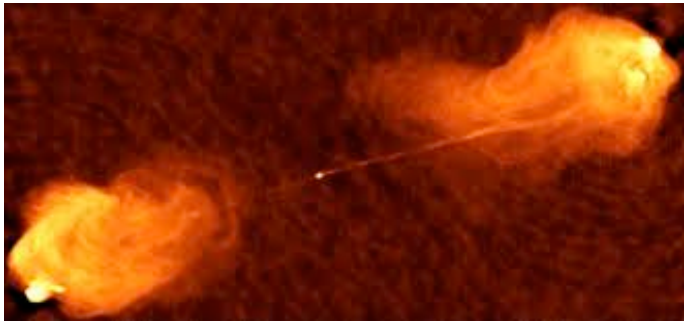
$I(\vec{s}, \nu, t)$

$P_0(\nu, t)$ $P_1(\nu, t)$ $P_2(\nu, t)$

Key questions for this lecture...

- How do we relate the brightness of the radio sky to the power received by the antennas?
- How do we turn this into a radio image?

Radio interferometric imaging



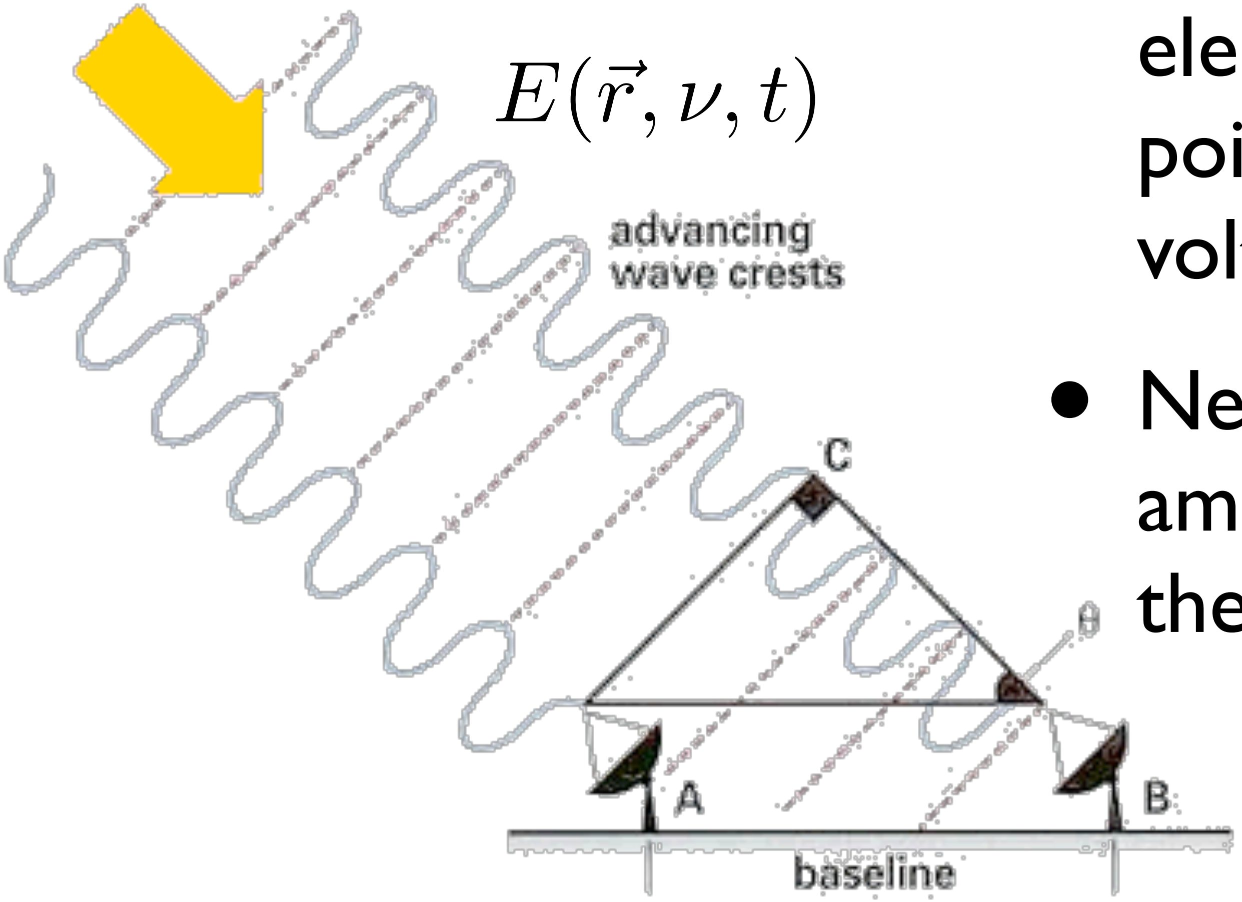
$I(\vec{s}, \nu, t)$

$P_0(\nu, t)$ $P_2(\nu, t)$ $P_1(\nu, t)$

- Need to correlate the received electric field (signal) at various geographically separate locations.

Simple interferometer

- Sample an incoming electric field at various points and convert to voltages.
- Need to register both amplitude and phase of the wave.

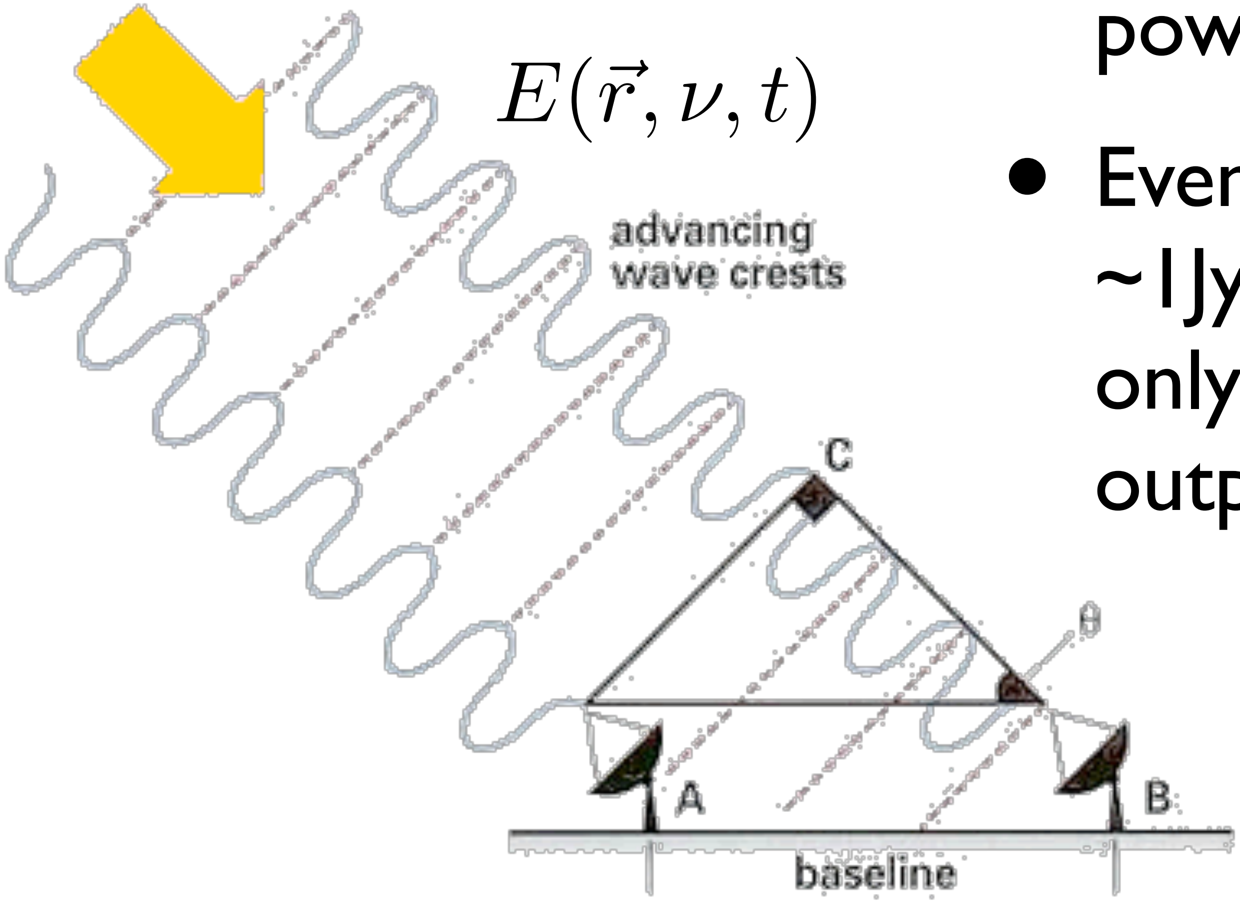


$$E(\vec{r}, \nu, t)$$

$$P(\nu, t) \propto V^2(\nu, t)$$

$$V_0(\nu, t) \qquad V_1(\nu, t)$$

Simple interferometer



$E(\vec{r}, \nu, t)$

$V_0(\nu, t)$ $V_1(\nu, t)$

- Antenna adds additional power to the signal.
- Even a bright source of ~1Jy will still constitute only ~0.5% of the outputted power.

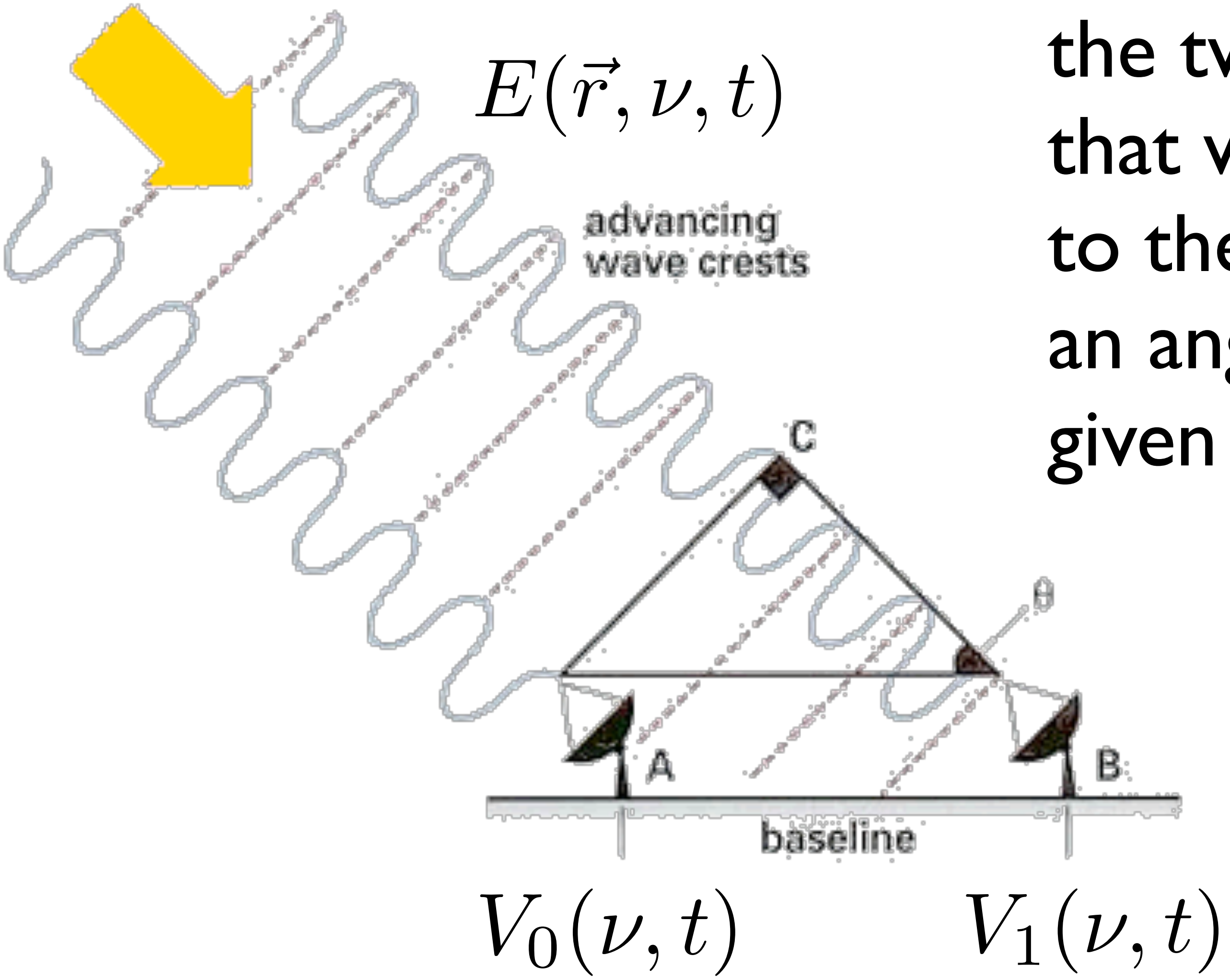
$$P(\nu, t) \propto V^2(\nu, t)$$

Simple interferometer

Start with an overly simplistic example

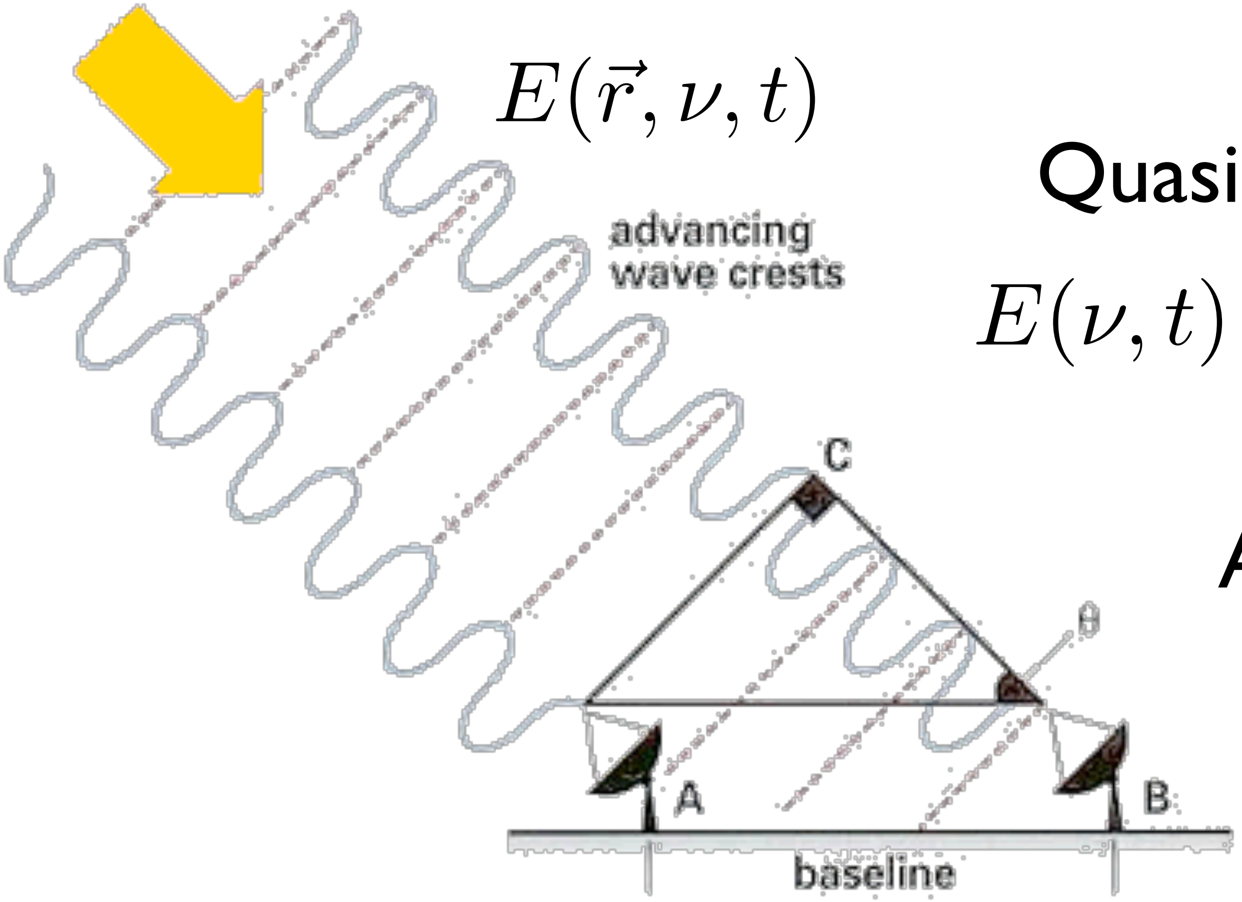
- Interferometer is fixed w.r.t. the sky (an instant in time).
- Quasi-monochromatic waves.
- Interferometer directly measures the sky frequency (“RF interferometer”).
- Single polarization.
- No distortions from ionosphere.
- Identical elements and perfect electronics.

Simple interferometer



- How can we combine the two voltages such that we can relate them to the sky brightness at an angular resolution given by the baseline?

Simple interferometer



$$E(\vec{r}, \nu, t)$$

Quasi-monotonic waves

$$E(\nu, t) = E\cos(2\pi\nu t + \phi)$$

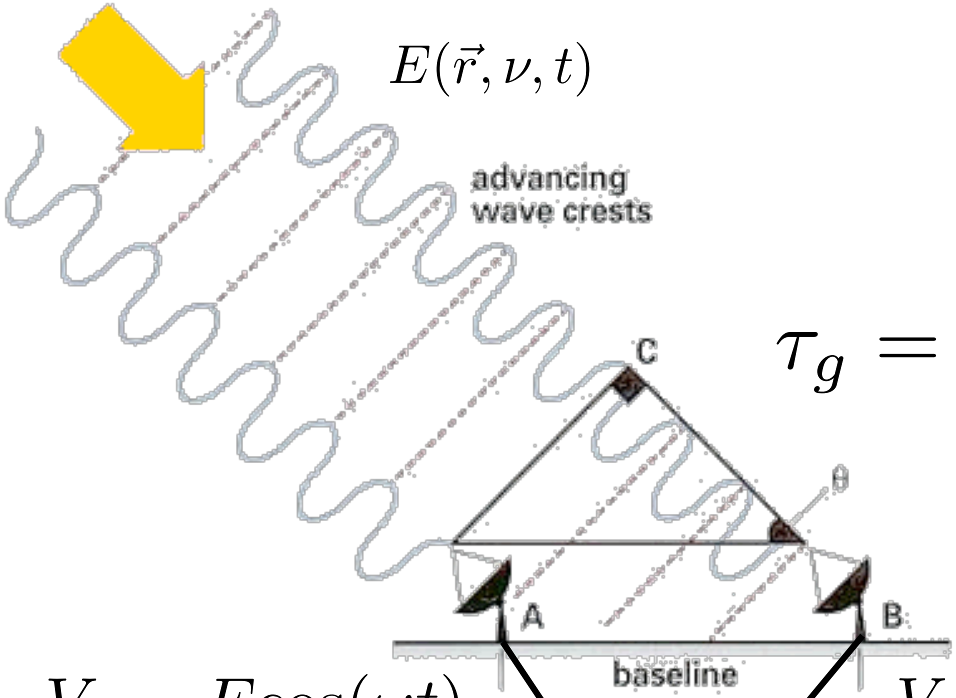
Amplitude and phase constant for

$$dt \sim 1/d\nu$$

$$V_0(\nu, t) \qquad V_1(\nu, t)$$

Simple interferometer

$E(\vec{r}, \nu, t)$



$$\tau_g = \frac{\vec{b} \cdot \vec{s}}{c}$$

Time delay between antennas

$V_0 = E\cos(\omega t)$

$V_1 = E\cos[\omega(t - \tau_g)]$

Multiply $\boxed{P[\cos(\omega\tau_g) + \cos(2\omega t - \omega\tau_g)]}$

Average $\boxed{R_C = P\cos(\omega\tau_g)}$

Varies rapidly, averages to zero

Simple interferometer

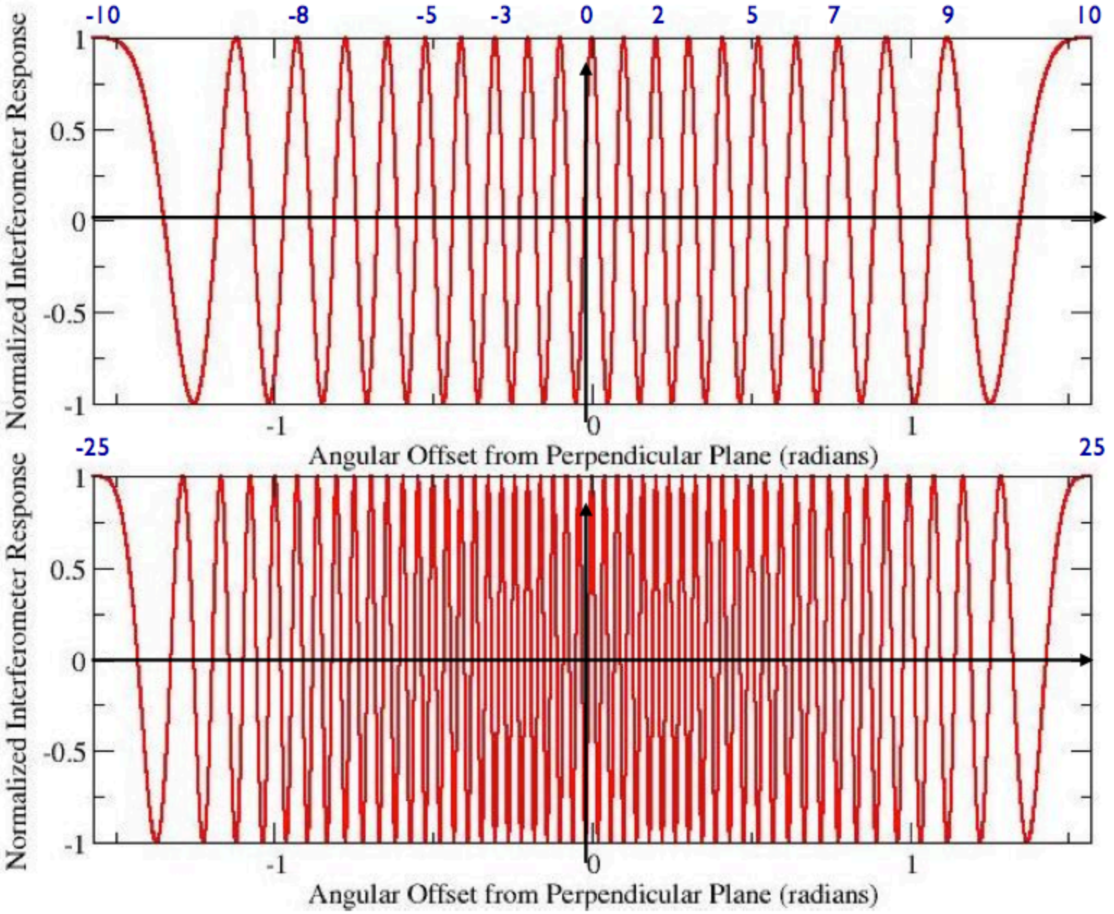
$$R_C = P\cos(\omega\tau_g) = \underset{\overset{\uparrow}{\text{Electric field}}}{P}\cos\left(2\pi\underset{\overset{\uparrow}{\text{Geometric delay}}}{\frac{\vec{b}\cdot\vec{s}}{\lambda}}\right)$$

- In other words, the response *Rc* depends on the received field and orientation of the baseline w.r.t. the source.

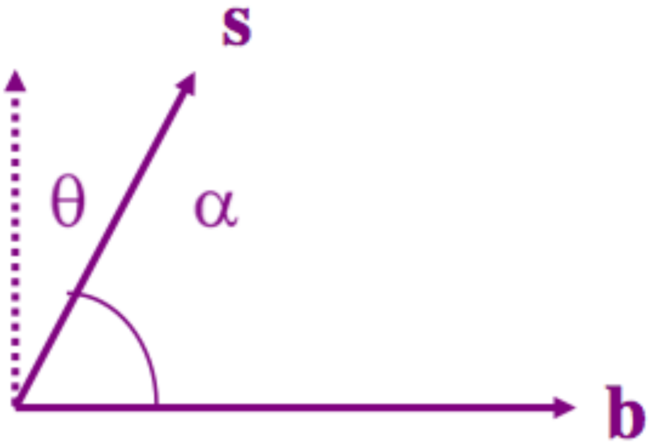
- Doesn’t depend on observation epoch, location of baseline (distance), or incoming signal phase (source is in the far field).

Simple interferometer

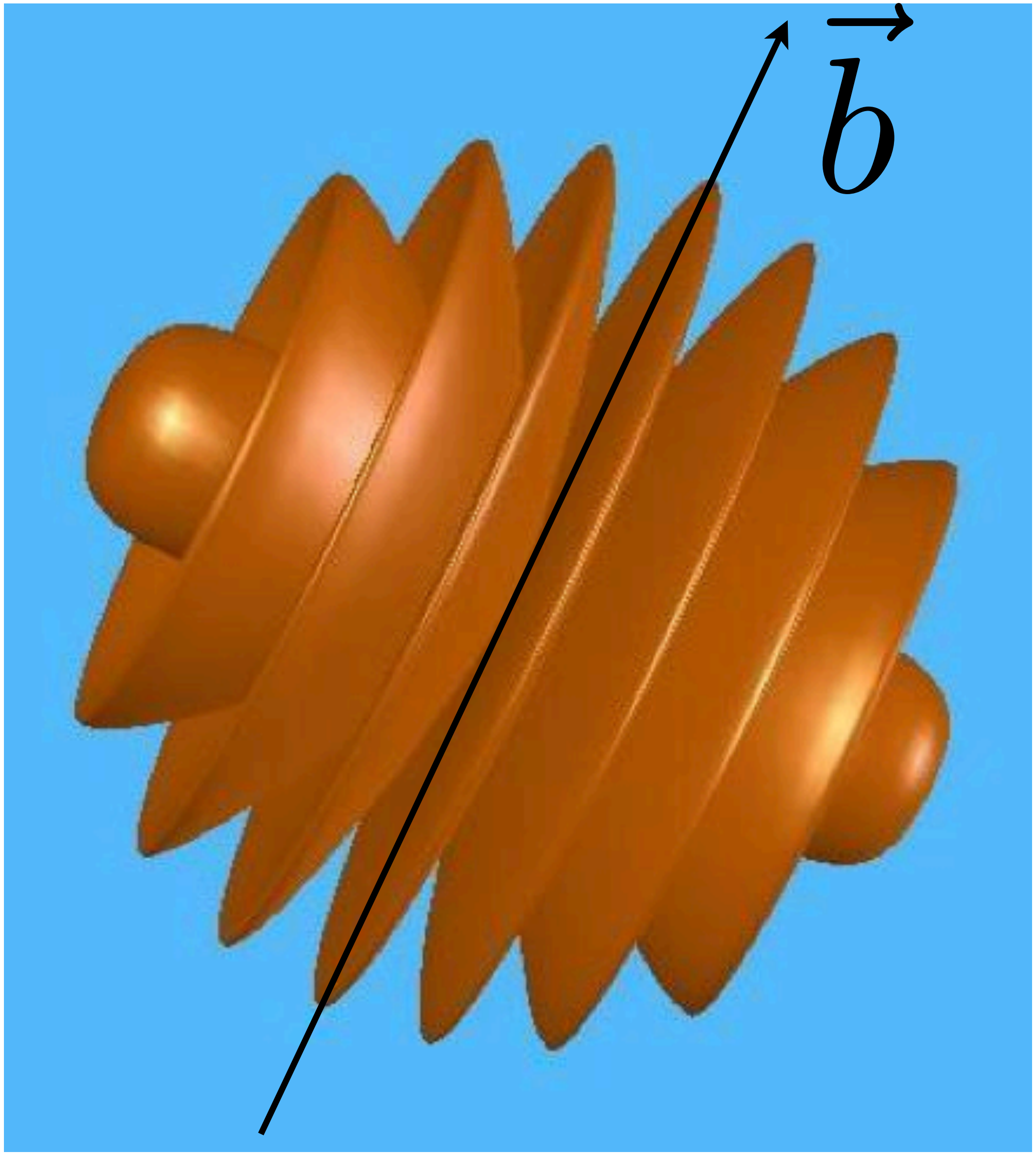
$$\frac{\vec{b}\cdot\vec{s}}{\lambda} = u\cos\alpha = u\sin\Theta = ul$$



Now consider the baseline in terms of the wavelength



Simple interferometer

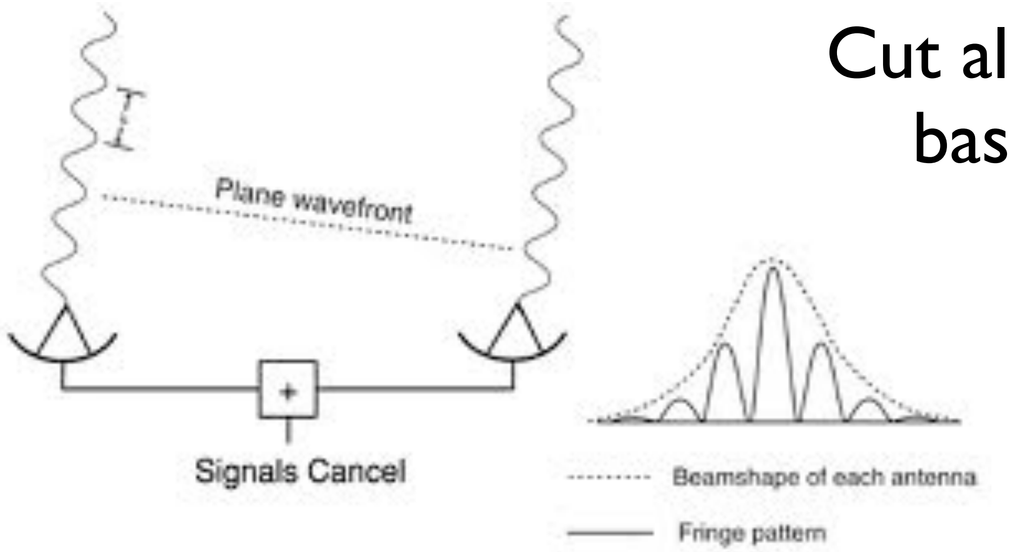


How things actually look in 2-D

Over the whole sky, u=4.

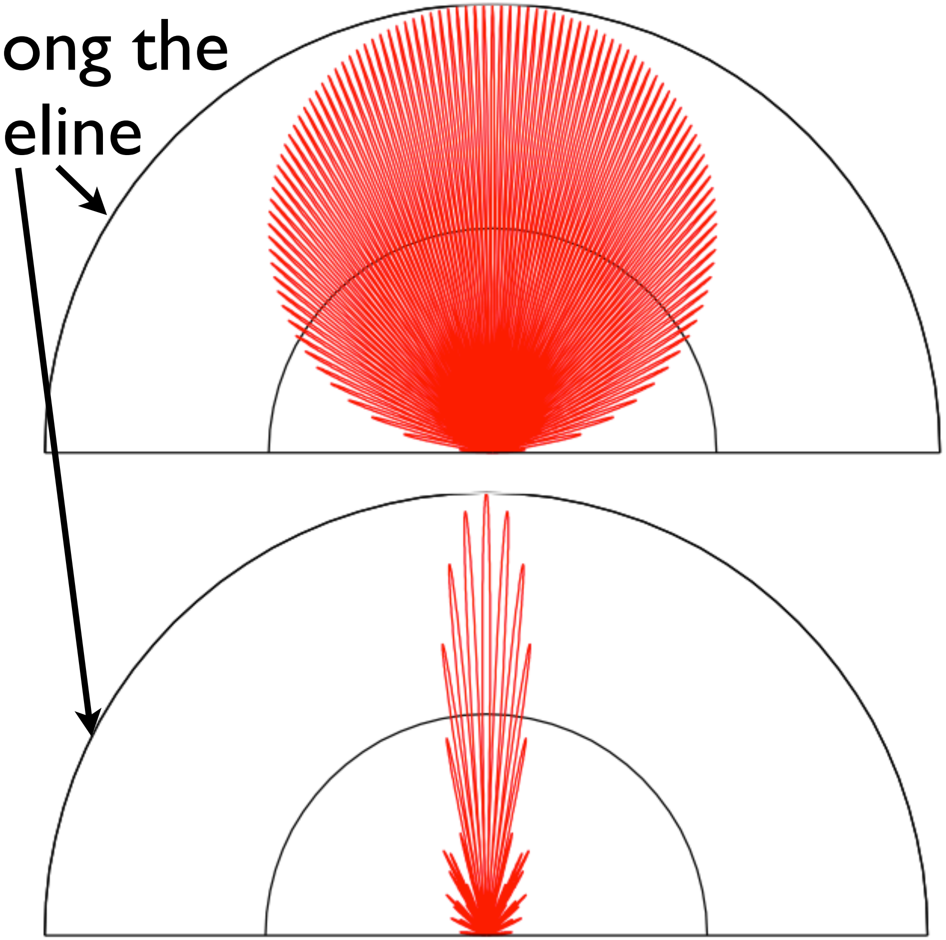
Simple interferometer

Don’t forget the effect of the aperture, which modulates the amplitude and phase of the received signal



LOFAR-like response

Cut along the baseline



More directional response

This adds an envelope to the response

Extended source response

For each element, sum over the sky and average:

$$R_C = \left\langle \int V_1 d\Omega_1 \int V_2 d\Omega_2 \right\rangle$$

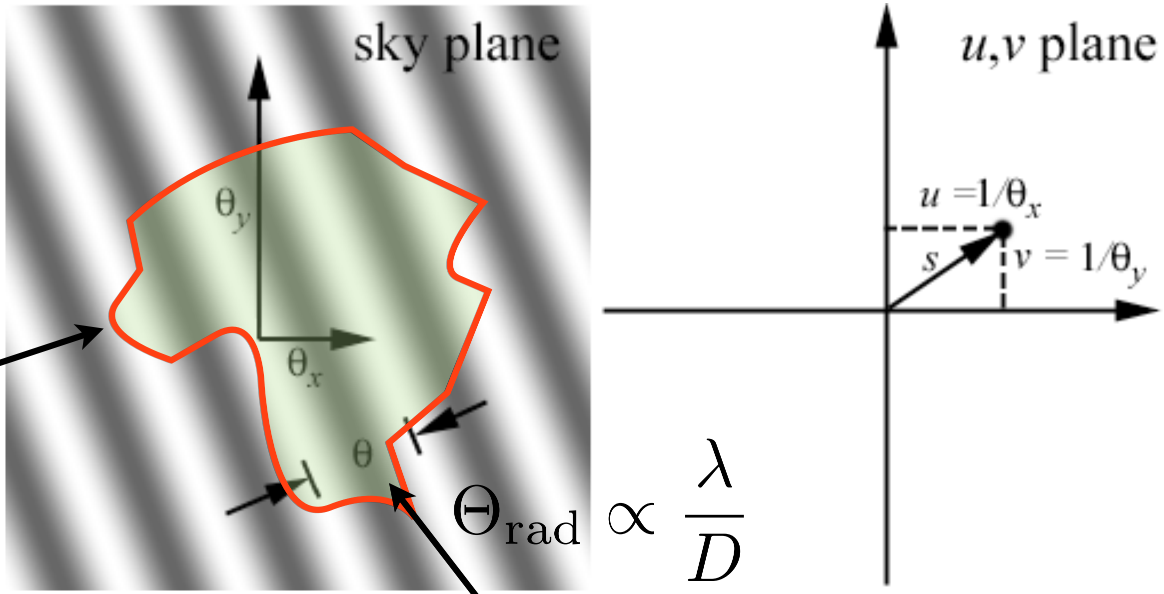
Switch order of integral/average (assume the emission is spatially incoherent):

$$R_C = \iint I(\vec{s}, \nu) \cos(2\pi\nu\vec{b} \cdot \vec{s}/c) d\Omega$$

We have now linked the interferometer response, *Rc*, and the sky intensity. But we'll need to invert to get *I*.

Extended source response

Schematically



$$\Theta_{\mathrm{rad}} \propto \frac{\lambda}{D}$$

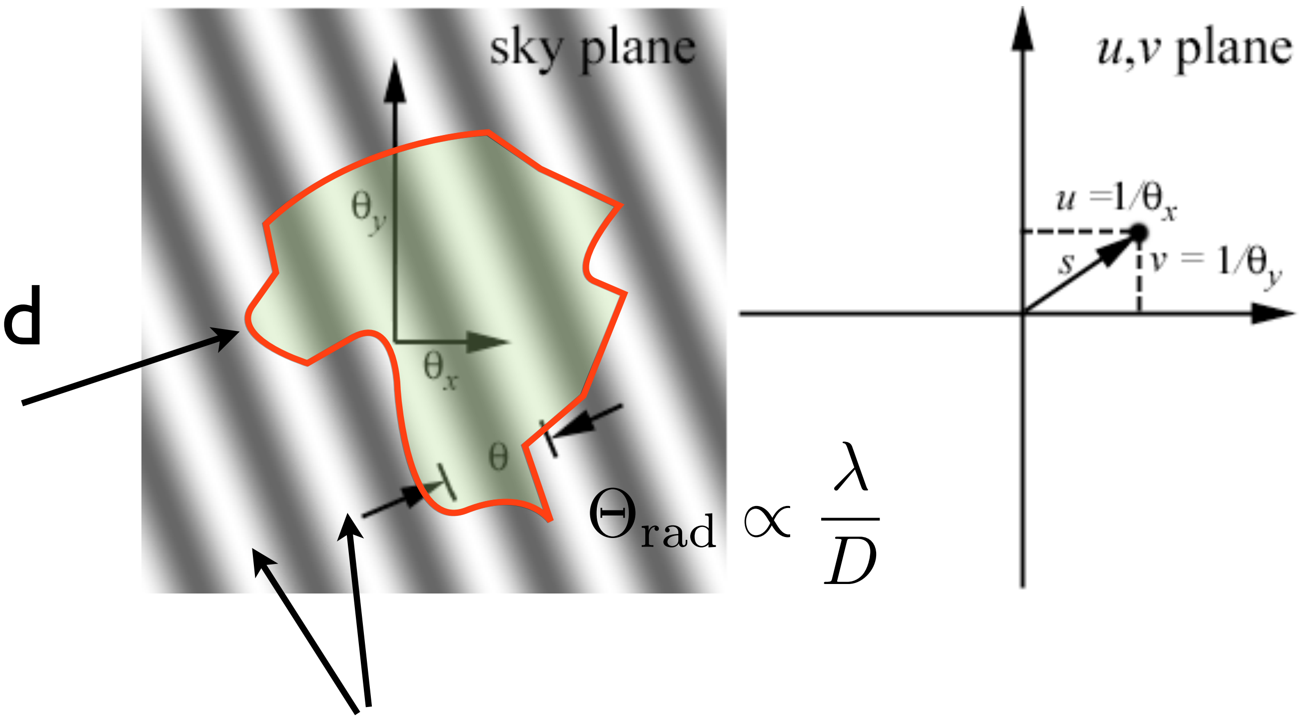
Extended Source

Orientation given by baseline geometry

Spacing given by (projected) baseline length and wavelength.

Extended source response

Schematically



Extended Source

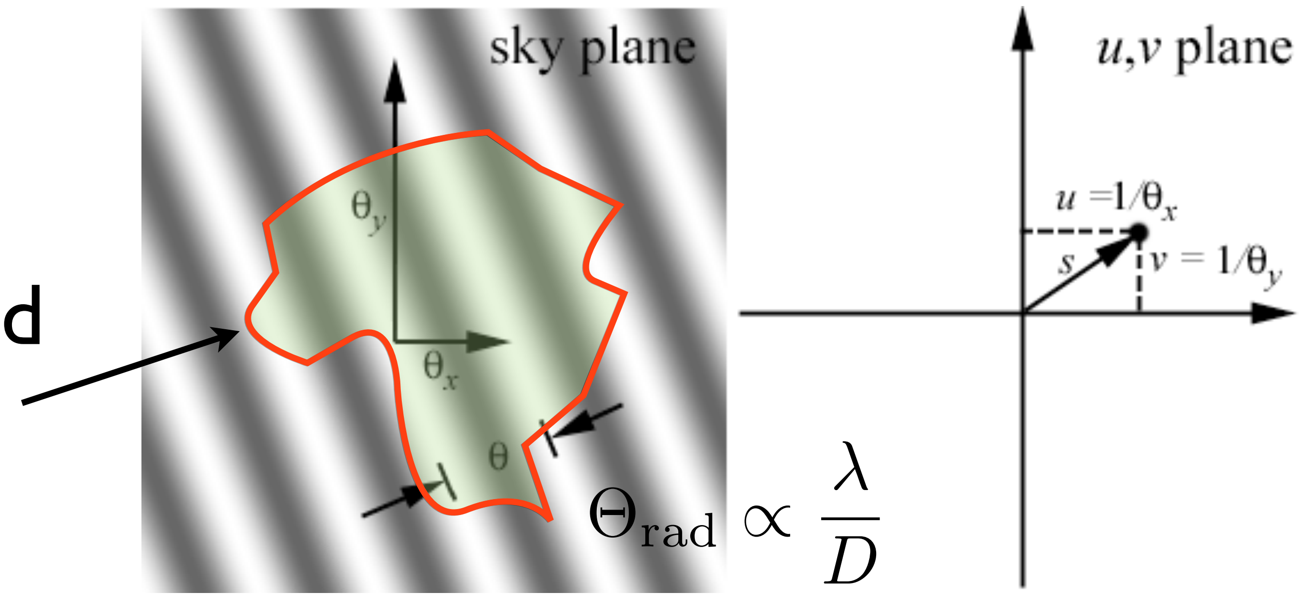
$$\Theta_{\text{rad}} \propto \frac{\lambda}{D}$$

“Fringes” (diffraction pattern): tightly packed for long baselines, widely separated for short baselines.

Extended source response

Schematically

Extended Source



$$\Theta_{\mathrm{rad}} \propto \frac{\lambda}{D}$$

- So the interferometer effectively places a cosine coherence pattern on the sky.
- The interferometer response is then the source brightness multiplied by this pattern and integrated over the whole sky.

Odd/even functions

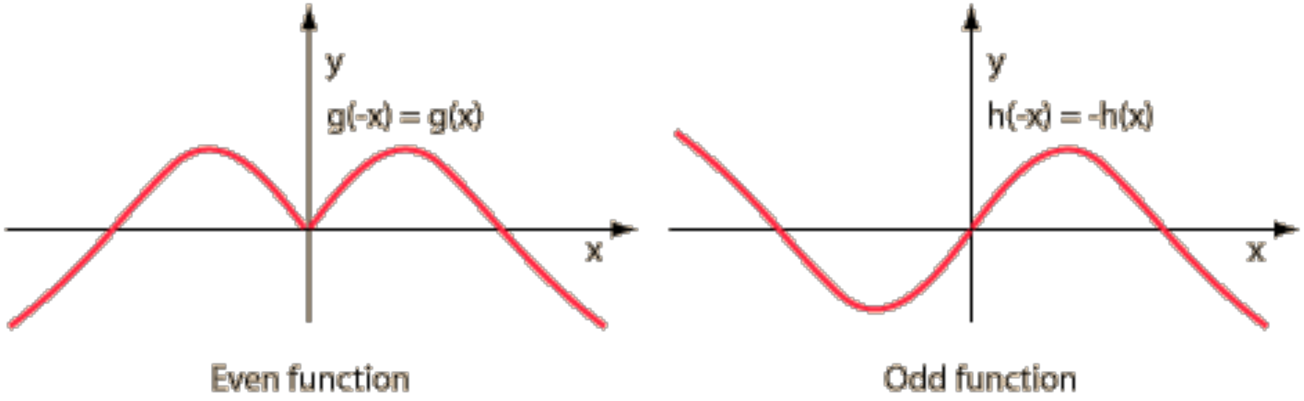
Any real function can be decomposed into an even and odd part:

$$I(x,y) = I_E(x,y) + I_O(x,y)$$

Such that:

$$I_E(x,y) = \frac{I(x,y) + I(-x,-y)}{2} = I_E(-x,-y)$$

$$I_O(x,y) = \frac{I(x,y) - I(-x,-y)}{2} = -I_O(-x,-y)$$



Correlator response

$I(\vec{s},\nu)$ is a real function with (potentially) both even and odd parts

Problem: *Rc* only samples the even part of $I(\vec{s},\nu)$

$$R_C = \iint I(\vec{s},\nu)\cos(2\pi\nu\vec{b}\cdot\vec{s}/c)d\Omega$$

$$R_C = \iint I_O(\vec{s},\nu)\cos(2\pi\nu\vec{b}\cdot\vec{s}/c)d\Omega = 0 \qquad I_O(\vec{s}) = -I_O(\vec{-}s)$$

We're missing some information about the source brightness!

Cosine/sine terms

To recover the full source brightness, we need:

$$R_C = \iint I(\vec{s},\nu)\cos(2\pi\nu\vec{b}\cdot\vec{s}/c)d\Omega = \iint I_E(\vec{s},\nu)\cos(2\pi\nu\vec{b}\cdot\vec{s}/c)d\Omega$$

$$R_S = \iint I(\vec{s},\nu)\sin(2\pi\nu\vec{b}\cdot\vec{s}/c)d\Omega = \iint I_O(\vec{s},\nu)\sin(2\pi\nu\vec{b}\cdot\vec{s}/c)d\Omega$$

Rc samples the even part of *I(s)* and *Rs* samples the odd part of *I(s)*.

To get Rs, we simply add a 90 deg phase shift into one of the signal paths before multiplying.

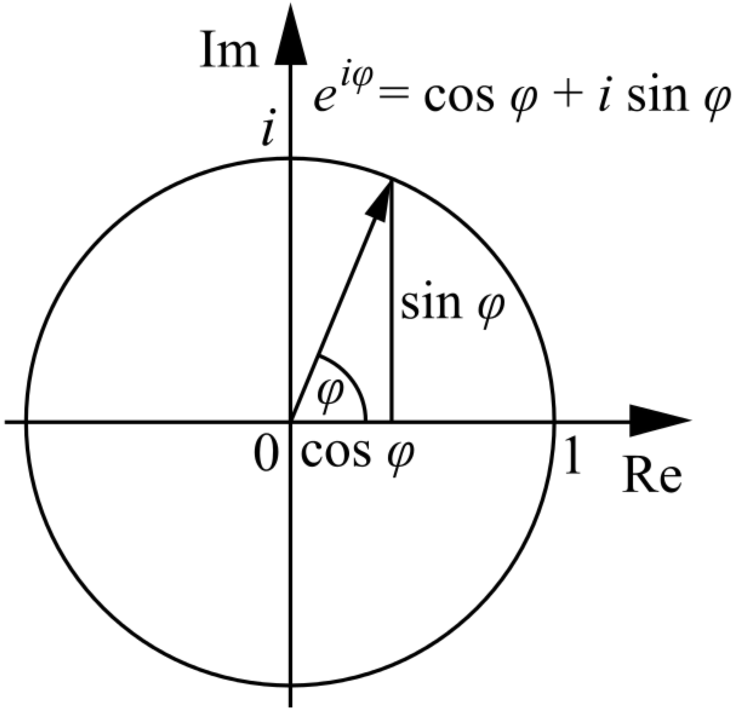
Visibilities and the uv-plane

Visibilities

Define a complex function called the "visibility", which contains all the information for that baseline:

$$V = R_C - iR_S = Ae^{-i\phi} \qquad A = \sqrt{R_C^2 - R_S^2} \qquad \phi = \tan^{-1}\left(\frac{R_S}{R_C}\right)$$

Recall Euler's Formula



Visibilities

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$$V = R_C - iR_S = Ae^{-i\phi} \qquad A = \sqrt{R_C^2 - R_S^2} \qquad \phi = \tan^{-1}\left(\frac{R_S}{R_C}\right)$$

$$V_\nu(\vec{b}) = R_C - iR_S = \iint I_\nu(\vec{s})e^{-2\pi i\nu\vec{b}\cdot\vec{s}/c}d\Omega$$

- *Complete* relation between the interferometer response and the source brightness.
- This is a 2-D Fourier relation under certain circumstances.

Complex correlator

- Machine to produce both the real and imaginary part of the visibilities (i.e. *Rc* and *Rs*).
- Effectively casts two sets of sinusoids on the sky, offset by 90deg.
- Again, both are needed if this pattern remains stationary w.r.t. the source.


Rc

“Fill-in the information gaps”


Rs

Complex correlator

To makes the math easier, let's use complex numbers. Re-write our antenna voltages as:

$$V_1 = A\cos(\omega(t)) = \mathrm{Re}\{Ae^{-i\omega(t)}\}$$

$$V_2 = A\cos(\omega(t - \vec{b}\cdot\vec{s}/c)) = \mathrm{Re}\{Ae^{-i\omega(t-\vec{b}\cdot\vec{s}/c)}\}$$

Correlated power becomes:

$$P_{corr} = \langle V_1 V_2^* \rangle = A^2 e^{-i\omega\vec{b}\cdot\vec{s}/c}$$

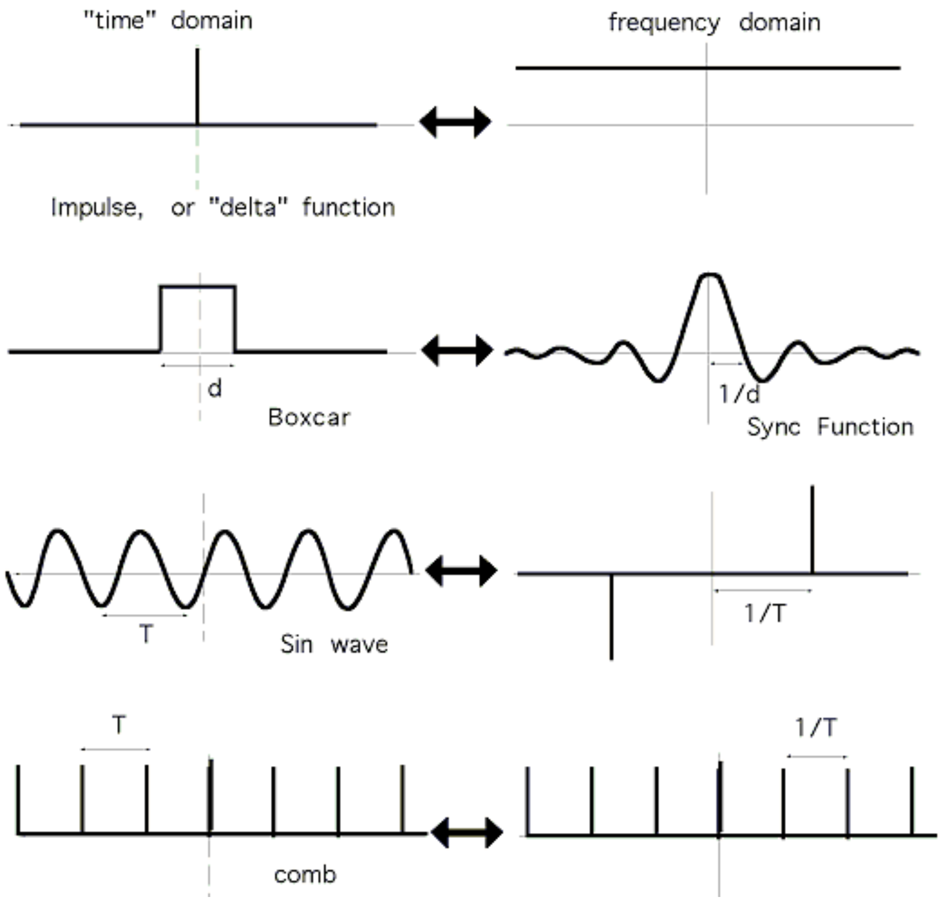
Time average

Complex conjugate

Visibility

$$V_\nu(u,v) \Leftrightarrow I_\nu(l,m)$$

Fourier Transform



1-D

Visibility

$$V_\nu(u,v) \Leftrightarrow I_\nu(l,m)$$

- *V* is a unique function of *I*.
- So far we've talked about a single baseline measuring a single frequency at a single time at a single (u,v) coordinate.
- This single visibility gives us limited information about the relevant spatial scales and morphology of the source we're observing.

Visibility

$$V_\nu(u, v) \Leftrightarrow I_\nu(l, m)$$

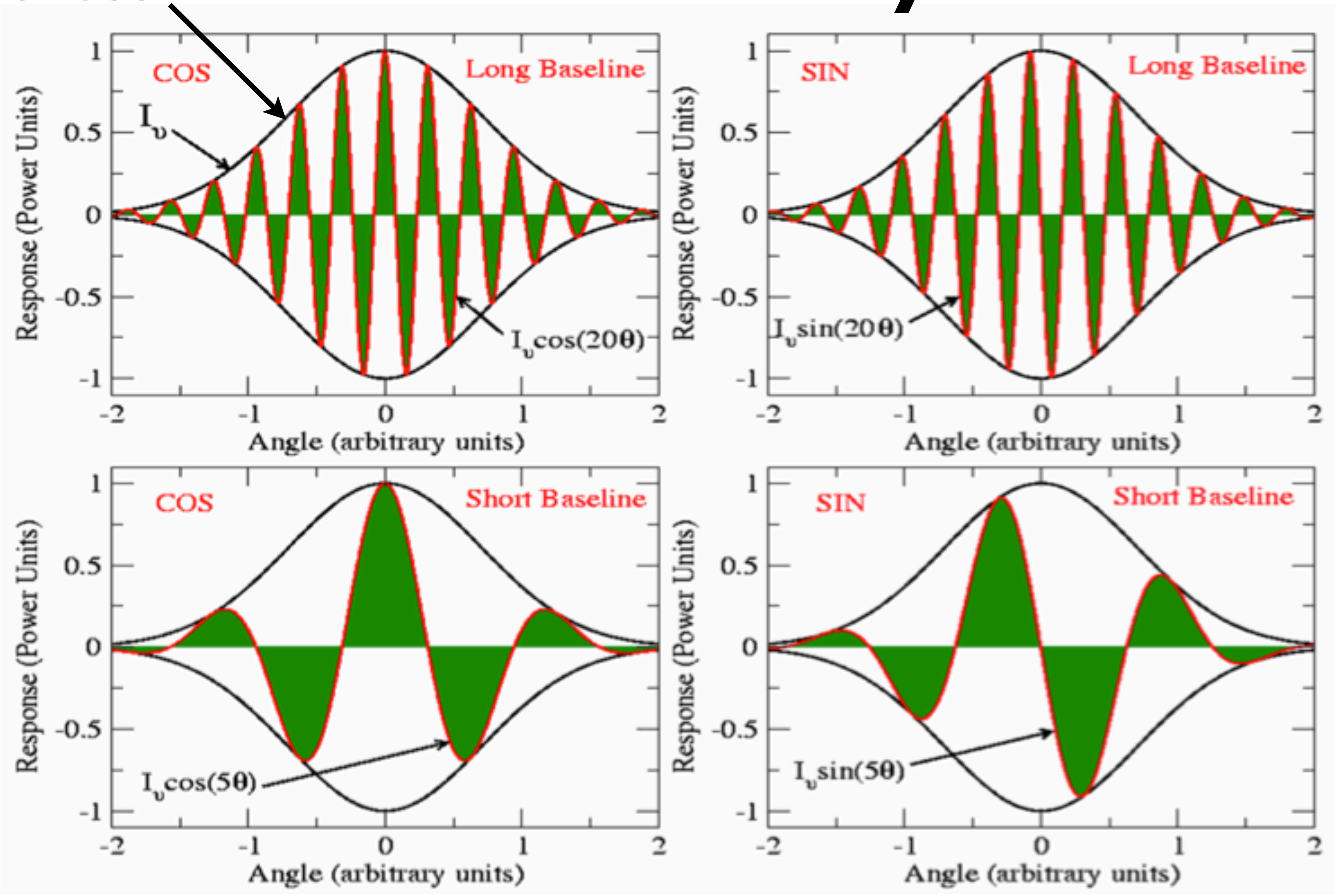
- Amplitude of visibility generally gets lower for increasingly long baseline.
- A source is "resolved out" when the visibility amplitude approaches zero.
- Visibility of "reversed baseline" is the complex conjugate of the original.

Visibility

Source brightness

R_C

R_S

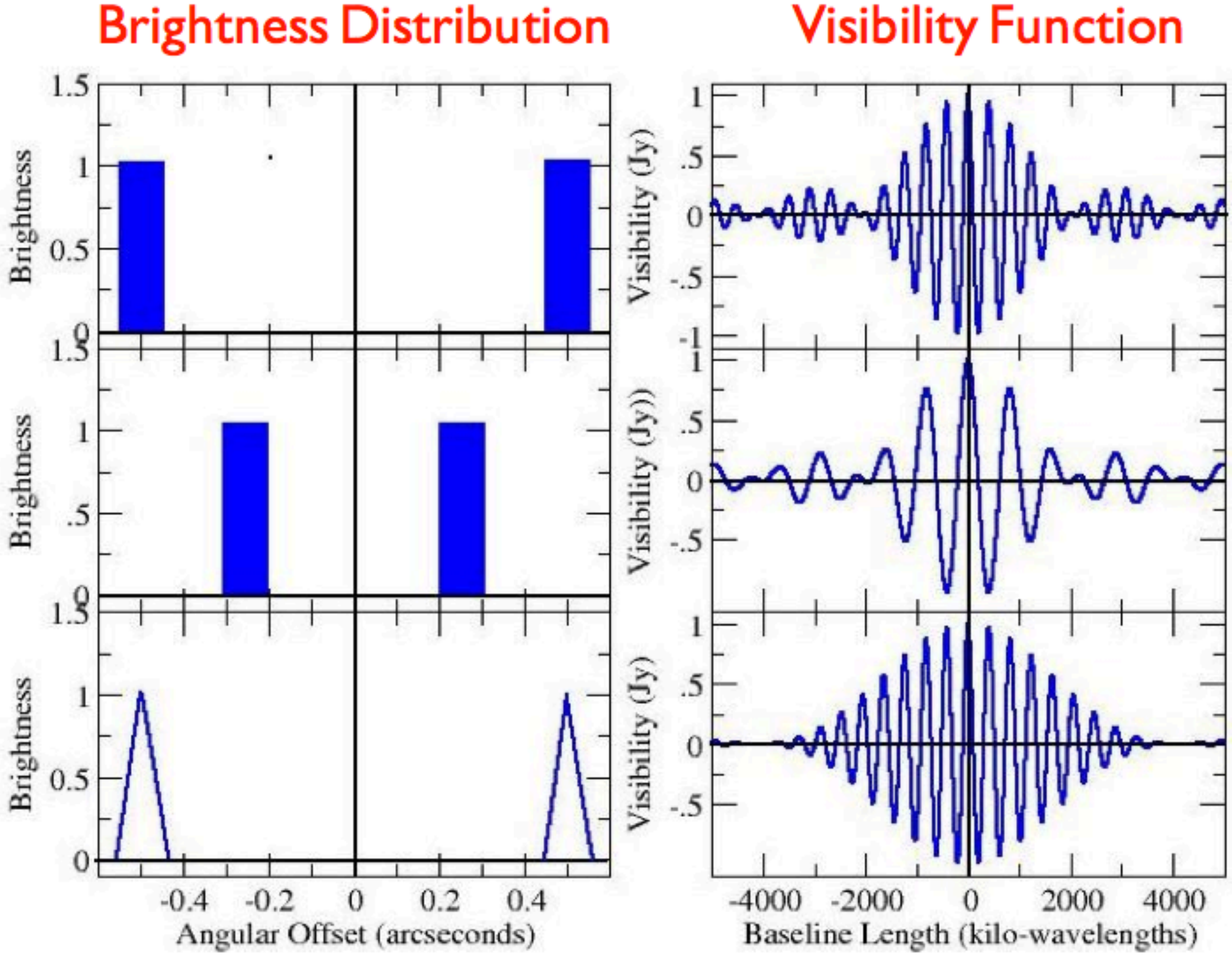


Long Baseline

Short Baseline

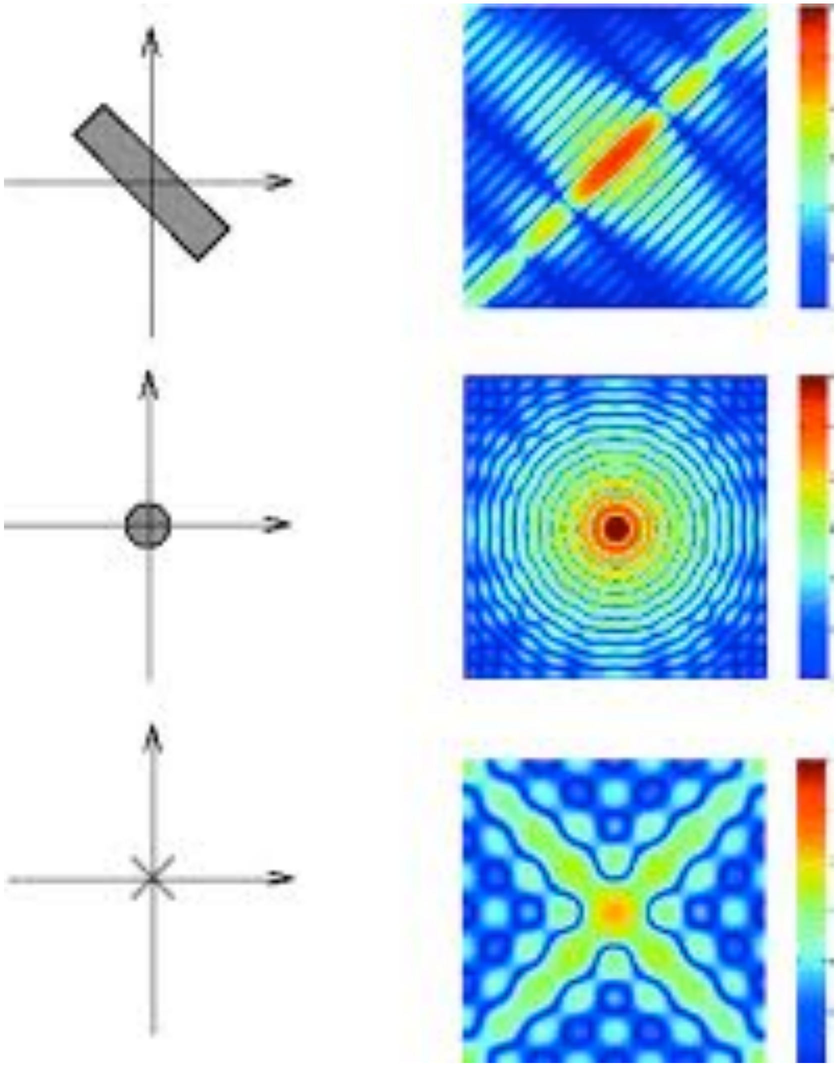
Red: fringes; green: response (visibility)

Visibility


Brightness Distribution
Visibility Function
Brightness
1.5
1
0.5
0
.5
Visibility (Jy)
Visibility (Jy))
-1
-.5
-0.4
-0.2
0
0.2
0.4
Angular Offset (arcseconds)
-4000
-2000
2000
4000
Baseline Length (kilo-wavelengths)

Moving to 2-D

- Of course, a more useful interferometer includes more than just two antennas and hence more than just 1 baseline.
- For a N-element interferometric array, there are N(N-1)/2 independent baselines.



2-D Fourier Transform

uv-Plane (2D version)

Assume all interferometric elements are in a plane.

$$\vec{b} = (\lambda u, \lambda v, \lambda w) = (\lambda u, \lambda v, 0)$$

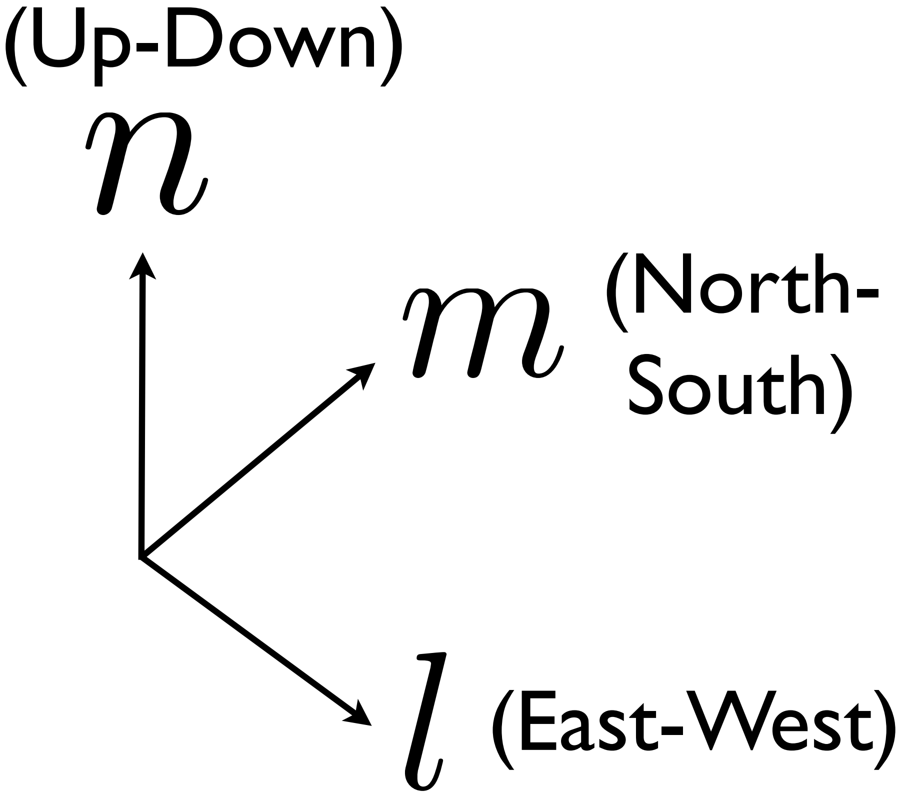
w is perpendicular to the observing plane. (u,v,w) in units of wavelengths.

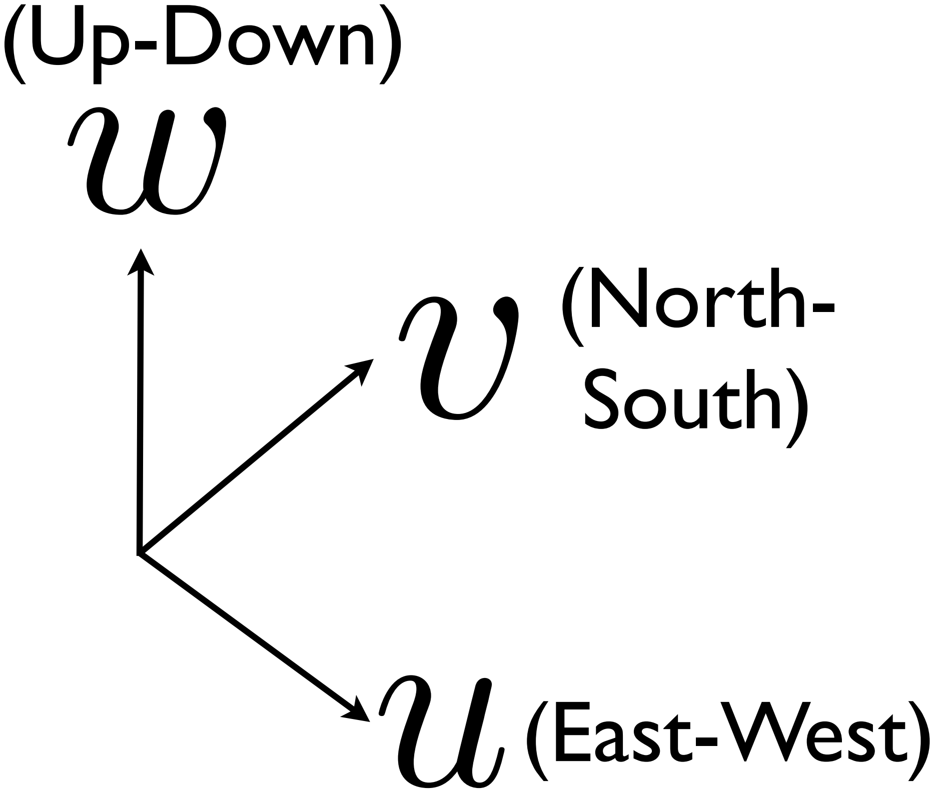
$$\vec{s} = (l, m, n) = (l, m, \sqrt{1 - l^2 - m^2})$$

(components of the unit direction vector)

uv-Plane (2D version)

"(*l*,*m*,*n*) and (*u*,*v*,*w*) coordinates"





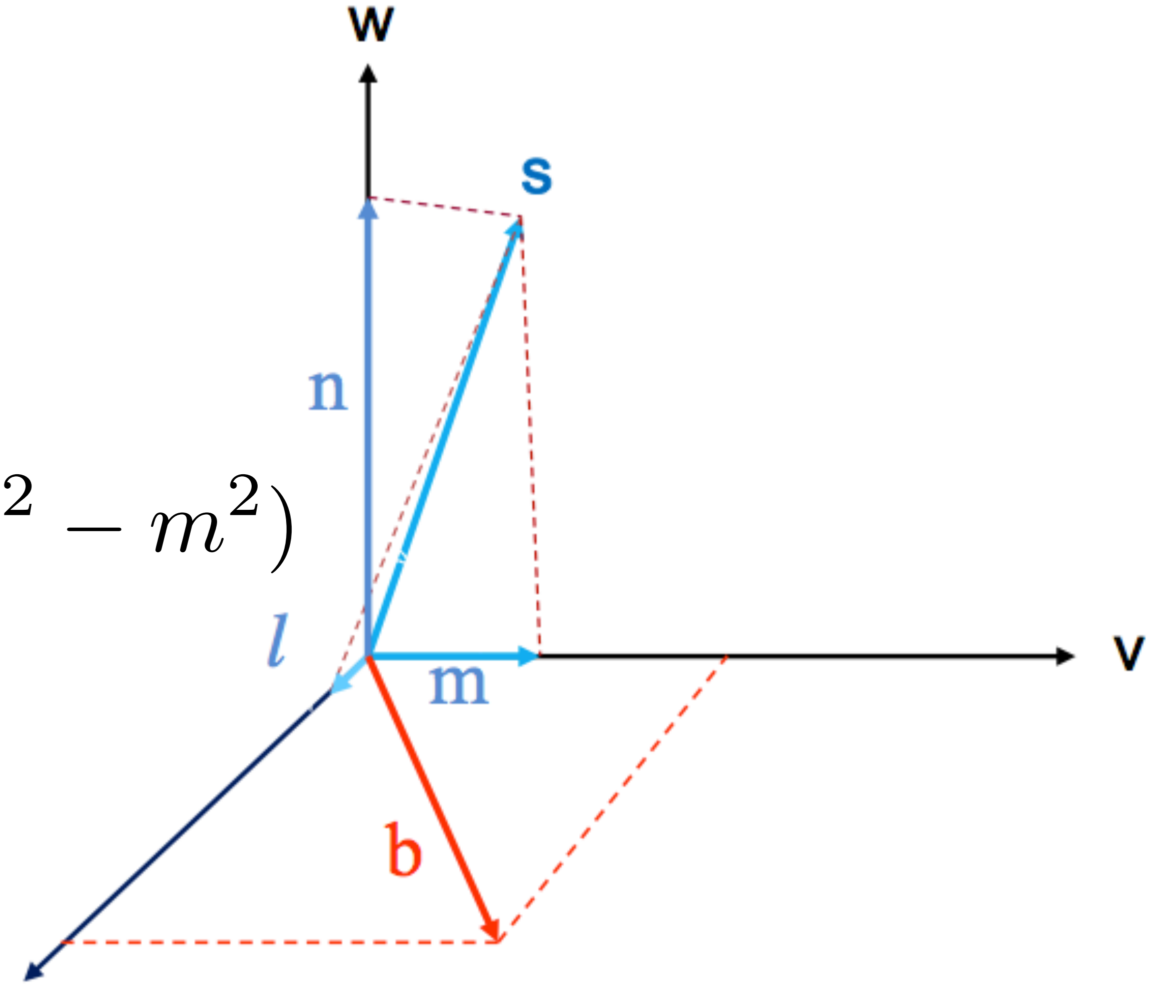
uv-Plane (2D version)

Unit vector s (direction) is defined by its projections (l,m,n) onto the (u,v,w) axes. We call (l,m,n) the direction cosines.

$$l = \cos(\alpha)$$

$$m = \cos(\beta)$$

$$n = \cos(\theta) = \sqrt{(1 - l^2 - m^2)}$$



2-D Fourier relation

Infer the source brightness from the visibilities, as before

$$V_\nu(u,v) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-i2\pi(ul+vm)} dldm$$

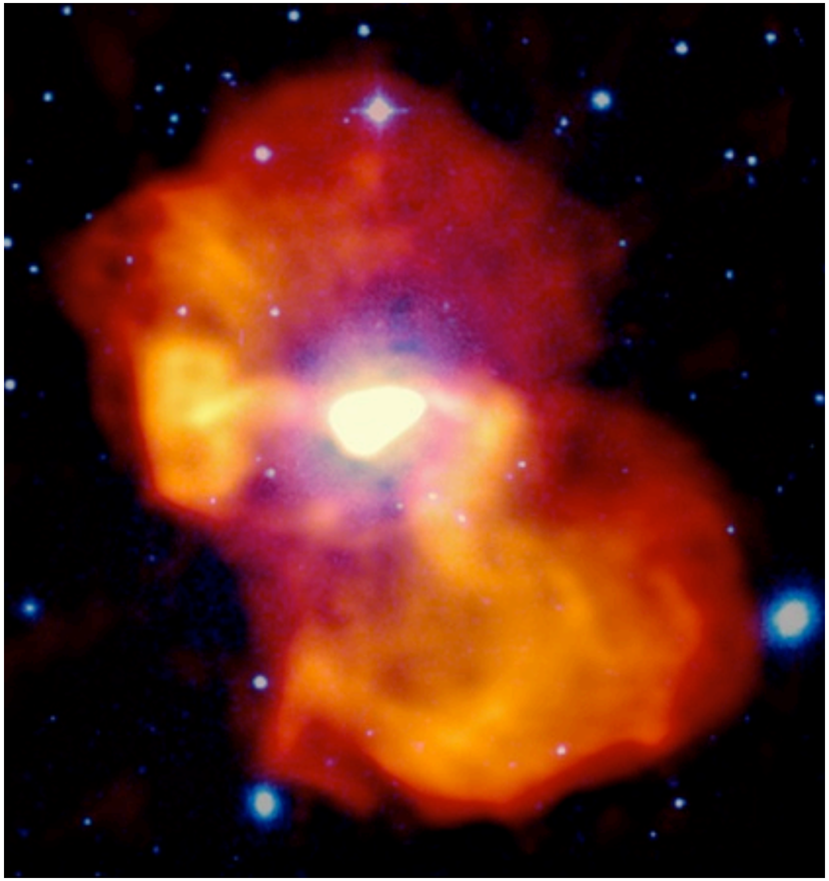
$$I_\nu(l,m)/\cos(\Theta) \Leftrightarrow V_\nu(u,v)$$

$$I_\nu(l,m) = \cos(\Theta) \iint V_\nu(u,v) e^{i2\pi(ul+vm)} dudv$$

Now a 2-D Fourier transform
Situation becomes more complicated if w not zero (not co-planar array **w.r.t. the source**)

uv-Coverage

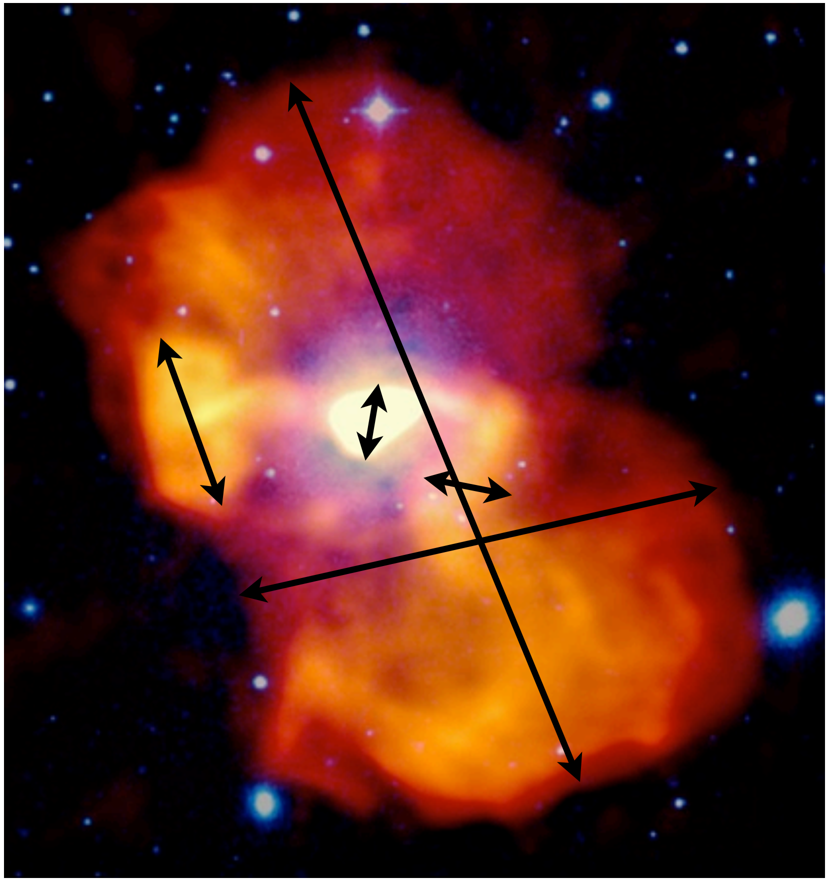
- Spatial sampling of the source brightness distribution is dictated by what baselines are recorded and their orientation w.r.t. the sky.
- More baselines means more spatial information on a variety of scales.
- Ideally the interferometric elements will be randomly distribution in order to disperse small errors.



Virgo A - LOFAR

uv-Coverage

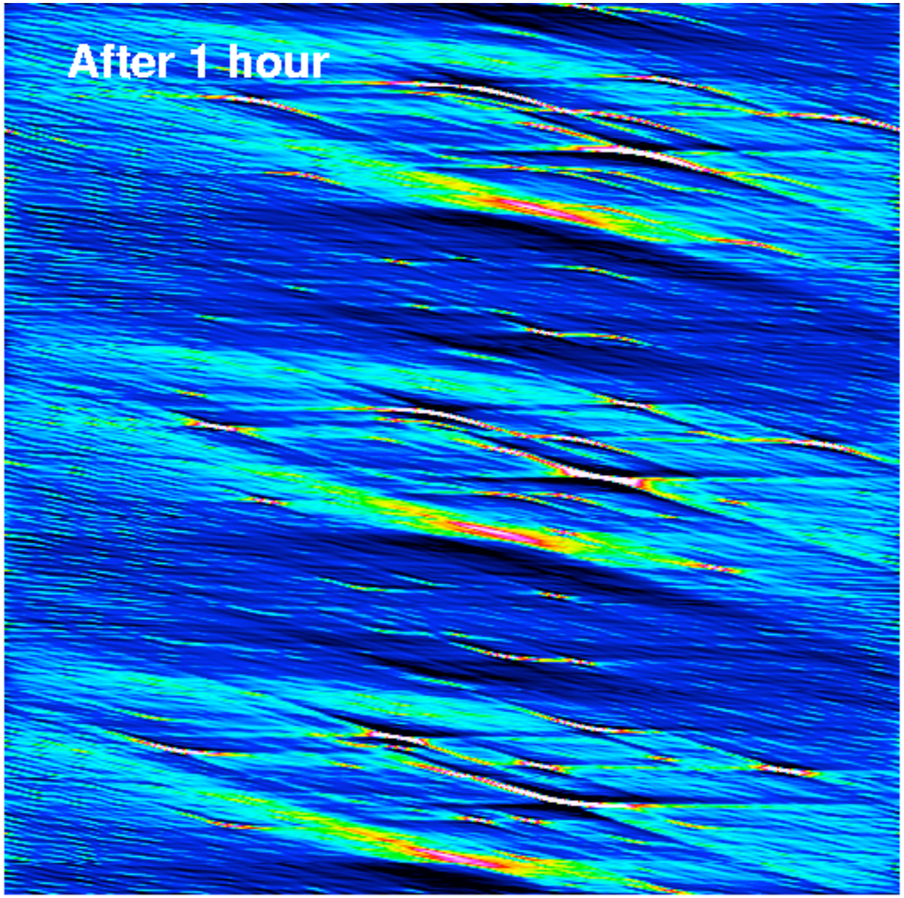
- Coverage is never 100% complete, but want to sample the relevant angular scales and orientations.



Virgo A - LOFAR

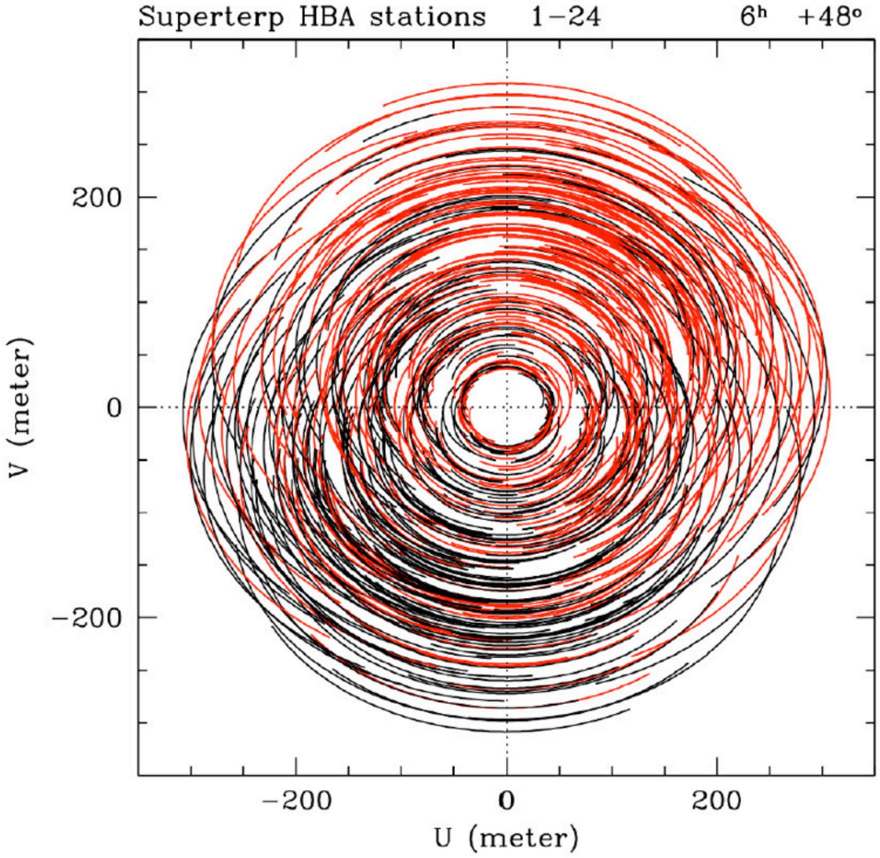
Earth rotation aperture synthesis

Baselines will rotate w.r.t. the sky as a function of time. Earth rotation "fills-in" the image.

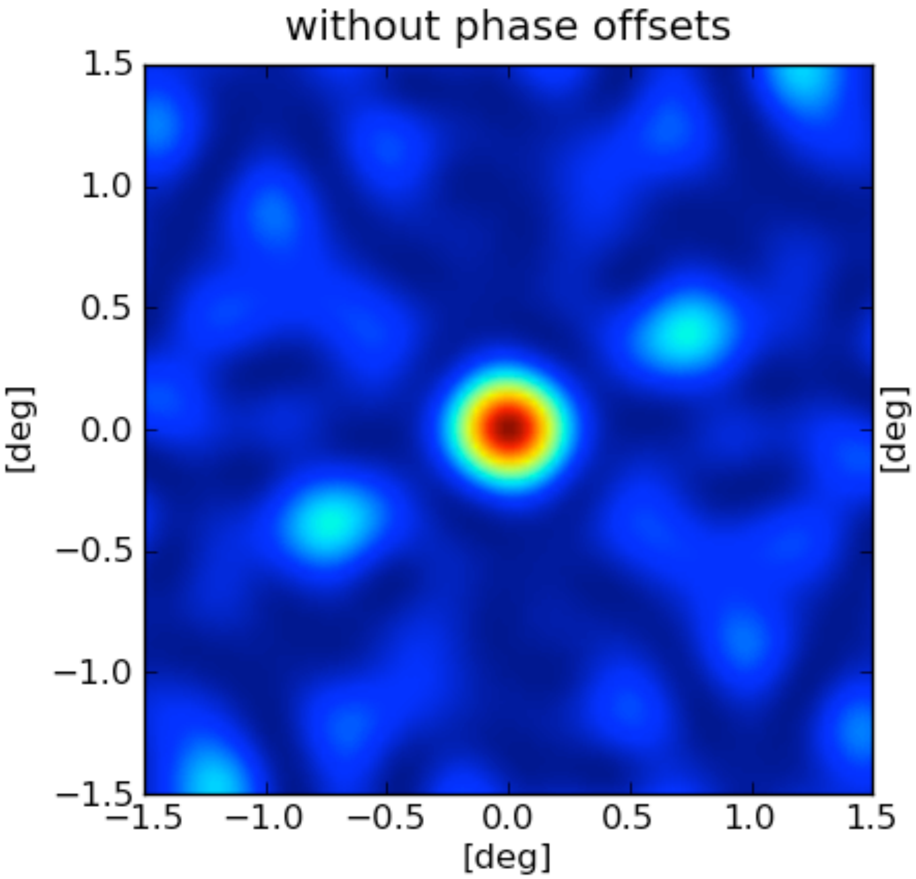


uv-Coverage and Beam

LOFAR Superterp (Inner core)

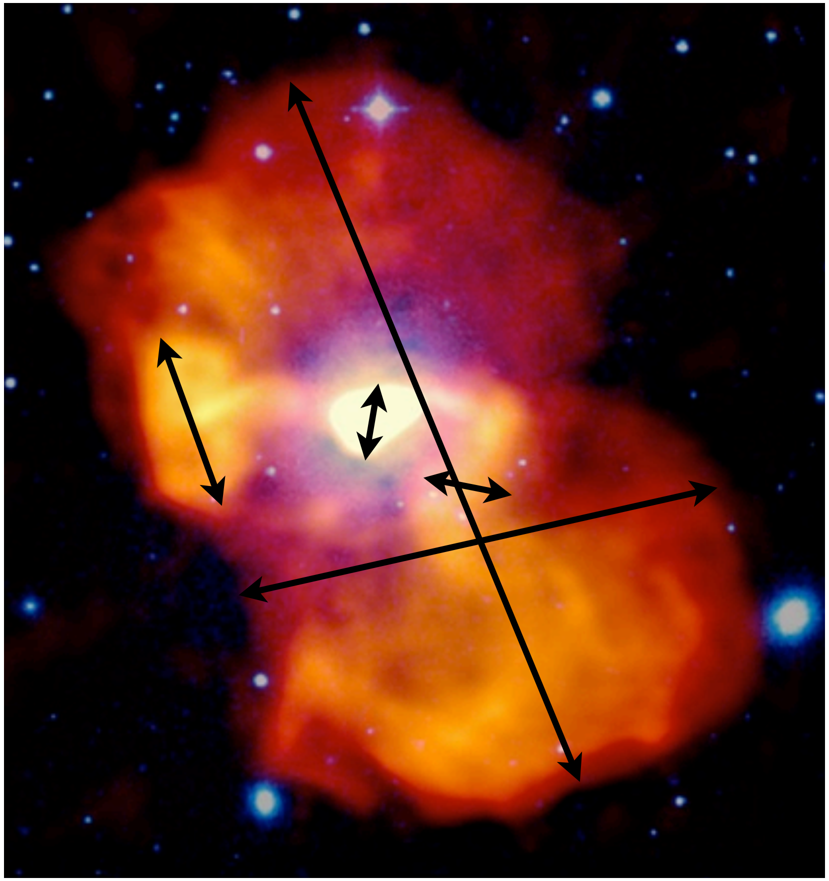


uv-coverage in 6 hours

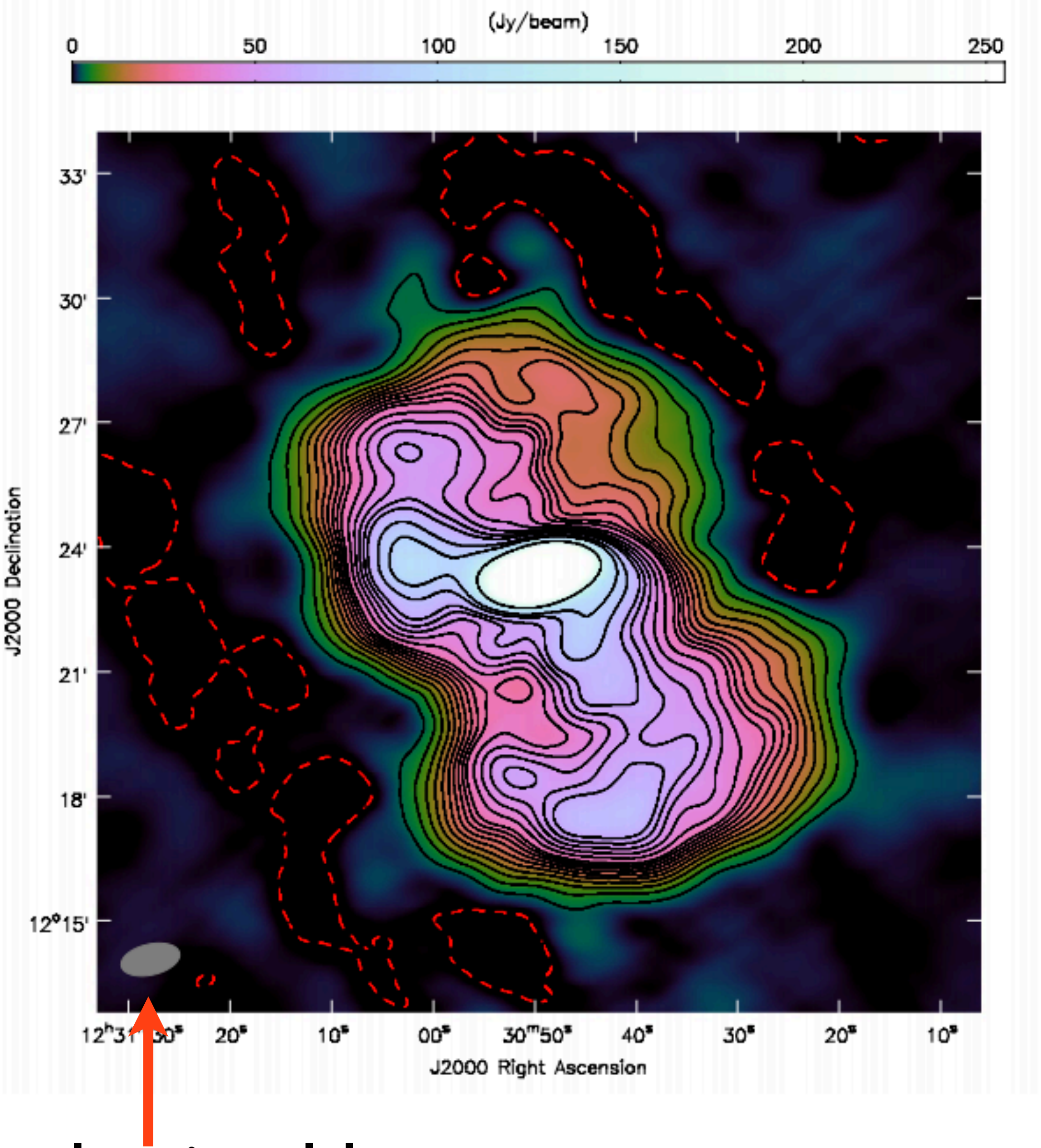


beam

Synthesized beam



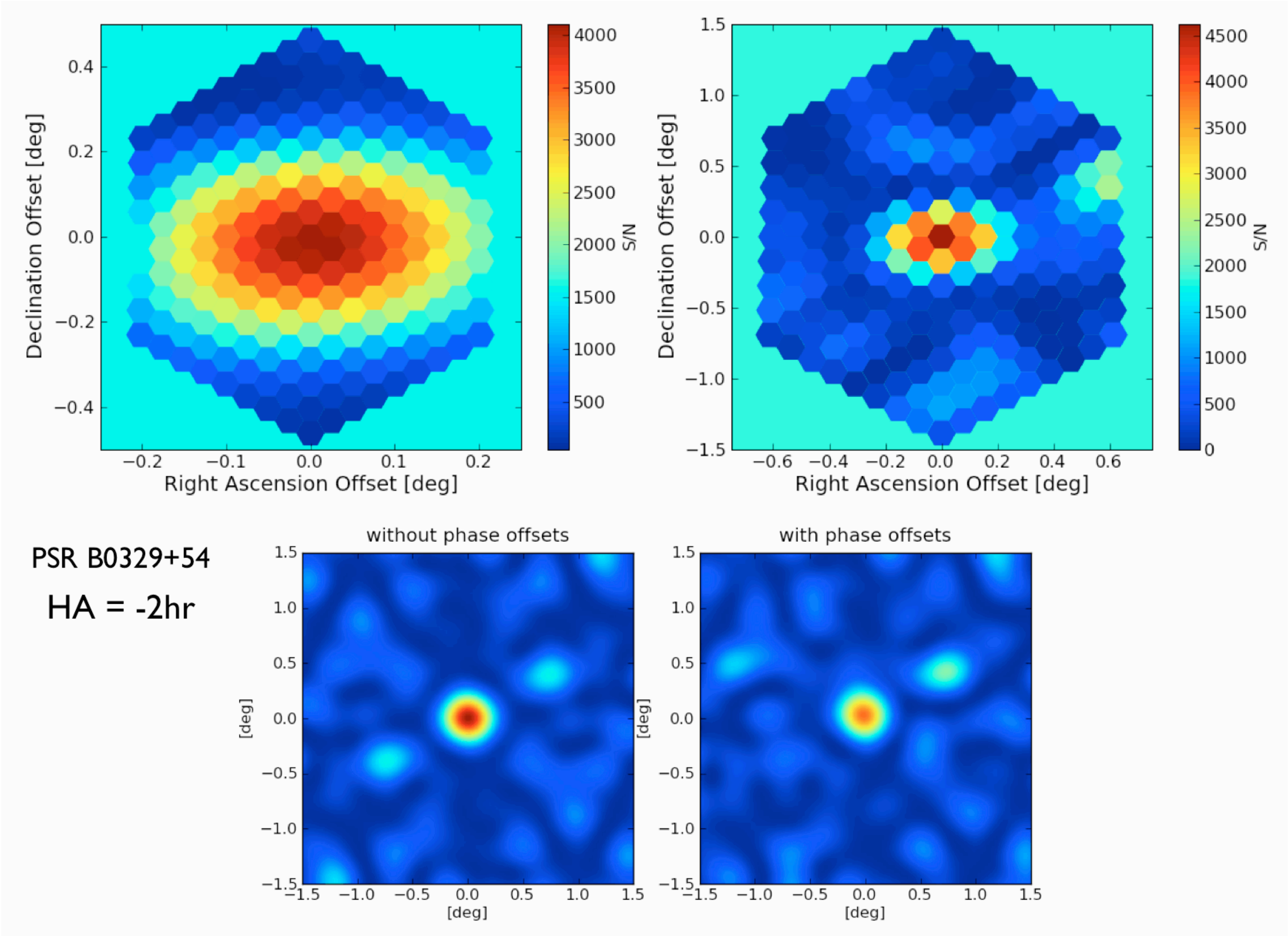
Virgo A - LOFAR



Synthesized beam

uv-Coverage and Beam

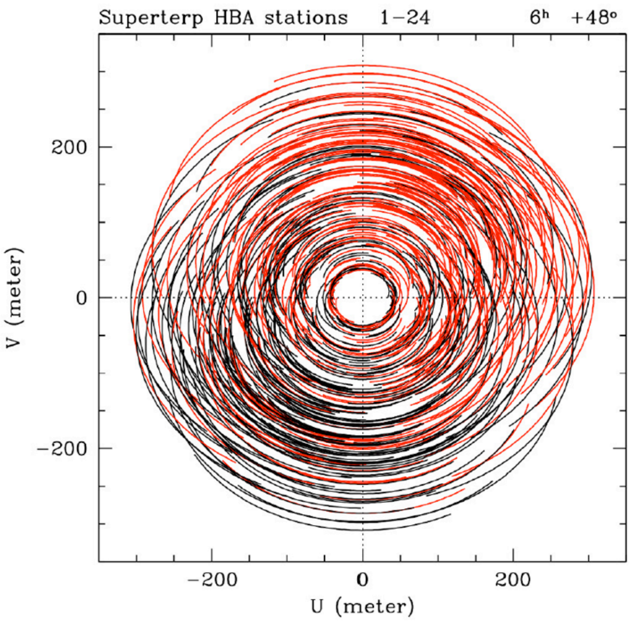
LOFAR Superterp (Inner core)



PSR B0329+54

HA = -2hr

(u,v,w) Coordinates



Define antenna positions in an Earth-based coordinate system

$$X \equiv H = 0, \delta = 0$$
$$Y \equiv H = -6, \delta = 0$$
$$Z \equiv \delta = 0(\mathrm{NCP})$$

(Bx,By,Bz) define - in number of wavelengths - the baseline in this coordinate system

(u,v,w) Coordinates

Source HA and DEC

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \sin H_\circ & \cos H_\circ & 0 \\ -\sin\delta_\circ \cos H_\circ & \sin\delta_\circ \sin H_\circ & \cos\delta_\circ \\ \cos\delta_\circ \cos H_\circ & -\cos\delta_\circ \sin H_\circ & \sin\delta_\circ \end{bmatrix} \times \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix}$$

- As before, the u and v coordinates describe E-W and N-S components of the projected interferometer baseline.
- The w coordinate is the delay distance in wavelengths between the two antennas.

$$\tau_g = \frac{\lambda}{c} w = \frac{w}{\nu}$$

Delay between antennas

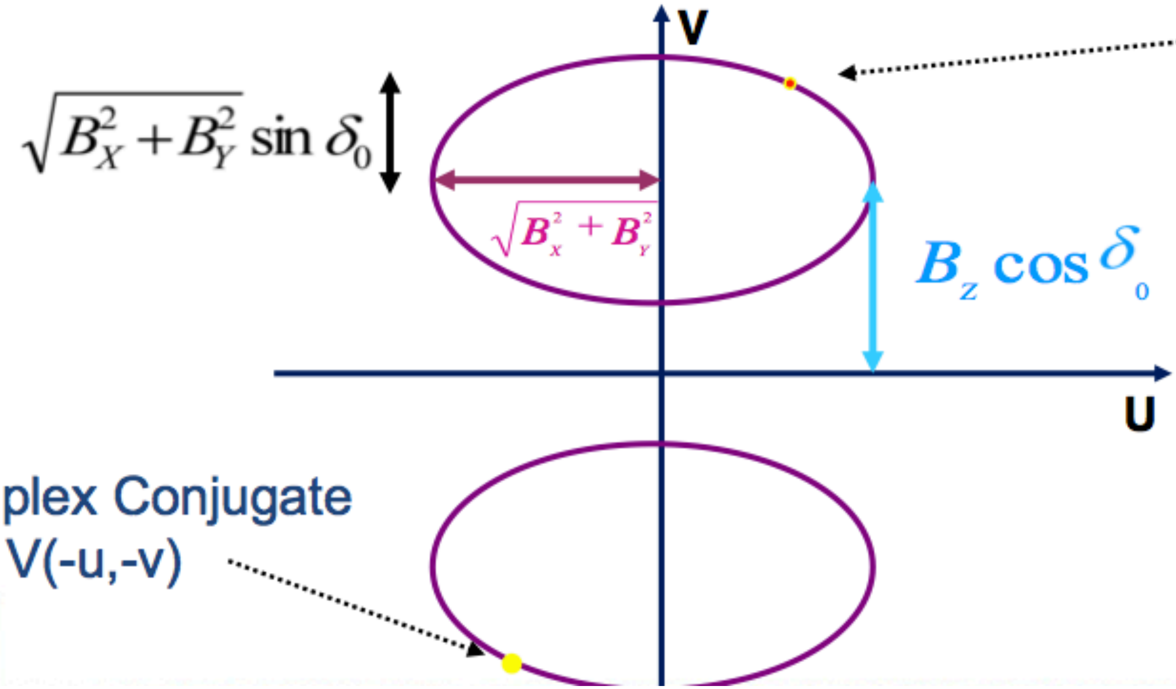
$$\nu_F = \frac{dw}{dt} = -\omega_E u \cos\delta_\circ \text{ Hz}$$

Fringe frequency

Baseline locus

$$u^2 + \left(\frac{v - B_z\cos\delta_\circ}{\sin\delta_\circ}\right)^2$$

- Traces out ellipse in 24hrs.
- Brightness is real so: $V(-u,-v) = V^*(u,v)$
- E-W baselines have no v offset.



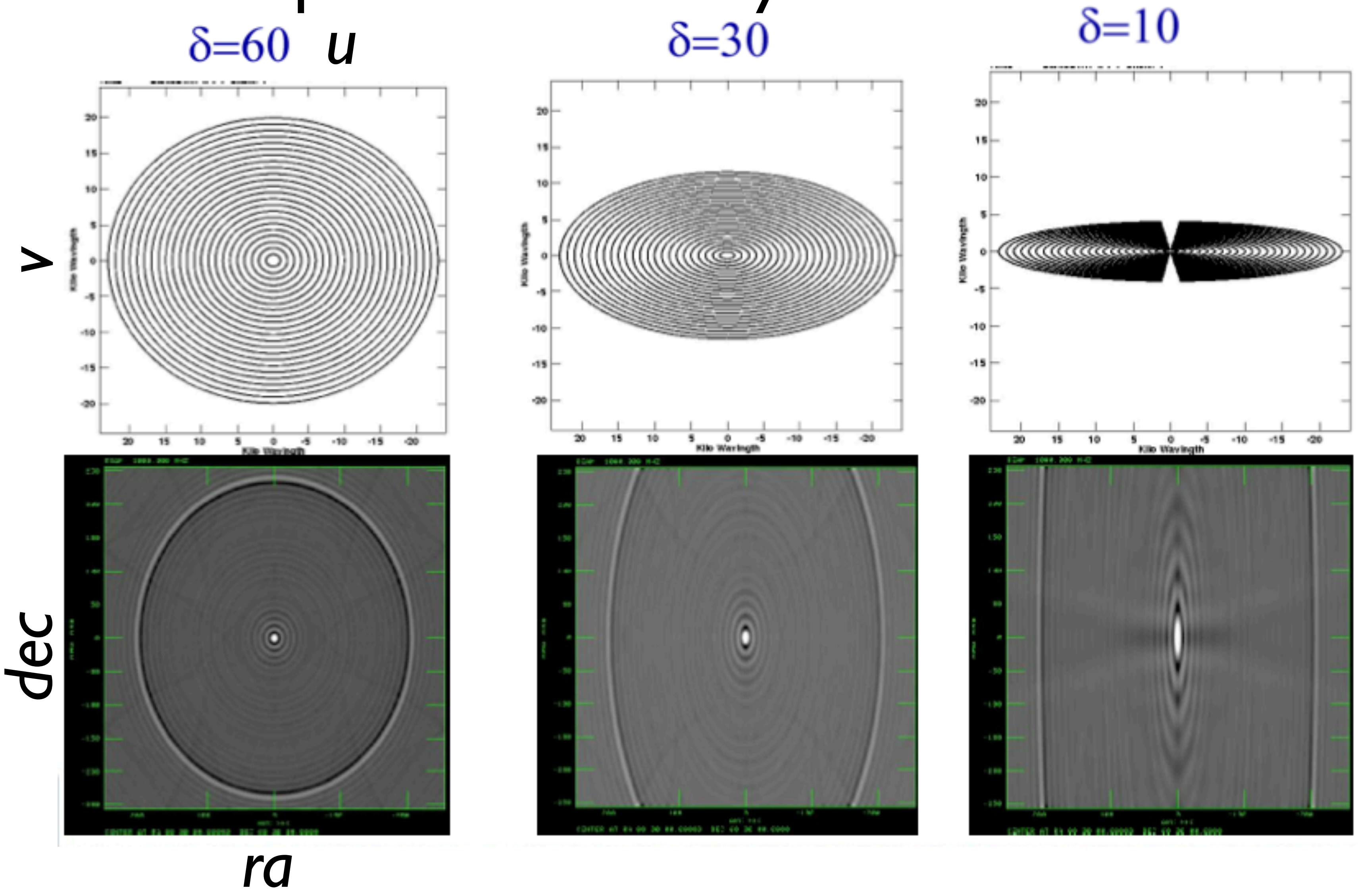
A single Visibility: V(u,v)

Good UV Coverage requires many simultaneous baselines amongst many antennas, or many sequential baselines from a few antennas.

Its Complex Conjugate V(-u,-v)

uv-Coverage and Beams

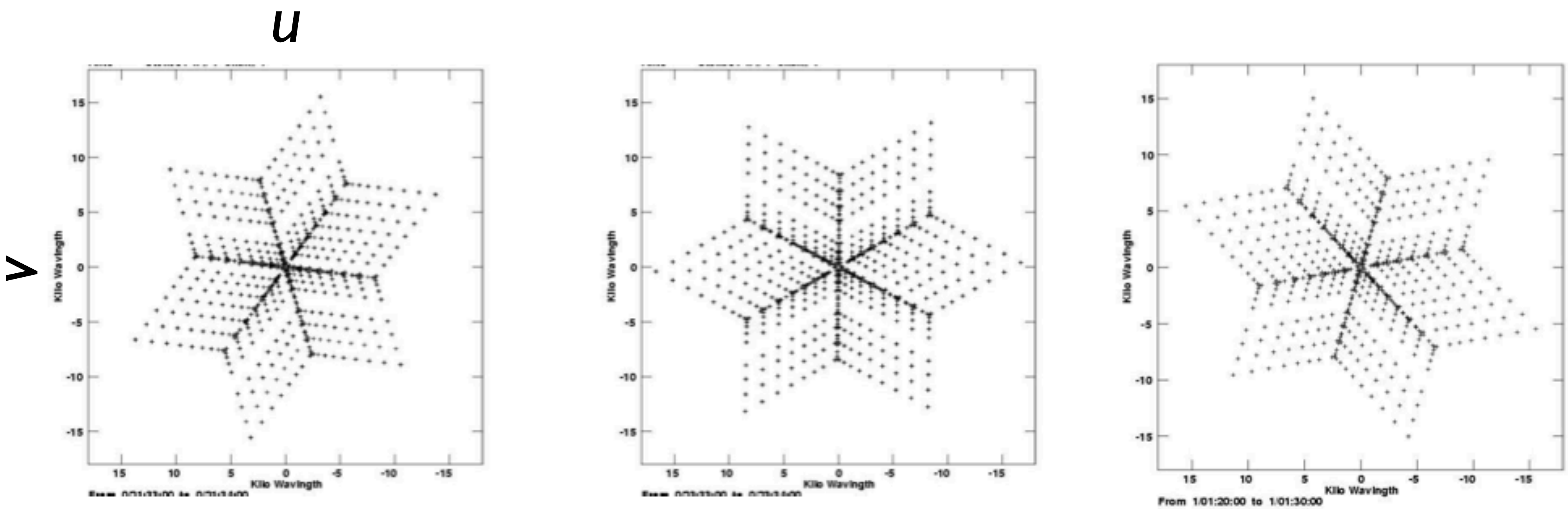
Example for E-W array like Westerbork



Notice that all uv-tracks are centered on (u,v) = (0,0).

Snapshot uv-Coverage

Example for VLA



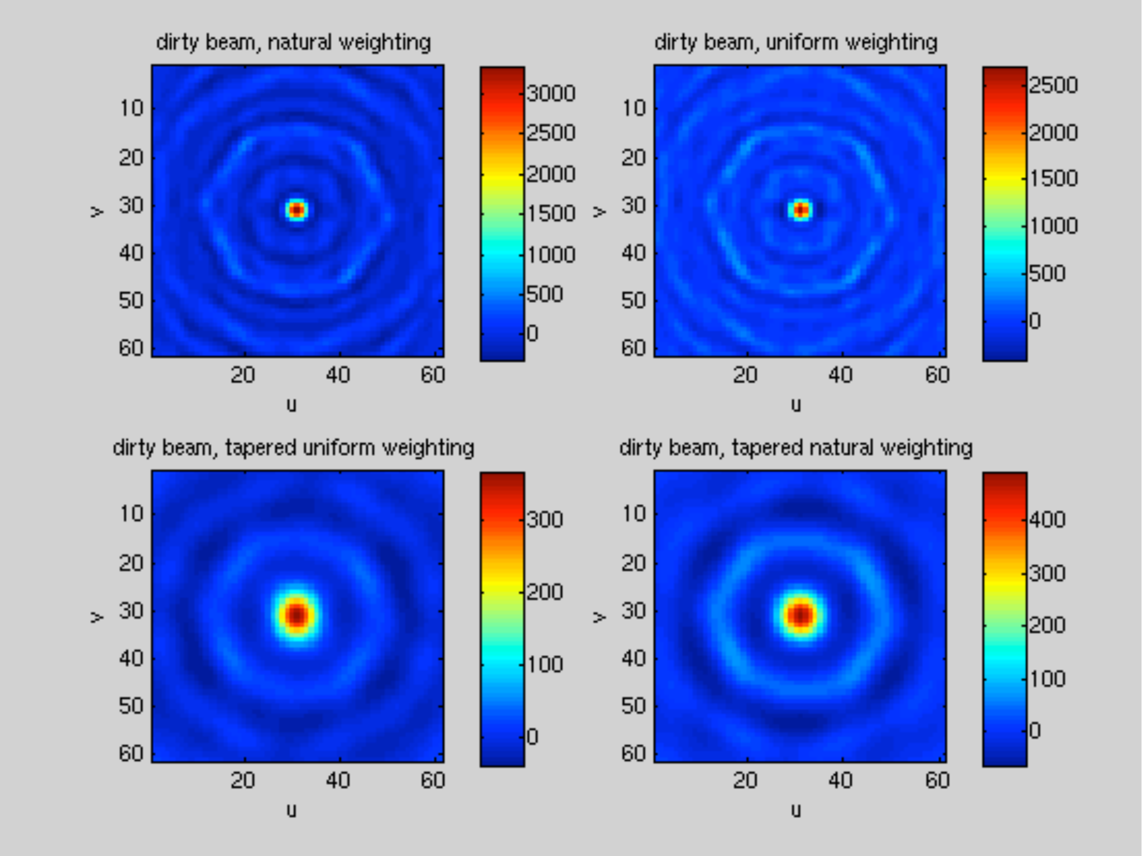
HA = -2h HA = 0h HA = 2h

North-South baselines mean that *no* uv-tracks are centered on (u,v) = (0,0).

Helps a lot with imaging around the celestial equator.

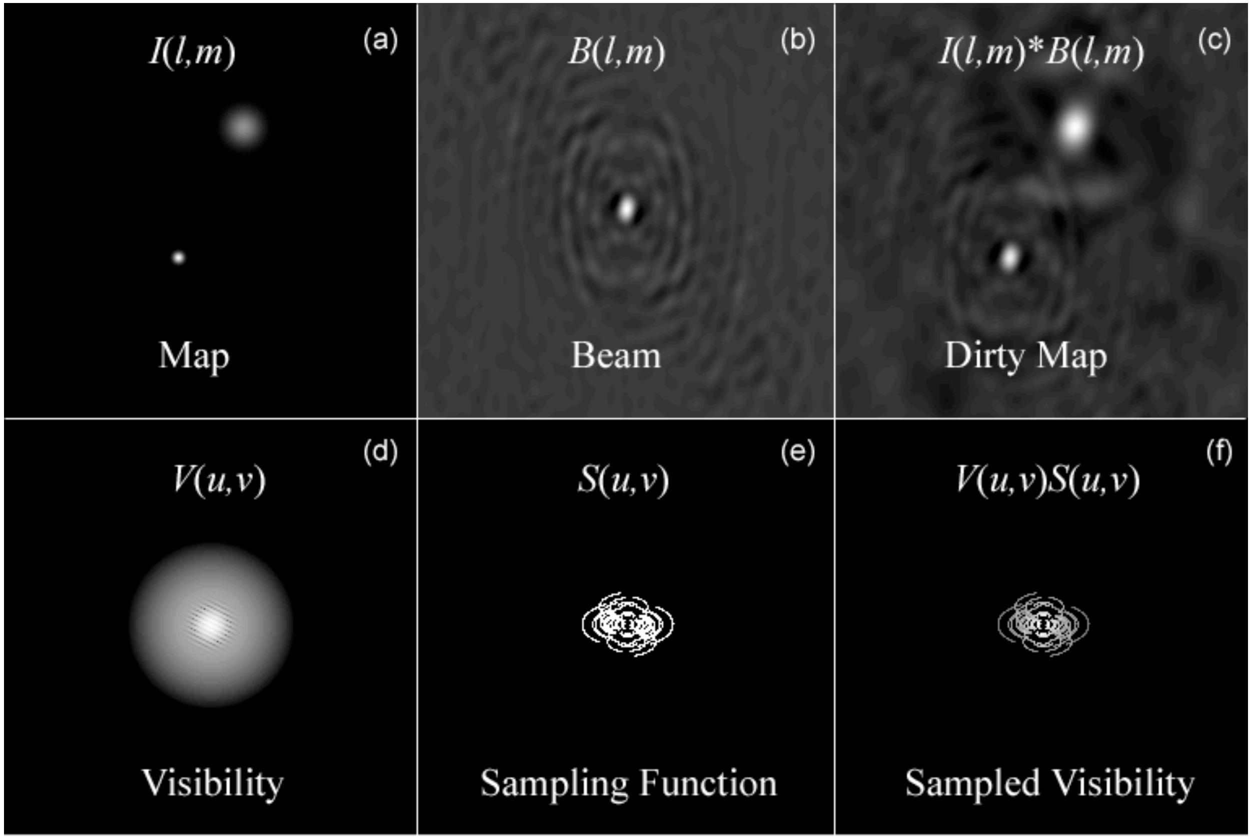
Basics of making an image

Dirty Beam



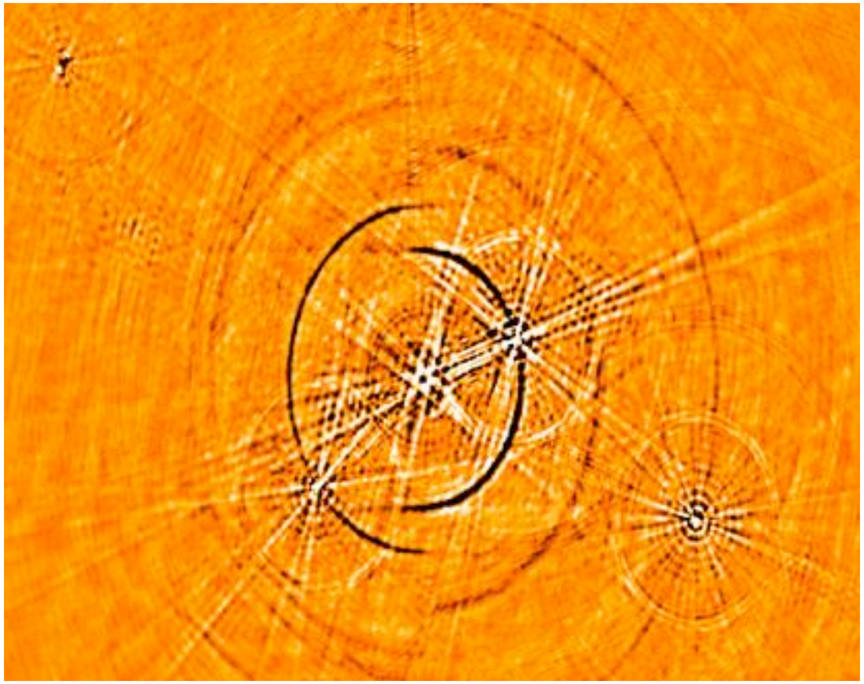
- Essentially the synthesized beam we have considered so far.
- Not unlike the "point spread function" of the interferometer.
- Weighting the baselines differently will give different dirty beams.

Dirty Image



- The sky brightness convolved with the dirty beam (instrument response).
- Need to deconvolve the dirty beam in order to get the “clean image”.
- More in the coming two lectures.

Dirty Image



- Highly symmetric arrays create very distinct patterns in the dirty beam. If these are not de-convolved from the image perfectly, then they lead to artifacts.
- Randomized array (like LOFAR) is better in this regard.

Other considerations

A more realistic interferometer

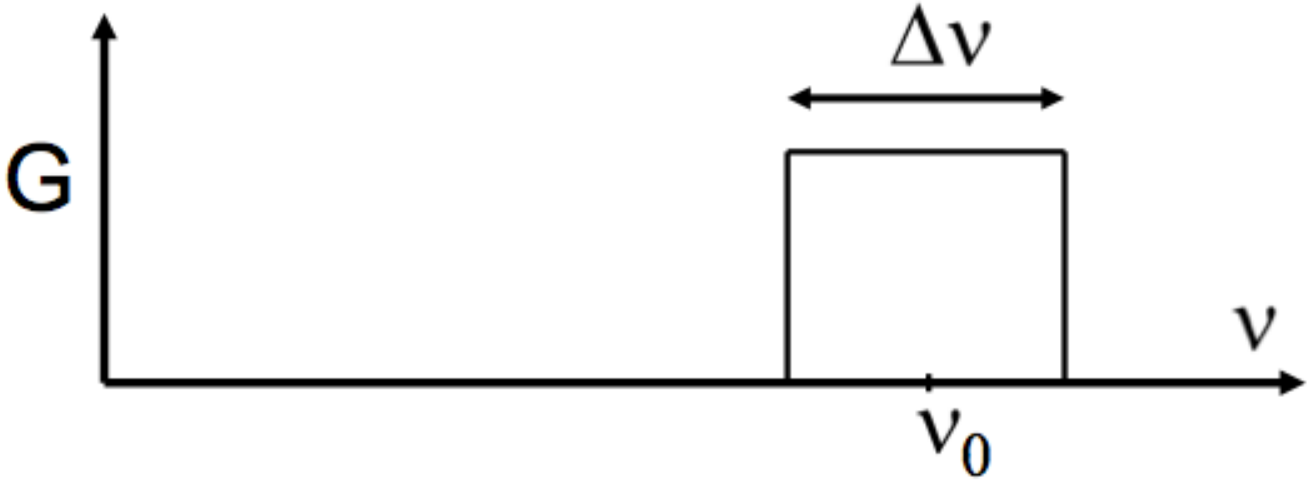
- Things aren’t static. Need “delay tracking” to steer the interference pattern towards the region of interest. The sine and cosine fringe patterns move with the source.
- Frequency down-conversion. Electronics do not necessarily work at the (higher) observed sky frequency (RF).
- Non-monochromatic waves. Finite bandwidth.
- Losses from time averaging. Delay tracking only works perfectly at the phase center. All other sources are moving (slightly) w.r.t. the fringe patterns.

Bandwidth response

- Need a finite bandwidth for good sensitivity etc.

$G(\nu)$ characterizes the amplitude and phase variation imparted on the signal by the instrument.

For example...



Bandwidth response

Plug in to get finite bandwidth visibilities....

$$V = \int \left(\frac{1}{\Delta\nu} \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} I(\vec{s},\nu) G_1(\nu) G_2^*(\nu) e^{-i2\pi\nu\tau_g} d\nu \right) d\Omega$$

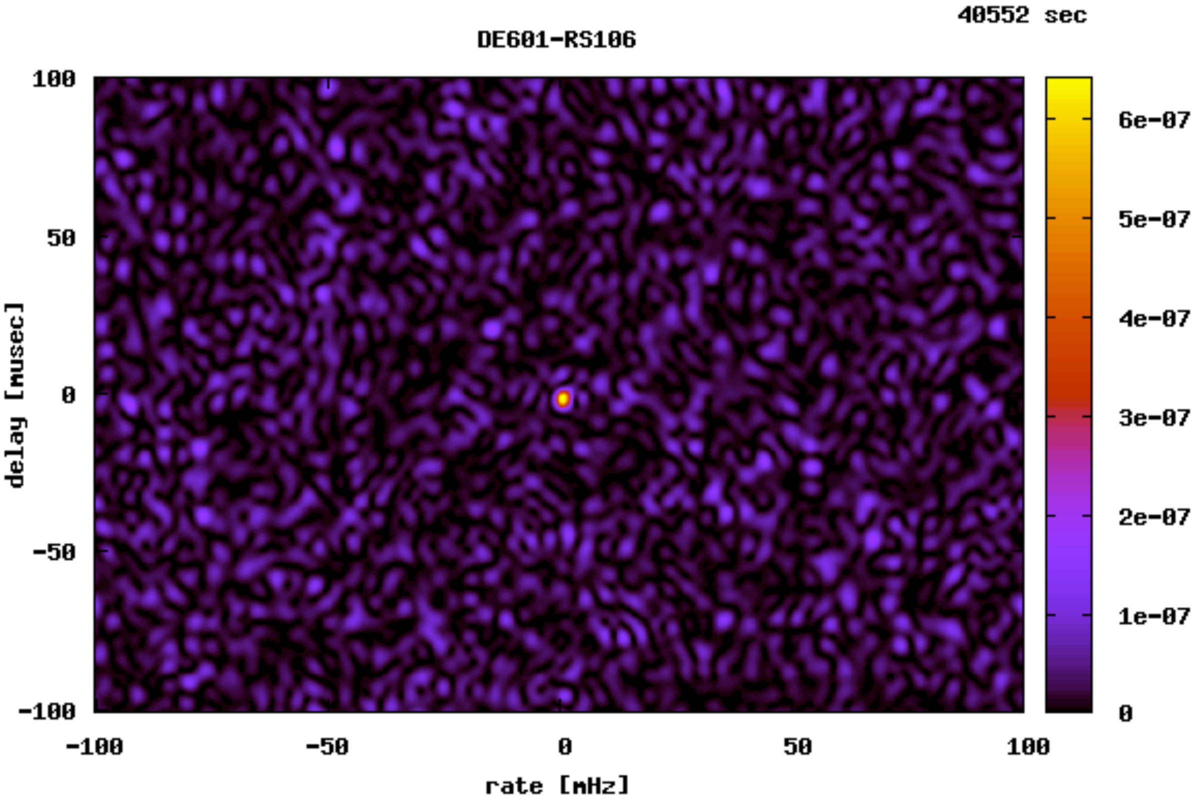
$$V = \iint I(\vec{s},\nu) \frac{\sin(\pi\tau_g\Delta\nu)}{\pi\tau_g\Delta\nu} e^{-i2\pi\nu_0\tau_g} d\Omega = \iint I(\vec{s},\nu) \mathrm{sinc}(\tau_g\Delta\nu) e^{-i2\pi\nu_0\tau_g} d\Omega$$

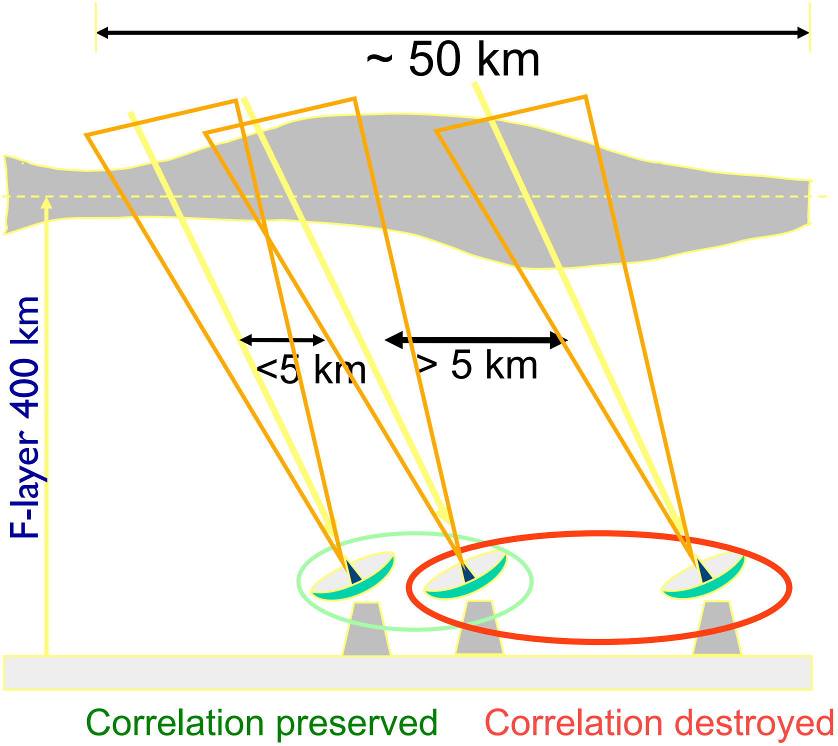
Assume source does not vary over this bandwidth and that the antennas provide the same response.

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \sim 1 - \frac{(\pi x)^2}{6} \quad (\mathrm{for} x << 1)$$

Fringe attenuation function (limits FoV off meridian)

Ionosphere





Greenhill

Introduces extra, dynamic phase delays between the antennas

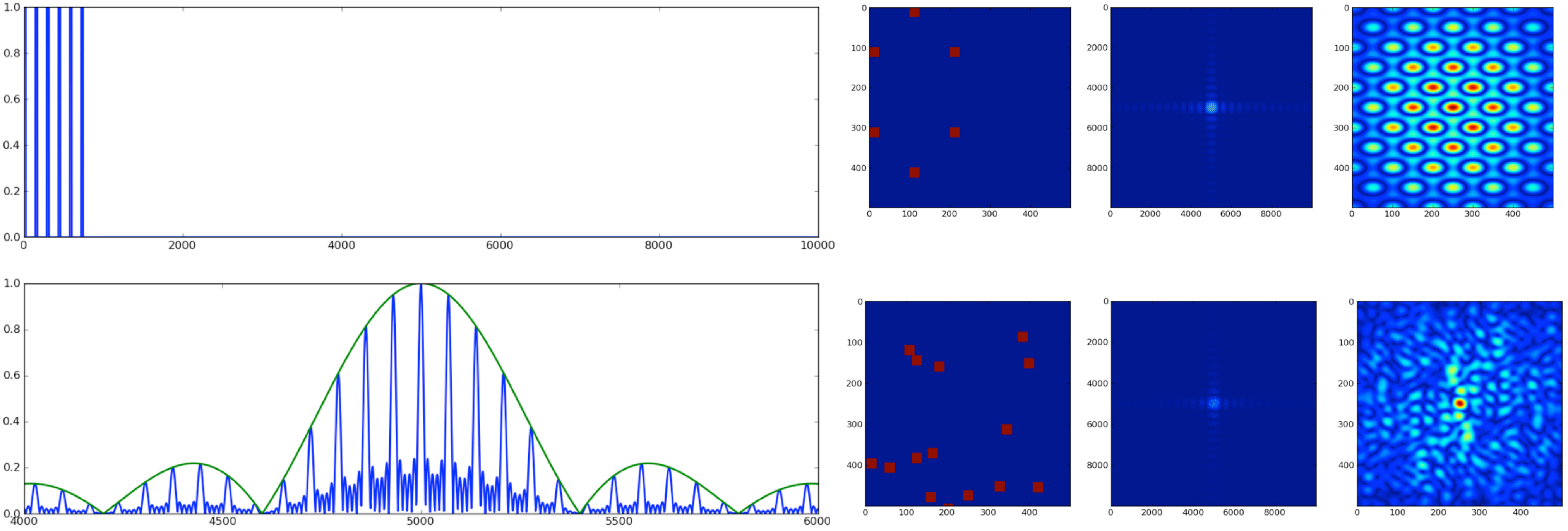
Next Two Lectures

Lots more on calibration and imaging

Questions?

Today's Practicum

Simulate your own radio interferometer!



Today's Practicum Suggestions

- Use Python (unless you're an expert with something else).
- Use the numpy and scipy Python packages.
- Start with 1-D case then expand to 2-D if there's time.
- Try both random and symmetric configurations.
- Get the units right on the axes.

Sources

NRAO Synthesis Summer School Lectures
(Perley)

New Jersey IT Lectures

http://web.njit.edu/~gary/728

Other course slides (see links on course wiki page):

http://www.astron.nl/astrowiki/doku.php?id=uva_msc_radioastronomy_2013

3-D Fourier relation

Infer the source brightness from the visibilities

$$V_\nu(u,v,w) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-i2\pi(ul+vm+wn)} dldm$$

$$V_\nu(u,v,w) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-i2\pi[ul+vm+w(\sqrt{1-l^2-m^2}-1)]} dldm$$