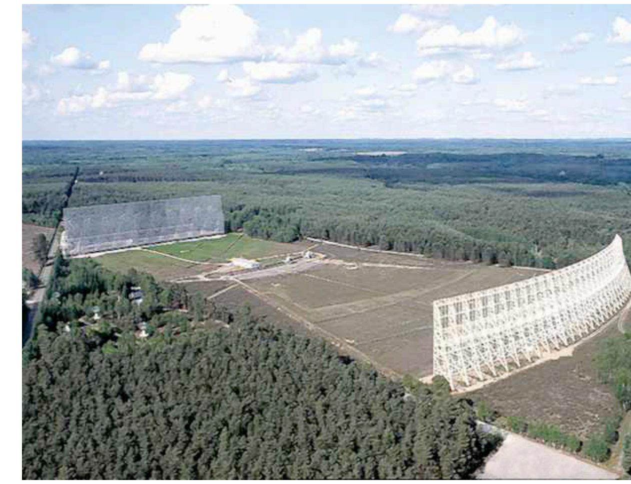


Radio Astronomy

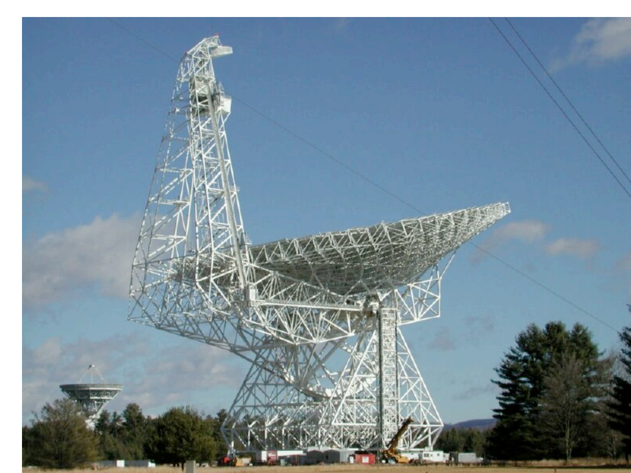
Lecture 8

The Techniques of Radio Interferometry III: Imaging

Lecturer: Michael Wise (wise@astron.nl)

April 25th, 2013



Westerbork/LOFAR Field Trip

Thursday May 2nd, 2013

Train to Beilen:

Depart Amsterdam Science Park: 10:48
Transfer Zwolle
Arrive Beilen: 13:04

Itinerary:

13:05 - Pick-up at Station Beilen
13:20 - Arrival at Westerbork
13:20 - 14:20: Tour of Westerbork
14:20 - Depart Westerbork
15:00 - Arrive LOFAR
15:00 - 16:00: Tour of LOFAR
16:00 - Depart LOFAR
17:00 - Arrive Station Beilen

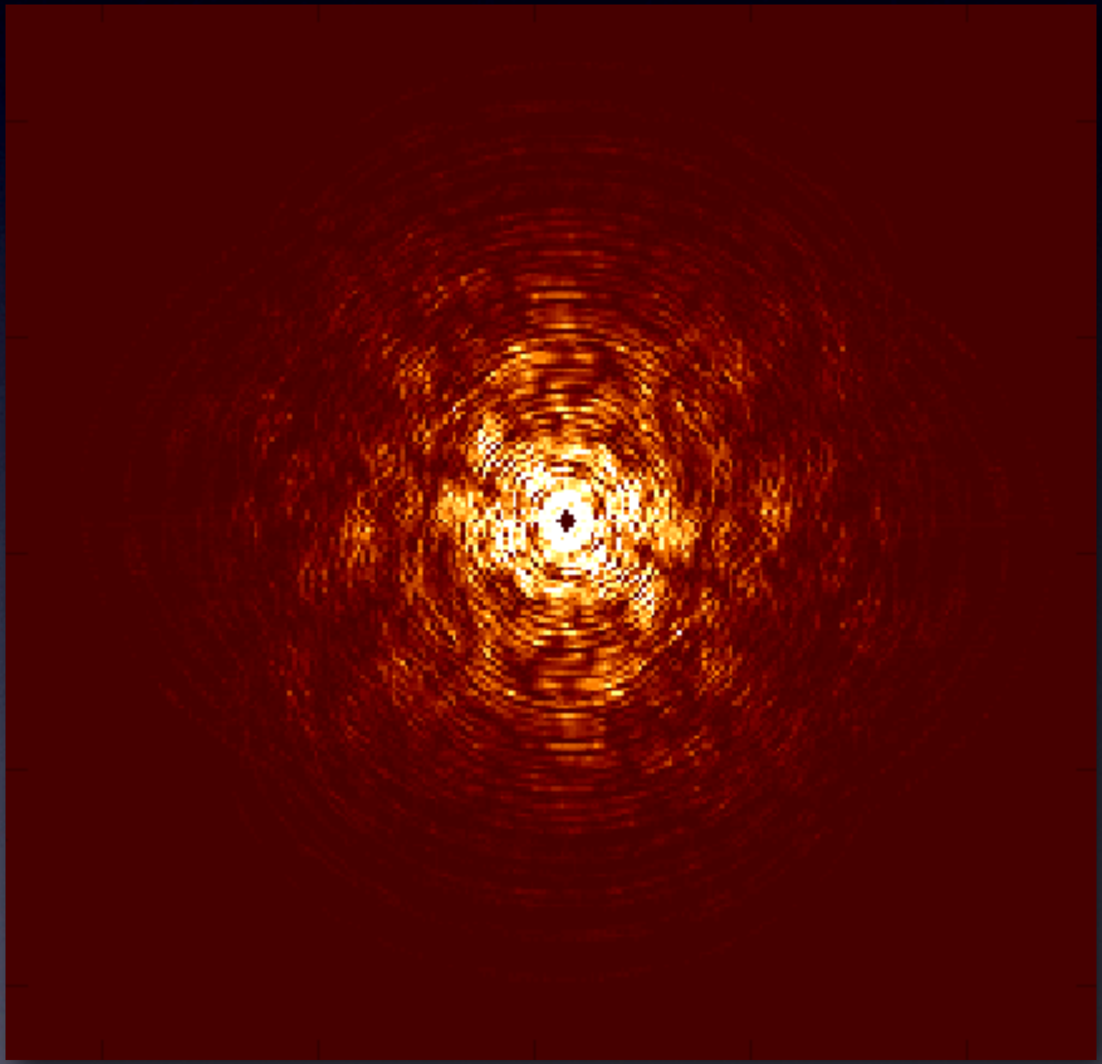
Outline

- Imaging and Deconvolution
- Image Quality, Noise, Dynamic Range
- Wide-band imaging
- Wide-field imaging
- Mosaicing

Imaging and Deconvolution

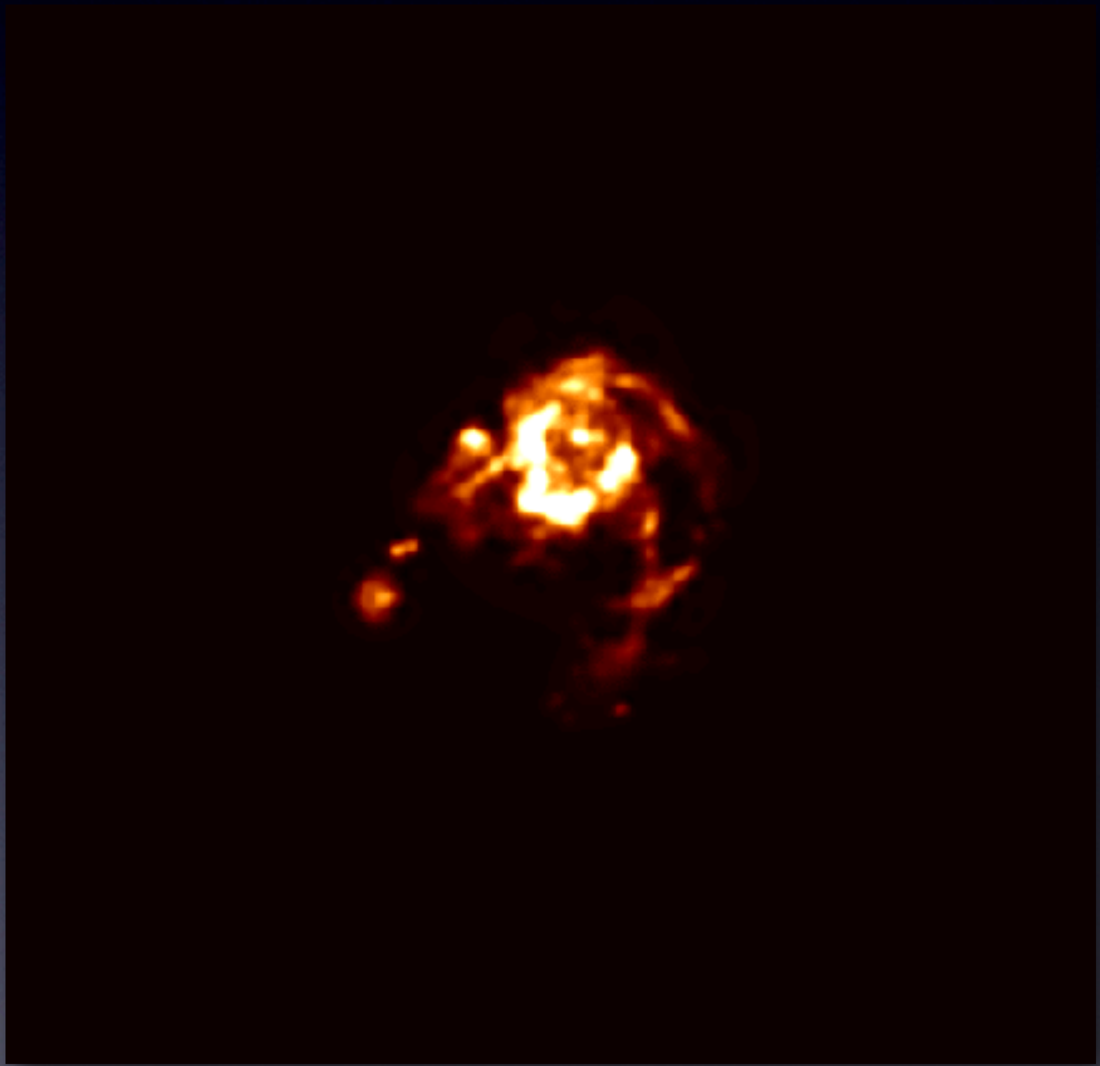
Basic Imaging

How do we go from the measurement of the visibility function to images of the sky?



$V(u,v)$

□

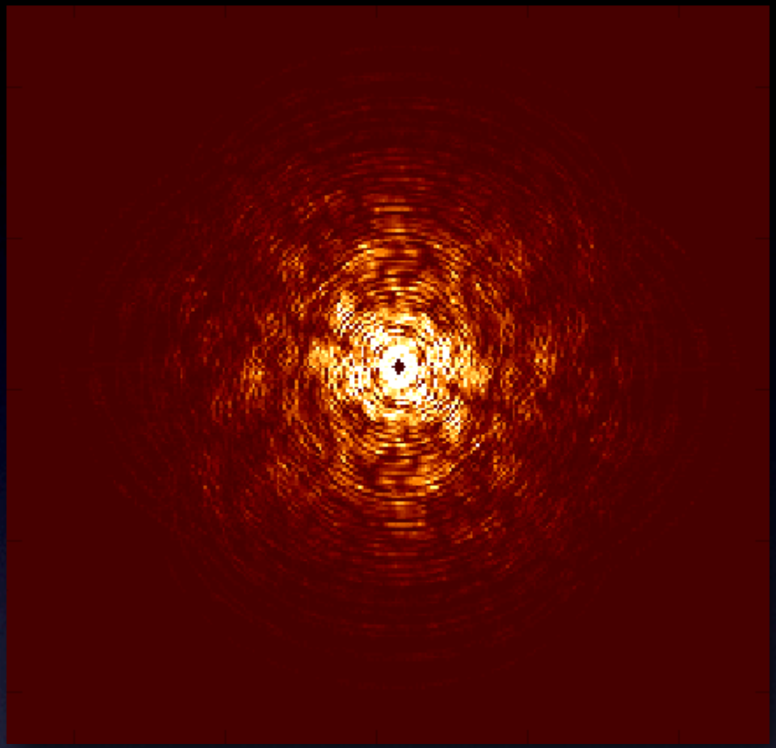


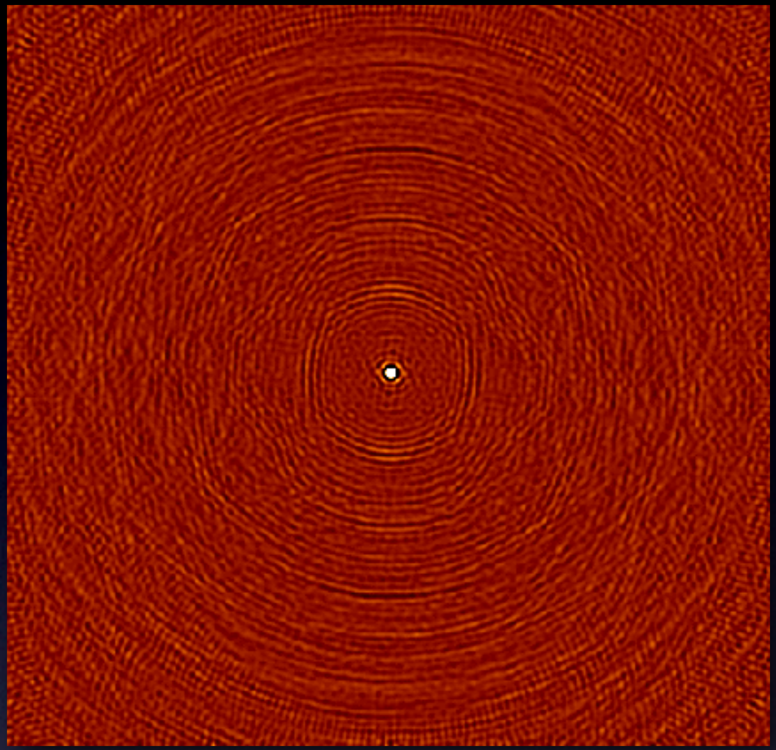
$I(x,y)$

Imaging Terminology

$V(u,v)$

Visibility



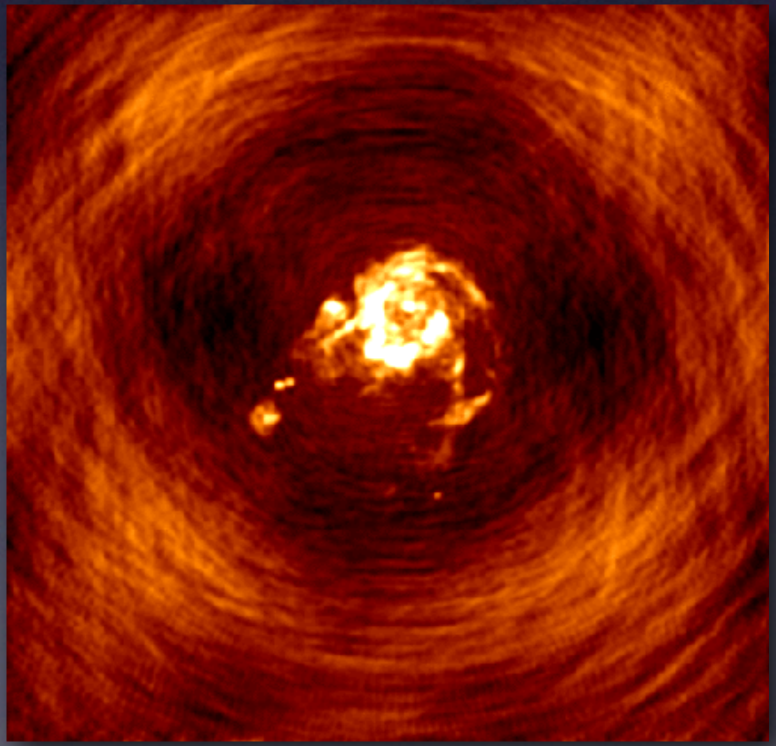


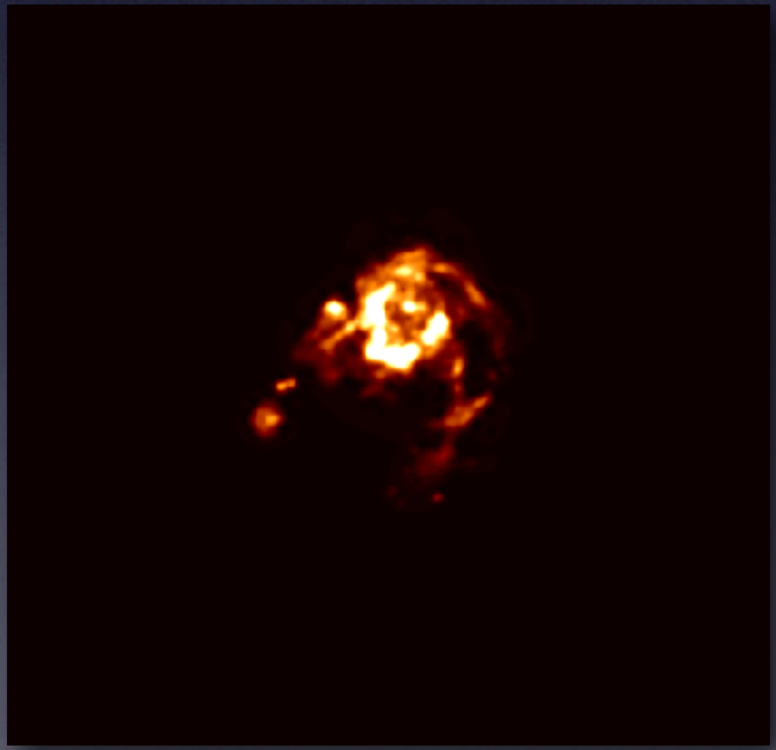
$B(x,y)$

Dirty Beam

$I_D(x,y)$

Dirty Image





$I(x,y)$

True Image

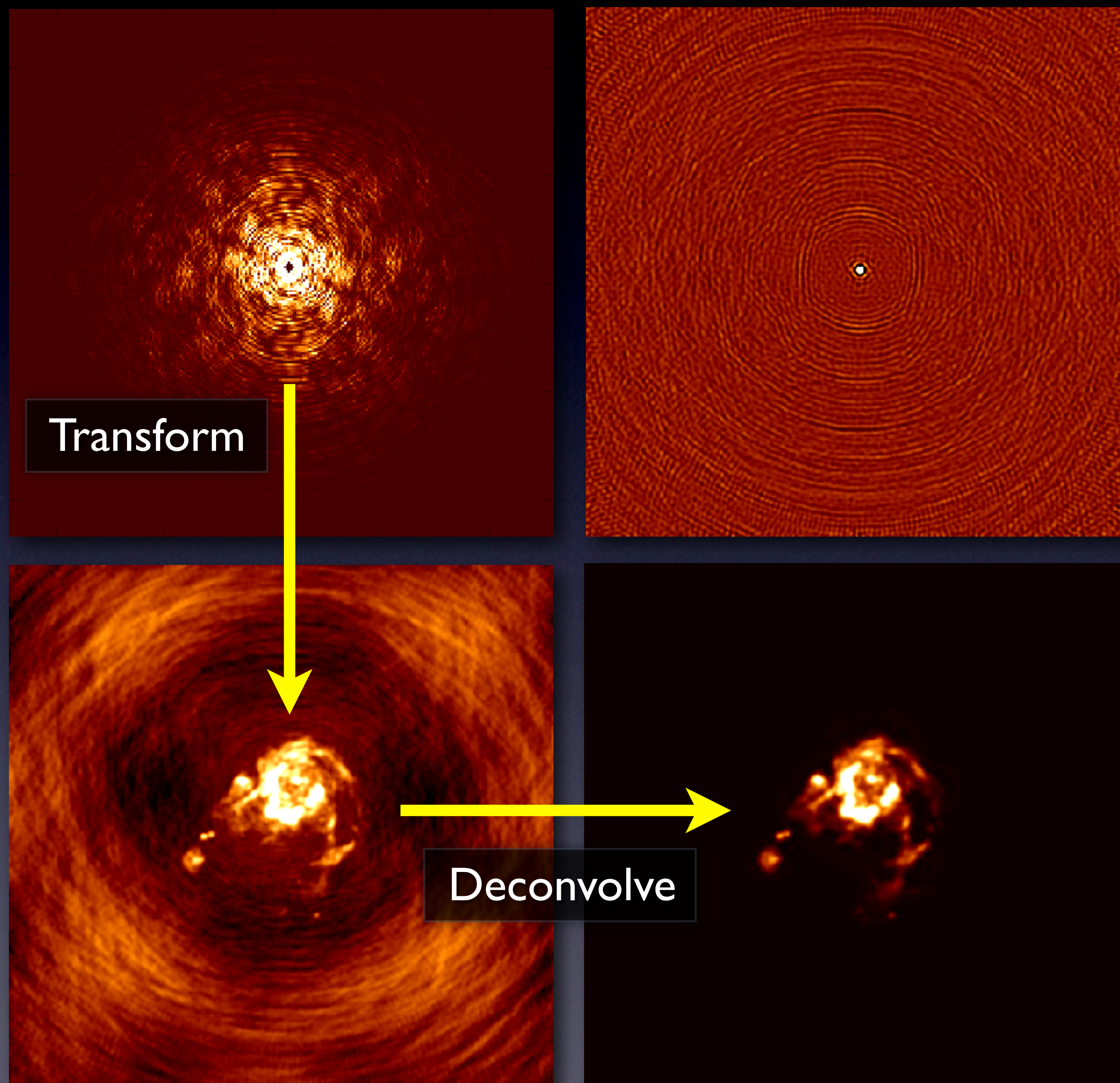
Imaging Terminology

V(u,v)

Visibility

B(x,y)

Dirty Beam



I_D(x,y)

Dirty Image

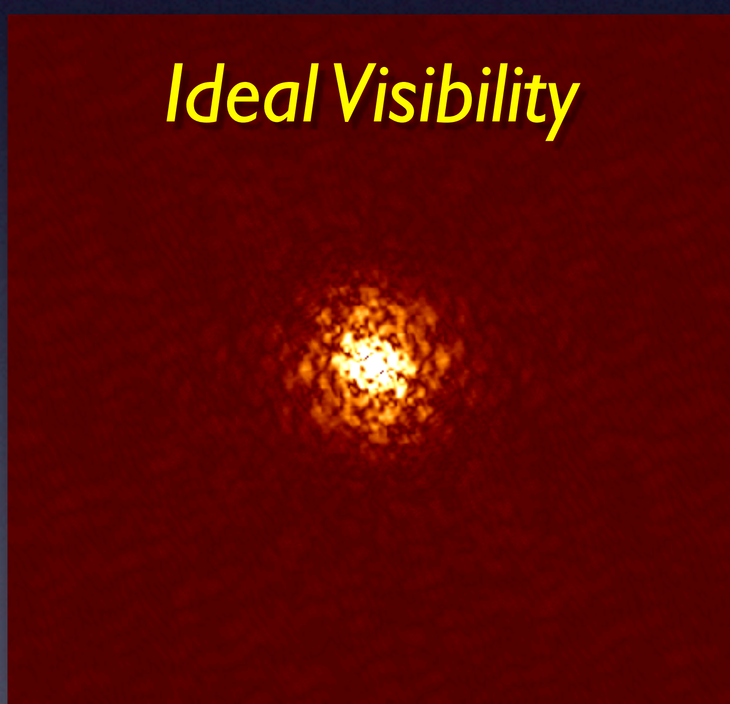
I(x,y)

True Image

Ideal Fourier Relationship

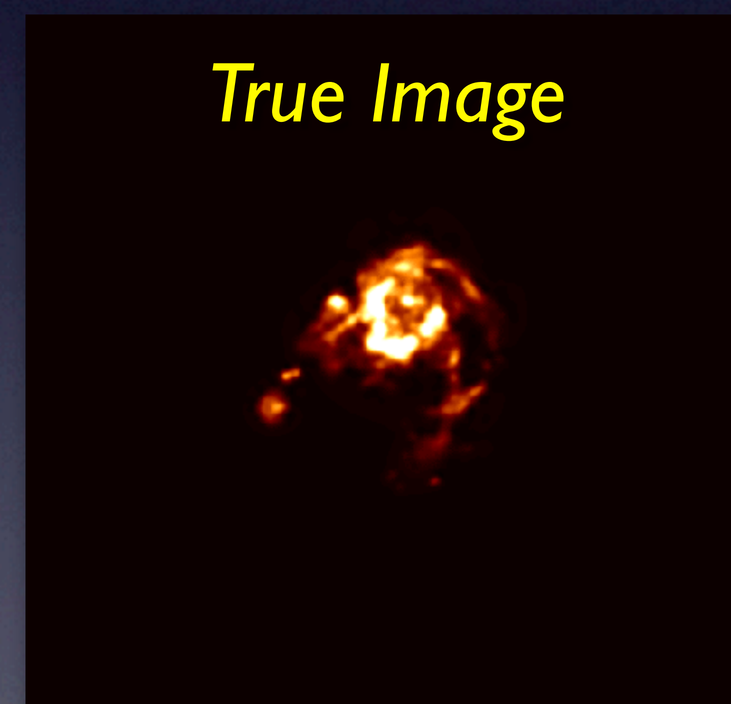
$$V(u,v) = \iint I(x,y) e^{2\pi i(ux+vy)} dx dy$$

- Interferometers are indirect imaging devices
- I(x,y) is 2D Fourier transform of V(u,v)



Fourier Transform

$$V(u,v) \Leftrightarrow I(x,y)$$



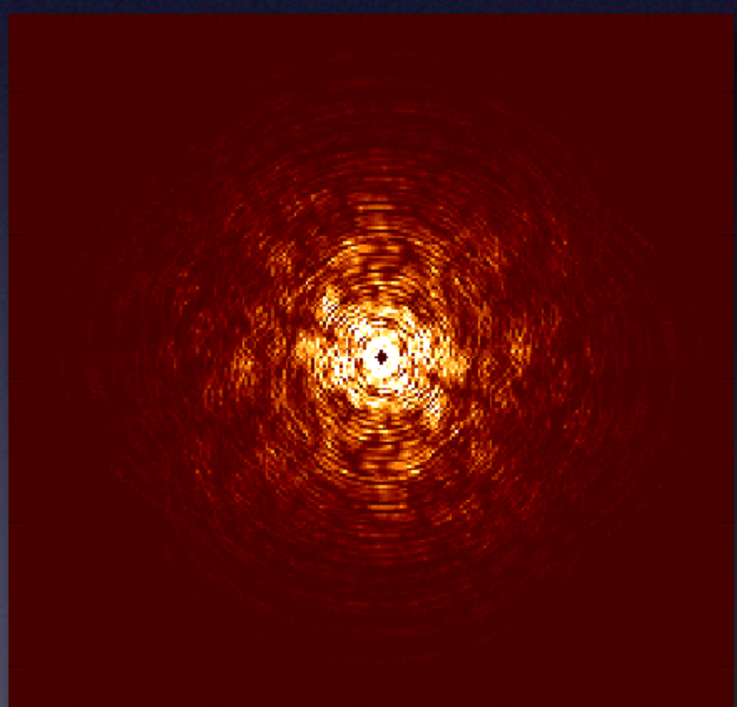
True ONLY if V(u,v) is measured for all (u,v)!

(u,v) Plane Sampling

- With a limited number of antennas, the uv-plane is sampled at discrete points:

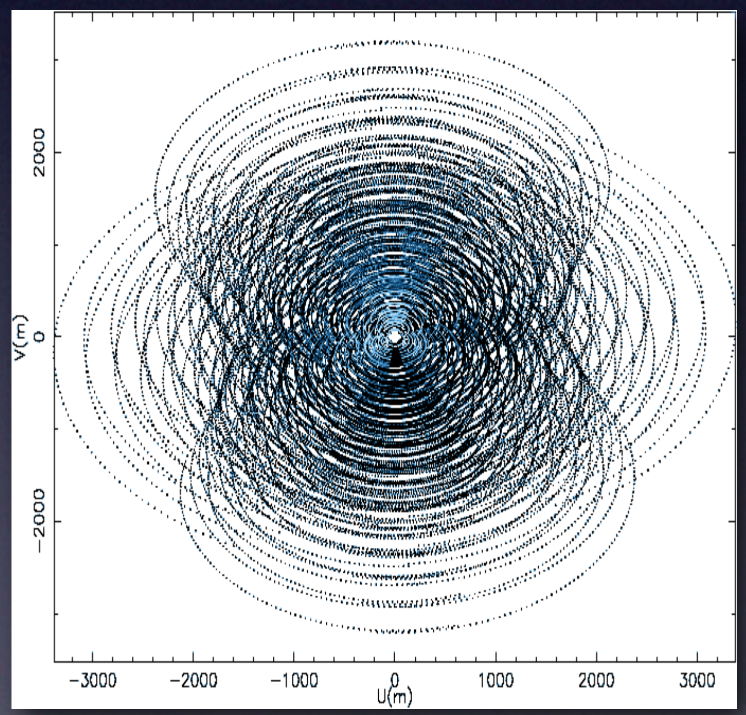
$$S(u,v) = \sum_k \delta(u_k, v_k) \qquad V_M(u,v) = S(u,v)V(u,v)$$

Measured



$V_M(u,v)$

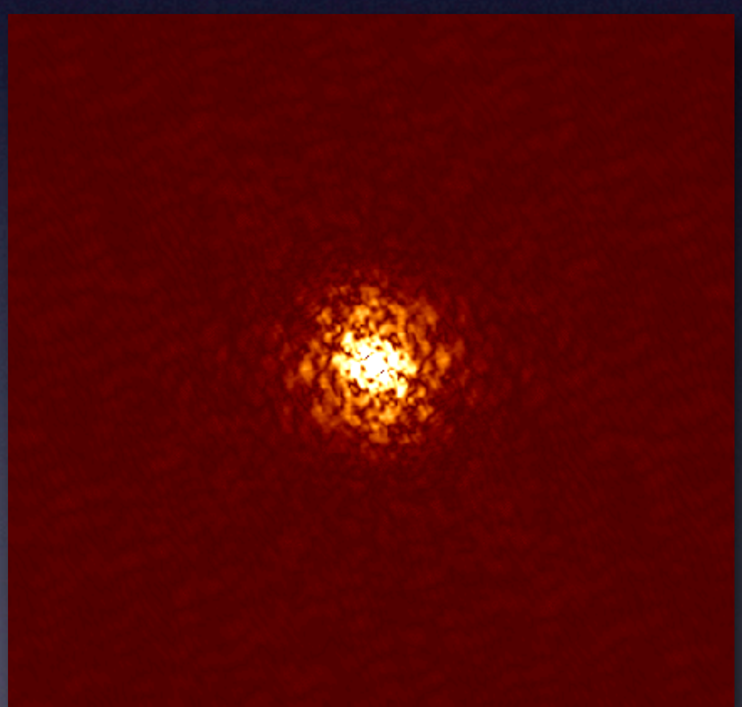
=



$S(u,v)$

x

Ideal

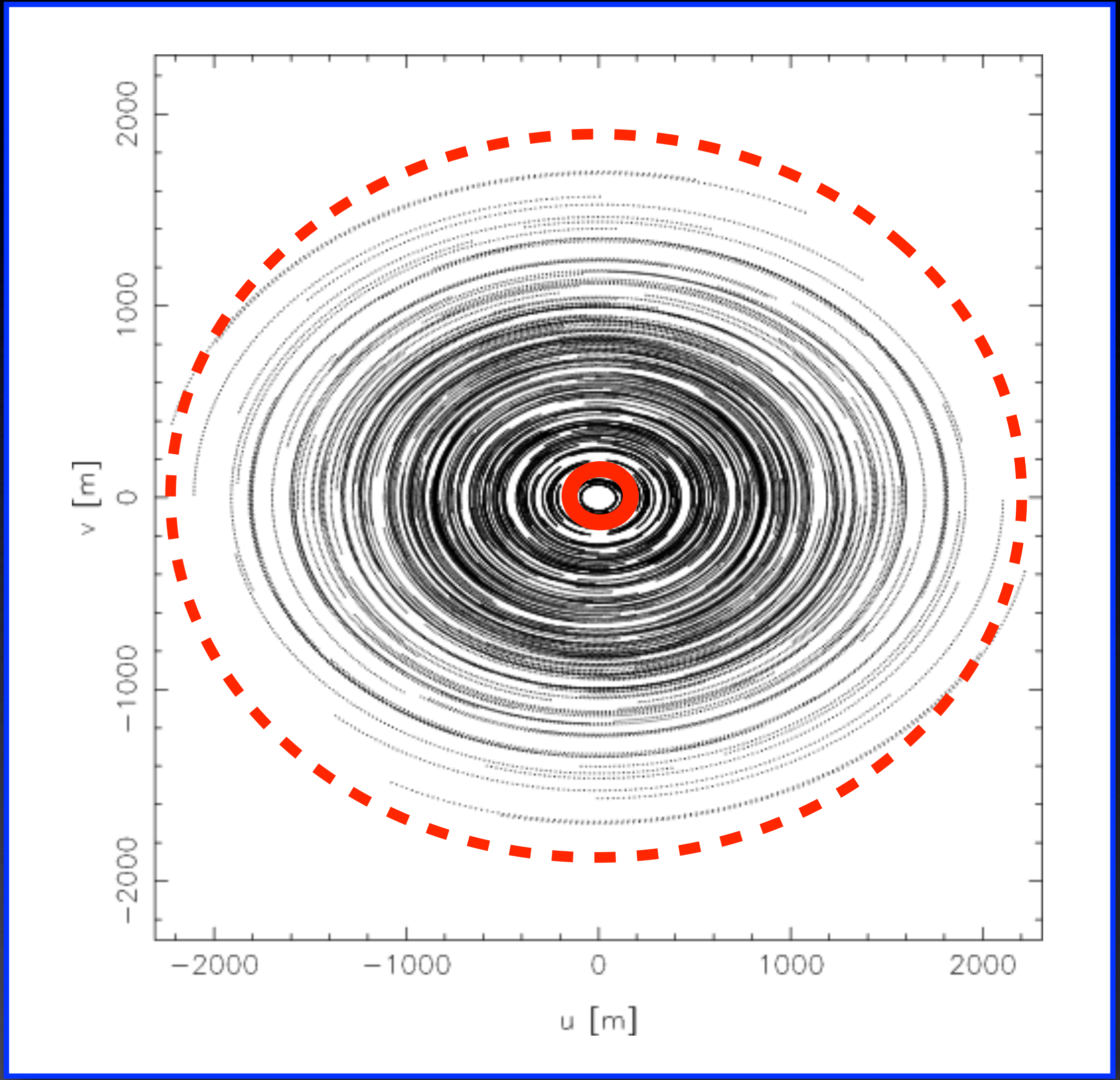


$V(u,v)$

(u,v) Plane Sampling

Incomplete (u,v) sampling means "missing information"

- Outer boundary
  - *No measurements beyond (u_{max}, v_{max})*
  - *Sets resolution limit of the array*
  - *No information on small scales*
- Inner boundary
  - *"Central hole" inside (u_{min}, v_{min})*
  - *Total integrated power is not measured*
  - *No information on large scales*
  - *Extended structures invisible*
- Sparse sampling
  - *Information missing over (u,v)*
  - *Contributes to side lobe structure in the beam*



Effect of (*u,v*) sampling

- Transforming gives the dirty image $I_D(x,y)$

$$I_D(x,y) = FT^{-1}[V_M(u,v)] = FT^{-1}[S(u,v)V(u,v)]$$

- Using the convolution theorem gives:

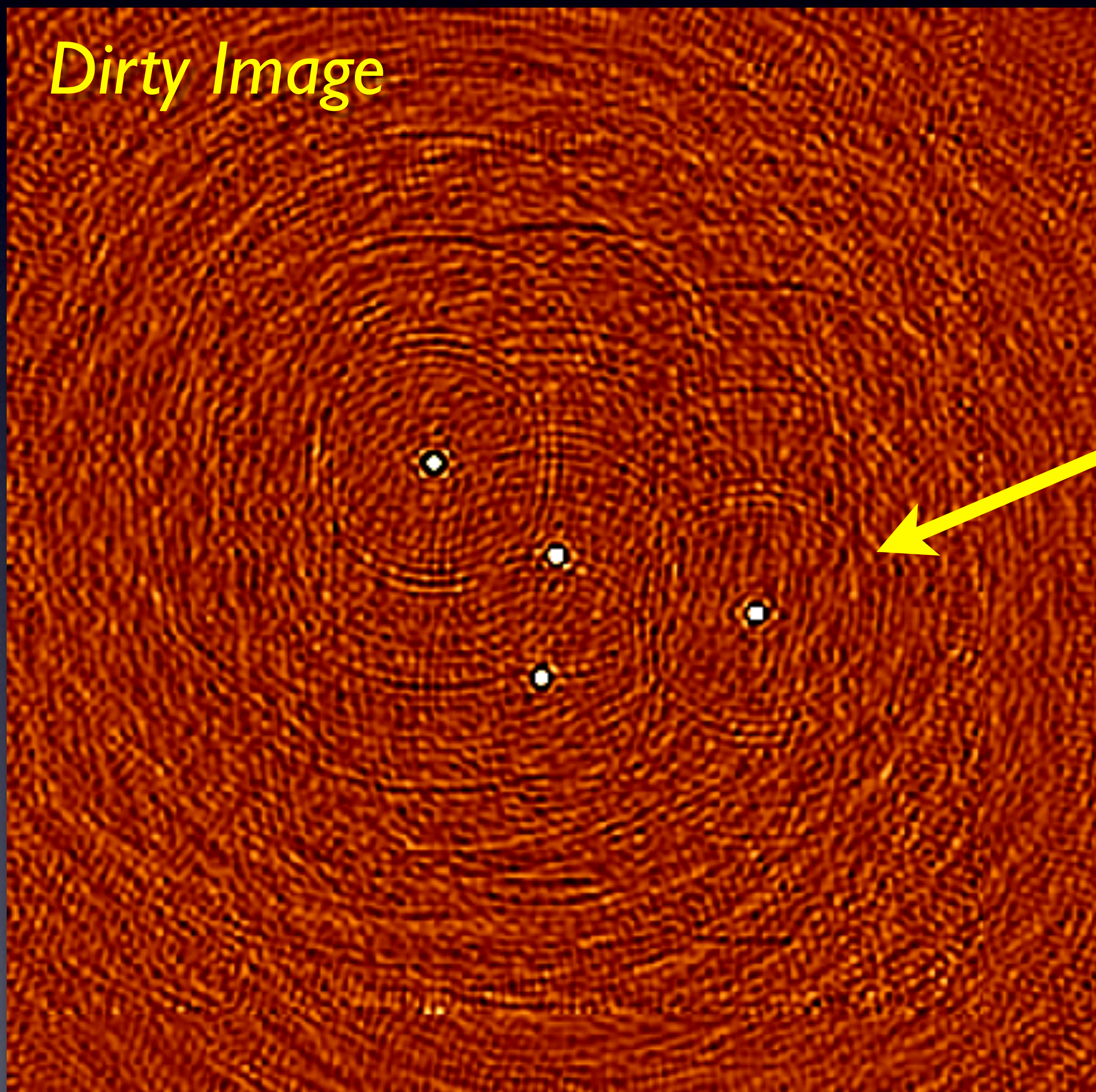
$$I_D(x,y) = B(x,y) * I(x,y) \qquad B(x,y) = FT^{-1}[S(u,v)]$$

- Dirty image is convolution of true image with dirty beam $B(x,y)$

To recover $I(x,y)$, we must deconvolve $B(x,y)$ from $I_D(x,y)$

Convolution with *B(x,y)*

$$I_D(x,y) = \sum_i B(x - x_i, y - y_i) * I(x_i, y_i)$$

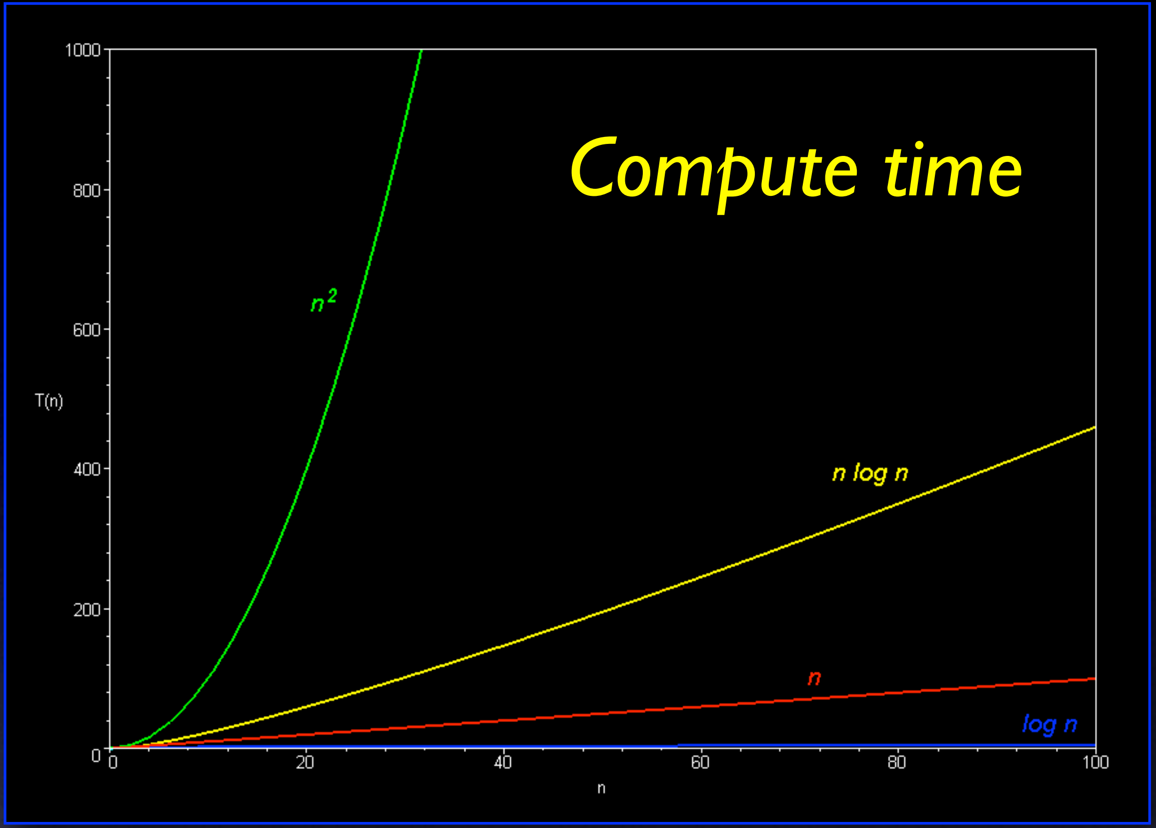


$$= I(x_0, y_0) * B(x - x_0, y - y_0) + I(x_1, y_1) * B(x - x_1, y - y_1) + \ldots$$

Dirty beam can vary with time and position across the field

Computing the Dirty Image

- "Fourier Transform"
  - *Use Fast Fourier Transform (FFT) algorithm*
  - *Compute scales as ~O(NlogN) for (N x N) image*
  - *FFT requires data on a regularly spaced grid*
  - *Radio arrays sample V(u,v) on irregular grids, so.....*
- "Gridding"
  - *Used to resample V(u,v) for FFT*
  - *Convolutional gridding used to resample $V_M(u,v)$*
  - *Gridding function affects resulting dirty image*
- "Weighting"
  - *Weighting function W_k can be chosen to modify the side lobes*
  - *Different weights □ different B(x,y)*
  - *Can "tune" for resolution or sensitivity*



Dirty beam is a weighted sum of the measured Fourier components

$$B(x,y) = \frac{\sum_k W_k cos(u_{kl} + v_{km})}{\sum_k W_k}$$

Weighting Schemes

Observed image is a weighted-average of the data

$$I_D(x,y) = \frac{\sum_k F^{-1}[W_k(u,v)\ S(u,v)\ V(u,v)]}{\sum_k W_k(u,v)}$$

$$W_k = \frac{1}{\sigma_k^2}$$

- Natural
  - *Maximizes the sensitivity, degrades angular resolution*

$$W_k = \frac{1}{\sigma_k^2\ \rho(u_k, v_k)}$$

- Uniform
  - *Best angular resolution, reduced point source sensitivity*

$$W_k = \frac{(1+s)}{\sigma_k^2\ [1 + \frac{s\rho(u_k,v_k)}{\sigma_k^2}]}$$

- Robust
  - *Smooth, tunable combination of natural and uniform*

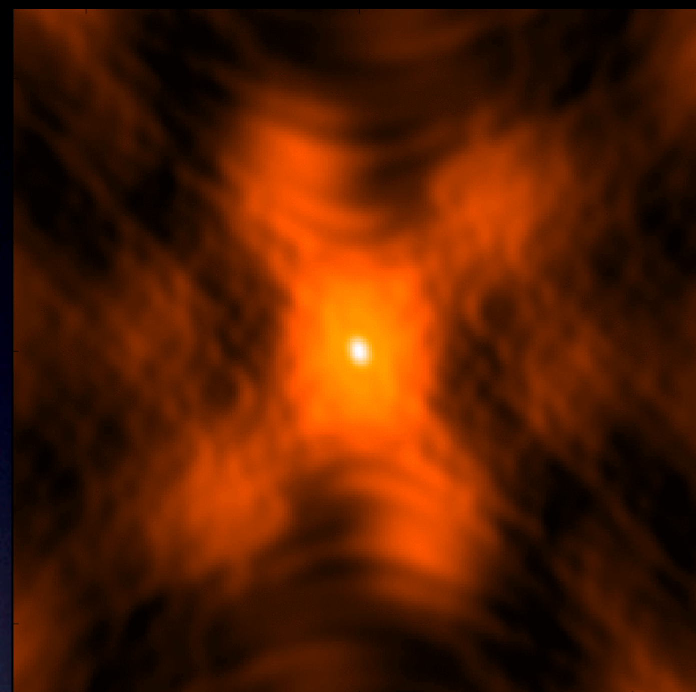
$$W_k = \frac{1}{\sigma_k^2} e^{-\frac{(u^2+v^2)}{t^2}}$$

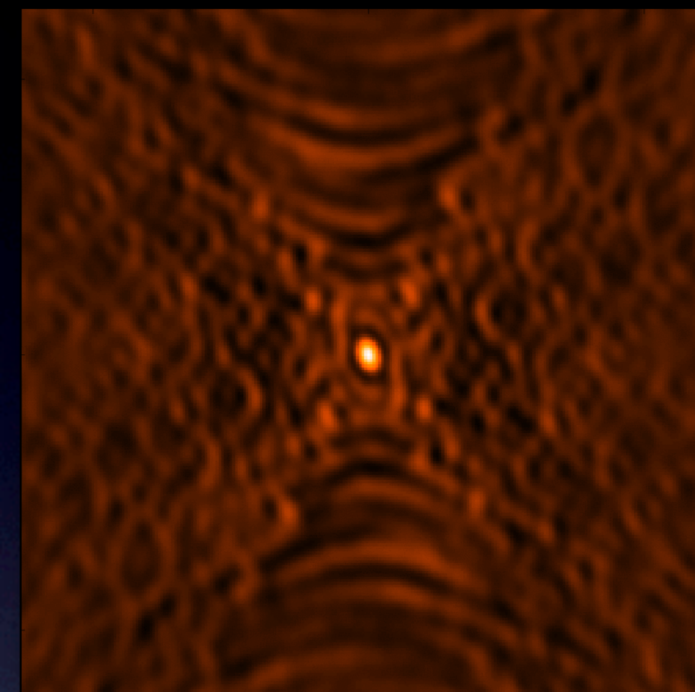
- Tapering
  - *Similar to smoothing, degrades angular resolution*

Weighting Schemes

Natural Weighting

0.77x0.62
$\sigma=1.0$



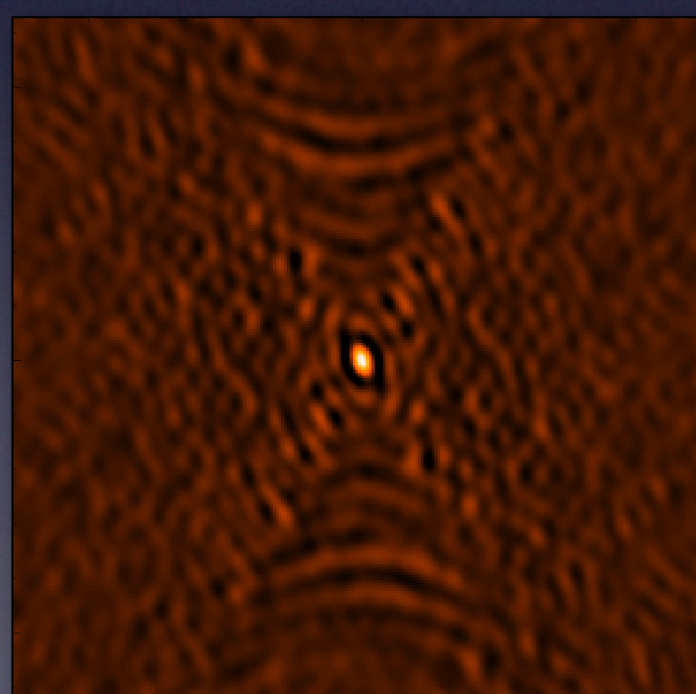


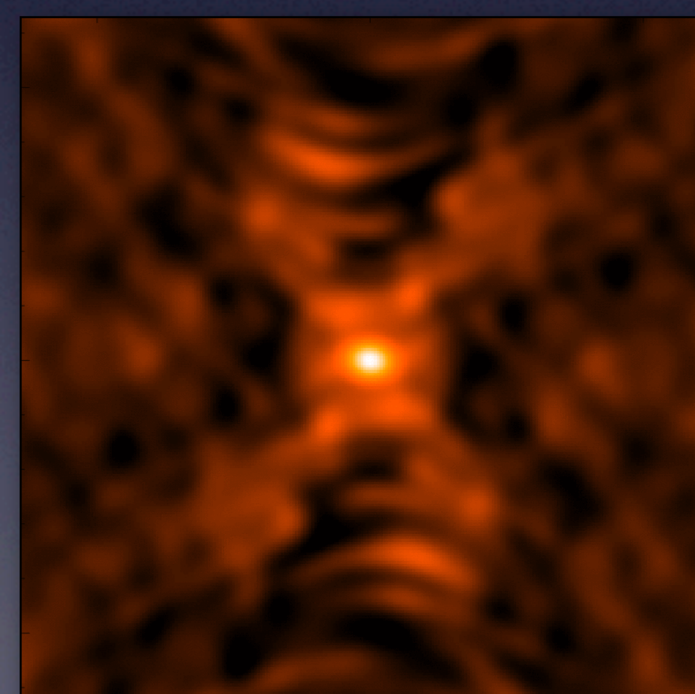
Robust Weighting

0.41x0.36
$\sigma=1.6$

Uniform Weighting

0.39x0.31
$\sigma=3.7$

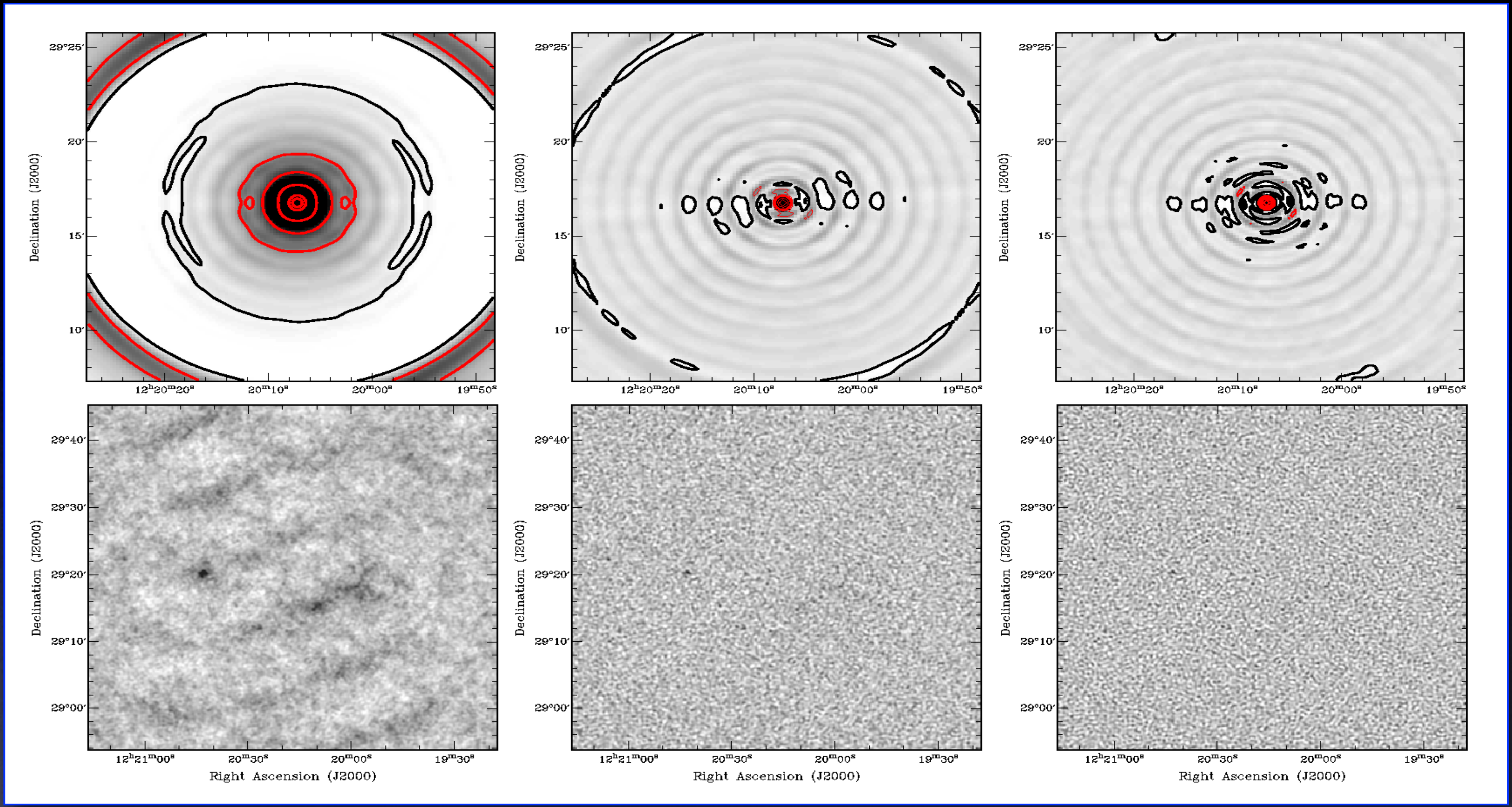




Robust + Taper

0.77x0.62
$\sigma=1.7$

Example: WSRT



Natural weighting
σ = 0.5

Robust = 0
σ = 0.6

Uniform weighting
σ = 0.7

Difference in noise 40% (factor 2 in observing time!)

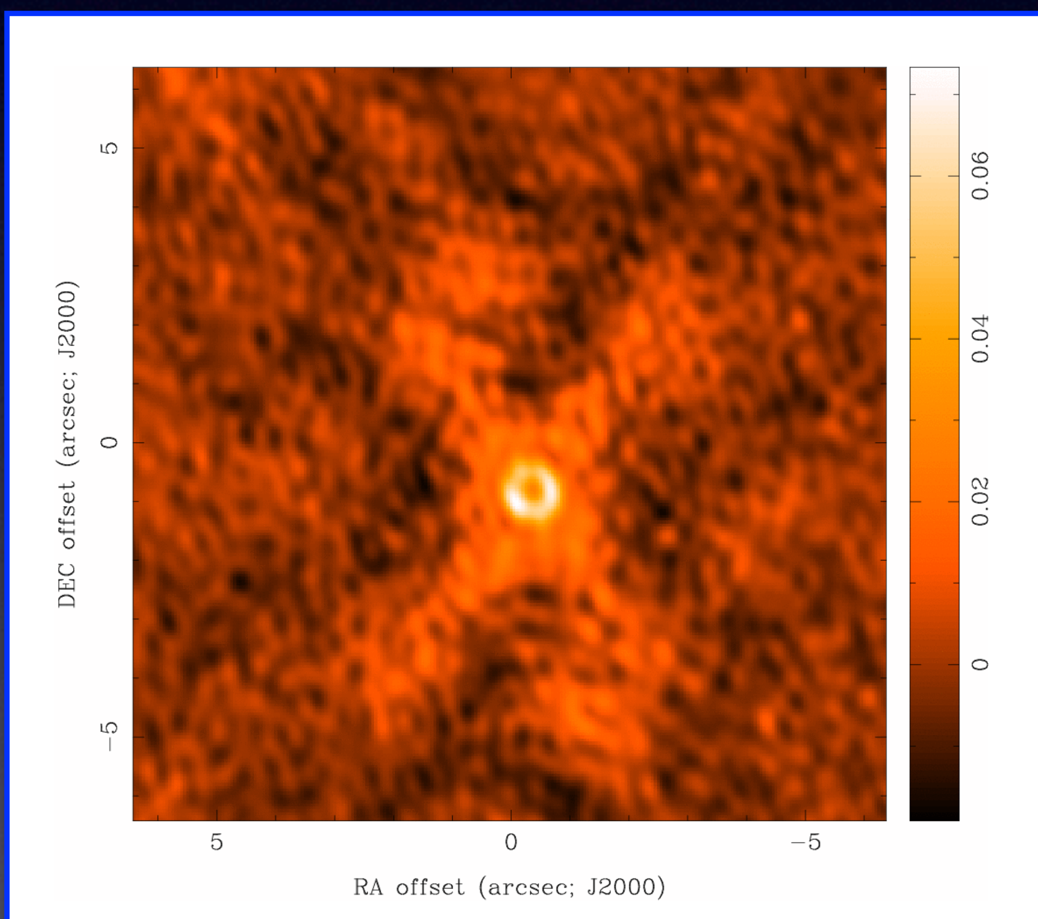
Weighting Summary

	Uniform/Robust *All spatial- frequencies get equal weight*	Natural/Robust *All data points get equal weight*	Tapering *Lower spatial freqs. get higher weight*
Resolution	Higher	Medium	Lower
Sidelobes	Lower	Higher	Depends
Point Source Sensitivity	Lower	Maximum	Lower
Extended Source Sensitivity	Lower	Medium	Higher

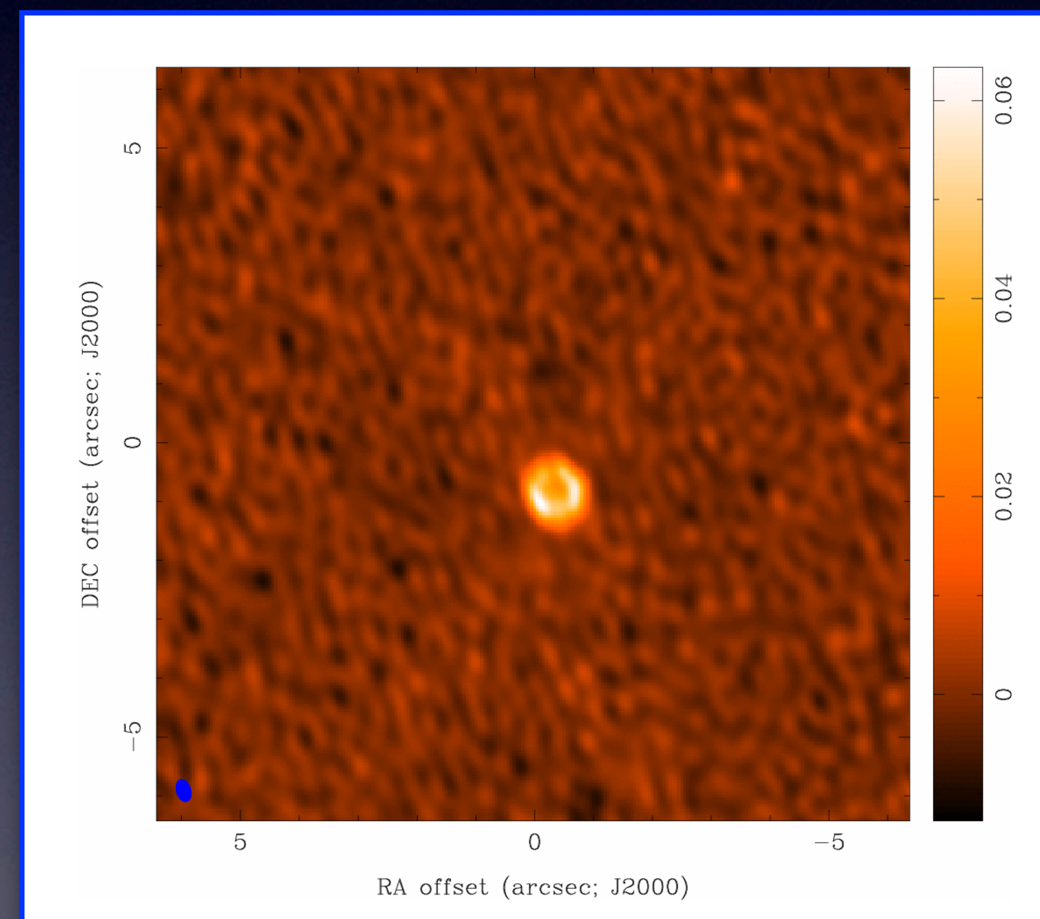
- Imaging parameters provide a lot of freedom
- Appropriate choice depends on science goals

Deconvolution

- Calibration and Fourier transform: $V(u,v) \Rightarrow I_D(x,y)$
- Deconvolve $B(x,y)$ from $I_D(x,y)$ to recover $I(x,y)$ for science
- Information is missing, so be careful (there's noise, too)



Dirty Image



Cleaned Image

Deconvolution Issues

Iteratively fit a sky-model to the observed visibilities

- Reconstruction Issues
  - No unique solution. In fact, there are infinite solutions.
  - There will always be un-resolved structure ⇒ Unphysical to believe structure < FWHM of beam
  - Total integrated power is never measured ⇒ Reconstruction of largest spatial scales is always an extrapolation
  - Requires iterative, non-linear fitting process ⇒ Compute intensive
  - No unique prescription for extracting optimal solution

⇒ *Constrain the solution using astrophysical plausibility*

Deconvolution Algorithms

Algorithms differ in choice of sky-model and optimization scheme

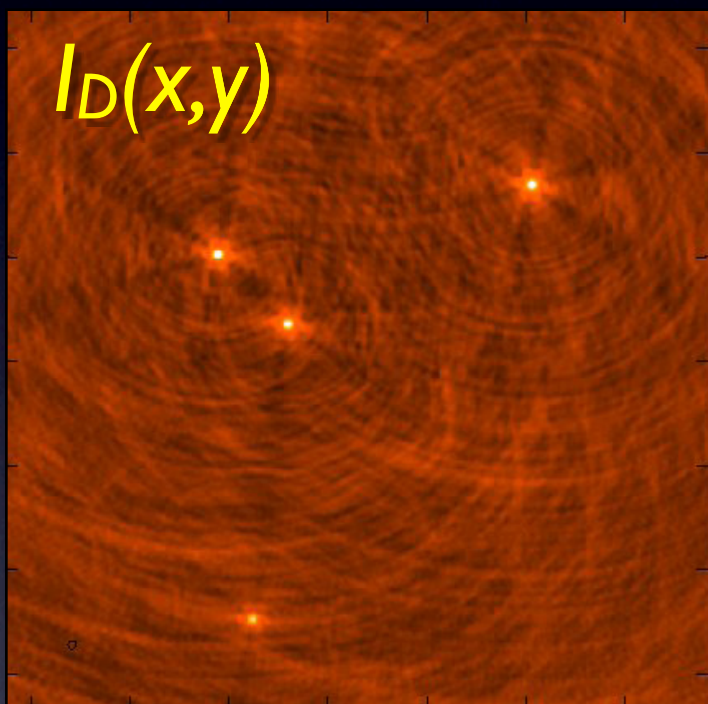
- Classic CLEAN
  - Point-source sky model
- Maximum Entropy Method
  - Assumes sky model is smooth and positive
- Multi-Scale CLEAN
  - Sky is linear combination of components of different shapes and sizes
- Adaptive-Scale-Pixel CLEAN
  - Sky is a linear combination of best-fit Gaussians

$\Rightarrow$ *Output of deconvolution is model image and residuals*

Classic Clean Deconvolution

Assume sky is sum of delta functions:
Developed by Högbom (1974)

$$I(x,y) = \sum_i a_i \; \delta(x_i, y_i)$$



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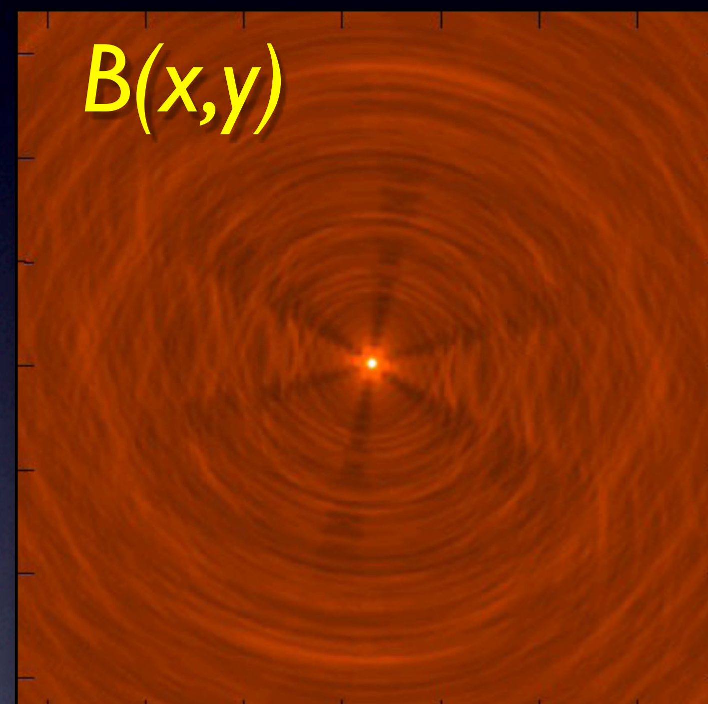

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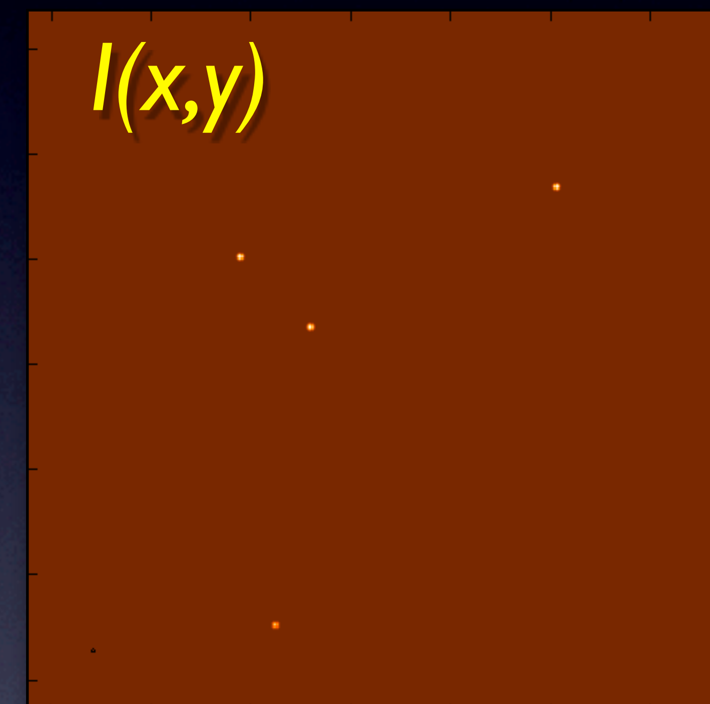


1. Construct the observed dirty image and dirty beam
2. Search for the location of peak amplitude
3. Add a delta-function of this peak at this location to the model
4. Subtract the contribution of this component from the dirty image
5. Repeat steps (2)-(4) until a stopping criterion is reached

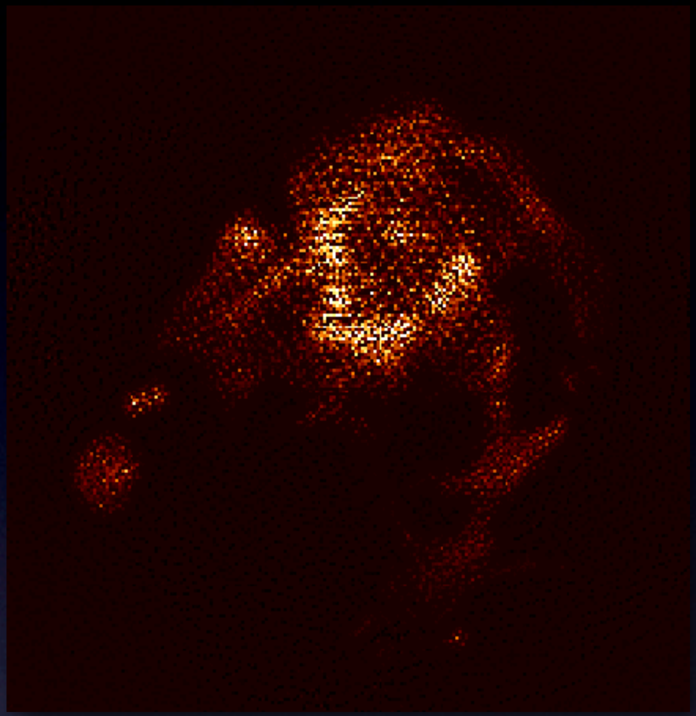
Restore the model using a "clean beam" and adding in final residuals

Clean in Action

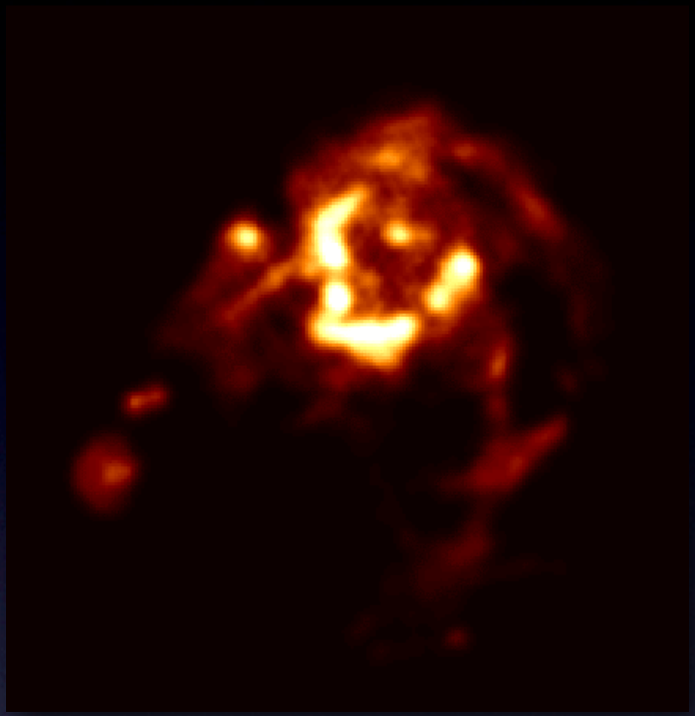
Clean in Action

Clean Example

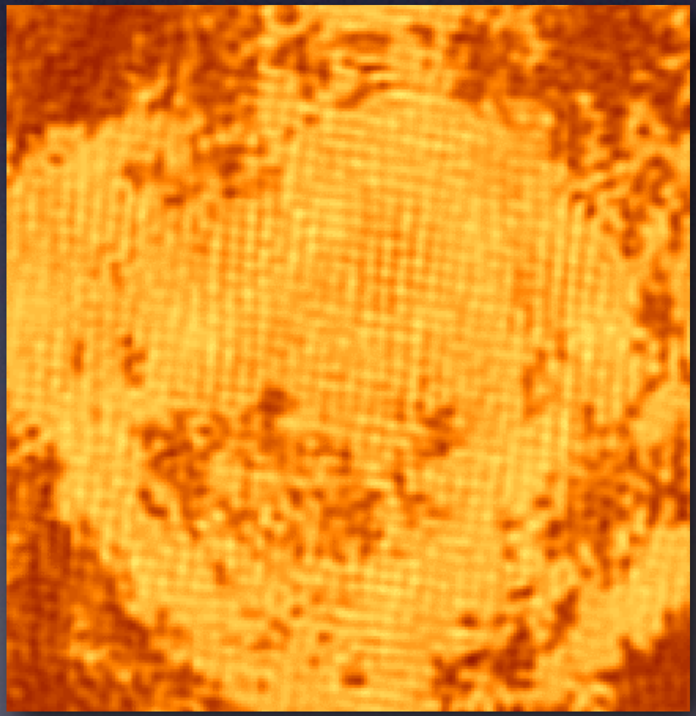
Sky Model



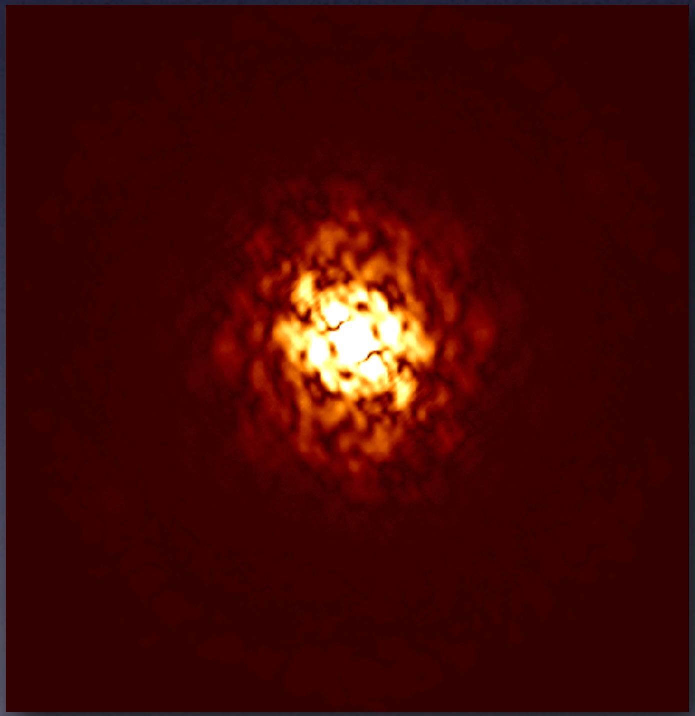
Restored Image



Residuals



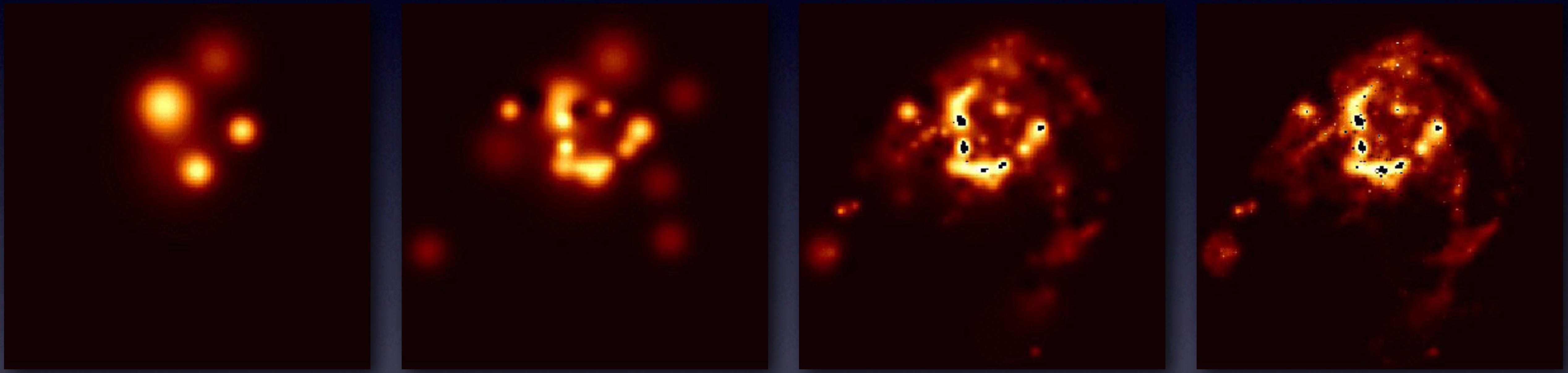
Visibilities



Adaptive Scale Pixel CLEAN

Assume sky is sum of Gaussian functions:
Bhatnagar & Cornwell (2004)

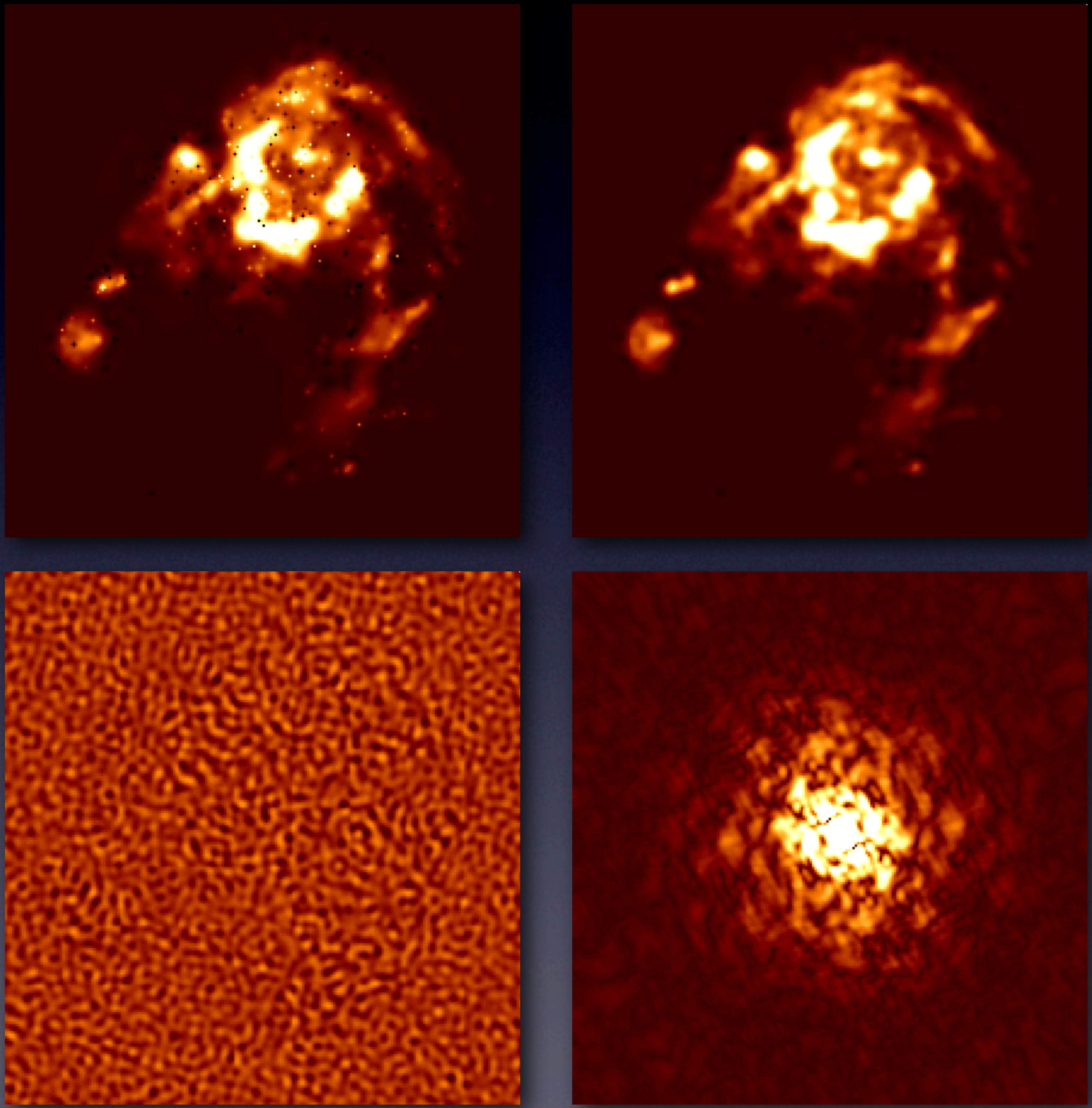
$$I(x,y) = \sum_i a_i \; e^{-\left[\frac{(x-x_i)^2}{\sigma_{xi}^2} + \frac{(y-y_i)^2}{\sigma_{yi}^2}\right]}$$



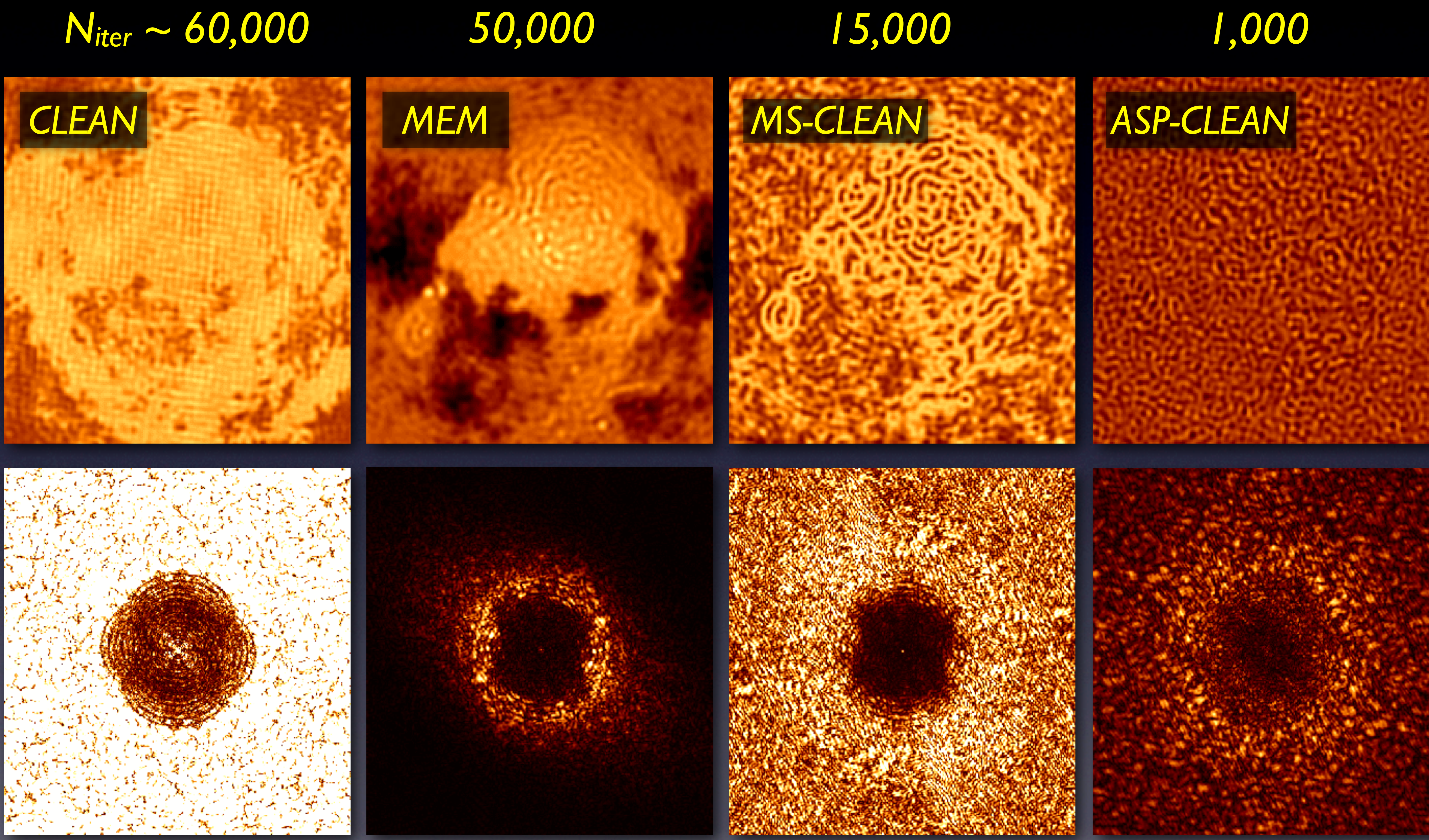
1. Calculate the dirty image, smooth to a few scales
2. Identify peak across scales to choose initial guess for new component
3. Add this new component to the list
4. Re-fit Gaussian parameters for new and old components together
5. Subtract the contribution of all updated components from the dirty image
6. Repeat steps (2)-(5) until a stopping criterion is reached

Adaptive Scale sizes leads to better image reconstruction

ASP-Clean Example



Comparison of Algorithms



Intermission

Image Quality

Measures of Image Quality

- "Dynamic Range"
  - *Defined as ratio of peak brightness to RMS noise in a region empty of emission*
  - *Alternatively, use ratio of peak brightness to peak error (residuals)*
  - *Easy to calculate lower limit to the error in brightness in a non-empty region*
  - *Values run from DR ~10^2 - 10^6*
- "Fidelity"
  - *Difference between the calculated image and the correct image*
  - *Convenient measure of how accurately image matches true I(x,y) on sky*
  - *Need a priori knowledge of the correct image for comparison*
  - *Fidelity image = input model / difference*
  - *Similar to a SNR map*

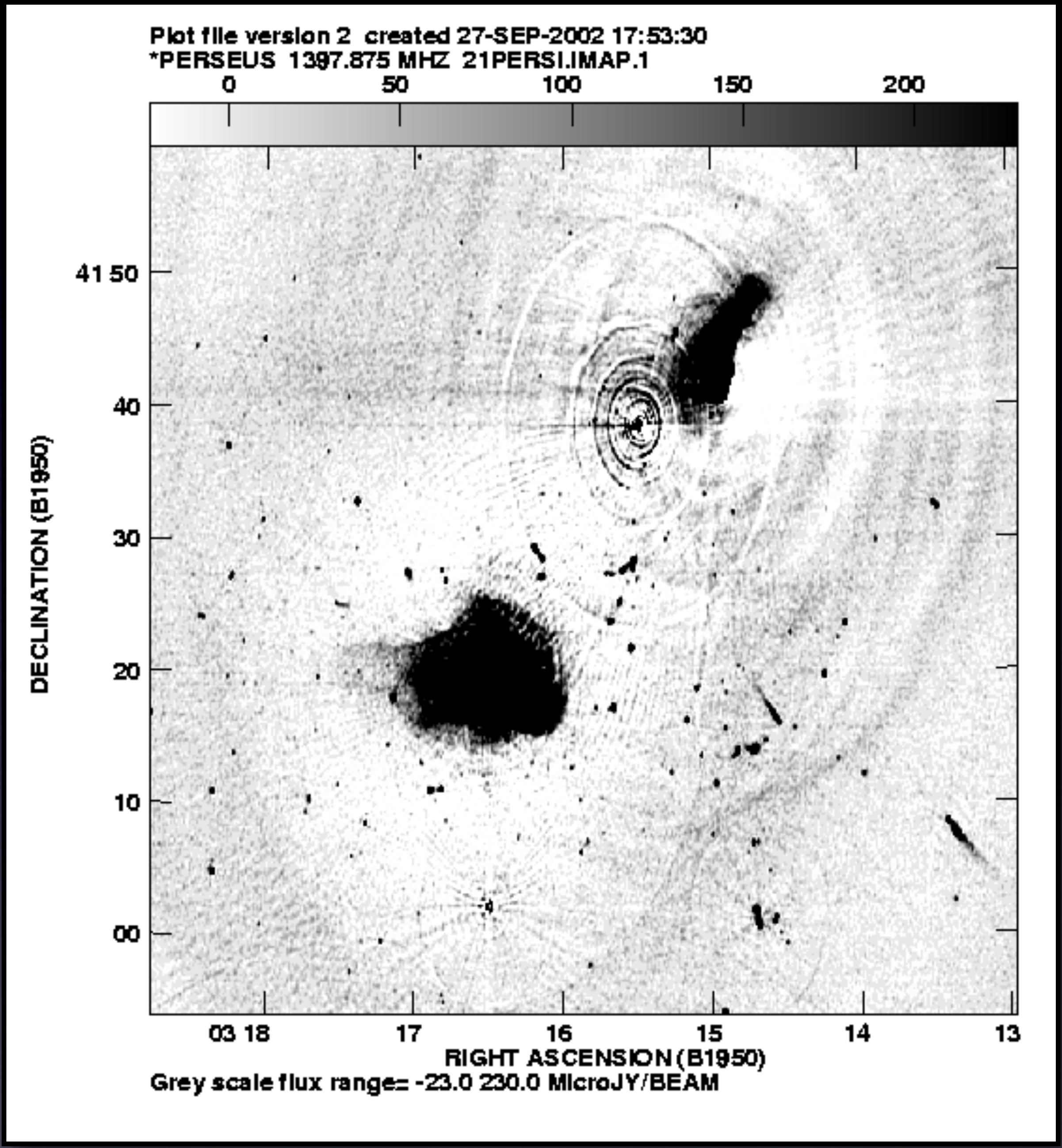


Image of the Perseus cluster showing details exposed at a dynamic range of 1,000,000:1 (de Bruyn & Brentjens 2010)

Recognizing Errors

Some Questions to ask:

Noise properties of image:

Is the rms noise about that expected from integration time?
Is the rms noise much larger near bright sources?
Are there non-random noise components (faint waves and ripples)?

Funny looking Structure:

Non-physical features; stripes, rings, symmetric or anti-symmetric
Negative features well-below a few times the rms noise
Does the image have characteristics that look like the dirty beam?

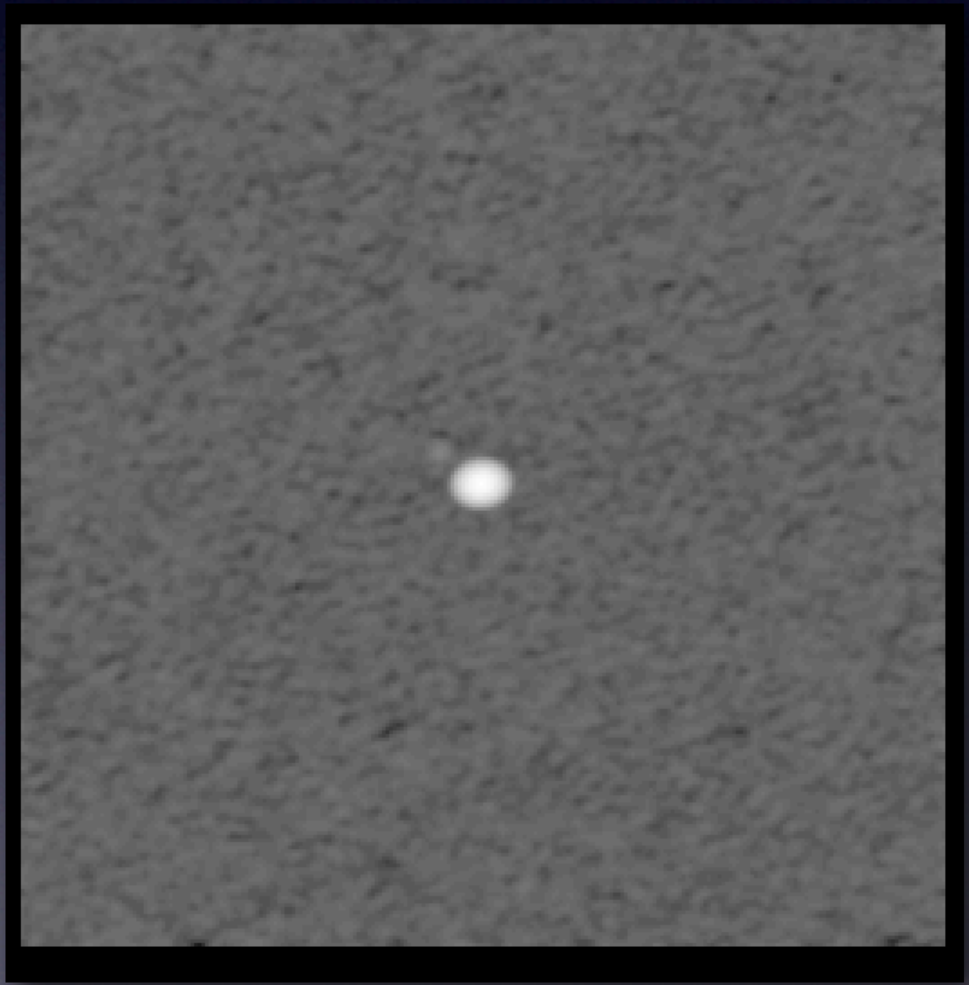
Image-making parameters:

Is the image big enough to cover all significant emission?
Is cell size too large or too small? ~4 points per beam okay
Is the resolution too high to detect most of the emission?

Example: Burst of Bad Data

Results for a point source using VLA, 13 x 5min observation over 10 hr
Images shown after editing, calibration and deconvolution.

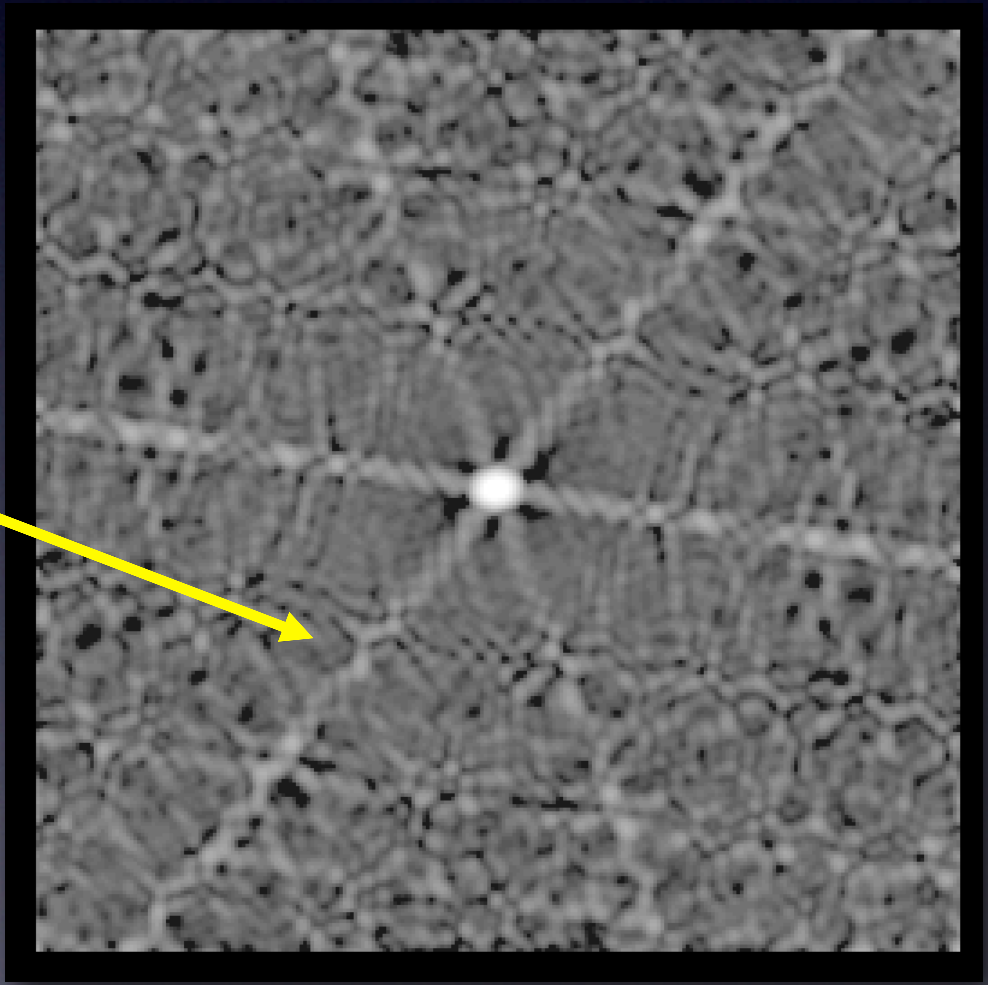
No errors
peak ~ 3.24 Jy, σ~ 0.11 mJy



6-fold symmetric pattern due to VLA "Y" configuration

Image has properties of dirty beam

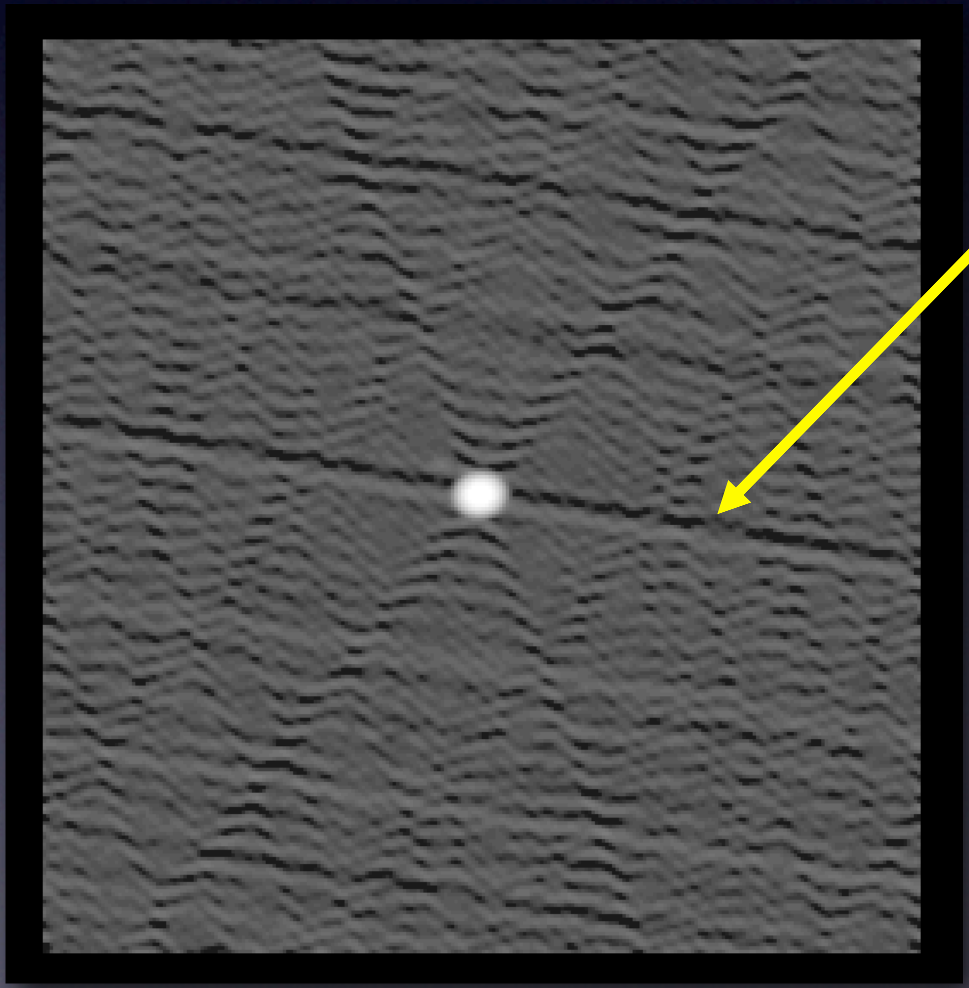
10% amp error for all antennas for 1 time period (σ~ 2.0 mJy)



Example: Bad Antenna

Typical effect from one bad antenna

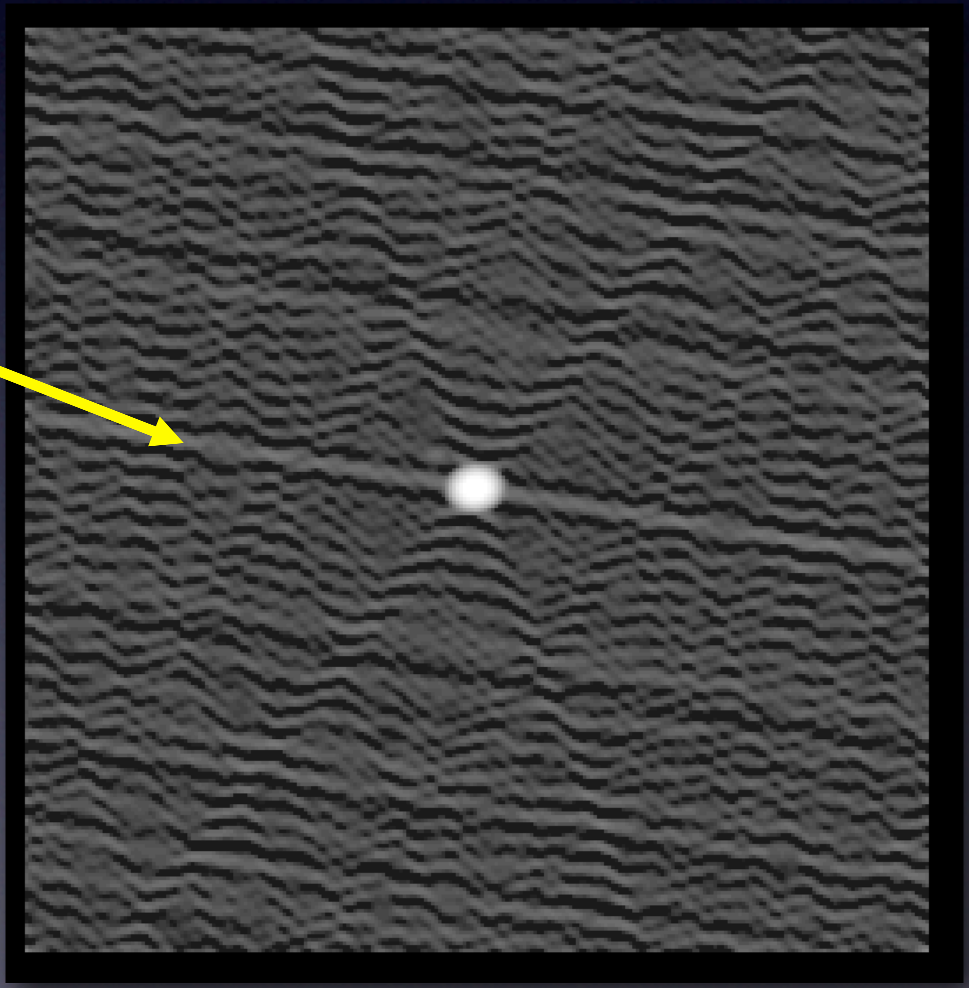
10 deg phase error for one antenna at one time (σ~ 0.49 mJy)



Anti-symmetric ridges

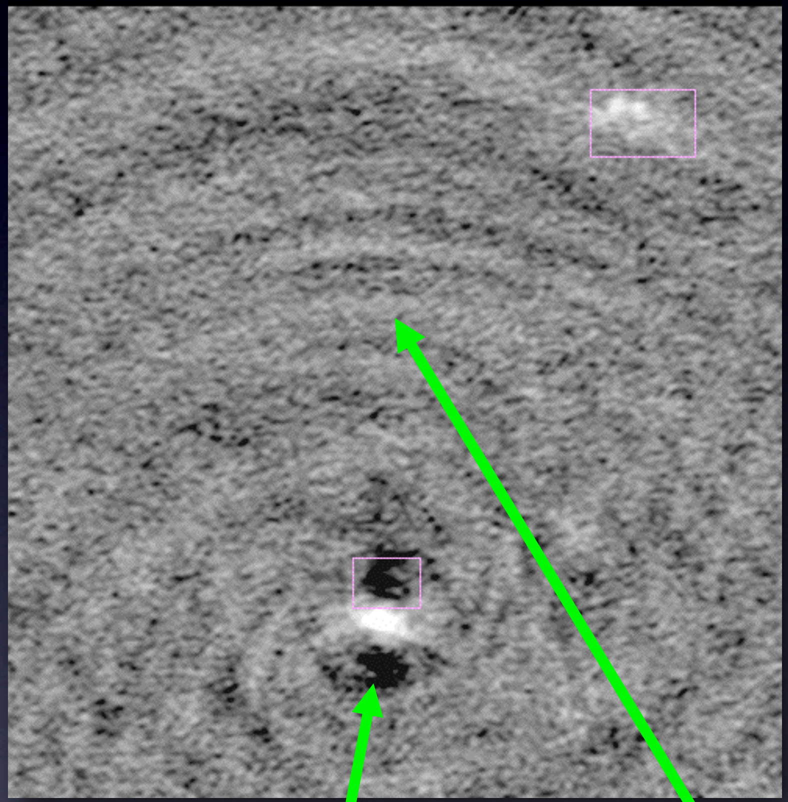
Symmetric ridges

20% amplitude error for one antenna at one time (σ~ 0.56 mJy)



Example: Clean Errors

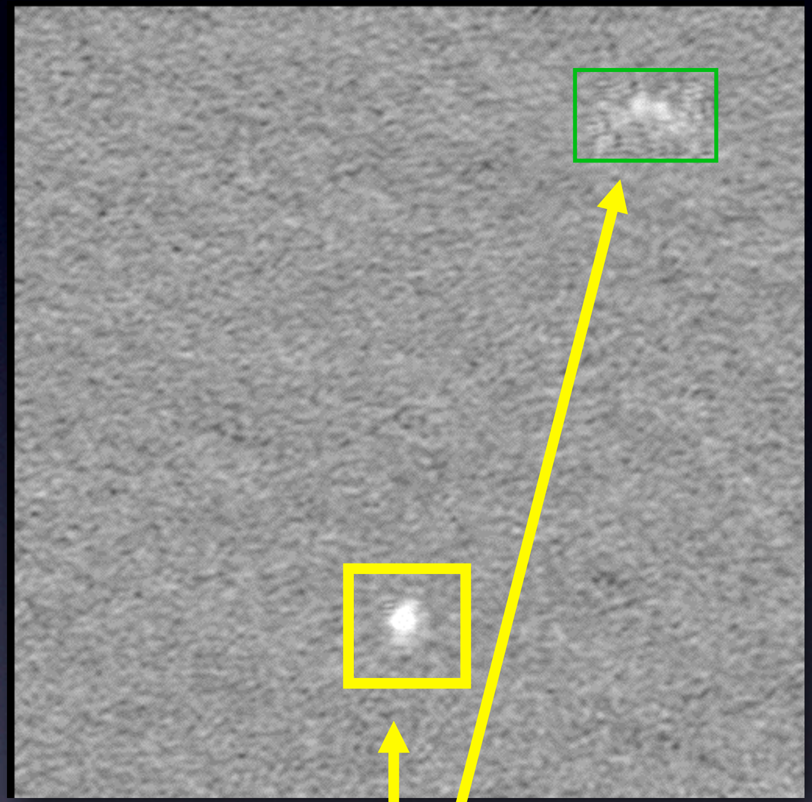
Under-cleaned



Residual sidelobes dominate the noise

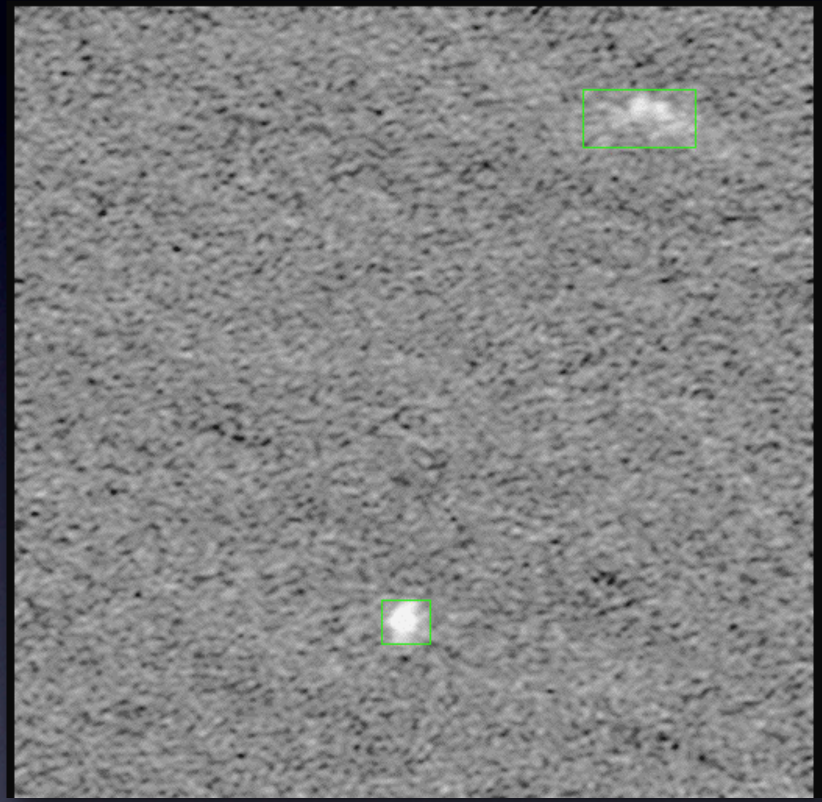
Emission from second source sits atop a negative "bowl"

Over-cleaned



Regions within clean boxes appear "mottled"

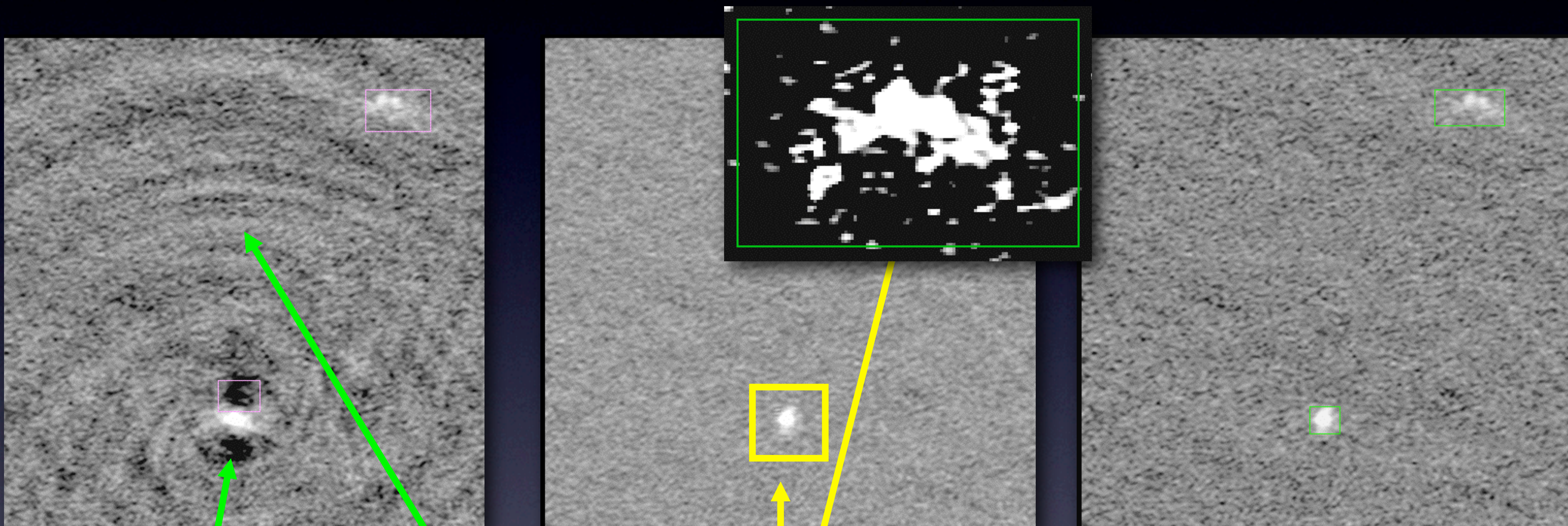
Properly cleaned



Background is thermal noise-dominated; no "bowls" around sources

Example: Clean Errors

Under-cleaned | Over-cleaned | Properly cleaned



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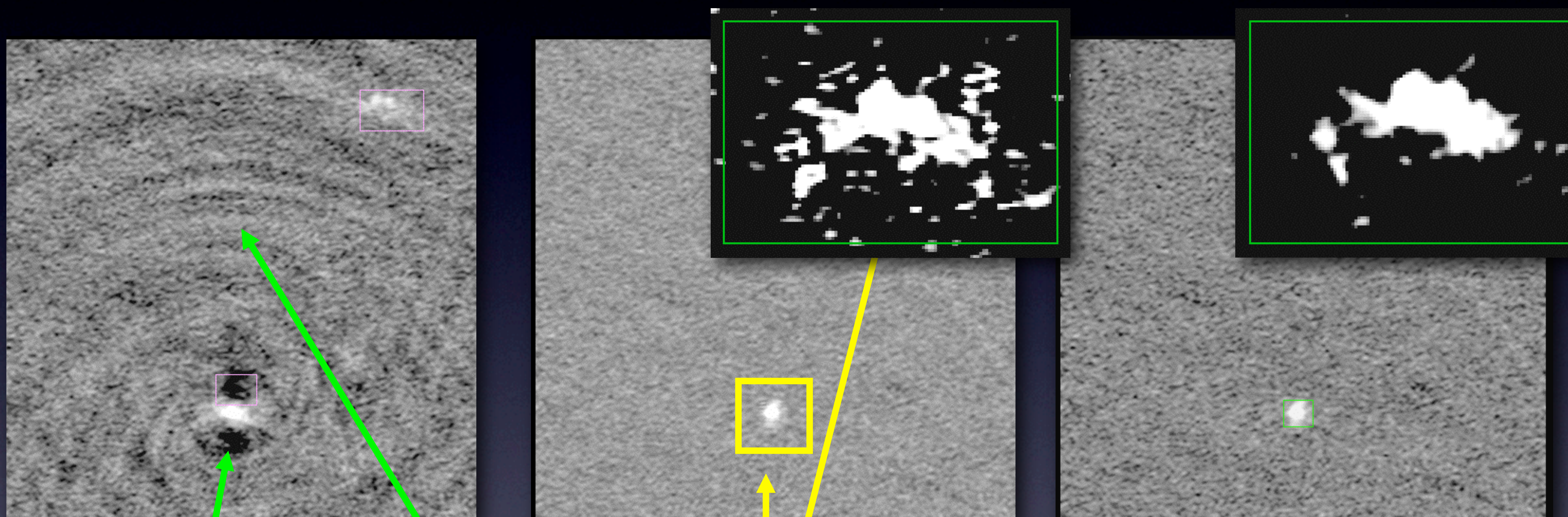
Example: Clean Errors

Under-cleaned | Over-cleaned | Properly cleaned



Residual sidelobes dominate the noise

Emission from second source sits atop a negative "bowl"

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Background is thermal noise-dominated; no "bowls" around sources

Recognizing Errors

Source structure should be "reasonable", the rms image noise as expected, and the background featureless. If not:

Examine (*u,v*) data

Look for outliers in (u,v) data using several plotting methods.

Check calibration gains and phases for instabilities.

Look at residual data (u,v data - clean components)

Examine image plane

Do defects resemble the dirty beam?

Are defect properties related to possible data errors?

Are defects related to possible deconvolution problems?

Are other corrections/calibrations needed?

Does the field-of-view encompass all emission?

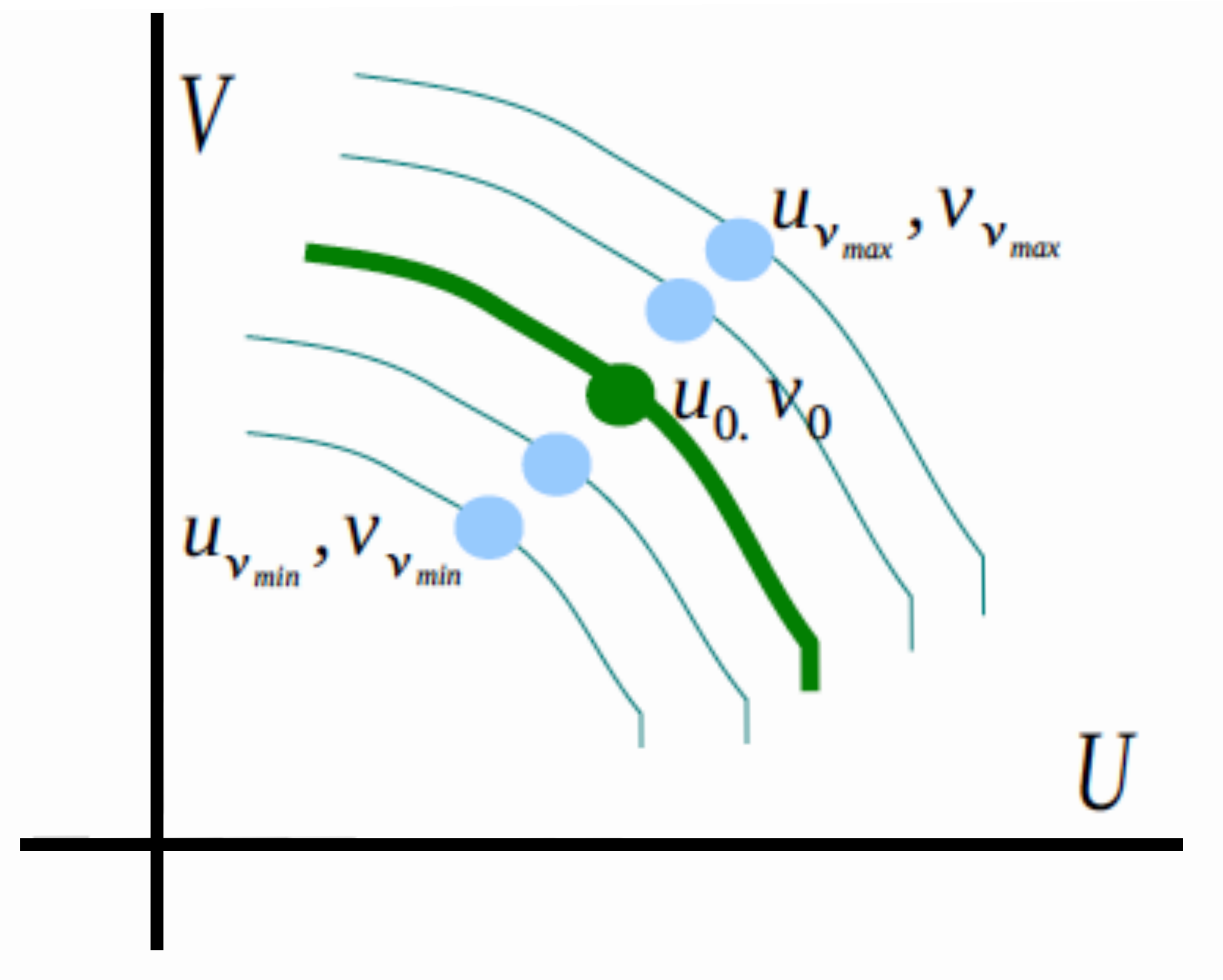
Advanced Imaging

Wide-band Imaging

- Radio telescopes suffer from chromatic aberration ⇒ "bandwidth smearing"
- Measure visibilities in many narrowband channels to avoid bandwidth-smearing
- Construct visibilities for multiple narrowband channels, each with its own delay-tracking

Max. channel width: $\delta\nu < \nu_0 \left(\frac{D}{b_{max}} \right)$

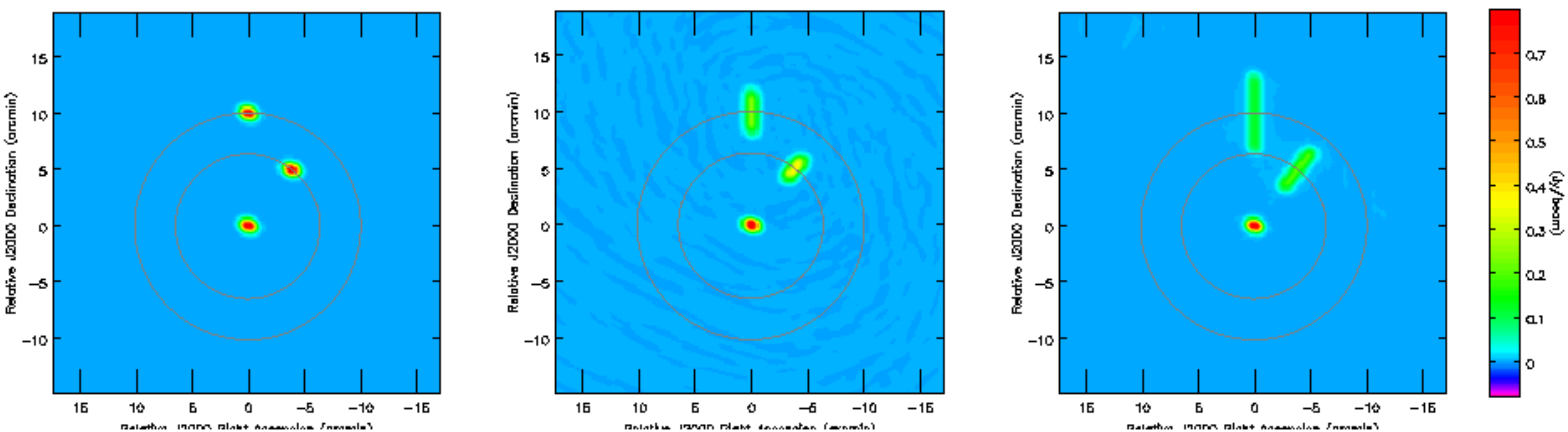
- Can use multi-frequency-synthesis to increase the uv-coverage used in deconvolution and image-fidelity
- Can make images at the angular-resolution allowed by the highest frequency
- Can take source spectrum into account



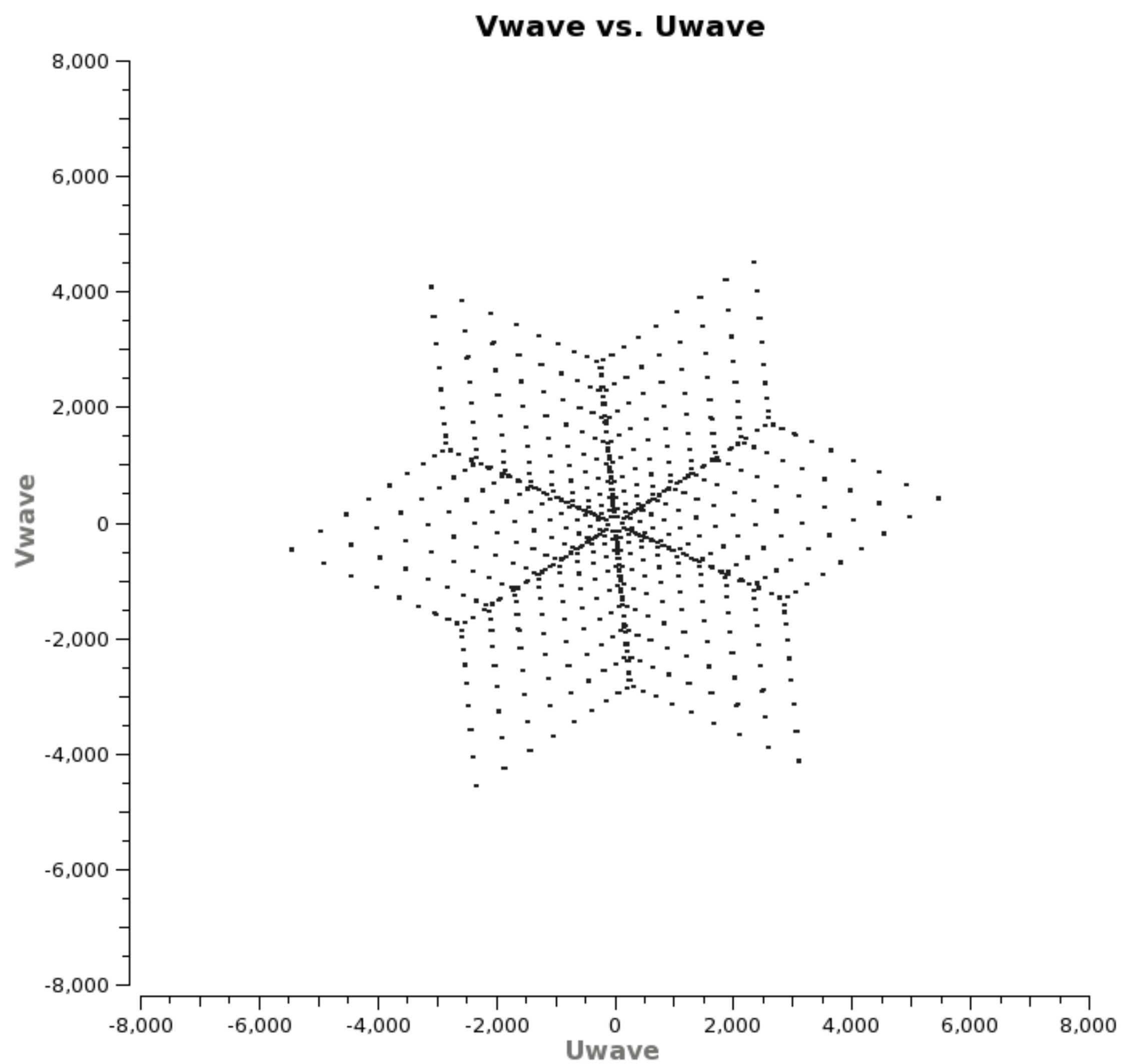
Spatial-frequency coverage changes with frequency

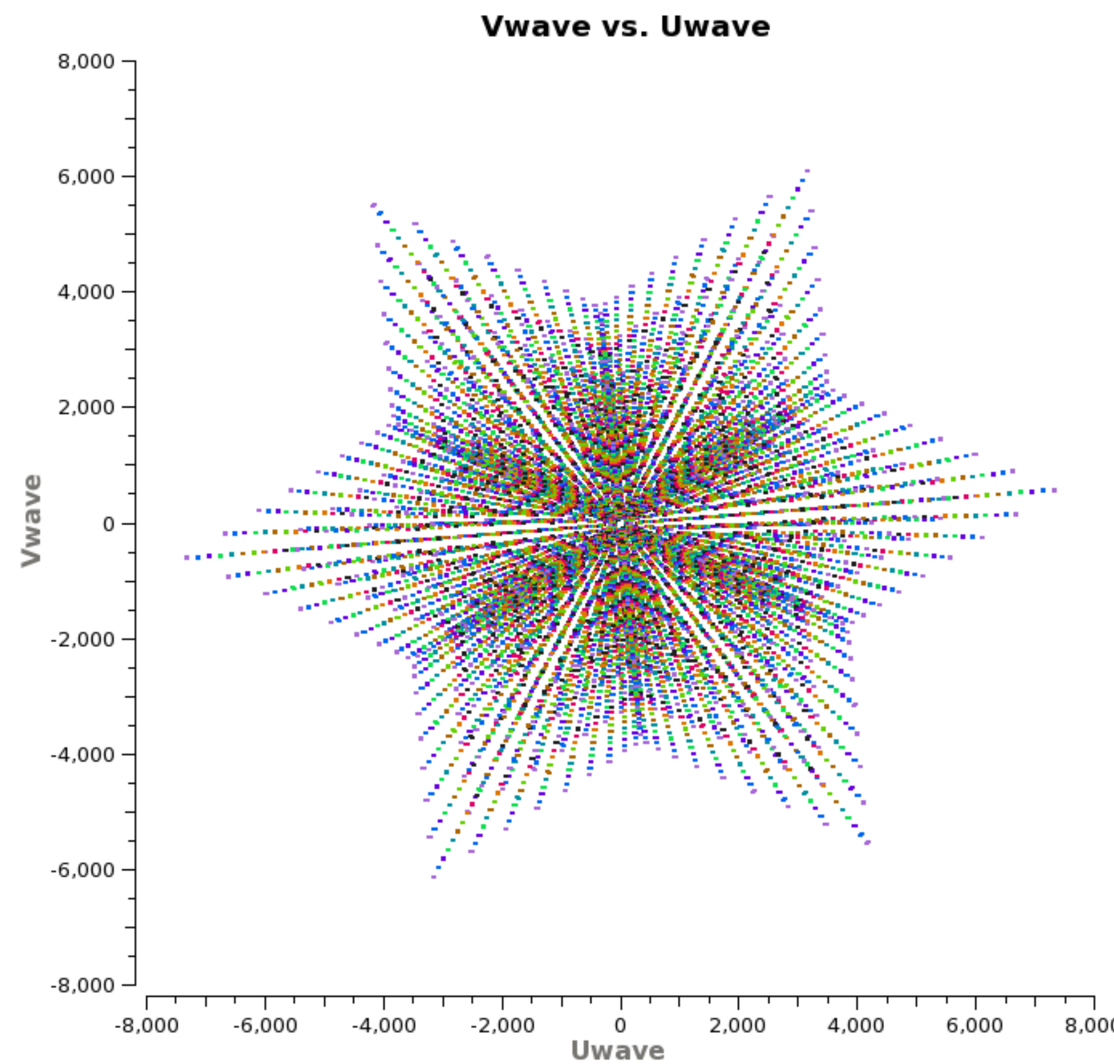
Bandwidth Smearing

$\Delta\nu$ = 2 MHz | $\Delta\nu$ = 200 MHz | $\Delta\nu$ = 1 GHz

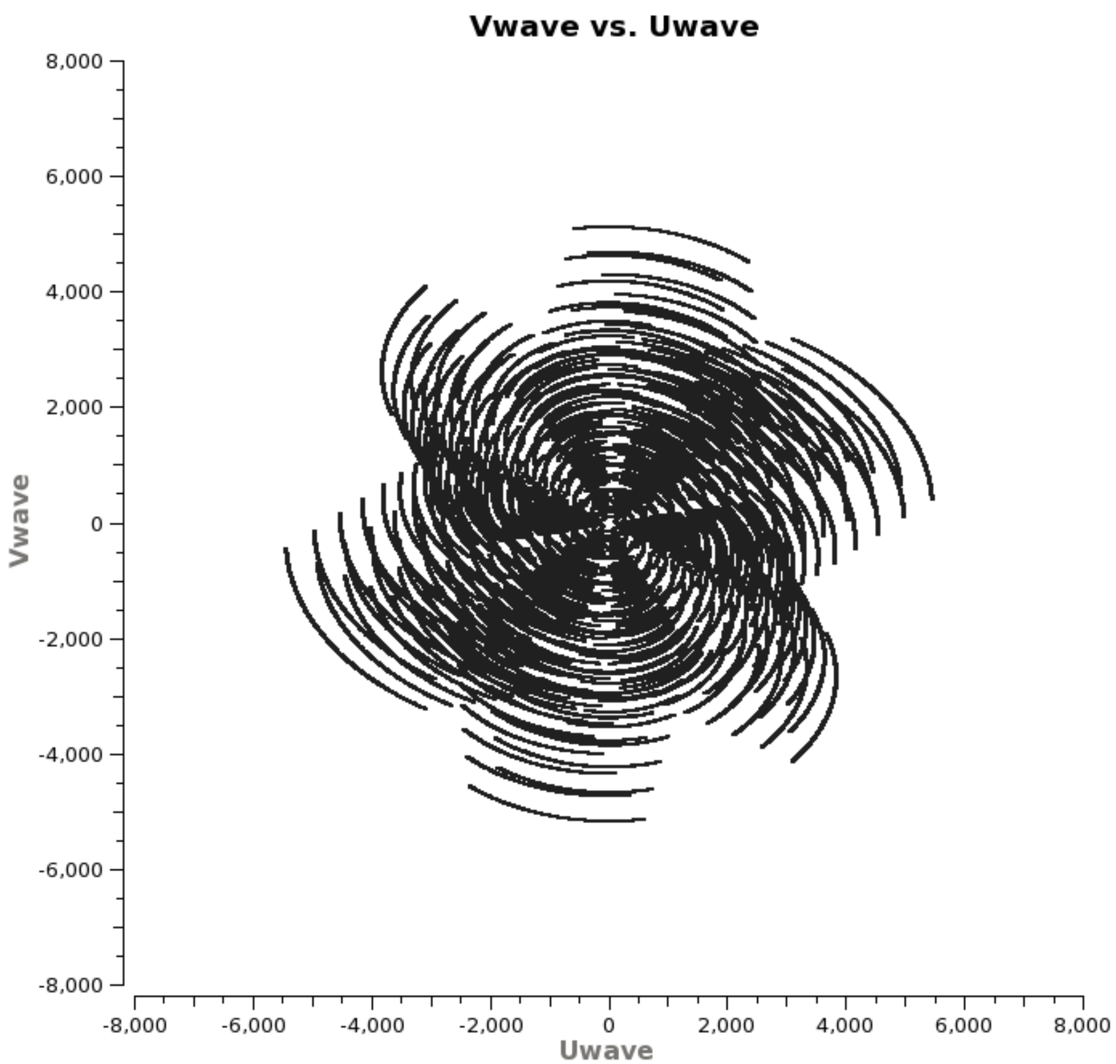


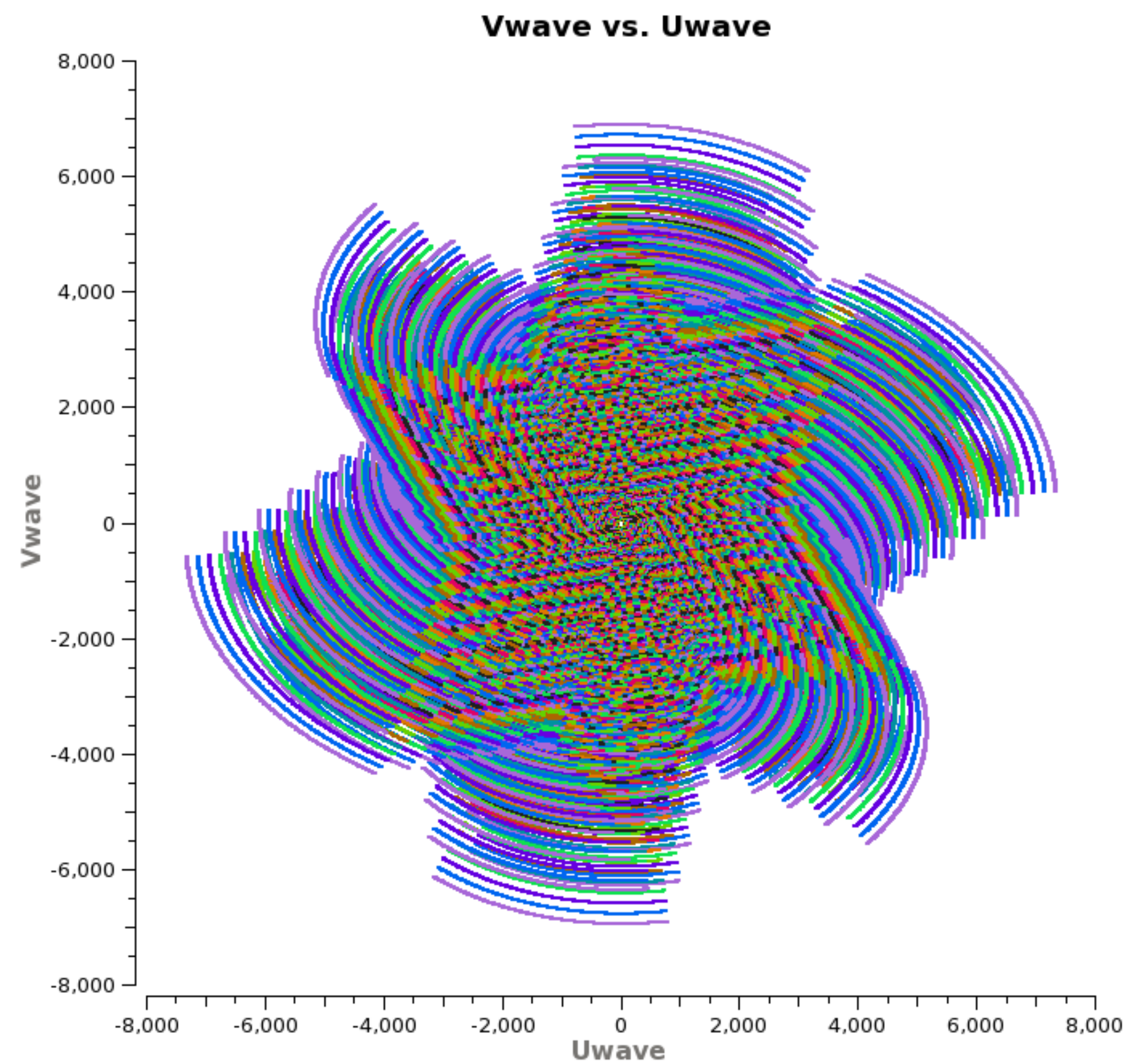
Multifrequency Synthesis





Multifrequency Synthesis

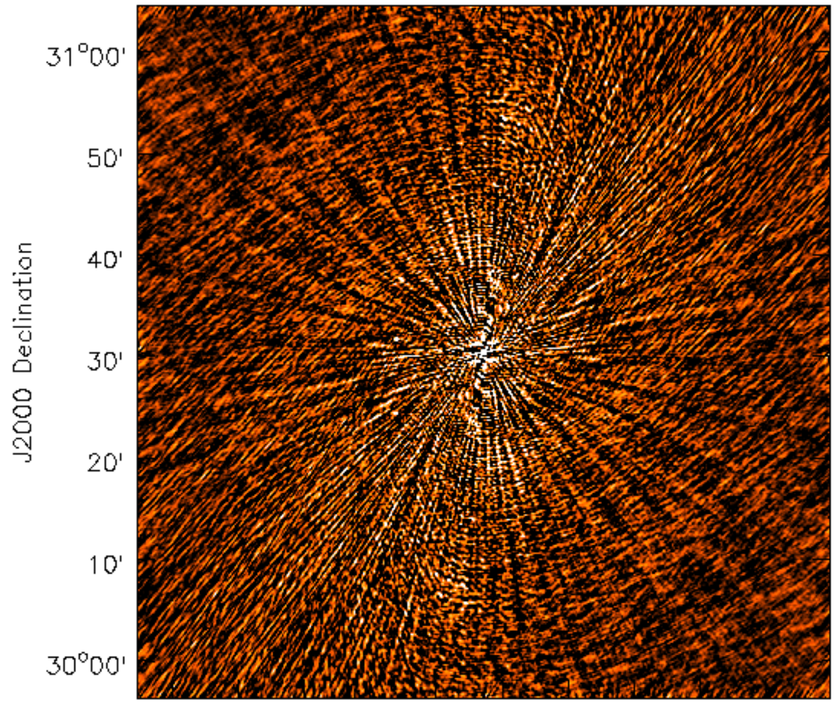


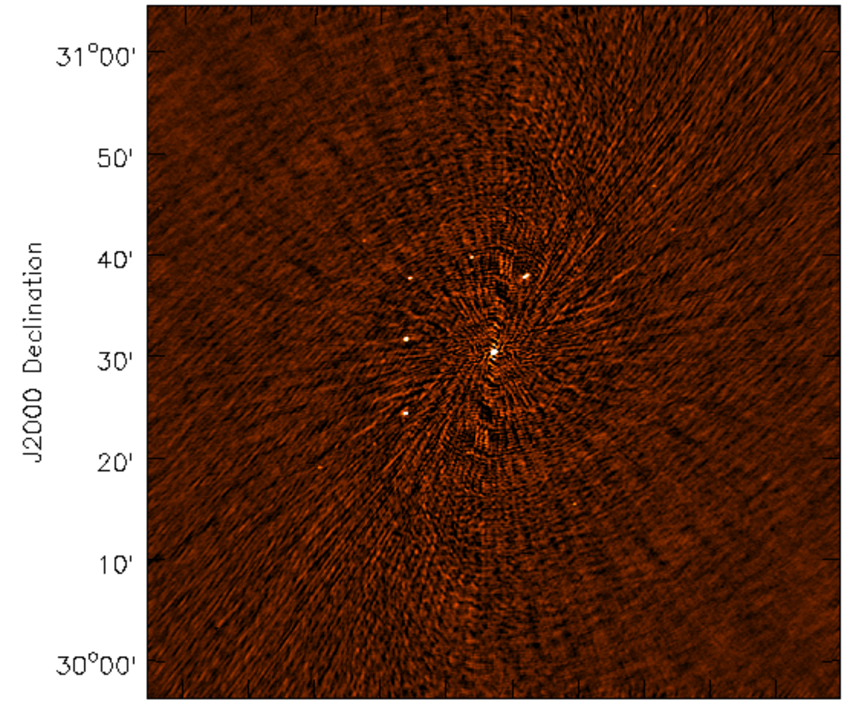


- Overlapping uv coverage ⇒ better sensitivity ⇒ $\sigma_{cont} = \frac{\sigma_{chan}}{\sqrt{N_{chan}}}$
- Increased uv filling ⇒ better imaging fidelity
- Larger spatial-frequency range ⇒ better angular resolution ⇒ $\frac{\lambda}{b_{max}}$

MFS Example: 3C286

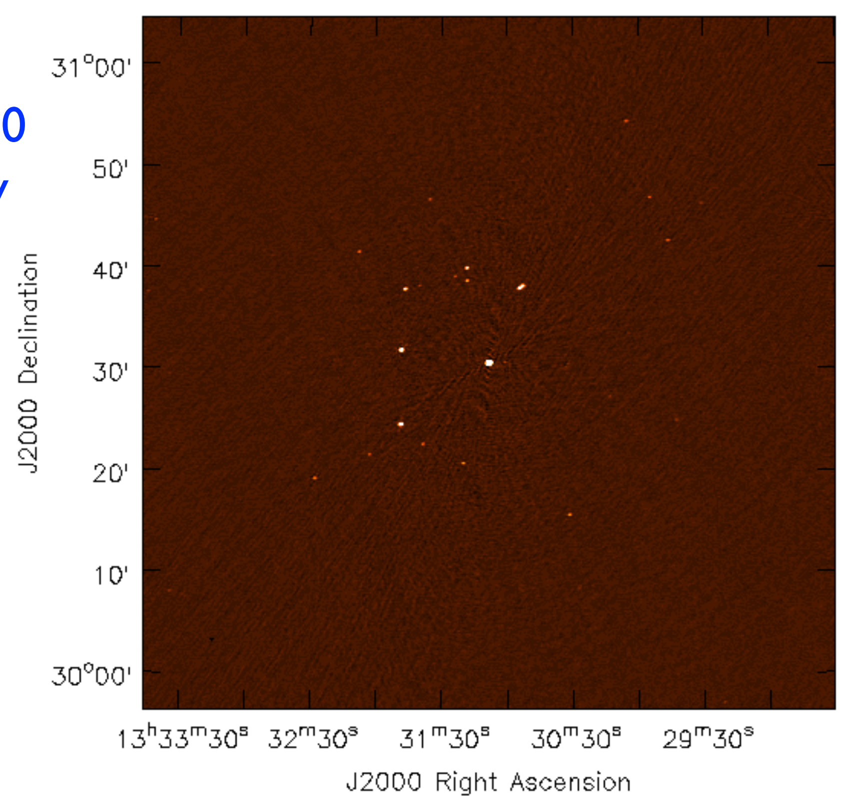
DR =1600 -13000
σ=9 mJy -1 mJy
NTerms = 1

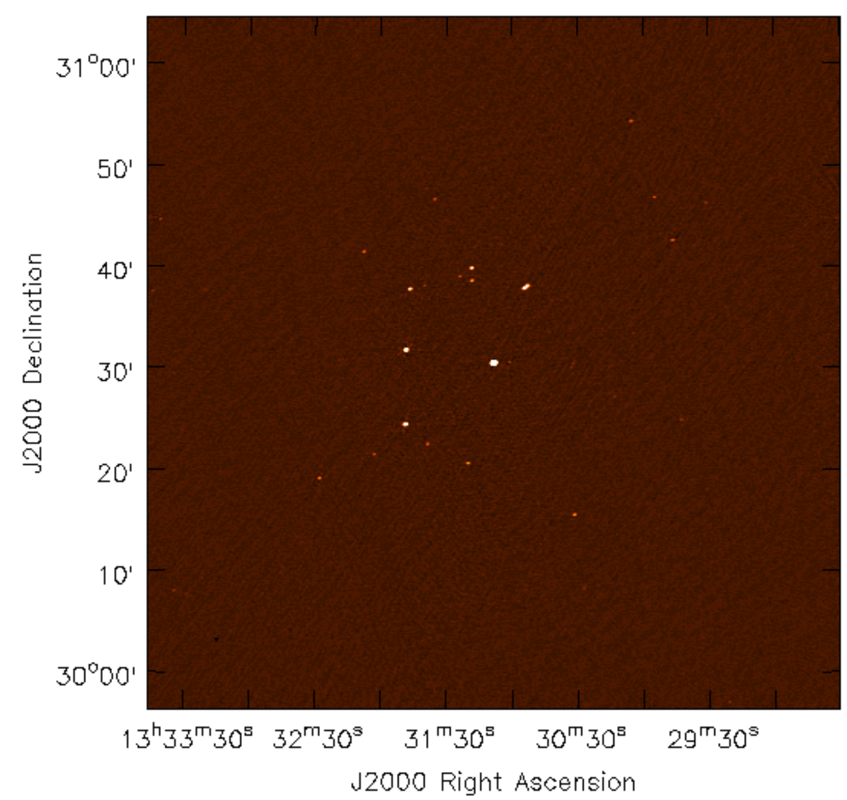




DR =10000 -17000
σ=1.0 mJy - 0.2 mJy
NTerms = 2

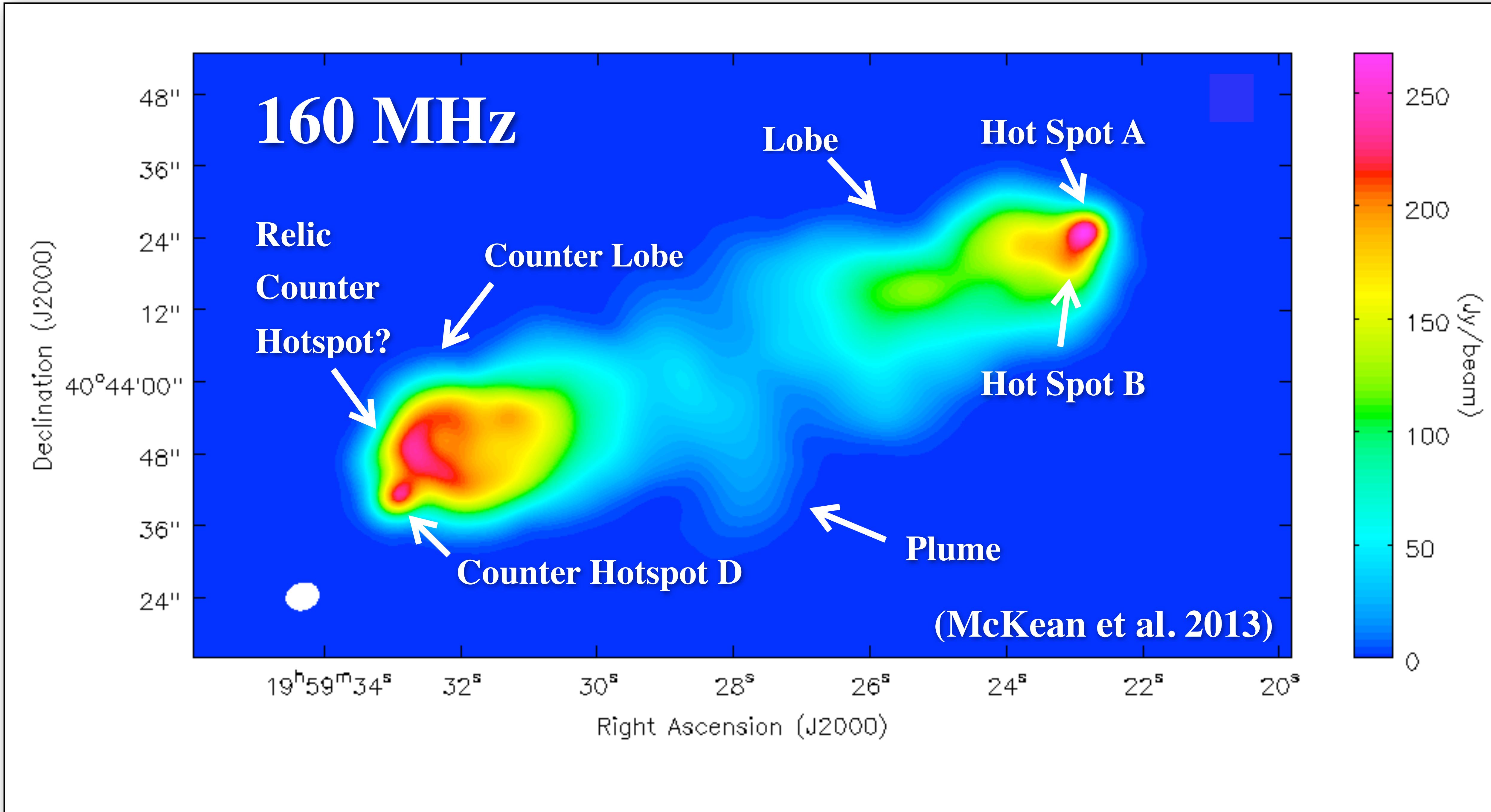
DR =65000 -170000
σ=0.2 mJy - 85 μJy
NTerms = 3





DR =110000 - 180000
σ=0.14 mJy - 80 μJy
NTerms = 4

MFS Example: Cygnus A

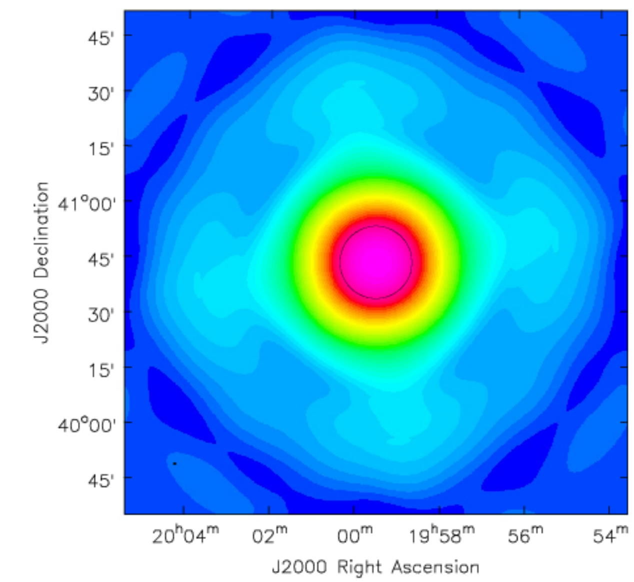


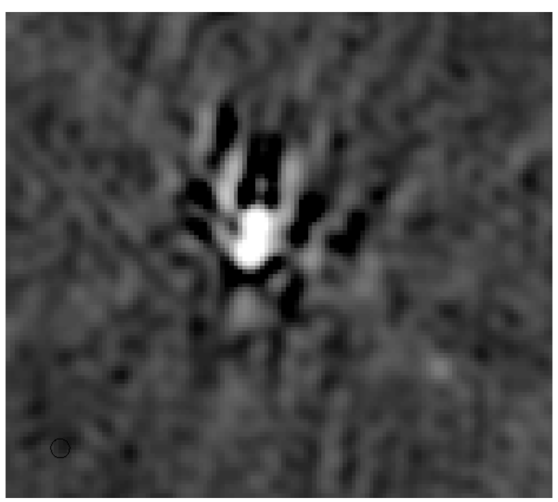
LOFAR HBA 6 hr / 110 - 182 MHz / 16 MHz
σ ~ 70 mJy / DR ~ 3000
NL baselines only, 3.0 arcsec resolution

Wide-band Imaging

Wide-band Imaging often requires wide-field imaging techniques

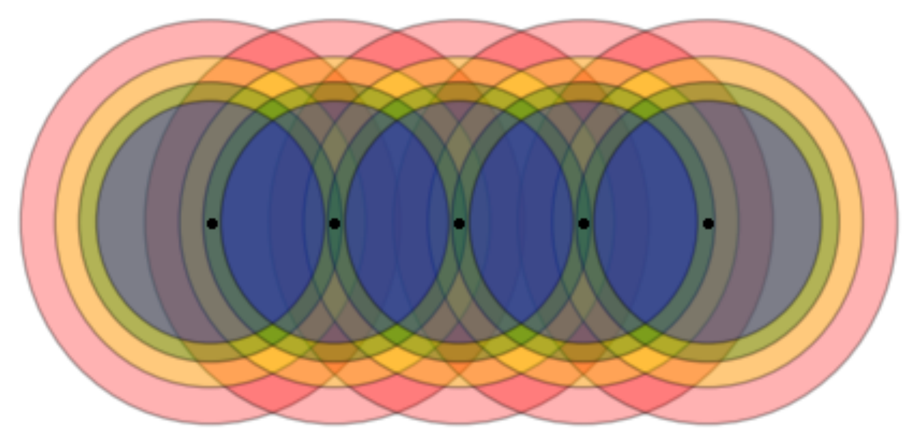
"Primary Beam" : The antenna-primary beam can introduces a time-varying spectrum in the data.





"W-term": Non-coplanar arrays also introduce a frequency-dependent instrumental effect. Narrow-band w-projection algorithm works for wide-band.

"Mosaicing": Make observations with multiple pointing and delay-tracking centers. Combine the data during (or after) image-reconstruction.



Wide-field Imaging

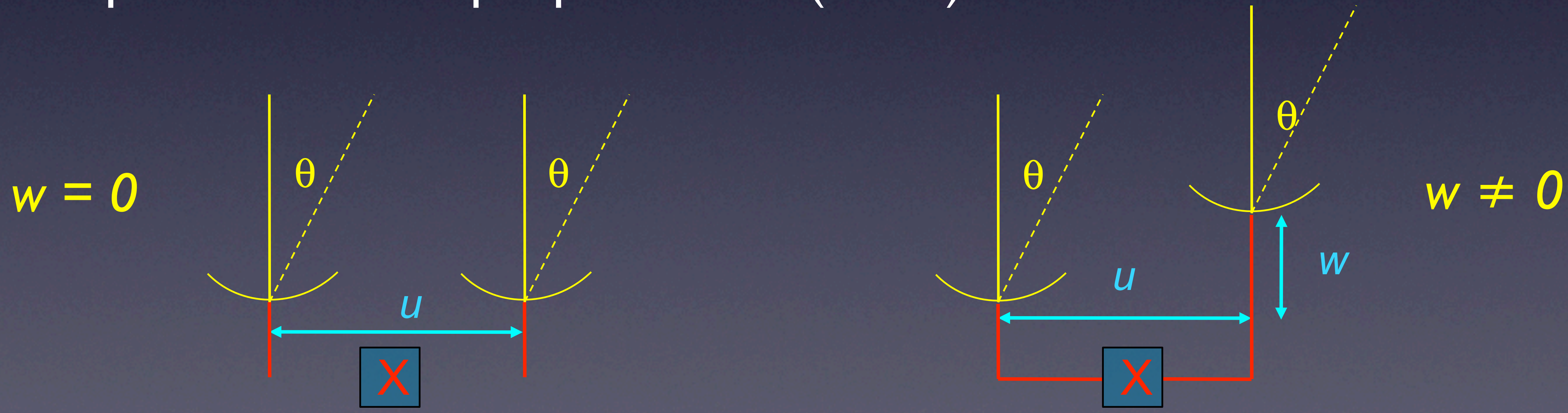
- New instruments are being built with wider fields of view (especially at lower frequencies): MeerKAT, ASKAP, Apertif, Allen Telescope Array, LOFAR
- Wide-field good for all-sky surveys and finding transients
- Traditional synthesis imaging assumes a flat sky and a visibility measurements lying on a (u,v) plane
- These approximations only hold near the phase center (implies small fields of view)
- To deal accurately with large fields of view requires more complicated algorithms (and much more computation)

Coplanar Arrays

- Recall the general relation between the complex visibility *V(u,v)*, and the sky intensity *I(l,m)*:

$$V'(u,v) = \iint I(l,m) e^{-i2\pi (ul+vm)} \, dl dm$$

- This equation is valid for $w = 0$
- For $w \neq 0$, any signal can easily be projected to the $w = 0$ plane with a simple phase shift ($e^{-2\pi i w}$)



Non-coplanar Arrays

- In the full form of this equation, the visibility *V(u,v,w)*, and the sky intensity *I(l,m,n)* are related by:

$$V(u,v,w) = \iint \frac{I(l,m)}{\sqrt{1-l^2-m^2}} e^{-i2\pi[ul+vm+w(n-1)]}\, dl\, dm$$

- This equation is valid for:
  - spatially incoherent radiation from the far field,
  - phase-tracking interferometer
  - narrow bandwidth: $\Delta\upsilon << \frac{\theta_{res}}{\theta_{offset}}\upsilon_0 \approx \frac{\lambda}{B}\frac{D}{\lambda}\nu_0 = \frac{D}{B}\nu_0$
  - short averaging time: $\Delta t << \frac{\lambda}{B\omega_e\theta_{offset}} \approx \frac{D}{B}\frac{1}{\omega_e}$

When Approximations Fail

- The 2-dimensional Fourier transform version applies when one of two conditions is met:
  - All the measures of the visibility are taken on a plane, or
  - The field of view is 'sufficiently small', given by:

$$\theta_{2D} < \sqrt{\frac{1}{w}} \leq \sqrt{\frac{\lambda}{B}} \sim \sqrt{\theta_{syn}}$$

Worst Case!

- We are in trouble when the "distortion-free" solid angle is smaller than the antenna primary beam solid angle.
- Define a ratio of these solid angles:

$$N_{2D} = \frac{\Omega_{PB}}{\Omega_{2D}} \sim \frac{\Omega_{PB}}{\theta_{syn}} \sim \frac{\lambda B}{D^2}$$

When $N_{2D} > 1$, 2-dimensional imaging is in trouble!

Example: VLA

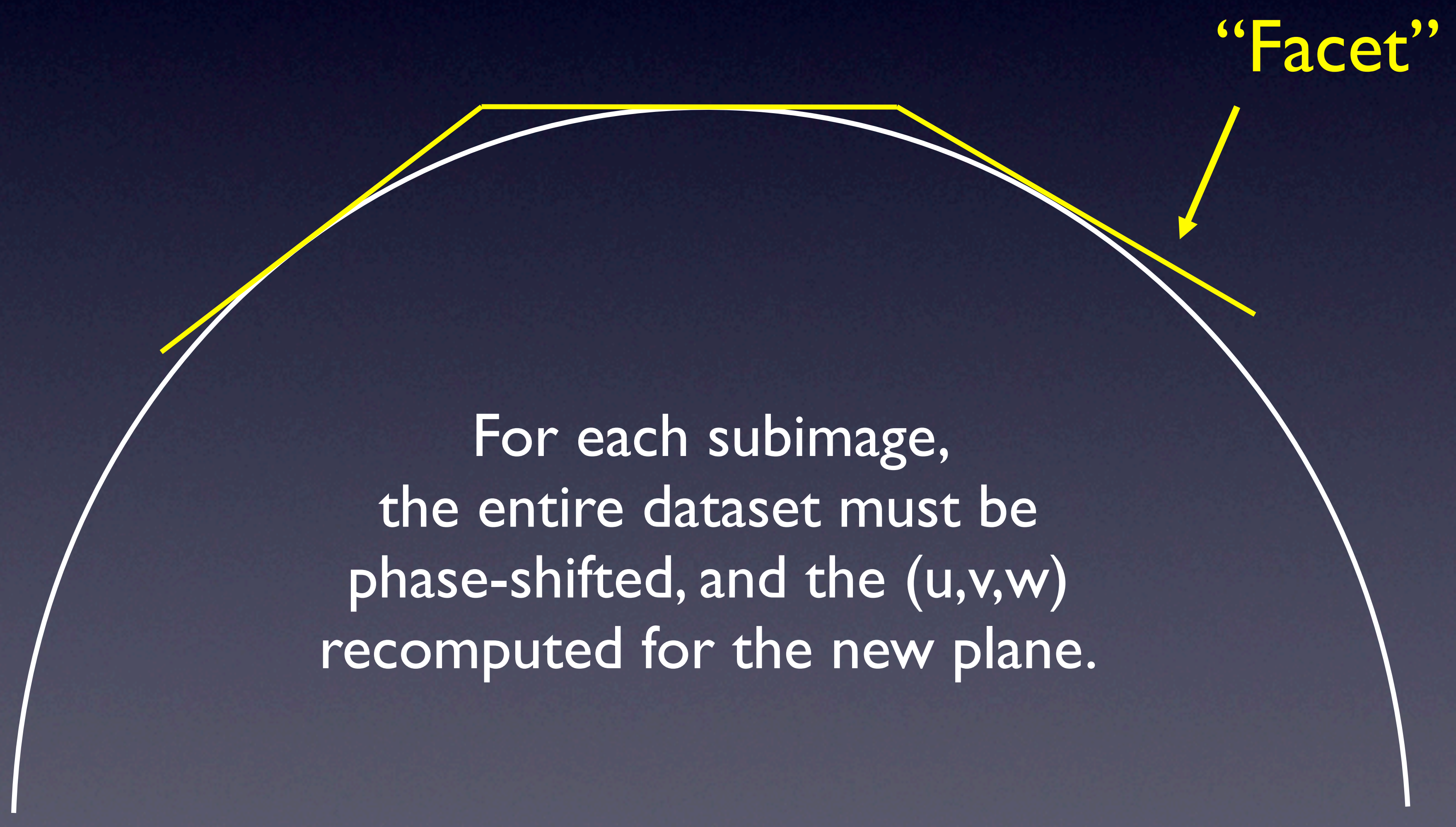
- The table below shows the approximate situation for the EVLA, when it is used to image its entire primary beam.
- Blue numbers show the respective primary beam FWHM
- Green numbers show situations where the 2-D approximation is safe.
- Red numbers show where the approximation fails totally.

	EVLA				MeerKAT	
λ	θ_{FWHM}	A	D		θ_{FWHM}	
6 cm	9'	6'	31'		17'	7'
20 cm	30'	10'	56'		56'	13'
90 cm	135'	21'	118'		249'	27'

Table showing the VLA's and MeerKAT's distortion free imaging range (green), marginal zone (yellow), and danger zone (red)

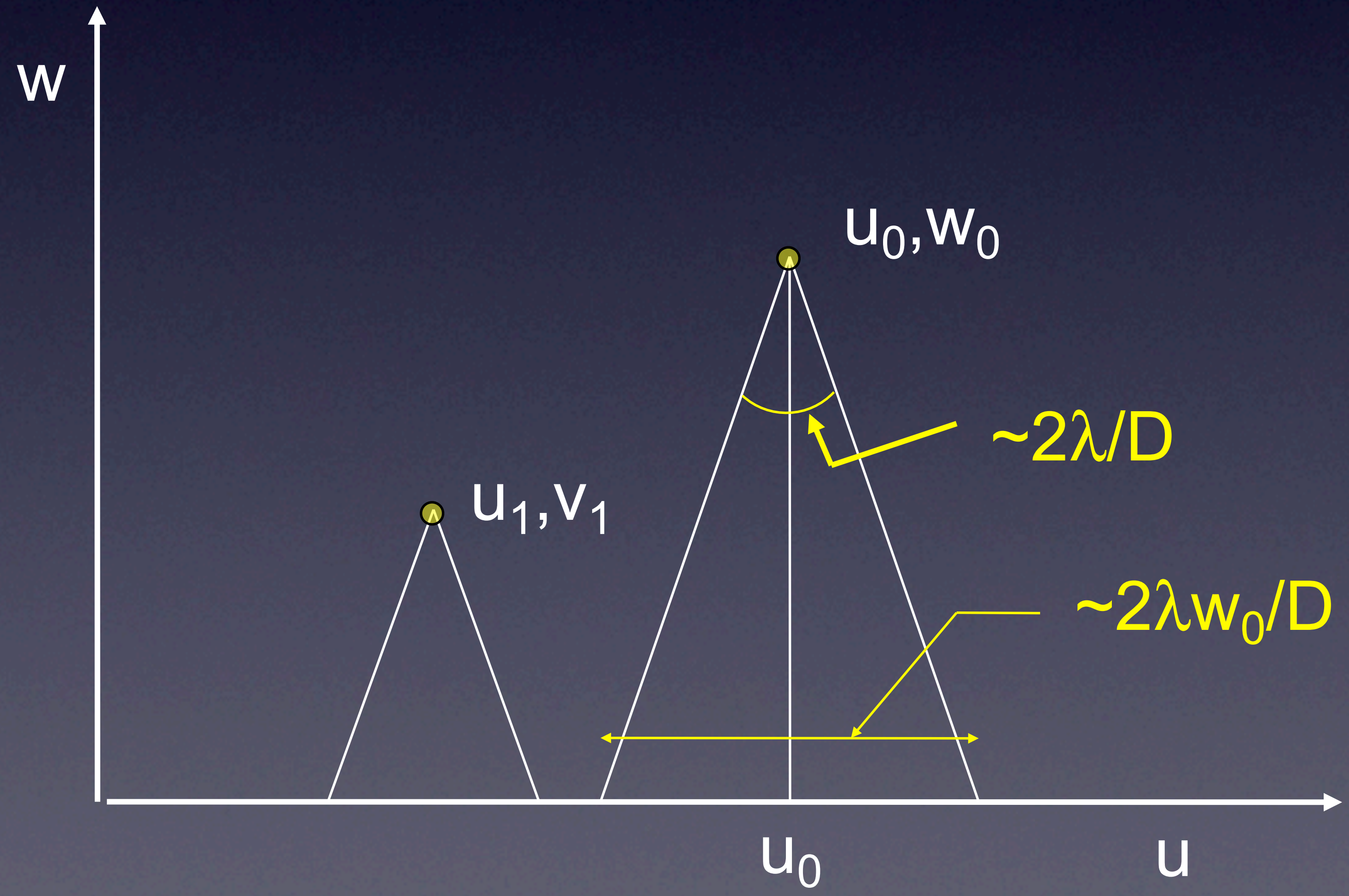
Faceted Imaging

- Approximates the unit sphere with series of small flat planes
- Within each facet, the 2D approximation applies
- Computing time scales with N of facets required
- Can produce artifacts at facet boundaries

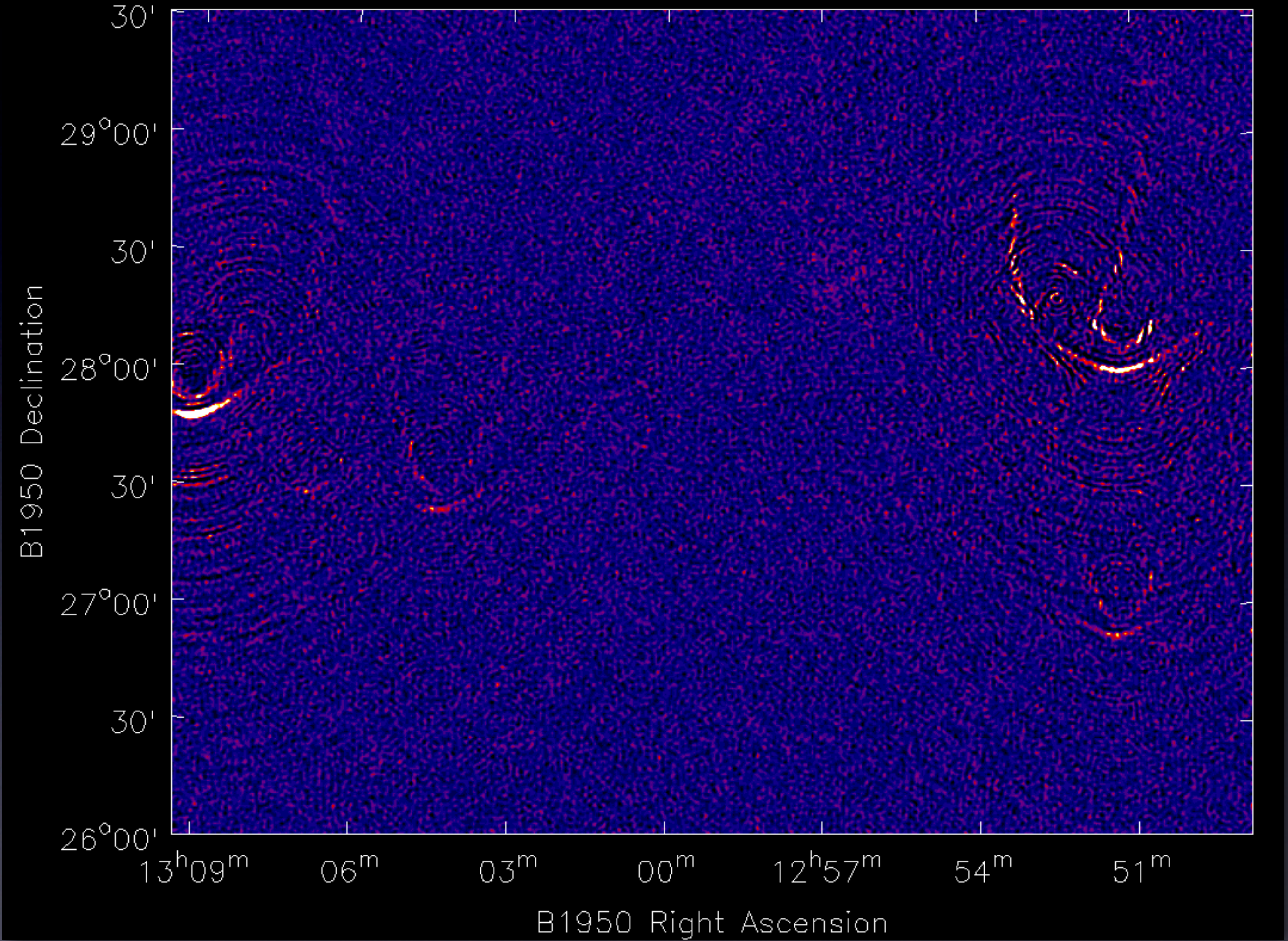


W-Projection

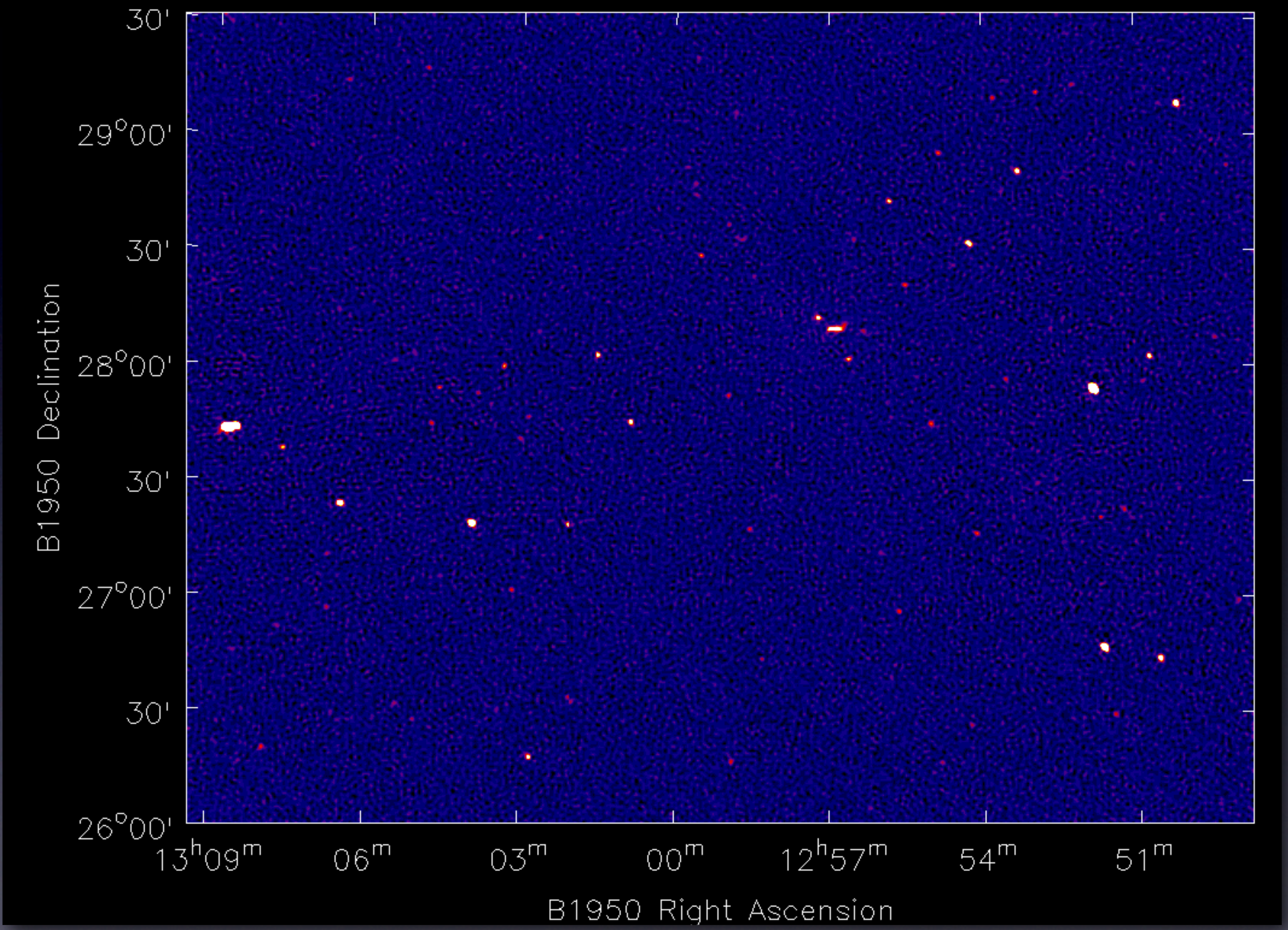
- Each visibility, at location (u,v,w) is mapped to the w=0 plane, with a phase shift proportional to the distance
- Each visibility is mapped to ALL the points lying within a cone whose full angle is the same as the field of view of the desired map (~$2\lambda/D$ for a full-field image)
- Area in the base of the cone is ~$4\lambda^2 w^2/D^2 < 4B^2/D^2$. Number of cells on the base which 'receive' this visibility is ~ $4w_0^2B^2/D^2 < 4B^4/\lambda^2D^2$



Without "3D" Processing

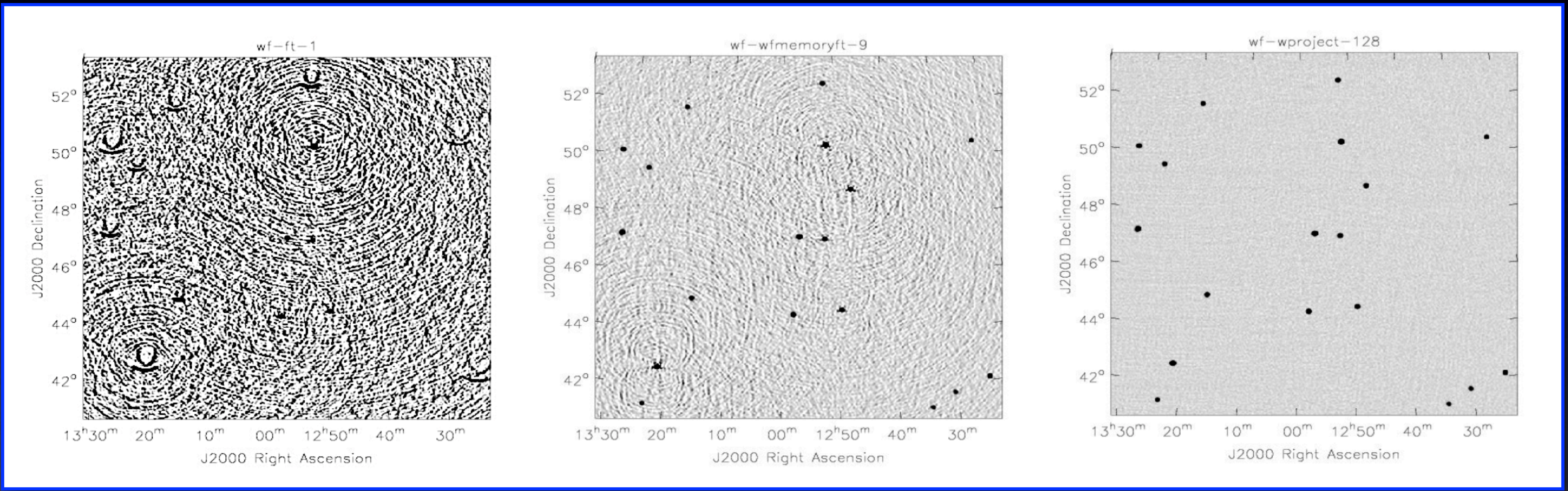


With "3D" Processing



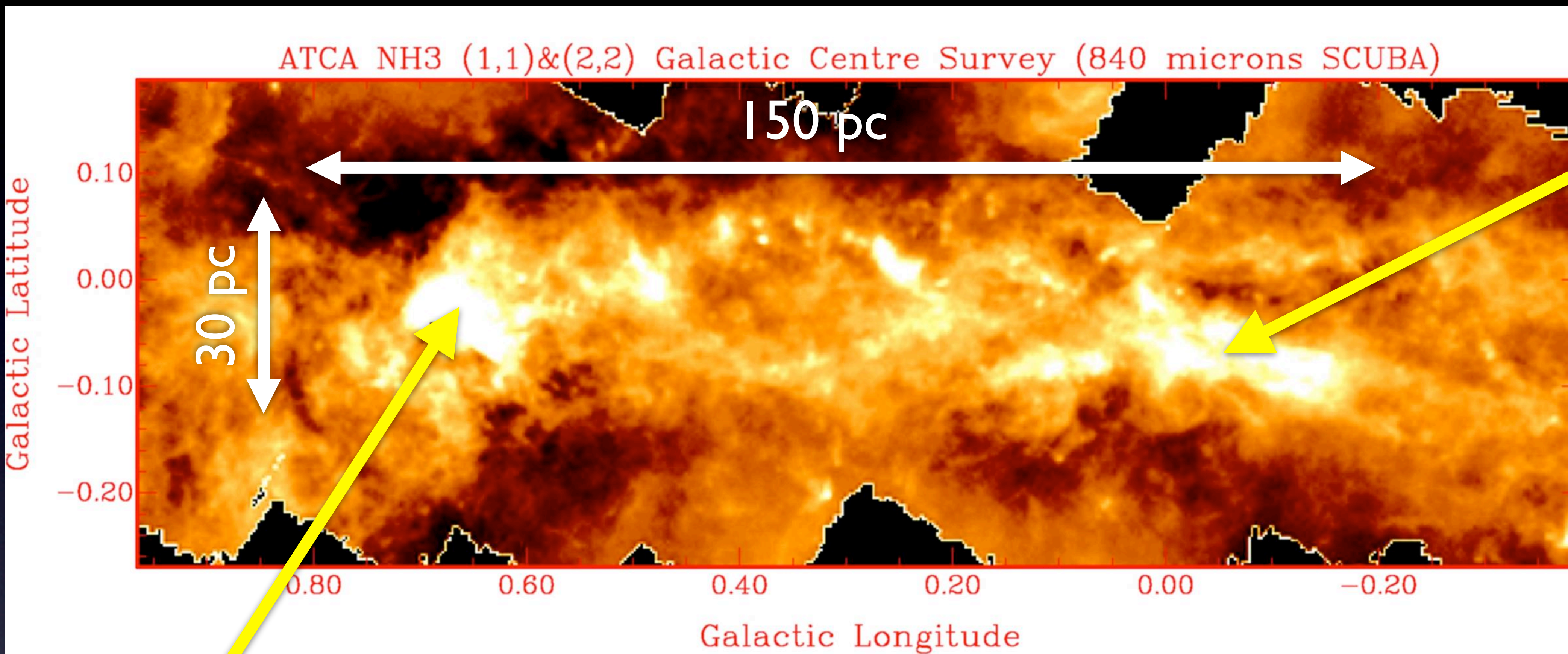
Comparison of Techniques

2D Imaging *Facet Imaging* *W-Projection*



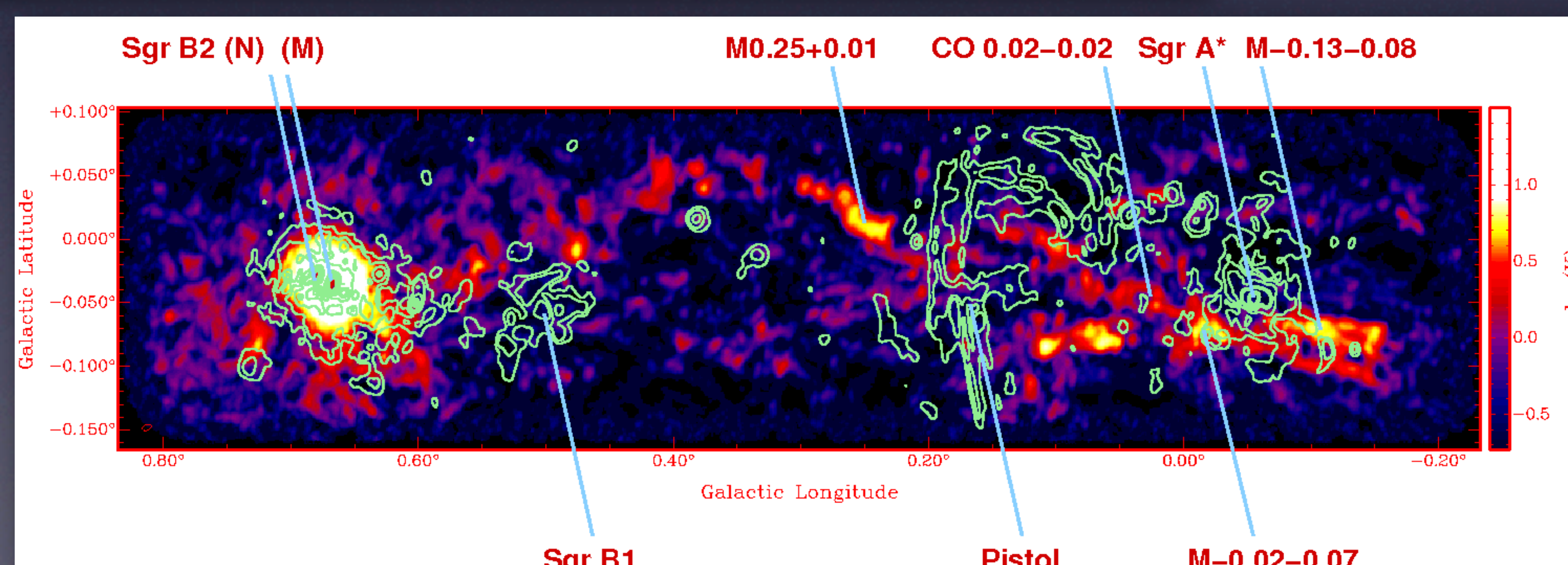
Mosaicing

Galactic Center



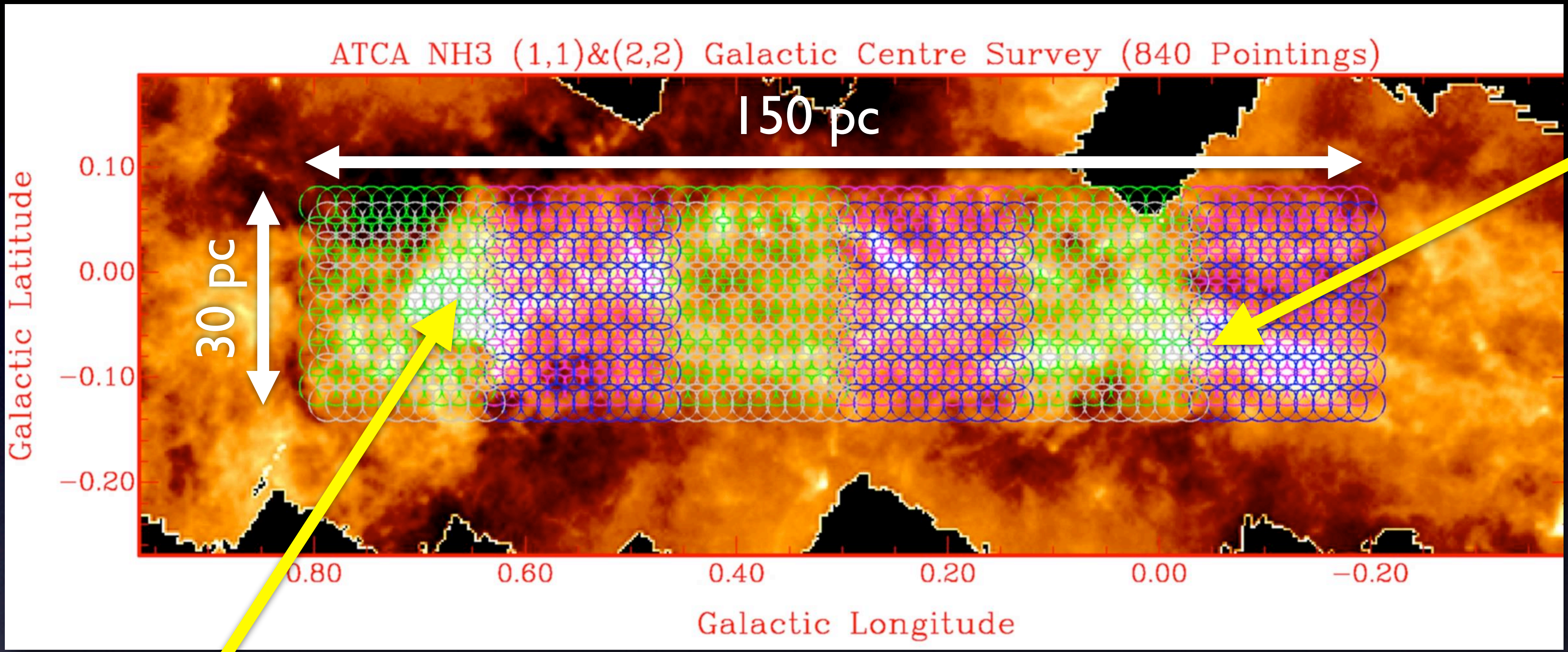
*Sgr A**

Sgr B2



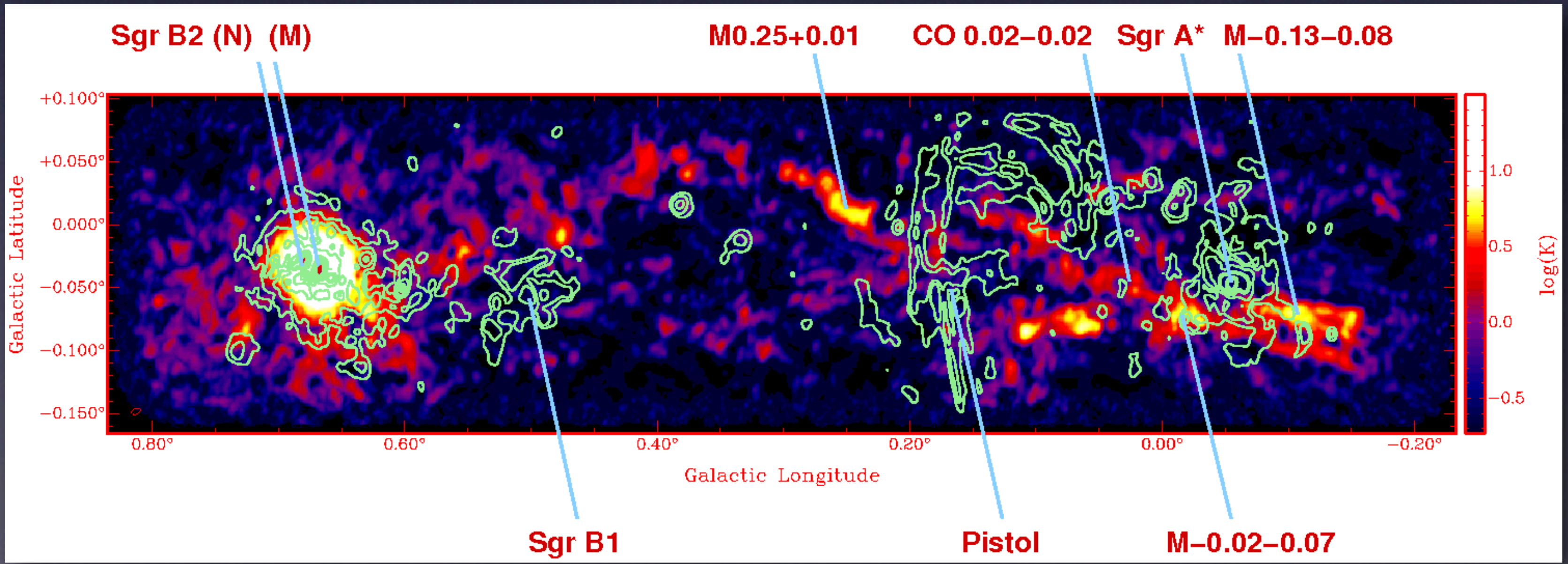
Mosaicing

Galactic Center

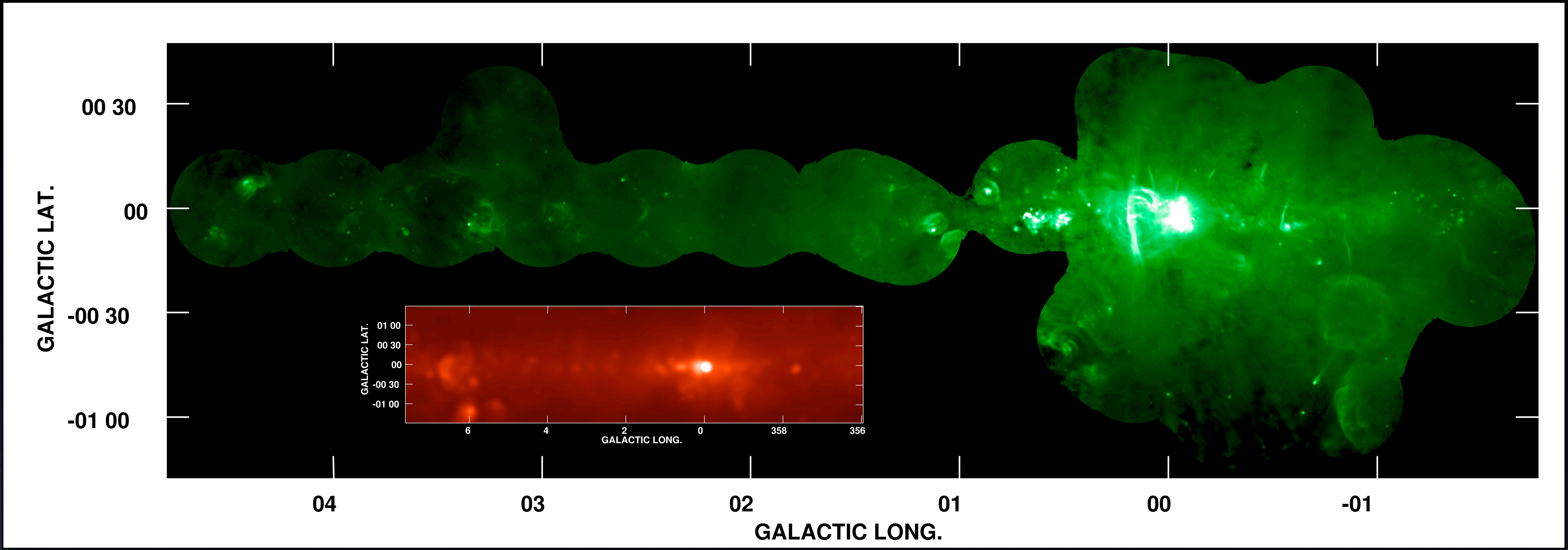


Sgr A*

Sgr B2



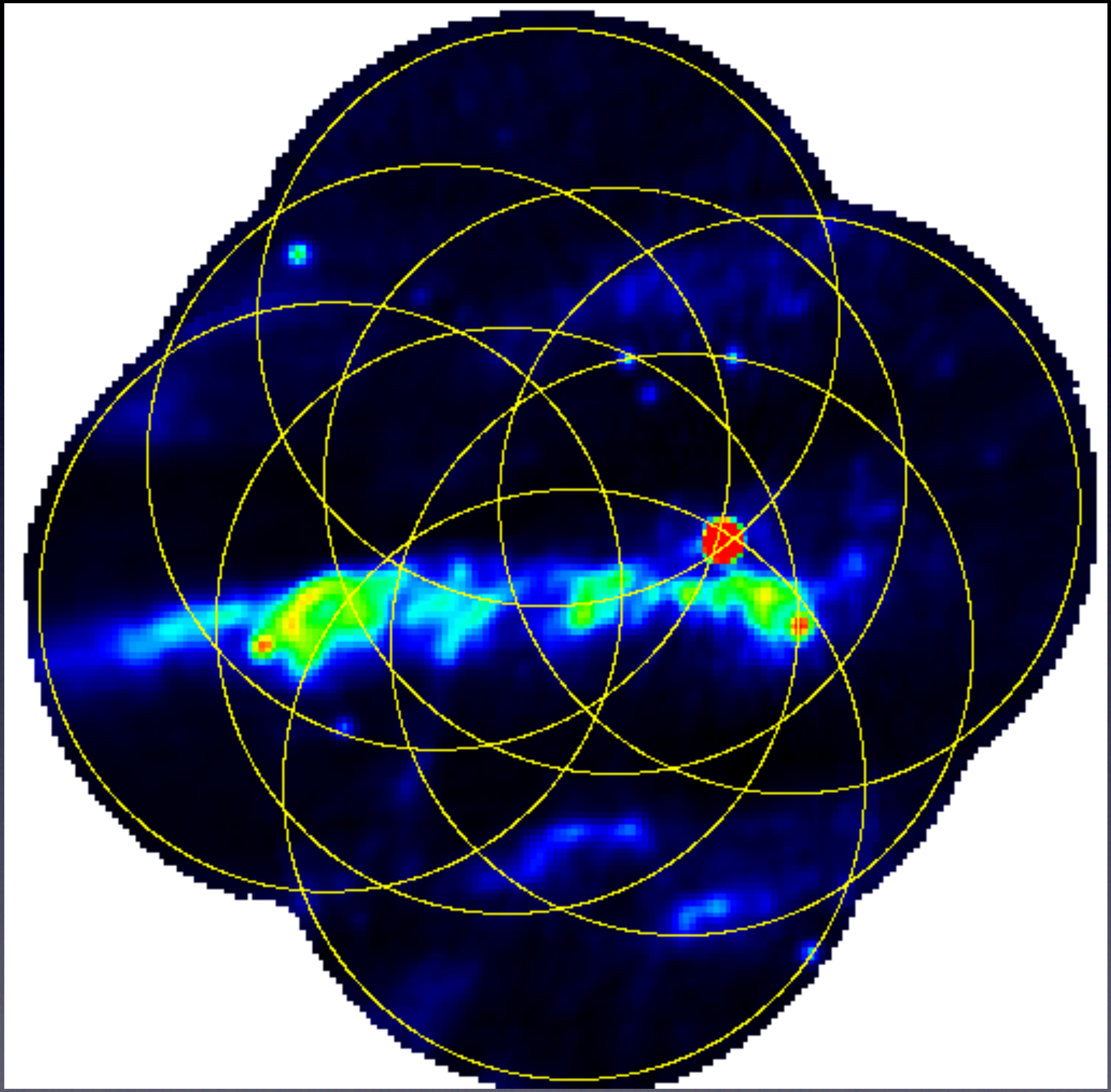
Mosaicing



- Effects of varying primary beams must be taken into account
- Adds complexity to the deconvolution process
- Need adequate sky coverage (try to keep Nyquist sampling)
- Can also be used to add single dish data and recover zero spacings

Mosaicing Techniques

- Primary Methods
  - Linear combination of deconvolved maps
  - Joint deconvolution
  - Regridding of all visibilities before FFT



Mosaicing Techniques

- Primary Methods
  - Linear combination of deconvolved maps
  - Joint deconvolution
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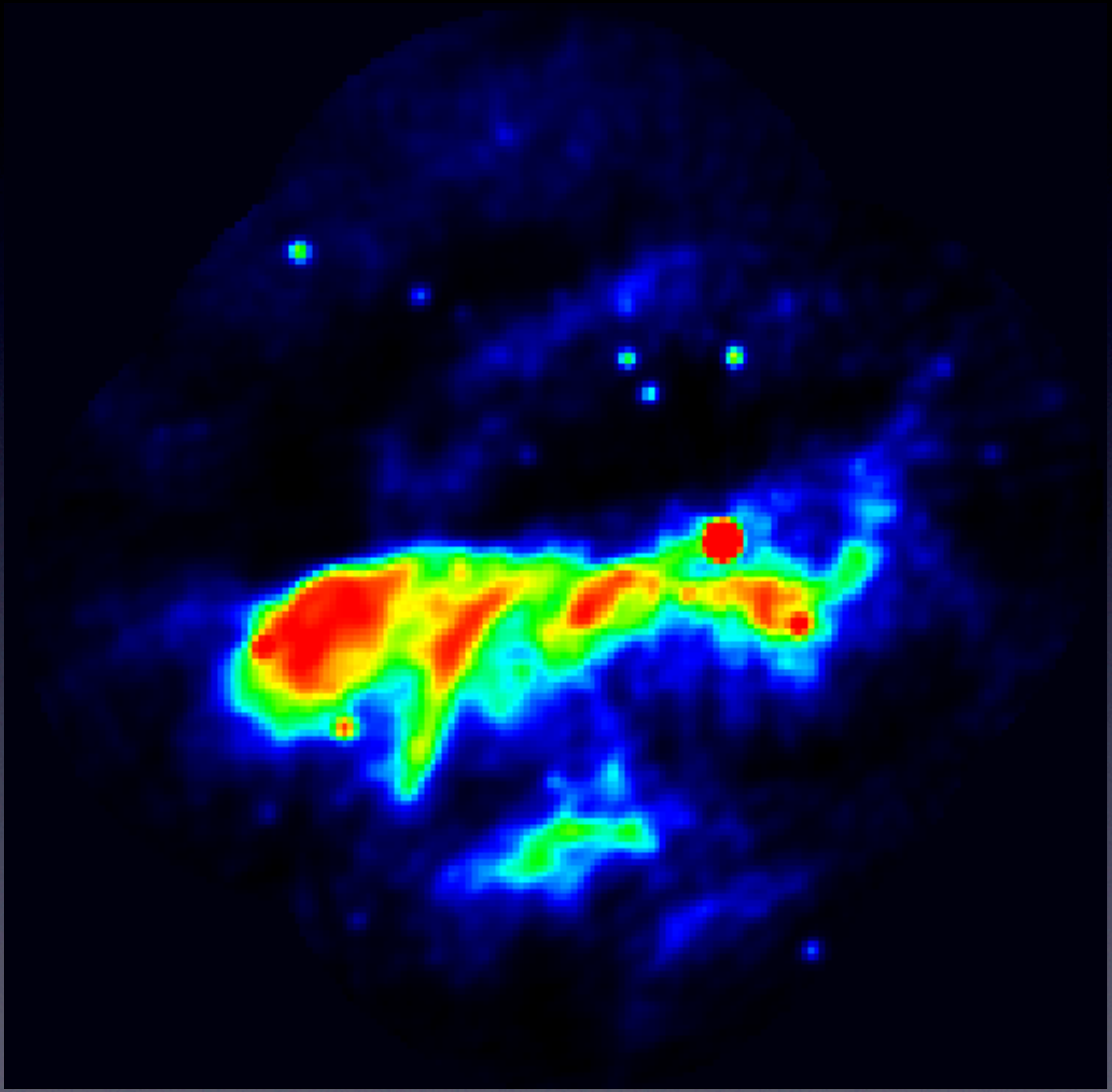
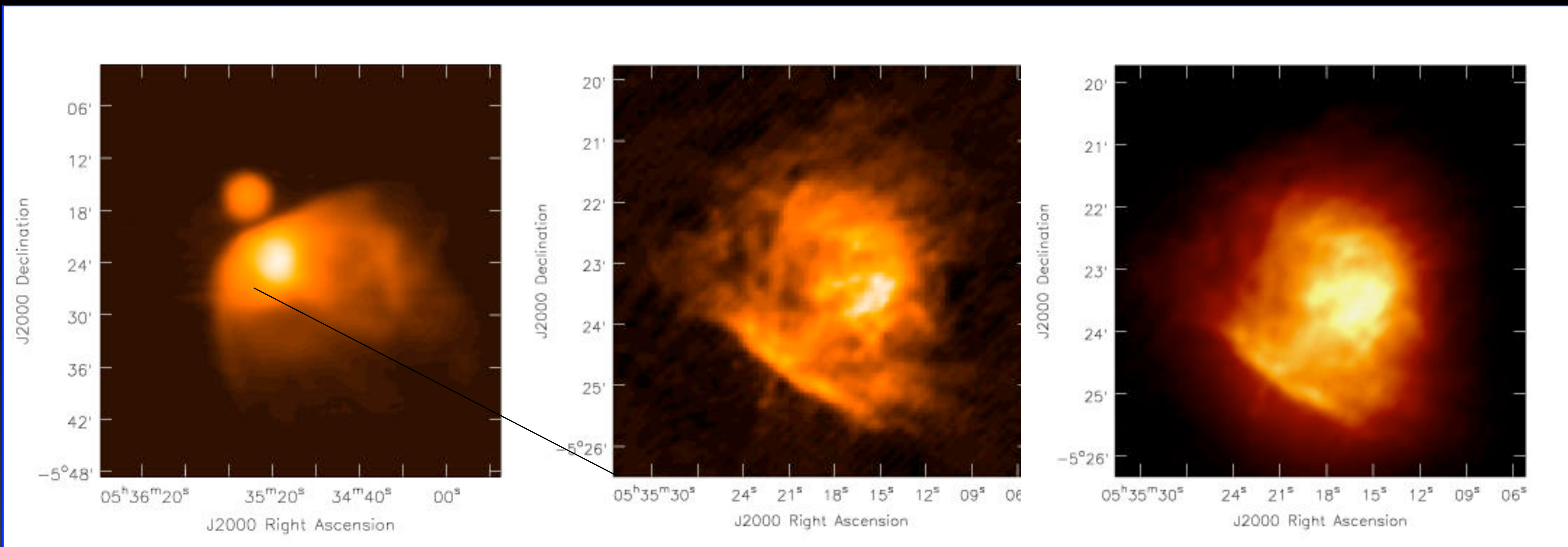


Zero Spacings

Orion Nebula

Shepherd, Maddalena, McMullin (2002)



GBT map of the large field
90” resolution

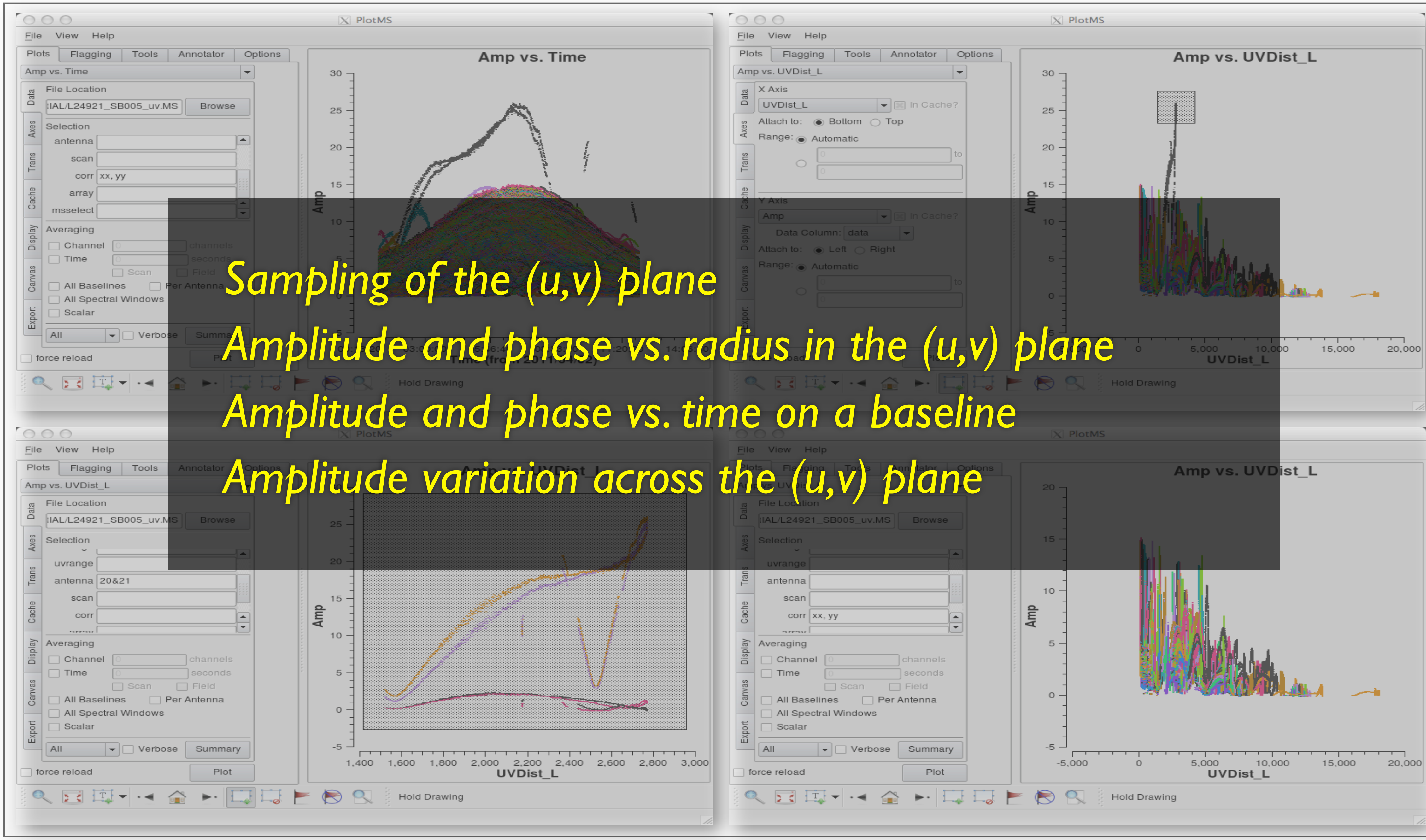
VLA mosaic of central region, 9 fields
8.4” resolution

Combined GBT+VLA mosaic deconvolved with multi-scale CLEAN

Final image fidelity significantly better

Questions?

Practicum Redux



- Examine the visibility data from a LOFAR observation
- Use the interactive CASA tool "casaplotms"