Full Polarization Calibration of Phased Array Systems

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Outline

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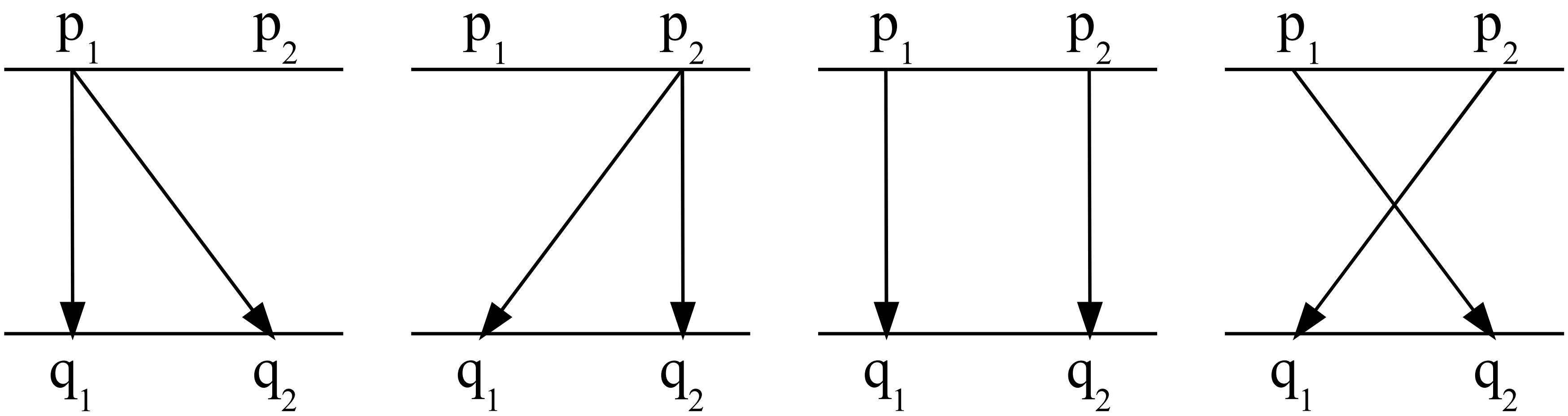
- The physics of phased arrays
- Phased array feeds
  - The optimal case
  - Practical methods
  - Conditions for polarimetric fidelity
- Aperture arrays
  - From bi-scalar to full polarization calibration
  - LOFAR example
  - Conditions for polarimetric fidelity
- Conclusions

Propagation of EM waves

Born & Wolf, *Principles of Optics*, 1980

Spatial correlation between q_1 and q_2 due to

1. common origin p_1
2. common origin p_2
3. coherence between field q_1 from p_1 and q_2 from p_2
4. coherence between field q_1 from p_2 and q_2 from p_1



The origin of spatial coherency

Born & Wolf, *Principles of Optics*, 1980

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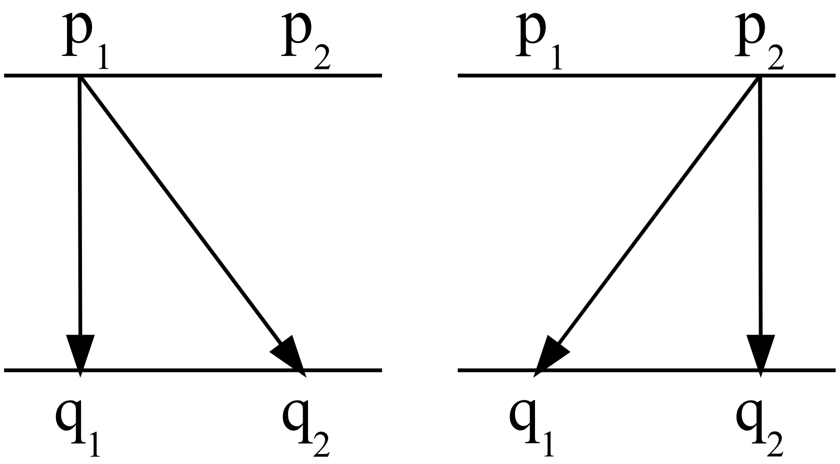
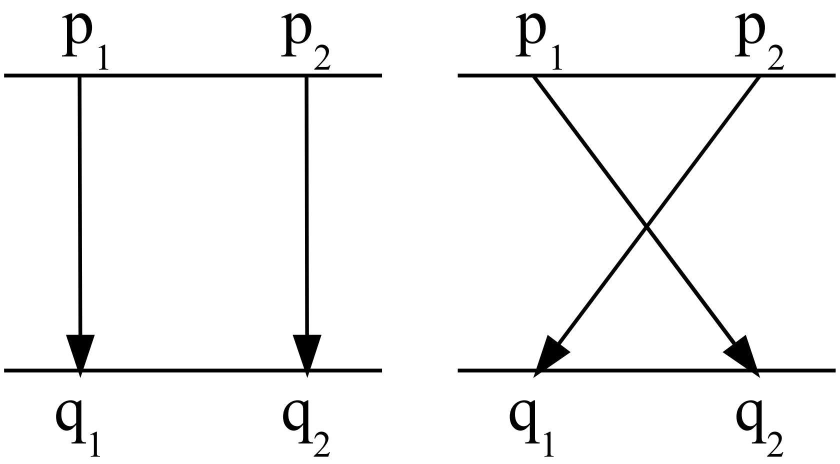
Spatial coherency in aperture plane

- Signal from common origin
- Mechanisms 1 and 2



Spatial coherency in focal plane

- Spat. coh. in aperture plane
- Imperfect focus
- Mechanisms 3 and 4

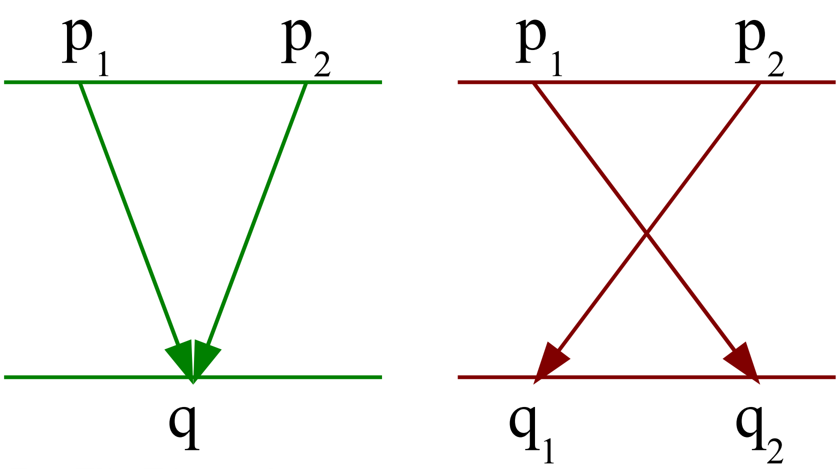


Spatial coherency in the focal plane

Cornwell & Napier, Radio Science, 1988

Physical relevance of spatial coherency

- Out of focus
- Aberrations
  - Diffraction
  - Imperfect reflector
  - atmo-/tropo-/ionosphere
  - …



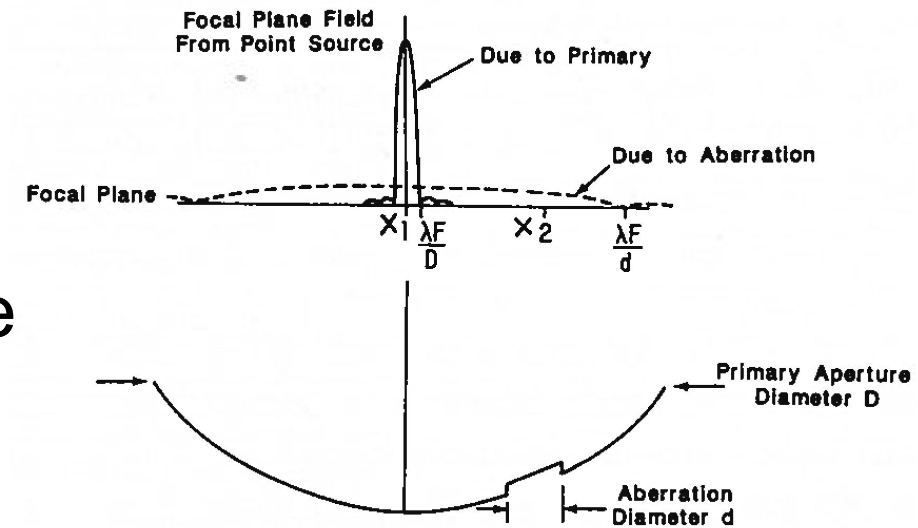


Image: Cornwell & Napier

Generic model of a phased array

Ivashina, Maaskant & Woestenburg, IEEE AWPL, 2008

Ivashina et al., IEEE TrAP, 2010, accepted

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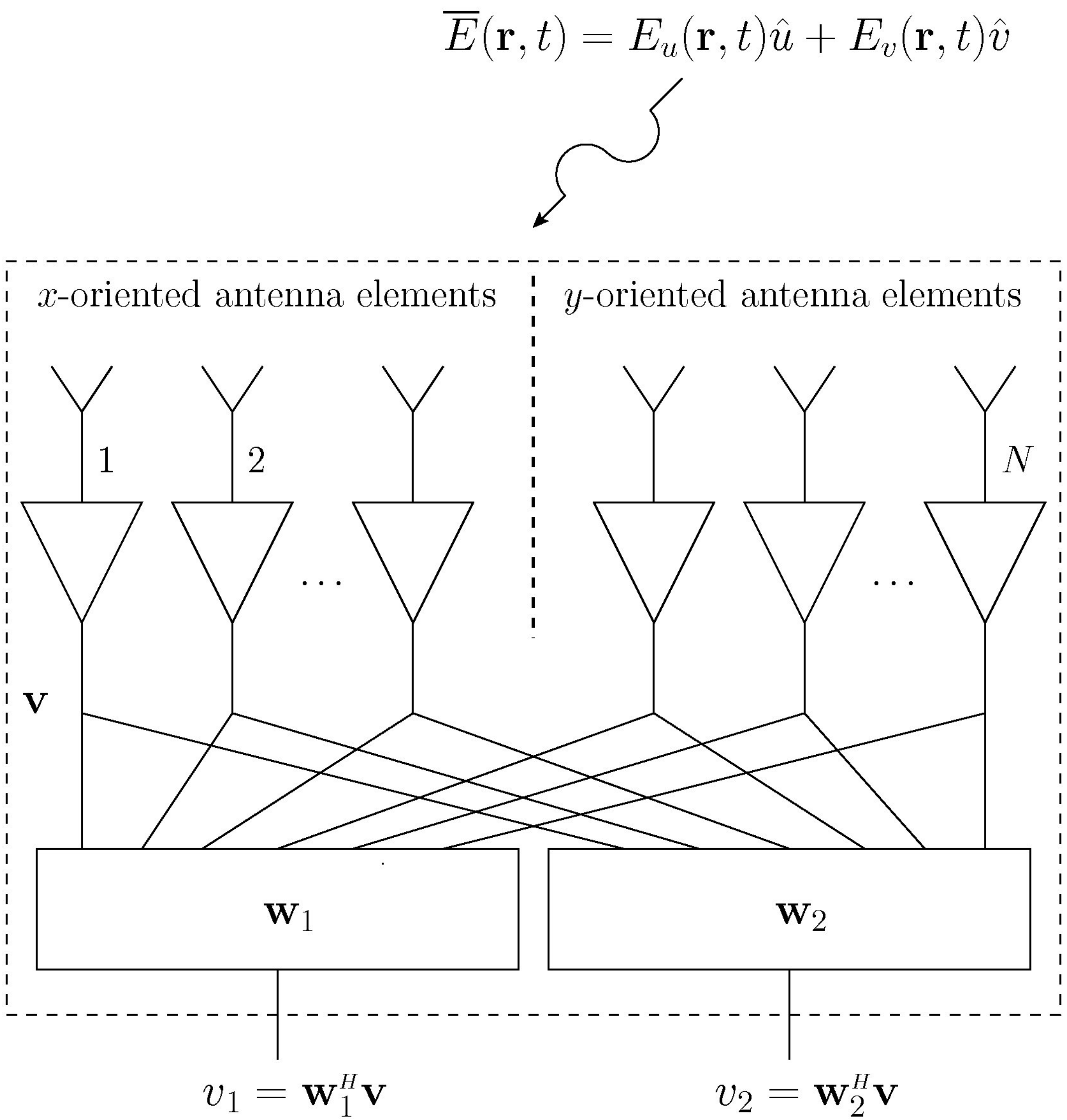
E(**r**, *t*)	incident field
E_u, E_v	*u*- and *v*-component
v	output voltage vector
$\mathbf{w}_1$, $\mathbf{w}_2$	BF weights
v_1, v_2	BF output voltages



Optimal polarimetric calibration (1)

Warnick, Jeffs, Ivashina, Maaskant & Wijnholds, Phased Array Workshop, April 2010

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$\mathbf{v}_u$, $\mathbf{v}_v$ voltage response to pure *u*- or *v*-polarized signal

Assume: $\mathbf{V} = [\mathbf{v}_u, \mathbf{v}_v]$ is known

BF output covariance matrix: $\mathbf{W}^H (\mathbf{R}_s + \mathbf{R}_n) \mathbf{W}$

where $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2]$

$\mathbf{R}_s$ is the signal covariance matrix

$\mathbf{R}_n$ is the noise covariance matrix

We want to:

1. minimize the noise: $\mathrm{argmin}_{\mathbf{W}}\ \mathbf{W}^H R_n \mathbf{W}$
2. preserve polarization: $\mathbf{W}^H \mathbf{V} = \mathbf{I}$

Optimal polarimetric calibration (2)

Warnick, Jeffs, Ivashina, Maaskant & Wijnholds, Phased Array Workshop, April 2010

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Steps to solution

- Reformulate using Lagrange multipliers
- Take derivatives and set them to zero
- Use contraint to find Lagrange multipliers

Solution

$$\mathbf{W} = \mathbf{R}_n^{-1} \mathbf{V} (\mathbf{V}^H \mathbf{R}_n^{-1} \mathbf{V})^{-1}$$

Interpretation

- Maximum sensitivity beam former
- Correction for optimal polarimetric fidelity

Practice (1): subspace method

Veidt, Phased Array Workshop, April 2010

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Problem: unknown $\mathbf{v}_u$ and $\mathbf{v}_v$

Calibration on an unpolarized source:

- on source measurement: $\mathbf{R}_{\mathrm{on}} = \mathbf{R}_{\mathrm{s}} + \mathbf{R}_{\mathrm{n}}$
- Off source measurement: $\mathbf{R}_{\mathrm{off}} = \mathbf{R}_{\mathrm{n}}$
- $\mathbf{R}_{\mathrm{s}} = \mathbf{R}_{\mathrm{on}} - \mathbf{R}_{\mathrm{off}}$
- Find dominant eigenvectors $\mathbf{v}_1$ and $\mathbf{v}_2$
- $\mathbf{W} = \mathbf{R}_{\mathrm{n}}^{-1} [\mathbf{v}_1, \mathbf{v}_2]$

Practice (2): interpreting eigenvectors

Wijnholds, Ivashina, Maaskant, Warnick & Jeffs, TrAP, in prep.

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Eigenvectors are orthogonal, $\mathbf{v}_u$ and $\mathbf{v}_v$ need not be

→ generally no one-to-one correspondence

Internal check

- Dominant eigenvalues: $\lambda_{1,2} = \sigma\,(1 \pm \varphi)$
- $\varphi = \mathbf{v}_u^H\,\mathbf{v}_v \,/\, ||\mathbf{v}_u||\;||\mathbf{v}_v||$
- Difference eigenvalues gives degree of orthogonality

Comparison with optimal method on poster Ivashina et al.

Practice (3): polarimetric requirement

Wijnholds, Ivashina, Maaskant, Warnick & Jeffs, TrAP, in prep.

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$\mathbf{v}_u$ and $\mathbf{v}_v$ span the same subspace as $\mathbf{v}_1$ and $\mathbf{v}_2$

$\rightarrow [\mathbf{v}_u, \mathbf{v}_v] = [\mathbf{v}_1, \mathbf{v}_2]\,\mathbf{T}$

$\rightarrow \mathbf{R}_s = [\mathbf{v}_1,\mathbf{v}_2]\,\mathbf{\Lambda}\,[\mathbf{v}_1,\mathbf{v}_2]^H = \mathbf{V}\,\mathbf{T}^{-1}\,\mathbf{\Lambda}\,\mathbf{T}^{-H}\,\mathbf{V}^H = \mathbf{V}\,\mathbf{T}'\,\mathbf{T}'^H\,\mathbf{V}^H$

$\rightarrow \mathbf{V}\,\mathbf{T}'\,\mathbf{T}'^H\,\mathbf{V}^H = \mathbf{V}\,\mathbf{T}'\,\mathbf{U}\,\mathbf{U}^H\,\mathbf{T}'^H\,\mathbf{V}^H$

$\rightarrow$ **T'** (and **T**) only known to a unitary matrix **U**

Physical significance:

- polrotation: rotation [Q, U, V]-vector in [Q, U, V]-space
- polconversion: conversion from I to [Q, U, V]

We need two distinctly polarized calibrators!

Practice (4): bi-scalar calibration

- Max-SNR BF for *u*- and *v*-array (separately)
- Pros
  - Allows full calibration on unpolarized source
  - Clear physical meaning of BF outputs
  - No unitary ambiguity at feed level
- Cons
  - Unitary ambiguity not solved but postponed
  - Needs identical polarimetric element response

Calibration of aperture arrays (1)

Optimal method is single source method

→ **problem: it does not work for aperture arrays!**

Reason: multiple source in FoV (2π sr)

Problem formulation:

$$\boldsymbol{\theta} = \mathrm{argmin}_{\boldsymbol{\theta}} \, ||\mathbf{R}_{obs} - R_{model}(\boldsymbol{\theta})||_F^2$$

where the parameter vector $\boldsymbol{\theta}$ includes, a.o.,

- electronic element gains (direction independent)
- apparent source Stokes vectors (direction dependent)

Calibration of aperture arrays (2)

Wijnholds, Ph.D. thesis, TU Delft, 2010

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Weighted alternating least squares (WALS):

1. initialize sky model using prior knowledge
2. estimate direction independent element gains

   Wijnholds & Van der Veen, TrSP, Sept. 2009
3. estimate apparent source Stokes vectors (DDEs)

   Wijnholds, Ph.D. thesis, TU Delft, 2010
4. estimate noise covariance matrix

   Wijnholds & Van der Veen, EuSiPCo, Aug. 2009
5. repeat 2 – 4 until convergence

Bi-scalar vs. full pol. calibration

Comparison of results with LBA-outer CS001 data

June 7, 2010, 14:00h

freq.: 45.3 MHz

BW: 195 kHz

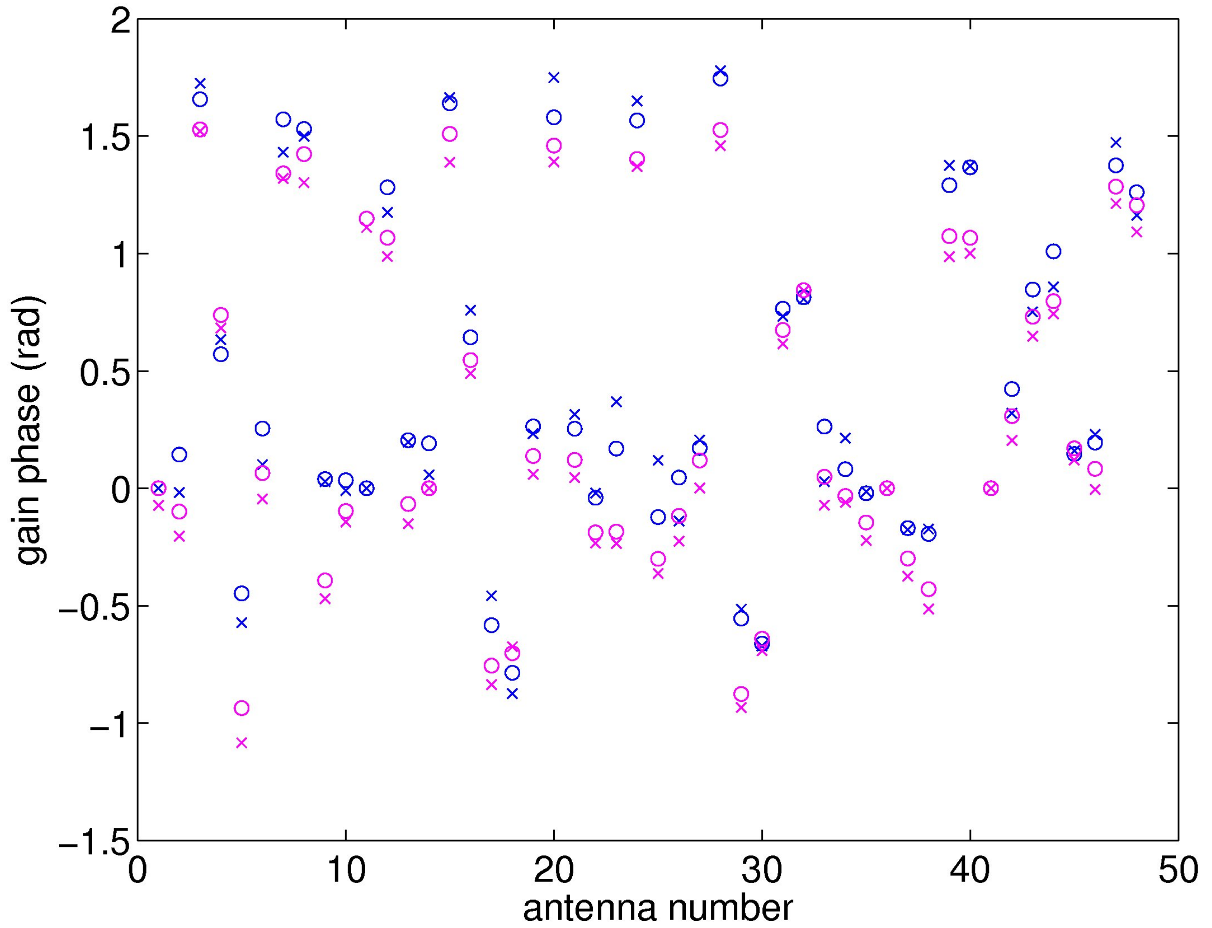
integration: 1 s

blue: x-elements

magenta: y-elements

circles: bi-scalar

crosses: full-pol



Observed Stokes vectors

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DDEs included in apparent source Stokes vectors

→ $\mathbf{E}_{app} = \mathbf{J}_1 \mathbf{E}_0 \mathbf{J}_2^H$

→ most sources are unpolarized, so $\mathbf{E}_0 = \mathbf{I}$

→ $\mathbf{E}_{app} = \mathbf{J}_1 \mathbf{J}_2^H = \mathbf{J}_1 \mathbf{U} \mathbf{U}^H \mathbf{J}_2^H$

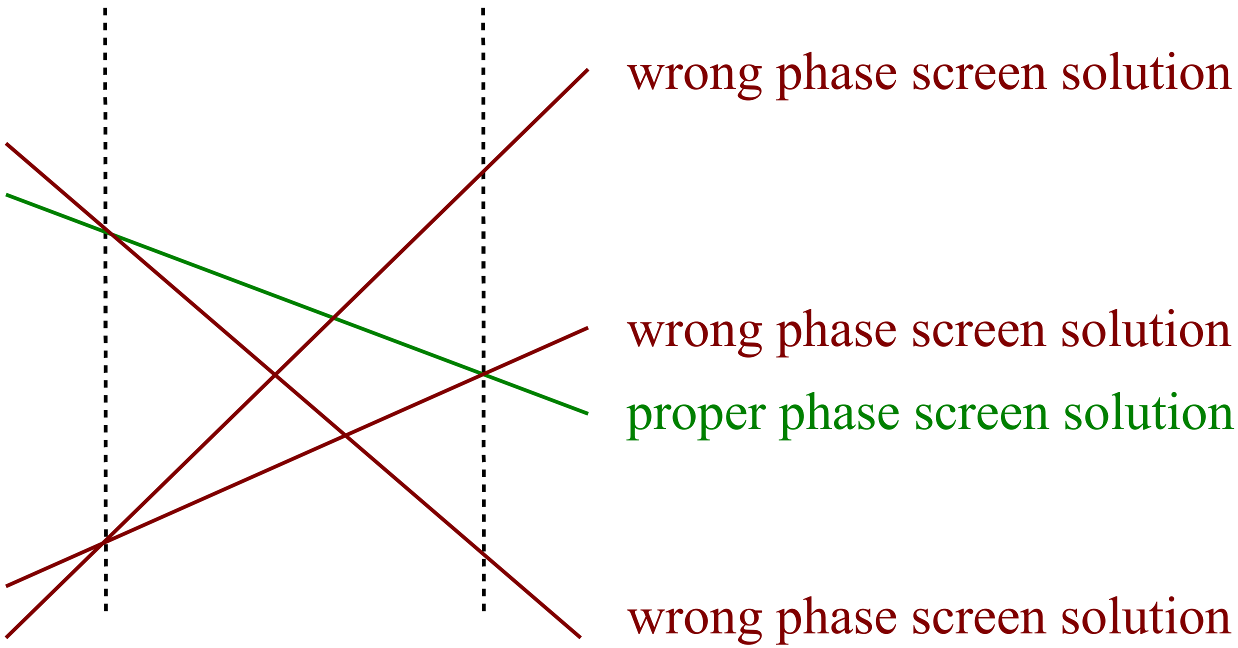
→ Polrotation and polconversion strike again!

Unitary ambiguity in each probed direction (source)

Understanding its implications

Scalar analog

- 2π phase ambiguity
- can be resolved by phase screen if enough sources
- leaves (irrelevant) common phase ambiguity



Resolving the unitary ambiguities

fit polarimetric model of DDEs

- atmo-/tropo-/ionospheric distortions
- beam patterns
- etc.

reduction to common ambiguity **if enough sources**

problem: common unitary matrix is physically significant

→ it should be determined

→ **we need two distinctly polarized sources per FoV per snapshot!**

Conclusions

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Phased array feeds

- optimal method provides bench mark
- practical methods: eigenvector and bi-scalar
- see poster MVI et al. for comparison
- calibration on unpol. sources gives unitary ambiguity

  → two measurements on distinctly polarized sources

Aperture arrays

- full polarization multi-source method
- needs sufficient sources within FoV for interpolation
- needs two polarized sources within FoV in snapshot