The Measurement Equation ...beyond the paraxial approximation

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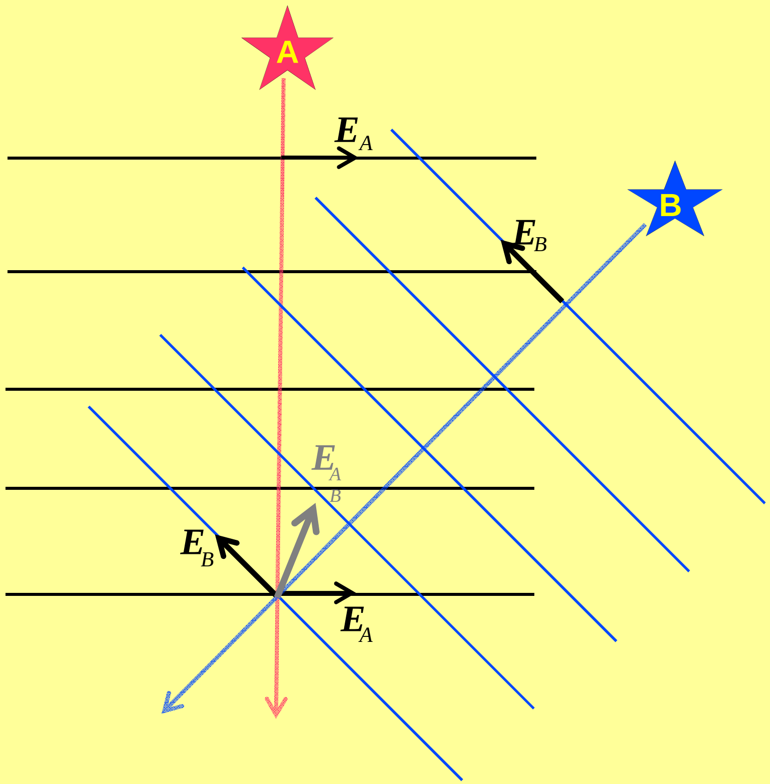
Background of Talk

- Next generation telescope will be polarimetric AND wide-field
- Measurement equation (MEq), lingua franca of radio interferometry, does not properly account for wide field
- Carozzi, Woan, "A generalized measurement equation and van Cittert-Zernike theorem for wide-field radio astronomical interferometry" MNRAS 395, 1558 (2009)
  - Not going to rederive it here
  - Just present the General Meq
  - and draw some interesting conclusions

Problem with paraxial



- Radiation from point sources or narrow fields is paraxial
- Paraxial approx
  - => planar, 2D formalism (i.e. Jones)
- But for wide enough fields, third electric field component is necessary
  - => 3D formalism

General MEq

MEq (Hamaker-Bregman-Sault formalism)

$$\mathcal{V} = \iint_{\mathcal{F}} \mathbf{J}\mathbf{B}\mathbf{J}^{\dagger} \mathrm{e}^{-\mathrm{i}2\pi\left[ul+vm+w\left(\sqrt{1-l^2-m^2}-1\right)\right]} \frac{1}{n} \, \mathrm{d}l\mathrm{d}m$$

Where J is the “Jones” matrix (2x2 complex)

$$\mathcal{V} \equiv \left\langle \begin{pmatrix} E_x \\ E_y \end{pmatrix} \otimes \begin{pmatrix} E_x^* & E_y^* \end{pmatrix} \right\rangle$$

$$\mathcal{V} = \iint_{\mathcal{F}} \mathbf{B} \mathrm{e}^{-\mathrm{i}2\pi\left[ul+vm+w\left(\sqrt{1-l^2-m^2}-1\right)\right]} \frac{1}{n} \, \mathrm{d}l\mathrm{d}m$$

For pure case (van Cittert-Zernike) i.e. no propagation or instrumental effects, J is 1

- This 2D in visibilities, so something must be missing!
- In fact if you do the full Electromagnetic theory (Maxwell's Eq) you get

General (3D) MEq

$$\mathcal{V}^{(3)} = \iint_{\mathcal{F}} \mathbf{T}\mathbf{B}\mathbf{T}^{\mathrm{T}} \mathrm{e}^{-\mathrm{i}2\pi\left[ul+vm+w\left(\sqrt{1-l^2-m^2}-1\right)\right]} \frac{1}{n} \, \mathrm{d}l\mathrm{d}m$$

For pure vCZ case

is the 3x3! visibility matrix

$$\mathcal{V}^{(3)} \equiv \left\langle \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \otimes \begin{pmatrix} E_x^* & E_y^* & E_z^* \end{pmatrix} \right\rangle$$

3D-MEq consequence:
Fully valid MEq doesn't have Jones matrices

- The transfer function for the Electric field at an interferometer is 3x2 not 2x2

$$\mathbf{T} = \frac{1}{\sqrt{1-(m\cos\Theta - n\sin\Theta)^2}} \begin{pmatrix} n\cos\Theta + m\sin\Theta & -lm\cos\Theta + ln\sin\Theta \\ -l\sin\Theta & (1-m^2)\cos\Theta + mn\sin\Theta \\ -l\cos\Theta & -mn\cos\Theta + (n^2-1)\sin\Theta \end{pmatrix}$$

- It depends on direction so can account some Direction Dependent Effect
  - Collary: DDEs are inherent in MEq

3D-MEq consequence:
No such thing as Stokes Visibilities

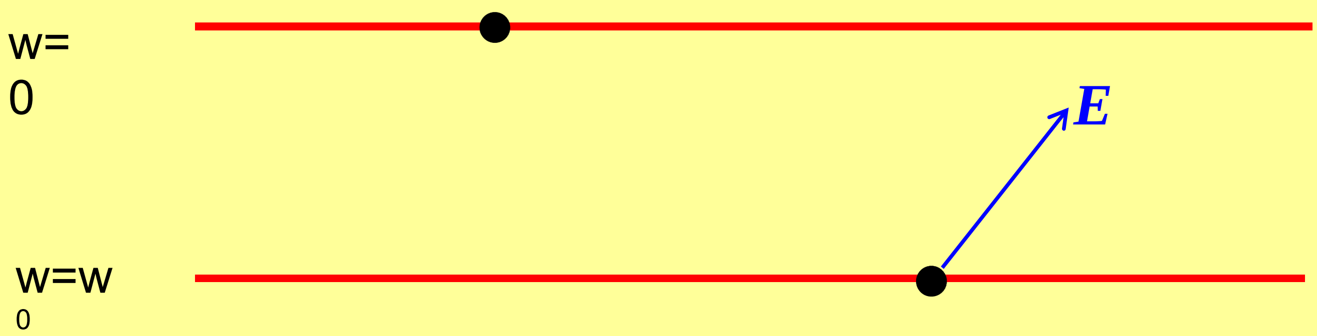
- Since visibility matrix is necessarily rank 3 (for anything but a single point source), the polarimetric visibility must have 9 complex parameters
- Stokes visibilities are 4 complex parameters
- => So Stokes visibilities *cannot* describe visibilities in general

3D-MEq consequence:
Brightness-Visibility relation is never a Fourier

- In the general MEq the relationship between the brightnesses and the visibilities is **Visibility 3x3 mat. <=!=> Brightness 2x2 mat.**
- This is *not* a Fourier transform (not even for planar arrays, so w=0) the dimensionalities are wrong (transformation matrix in the way)
- Therefore NO reciprocity between brightnesses and visibilities (as in vCZ)

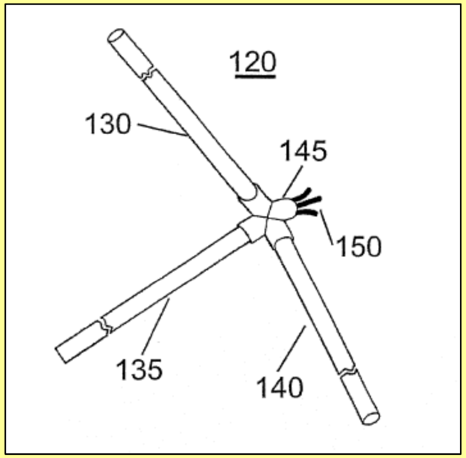
3D-MEq consequence:
Std w-projection isn't fully correct

- Diffraction interpretation cannot generally be scalar, rather it must be vector

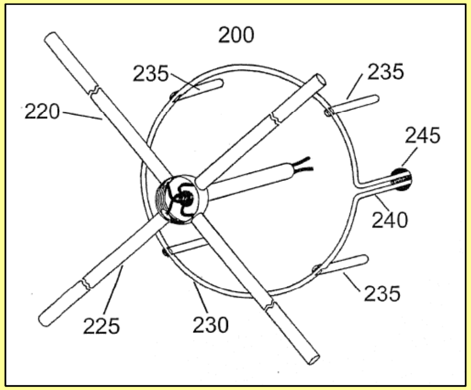


3D-MEq consequence:
Full EM field is important

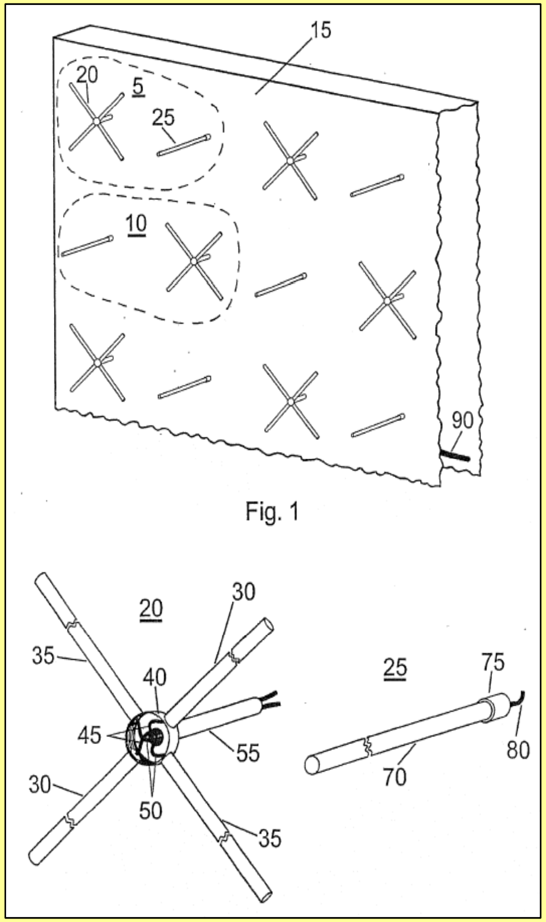
Electric tripole
(3 elements)



Electromagnetic
tripole (flat)



3D polarization diverse array



These types of interferometers are (in principle) not polarimetrically aberrated

Can't (currently) be modelled by Oleg's MeqTrees :-(

Bergman, Carozzi, Karlsson
International patent (2003)

Conclusions

- Fully general MEq (wide-field polarimetric) is not achieved by simple prescription

  Scalar --> Matrix

  It can only be obtained by a full electromagnetic formalism (3Dx2)
- Some Direction Dependent Effects are purely due to use of paraxial approx