

Polarization in Interferometry

A basic introduction

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Onsala Space Observatory
Chalmers University of Technology (Sweden)

European Radio Interferometry School
Dwingeloo (October 2017)



???



???



Departures

Airport	Flight	Arrival Time	Status	
Atlanta	WN 751	1:35 PM	On Time	
Austin	WN 355	1:35 PM	On Time	
Austin	WN 517	1:45 PM	On Time	
Austin	WN 2355	1:45 PM	On Time	
Baltimore	UA 985	1:30 PM	On Time	
Baton Rouge	AS 2484	1:15 AM	On Time	
Bethel	WN 3330	1:35 PM	On Time	
Bethel	WN 333	2:25 PM	On Time	
Chicago-Midway	WN 291	1:35 PM	On Time	
Chicago-O'Hare	AA 1538	1:50 PM	On Time	
Dallas Love Field	WN 3505	1:35 PM	On Time	
Dallas Love Field	WN 517	1:45 PM	On Time	
Des Moines	AA 1184	1:35 PM	On Time	
Des Moines	WN 5518	1:15 AM	On Time	
Des Moines	WN 2160	1:30 PM	New 12:35PM	
Edmonton	volaris	Y4 931	1:30 PM	On Time
Edmonton	volaris	Y4 930	1:35 PM	On Time
Edmonton	WN 2390	1:35 PM	On Time	
Edmonton	WN 453	10:50 AM	Boarding	
Edmonton	WN 453	10:50 AM	Boarding	
Edmonton	WN 3939	12:35 PM	On Time	
Edmonton	WN 2380	1:55 PM	On Time	
Edmonton	WN 282	1:15 AM	On Time	
Edmonton	AA 2854	11:55 AM	On Time	
Edmonton	UA 8311	11:56 AM	On Time	
Edmonton	WN 517	1:45 PM	On Time	
Edmonton	WN 1894	11:45 AM	On Time	
Edmonton	WN 2338	1:25 PM	On Time	

Jun 1, 2013
10:44 AM

Departures

Airport	Flight	Arrival Time	Status
Phoenix	UV 985	1:30 PM	On Time
Phoenix	WN 2385	1:45 PM	On Time
Raleigh-Durham	WN 2160	1:35 PM	New 12:35PM
Salt Lake City	DL 4512	1:15 PM	On Time
San Diego	AA 1538	1:50 PM	On Time
San Diego	WN 2639	12:35 PM	On Time
San Diego	WN 1956	1:55 PM	On Time
Tokyo-Narita	NH 1072	1:05 PM	On Time
Tucson	WN 762	11:25 AM	On Time

Jun 1, 2013
10:44 AM

Arrivals

Airport	Flight	Arrival Time	Status
Berkeley	WN 1483	11:35 AM	New 11:25AM
Berkeley	WN 769	1:05 PM	On Time
Chicago-Midway	WN 2336	12:35 PM	New 12:00PM
Denver	WN 533	1:45 PM	On Time
Las Vegas	WN 2774	11:10 AM	On Time
Las Vegas	WN 2160	11:45 AM	New 12:00PM
Las Vegas	WN 3855	1:15 PM	On Time
Los Angeles	AS 2484	10:40 AM	Arrived 10:24AM
Los Angeles	WN 2774	11:10 AM	On Time
Los Angeles	WN 1183	2:10 PM	On Time
Louisville	WN 2336	12:35 PM	New 12:00PM
Oklahoma City	WN 2160	11:45 AM	New 12:00PM
Omaha	WN 731	12:25 PM	On Time
Ontario	WN 1511	12:05 PM	On Time
Orange County	WN 1326	10:50 AM	Arrived
Orange County	WN 2360	1:20 PM	On Time
Phoenix	WN 1484	11:35 AM	New 11:25AM
Phoenix	WN 731	12:25 PM	On Time
Phoenix	WN 769	1:25 PM	On Time
Portland	AS 2185	11:00 AM	New 11:05AM
Portland	WN 2986	11:35 AM	New 11:20AM
Raleigh-Durham	WN 533	1:45 PM	On Time
Salt Lake City	WN 517	1:15 PM	On Time
San Antonio	WN 1183	2:10 PM	On Time
San Diego	WN 2055	1:10 PM	On Time

Terminal 8 Flights Only Information on Other Flights in Terminal
Jun 1, 2013
10:44 AM

Goals of this lecture

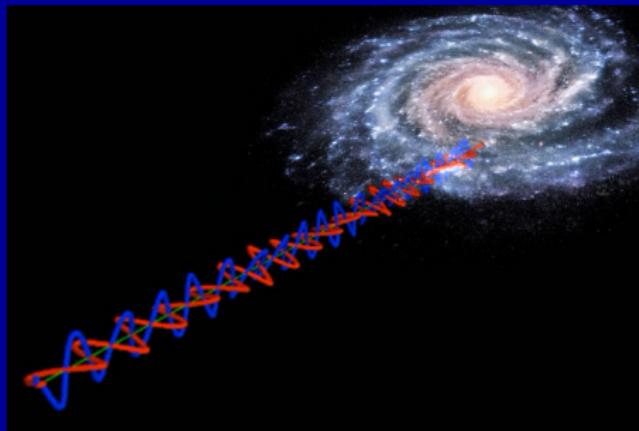
- Get familiar with some basics of polarimetry.
 - ▶ The different states of polarization.
 - ▶ The Stokes parameters.
- Understand radioastronomical polarizers.
 - ▶ Linear dipoles and quarter waveplates.
 - ▶ Polarization in interferometry: the Measurement Equation.
- Learn the basic calibration procedures.
 - ▶ Calibration with the Measurement Equation.
 - ▶ The effects of cross-delay (phase), amplitude, and leakage.
- Calibrate and process real observations (Tutorial: ALMA Band 5).

“Ordering” photons: The polarization of light

Light polarization in the Universe.

Photons are produced at different source locations and under different conditions. As a result, light can have two components: one with deterministic \vec{E} directions and another with a “stochastic” variability.

Light polarization in the Universe.

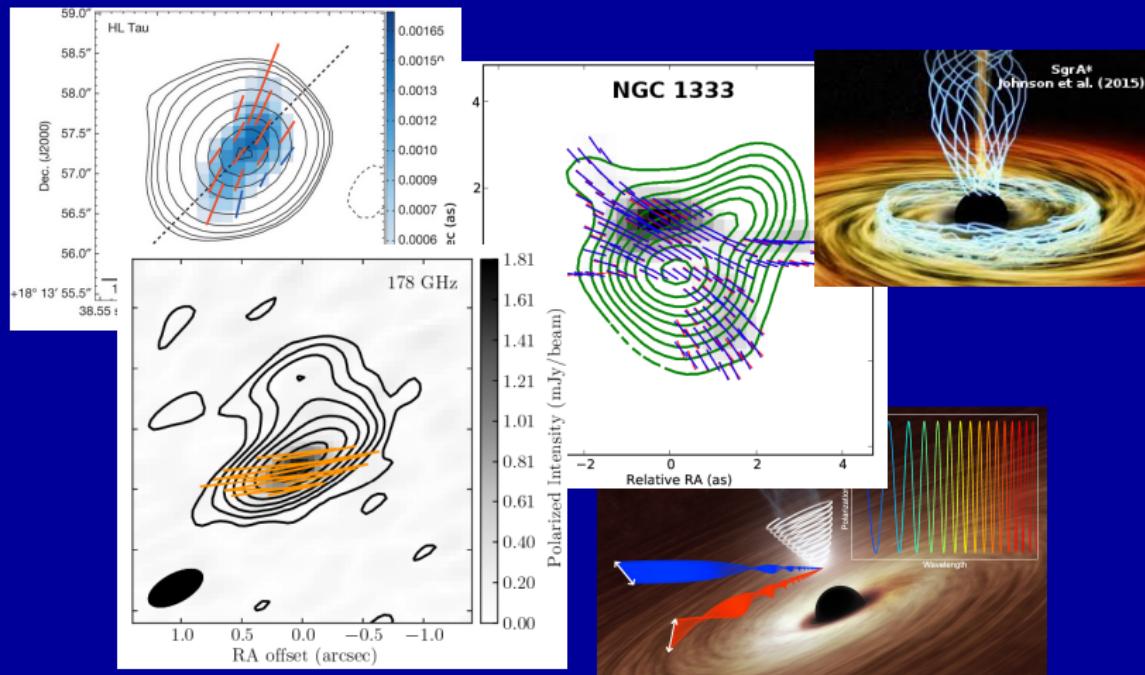


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Light polarization in the Universe.



RadioNet



“Ordered” light comes from large-scale structures
with consistent properties (e.g., \vec{B})

A random orientation of \vec{E}



\vec{E} as seen on the wave-front plane

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A (less) random orientation of \vec{E}



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Polarization modes

- LINEAR
- CIRCULAR
- ELLIPTIC (i.e., LINEAR + CIRCULAR)

The Stokes parameters



The Stokes parameters

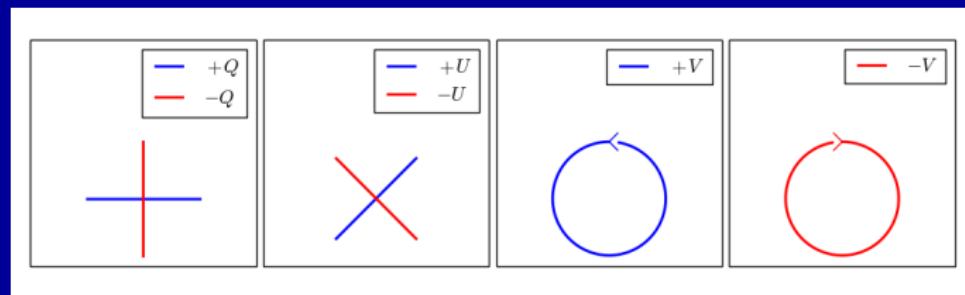
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The Stokes parameters

- We need **four** quantities to **fully** describe the polarization state:
 - ▶ How much polarized vs. unpolarized light do we have?
 - ▶ What is the **strength** and **direction** of the **linear** polarization?
 - ▶ How much **circular** polarization do we have?

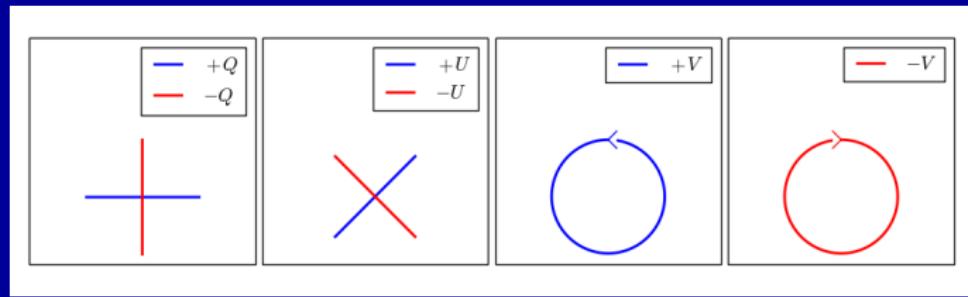
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The Stokes parameters

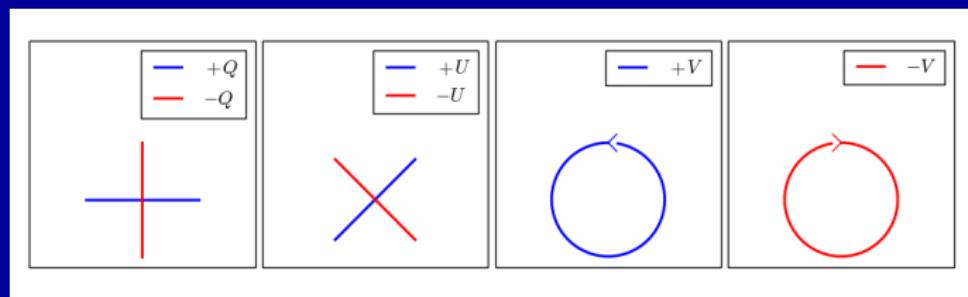
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- Linear polarization: $I_p = \frac{\sqrt{Q^2 + U^2}}{I}$, $\theta = \frac{1}{2} \arctan \left(\frac{U}{Q} \right)$

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- Linear polarization: $I_p = \frac{\sqrt{Q^2 + U^2}}{I}$, $\theta = \frac{1}{2} \arctan \left(\frac{U}{Q} \right)$
- Unpolarized intensity: $I_u = \sqrt{I^2 - Q^2 - U^2 - V^2}$

Polarizers in Radio Astronomy

Detecting source polarization



- The Stokes parameters describe the polarization state of light.
But how do we measure them?

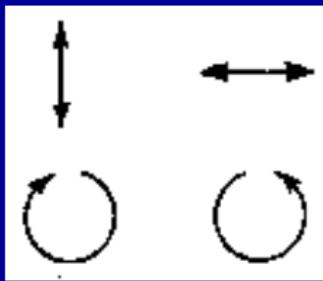
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Detecting source polarization

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But how do we measure them?
- Polarizing receivers (polarizers). The signal is split coherently into two orthogonal polarization states.
 - ▶ Linear polarizers (horizontal / vertical linear polarization).
 - ▶ Circular polarizers (left / right circular polarization).



Linear polarizers

Decomposing linear pol. with linear polarizers (no phase offset)

Linear polarizers



Decomposing **circular** pol. (left) with linear polarizers (90° offset)

Linear polarizers

Decomposing **circular** pol. (right) with **linear** polarizers (270° offset)

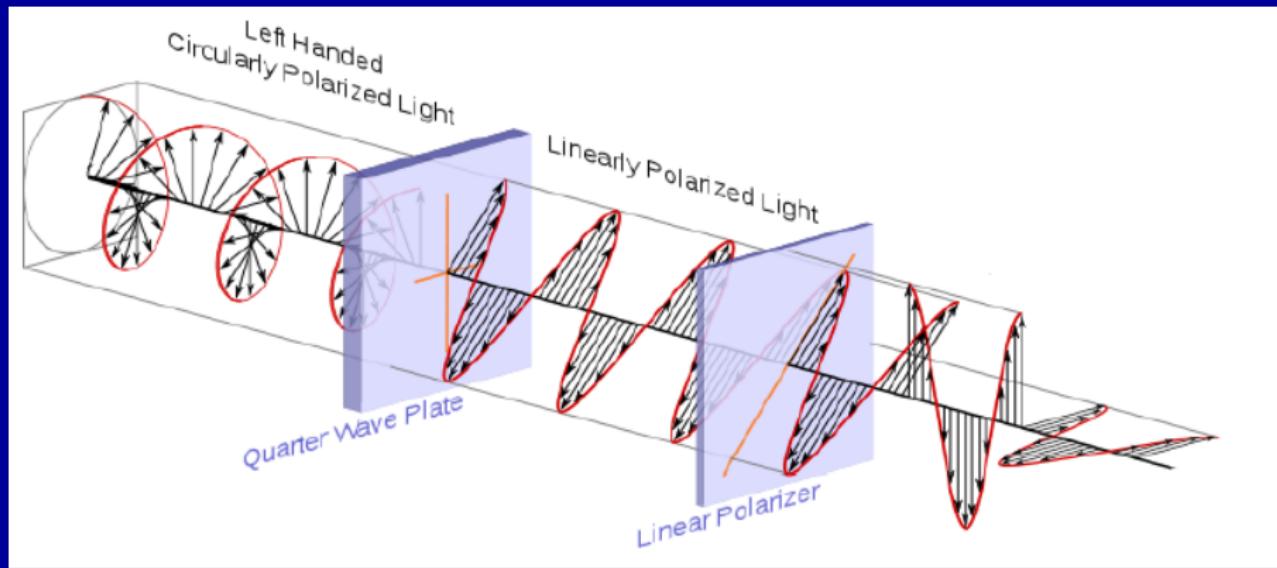
Linear polarizers

Decomposing **elliptical** pol. (right) with **linear** polarizers (generic phase offset)

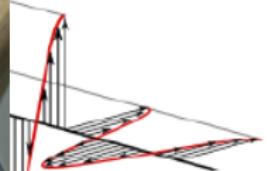
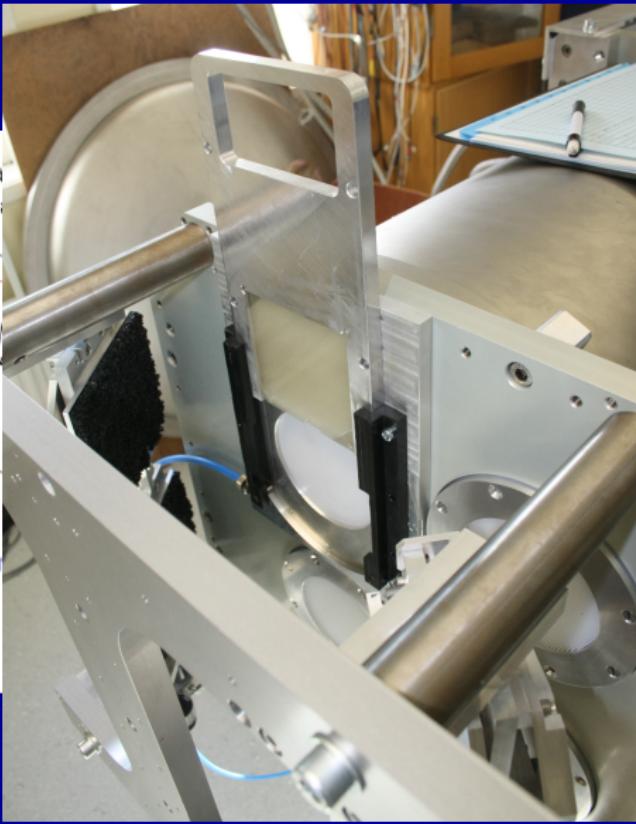
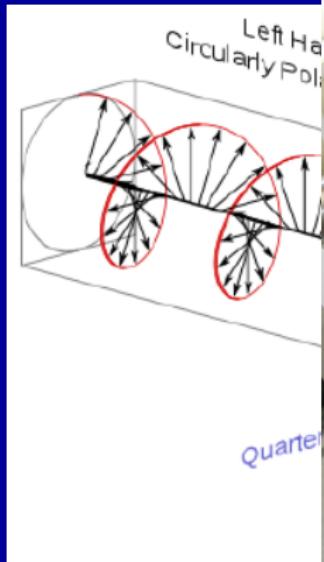
Linear polarizers

- $I = |E_x|^2 + |E_y|^2$
- $Q = |E_x|^2 - |E_y|^2$
- $U = 2 \operatorname{Re}(E_x E_y^*)$
- $V = 2 \operatorname{Im}(E_x E_y^*)$

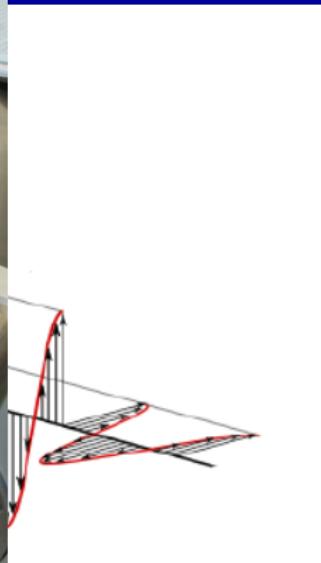
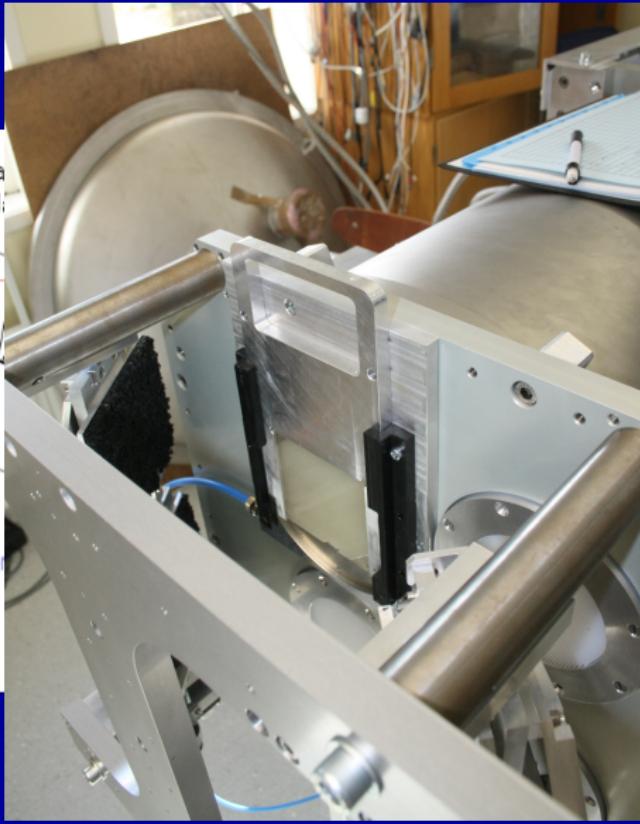
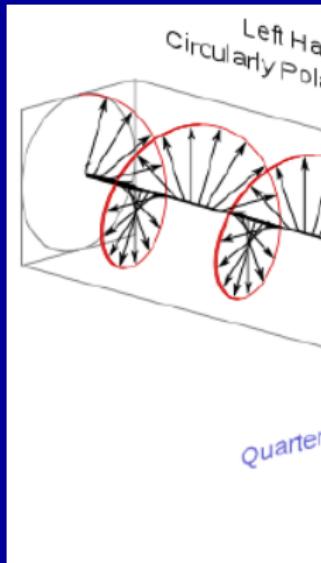
Circular polarizers



Circular polarizers



Circular polarizers



Circular polarizers

Decomposing linear pol. with circular polarizers (phase offset gives inclination)

Circular polarizers

Decomposing **elliptical** pol. with **circular** polarizers (R/L amplitude difference)

Circular polarizers

- $I = |E_I|^2 + |E_r|^2$
- $V = |E_I|^2 - |E_r|^2$
- $Q = 2 \operatorname{Re}(E_I^* E_r)$
- $U = -2 \operatorname{Im}(E_I^* E_r)$

Advanced formulation: The Measurement Equation

The classical formalism

$$V_{obs}^{AB} = G_A G_B^* \int_{\alpha, \delta} I(\alpha, \delta) e^{-\frac{2\pi j}{\lambda}(u\alpha + v\delta)} \frac{d\alpha d\delta}{z}$$

The MEq. Coherency matrix

- Electric field seen by antenna A : \vec{E}^A .

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- In the x-y polarization basis, the coherency matrix for baseline AB is:

$$E^{AB} = \begin{pmatrix} \left\langle E_x^A (E_x^B)^* \right\rangle & \left\langle E_x^A (E_y^B)^* \right\rangle \\ \left\langle E_y^A (E_x^B)^* \right\rangle & \left\langle E_y^A (E_y^B)^* \right\rangle \end{pmatrix}$$

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- We also define the brightness matrix, S . For x-y polarizers, it is

$$S = \begin{pmatrix} I + Q & U + j V \\ U - j V & I - Q \end{pmatrix}$$

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- The coherency matrix is related to the Fourier transform of the brightness matrix!

$$E^{AB} = \mathcal{F}[S]|_{(u,v)}$$

Coherency matrix and Visibility matrix.

- Voltage for antenna A with an x-y polarizer is: $\vec{v}^A = J^A \vec{E}^A$, where \vec{E}^A is the electric field in the x-y base and J^A is the **Jones matrix** that calibrates antenna A .

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- The visibility matrix (i.e., voltage cross-correlations) is:

$$V^{AB} = \vec{v}_A (\vec{v}_B)^H = \begin{pmatrix} \left\langle v_x^A (v_x^B)^* \right\rangle & \left\langle v_x^A (v_y^B)^* \right\rangle \\ \left\langle v_y^A (v_x^B)^* \right\rangle & \left\langle v_y^A (v_y^B)^* \right\rangle \end{pmatrix}$$

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- Since $\vec{v}_i = J_i \vec{E}_i$,

$$V^{AB} = J_A \vec{E}_A \left(\vec{E}_B \right)^H J_B^H = J_A \begin{pmatrix} \langle E_x^A (E_x^B)^* \rangle & \langle E_x^A (E_y^B)^* \rangle \\ \langle E_y^A (E_x^B)^* \rangle & \langle E_y^A (E_y^B)^* \rangle \end{pmatrix} J_B^H$$

The MEq. A full Stokes formalism



For a source with a generic structure, the visibility matrix for antennas A and B (with no direction-dependent calibration) will be

$$V_{AB}^{obs} = J_A \left[\int_{\alpha, \delta} S e^{-\frac{2\pi j}{\lambda}(u\alpha + v\delta)} \frac{d\alpha d\delta}{z} \right] (J_B)^H,$$

where (α, δ) are the (normalized) sky coordinates in the source plane, and $z = \sqrt{1 - \alpha^2 - \delta^2}$.

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Let us remember the classical interferometer equation:

$$V_{AB}^{obs} = G_A G_B^* \int_{\alpha, \delta} I(\alpha, \delta) e^{-\frac{2\pi j}{\lambda}(u\alpha + v\delta)} \frac{d\alpha d\delta}{z}$$

Jones calibration matrices. Examples

- Gain, $G = \begin{pmatrix} A_x(t) e^{j\phi_x(t)} & 0 \\ 0 & A_y(t) e^{j\phi_y(t)} \end{pmatrix}$
- Delay, $K = \begin{pmatrix} e^{j\tau_x(\nu - \nu_0)} & 0 \\ 0 & e^{j\tau_y(\nu - \nu_0)} \end{pmatrix}$
- Bandpass, $B = \begin{pmatrix} A_x(\nu) e^{j\phi_x(\nu)} & 0 \\ 0 & A_y(\nu) e^{j\phi_y(\nu)} \end{pmatrix}$

The Jones matrices are multiplicative, e.g.: $J = G \times B \times K$, but care must be taken, since matrices generally do not commute.

Polarization calibration

Polarization calibration

- Parallactic angle.
- Polarization leakage.
- Cross-Delay/phase.
- Amplitude ratio.

Pol. calibration I. Parallactic angle



$$P_{xy} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \quad P_{rl} = \begin{pmatrix} e^{j\phi} & 0 \\ 0 & e^{-j\phi} \end{pmatrix}$$

- Is the rotation of the antenna mount axis w.r.t. the sky.
- Is deterministic. It's good to apply it before the phase (and delay/rate) calibration.
- It does not commute with the gains for linear polarizers.
- In VLBI, it also mixes V_{xx} and V_{yy} with V_{xy} and V_{yx} .

Pol. calibration II. Leakage



$$D_{xy} = \begin{pmatrix} 1 & D_x(\nu) \\ D_y(\nu) & 1 \end{pmatrix}$$

- Is caused by cross-talking between the polarizer channels
- Each leaked signal is modified by an amplitude and a phase.
- Introduces spurious ellipticity and linear polarization.

LIN. + LEAK

CIRC. + LEAK

Pol. calibration III. Cross-hand delay/phase



$$K_c = \begin{pmatrix} 1 & 0 \\ 0 & e^{j(\tau_c(\nu - \nu_0) + \phi_c)} \end{pmatrix}$$

- Is caused by a delay between the polarizer channels at the reference antenna.
- In **linear** polarizers, introduces **ellipticity** and **spurious V**.
- In **circular** polarizers, just **rotates** the PA of the linear polarization.

OFFSET: 0°

OFFSET: 45°

LINEAR:

CIRCULAR:

Pol. calibration IV. Amplitude ratio

$$G_a = \begin{pmatrix} 1 & 0 \\ 0 & A_c \end{pmatrix}$$

- Is caused by different T_{sys} , gain and/or bandpass between polarizer channels.
- In linear polarizers, introduces spurious linear polarization.
- In circular polarizers, introduces spurious Stokes V .
- Not explicitly calibrated, but implicit in the gain calibration.

Calibration strategy

The right order for matrix product is: $J = (G_a K_c) \times D \times P$

$$\text{i.e.: } V^{cal} = P^{-1} \times D^{-1} \times (G_a K_c)^{-1} \times V^{obs}$$

- STEP 1 (optional): Calibrate the cross-delay using a strong polarized source.

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- STEP 2: Calibrate the leakage using an unpolarized source.
 - ▶ If all calibrators are polarized, solve for leakage and source polarization simultaneously.
 - ▶ Need good parallactic-angle coverage.

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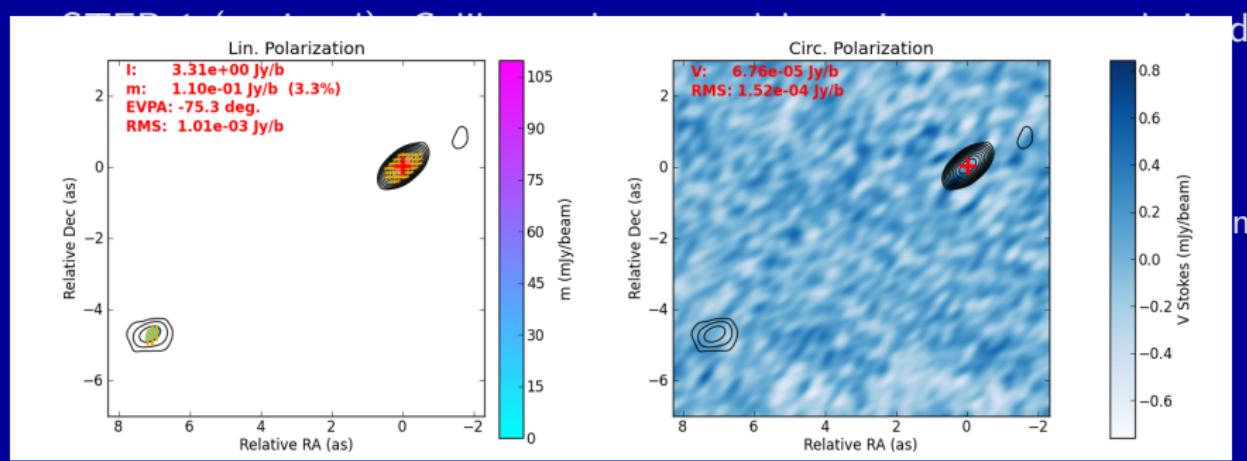
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- STEP 4: Image each Stokes parameter separately. Combine images: $(Q, U) \rightarrow (I_p, \theta)$.

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SUMMARY

- We have reviewed basic concepts of polarization.
 - ▶ Modes of polarization.
 - ▶ Stokes parameters.
- We have discussed about the different kinds of polarizers in radioastronomical receivers.
 - ▶ Linear polarizers (X-Y).
 - ▶ Circular polarizers (R-L).
- We have studied how to deal with polarization in interferometric observations.
 - ▶ The Measurement Equation.
 - ▶ The matrices for polarization calibration.
 - ▶ Calibration effects on X-Y vs. R-L polarizers.
 - ▶ Overview of calibration procedure.

RadioNet has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 730562

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