Polarization in Interferometry

A basic introduction

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Goals of this lecture

- Get familiar with some basics of polarimetry.
  - The different states of polarization.
  - The Stokes parameters.
- Understand radioastronomical polarizers.
  - Linear dipoles and quarter waveplates.
  - Polarization in interferometry: the Measurement Equation.
- Learn the basic calibration procedures.
  - Calibration with the Measurement Equation.
  - The effects of cross-delay (phase), amplitude, and leakage.
- Calibrate and process real observations (Tutorial: ALMA Band 5).

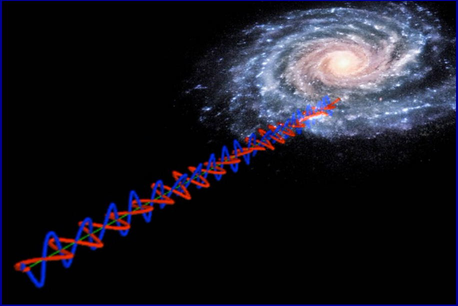
“Ordering” photons:

The polarization of light

Light polarization in the Universe.

Photons are produced at different source locations and under different conditions. As a result, light can have two components: one with deterministic $\vec{E}$ directions and another with a "stochastic" variability.

Light polarization in the Universe.



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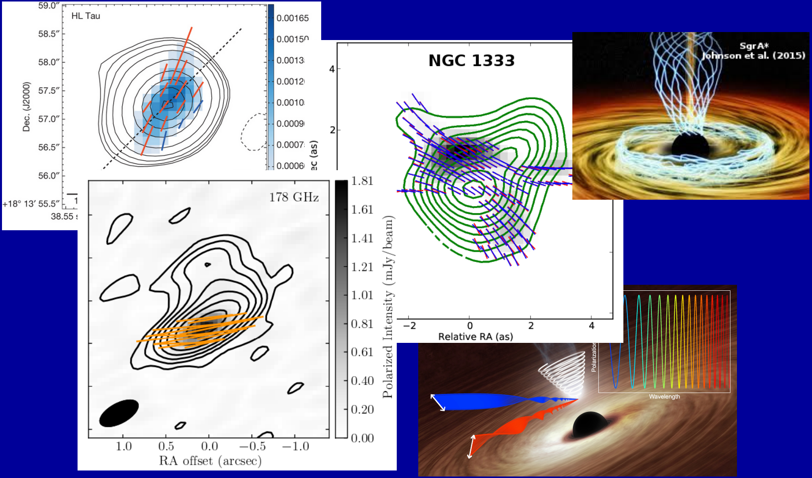

"Ordered" light comes from large-scale structures with consistent properties (e.g., $\vec{B}$)

A random orientation of $\vec{E}$

$\vec{E}$ as seen on the wave-front plane

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Polarization modes

- LINEAR

- CIRCULAR

- ELLIPTIC (i.e., LINEAR + CIRCULAR)

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  - What is the strength and direction of the linear polarization?
  - How much circular polarization do we have?

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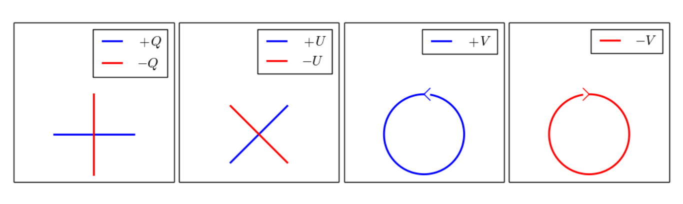

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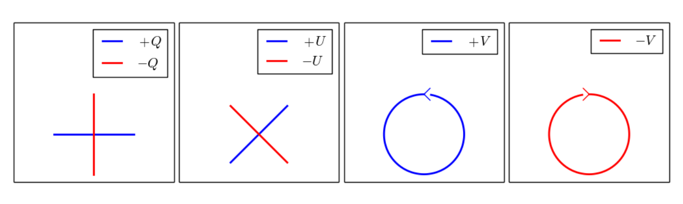

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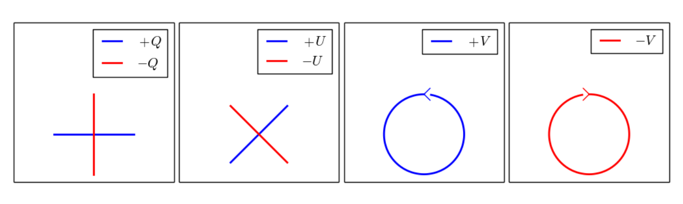

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- Unpolarized intensity: $I_u = \sqrt{I^2 - Q^2 - U^2 - V^2}$

Polarizers in Radio Astronomy

Detecting source polarization

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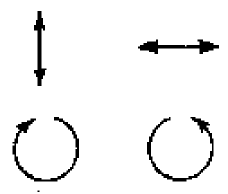
- Polarizing receivers (polarizers). The signal is split coherently into two orthogonal polarization states.
  - Linear polarizers (horizontal / vertical linear polarization).
  - Circular polarizers (left / right circular polarization).



Linear polarizers

Decomposing linear pol. with linear polarizers (no phase offset)

Linear polarizers

Decomposing circular pol. (left) with linear polarizers (90° offset)

Linear polarizers

Decomposing circular pol. (right) with linear polarizers (270° offset)

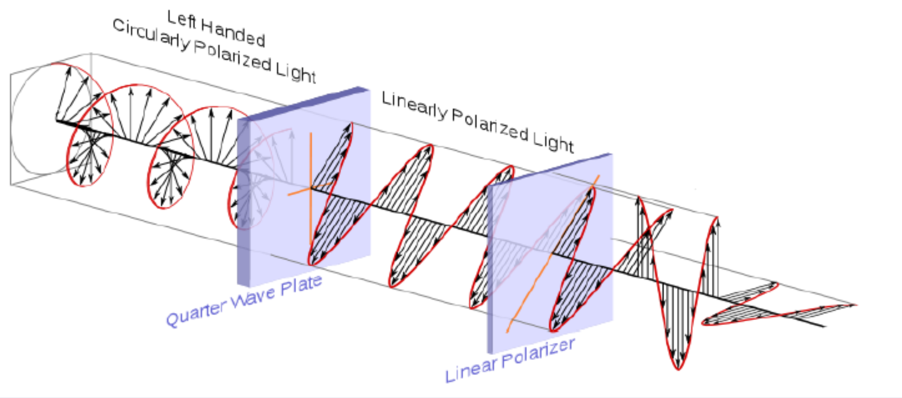
Linear polarizers

Decomposing elliptical pol. (right) with linear polarizers (generic phase offset)

Linear polarizers

- $I = |E_x|^2 + |E_y|^2$
- $Q = |E_x|^2 - |E_y|^2$
- $U = 2\,Re(E_x\,E_y^*)$
- $V = 2\,Im(E_x\,E_y^*)$

Circular polarizers



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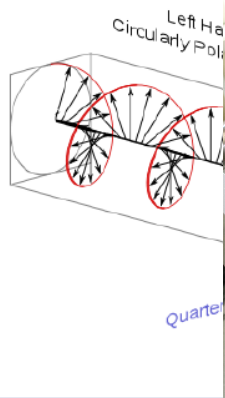
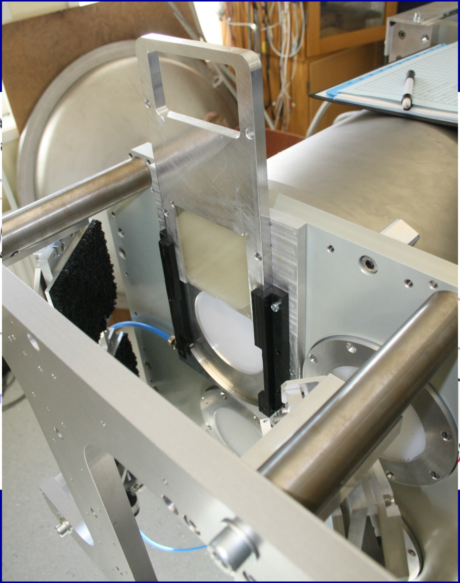
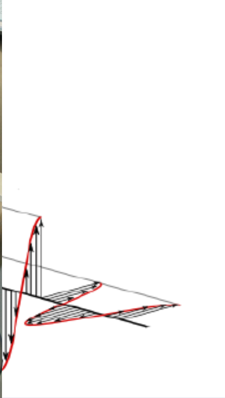
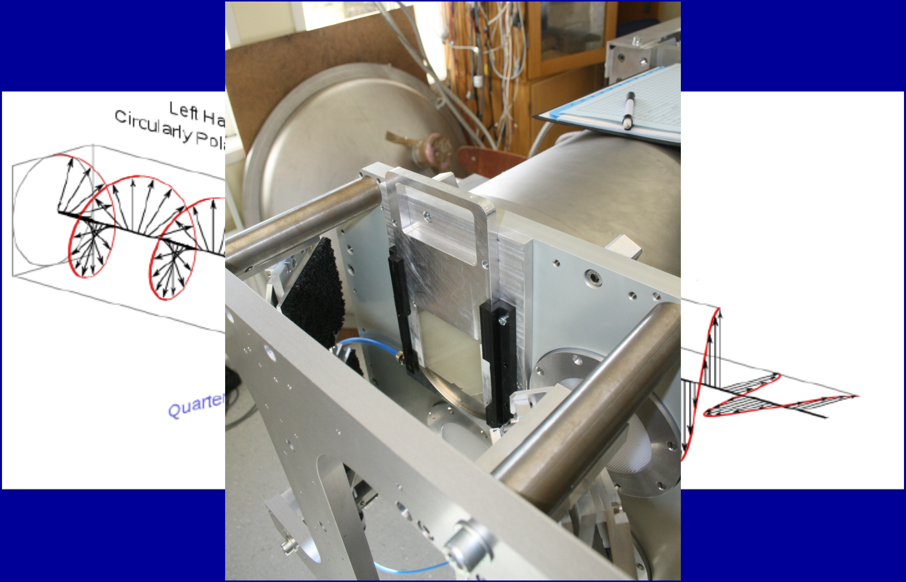






Circular polarizers



Circular polarizers

Decomposing linear pol. with circular polarizers (phase offset gives inclination)

Circular polarizers

Decomposing elliptical pol. with circular polarizers (R/L amplitude difference)

Circular polarizers

- $I = |E_l|^2 + |E_r|^2$
- $V = |E_l|^2 - |E_r|^2$
- $Q = 2\,Re(E_l^*\,E_r)$
- $U = -2\,Im(E_l^*\,E_r)$

Advanced formulation:

The Measurement Equation

The classical formalism

$$V_{obs}^{AB} = G_A\, G_B^* \int_{\alpha,\delta} I(\alpha,\delta)\, e^{-\frac{2\pi j}{\lambda}(u\,\alpha + v\,\delta)}\, \frac{d\alpha\, d\delta}{z}$$

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$$E^{AB} = \begin{pmatrix} \left\langle E_x^A \, (E_x^B)^* \right\rangle & \left\langle E_x^A \, (E_y^B)^* \right\rangle \\ \left\langle E_y^A \, (E_x^B)^* \right\rangle & \left\langle E_y^A \, (E_y^B)^* \right\rangle \end{pmatrix}$$

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- We also define the brightness matrix, S. For x-y polarizers, it is

$$S = \begin{pmatrix} I + Q & U + j\,V \\ U - j\,V & I - Q \end{pmatrix}$$

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- The coherency matrix is related to the Fourier transform of the brightness matrix!

$$E^{AB} = \mathcal{F}\left[S\right]\big|_{(u,v)}$$

Coherency matrix and Visibility matrix.

- Voltage for antenna A with an x-y polarizer is: $\vec{v^A} = J^A \vec{E^A}$, where $\vec{E^A}$ is the electric field in the x-y base and J^A is the Jones matrix that calibrates antenna A.

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- The visibility matrix (i.e., voltage cross-correlations) is:

$$V^{AB} = \vec{v_A} \left(\vec{v_B}\right)^H = \begin{pmatrix} \left\langle v_x^A \left(v_x^B\right)^* \right\rangle & \left\langle v_x^A \left(v_y^B\right)^* \right\rangle \\ \left\langle v_y^A \left(v_x^B\right)^* \right\rangle & \left\langle v_y^A \left(v_y^B\right)^* \right\rangle \end{pmatrix}$$

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- Since $\vec{v_i} = J_i \vec{E_i}$,

$$V^{AB} = J_A \vec{E_A} \left(\vec{E_B}\right)^H J_B^H = J_A \begin{pmatrix} \left\langle E_x^A \left(E_x^B\right)^* \right\rangle & \left\langle E_x^A \left(E_y^B\right)^* \right\rangle \\ \left\langle E_y^A \left(E_x^B\right)^* \right\rangle & \left\langle E_y^A \left(E_y^B\right)^* \right\rangle \end{pmatrix} J_B^H$$

The MEq. A full Stokes formalism

For a source with a generic structure, the visibility matrix for antennas A and B (with no direction-dependent calibration) will be

$$V_{AB}^{obs} = J_A \left[\int_{\alpha,\delta} S\, e^{-\frac{2\pi j}{\lambda}(u\,\alpha + v\,\delta)} \, \frac{d\alpha\, d\delta}{z} \right] (J_B)^H,$$

where (α, δ) are the (normalized) sky coordinates in the source plane, and $z = \sqrt{1 - \alpha^2 - \delta^2}$.

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Let us remember the classical interferometer equation:

$$V_{AB}^{obs} = G_A\, G_B^* \int_{\alpha,\delta} I(\alpha,\delta)\, e^{-\frac{2\pi j}{\lambda}(u\,\alpha + v\,\delta)}\, \frac{d\alpha\, d\delta}{z}$$

Jones calibration matrices. Examples

- Gain, $G = \begin{pmatrix} A_x(t)\, e^{j\phi_x(t)} & 0 \\ 0 & A_y(t)\, e^{j\phi_y(t)} \end{pmatrix}$
- Delay, $K = \begin{pmatrix} e^{j\tau_x(\nu-\nu_0)} & 0 \\ 0 & e^{j\tau_y(\nu-\nu_0)} \end{pmatrix}$
- Bandpass, $B = \begin{pmatrix} A_x(\nu)\, e^{j\phi_x(\nu)} & 0 \\ 0 & A_y(\nu)\, e^{j\phi_y(\nu)} \end{pmatrix}$

The Jones matrices are multiplicative, e.g.: $J = G \times B \times K$, but care must be taken, since matrices generally do not commute.

Polarization calibration

Polarization calibration

- Parallactic angle.
- Polarization leakage.
- Cross-Delay/phase.
- Amplitude ratio.

Pol. calibration I. Parallactic angle

$$P_{xy} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \qquad P_{rl} = \begin{pmatrix} e^{j\phi} & 0 \\ 0 & e^{-j\phi} \end{pmatrix}$$

- Is the rotation of the antenna mount axis w.r.t. the sky.
- Is deterministic. It's good to apply it before the phase (and delay/rate) calibration.
- It does not commute with the gains for linear polarizers.
- In VLBI, it also mixes V_{xx} and V_{yy} with V_{xy} and V_{yx}.

Pol. calibration II. Leakage

$$D_{xy} = \begin{pmatrix} 1 & D_x(\nu) \\ D_y(\nu) & 1 \end{pmatrix}$$

- Is caused by cross-talking between the polarizer channels
- Each leaked signal is modified by an amplitude and a phase.
- Introduces spurious ellipticity and linear polarization.

LIN. + LEAK CIRC. + LEAK

Pol. calibration III. Cross-hand delay/phase

$$K_c = \begin{pmatrix} 1 & 0 \\ 0 & e^{j(\tau_c(\nu-\nu_0)+\phi_c)} \end{pmatrix}$$

- Is caused by a delay between the polarizer channels at the reference antenna.
- In linear polarizers, introduces ellipticity and spurious V.
- In circular polarizers, just rotates the PA of the linear polarization.

OFFSET: 0° OFFSET: 45°

LINEAR:

CIRCULAR:

Pol. calibration IV. Amplitude ratio

$$G_a = \begin{pmatrix} 1 & 0 \\ 0 & A_c \end{pmatrix}$$

- Is caused by different T_{sys}, gain and/or bandpass between polarizer channels.
- In linear polarizers, introduces spurious linear polarization.
- In circular polarizers, introduces spurious Stokes V.
- Not *explicitely* calibrated, but implicit in the gain calibration.

Calibration strategy

The right order for matrix product is: $J = (G_a\,K_c) \times D \times P$

i.e.: $V^{cal} = P^{-1} \times D^{-1} \times (G_a\,K_c)^{-1} \times V^{obs}$

- STEP 1 (optional): Calibrate the cross-delay using a strong polarized source.

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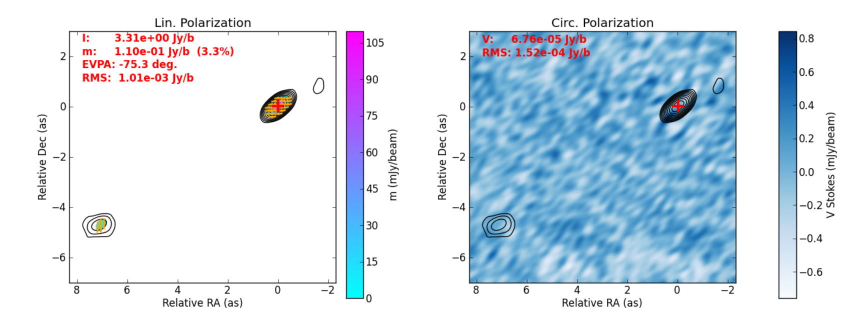

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SUMMARY

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  - Modes of polarization.
  - Stokes parameters.
- We have discussed about the different kinds of polarizers in radioastronomical receivers.
  - Linear polarizers (X-Y).
  - Circular polarizers (R-L).
- We have studied how to deal with polarization in interferometric observations.
  - The Measurement Equation.
  - The matrices for polarization calibration.
  - Calibration effects on X-Y vs. R-L polarizers.
  - Overview of calibration procedure.

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