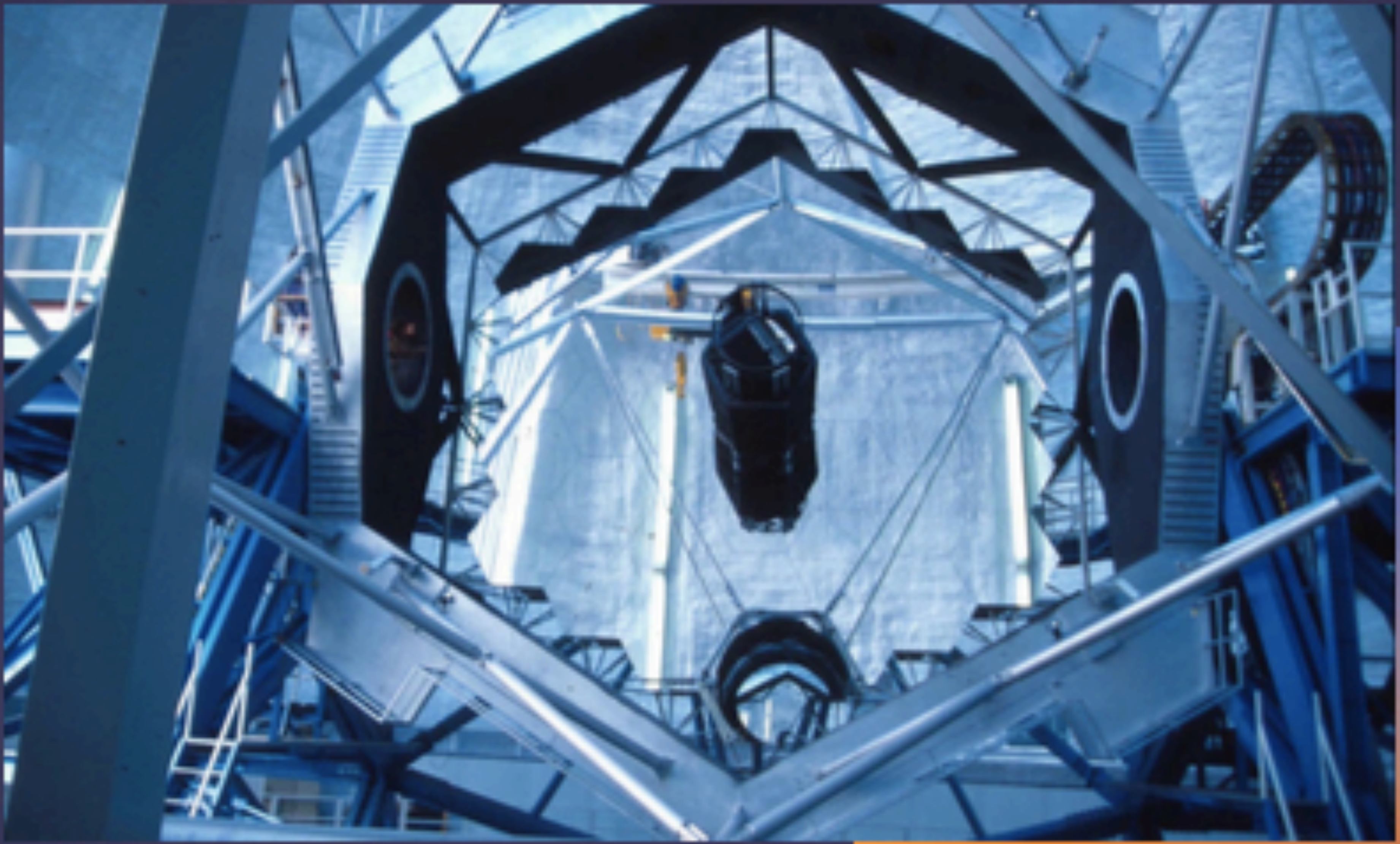
ANNA SCAIFE
JODRELL BANK CENTRE FOR ASTROPHYSICS
@RADASTRAT

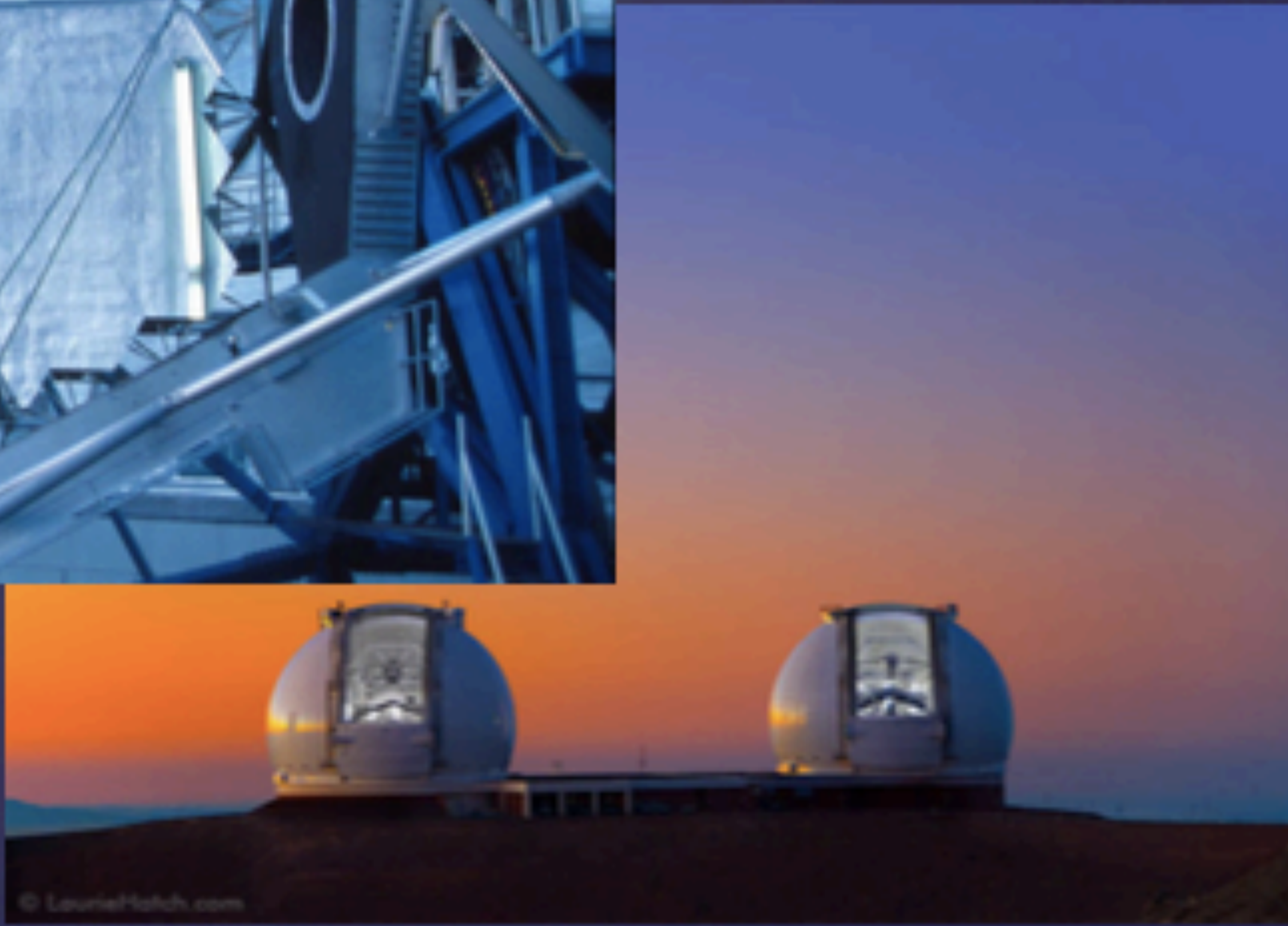
(GENTLE) INTRODUCTION TO INTERFEROMETRY

ANNA SCAIFE
JODRELL BANK CENTRE FOR ASTROPHYSICS
@RADASTRAT

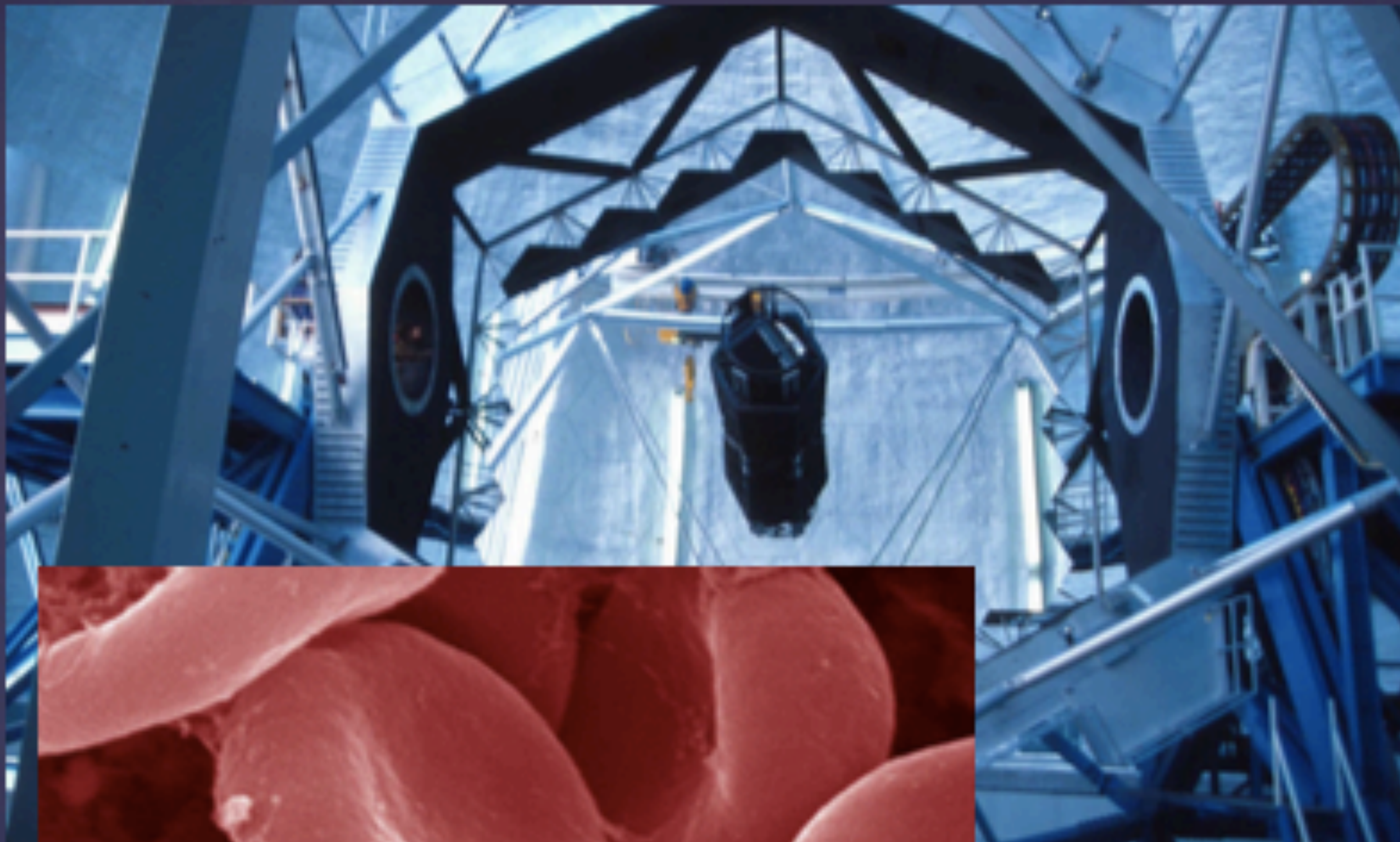
(GENTLE) INTRODUCTION TO INTERFEROMETRY

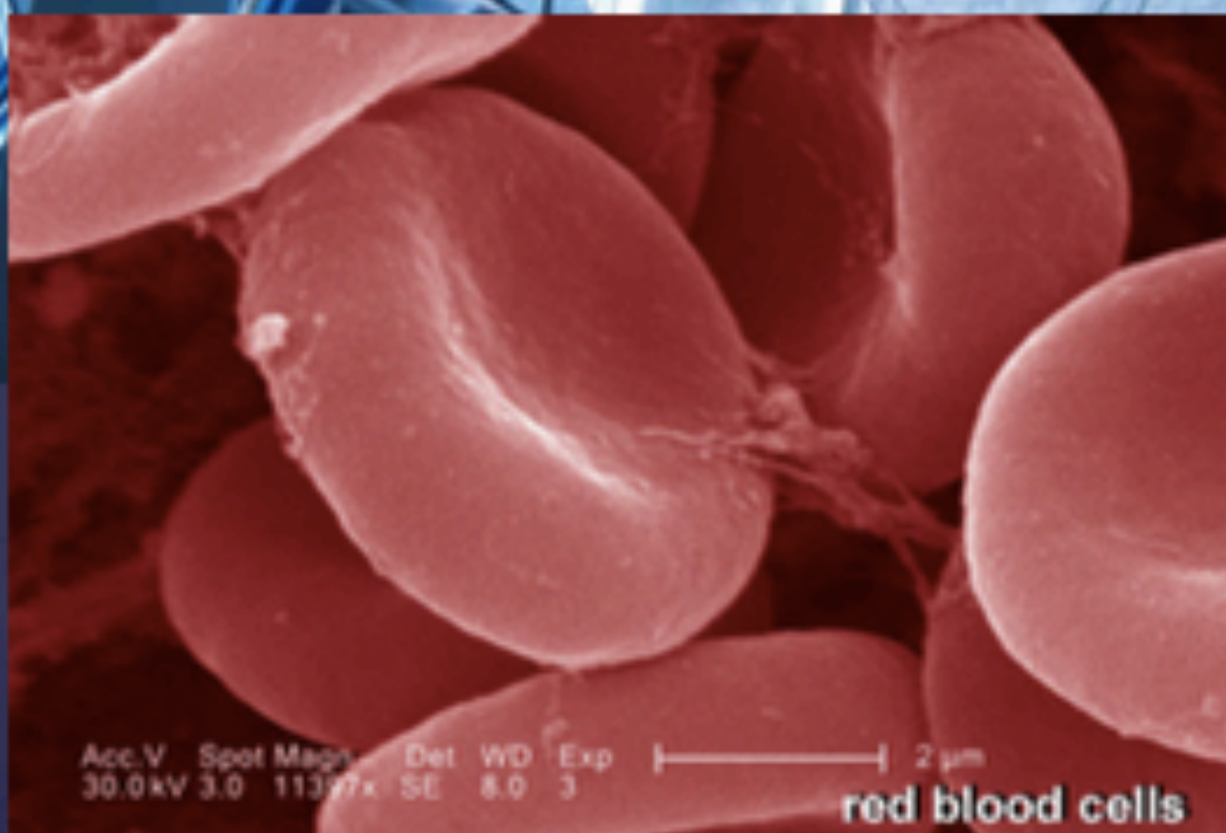
WHAT IS A TELESCOPE?



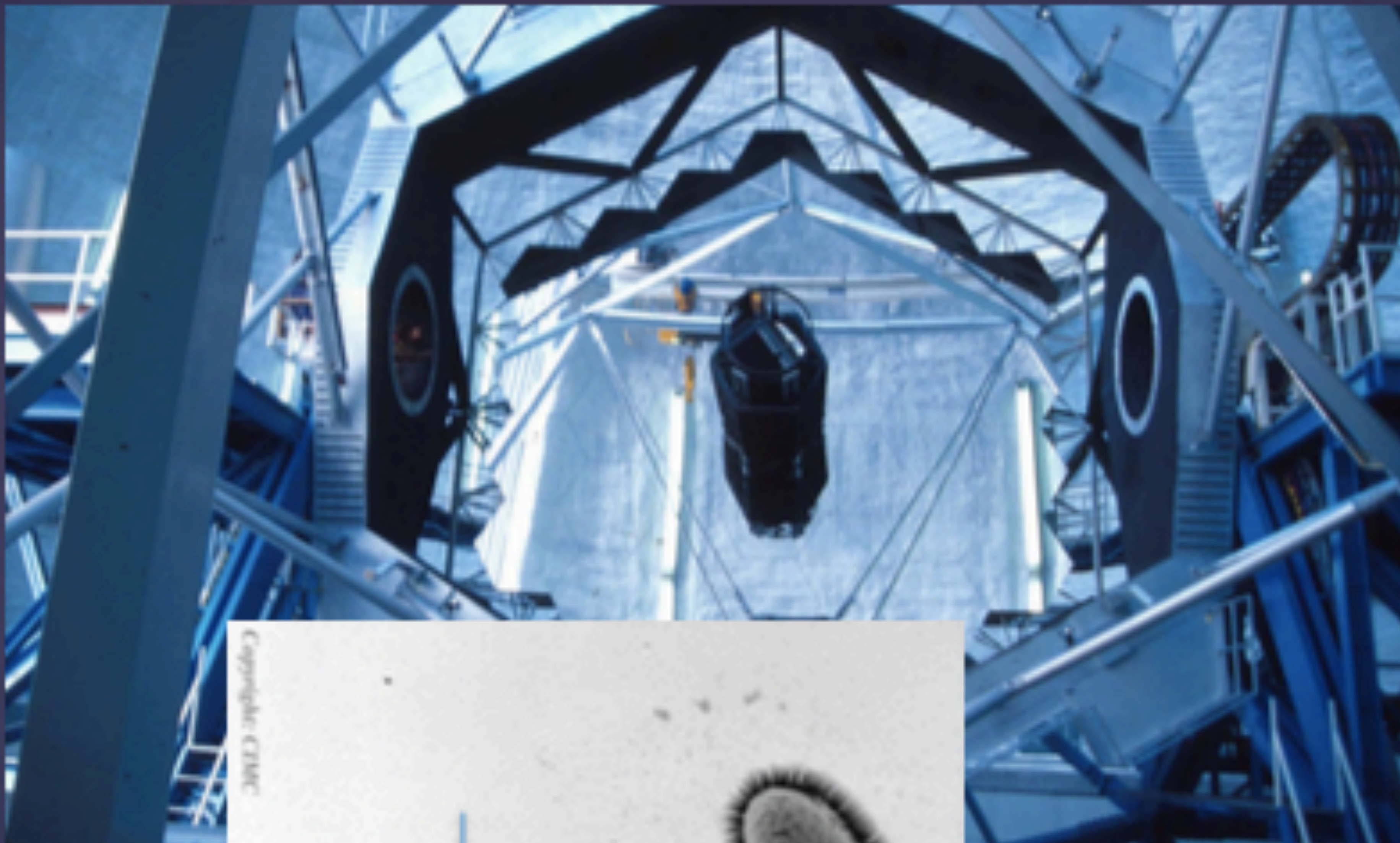


Optical: λ = 350-750nm



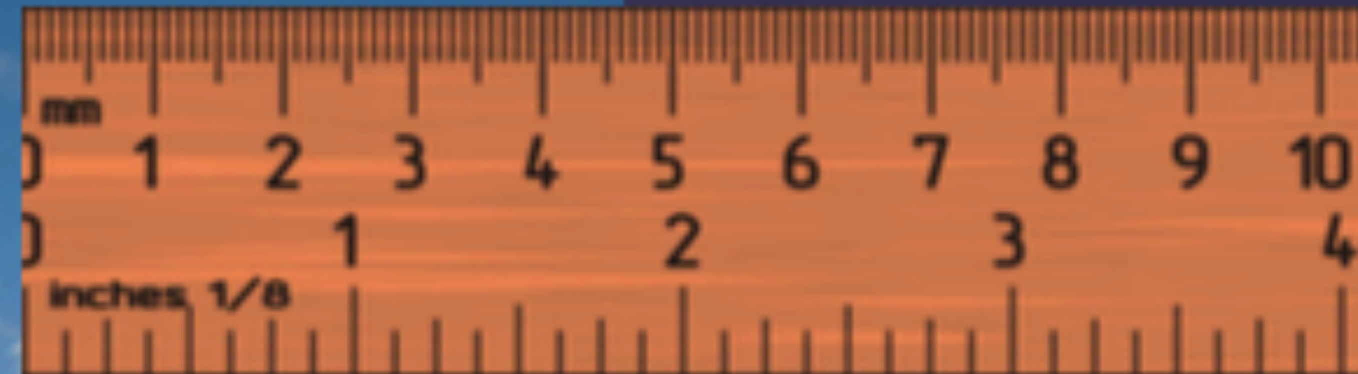


Optical: λ = 350-750nm

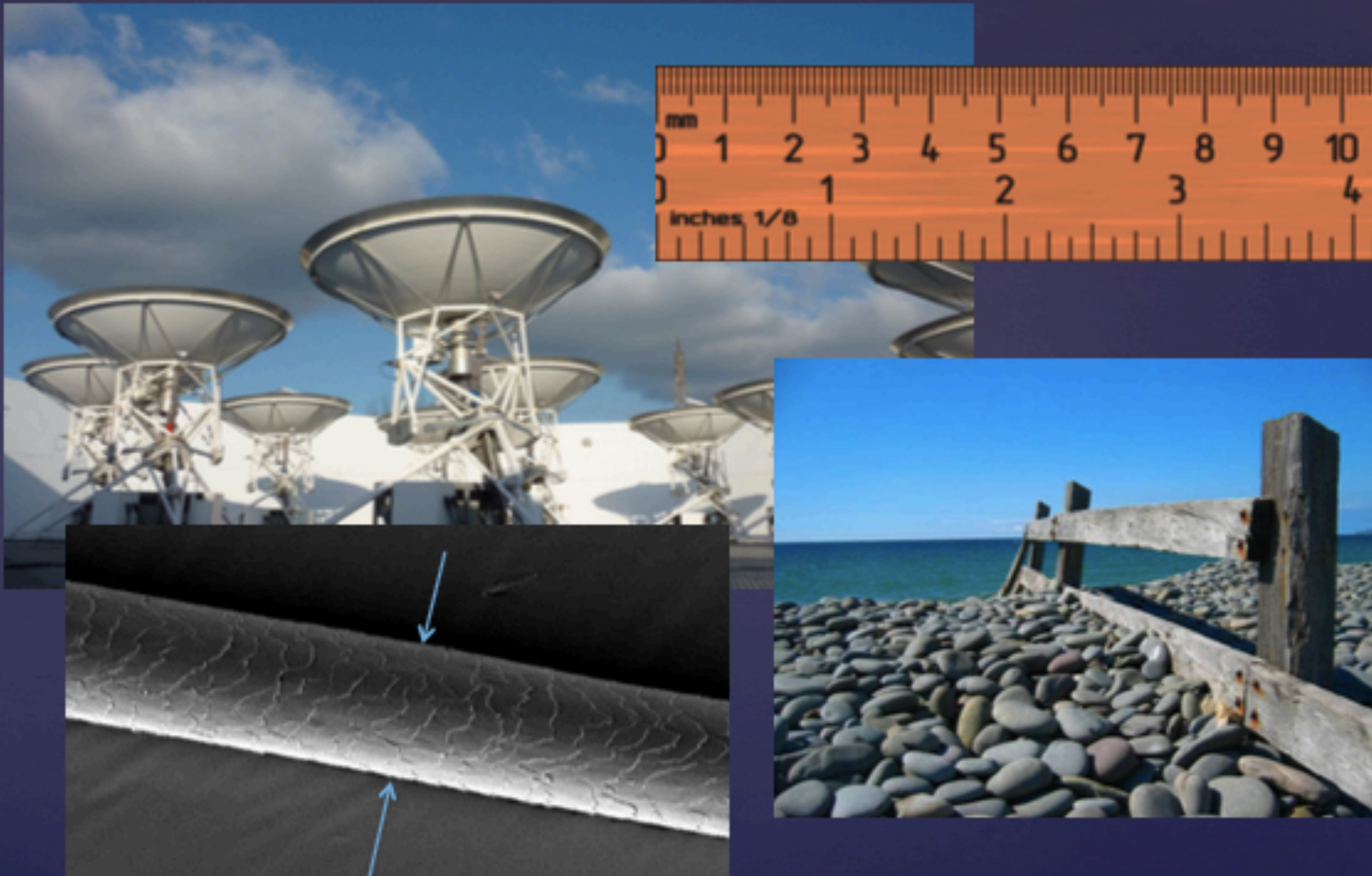


Optical: λ = 350-750nm

High Frequency Radio: $\lambda = 1 - 21$ cm



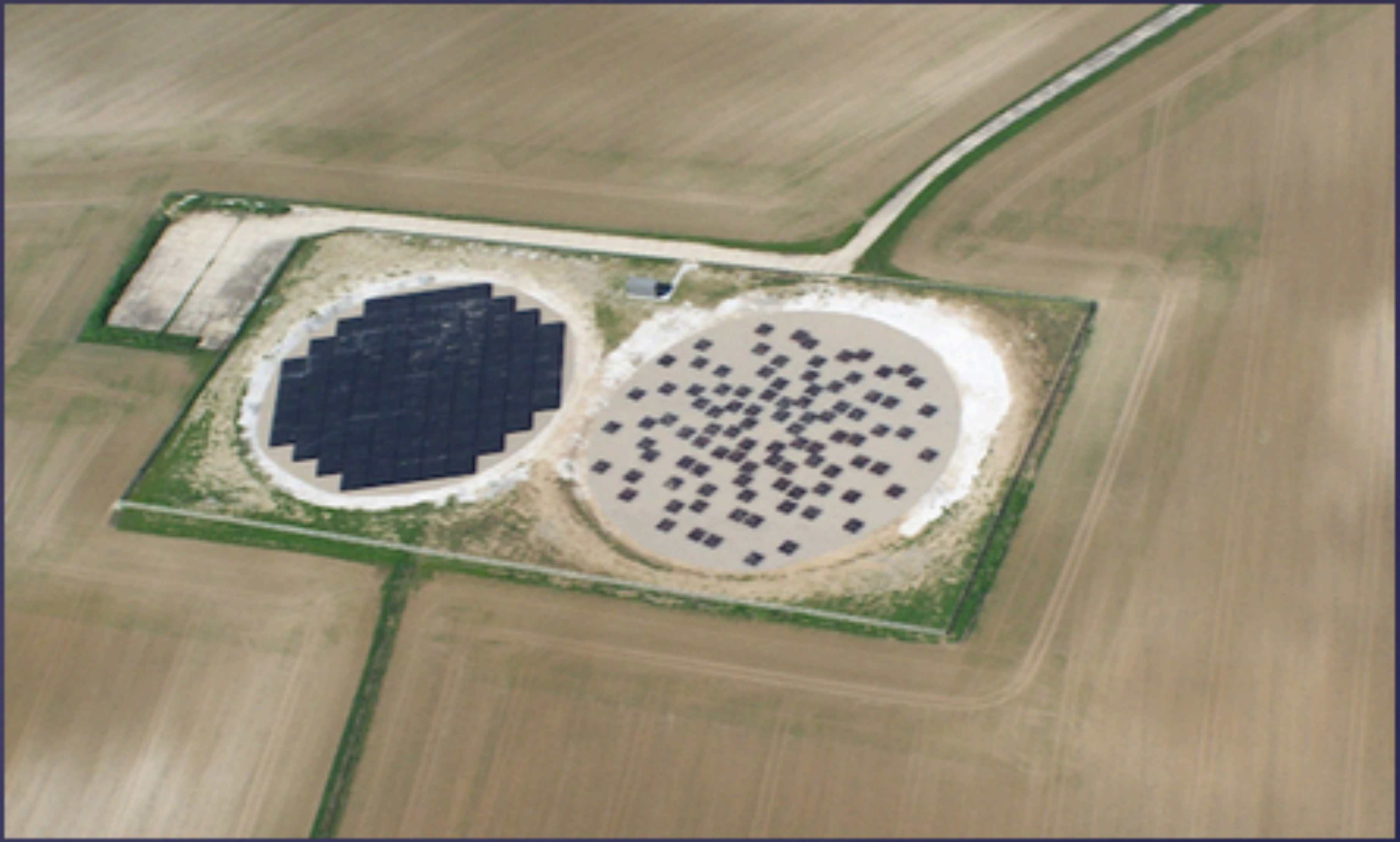
High Frequency Radio: $\lambda = 1 - 21$ cm



High Frequency Radio: $\lambda = 1 - 21$ cm

Lower Radio Frequency…

LOFAR-UK @
Chilbolton
Observatory



Low Radio Frequency!
$\lambda = 1\text{-}30\text{m}$

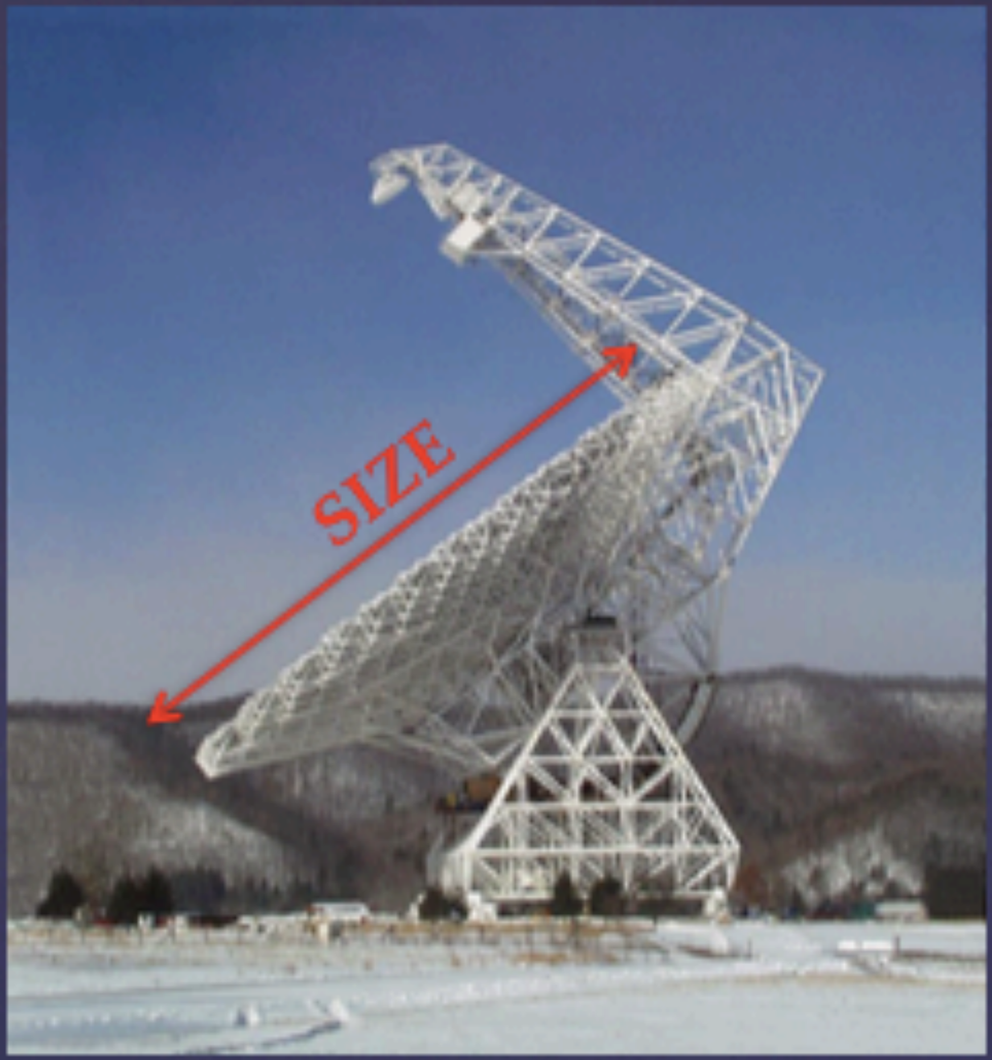
x 3

Low Radio Frequency!

λ = 1-30m

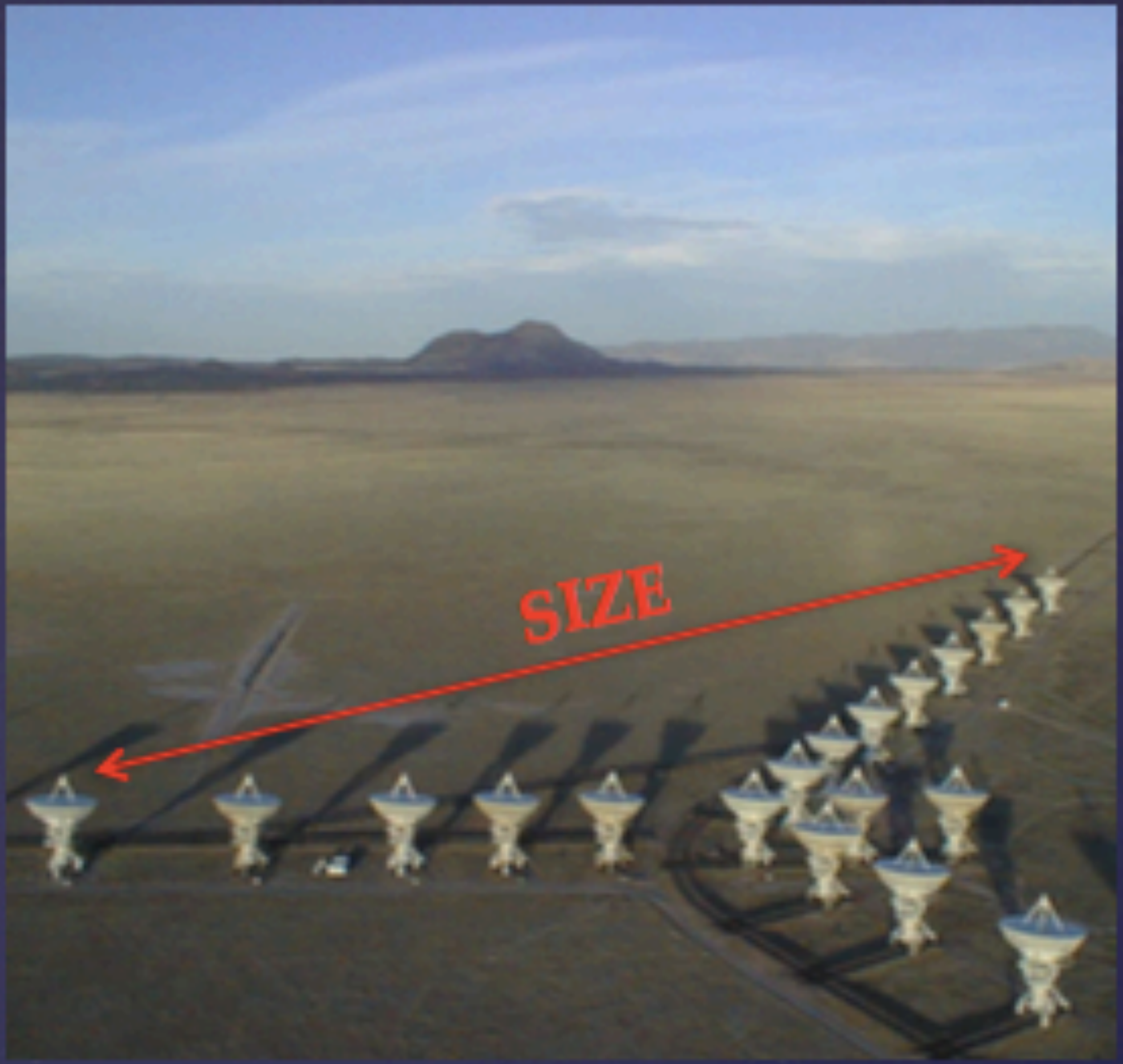
RESOLUTION:

$$\sim \frac{\lambda}{\mathrm{SIZE}}$$



Green Bank Telescope, West Virgina, USA

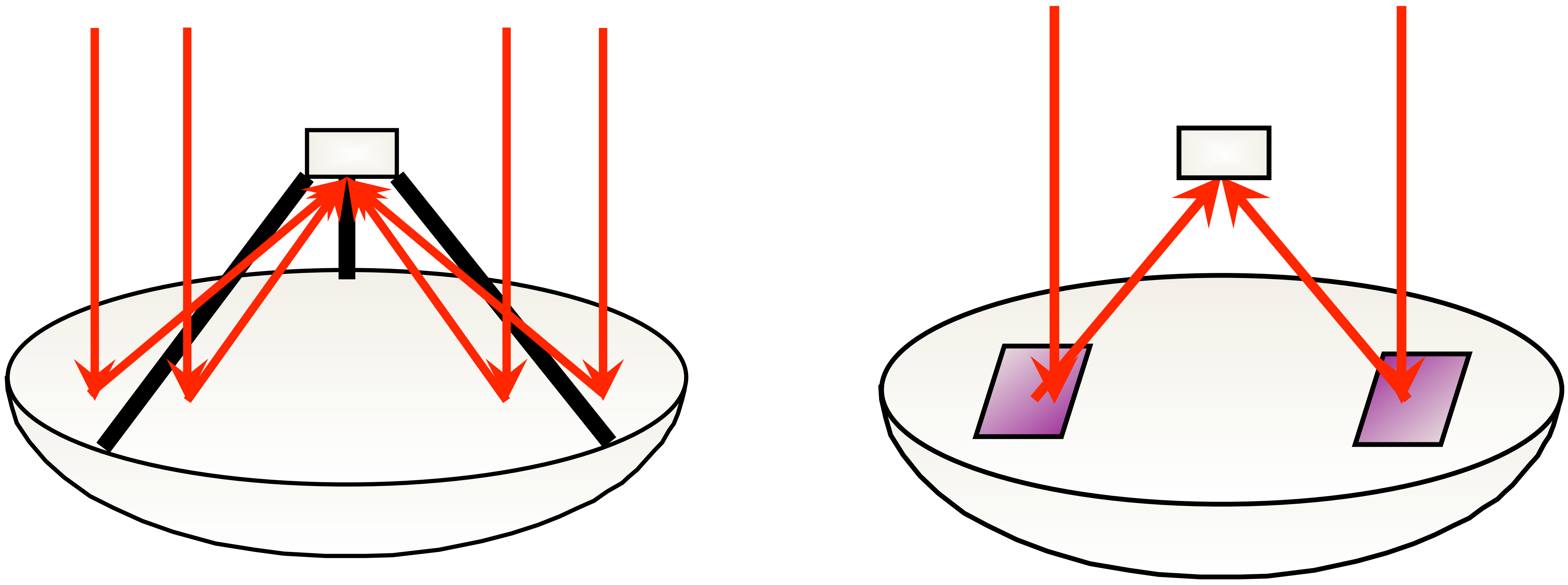
Single Dish

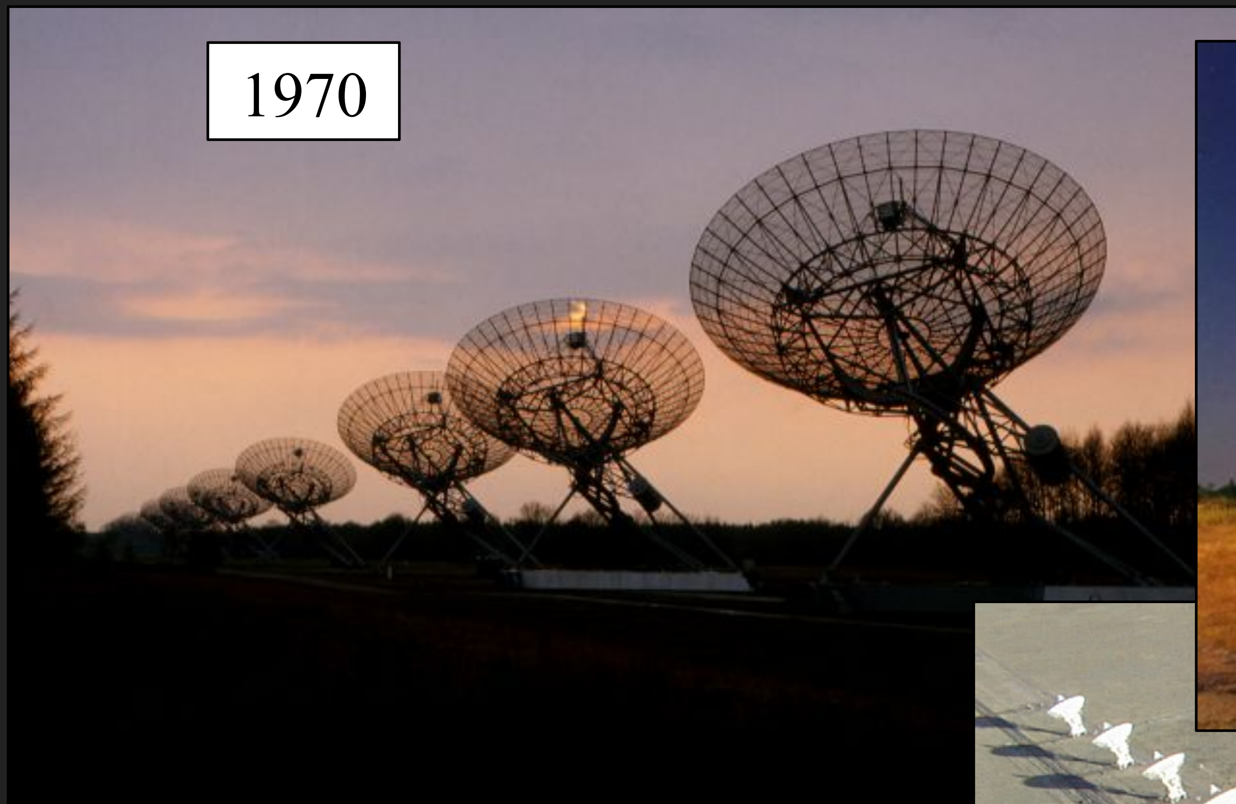


Very Large Array (VLA), New Mexico, USA

Array

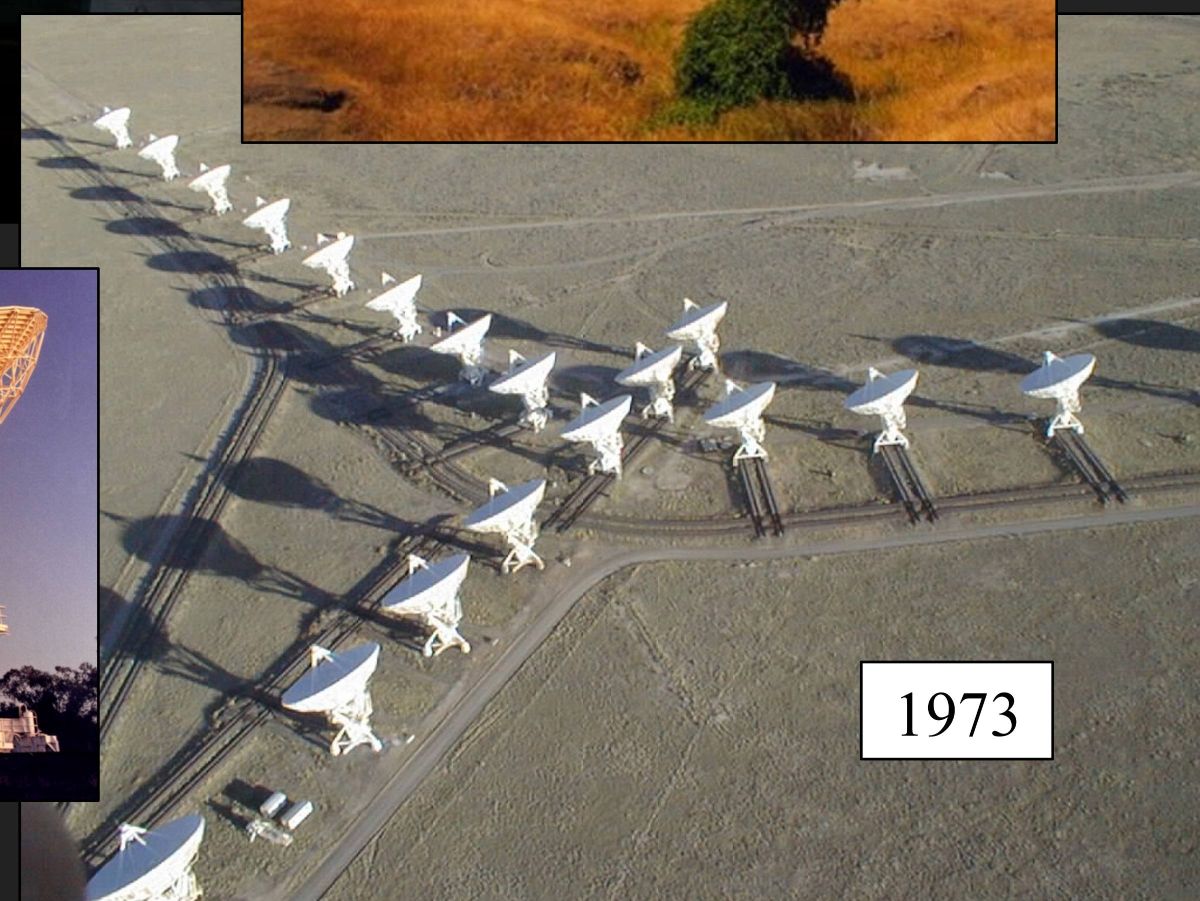
FILLED APERTURES



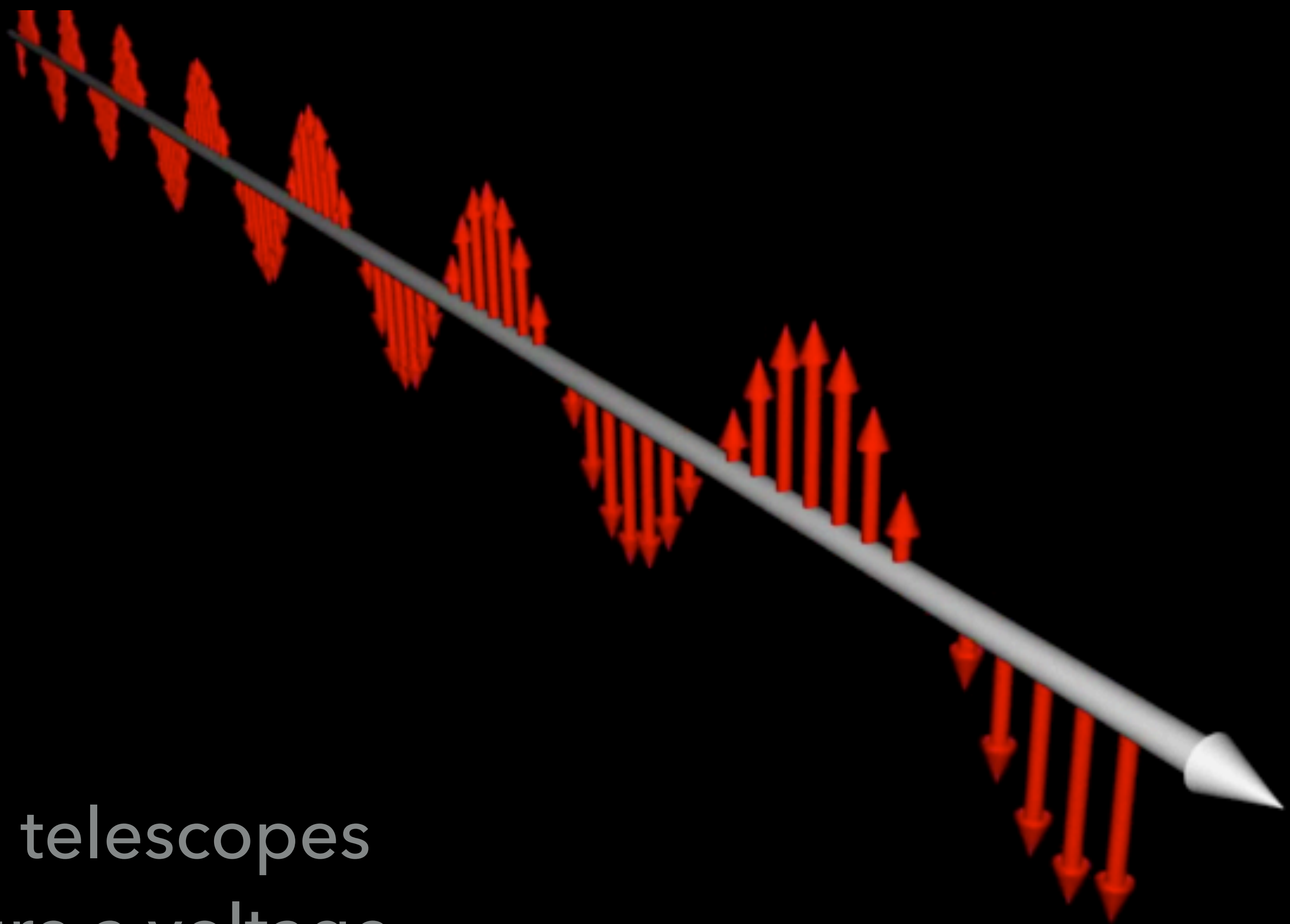

1970


1995


1988

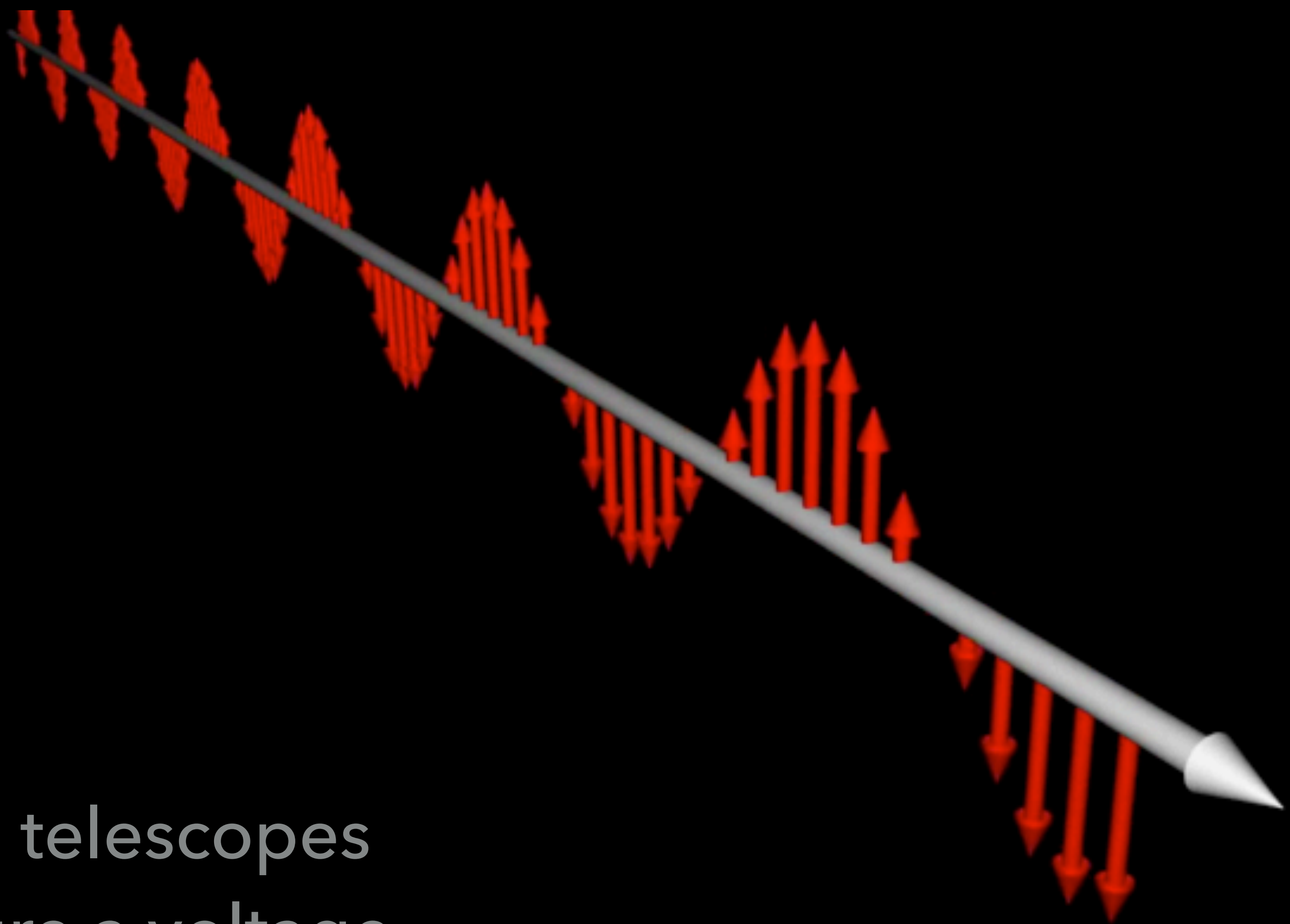

1973

Animation by FilmSense



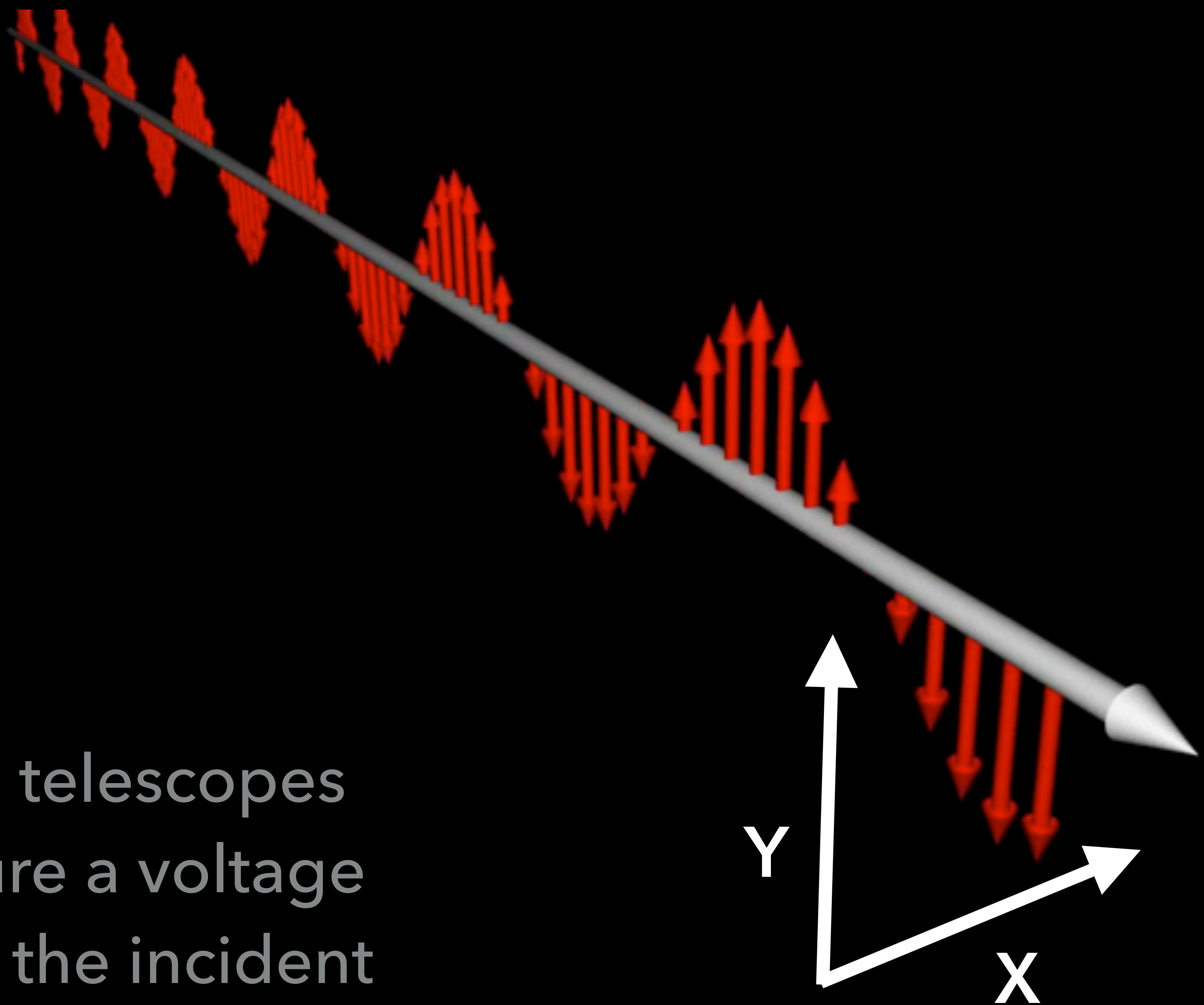
Radio telescopes measure a voltage due to the incident EM radiation

Animation by FilmSense



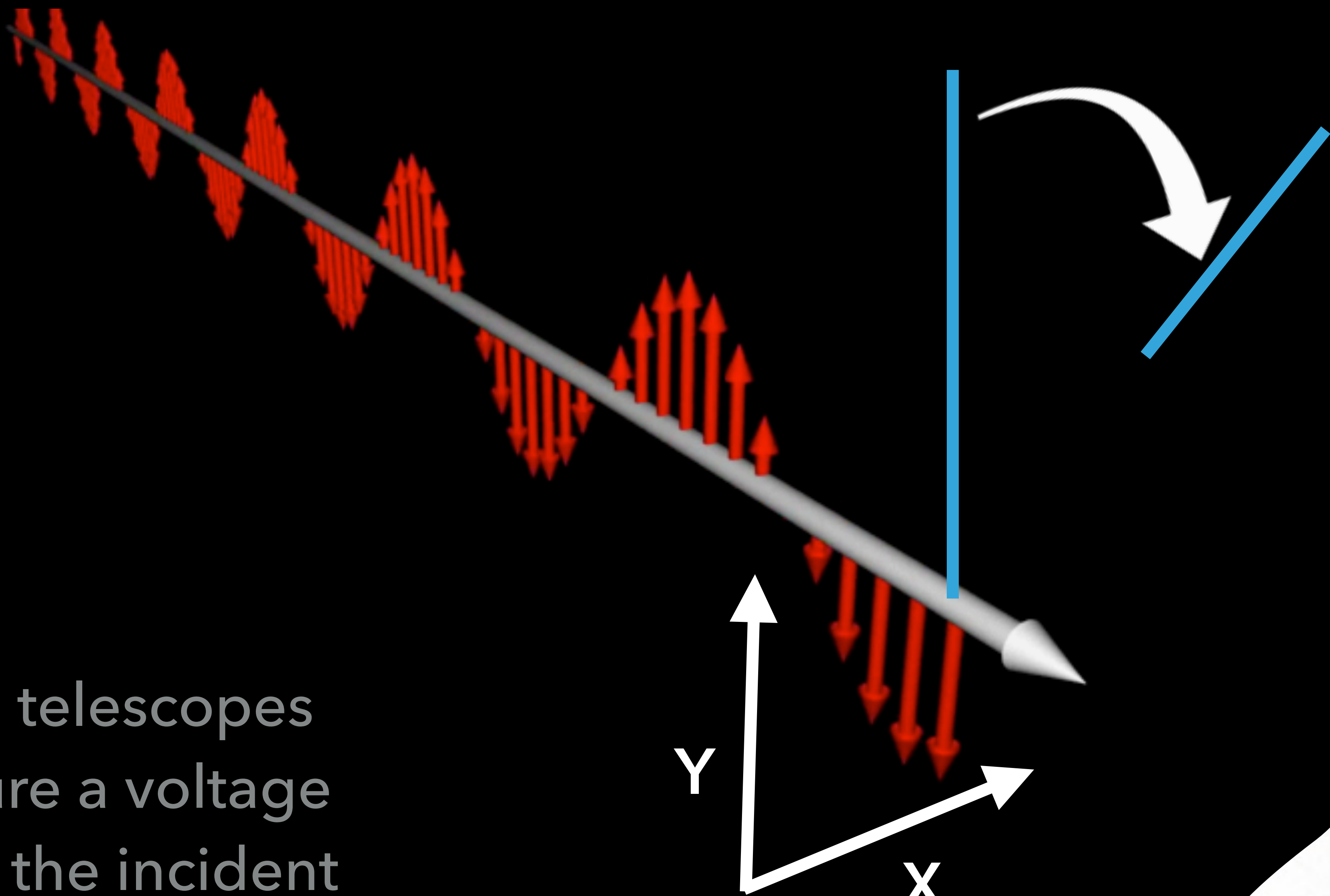
Radio telescopes measure a voltage due to the incident EM radiation

Animation by

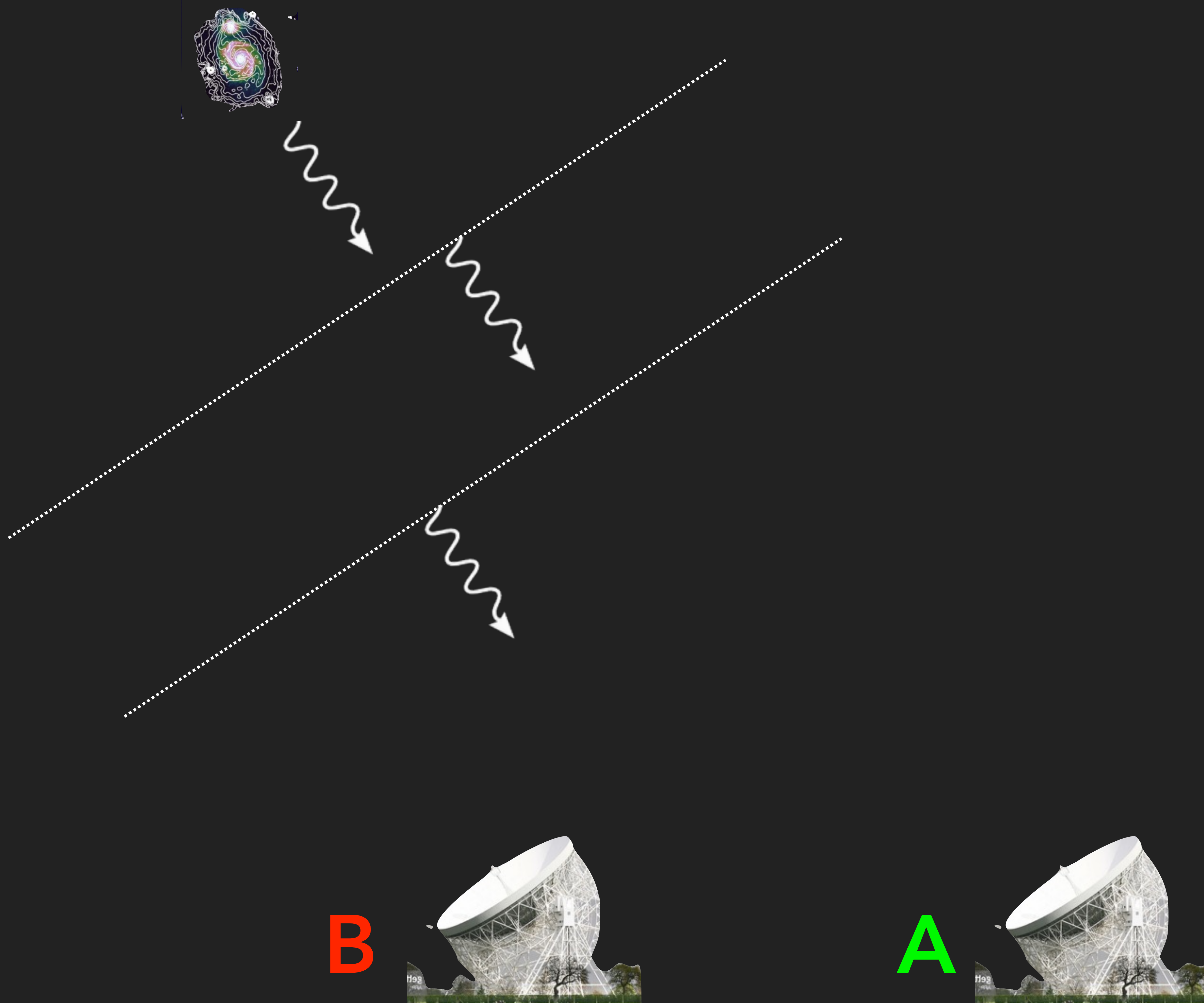


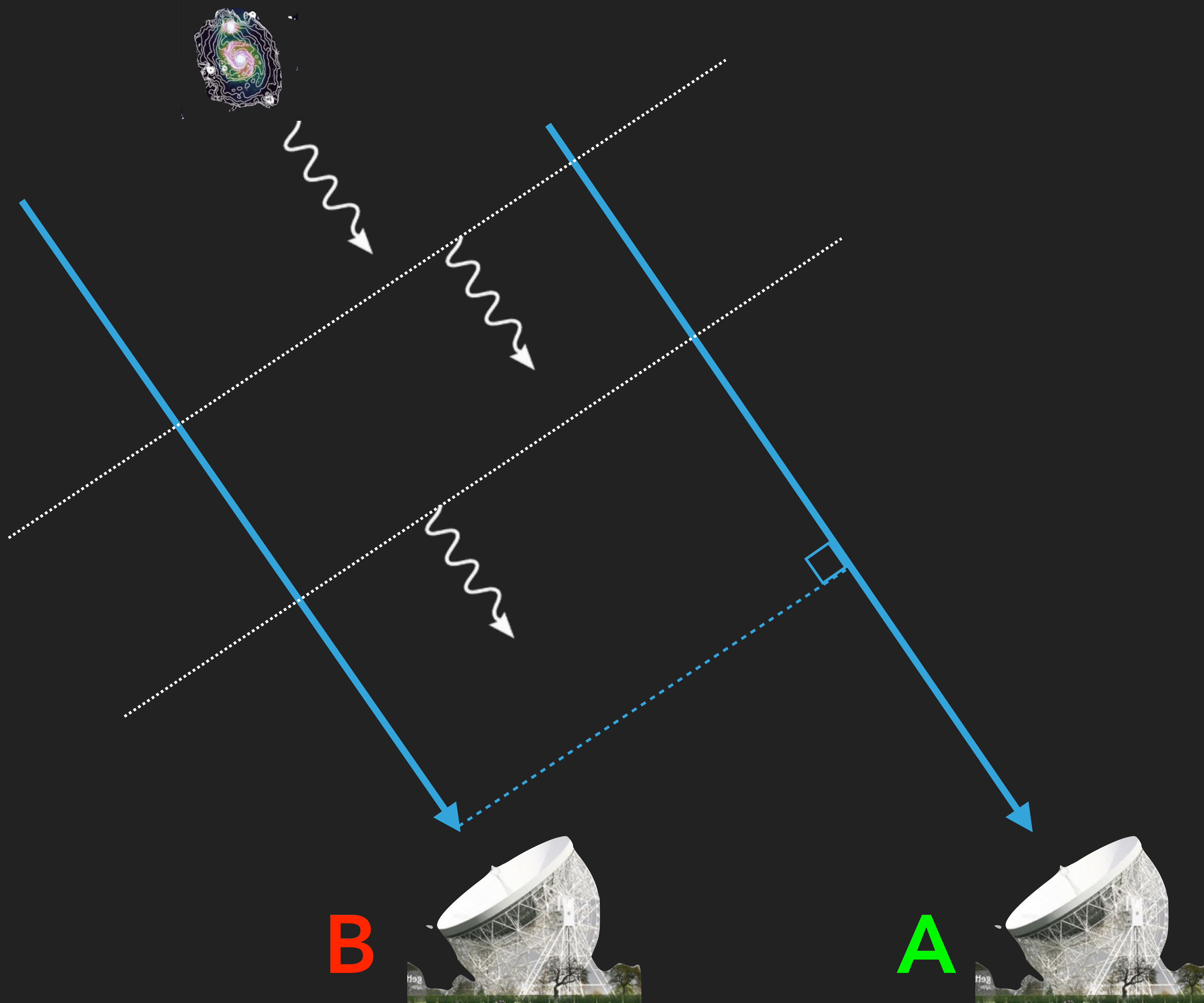
Radio telescopes measure a voltage due to the incident EM radiation

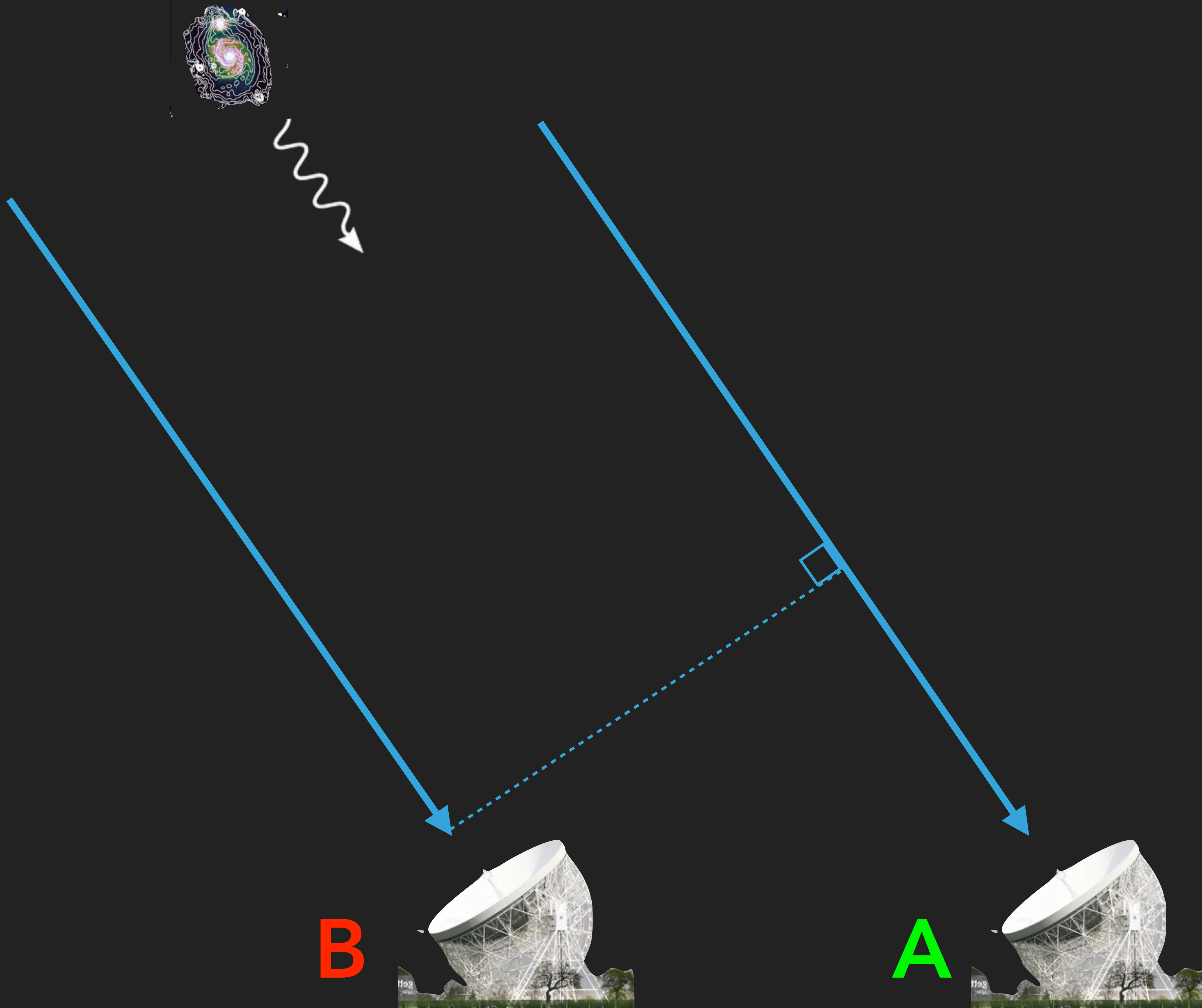
Animation by FilmSense

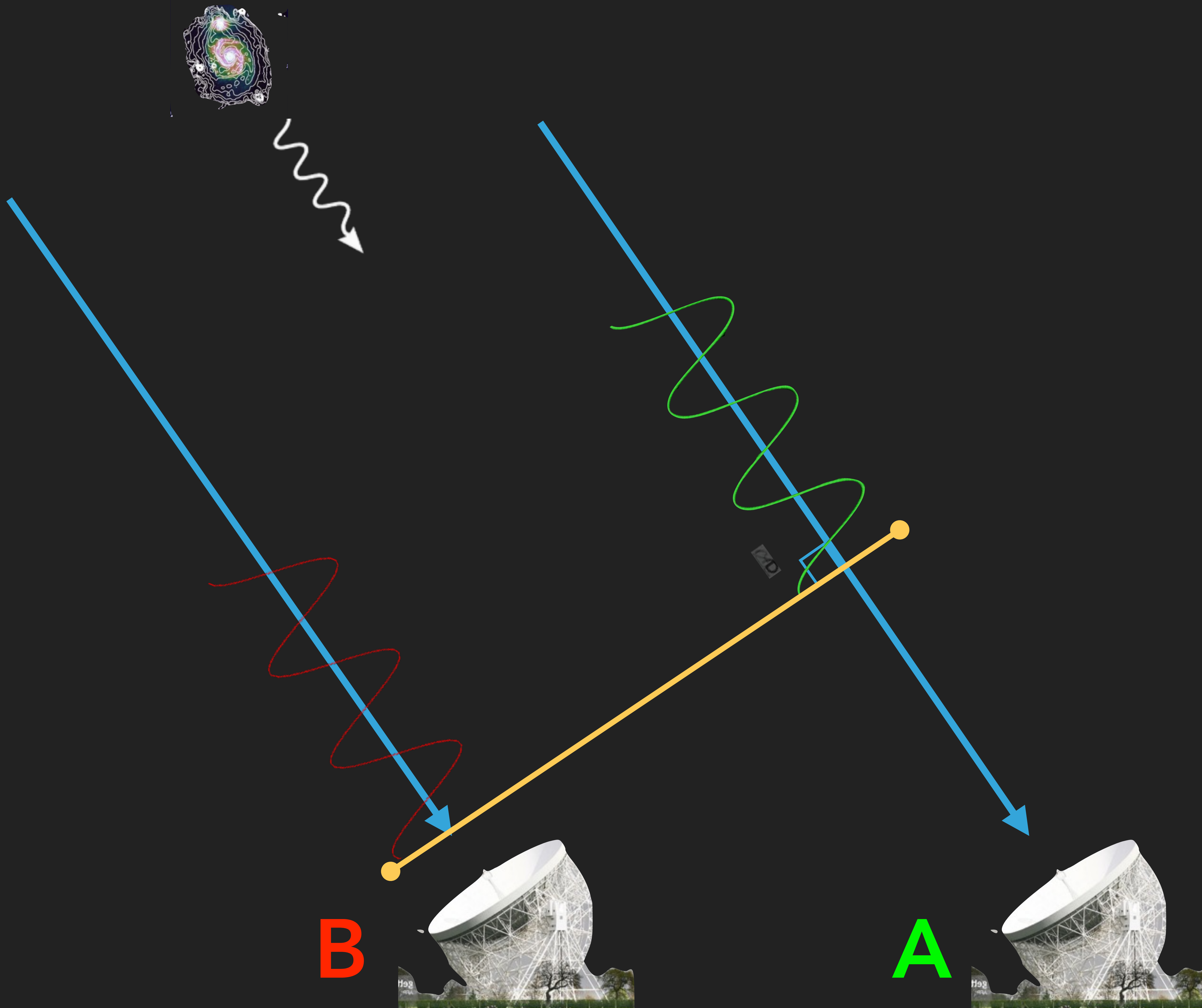


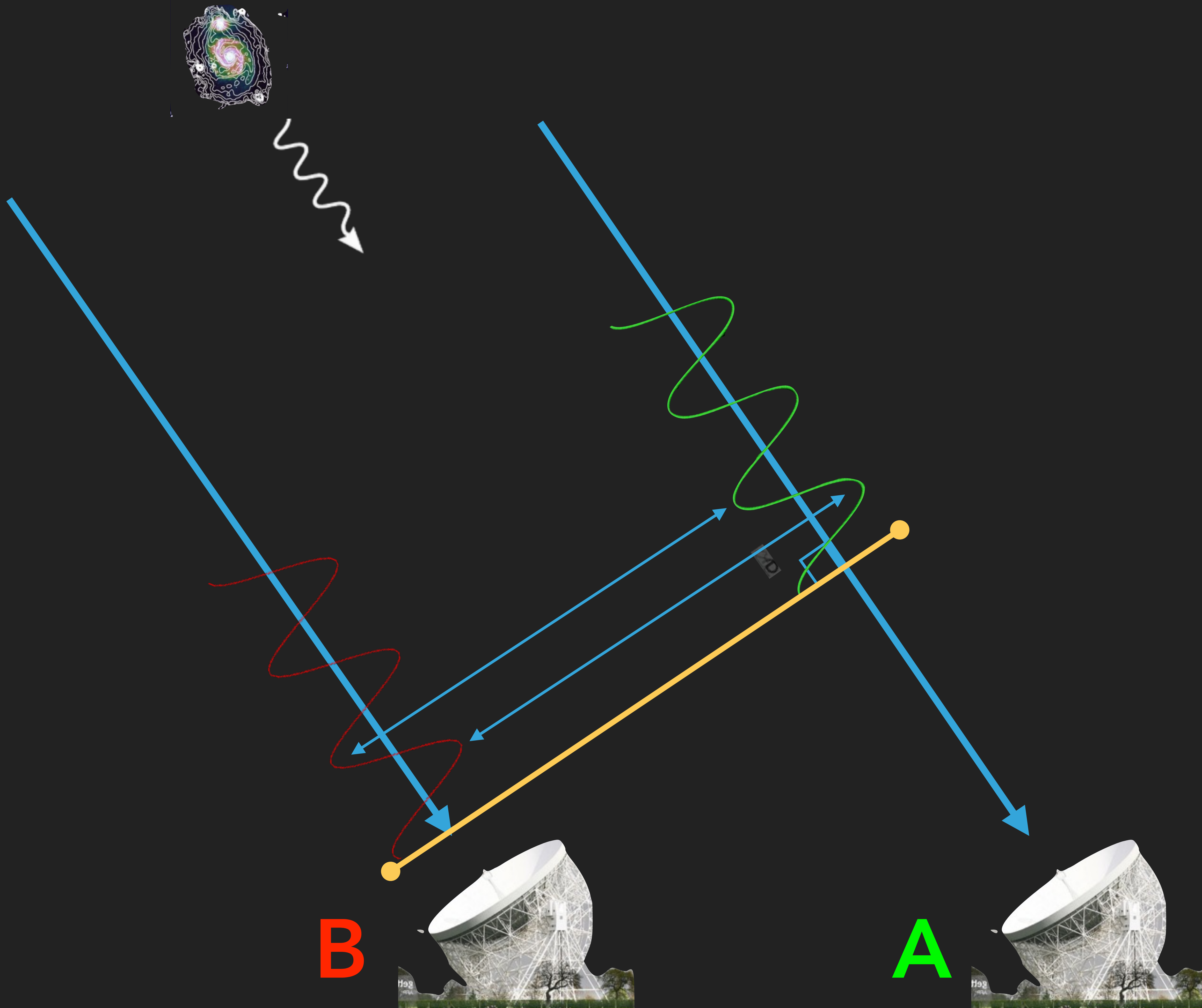
Radio telescopes measure a voltage due to the incident EM radiation

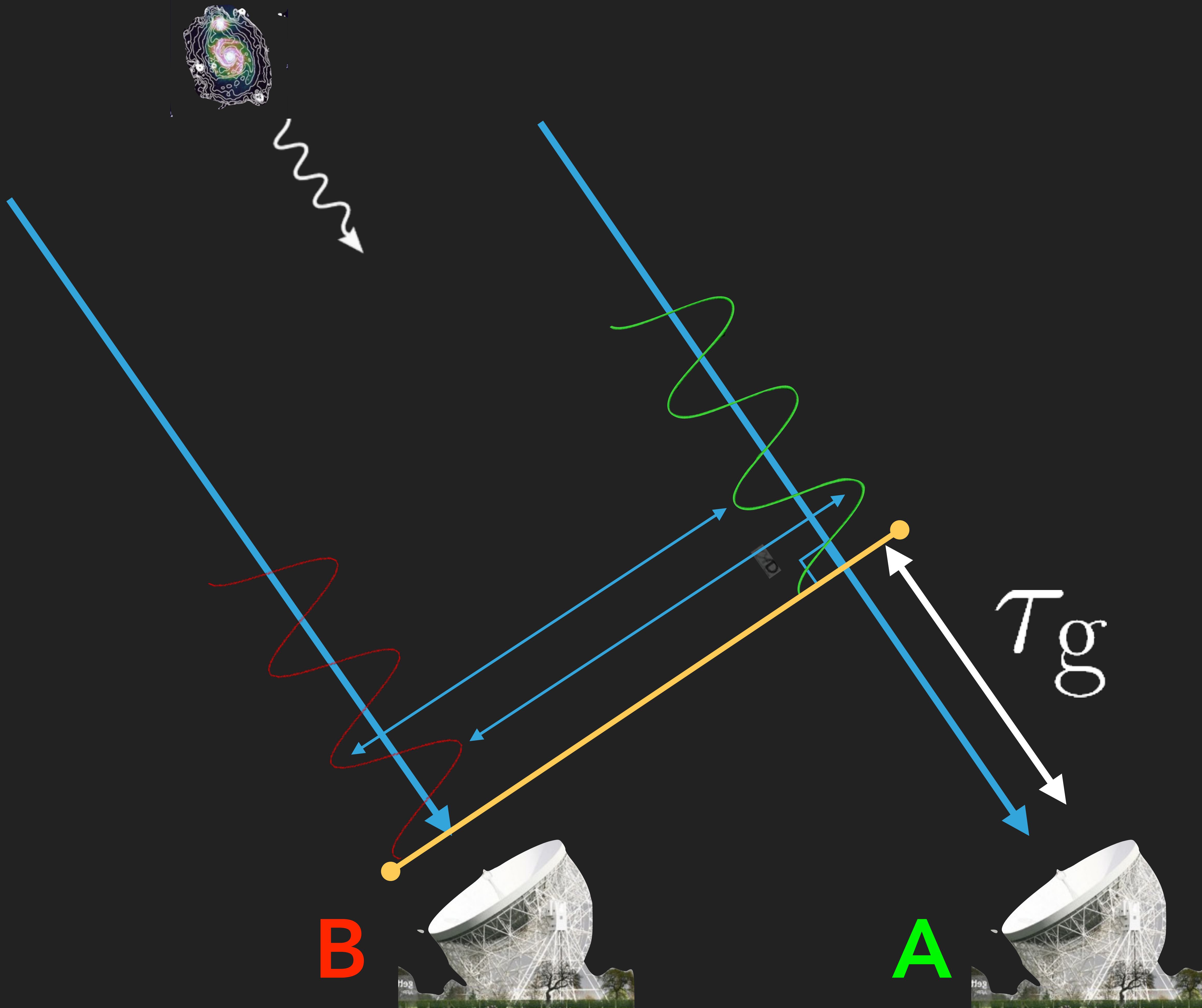

B
A


B
A

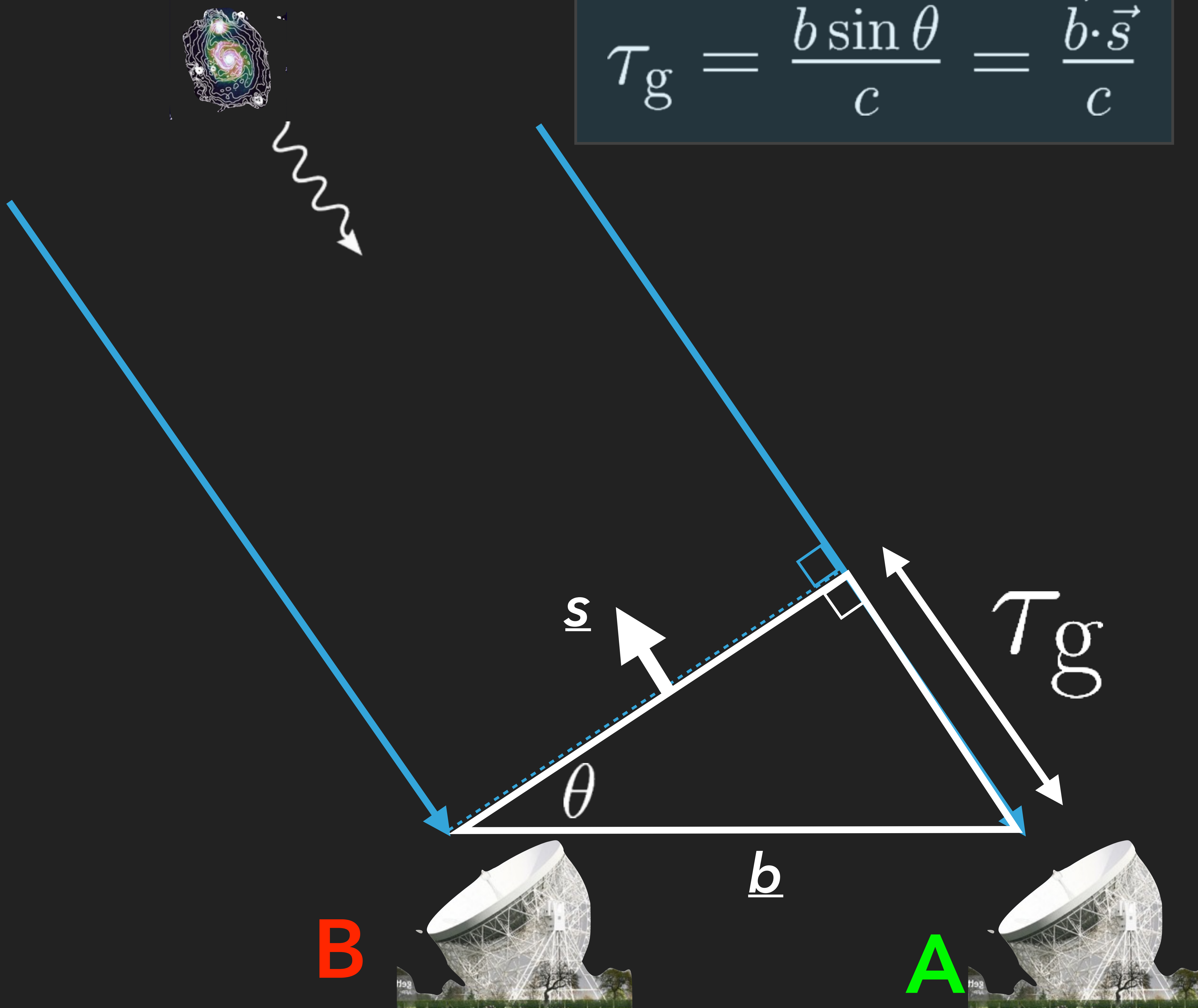

B
A

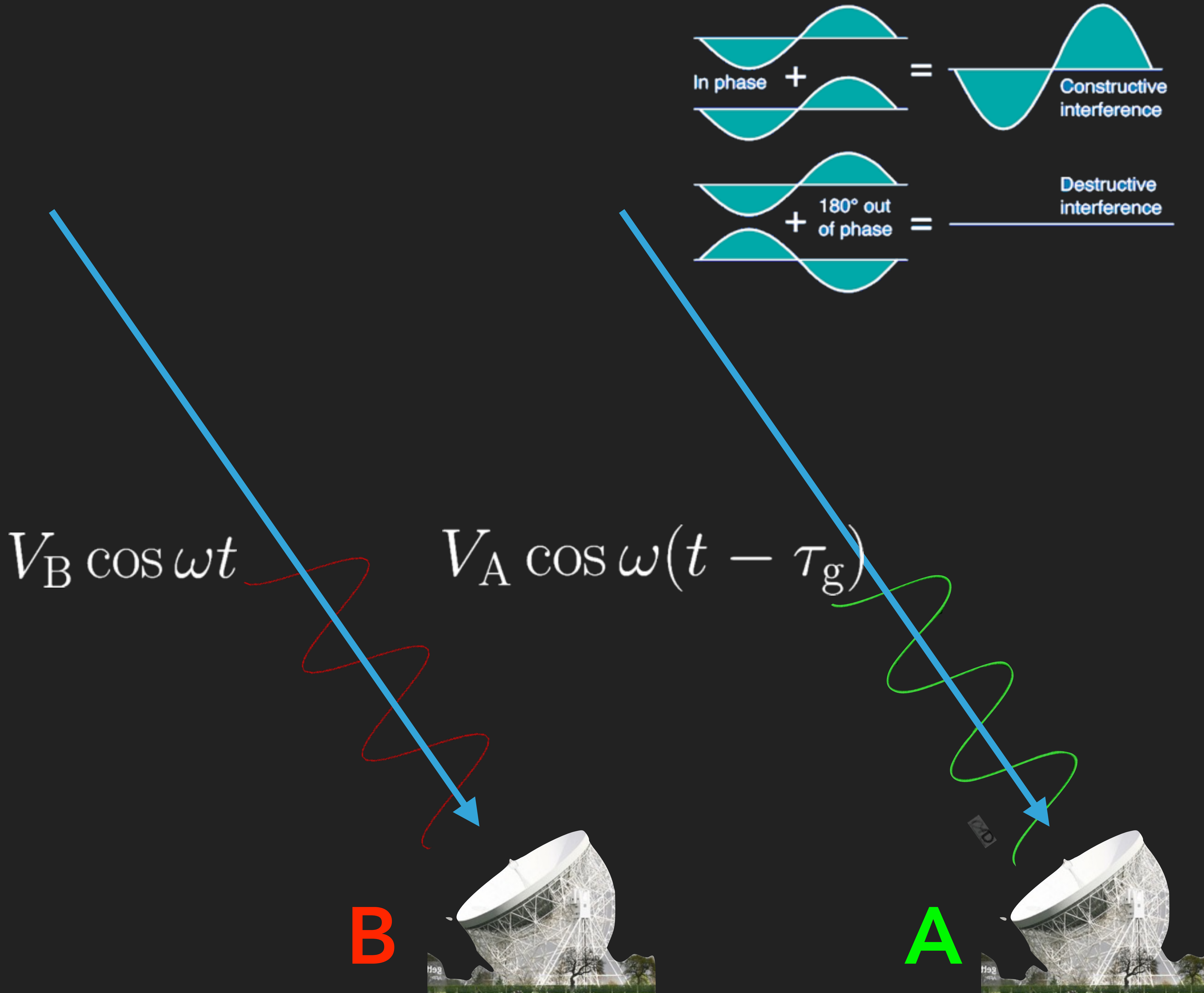

B
A


B
A

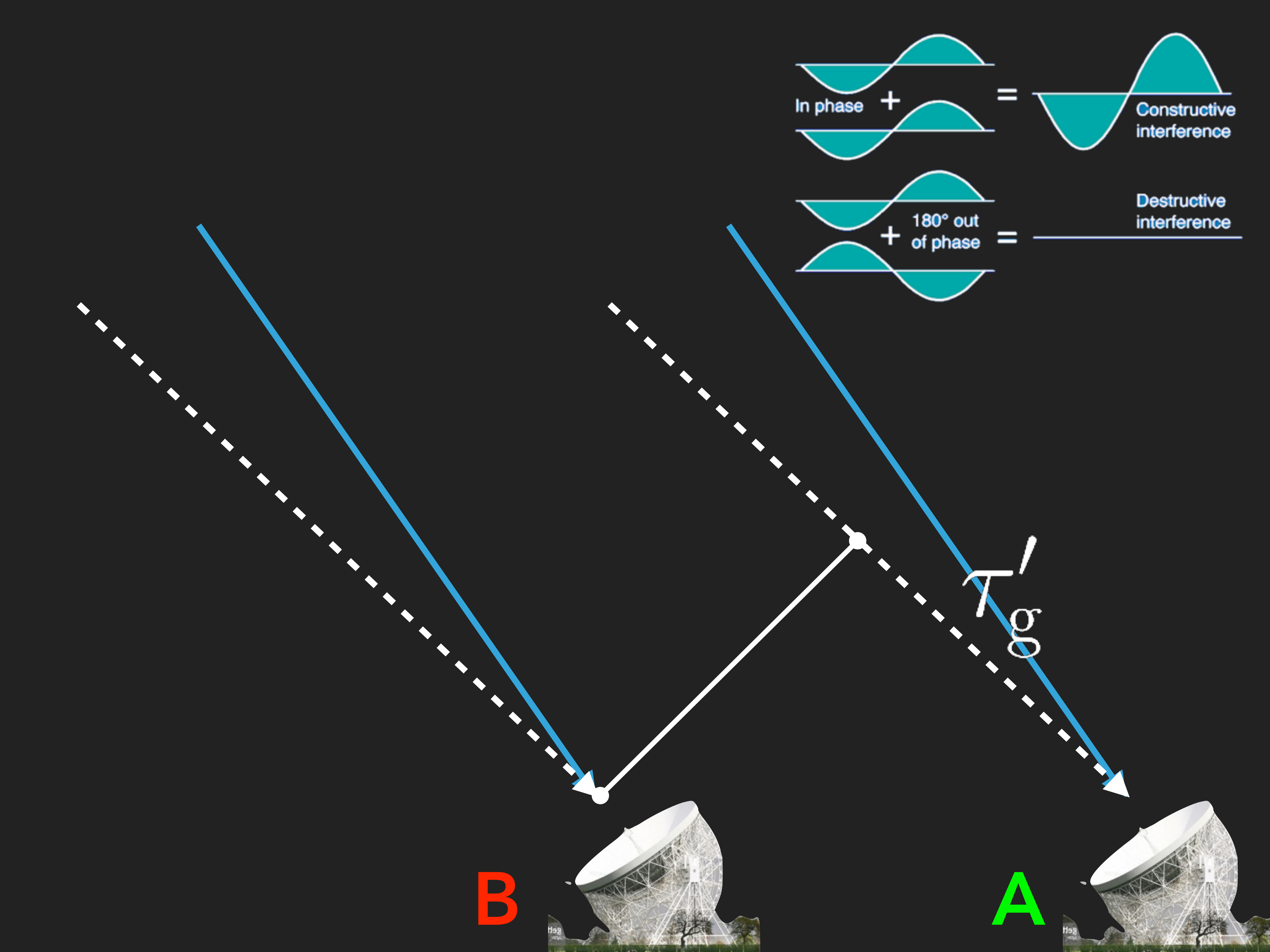

τ_g
B
A

$$\tau_g = \frac{b \sin\theta}{c} = \frac{\vec{b} \cdot \vec{s}}{c}$$

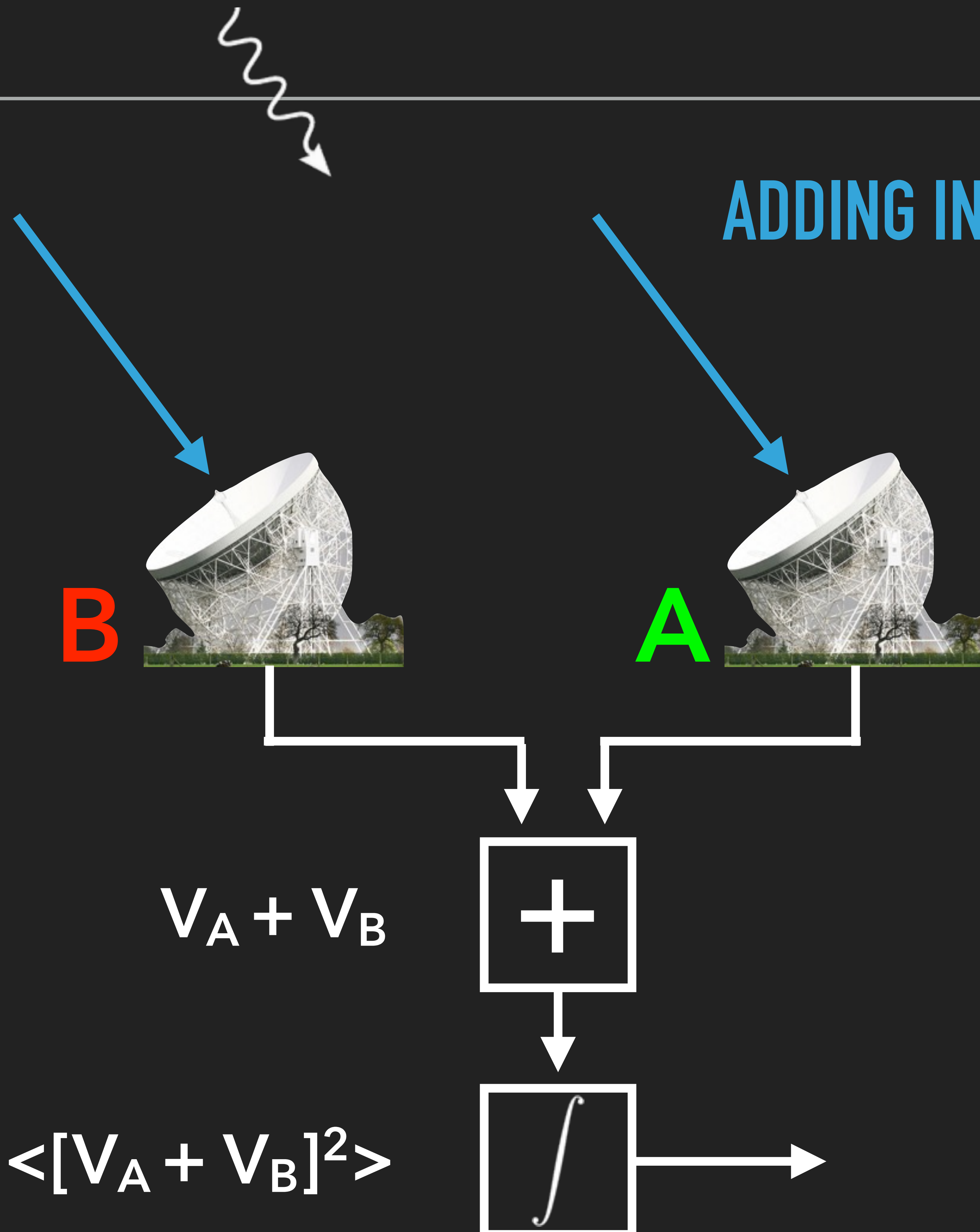




In phase
Constructive interference
180° out of phase
Destructive interference
$V_{\mathrm{B}} \cos \omega t$
$V_{\mathrm{A}} \cos \omega (t - \tau_{\mathrm{g}})$
B
A


In phase
Constructive interference
180° out of phase
Destructive interference
τ_g'
B
A

ADDING INTERFEROMETER



ADDING INTERFEROMETER

$$R \propto \langle [V_\mathrm{A} \cos \omega(t - \tau_\mathrm{g}) + V_\mathrm{B} \cos(\omega t) + V_\mathrm{rec,A} + V_\mathrm{rec,B}]^2 \rangle$$

$$R \propto \frac{1}{2} \langle [V_\mathrm{A}^2 + V_\mathrm{B}^2] + [V_\mathrm{rec,A}^2 + V_\mathrm{rec,B}^2] + [V_\mathrm{A} V_\mathrm{B} \cos(\omega \tau_\mathrm{g})] \rangle$$

1 — Antenna noise powers - usually small

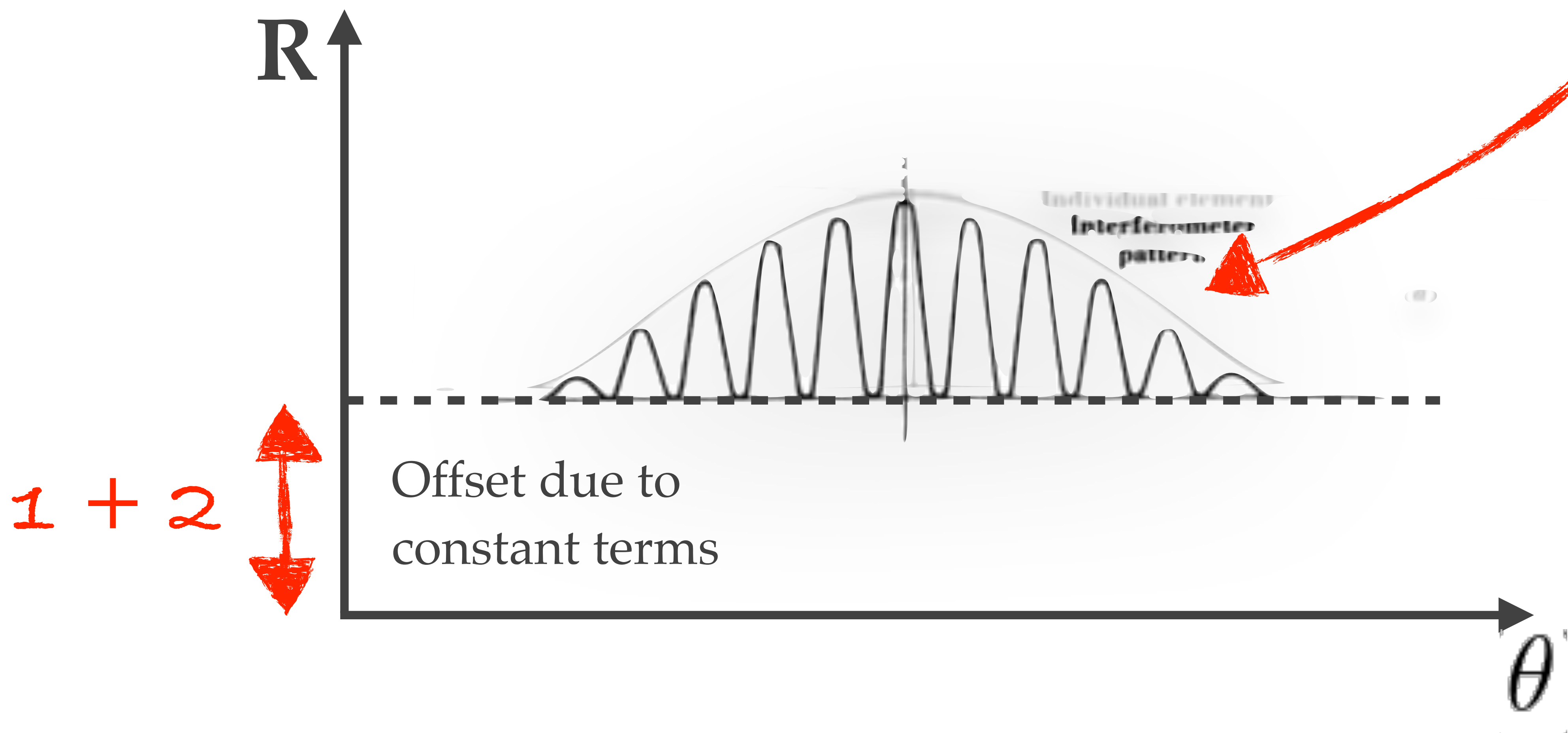
2 — Receiver noise powers - usually dominate

3 — Interference fringes vary with τ_g

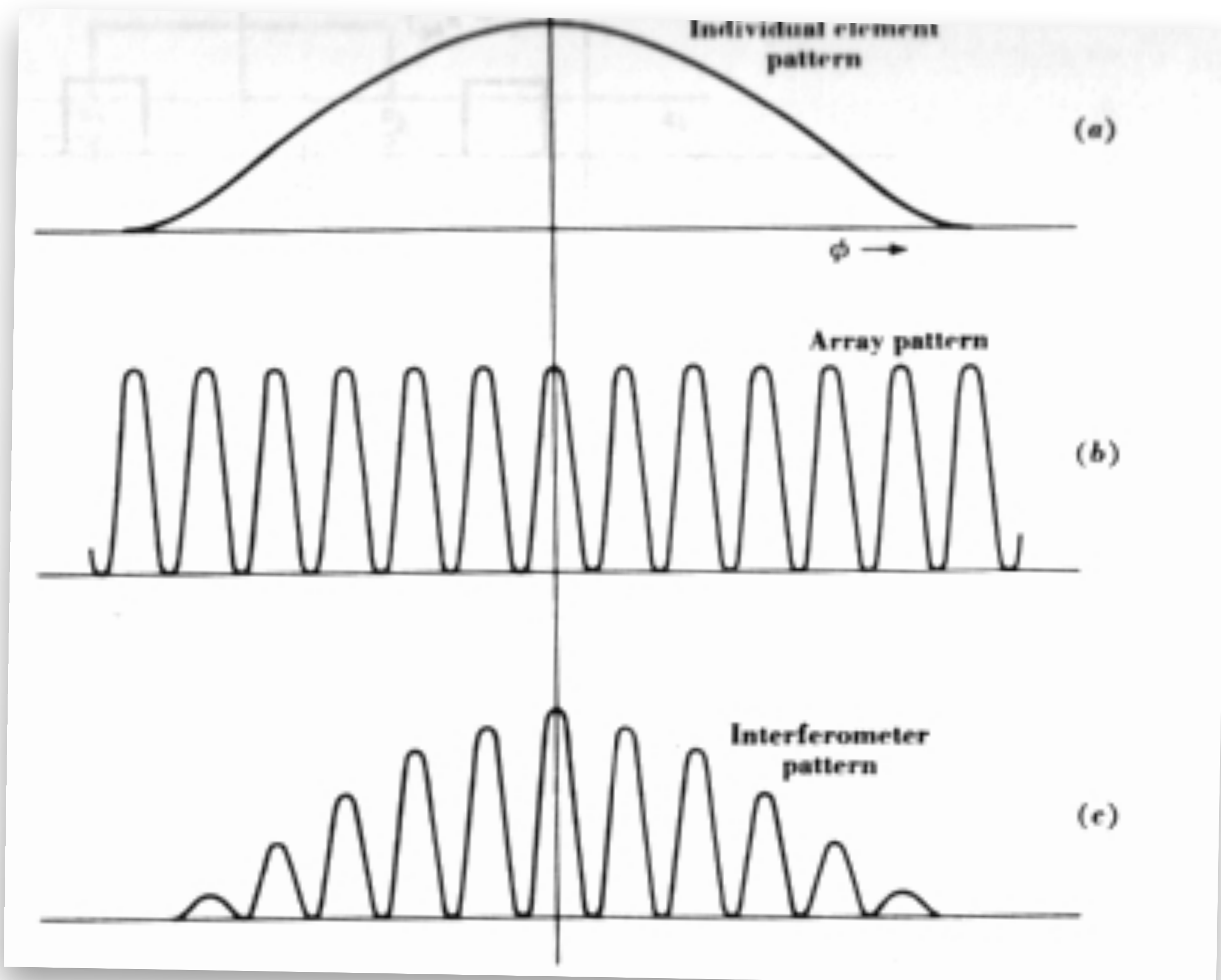
ADDING INTERFEROMETER

$$R \propto \frac{1}{2}\langle [V_A^2 + V_B^2] + [V_{rec,A}^2 + V_{rec,B}^2] + [V_A V_B \cos(\omega\tau_g)] \rangle$$

1 2 3



ADDING INTERFEROMETER

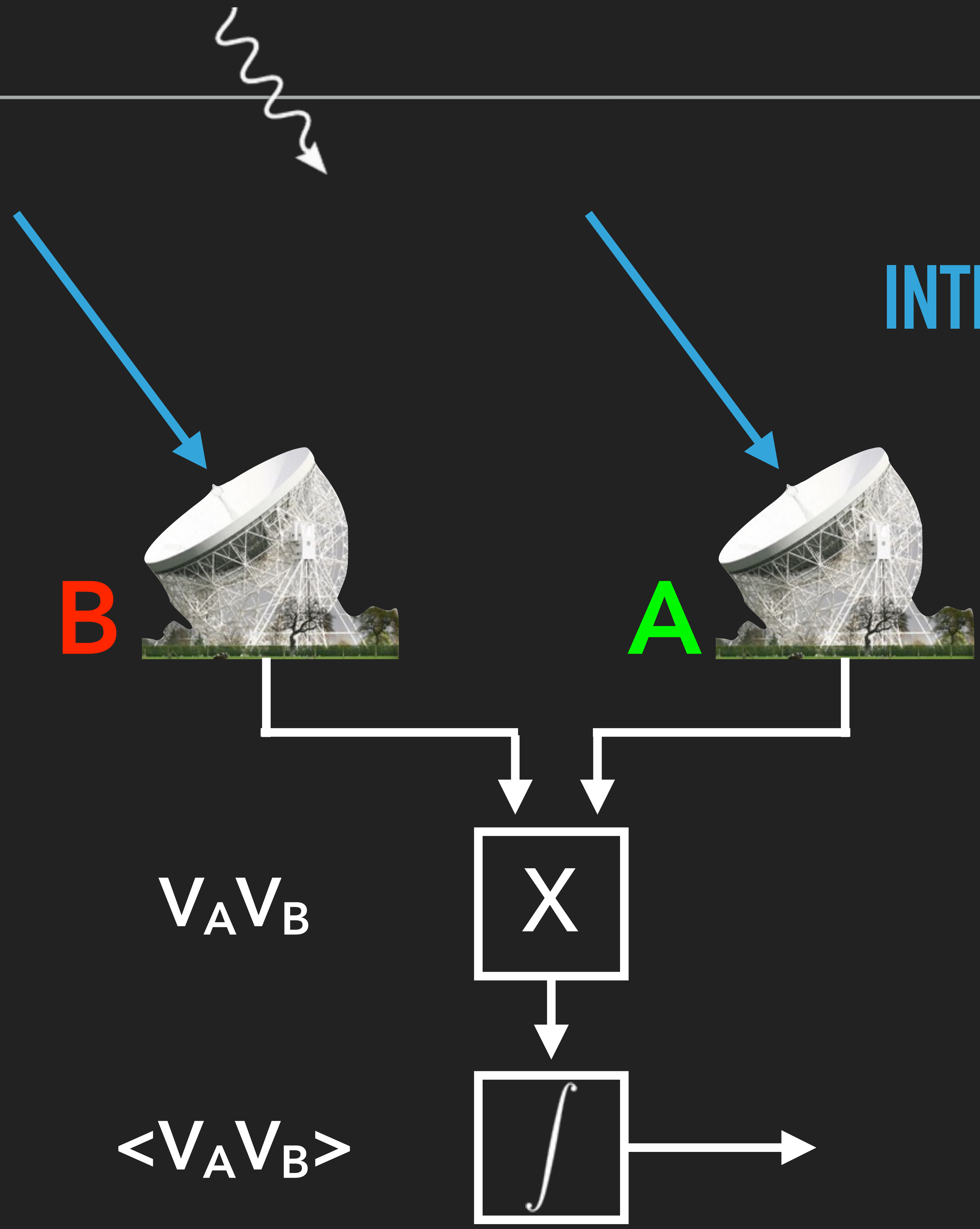


Envelope due to primary beam $\frac{\lambda}{D}$

Fringes due to interference pattern $\frac{\lambda}{b}$

Combination

MULTIPLYING INTERFEROMETER

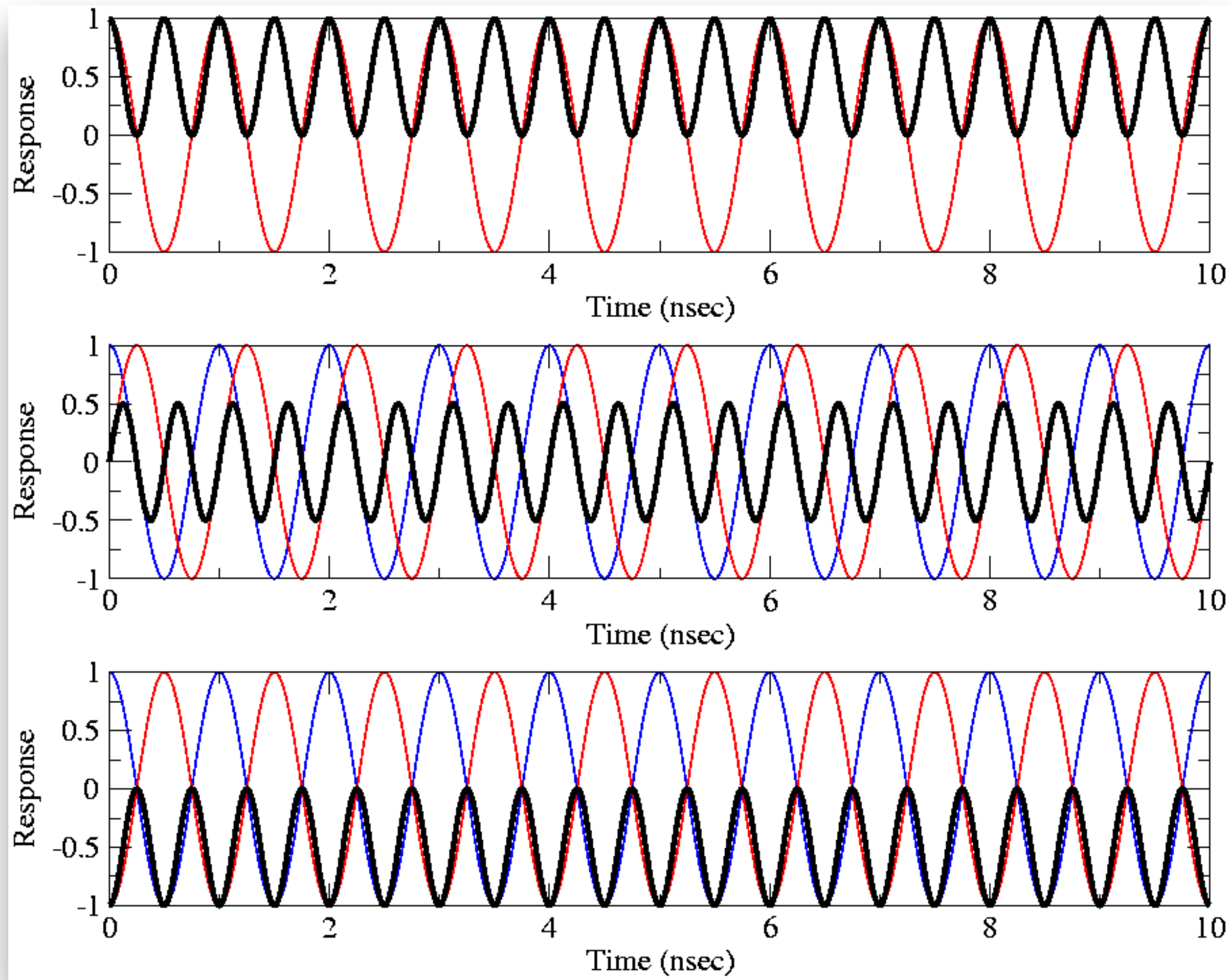


MULTIPYING INTERFEROMETER

IN PHASE

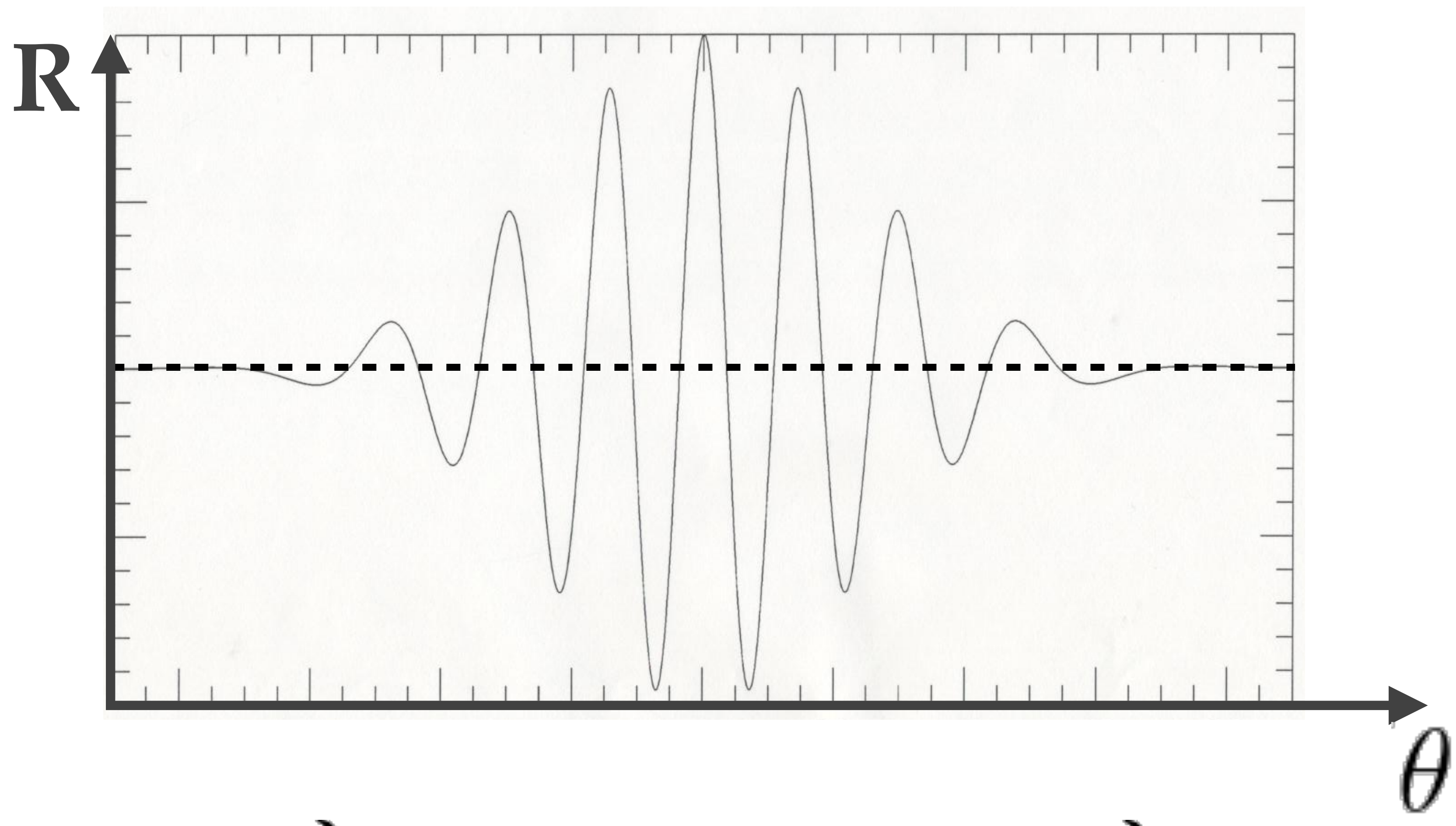
QUADRATURE PHASE

OUT OF PHASE



MULTIPLYING INTERFEROMETER

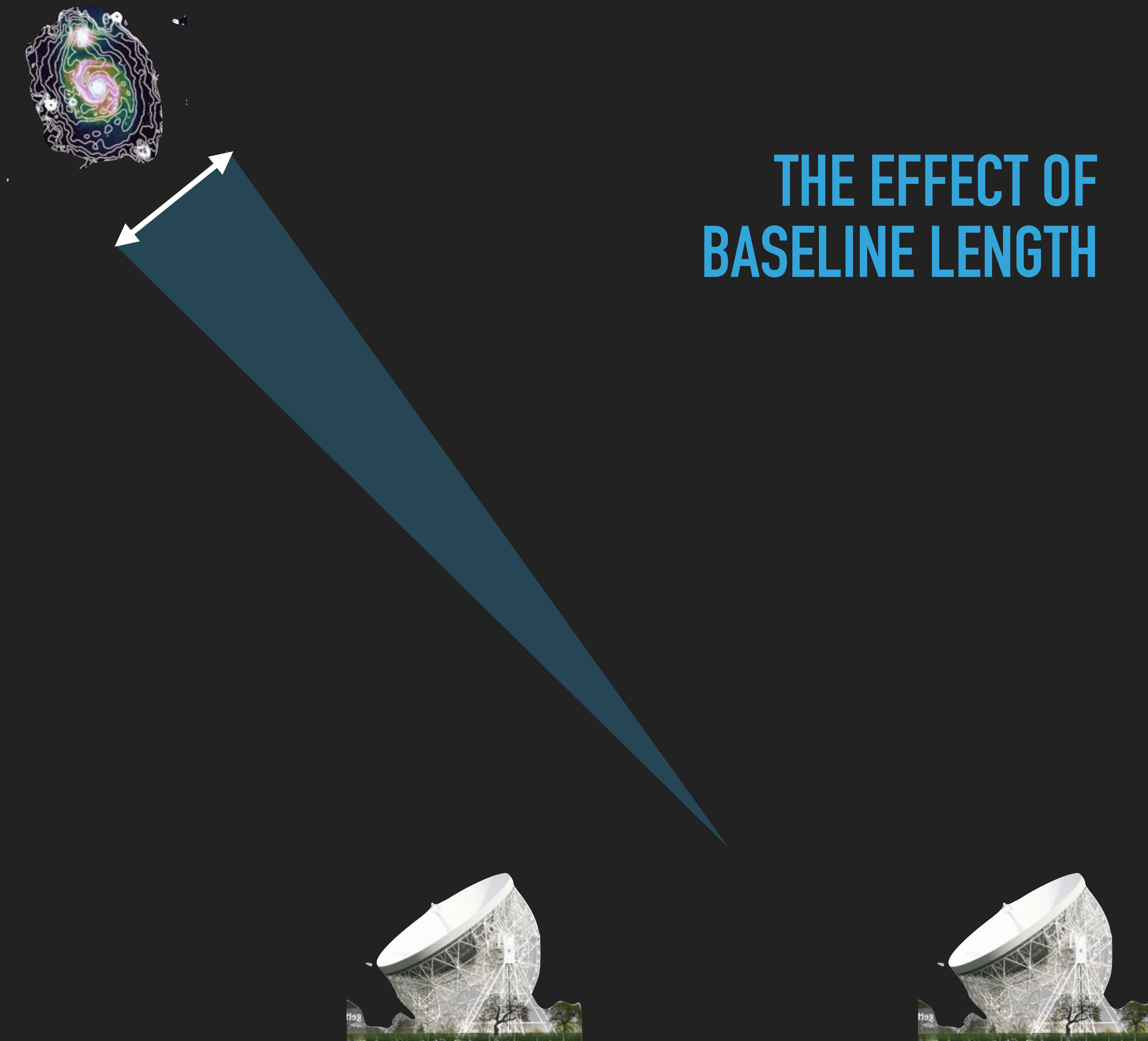
$$R \propto \langle V_{\rm A} \cos \omega(t - \tau_{\rm g}) \cdot V_{\rm B} \cos \omega t \rangle = \tfrac{1}{2} V_{\rm A} V_{\rm B} \cos \tau_{\rm g}$$

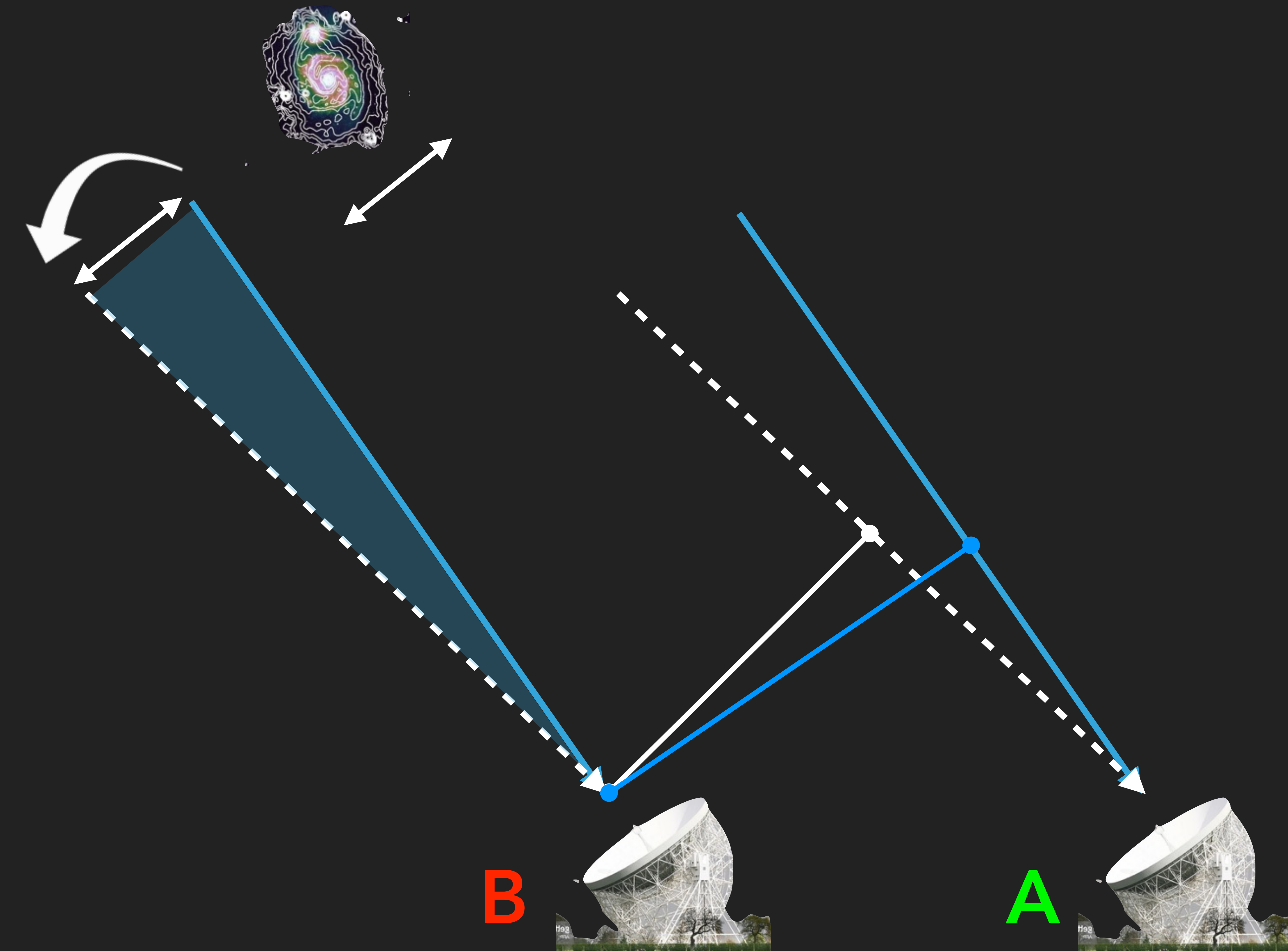


primary beam envelope: $\dfrac{\lambda}{D}$

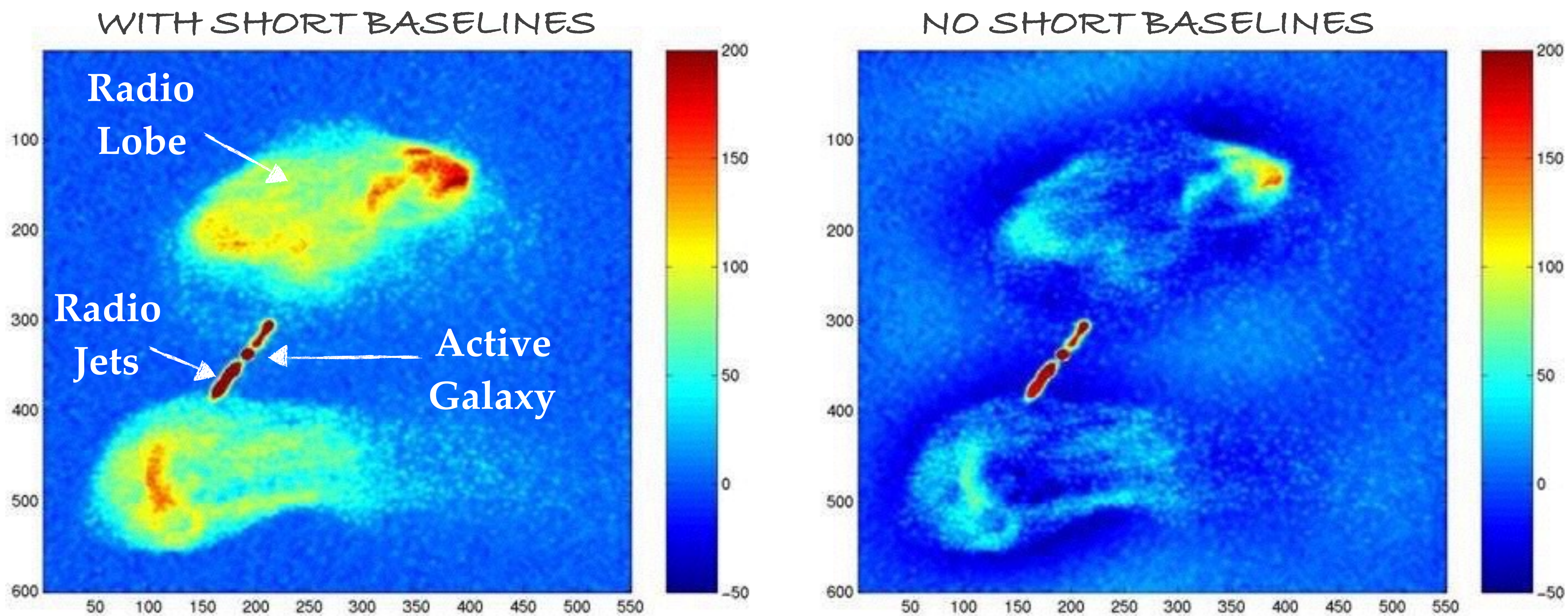
fringe spacing: $\dfrac{\lambda}{b}$

THE EFFECT OF BASELINE LENGTH

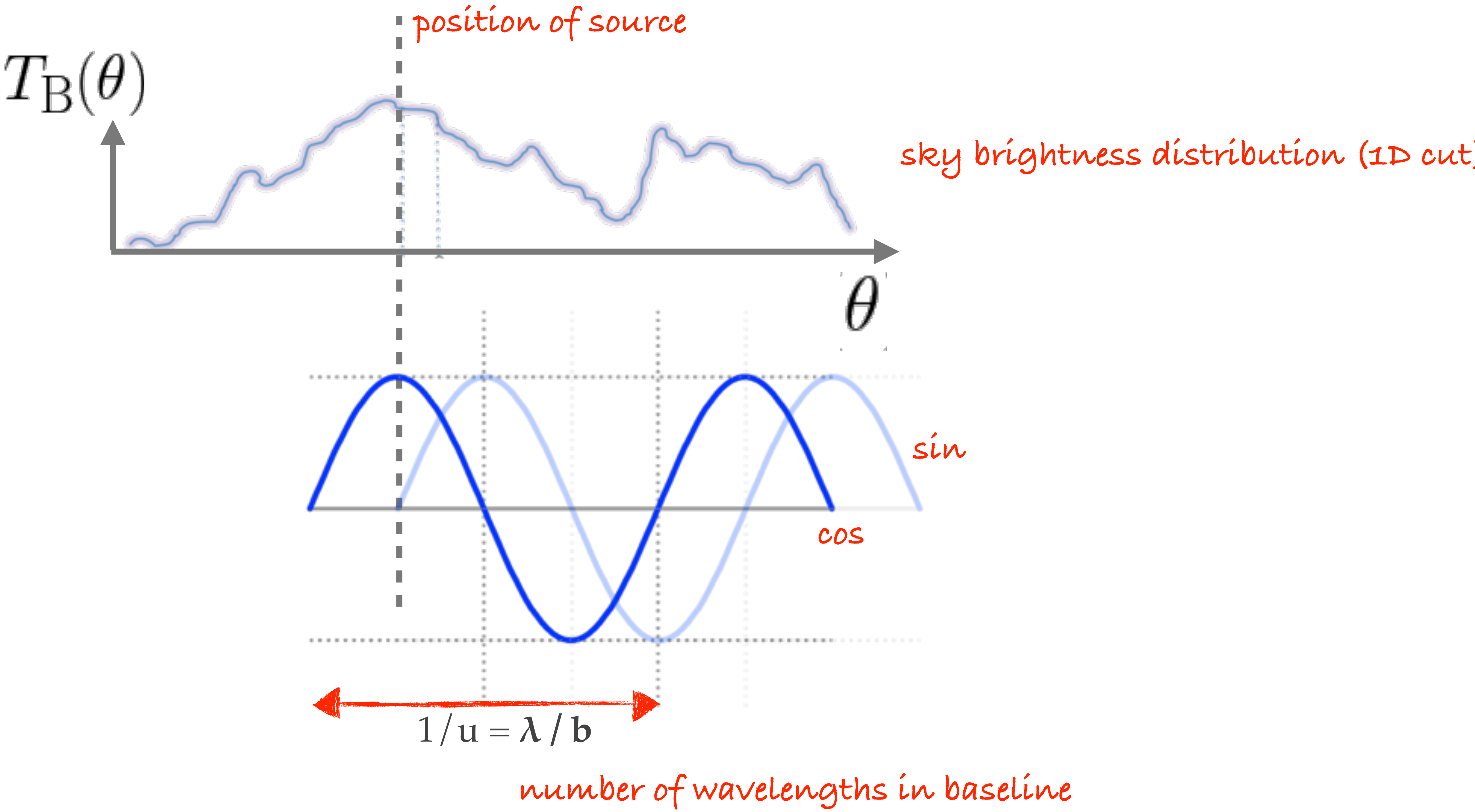



B
A

BASELINE LENGTH



Missing short baselines results in poor sensitivity to low brightness extended structure. Notice the negative "holes" (darker blue) underlying the bright structure.


position of source
$T_B(\theta)$
sky brightness distribution (1D cut)
θ
sin
cos
$1/u = \lambda / b$
number of wavelengths in baseline

COMPLEX VISIBILITIES

In reality the response will be 2D, but in 1D for simplicity:

power out as a function of baseline

$$R_{\cos}(u) = \int_{\mathrm{src}} B(\theta)\cos(2\pi\, u\, \theta)\mathrm{d}\theta$$

$$R_{\sin}(u) = \int_{\mathrm{src}} B(\theta)\sin(2\pi\, u\, \theta)\mathrm{d}\theta$$

The sky brightness distribution is **not an even function**. If we want to reconstruct it from its Fourier components then we need **both the cos and sin terms**.

VAN CITTERT ZERNIKE FUNCTION

The (2-D) lateral coherence function of the radiation field in space is the Fourier Transform of the (2-D) brightness (or intensity) distribution of the source.

$$\langle V(x_1, t) V(x_2, t) \rangle = \int \int B(\theta, \phi) \mathrm{e}^{-2\pi i (u\theta + v\phi)} \mathrm{d}\theta \mathrm{d}\phi$$

$$u = \frac{(x_1 - x_2)}{\lambda} \qquad v = \frac{(y_1 - y_2)}{\lambda}$$

The Visibility Function is therefore another name for the spatial correlation function.

VISIBILITY

From these components we define the **complex fringe visibility** for a particular baseline:

$$Ae^{i\Psi}$$

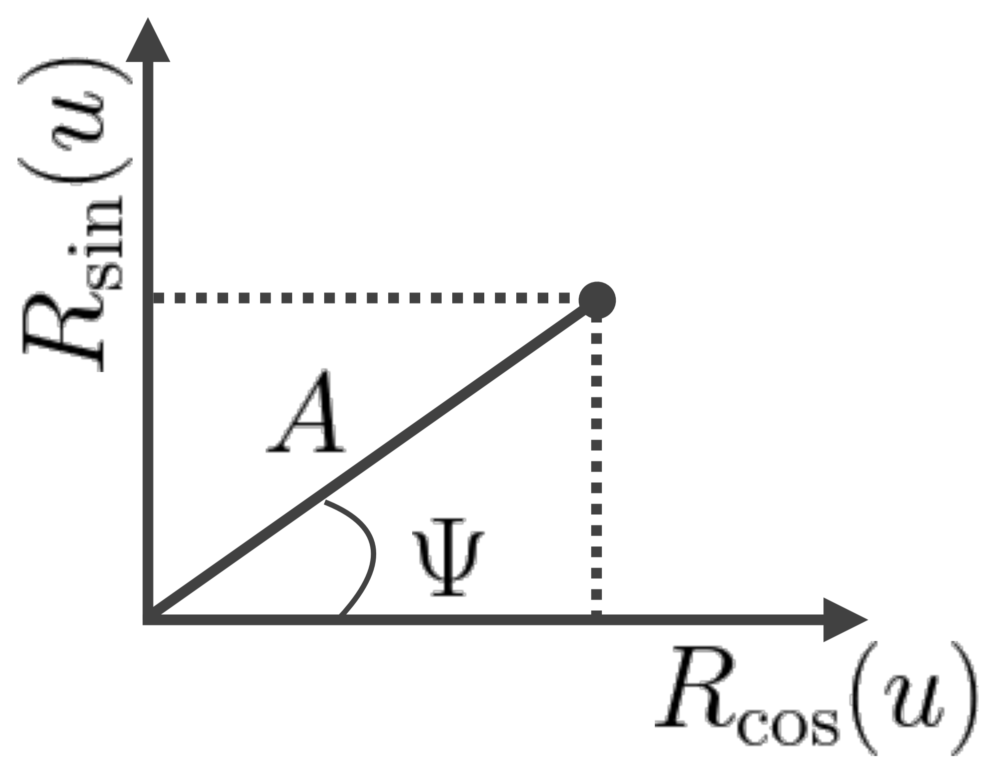
phase

amplitude

$$\Psi = \tan^{-1}\left[\frac{R_{\sin}}{R_{\cos}}\right]$$

$$A = \left[R_{\cos}^2 + R_{\sin}^2\right]^{1/2}$$

Ψ is the **shift** (in radians) of the Fourier component with respect to the reference position.



VISIBILITIES & IMAGES

$$I_{meas}(l,m) = \frac{1}{M}\sum_{i=1}^{M} V(u_i,v_i)e^{2\pi i(u_i l + v_i m)}$$

$$= \frac{1}{M}\sum_{i=1}^{M} V(u_i,v_i)\left[\cos[2\pi(u_i l + v_i m)] + i\sin[2\pi(u_i l + v_i m)]\right]$$

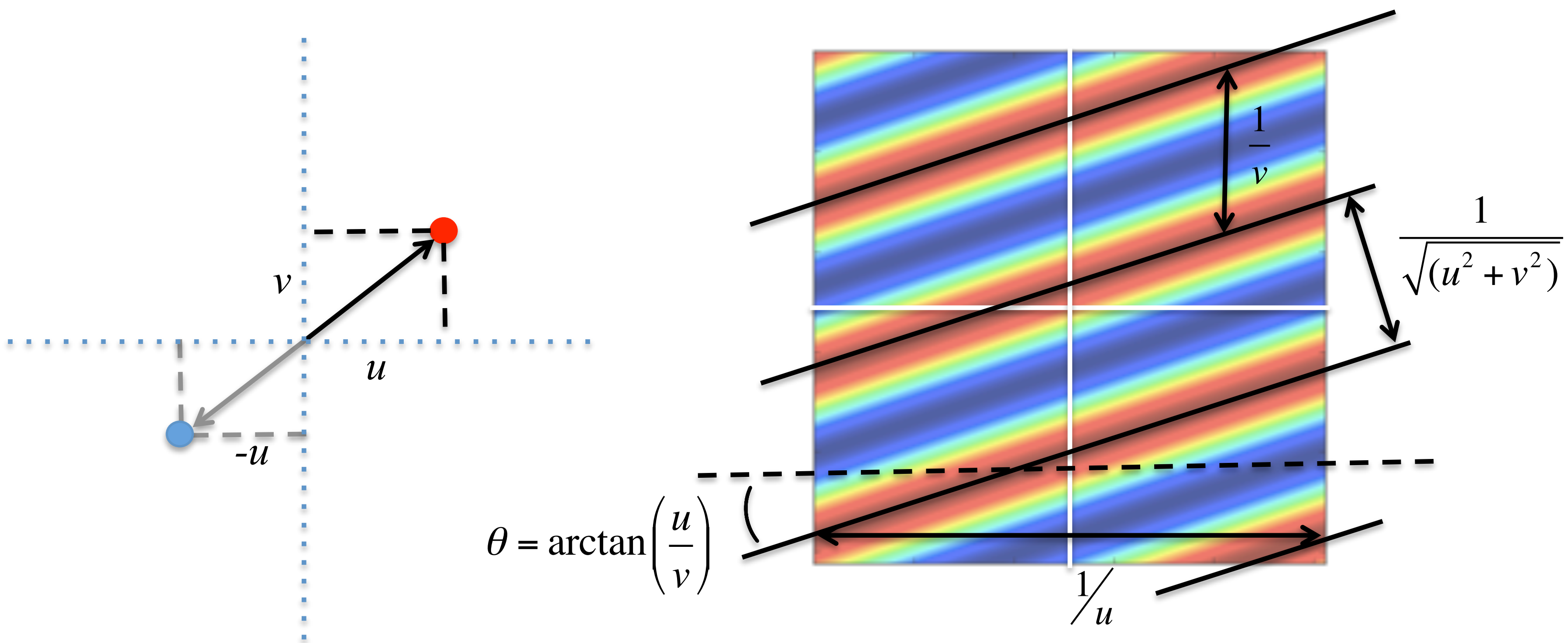
This is a **complex** quantity, but the sky intensity is **real**.

$$\boxed{V(-u,-v) = V^*(u,v)}$$

If we change our notation slightly, so that $V = Ae^{i\phi}$, we can write:

$$I_{meas}(l,m) = \frac{1}{M}\sum_{i=1}^{M} A(u_i,v_i)\cos[2\pi(u_i l + v_i m) + \phi_i]$$

FOURIER COMPONENTS



Writing the equation in this way allows us to visualise how our image is composed.

$$I_{meas}(l,m) = \frac{1}{M}\sum_{i=1}^{M} A(u_i,v_i)\cos[2\pi(u_i l + v_i m) + \phi_i]$$

CONCLUSIONS

- The key to interferometry is the geometric delay;
- A 2-element adding interferometer is a direct analogue of Young's slits;
- The sky is not symmetric - we need both cosine & sine waves to make a picture of it;
- Interferometers measure complex visibilities, which are the Fourier components of the sky brightness.

RadioNet has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 730562

The University of Manchester
Jodrell Bank Observatory

RadioNet

MANCHESTER 1824

CONCLUSIONS

- The key to interferometry is the geometric delay;
- A 2-element adding interferometer is a direct analogue of Young's slits;
- The sky is not symmetric - we need both cosine & sine waves to make a picture of it;
- Interferometers measure complex visibilities, which are the Fourier components of the sky brightness.

RadioNet has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 730562