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Netherlands Institute for Radio Astronomy

Calibration

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ASTRON is part of the Netherlands Organisation for Scientific Research (NWO)

RadioNet

Preamble

- **AIM:** This lecture aims to give a general introduction to advanced calibration techniques, focusing on conceptual knowledge.

- **OUTLINE:**

1. Revision of an ideal interferometer and calibration philosophy.
2. Self-calibration (self-cal).
3. Direction dependent effects.
    1. The beam
    2. The atmosphere
4. Spectral dependence of calibration

Contents

1. Revision of an ideal interferometer and calibration philosophy

Revision of an ideal interferometer

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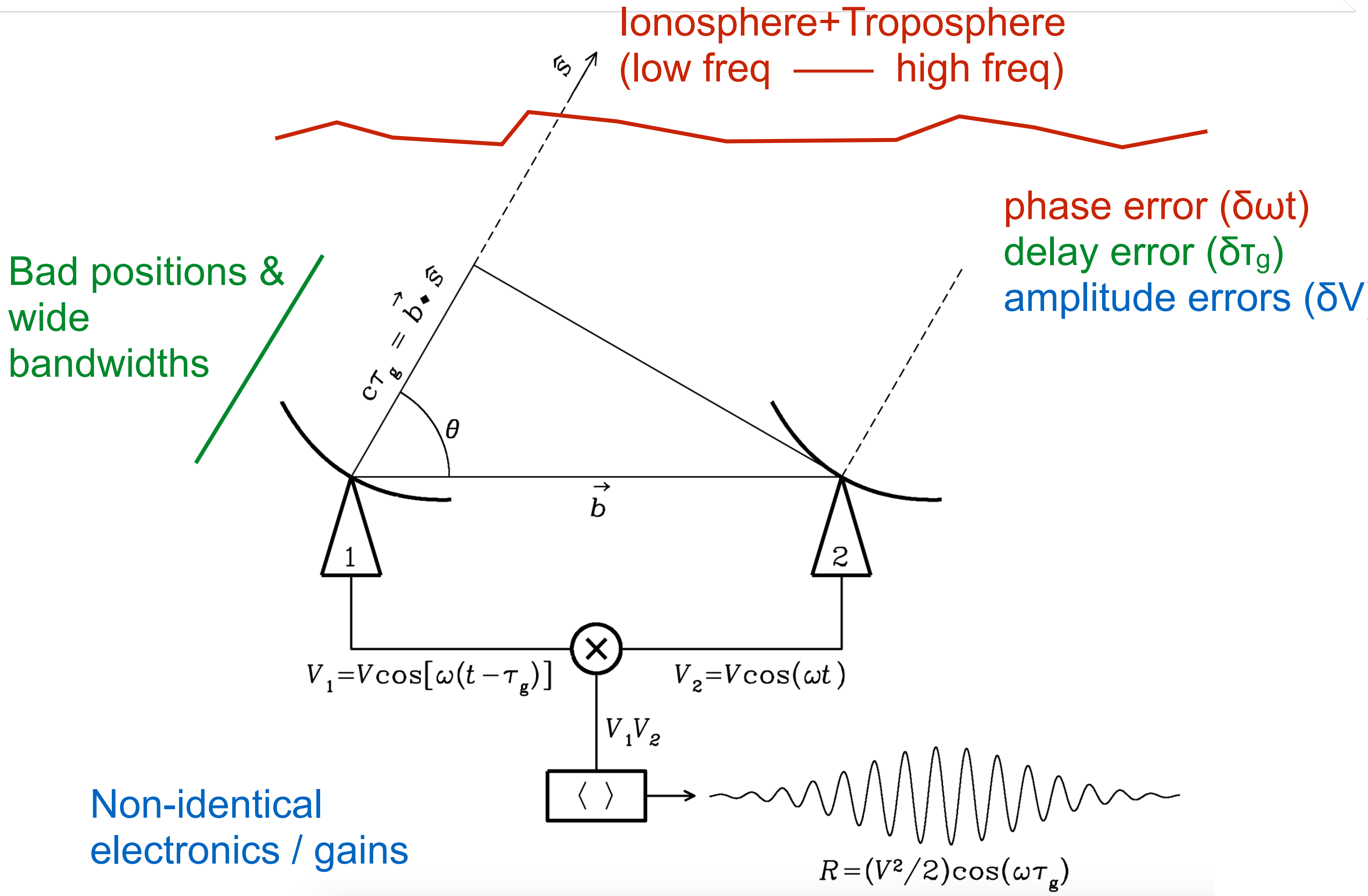


Solve for these issues using calibration

Revision of the RIME

The **radio interferometry measurement equation** (RIME) relates the **observed** (perturbed) visibility to the **ideal** (unperturbed) **visibility.**

Jones matrix for antenna *i*

$$\vec{V}_{ij} = J_{ij} \vec{V}_{ij}^{\mathrm{IDEAL}}$$ where $$J_{ij} = J_i \times J_j^*$$

observed visibility

Combined Jones matrix

ideal visibility

A Jones matrix is a 2 x 2 matrix that describes the *antenna* based calibrations, for each correlation, for a given correction (gain, bandpass, delay, etc.), for example,

$$J_{\mathrm{gain}} = \begin{pmatrix} g_{\mathrm{R}} & 0 \\ 0 & g_{\mathrm{L}} \end{pmatrix} \qquad J_{\mathrm{leakage}} = \begin{pmatrix} 1 & D_{\mathrm{R}} \\ D_{\mathrm{L}} & 0 \end{pmatrix}$$ such that $$J_{\mathrm{overall}} = J_1 J_2 J_3$$

Revision of the RIME

Within, for example **CASA**, the full radio interferometry measurement equation can be written as,

Baseline based, non closing errors

Gain amplitude and phase

Errors due to elevation

Opacity and path length variation

Observed visibility for ant. *i* and *j*

$$V_{ij}^{\rm obs} = M_{ij} B_{ij} G_{ij} D_{ij} E_{ij} P_{ij} T_{ij} V_{ij}^{\rm true}$$

true visibility for ant. *i* and *j*

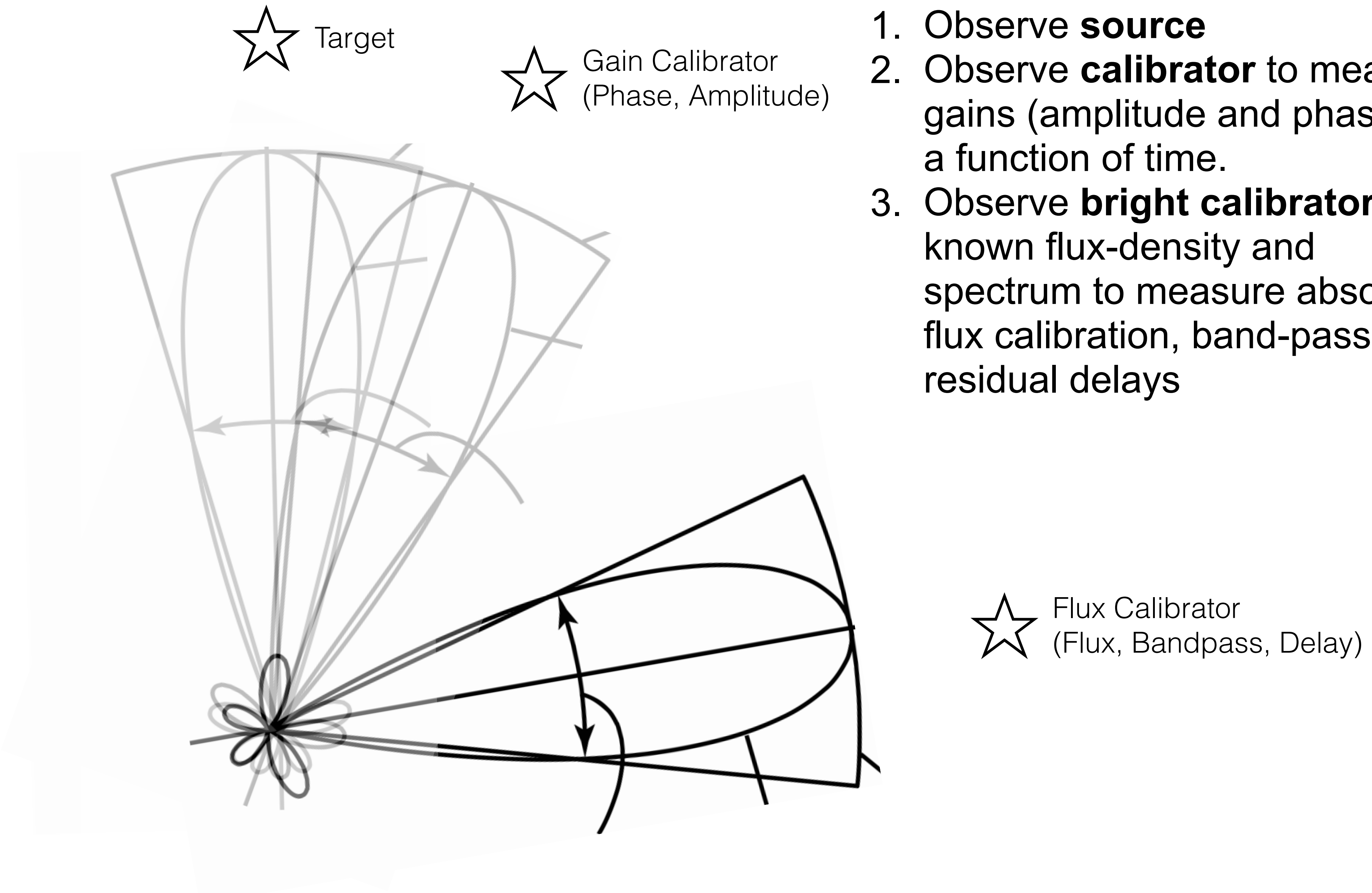
Bandpass response

Instrumental polarisation

Change in paralactic angle

Calibration solves for each Jones matrix (when required) given a **model** for the sky.

Calibration strategy

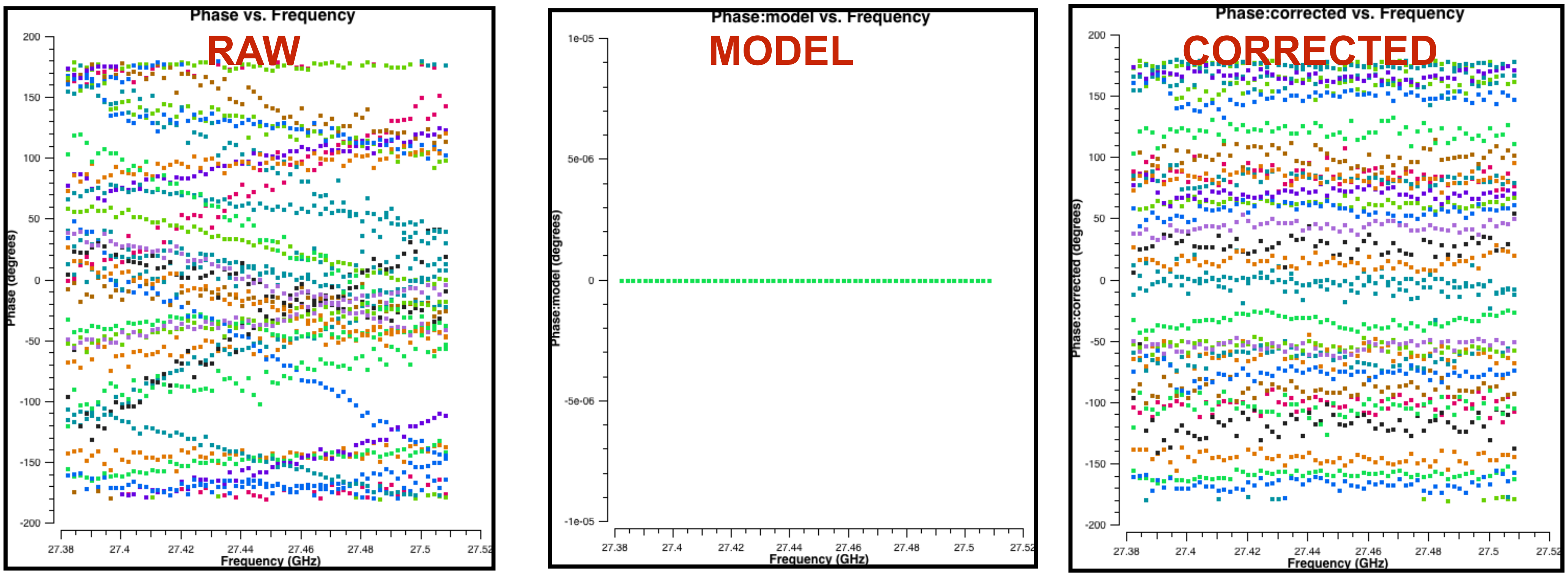


1. Observe **source**
2. Observe **calibrator** to measure gains (amplitude and phase) as a function of time.
3. Observe **bright calibrator** of known flux-density and spectrum to measure absolute flux calibration, band-pass and residual delays

Example of delay calibration

Here is an observed visibility function (delay), the ideal visibility function and the calibrated data (after solving the K_{ij} in the the measurement equation).

Main source of delay error: Large fractional bandwidths.

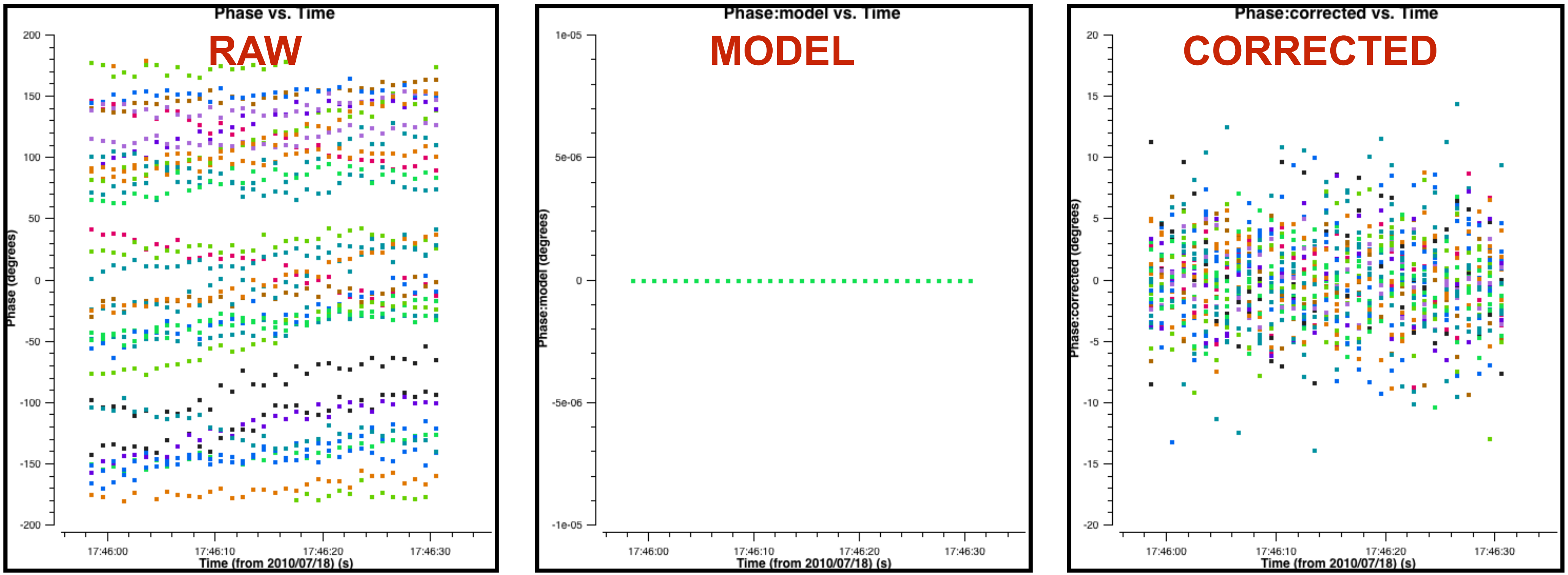


More complex delay corrections require ‘fringe fitting’ : see VLBI lecture.

Example of phase calibration

Here is an observed visibility function (phase), the ideal visibility function and the calibrated data (after solving the G_{ij} in the the measurement equation).

Main source of phase error: Variable ionosphere or troposphere + electronics.

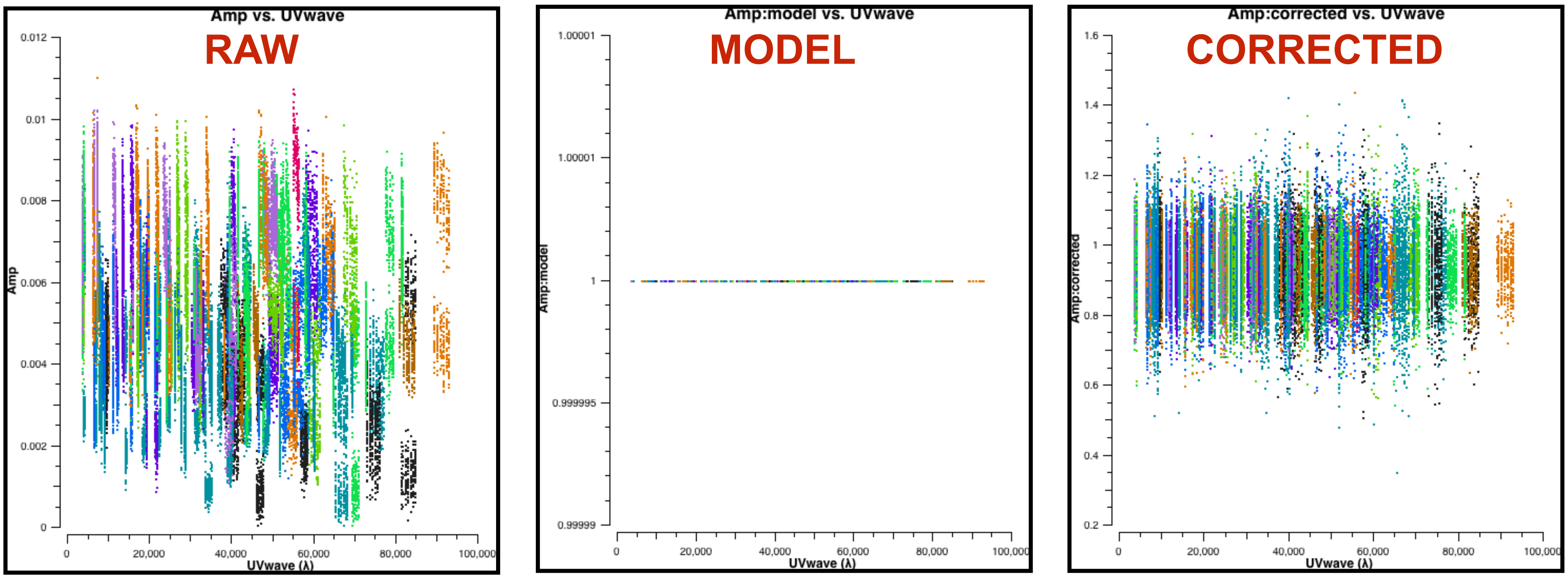


More complex delay corrections require ‘fringe fitting’ : see VLBI lecture.

Example of amplitude calibration

Here is an observed visibility function (amplitude), the ideal visibility function and the calibrated data (after solving the G_{ij} in the the measurement equation).

Main source of amplitude error: Variable gain in the amplifiers of the system.



Each colour represents visibilities with a common antenna.

Looking for closure

Calibration works due to **closure** (the expectation that the phase, φ_{ij}, and amplitude, V_{ij}, of groups of baselines have certain properties given the source structure).

Phase:

$$C_{ijt}(t) = \phi_{ij}(t) + \phi_{jk}(t) + \phi_{ki}(t)$$

Amplitude:

$$\Gamma_{ijkl}(t) = \frac{|V_{ij}(t)||V_{kl}(t)|}{|V_{ik}(t)||V_{jl}(t)|}$$

An error in your model can be absorbed in the calibration

e.g. for a point source the closure phase is 0.

$$\vec{V}_{ij} = J_{ij}\vec{V}_{ij}^{\mathrm{IDEAL}}$$

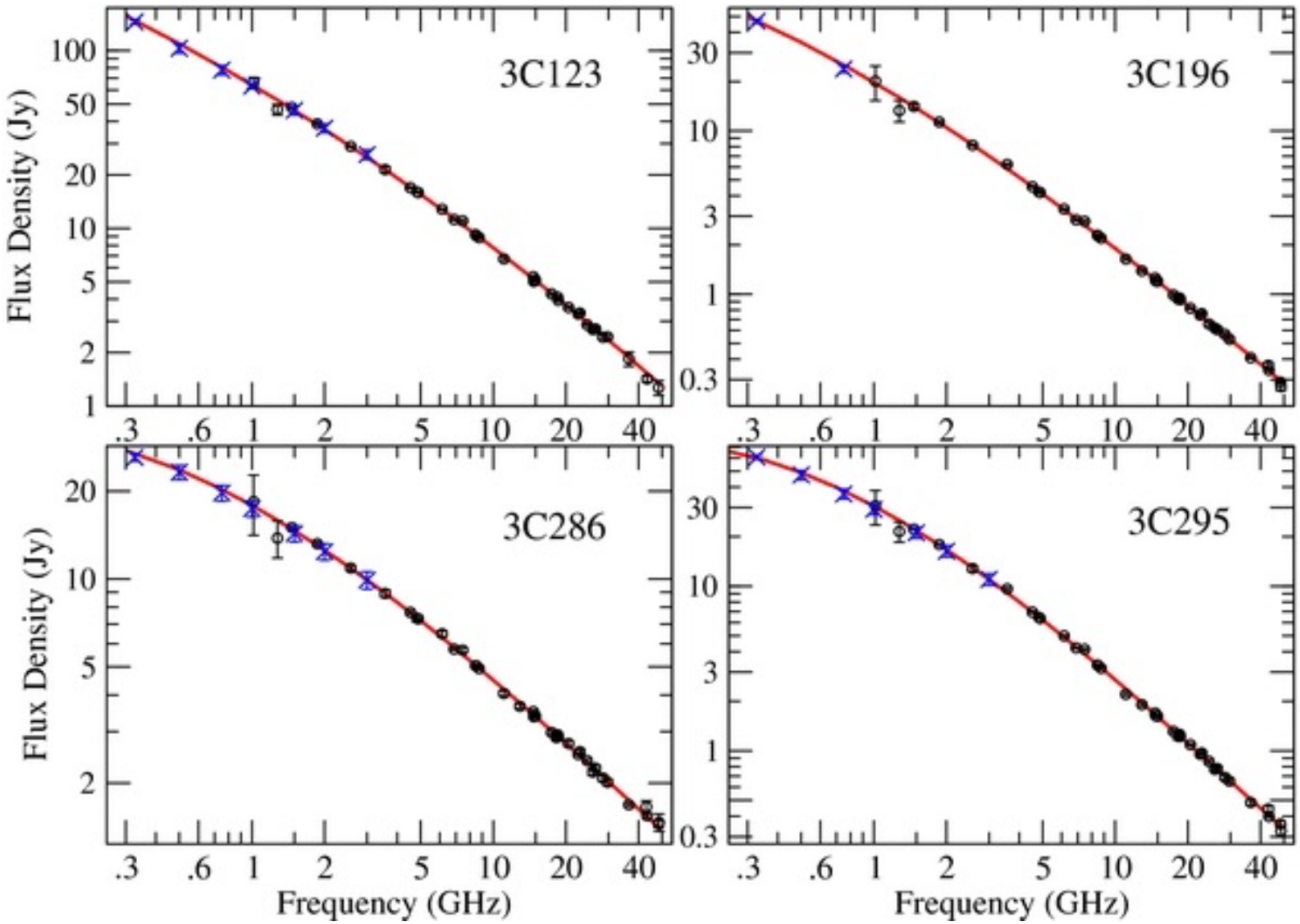
Aren't we just forcing the data to fit the model? No as long as,

i) You have a good model for the sky (point-source)
ii) You have sufficient signal-to-noise ratio on each baseline (> 3σ)

There are *N* free-parameters for each (close to) N*(N-1) / 2 constraints from the total number of baselines.

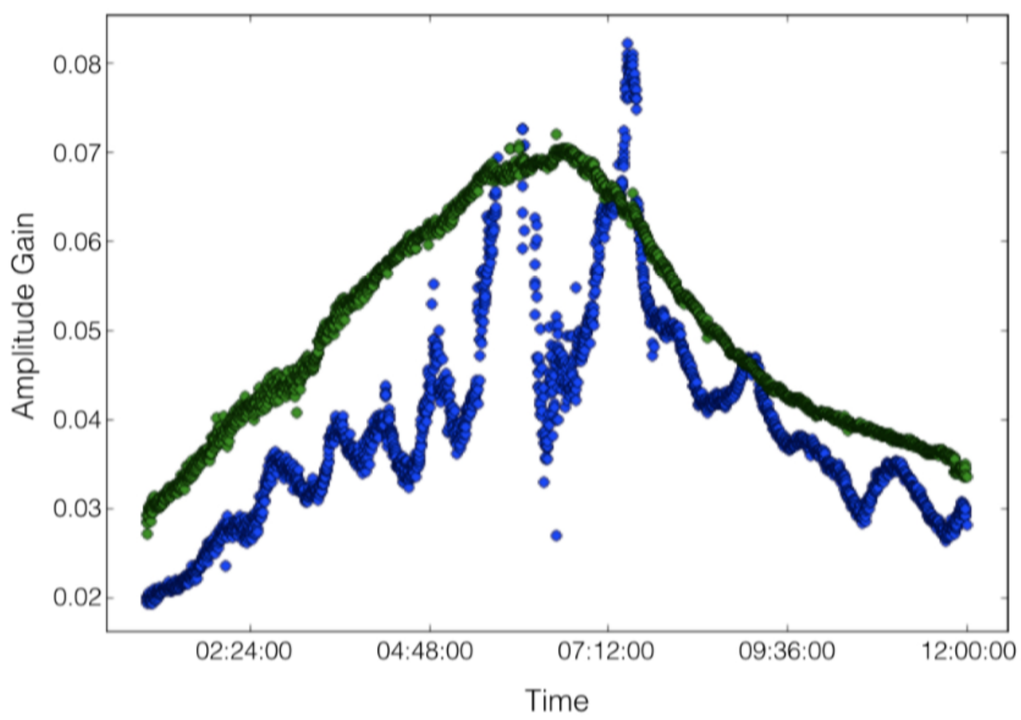
Absolute flux-density calibration

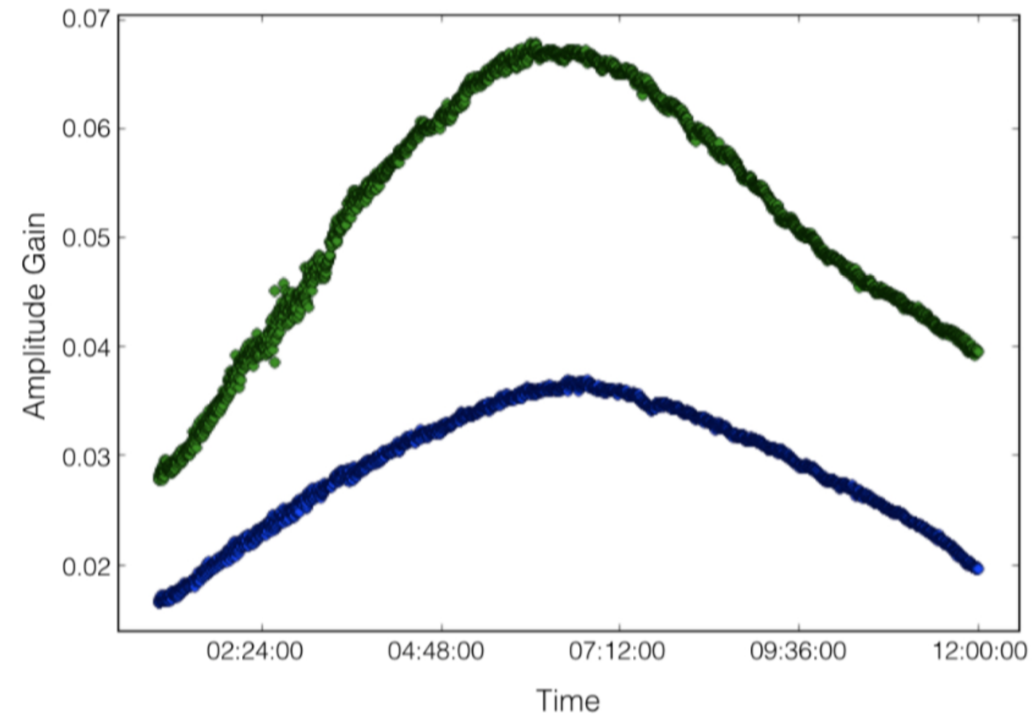
By comparing with an object with a known flux-density (compare the amplitude gains), it is possible to move from relative amplitudes to absolute amplitudes (~5 % repeatability).



Inspect your solutions

Always inspect your solutions to see if the variations as a function of time and frequency are as expected.





(left) A point-source model use to calibration antennas where the baselines see a point (green) and resolved (blue) source. (right) A proper model is used for all baselines.

An error in your model can be absorbed in the calibration

$$\vec{V}_{ij} = J_{ij}\vec{V}_{ij}^{\mathrm{IDEAL}}$$

Contents

2. Self-calibration (self-cal)

Self-calibration philosophy

After **transferring the solutions** from a calibrator we may find that there are **residual errors** in our data.

Why?

Our calibrators are observed at a **different time** (except for simultaneous observations; in beam-calibration) and **position** on the sky than our target.

Use the process of self-calibration:

1) Make an image of your target (after applying calibrator solutions).
2) Use this model to calibrate the data over some solution interval.
3) Make an image of your target (after applying self-calibration solutions).
4) Use this model to calibrate the data over some solution interval.
5) Iterate this process until no major improvement on image quality.

Advantages:

1) Can correct for residual amplitude and phase errors.
2) Can correct for direction dependent effects (see later).

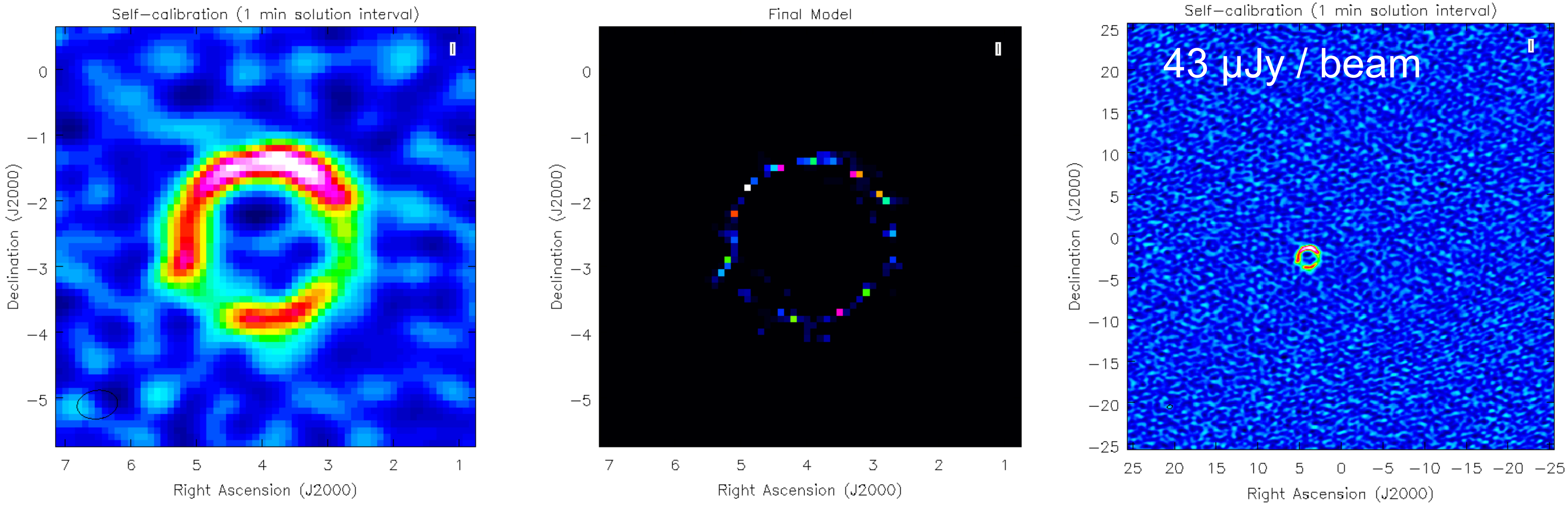
Disadvantages:

1) Errors in the model or low SNR can propagate into your self-calibration solutions, and you can diverge from the correct model.

Phase errors

Our calibration of the instrumentation + propagation phase shifts relies on our calibrator giving a good estimate of these corrections.

Assume: They do not change as a function of (short) time and (small) position on the sky (otherwise we loose coherence).



Example of ALMA (1 min solution interval) self-calibration of a gravitational lens.

When self-calibration goes bad (phase)

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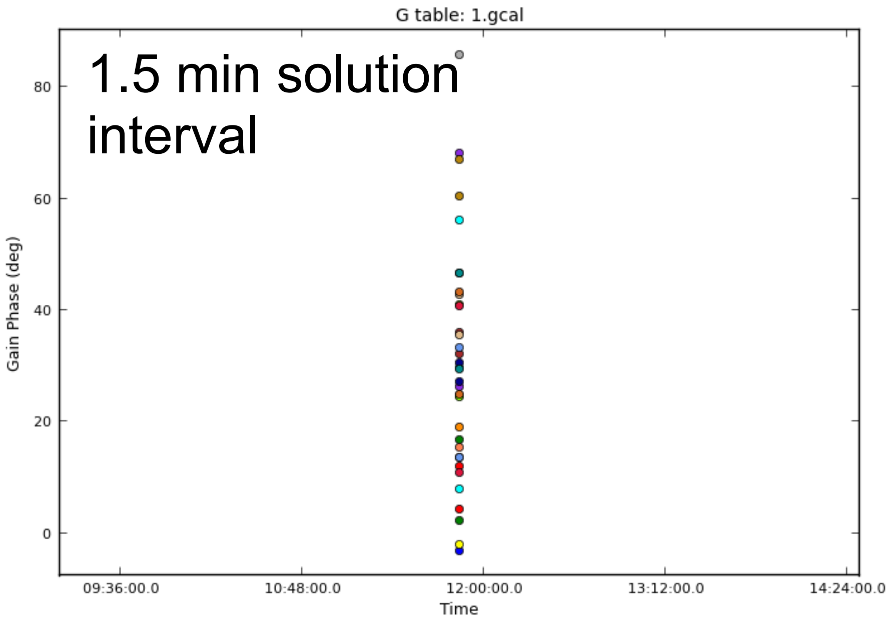
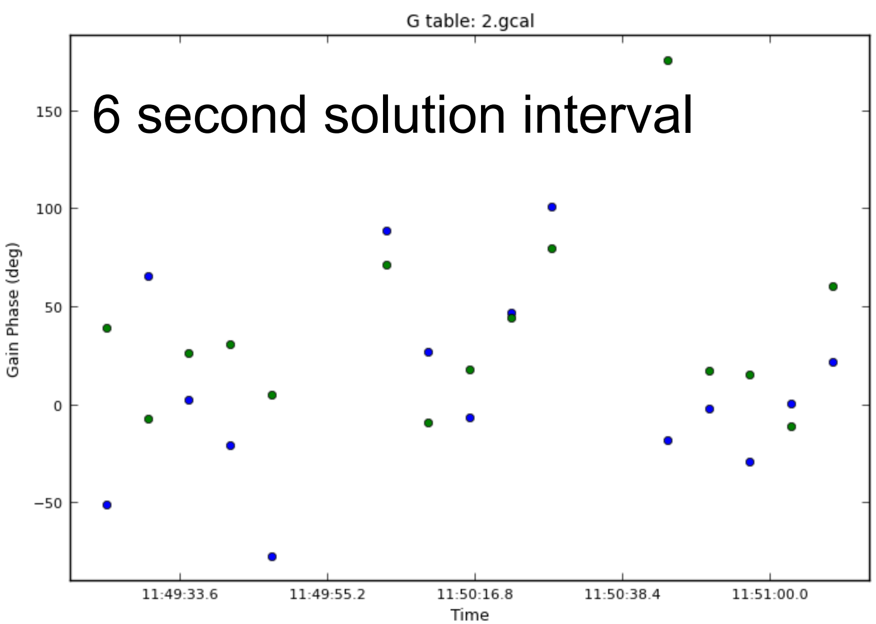
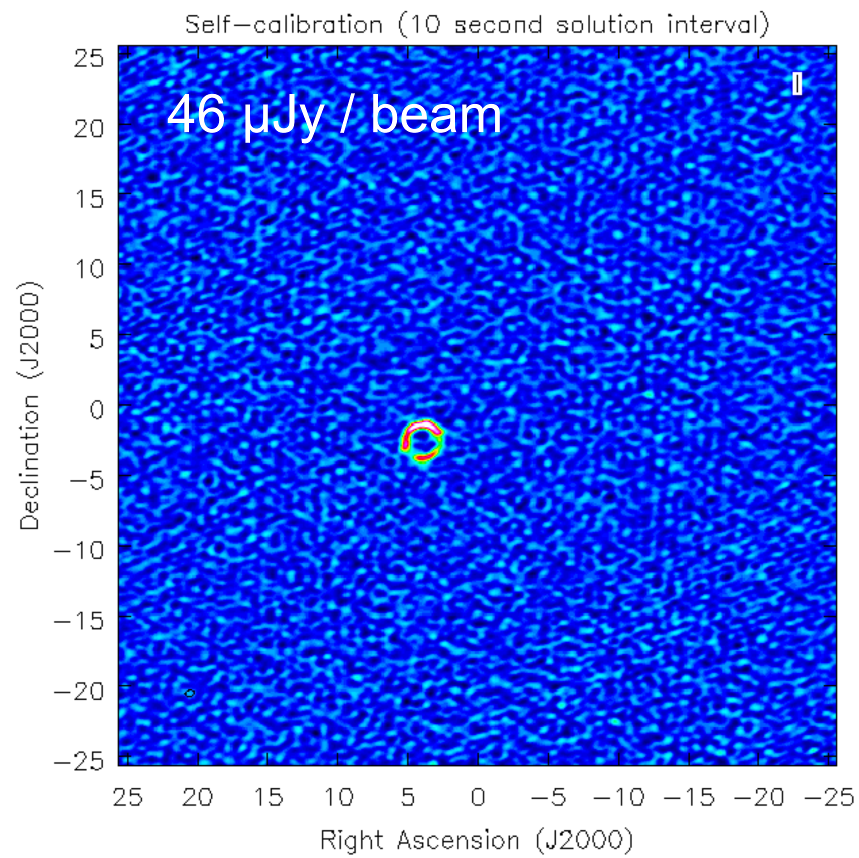


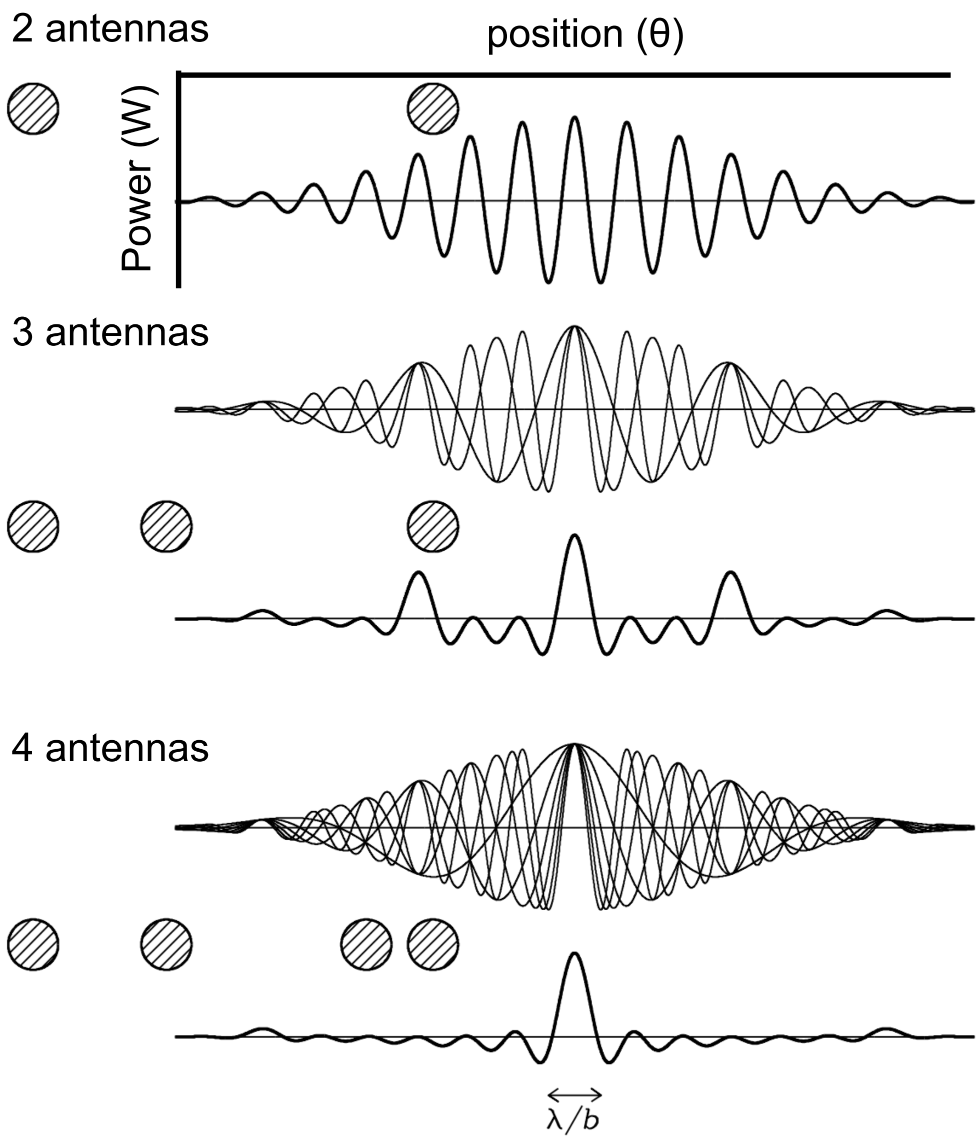
If the solution interval is too low, solutions become noisy (start fitting the noise).

Source flux goes up.
image rms goes up.





Amplitude errors



Ideal situation with no errors.

Our point spread function comes from the response our interferometer.

By adding just one more antenna we suppress the side-lobes.

By adding another antenna we suppress the side-lobes further.

But, what happens if one antenna has an amplitude error?

When self-calibration goes bad (amp)

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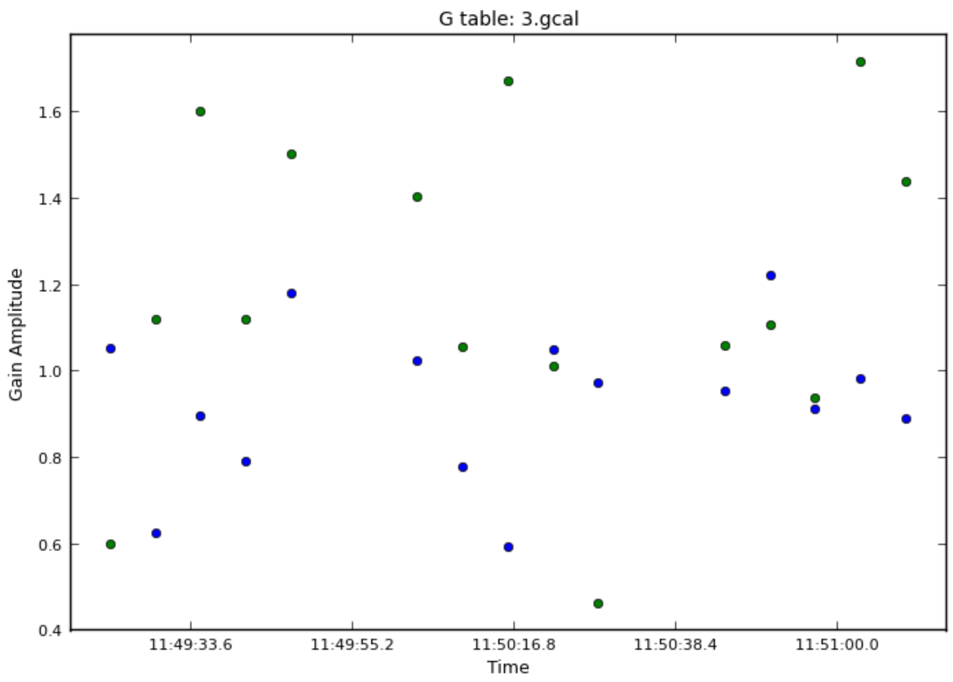
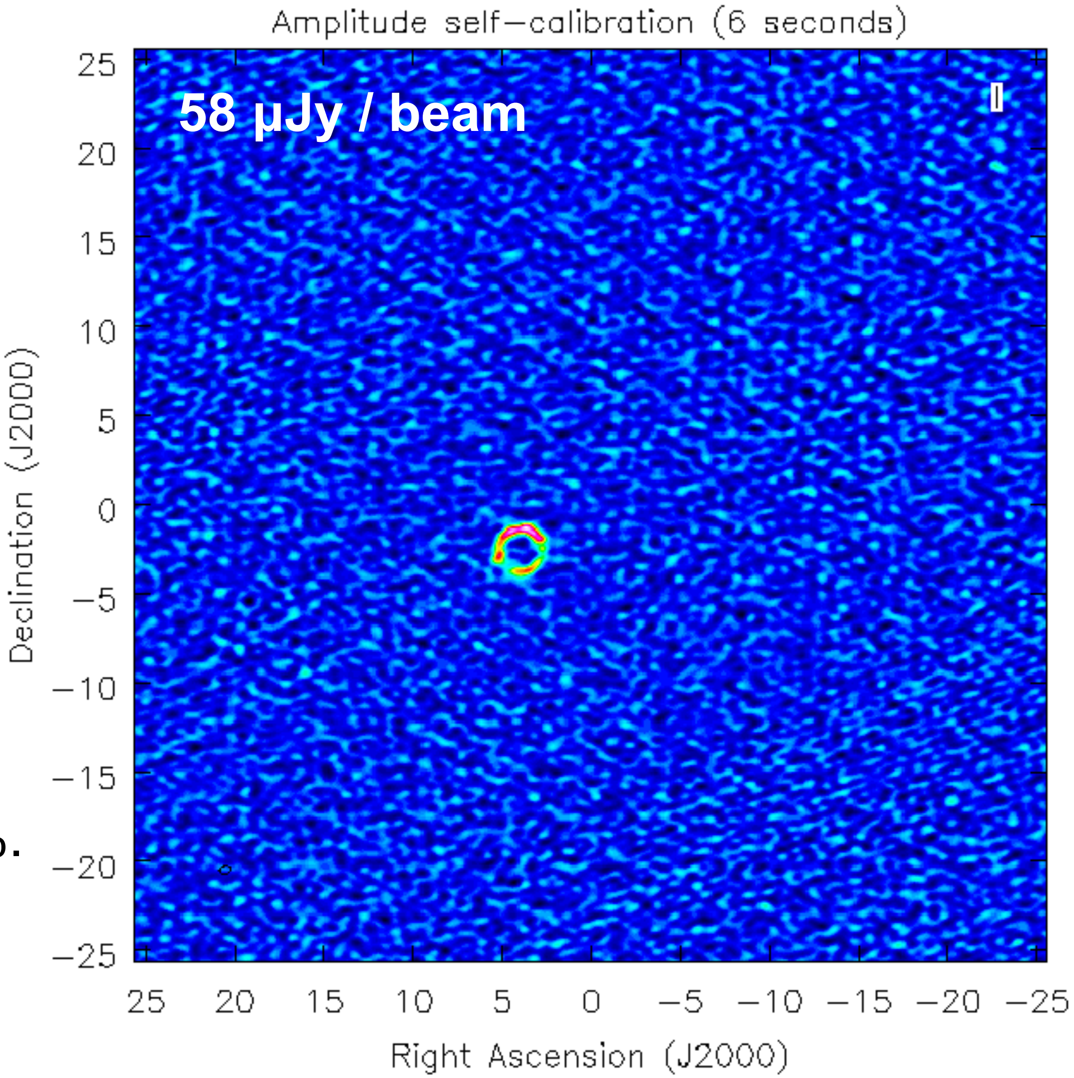
CAUTION: Amplitude self-calibration should be handled with care.

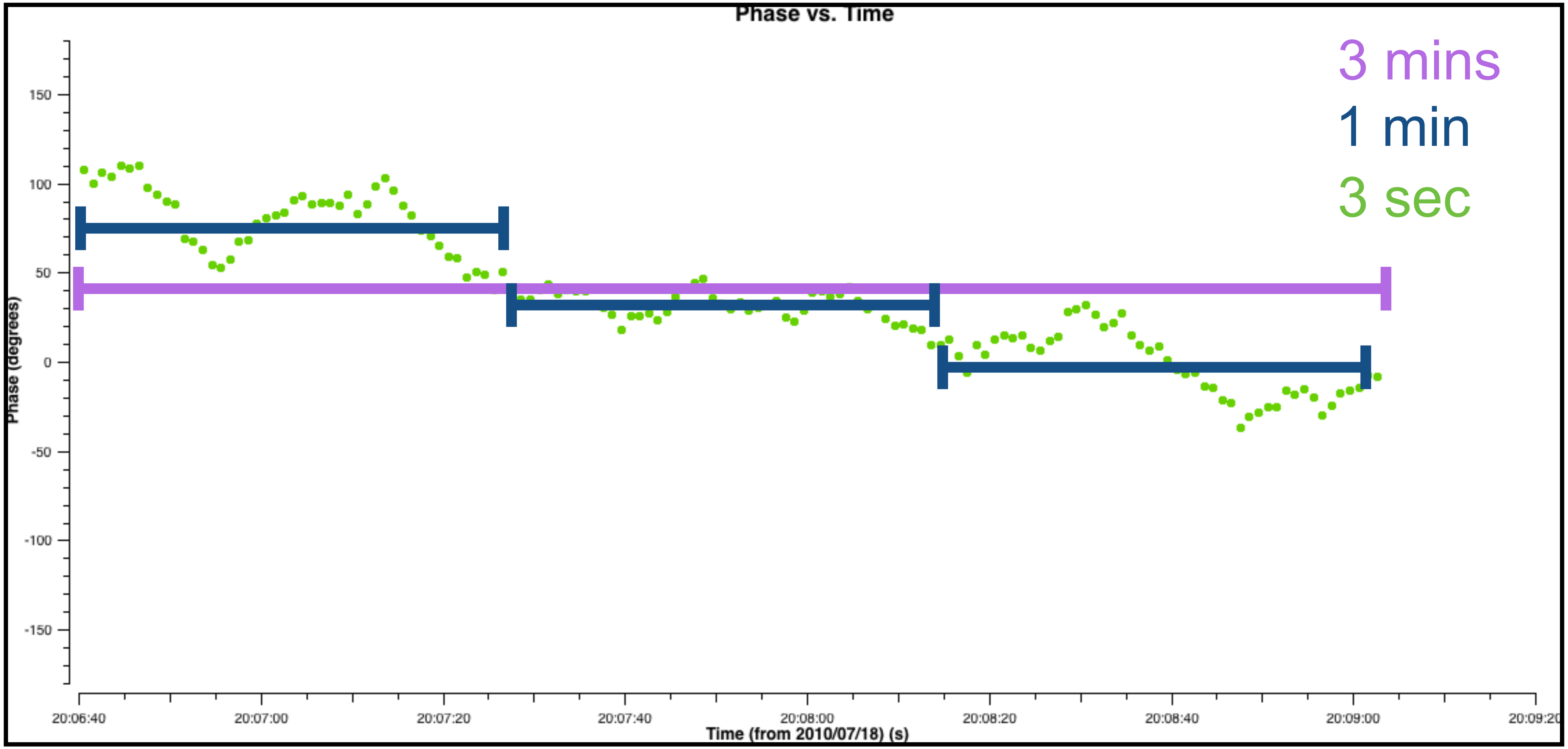




6 seconds amplitude self-calibration changes the relative gains by 40 to 60%.

Strategy: Start long (hours) and work down until the solutions get noisy.

What is an appropriate solution time?



Want to have,

Shortest possible time-scale to **track** the gain variations, whist being **long enough** to have a sufficient **signal-to-noise ratio.**

Student exercise

Q. What is the minimum solution interval to achieve a 3σ baseline-sensitivity for a 100 mJy point-source that is detected at the 100 sigma level in 10 minutes for an array with 10 identical antennas?

A. For a 100 sigma detection the *image* sensitivity is 1 mJy / beam.

The number of baselines is 10 * 9 / 2 = 45 baselines.

Therefore the *baseline* sensitivity in 10 minutes is 1 * sqrt(45) = 6.7 mJy / beam or 15 sigma (= 100 / 6.7 sigma).

For a 3 sigma detection, the baseline sensitivity can go up by factor of 5, so the time must go down by factor $5^2 = 25$ (recall that $\sigma \sim 1 / \sqrt{\Delta v * t}$).

So the solution interval we need is 10 * 60 / 25 ~ **25 seconds**.

We will be able to track the phase variations over ~ 20 time intervals that are 25 seconds each.

3. Direction dependent effects.

Phased arrays

Phased arrays

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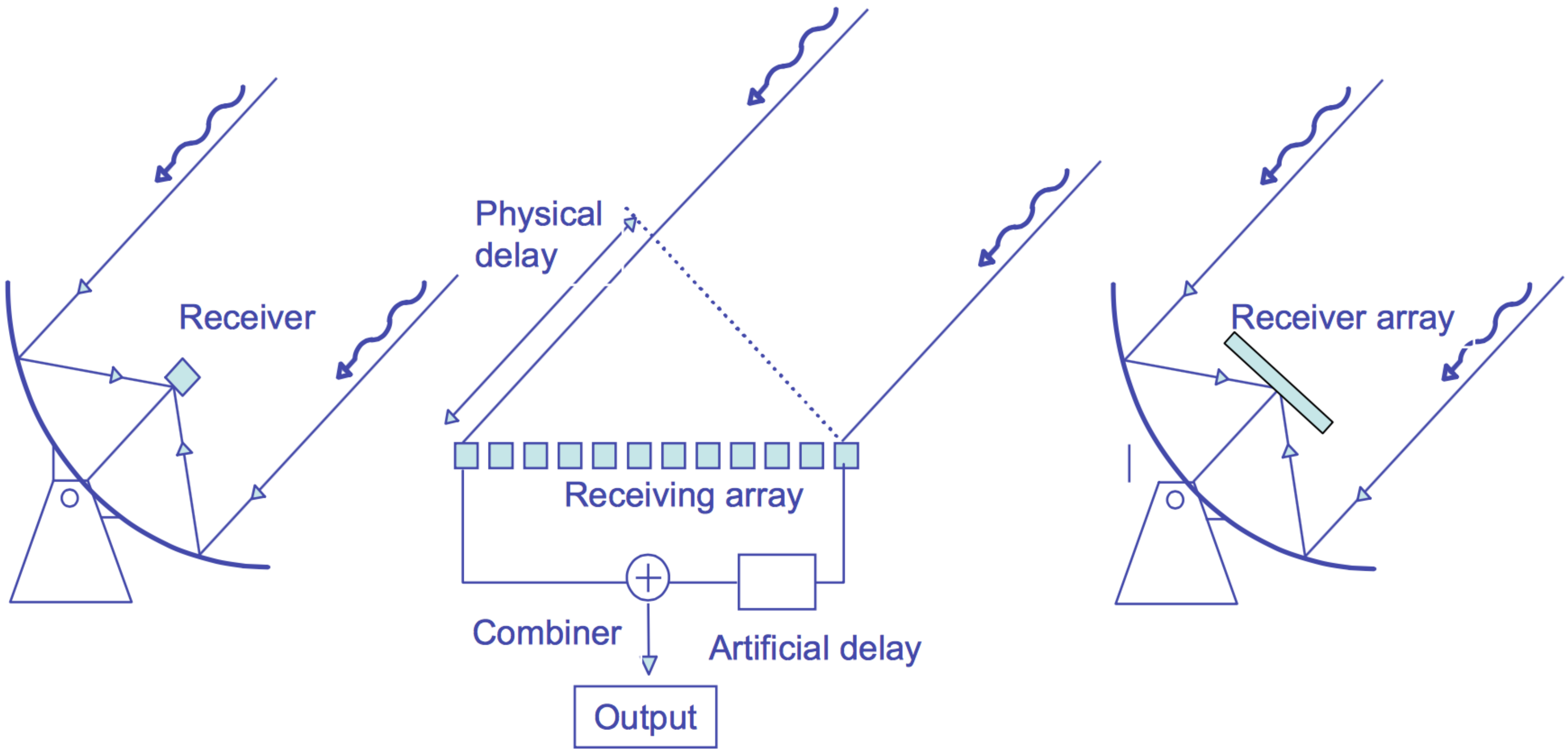
How do you point with a phased array?

Pointing a phased array



Parabolic reflector
(mechanical)

Aperture array
(electronic)

Reflector + receiver array
(mechanical + electronic)

The delay that we add will coherently add the different elements of an aperture array in one direction, and suppress the emission from other directions.

Wide field imaging is fun!

Imaging wide-fields is useful for,

1) Efficient all-sky survey
2) Looking for rare objects

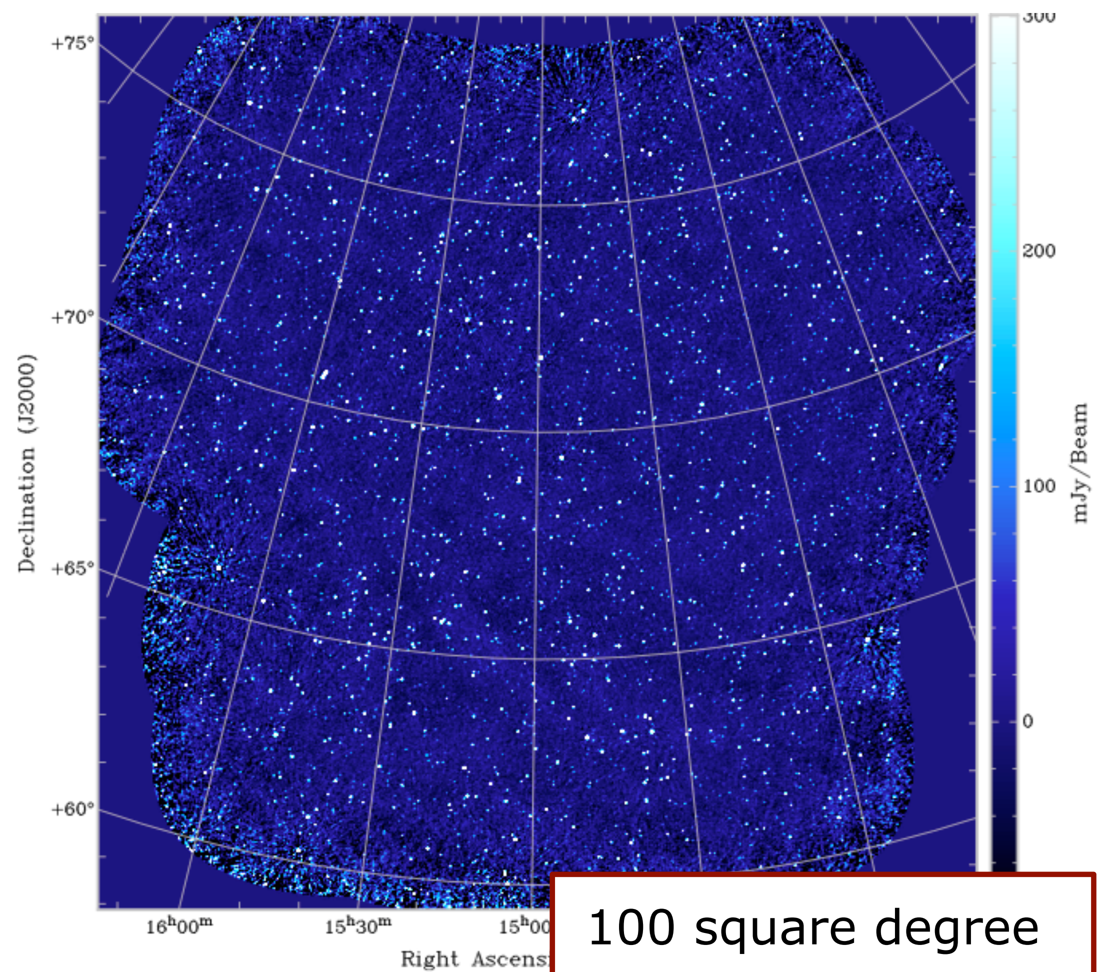
Wide-fields introduce many issues for a good calibration,

1) Variable beam power as a function of position results in a more complicated amplitude calibration.
2) The phase solutions in one direction cannot be applied to another.
3) Sky model is more complicated (many sources).

An error in your model can be absorbed in the calibration

$$\vec{V}_{ij} = J_{ij}\vec{V}_{ij}^{\rm IDEAL}$$

LOFAR MSSS SVF; *George Heald*

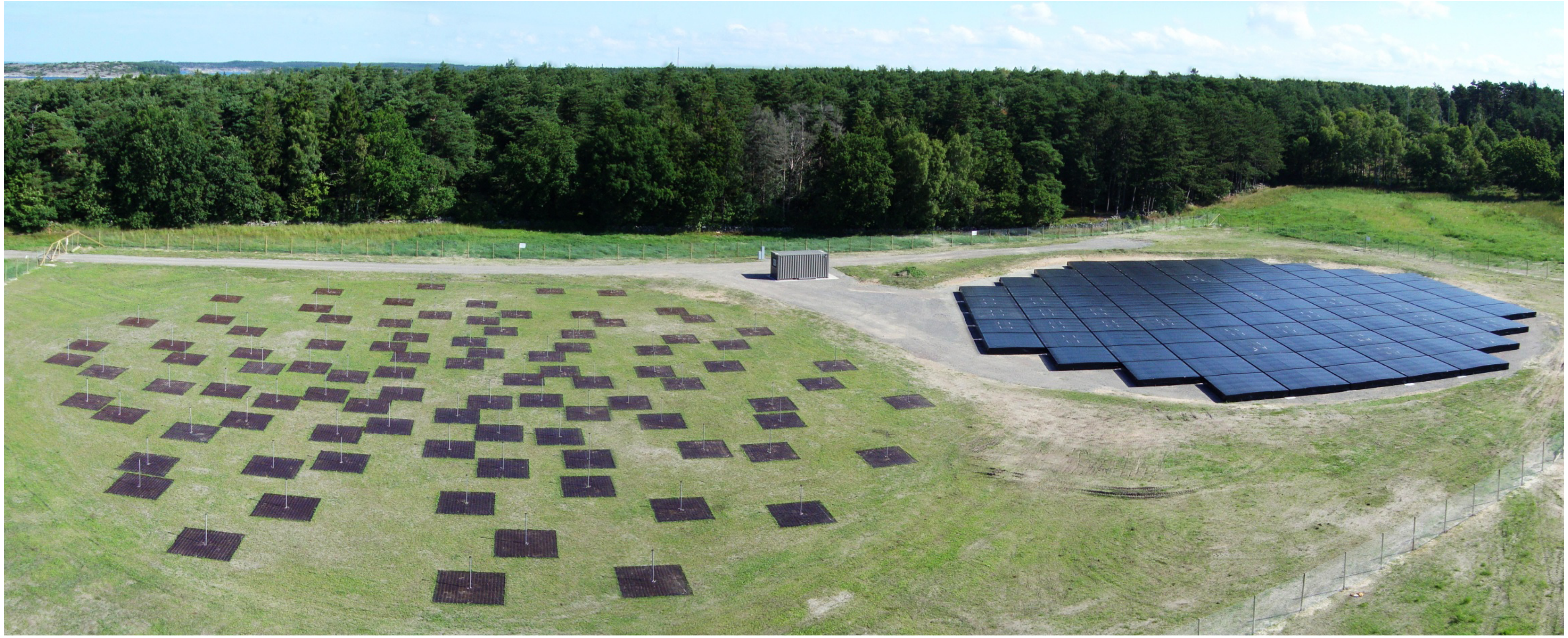


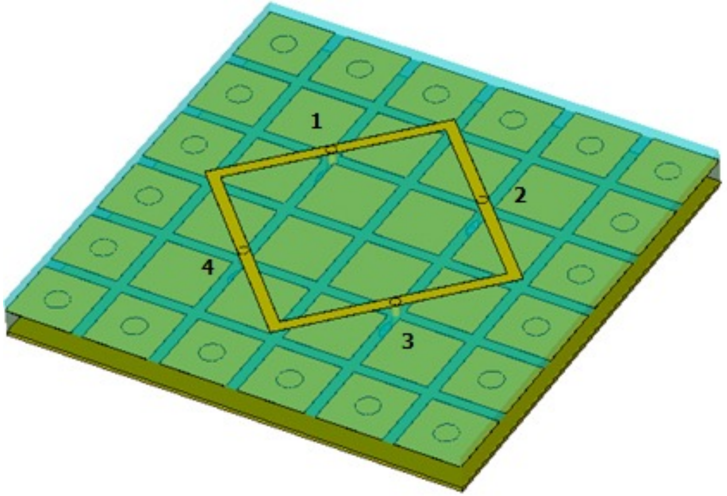
100 square degree

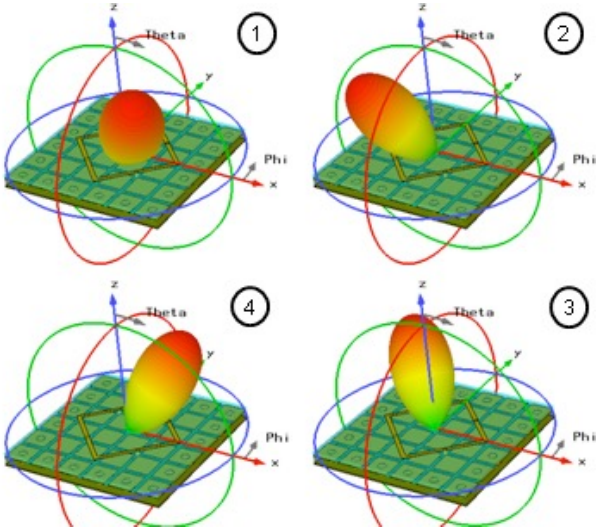
~10 mJy /beam [2 arcmin resolution]

3. Direction dependent effects. I - The beam

Beam-forming

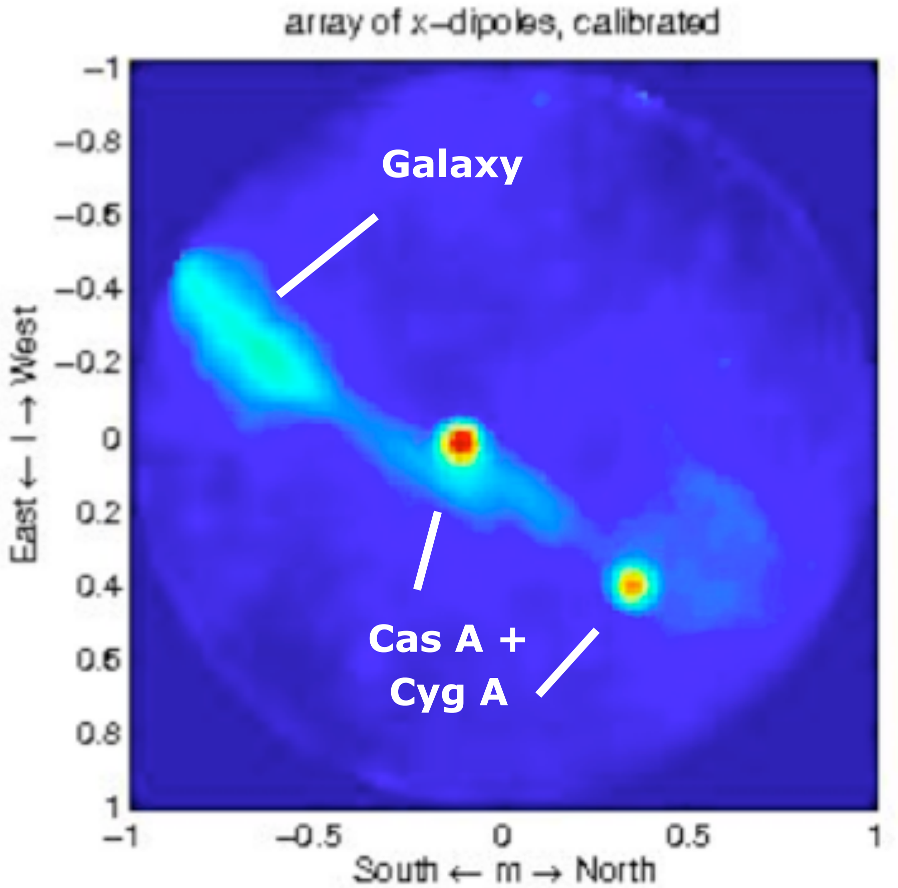






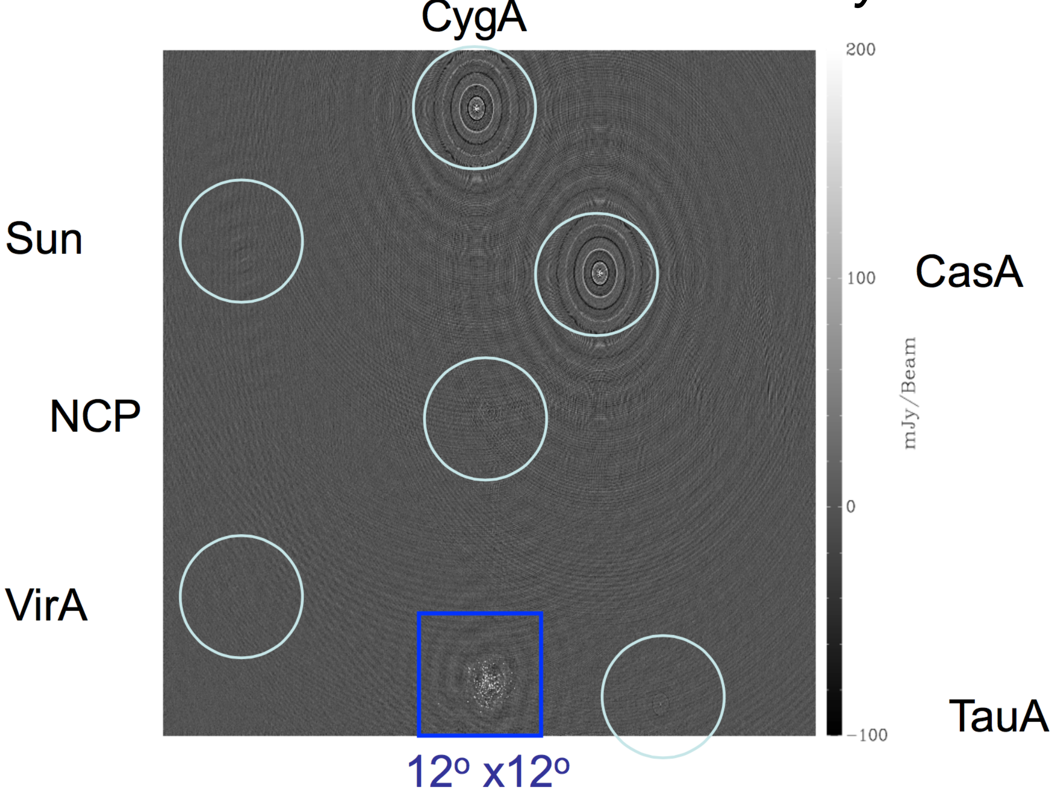
What happens with a large beam?

A large beam means that you can survey much larger areas of the sky
- **Great** for surveys, transients
- **Bad** if you are not interested in the sky that is off-axis



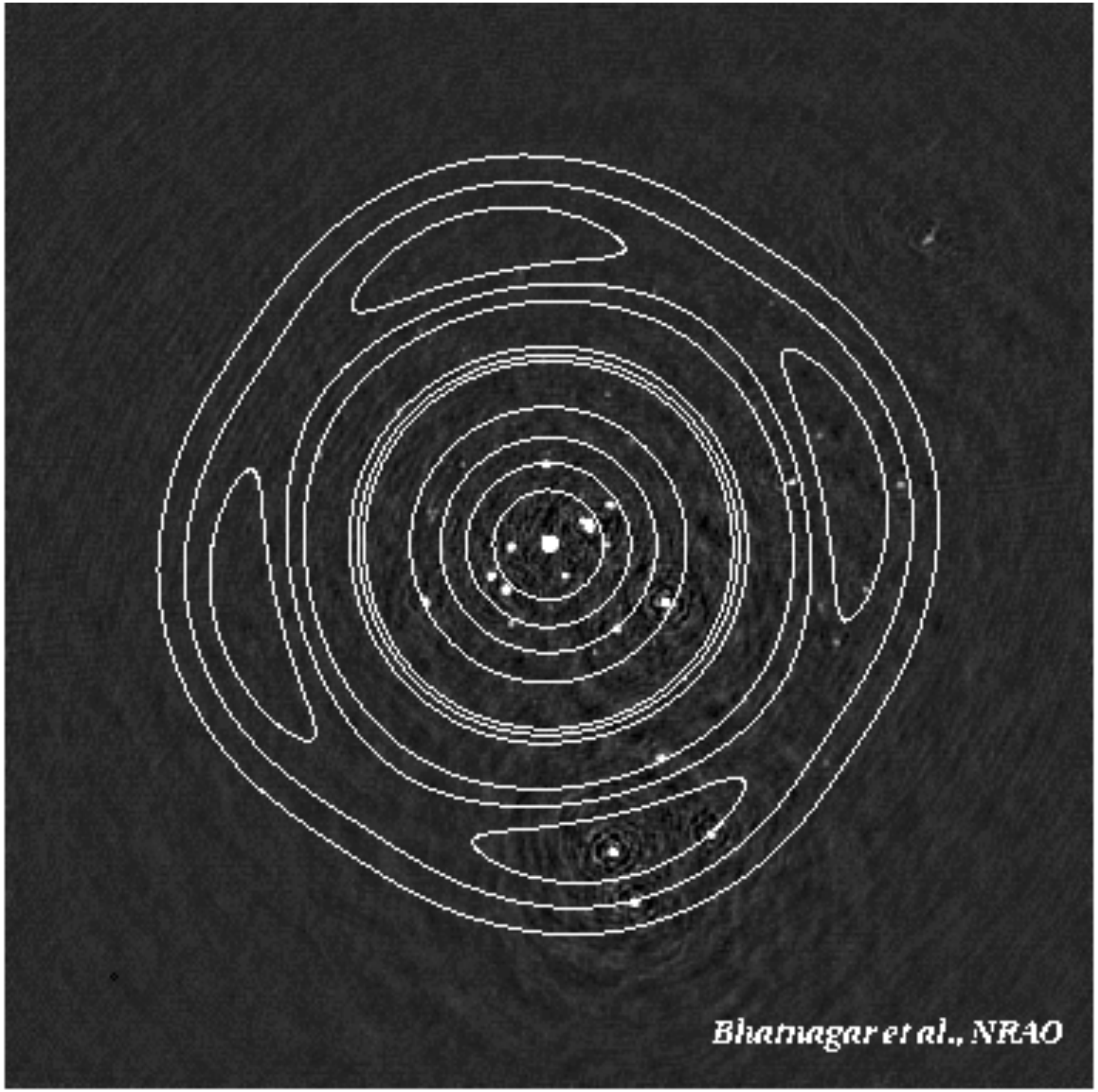
Single LBA station image

Ger de Bruyn



WSRT (25-m dish array) at 150 MHz

The beam is not constant with time



Correcting for the beam

Variable beams as a function of time mean that the contribution from each source will vary over time to the visibilities (must convolve sky model with beam model).

$$V_\nu(u,v) = \int\int A_\nu(l,m) I_\nu(l,m) e^{-2\pi i(ul+lm)} dl dm$$

More sophisticated calibration that includes the beam (a-projection is being implemented in CASA for the JVLA).

Issues:

1) How well do we know the beam? Recall, the beam is the FT of the aperture. What happens if a dipole stops working?

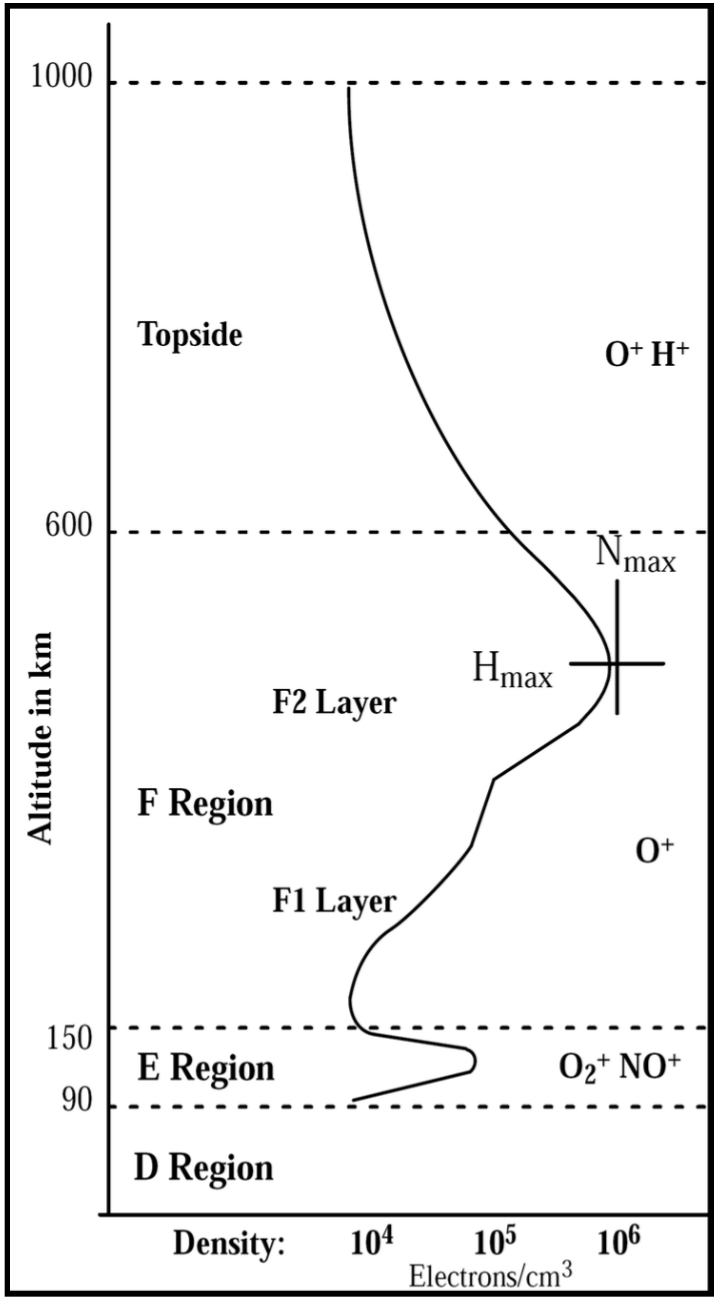
2) The beam changes as a function of frequency (FWHM ~ λ / D).

An error in your model can be absorbed in the calibration

$$\vec{V}_{ij} = J_{ij} \vec{V}_{ij}^{\mathrm{IDEAL}}$$

3. Direction dependent effects. II - The atmosphere

The ionosphere

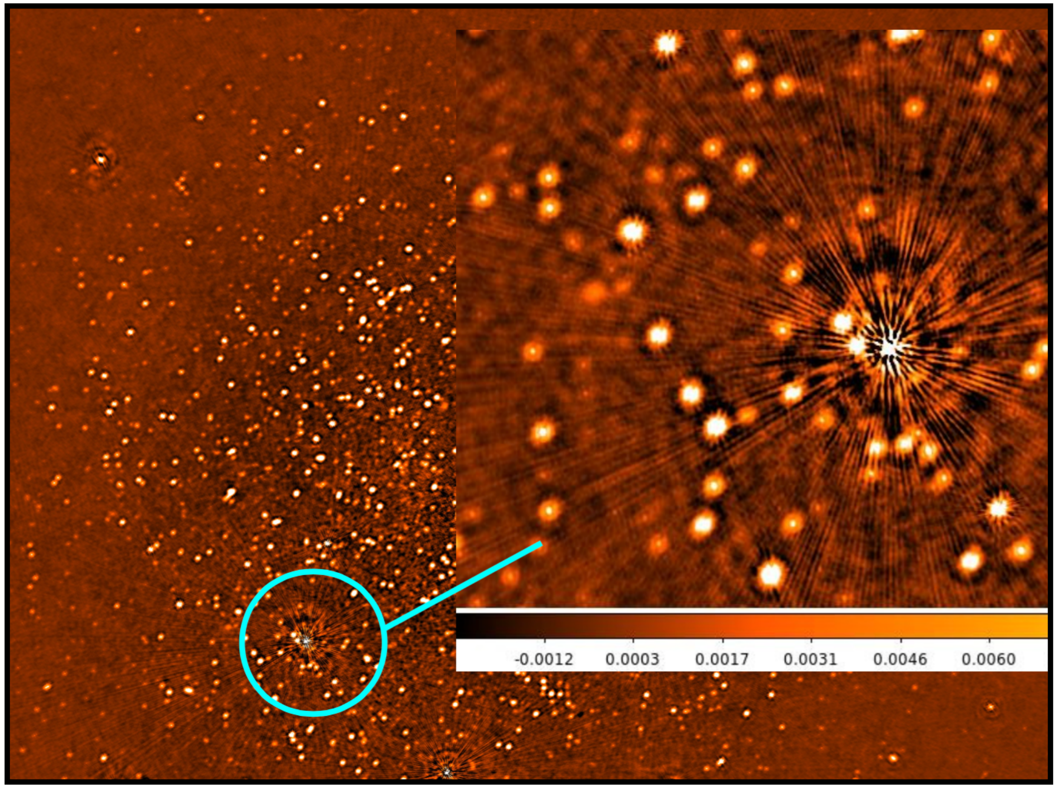


The ionosphere is a reflecting (to long wavelengths) layer of the atmosphere at ~ 125 km.

Structure and electron density changes with altitude.

Effects radio waves through:

1) Reflection (transparency)
2) Scintillation (continuum imaging)
3) Faraday rotation (polarisation)

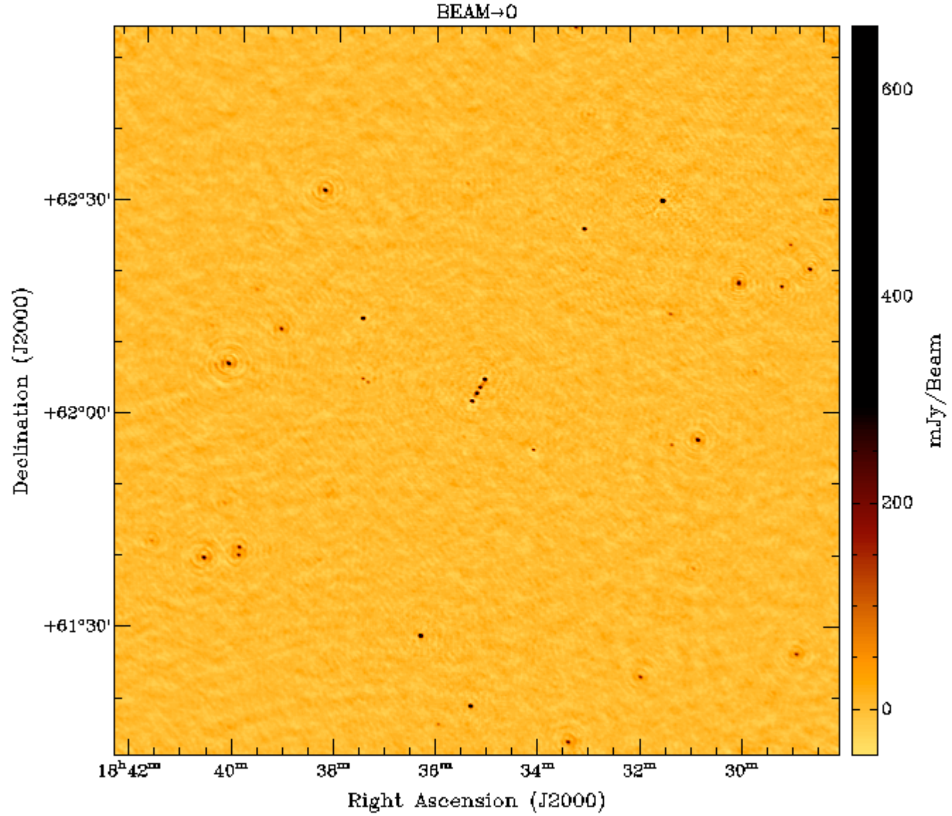


Direction dependent calibration

The solution to these issues is to calibrate of gains, not in a single position, but over several positions (10s to 100s) across the sky.

$$\vec{V}_{ij} = \sum_s J_{ij,s} \vec{V}_{ij}^{\text{IDEAL}}$$

Sarod Yattawatta

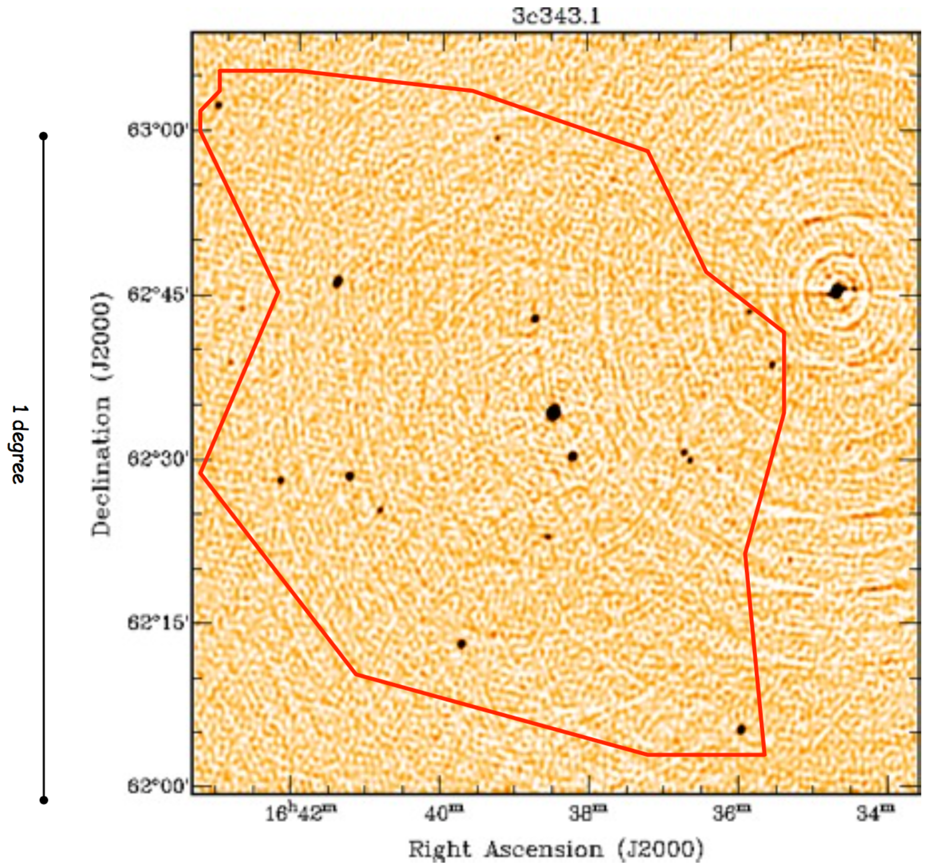


Computationally expensive and the robustness is a matter of (current) debate.

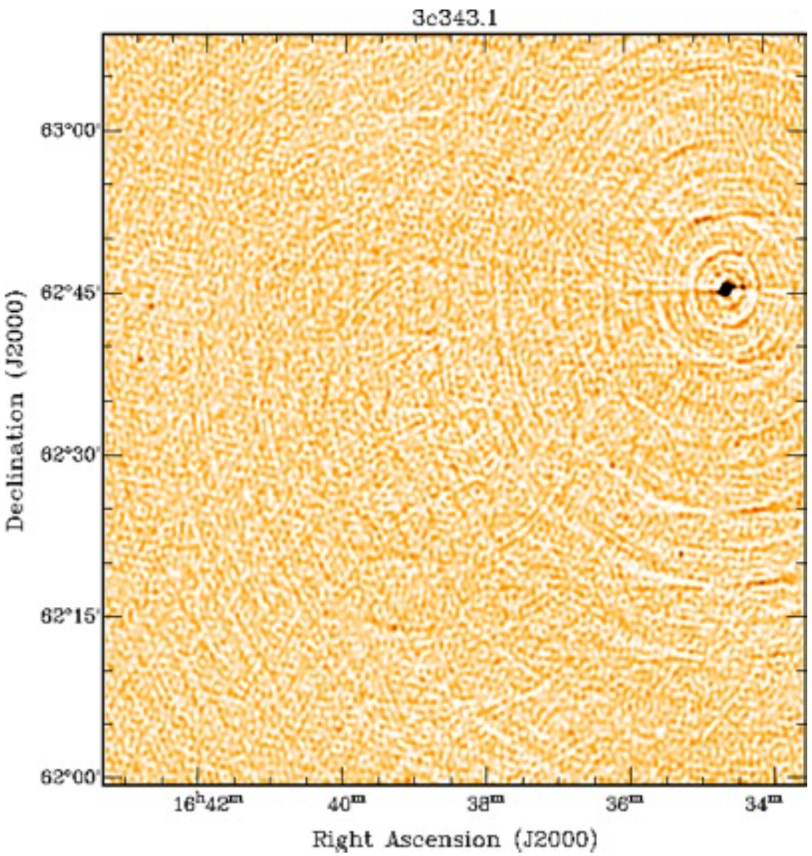
Direction dependent calibration

Alternatively, calibrate in one direction at a time and remove the troublesome sources (called peeling)

Tom Oosterloo



Full-field self-calibration



Subtract central sources only, leave off-axis source.

Direction dependent calibration

Alternatively, calibrate in one direction at a time and remove the troublesome sources (called peeling)

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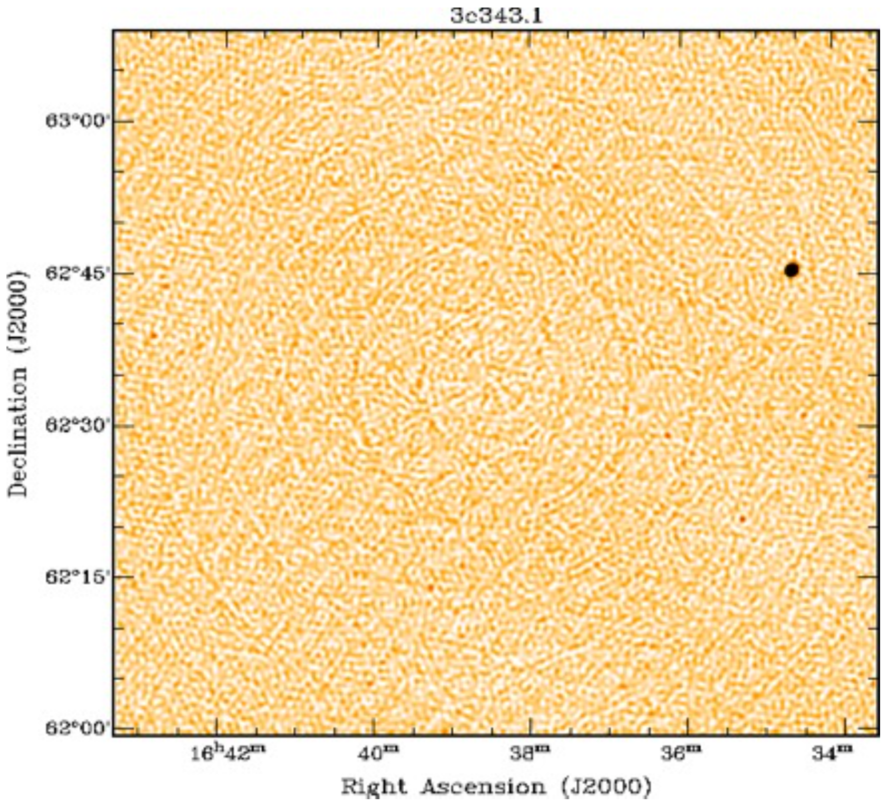
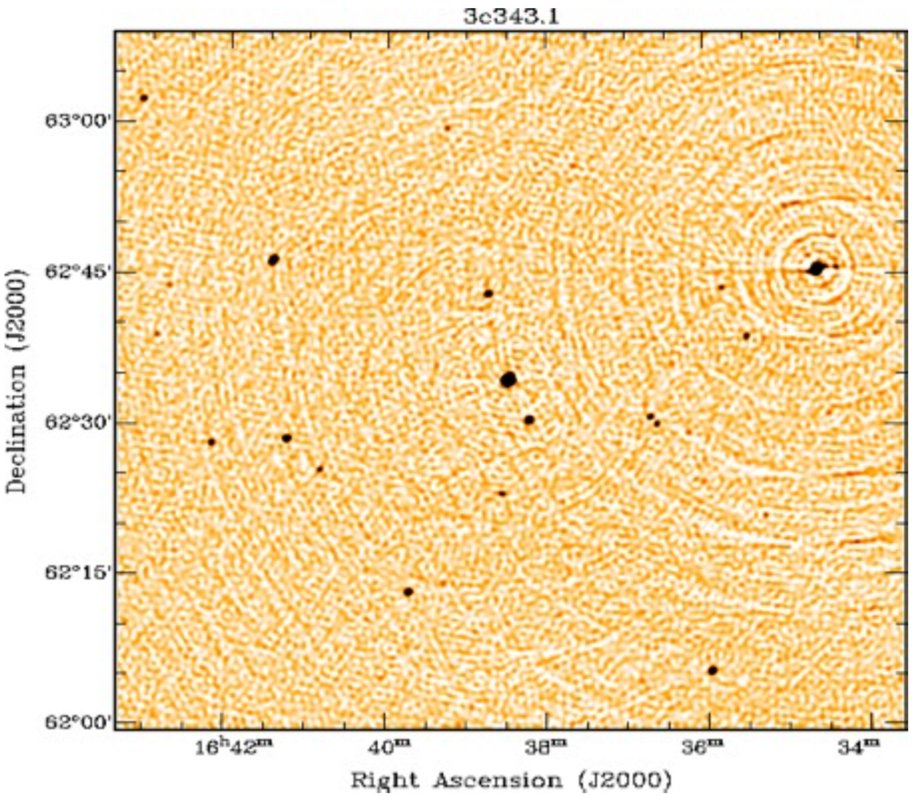


Self-Calibrate using model of off-axis source, apply calibrations and image

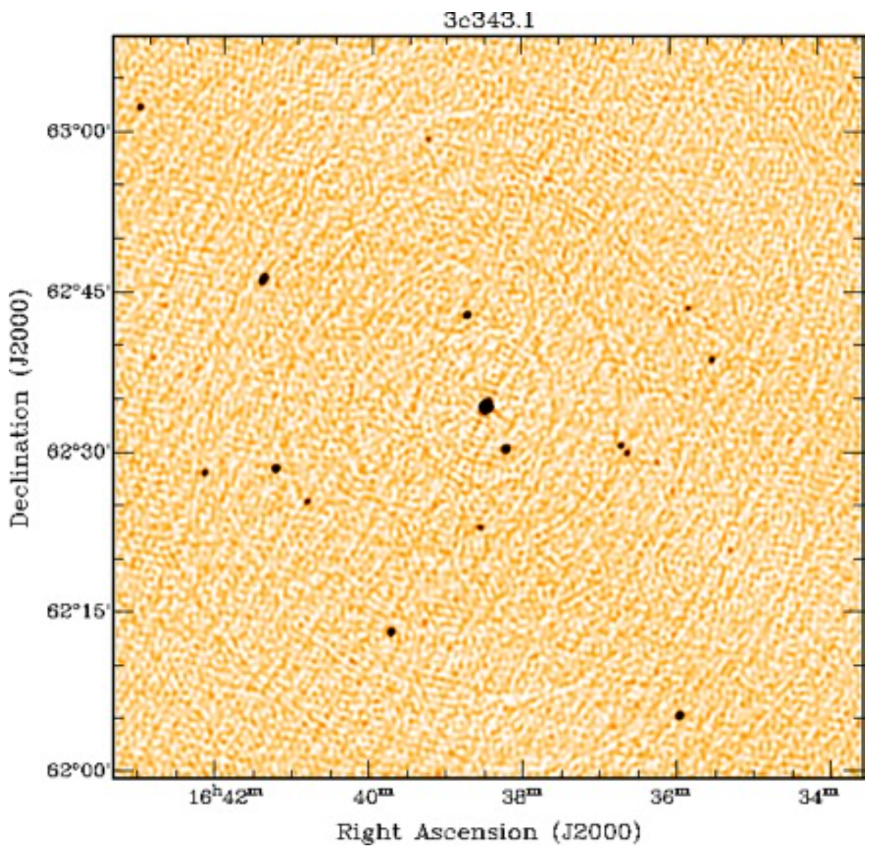


Apply-corrections to whole dataset and remove off-axis source. Remove any corrections.

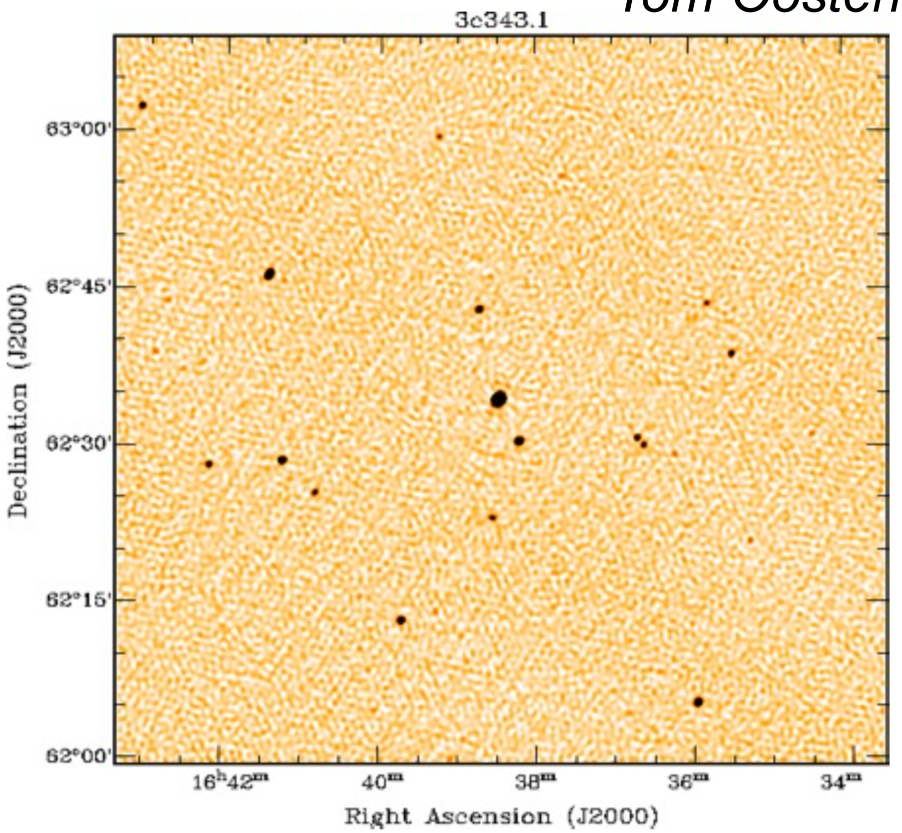
Direction dependent calibration

Alternatively, calibrate in one direction at a time and remove the troublesome sources (called peeling)

Tom Oosterloo



Make new image of the sky (without off-axis source).



Use self-calibration, apply calibration and make new image.

Direction dependent calibration

Alternatively, calibrate in one direction at a time and remove the troublesome sources (called peeling)

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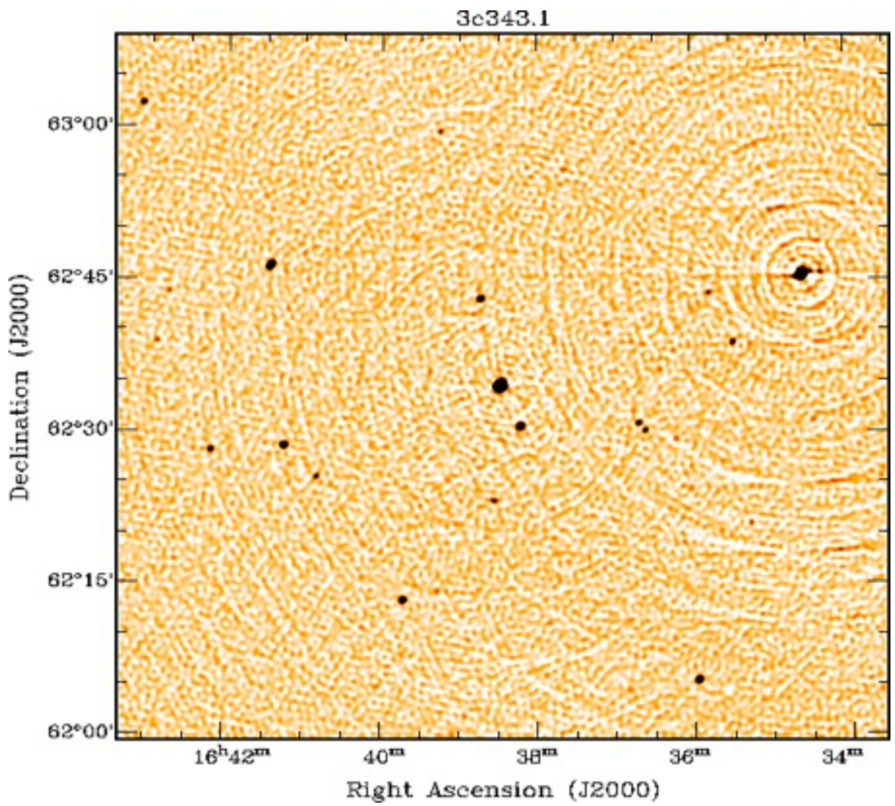
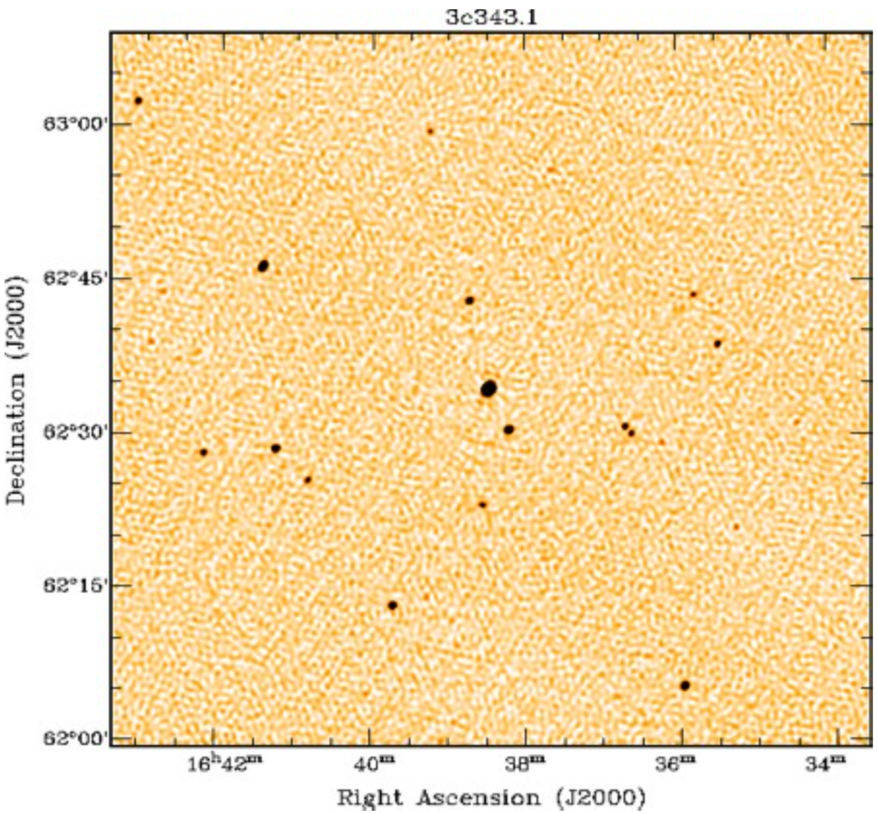


Before.



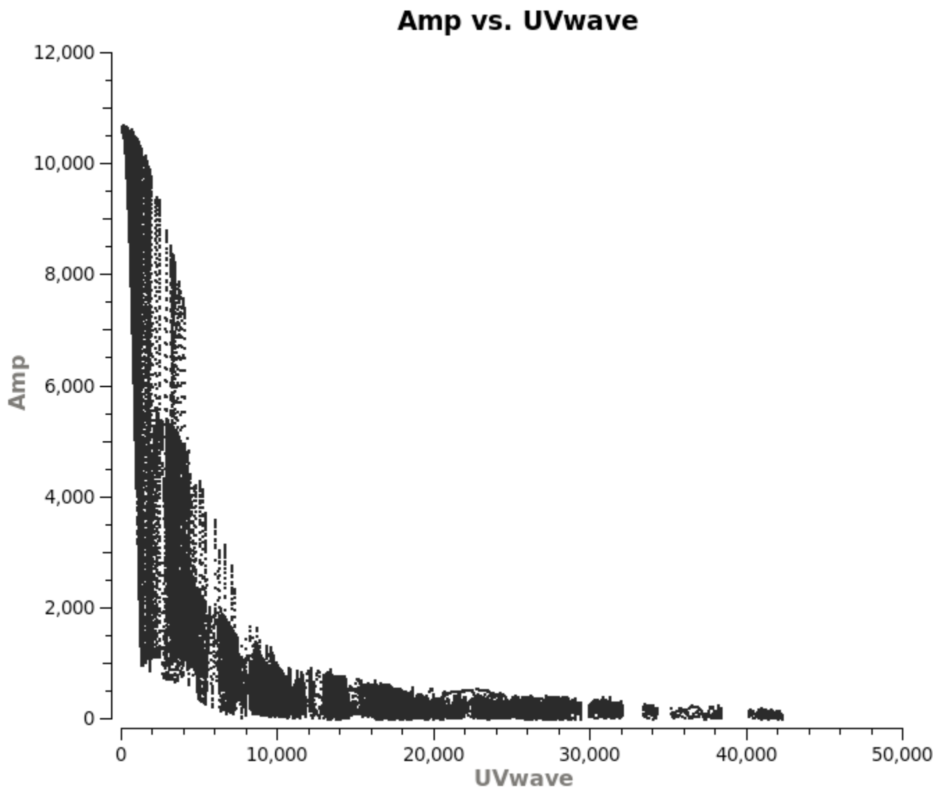
After peeling.

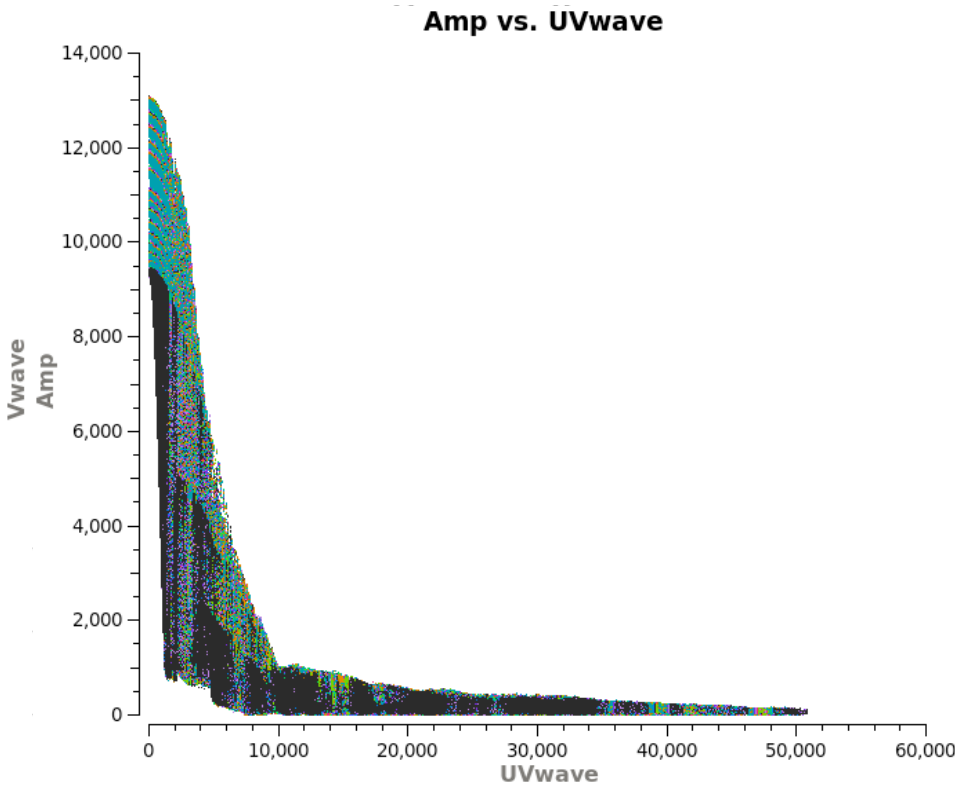
4. Spectral dependence of calibration

Dealing with large bandwidths

New interferometers have (fractional) large bandwidths.
Good for sensitivity: $\sigma_T \sim (\Delta\nu)^{-0.5}$
Better for image fidelity: good uv-coverage.

Must know the surface brightness distribution as a function of frequency.





Multi-Frequency Synthesis (MFS)

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We can represent the sky in emission interms of a Taylor expansion about some reference frequency (see Rau & Cornwell 2011).

Taylor co-efficient images

MS model image

Build *I(v)* model: $$\boldsymbol{I}^{\rm m}_{\nu} = \sum_{t=0}^{N_{\rm t}-1} w^t_{\nu} \boldsymbol{I}^{\rm sky}_t \quad \text{where} \quad w^t_{\nu} = \left(\frac{\nu - \nu_0}{\nu_0}\right)^t$$

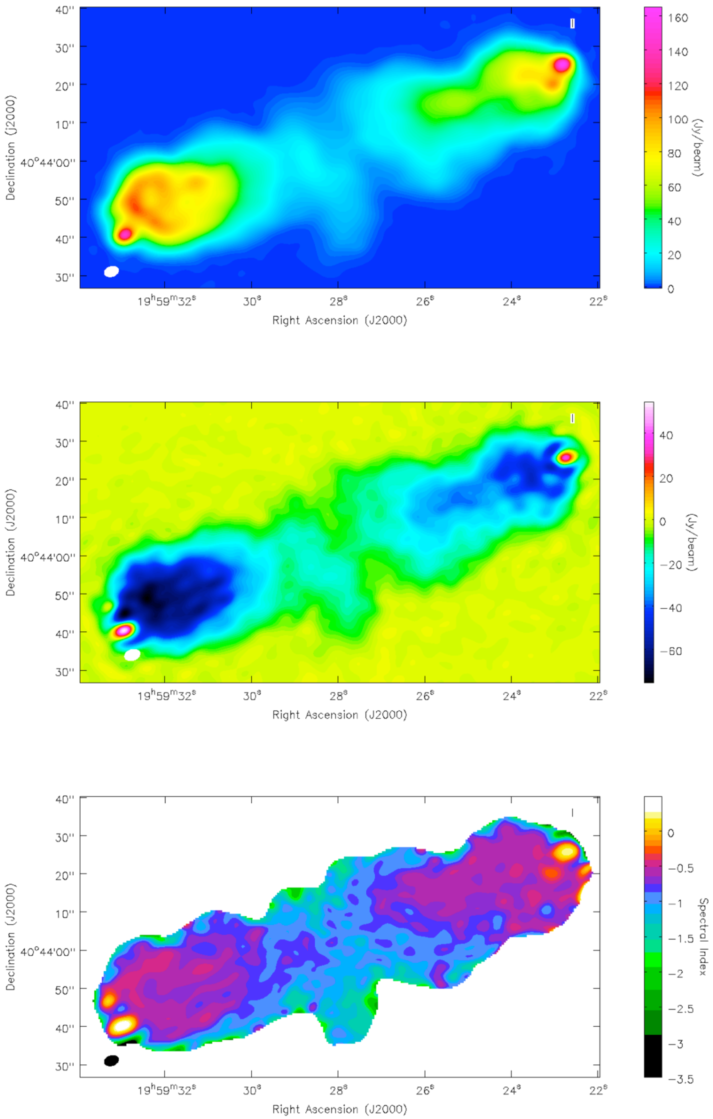
A power-law model is used to describe the spectral dependence of the sky emission.

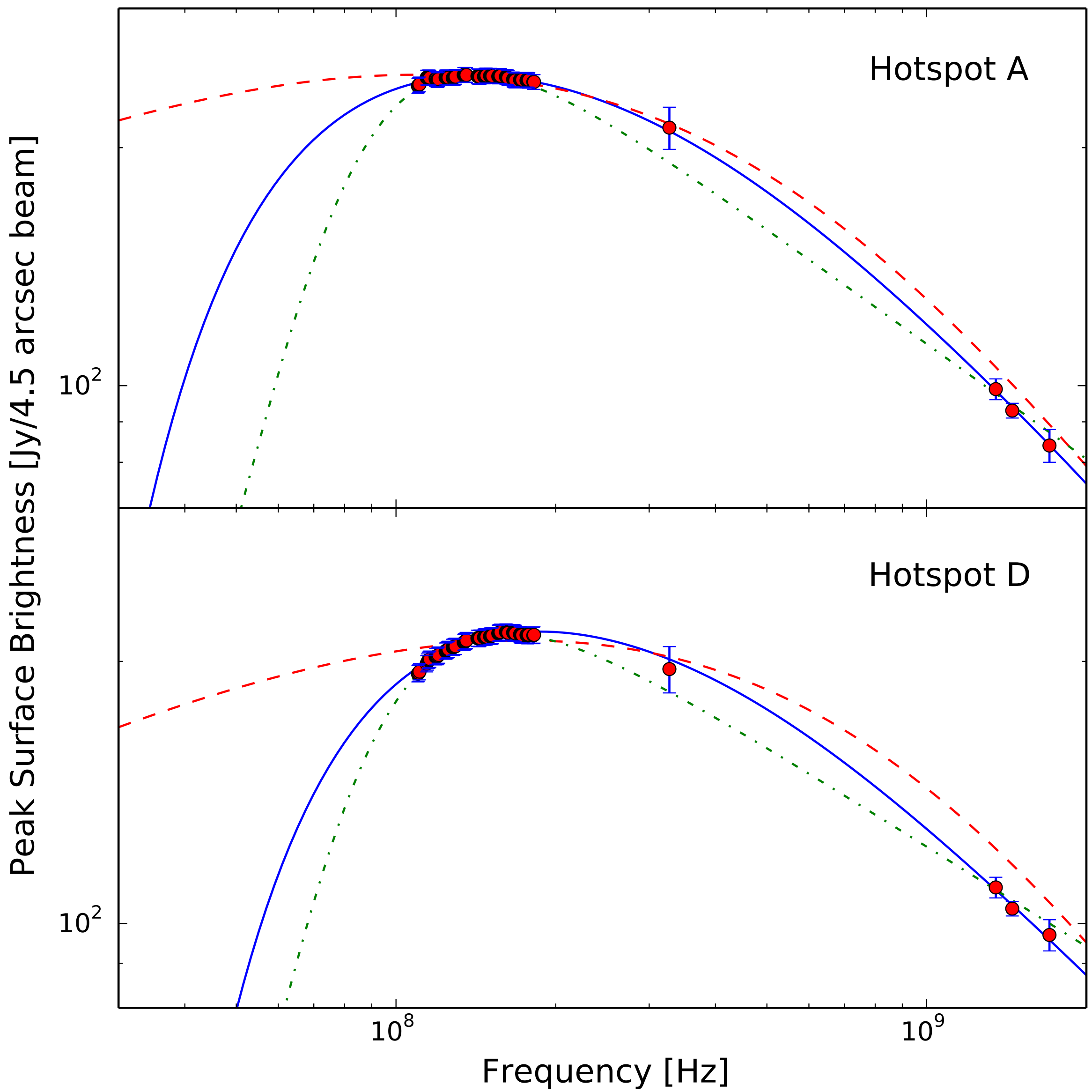
Parameterise: $$\boldsymbol{I}^{\rm sky}_{\nu} = \boldsymbol{I}^{\rm sky}_{\nu_0} \left(\frac{\nu}{\nu_0}\right)^{\boldsymbol{I}^{\rm sky}_{\alpha} + \boldsymbol{I}^{\rm sky}_{\beta} \log\left(\frac{\nu}{\nu_0}\right)}$$

Sky images: $$\boldsymbol{I}^m_0 = \boldsymbol{I}^{\rm sky}_{\nu_0} \;;\; \boldsymbol{I}^m_1 = \boldsymbol{I}^{\rm sky}_{\alpha} \boldsymbol{I}^{\rm sky}_{\nu_0} \;;\; \boldsymbol{I}^m_2 = \left(\frac{\boldsymbol{I}^{\rm sky}_{\alpha}(\boldsymbol{I}^{\rm sky}_{\alpha} - 1)}{2} + \boldsymbol{I}^{\rm sky}_{\beta}\right) \boldsymbol{I}^{\rm sky}_{\nu_0}$$

Imaging example: Cygnus A

LOFAR imaging at 109 to 183 MHz for 8 h on source.





Summary

1. The atmosphere, delay errors and electronics of the receiver systems will corrupt the signal from your target of interest.
2. Standard calibration transfer techniques, using bright and simple sources can eliminate most of these effects.
3. Residual errors can be removed using self-calibration providing you have sufficient signal-to-noise ratio, enough baselines, and an accurate model for your source.

Your calibration is only as good as your model since model errors will be absorbed into your calibration solutions.

4. Direction dependent effects will limit the quality of wide-field imaging due to time variable beam patterns, time variable ionosphere and our limited knowledge of the sky model.
5. New advanced calibration techniques are being tested and already show promise in reaching the thermal noise in the images, but careful study of the effects of direction dependent calibration need to be better understood.
6. Spectral variation in the sky model must also be taken into account due to the large bandwidths of the new telescope systems.

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