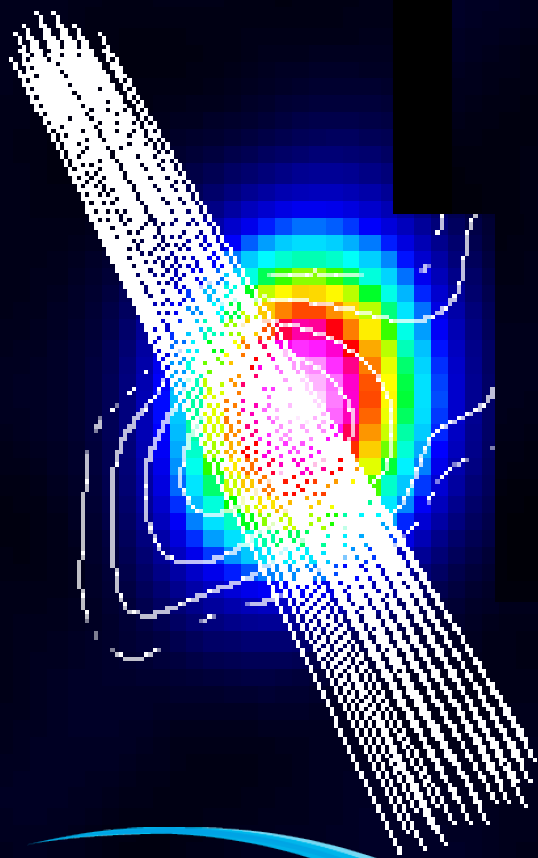
Polarisation Calibration



Anita Richards
Thanks to Ivan Marti-Vidal, Robert Laing, Rosita Palladino, Michiel Brentjens et al.

Main example:
Linearly polarized continuum target
Alt-az array with circular feeds

Summary: Polarisation

- Recap: Origins of hazards to be corrected
- Polarization calibration of 3C277.1
  - Circular feeds (L, R), linearly polarised target
- Practical approach for calibration
- Polarisation image products and analysis
- Other cases including:
  - Circular polarization e.g. Masers
  - Linear feeds (X, Y), e.g. ALMA

Hazards

- Above the telescope
  - Mostly high frequency
  - Mostly low frequency

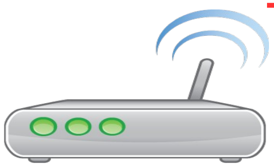
Close to AGN: Scintillation

Lobes Faraday rotation

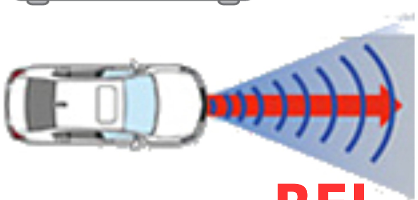
Galactic CO etc.

Ionospheric refraction/Faraday rotation

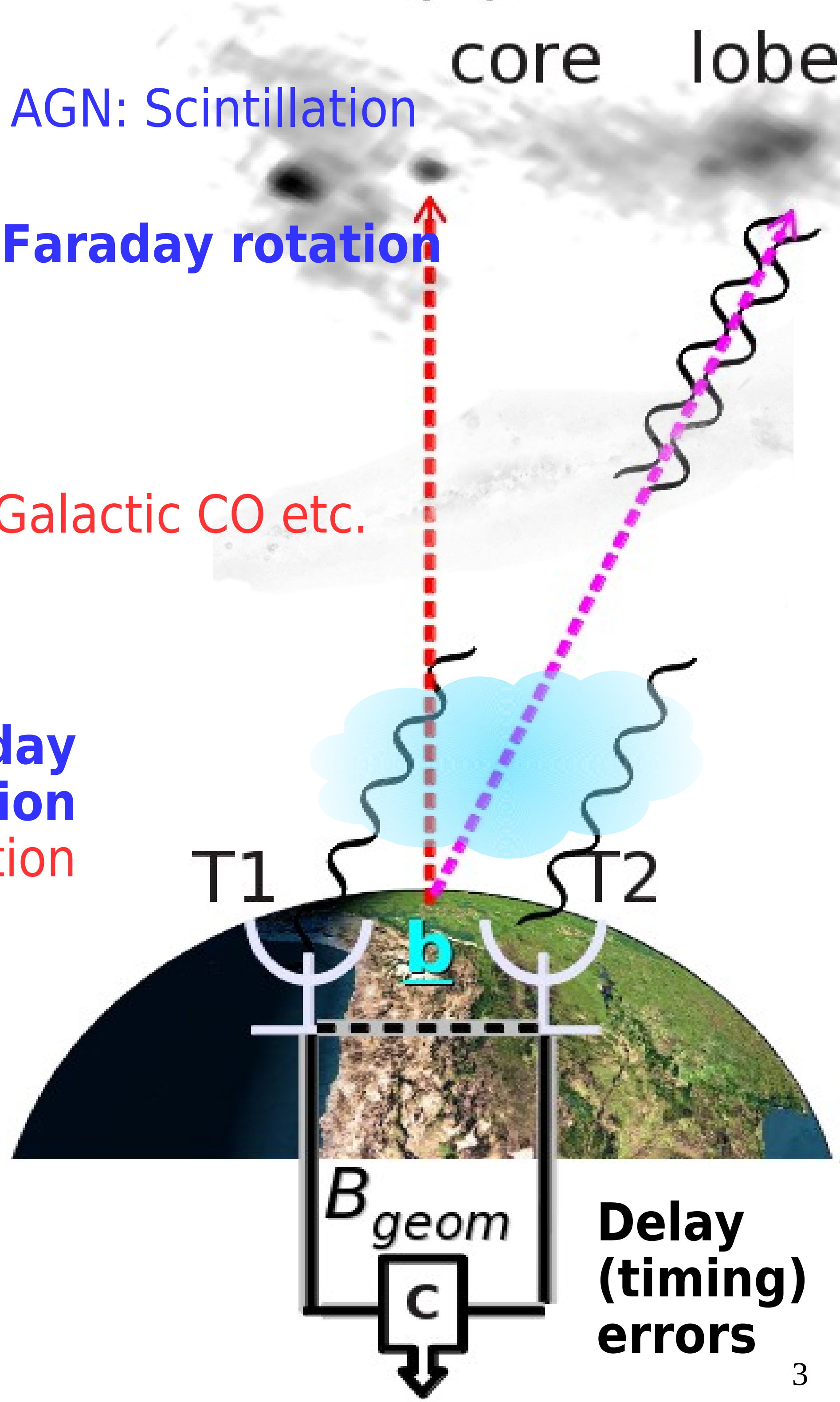
Tropospheric refraction/absorption



RFI



RFI



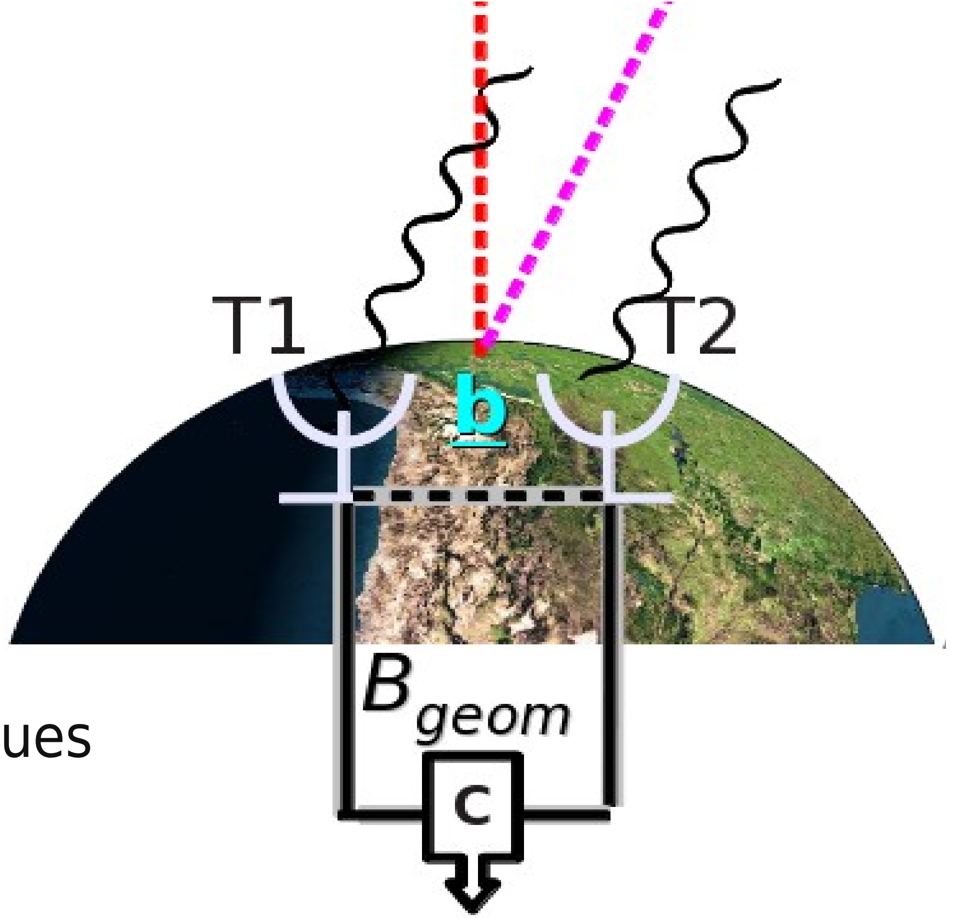
Delay (timing) errors

Hazards

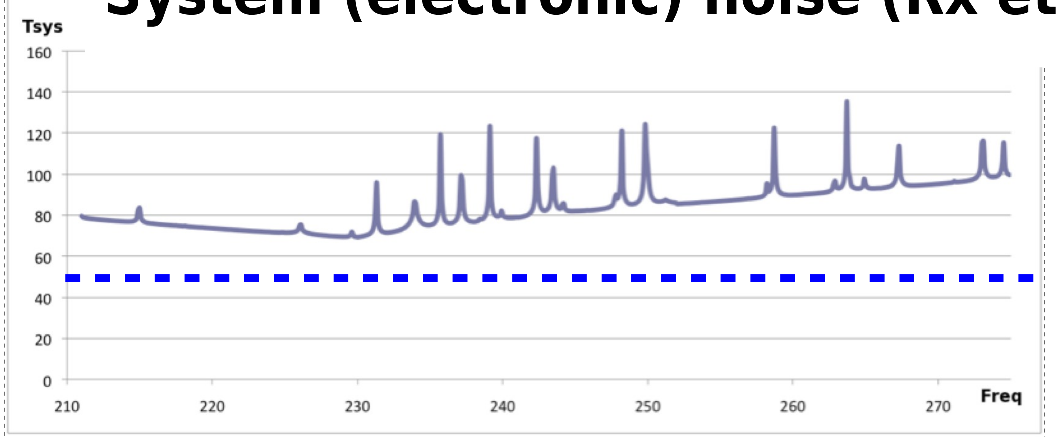
- At the telescope and later

Antenna positions
Pointing, Focus
Efficiency (**surface**)

Timing and frequency information issues
(station clock, local oscillator...)



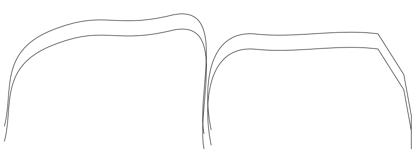
System (electronic) noise (Rx etc.)



Receiver leakage

Insufficient corrections for
delay tracking

Bandpass response



Measurement Equation

$$\underline{V}_{ij} = \mathbf{M}_{ij}\mathbf{B}_{ij}\mathbf{G}_{ij}\mathbf{D}_{ij}\int \mathbf{E}_{ij}\mathbf{P}_{ij}\mathbf{T}_{ij}\mathbf{F}_{ij}\mathbf{S}_{ij} S\, \underline{I}_n\,(x,y)\, e^{i2\pi\,(u_{ij}x+v_{ij}y)]}\, dx dy + \underline{A}_{ij}$$

Vectors

Starting point

$\underline{V}$ isibility = *f(u,v)*

$\underline{I}$ mage

Goal

$\underline{A}$dditive baseline error

Scalars

Methods

S (mapping $\underline{I}$ to observer polarization)

x,y image plane coords

u,v Fourier plane coords

i,j telescope pair

Jones Matrices

Hazards

Multiplicative baseline error

Bandpass response

Generalised electronic gain

Dterm (pol. leakage)

E (antenna voltage pattern)

Parallactic angle

Tropospheric effects

Faraday rotation (ionospheric)

S Faraday rotation (astronomic)

Calibration and polarisation

- Basic calibration derives corrections per-antenna
  - Phase and amplitude v. frequency (delay, bandpass) **B**
  - and v. time (phase calibrator solutions, self-cal) **G**
    - Incl. ionospheric and tropospheric refraction corrections
- Separate R and L corrections applied to all correlations
- These were performed in Calibration and Self-calibration
- Now correct RL and LR
  - Parallactic Angle (instrumental) **P**
  - Cross-hand delays (mostly instrumental) **B**
  - Polarisation leakage in receiver systems (instrumental) **D**
  - Faraday rotation of polarisation angle (ionosphere etc.) **F**,**S**

Stokes parameters

Circular feeds, correlated visibilities (see Brentjens talk)

$$I = \left(R_1\ R_2^{*} + L_1\ L_2^{*}\right)/\,2$$

$$Q = \left(R_1\ L_2^{*} + L_1\ R_2^{*}\right)/\,2$$

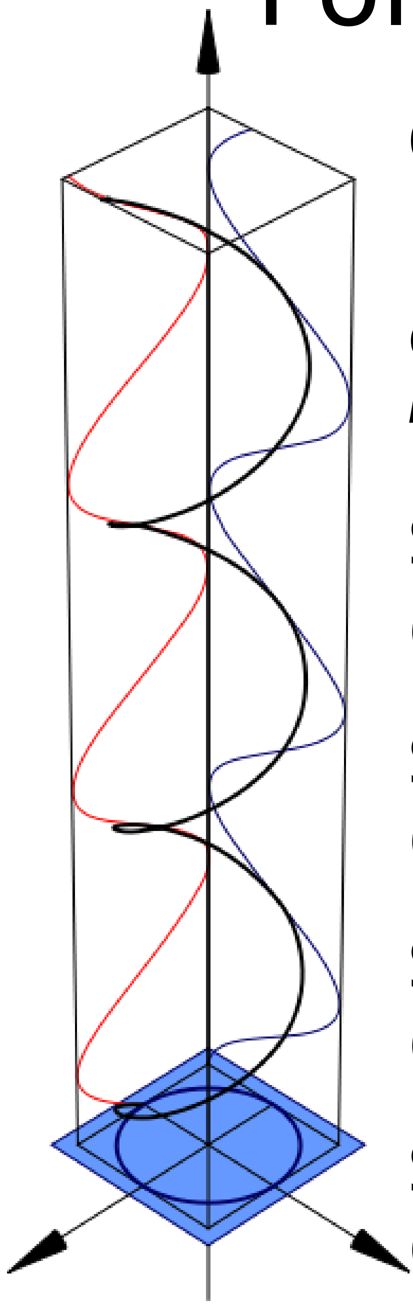
$$U = \left(R_1\ L_2^{*} - L_1\ R_2^{*}\right)/\,2\ i$$

$$V = \left(R_1\ R_2^{*} - L_1\ L_2^{*}\right)/\,2$$

- Extra-galactic continuum sources usually only show linear cm-wave polarisation at ≳10-mas resolution
  - *I, Q, U* of interest; *V* reveals residual leakage
- Polarised intensity $P = \sqrt{(Q^2+U^2)}$
- Polarisation angle χ = ½ atan(U/Q)
  - Image polarisation angle half visibility R-L phase offset
    - You can prove using formal definition of Stokes parameters in terms of electrical vector angle (Brentjens); recall:

$$Ae^{i\phi} = A\left(\cos\phi + i\sin\phi\right) \qquad \sin 2\phi = 2\sin\phi\cos\phi \qquad \cos 2\phi = \cos^2\phi - \sin^2\phi$$

Polarisation jargon: Rx feeds



CIRCULAR feeds

Left/Right/cross correlations
LL RR LR RL

Stokes I = *(RR+LL)/2*

Stokes Q = *(RL + LR)/2*

Stokes U = *(RL – LR)/2i*

Stokes V = *(RR-LL)/2*

Diagrams thanks to Wikipaedia

LINEAR feeds

Correlations
XX, YY, XY, YX

Stokes I = *(XX+YY)/2*

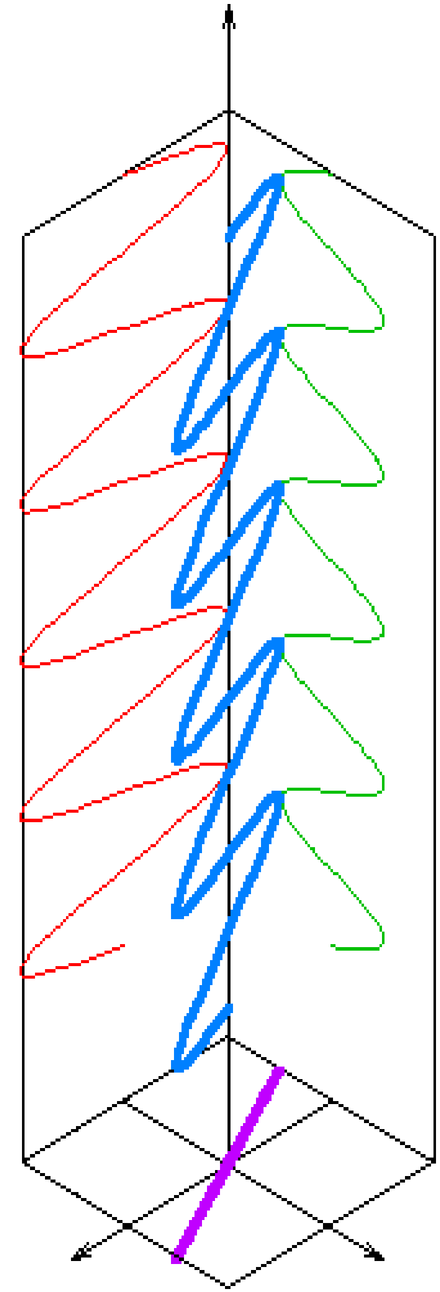
Stokes Q = *(XX – YY)/2*

Stokes U = *(XY – YX)/2*

Stokes V = *(XY – YX)/2i*

Polarised intensity P $= \sqrt{(Q^2+U^2+V^2)}$

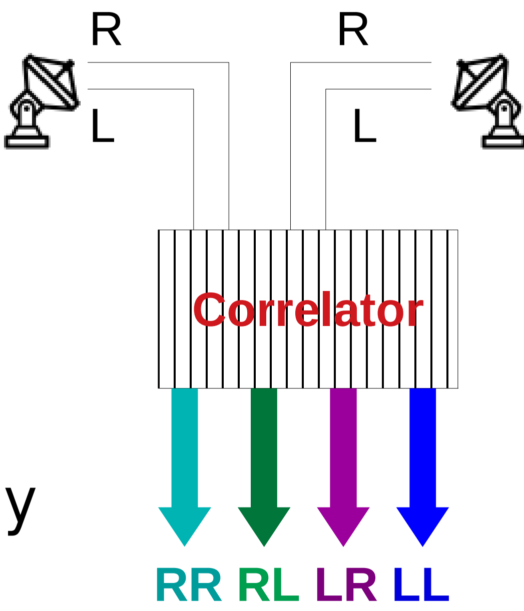
Polarisation angle χ $= ½ \operatorname{atan}(U/Q)$



3C277.1 total intensity prior calibration

- 3C277.1 aka 1252+5634 and calibrators as used for Calibration and Self-calibration tutorial
- Calibration and Self-calibration scripts run
  - `refantmode='strict'` ensures consistent R-L phase
- Apply all flagging & calibration to the relevant sources in the **full** data set **`all.avg.ms`**
  - Including self-calibration of target
    - So, slightly different approach from Imaging script
- Split out 'CORRECTED' column into **`all.avg.pol.ms`**
  - New `'DATA'` column visibilities have LL RR calibrated
    - Polarization calibration for RL and LR and relative R-L

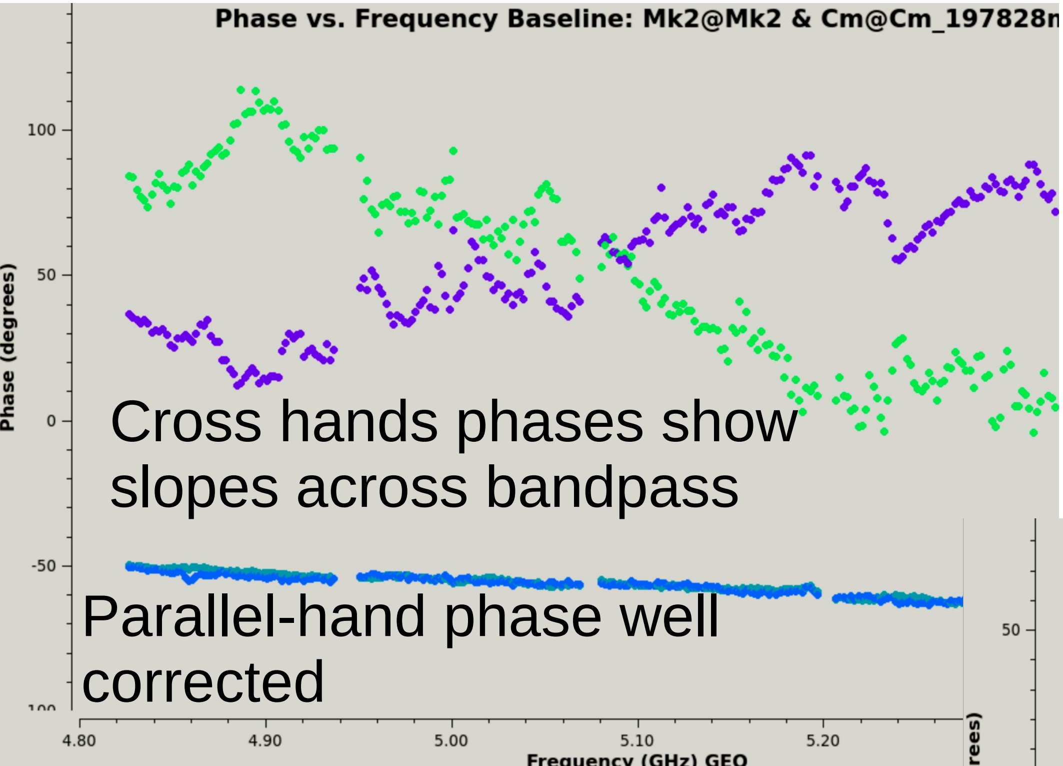
Polarization calibration
(for circular feeds)



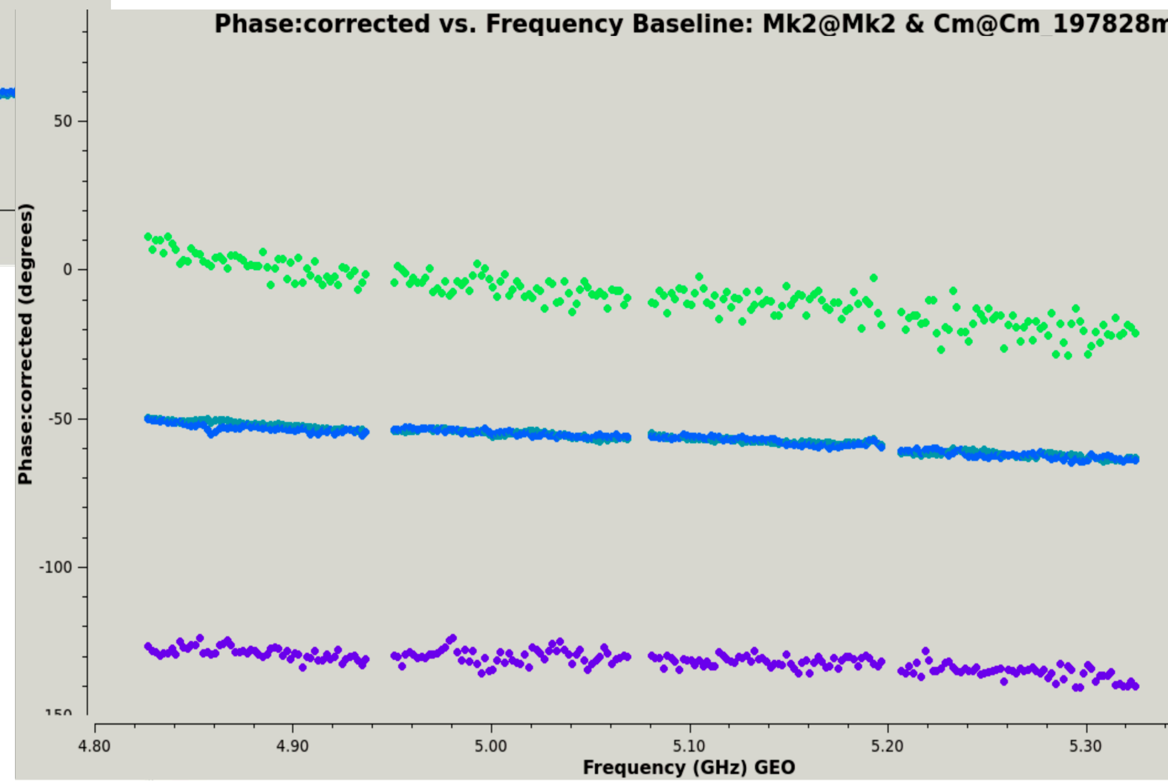
1) Correct cross-hand delay
   - Strongly-polarised calibrator
     - Fit to cross-hand phase slope with frequency
2) Correct leakage ('D-terms')
   - Calibrator model – here, 3C84 is known to be unpolarised
   - If calibrator has non-zero polarisation, calculate:
     - Need at least 3 scans over 6 hr
     - Use parallactic angle rotation to deduce source polarization
3) Apply parallactic angle correction
4) Correct rotation of pol. angle (L-R phase difference)
   - Calibrator with known angle

Noun project

Cross-hand delay corrections



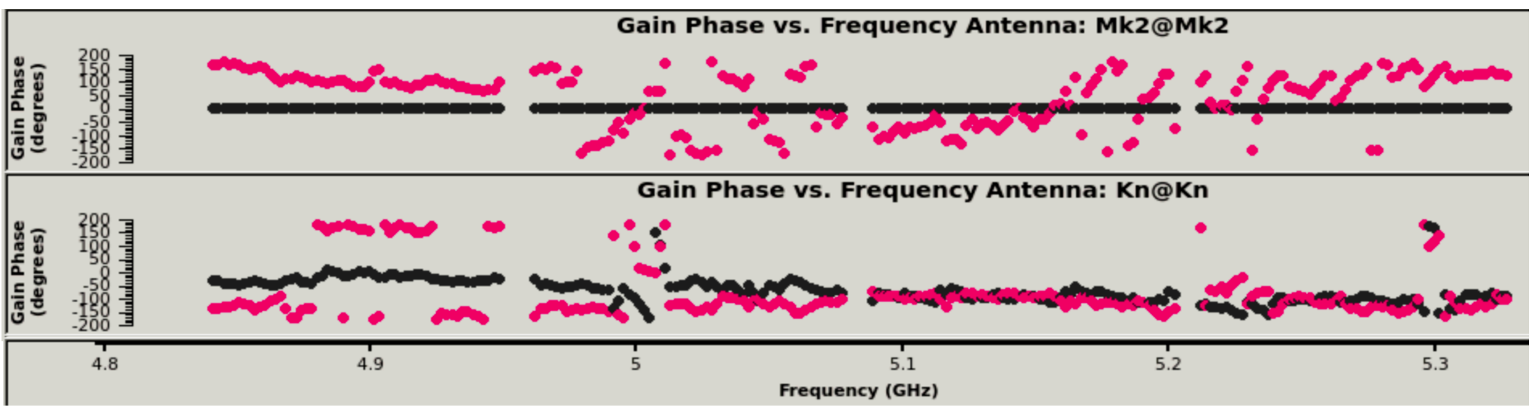
- Use 3C286 model to correct cross-hand delay (remove slope across band



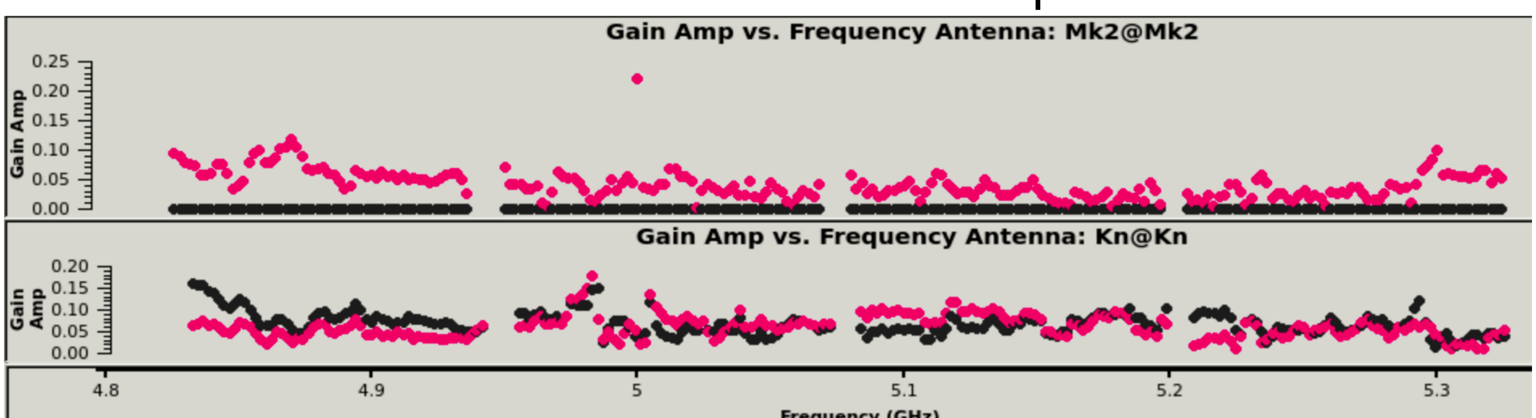
- After cross-hand delay, parallactic angle and other corrections, phase is flat across band
  - Slight slope due to source structure/ spectral index

Leakage corrections

- Rx system does not separate polarisations perfectly
  - Leakage constant during observations, frequency-dependent
- Derive corrections using source of known polarisation
  - Here, 3C84 has known, zero polarisation

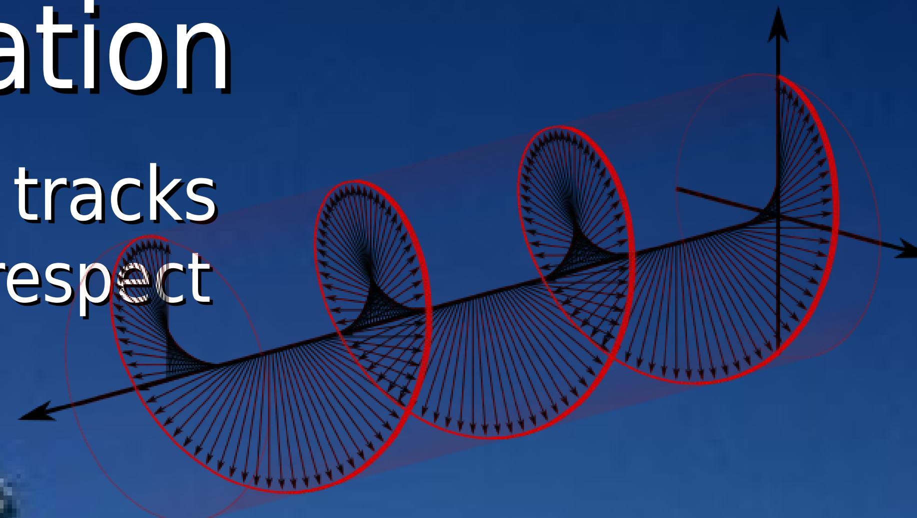


Derived corrections for L and R polarisations

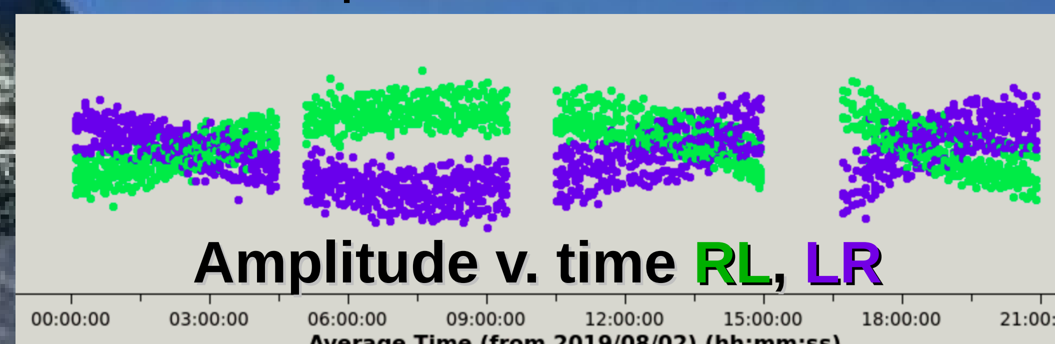


Parallactic angle rotation

- An alt-az telescope rotates as it tracks
  - The receiver feeds rotate with respect to a celestial source



- Cross hands of polarisation undulate diurnally
  - Geometric correction required

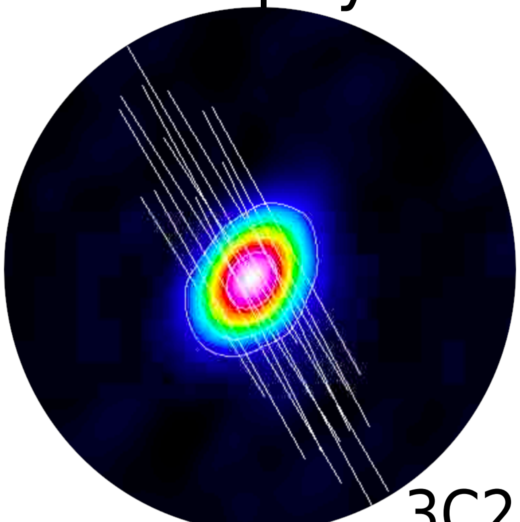


~9-hr track.
Phase similarly rotates

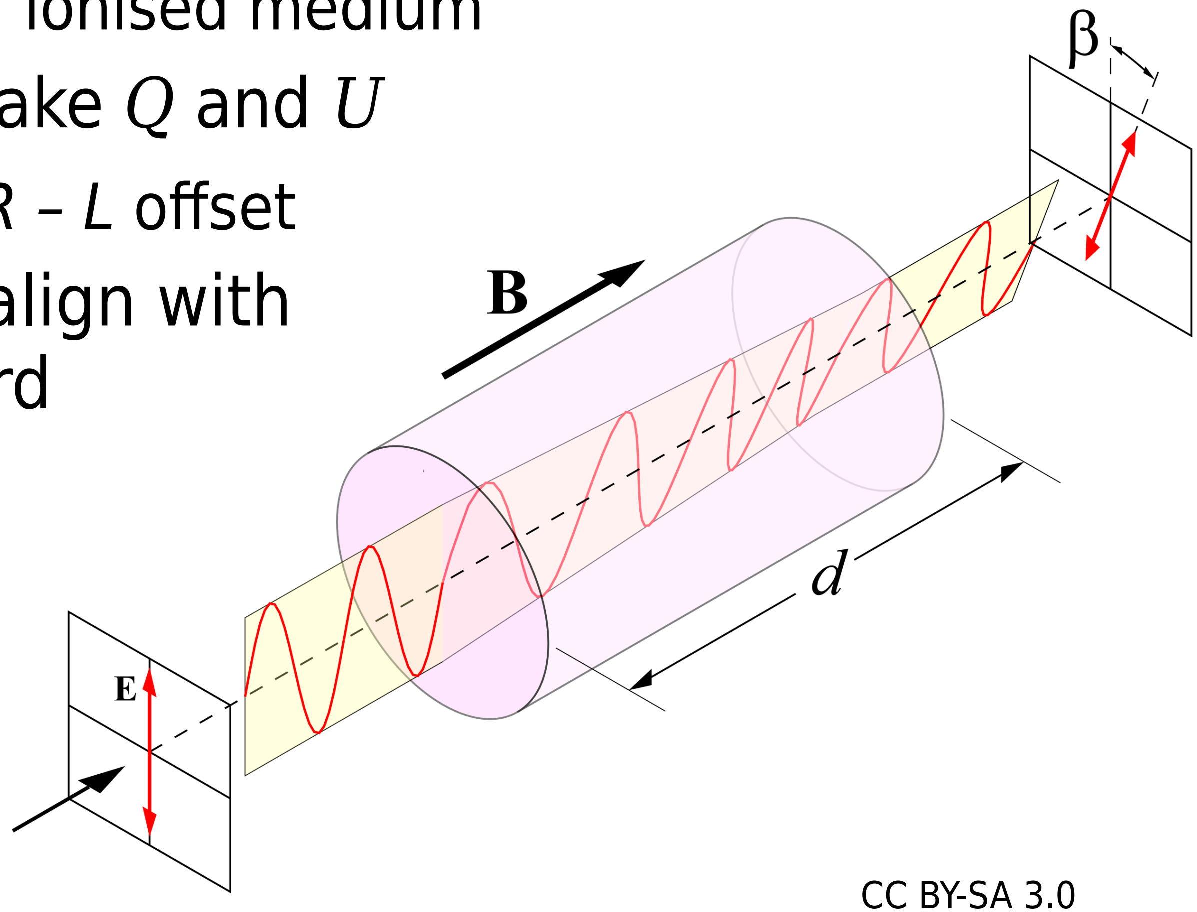
Bill Petrachenko NRCan
Alex Dunning, ATNF
Wikipedia

Polarisation angle calibration

- Ionosphere/ISM Faraday rotation of linear pol. angle χ by angle β
  - Depends on magnitude and direction of magnetic field B and depth/density of ionised medium
- Correlations *RL, LR* make *Q* and *U*
  - Based on observed *R* – *L* offset
- Derive correction to align with astrophysical standard

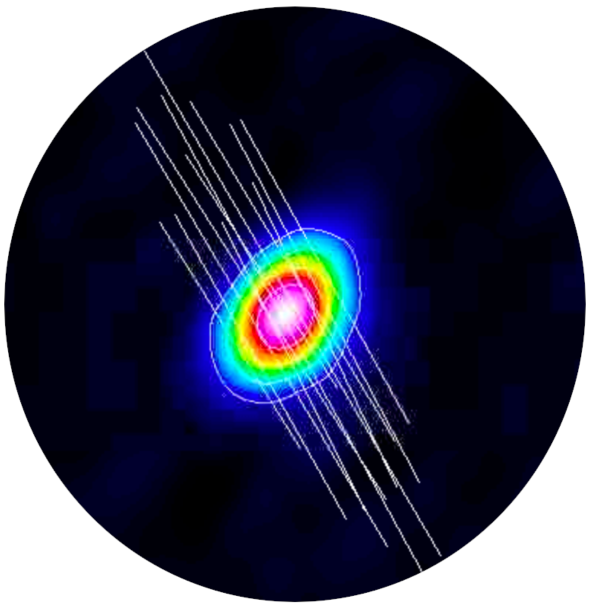


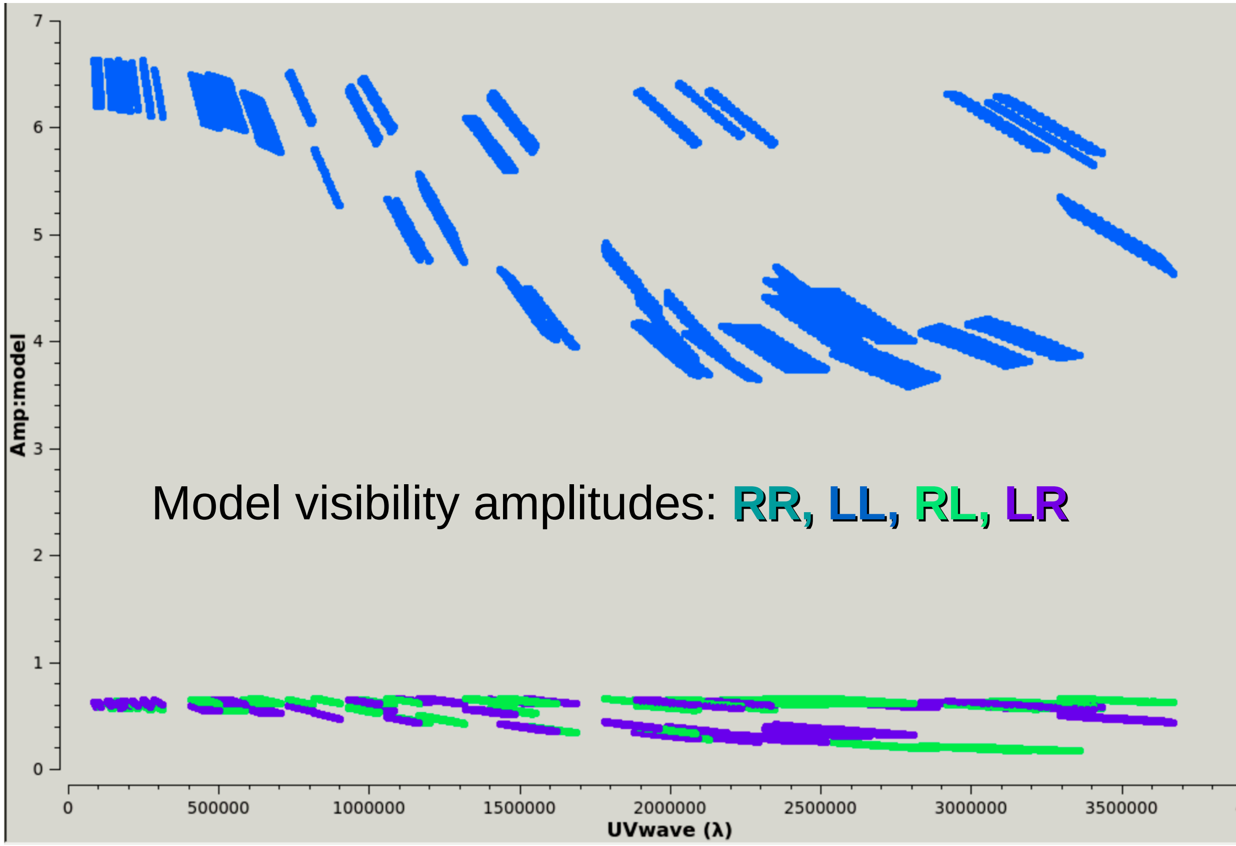
3C286



3C286 polarised model

- 3C286 has strong, well-studied, stable polarisation
  - see VLA etc. catalogues
  - e-MERLIN model (slightly resolved around 5 GHz)
    - Position angle χ 33^0
    - ~10% polarised





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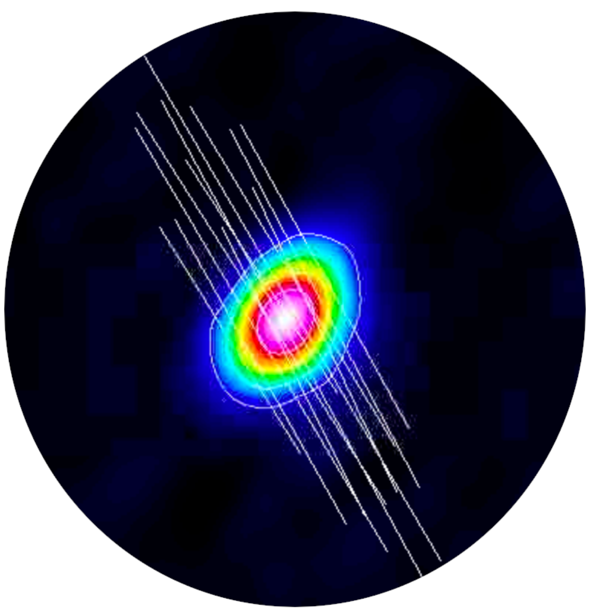
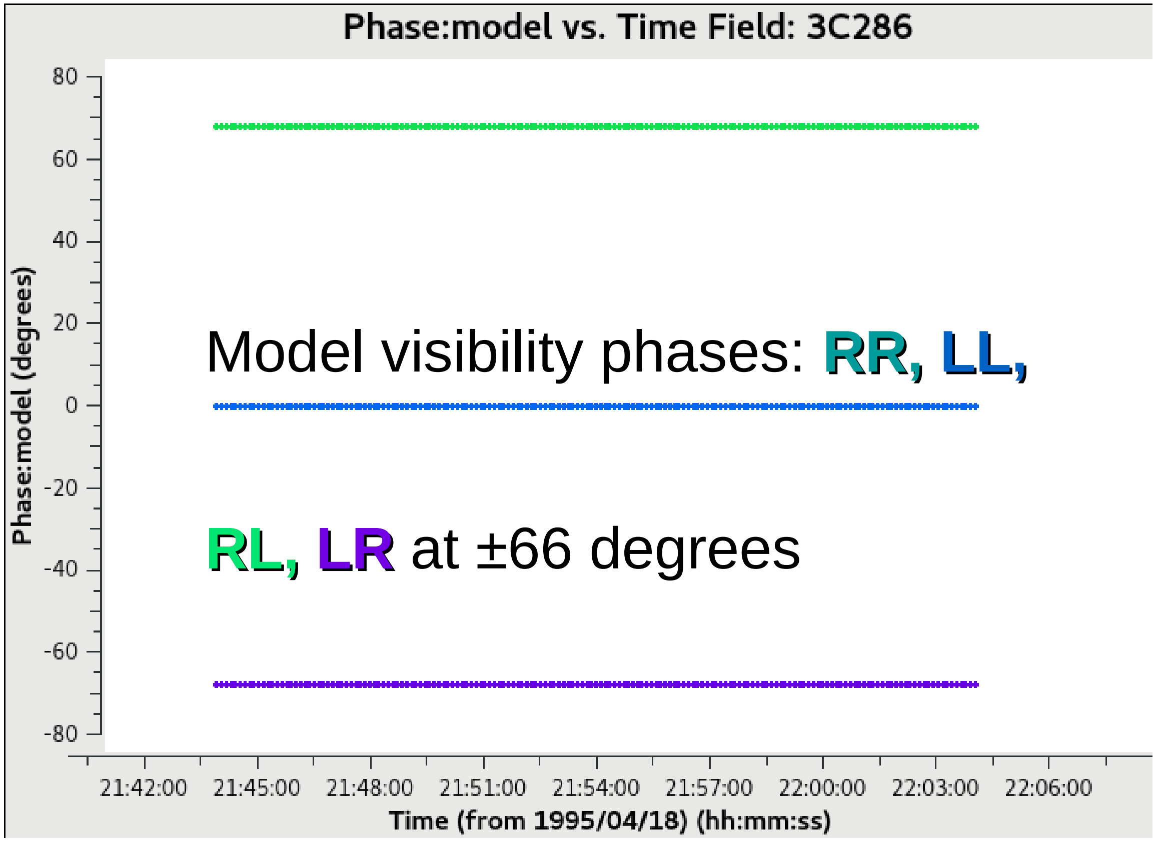
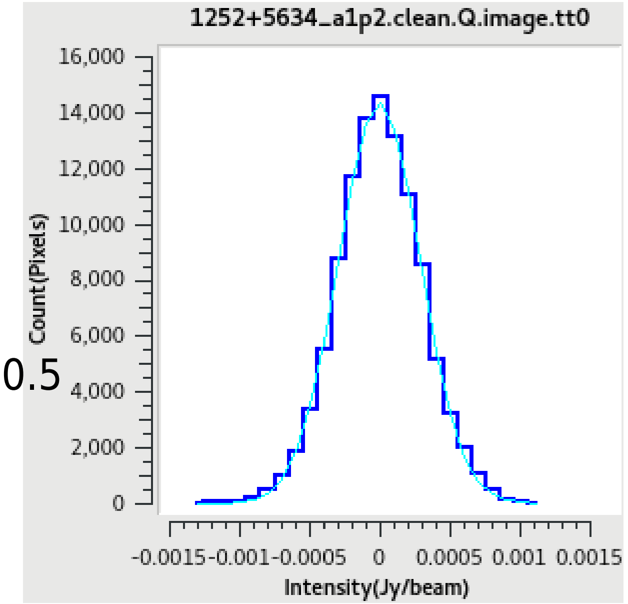




Polarization imaging

- Apply total intensity calibration and:
  - Cross-hand delay corrections KCROSS (if required)
  - Leakage 'D' terms
  - Parallactic angle correction
  - Pol. angle correction
- In `tclean: stokes='IQUV'` (or chosen products)
  - Even if you expect no circular pol., V useful diagnostic
- Interactive masking: click 'all polarizations' if you expect the same total I and polarization distributions
  - I may be dynamic range limited, lower noise in QUV
  - After cleaning, can make:
    - (linear) polarized intensity image $P = (Q^2 + U^2)^{0.5}$
    - polarization angle $\chi = ½\text{atan}(U/Q)$ (use atan2 to remove ambiguity)

De-biasing and blanking



- Linear polarized intensity $P = (Q^2 + U^2)^{0.5}$
  - *Q* and *U* image noises sums to zero
    - But *P* image must be entirely positive
  - *P* will appear too high due to 'Rician bias'
- Rician bias is complicated depending on S/N
  - See e.g. *Wardle & Kronberg 1974; Bon Wong Sohn 2011*
  - Weak polarization: $P_{corrected} \sim (P_{obs}{}^2 - \sigma_P{}^2)^{0.5}$
    - Rayleigh approximation at low S/N
  - CASA task `immath` can de-bias using parameter `sigma`
    - Or viewer estimates bias to remove for suitable images
- Also blank pol. angle image input maps at $\sim\sigma_P$

Polarisation accuracy

- Leakage: We assumed 0319+4130 is unpolarised.
  - Apply the polarisation calibration
  - Image 0319+4130 and make Poli P image
    - At I peak position, P / I is fractional linear leakage
- Polarization Angle: 1331+3030 core has constant χ
  - In aperture enclosing I peak, measure Pola (χ) rms
- Circular polarization
  - At cm λ, resolution ≳few mas, QSO cores have V~zero
    - Apparent V tells you circular leakage
    - Measure 0319+4130 V / I

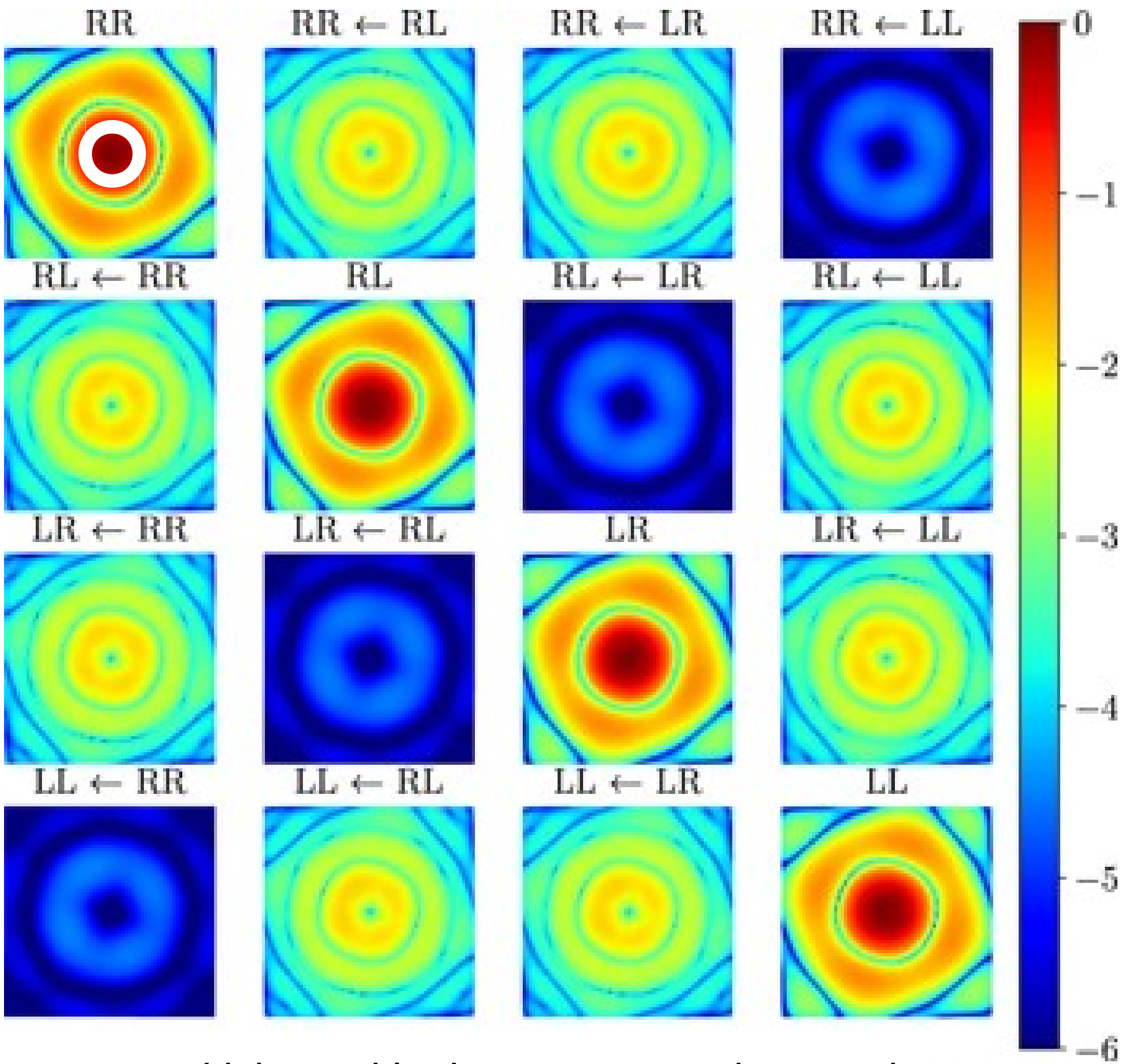
Additional considerations

- Bad data, especially RFI can affect polarisations selectively
  - Make sure all correlations are flagged if one is to avoid biasing polarisation
- Even if you are not interested in polarization, it should be calibrated for high dynamic range, wide-field imaging
  - The *I, Q, U, V* primary beam responses differ
- For high accuracy on long baselines, parallactic angle rotation should be corrected to align the L and R phases
  - Position error ~1% interferometric beam at 1.6 GHz
    - e.g. 2 mas at 200 mas resolution - significant at S/N >100
- *I* is affected by leakage from *V* for circular feeds and (worse) from *Q* for linear feeds

https://www.alma.inaf.it/index.php/European_ARC_polarization_internal_workshop

Primary beam response

- Primary beam affected by surface setting, asymmetry, relative feed orientation, receiver optics, reflections...
  - Colour log scale of fractional sensitivity (diagonal plots)
- Off-diagonal plots show asymmetries and leakage
  - <few% within FWHM (white circle), as for 3C277.1
- Direction-dependent calibration essential for wider fields



VLA sensitivity and leakage *Jagannathan et al. 2017*

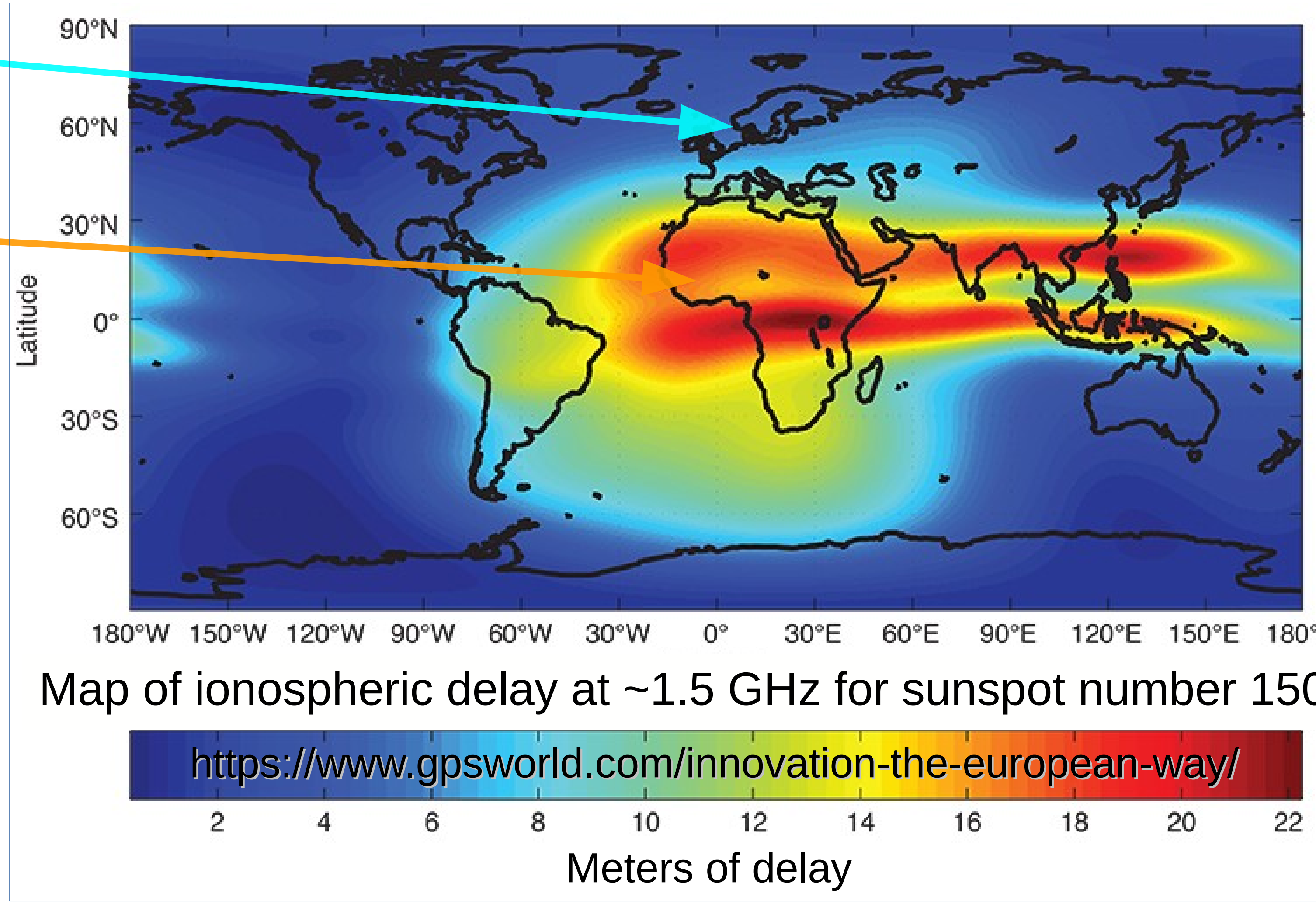
Ionospheric variability

1) Effective path to Dwingeloo ~10m shorter than to Kutunse

2) Models to correct delays during observations

3) TEC monitoring to correct after data recorded

4) Final corrections derived from astrophysical standards

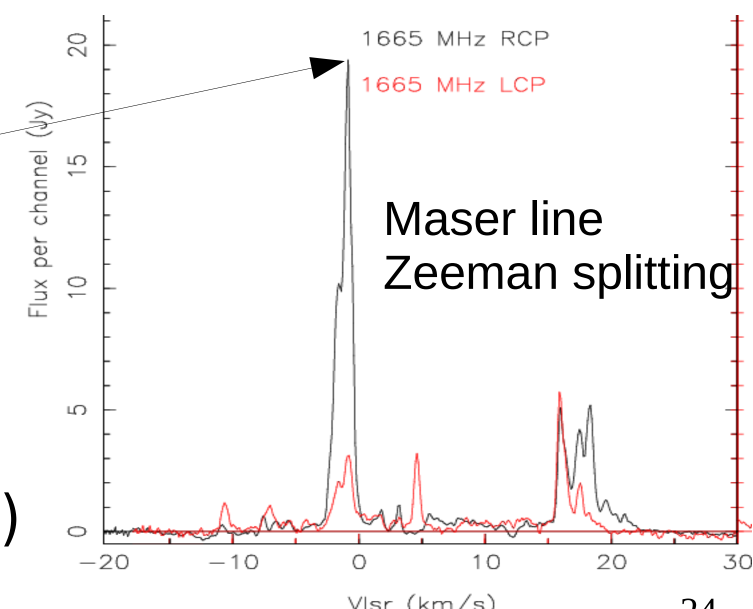


Map of ionospheric delay at ~1.5 GHz for sunspot number 150

- For cm-wave, small field, assume per observation corrections suffice
- Additional issues for VLBI
  - Polarisation calibrators may be highly resolved
  - Can use simultaneous short-spacing (e.g. VLA for VLBA) model

Targets with circular polarisation

- Circular feeds: same use of calibration sources
  - Including polarization calibration if all 4 pol products
- Target self-calibration: Do not use total intensity model as that forces RR=LL and removes V
- RR and LL both strong (bright source &/or small V)
  - Make RR, LL 2-plane image
    - Use model for calibration, `gaintype G`
- If only one of RR or LL has good S/N
  - Split out RR and derive solutions
    - Apply to all polarisations (and chans)



Linear Feeds (XY e.g. ALMA, ATCA)

Leakage **D** in the Linear Basis: **V= DPV^true** :

Visibility **V**; Stokes $\mathcal{IQUV}$; parallactic angle **P** effect ψ

Contaminating fractions $\mathcal{U}d$ etc

- Linearized, sorted, $d\,\mathcal{V} \sim 0$, regrouped Stokes

$$V_{XX} = (\mathcal{I}+\mathcal{Q}_\psi) + \underline{\mathcal{U}_\psi(d_{Xj}^*+d_{Xi})}$$

$$V_{XY} = (\mathcal{U}_\psi+i\mathcal{V}) + \underline{\mathcal{I}\,(d_{Yj}^*+d_{Xi})} + \underline{\mathcal{Q}_\psi\,(d_{Yj}^*-d_{Xi})}$$

$$V_{YX} = (\mathcal{U}_\psi-i\mathcal{V}) + \underline{\mathcal{I}\,(d_{Yi}+d_{Xj}^*)} + \underline{\mathcal{Q}_\psi\,(d_{Yi}-d_{Xj}^*)}$$

$$V_{YY} = (\mathcal{I}-\mathcal{Q}_\psi) + \underline{\mathcal{U}_\psi(d_{Yi}+d_{Yj}^*)}$$

Cross-hands complex offset proportional to I , constant in time

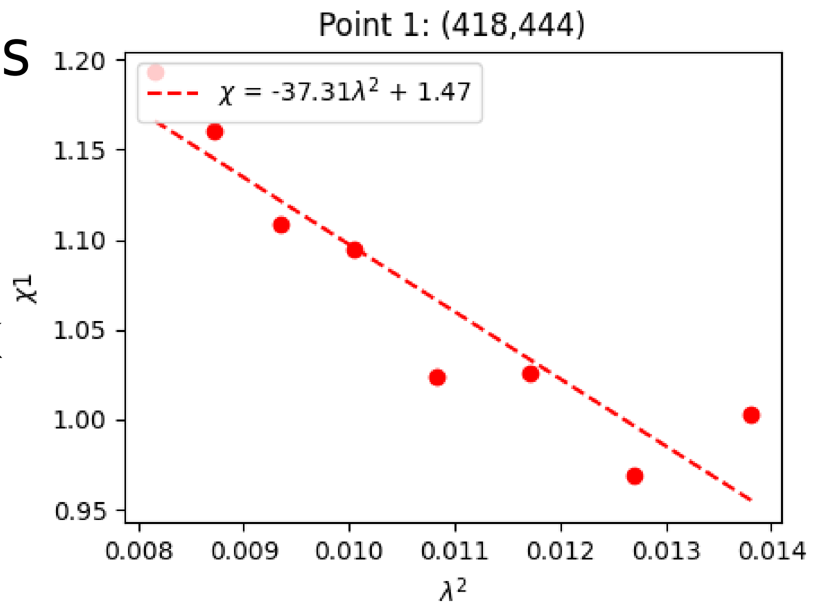
Leakage in all correlations, frequency-dependent

Linear Feed Data Calibration (dish)

- Observe polarization calibrator 3 times over at least 3 hr
  - Significant but unknown polarization
- Calibrate total intensity of bandpass and phase-ref cals
  - Bandpass, time-dependent phase, amplitude `gaintype T`
    - i.e. average XX, YY (no assumptions about polarization)
- Time-dependent cal of pol. cal.: `gaintype G` to keep X, Y separate
- Polarization calibration
  - Cross-hand delay
  - XY phase offset
  - Estimate Q and U from calibration gain variation with parallactic angle
  - Remove parallactic angle, re-calibrate XY YX using improved QU model
    - Remove residual time-dependent errors
  - Solve for leakage
- The good news: known feed orientation properties provide good estimate of 'true' polarization angle
- See e.g. ALMA 3C286 CASA Guide, Ivan's 2017 ERIS tutorial

Additional resources

- VLA polarisation CASA guide No. 2.3
  https://casaguides.nrao.edu/index.php?title=Karl_G._Jansky_VLA_Tutorials
  - Including Rotation Measure synthesis
    - Change of pol. angle χ with λ^2 due to source plasma



- References as at end of Calibration talk
- Brentjen's talk (LOFAR)
- Hales 2017AJ....154...54H
  Calibration Errors in Interferometric Radio Polarimetry
- VLBI polarization calibration in CASA
  - Marti-Vidal et al. A&A, 646 (2021) A52
  - Janssen et al. A&A, 626 (2019) A75