Introduction to Interferometry

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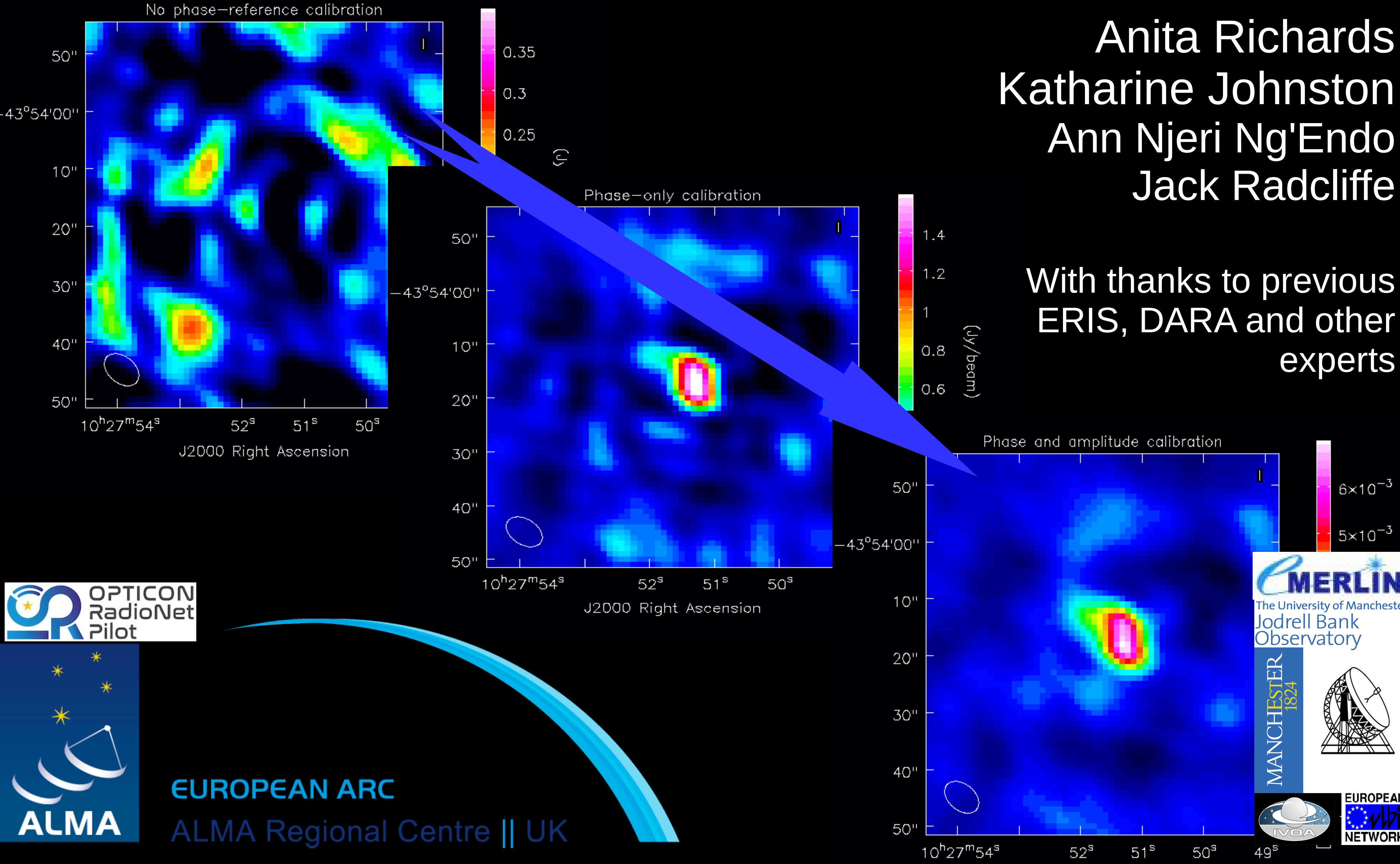
With thanks to previous ERIS, DARA and other experts



OPTICON RadioNet Pilot

EUROPEAN ARC
ALMA Regional Centre || UK

e-MERLIN
The University of Manchester
Jodrell Bank Observatory

EUROPEAN VLBI NETWORK

Have you got:

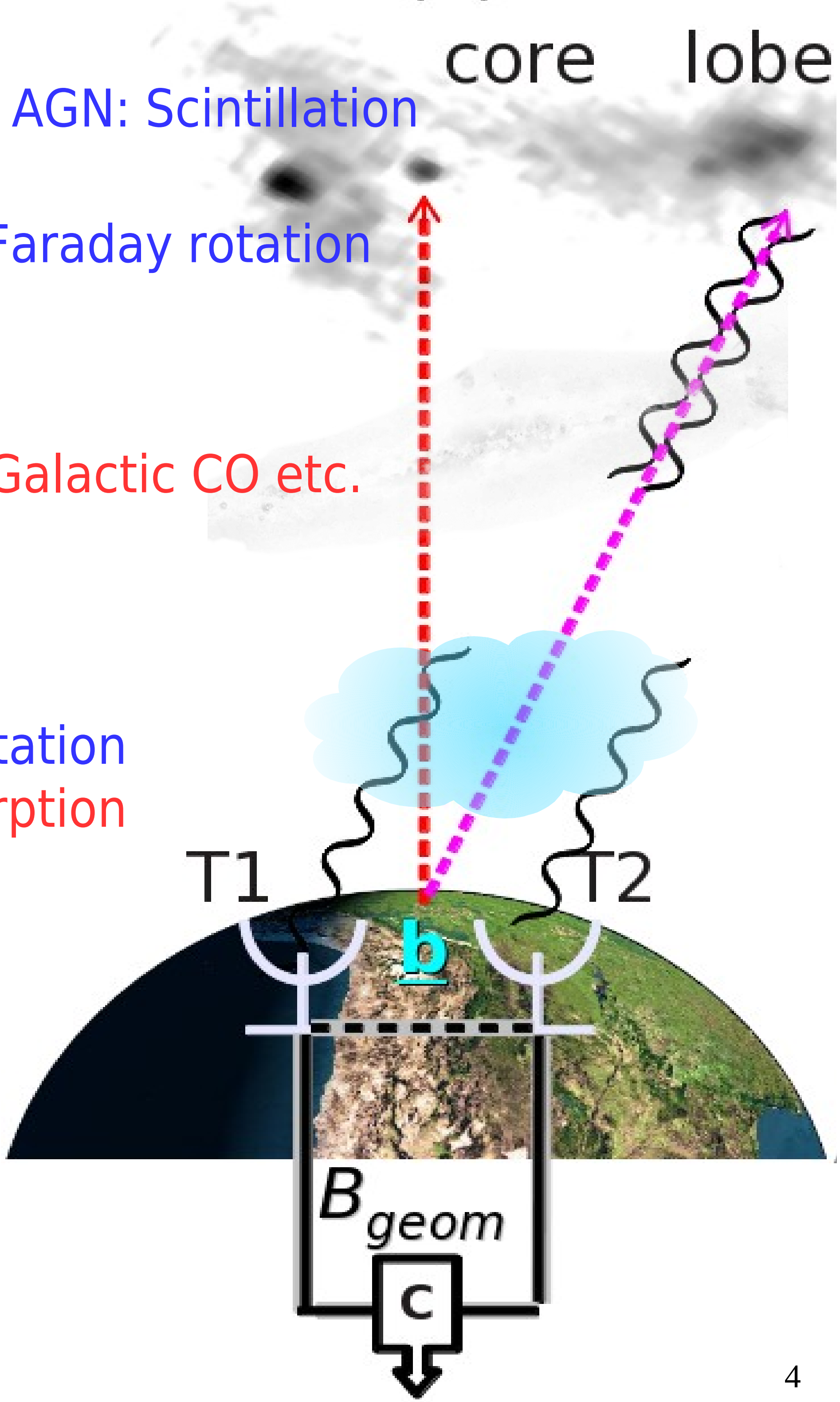
- CASA installed and working
  - https://www.jb.man.ac.uk/DARA/ERIS22/index.html
- Data downloaded into an area with 20 G space
  - Start with `ERIS22_calibration_tutorial.tar.gz`

Summary

- Hands-on *Introduction to Interferometric Data* will cover inspecting the data and flagging (removing) bad data
- The first part of this talk summarises the issues affecting the route from astronomical radio waves to data on your disk:
  - What attacks the data
  - Calibration before/during observations ('on-line')
  - What’s in your data
    - How to recognise bad data v. data which just needs calibrating
- The second part of the talk and set of tutorials *Introduction to Calibration* will cover:
  - Deriving corrections from astrophysical standards
    - Correcting amplitude and phase v. frequency (delay, bandpass)
    - Setting the flux scale
    - Correcting amplitude and phase v. time (including phase referencing)

Hazards

- Above the telescope
  - Mostly high frequency
    - Atmosphere: water vapour and other molecules
  - Mostly low frequency
    - Propagation through plasma



Close to AGN: Scintillation

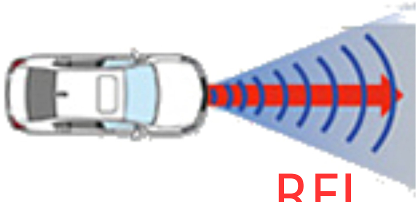
Lobes Faraday rotation

Galactic CO etc.

Ionospheric refraction/Faraday rotation

Tropospheric refraction/absorption

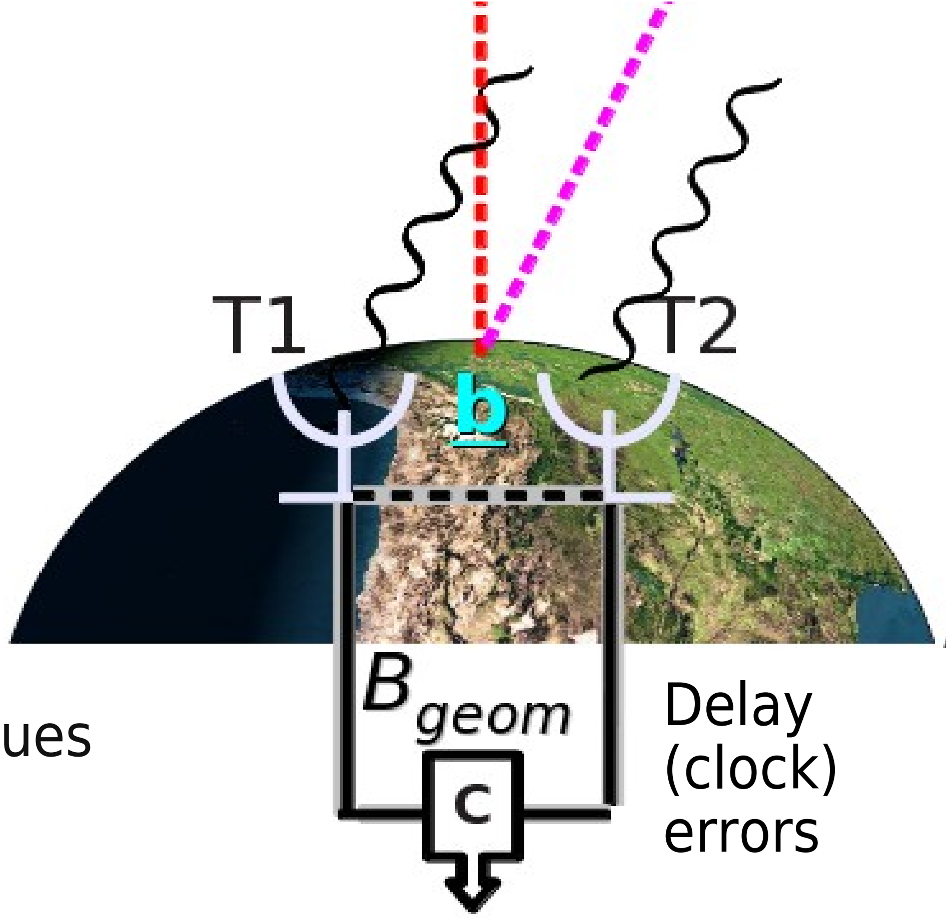
RFI



RFI

Hazards

- At the telescope and later

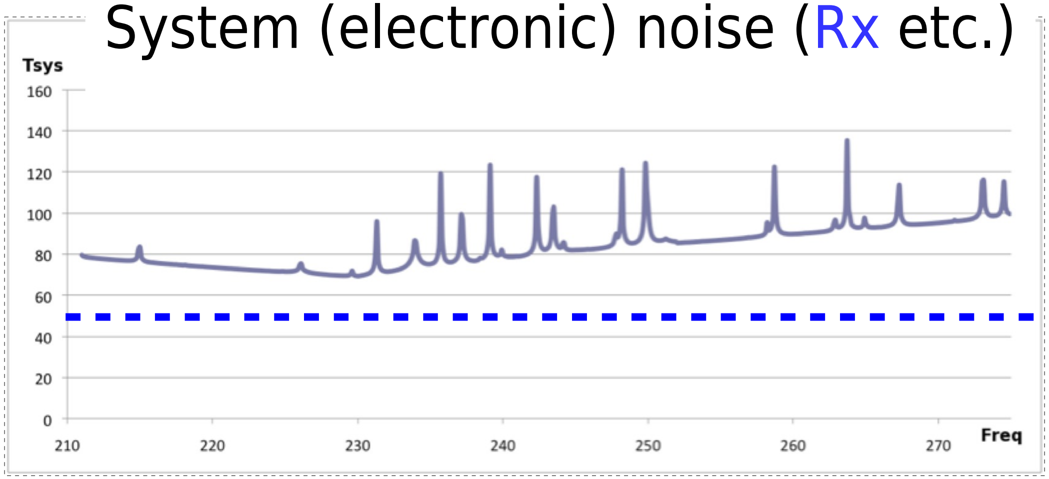


Delay (clock) errors

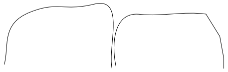
Antenna positions
Pointing, Focus
Efficiency (surface)

Timing and frequency information issues
(station clock, local oscillator...)

Atmospheric emission noise
System (electronic) noise (Rx etc.)

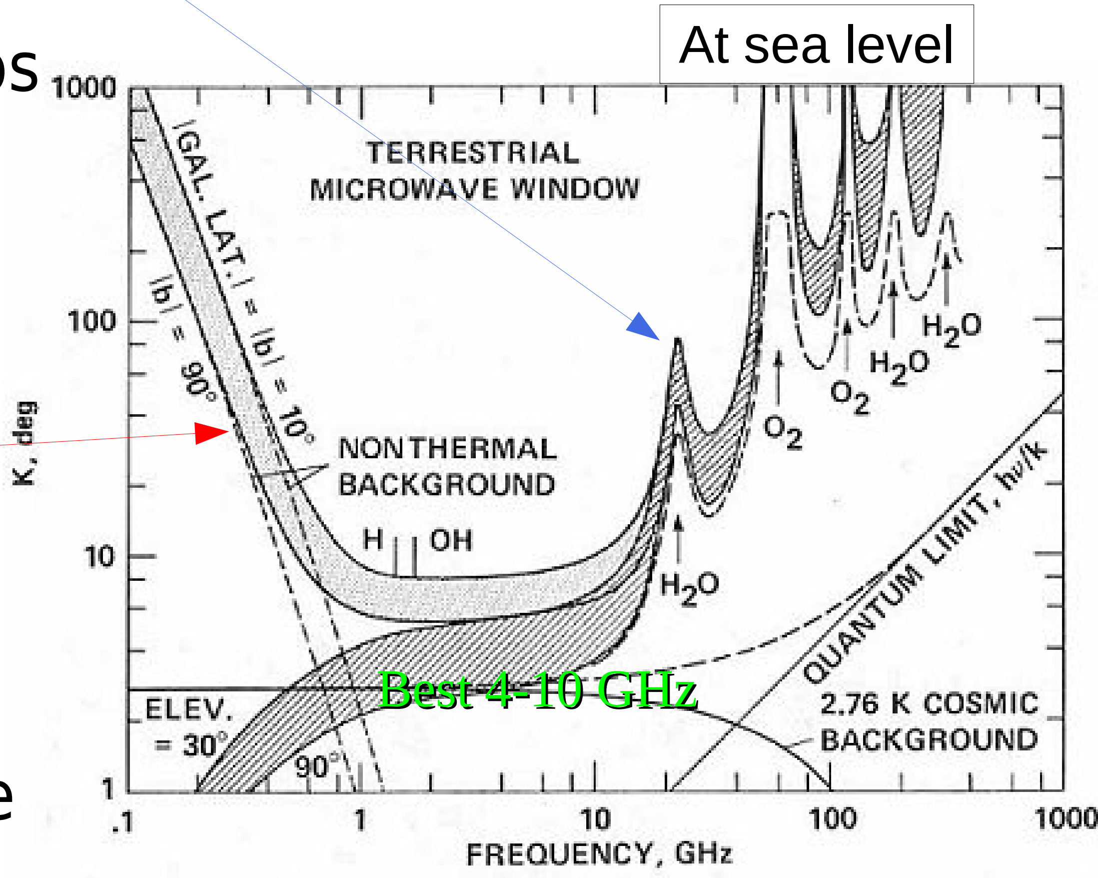


Insufficient corrections for delay tracking

Bandpass response 

Atmospheric errors

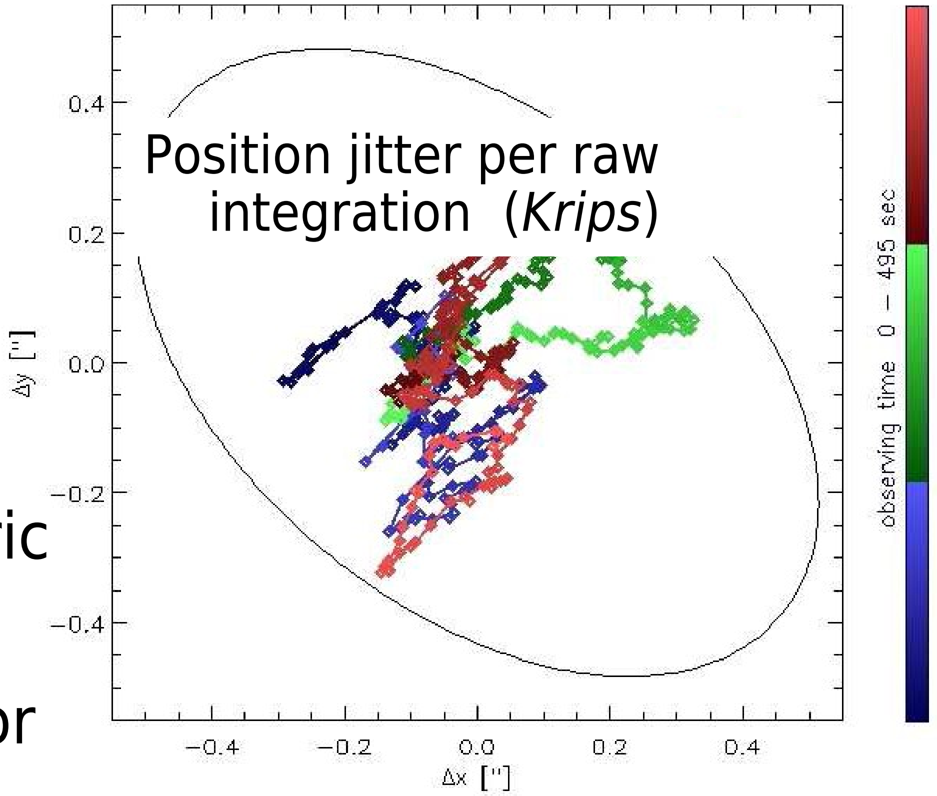
- Tropospheric errors: $\propto \nu$
  - Refractive phase errors: decorrelation, position jitter
  - Absorption reduces amps
  - Emission adds noise
- Ionospheric errors $\propto 1/\nu^2$
  - Phase errors
  - Rotation of polarisation
- Gain-elevation dependence
- Highest and lowest frequencies worst affected


At sea level

Best 4-10 GHz

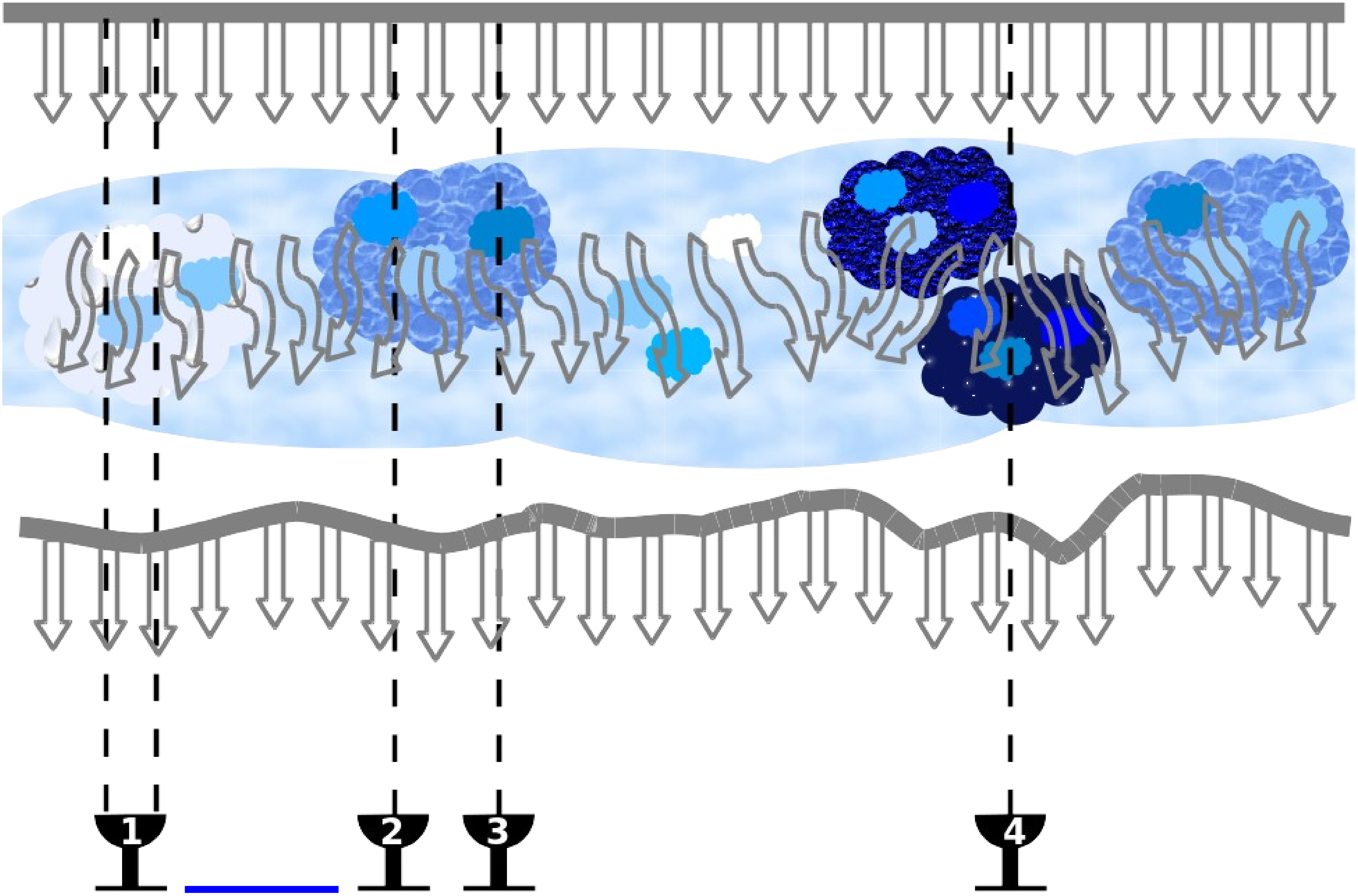
Phase/refractive errors

- Averaging over phase fluctuations causes decorrelation of amplitudes
  - Visibility $V = V_o e^{i\phi}$ so $\langle V \rangle = V_o \langle e^{i\phi} \rangle = V_o e^{-(\phi_{rms}^2)/2}$
    - ϕ_{rms} in radians
  - Lose 2% amplitude for 10° ϕ_{rms}
    - Plus absorption effects
- Precision limit to resolution
  - Like optical 'seeing'
  - Raw data position jitter
- At <1 GHz or >30 GHz atmospheric fluctuation time-scales few sec
- Intermediate frequencies stable for minutes



Position jitter per raw integration (*Krips*)

Troposphere variability scales



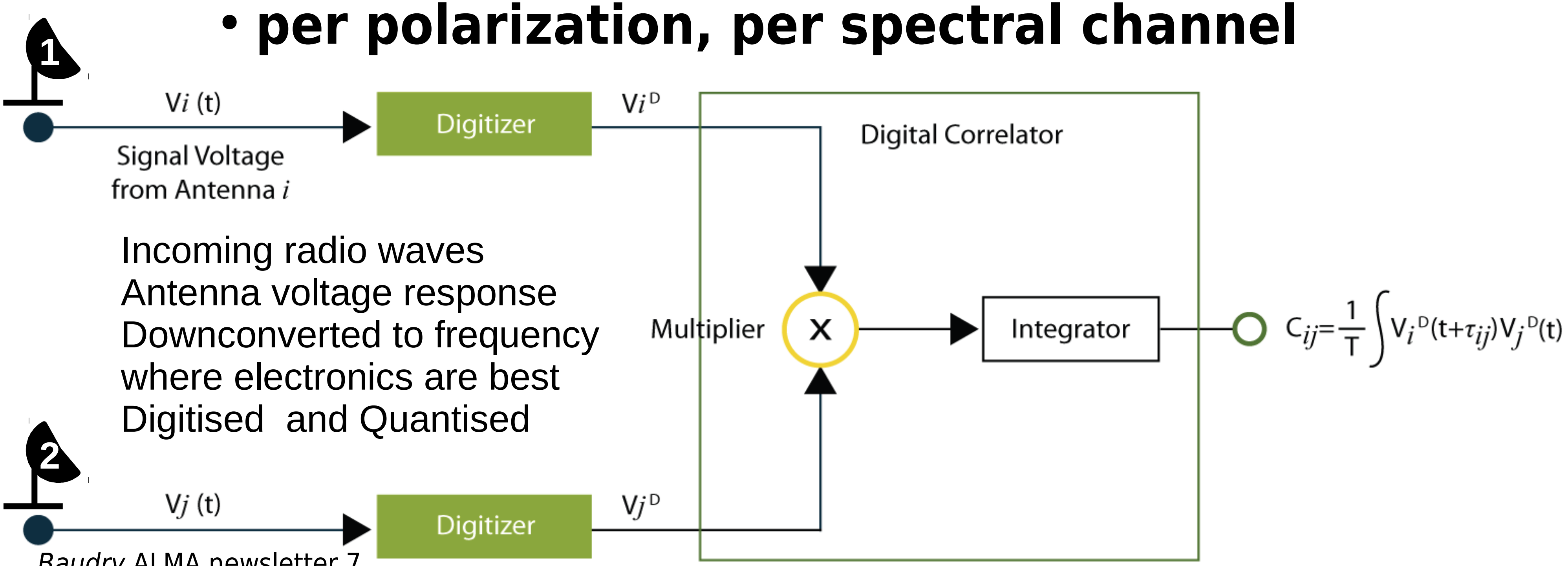
Width of turbulent layer, W ~ 800m

Wind
75 km/hr
~ 21 m/s

- Isoplanatic patch > single mm/cm antenna Field of View
  - (Long wavelengths: FoV may be anisoplanatic)
- Antennas 1, 2, 3 see slightly different disturbances
- Sky above antenna 4 very different, varies independently

Correlation

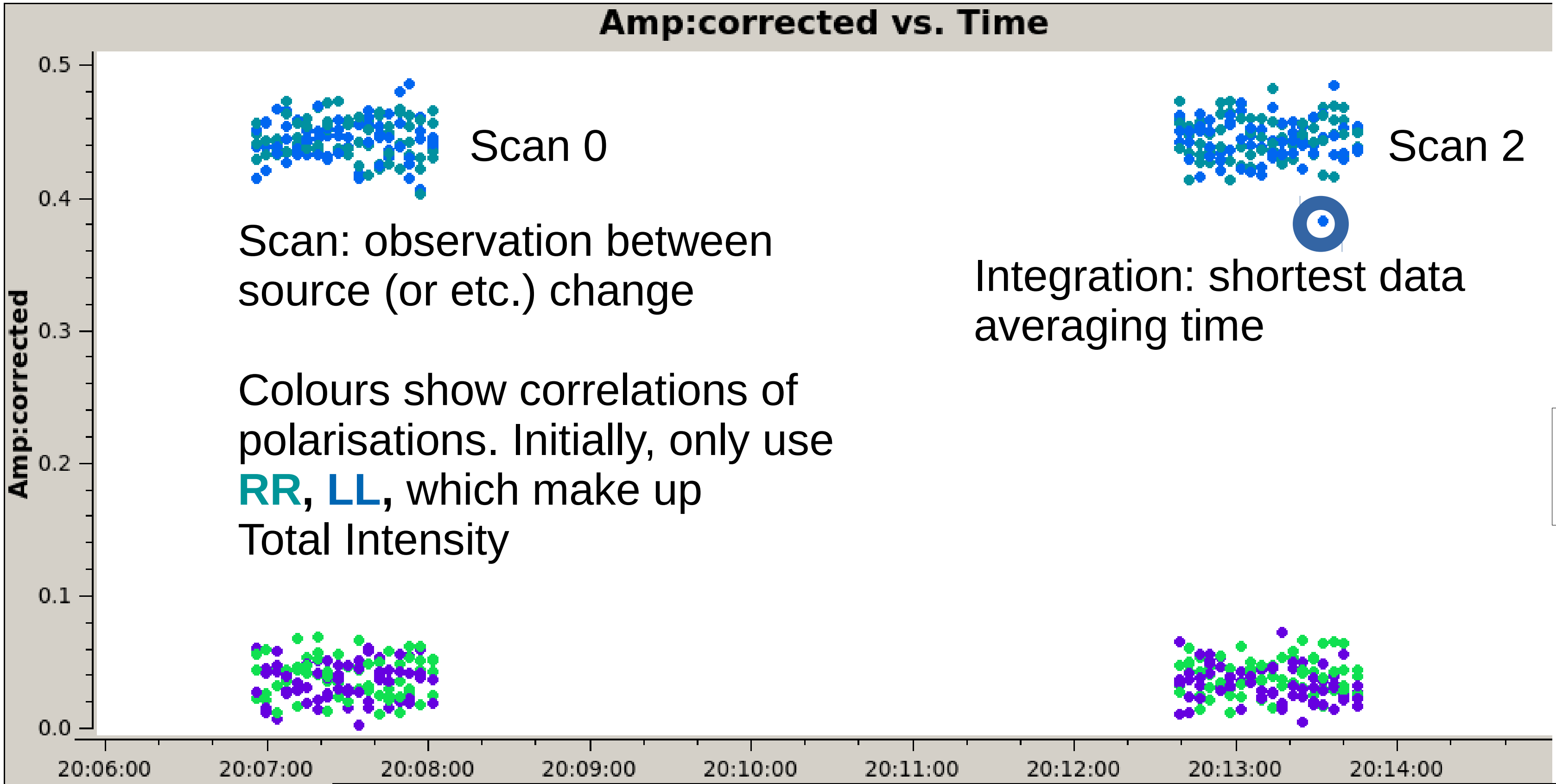
- Digitise and combine signals in correlator
  - Create spectral channels by adding ~msec time lags
  - Make parallel (and cross) polarizations
    - (another) FT into frequency domain
      - Output averaging determines integration time
- **Produces complex visibility data $V = V_o e^{i\phi}$**
  - **Time series of amplitudes & phases per baseline**
    - **per polarization, per spectral channel**



Incoming radio waves
Antenna voltage response
Downconverted to frequency
where electronics are best
Digitised and Quantised

Baudry ALMA newsletter 7

Data description



Scan 0

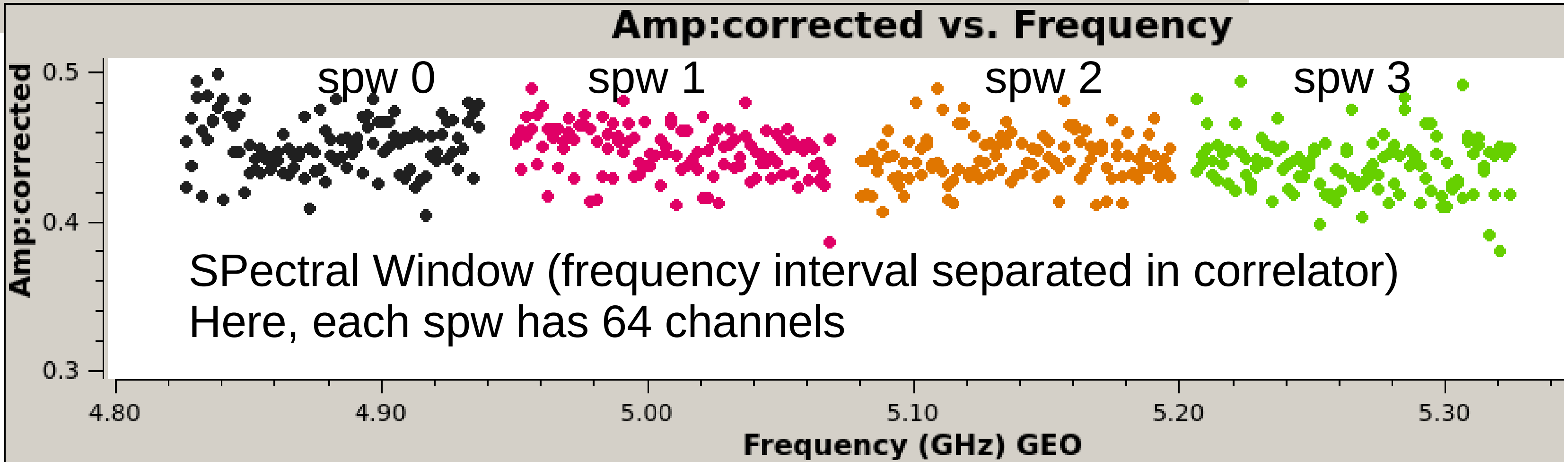
Scan 2

Scan: observation between source (or etc.) change

Integration: shortest data averaging time

Colours show correlations of polarisations. Initially, only use **RR**, **LL**, which make up Total Intensity

NB all zero-indexed



spw 0

spw 1

spw 2

spw 3

SPectral Window (frequency interval separated in correlator)
Here, each spw has 64 channels

Visibility data: Measurement Set format

MAIN	**Model, e.g.:**	**Corrected data**	**Flags**
DATA *Original visibilities*	*FT of image made from MS* *FT of supplied model image* *FT of point flux density*	*Copy of visibilities with calibration tables applied* (Used in imaging not calibration)	(Edits are stored here first; backup tables can be made and used to modify)

- Instrumental calibration in tables inside MS
- Calibration derived during data reduction stored in external tables (similar format)
- Apply calibration to Data table to write Corrected
  - Corrected and Model can be re-initialised if you mess up!

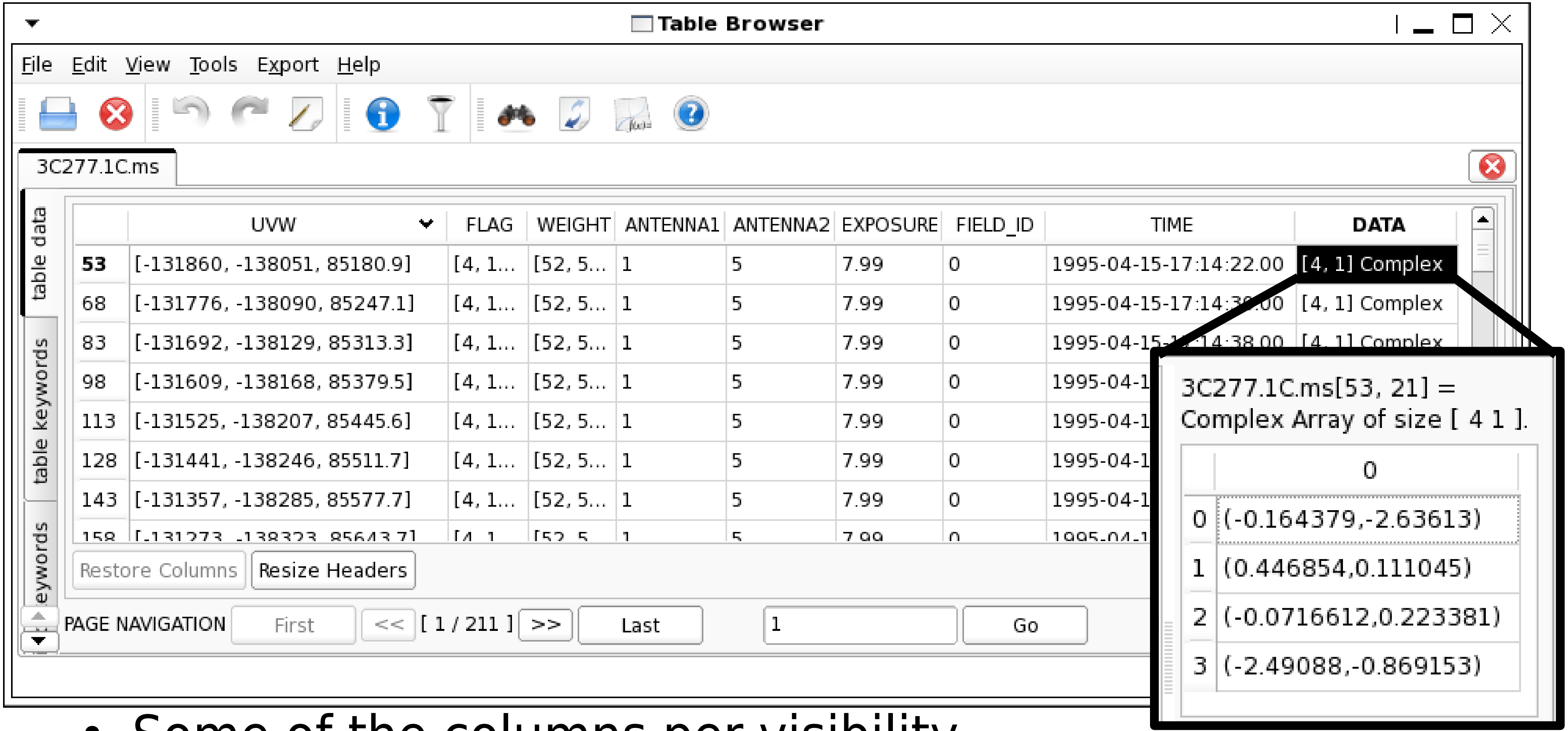
Measurement Set visibility data

- Directory of Tables
  - MAIN Data
    - Binary visibilities
  - Observational properties
  - Metadata
- Similar format for images
- Easy to access
- http://casa.nrao.edu/Memos/229.html

```
> tree jupiterallcal.split.ms
```

```
jupiterallcal.split.ms
|-- ANTENNA
|   |-- table.dat
|   |-- table.f0
|   |-- table.info
|   `-- table.lock
|-- DATA_DESCRIPTIO
|   |-- table.dat
|   |-- table.f0
|   |-- table.info
|   `-- table.lock
|-- FEED
|   |-- table.dat
|   |-- table.f0
|   |-- table.f0i
|   |-- table.info
|   `-- table.lock
|-- FIELD
|   |-- table.dat
|   |-- table.f0
|   |-- table.f0i
|   |-- table.info
|   `-- table.lock
|-- FLAG_CMD
|   |-- table.dat
|   |-- table.f0
|   |-- table.info
|   `-- table.lock
|-- HISTORY
|   |-- table.dat
|   |-- table.f0
|   |-- table.info
|   `-- table.lock
|-- OBSERVATION
|   |-- table.dat
|   |-- table.f0
|   |-- table.info
|   `-- table.lock
|-- POINTING
|   |-- table.dat
|   |-- table.f0
|   |-- table.f0i
|   |-- table.f1
|   |-- table.info
|   `-- table.lock
|-- POLARIZATION
|   |-- table.dat
|   |-- table.f0
|   |-- table.f0i
|   |-- table.info
|   `-- table.lock
|-- PROCESSOR
|   |-- table.dat
|   |-- table.f0
|   |-- table.info
|   `-- table.lock
|-- SOURCE
|   |-- table.dat
|   |-- table.f0
|   |-- table.f0i
|   |-- table.info
|   `-- table.lock
|-- SPECTRAL_WINDOW
|   |-- table.dat
|   |-- table.f0
|   |-- table.f0i
|   |-- table.info
|   `-- table.lock
|-- STATE
|   |-- table.dat
|   |-- table.f0
|   |-- table.info
|   `-- table.lock
|-- table.dat
|-- table.f0
|-- table.f1
|-- table.f2
|-- table.f2_TSM1
|-- table.f3
|-- table.f3_TSM1
|-- table.f4
|-- table.f5
|-- table.f6
|-- table.f6_TSM0
|-- table.f7
|-- table.f7_TSM1
|-- table.f8
|-- table.f8_TSM1
|-- table.info
`-- table.lock
```

Measurement Set MAIN table



	UVW	FLAG	WEIGHT	ANTENNA1	ANTENNA2	EXPOSURE	FIELD_ID	TIME	DATA
53	[-131860, -138051, 85180.9]	[4, 1...	[52, 5...	1	5	7.99	0	1995-04-15-17:14:22.00	[4, 1] Complex
68	[-131776, -138090, 85247.1]	[4, 1...	[52, 5...	1	5	7.99	0	1995-04-15-17:14:30.00	[4, 1] Complex
83	[-131692, -138129, 85313.3]	[4, 1...	[52, 5...	1	5	7.99	0	1995-04-15-17:14:38.00	[4, 1] Complex
98	[-131609, -138168, 85379.5]	[4, 1...	[52, 5...	1	5	7.99	0	1995-04-1	
113	[-131525, -138207, 85445.6]	[4, 1...	[52, 5...	1	5	7.99	0	1995-04-1	
128	[-131441, -138246, 85511.7]	[4, 1...	[52, 5...	1	5	7.99	0	1995-04-1	
143	[-131357, -138285, 85577.7]	[4, 1...	[52, 5...	1	5	7.99	0	1995-04-1	
158	[-131273, -138323, 85643.7]	[4, 1...	[52, 5...	1	5	7.99	0	1995-04-1	

3C277.1C.ms[53, 21] =
Complex Array of size [4 1].

	0
0	(-0.164379,-2.63613)
1	(0.446854,0.111045)
2	(-0.0716612,0.223381)
3	(-2.49088,-0.869153)

- Some of the columns per visibility
  - **Data**: Complex value for each of 4 correlations (RR RL LR LL) per spectral channel
    - Inspect in CASA `browsetable` or write to file
      - Not usually needed in simple calibration!

Hands-on

Now go to

Introduction to Interferometric Data

steps 1A,B, 2A

https://www.jb.man.ac.uk/DARA/ERIS22/intro_data.html

System temperature T_{sys}

- Final amplitudes made up of source, atmosphere & instrumental contributions

$$T_{sky} = T_{source}\, e^{\tau_{atm}/\cos z} + T_{atm}\left(1 - e^{\tau_{atm}/\cos z}\right)$$

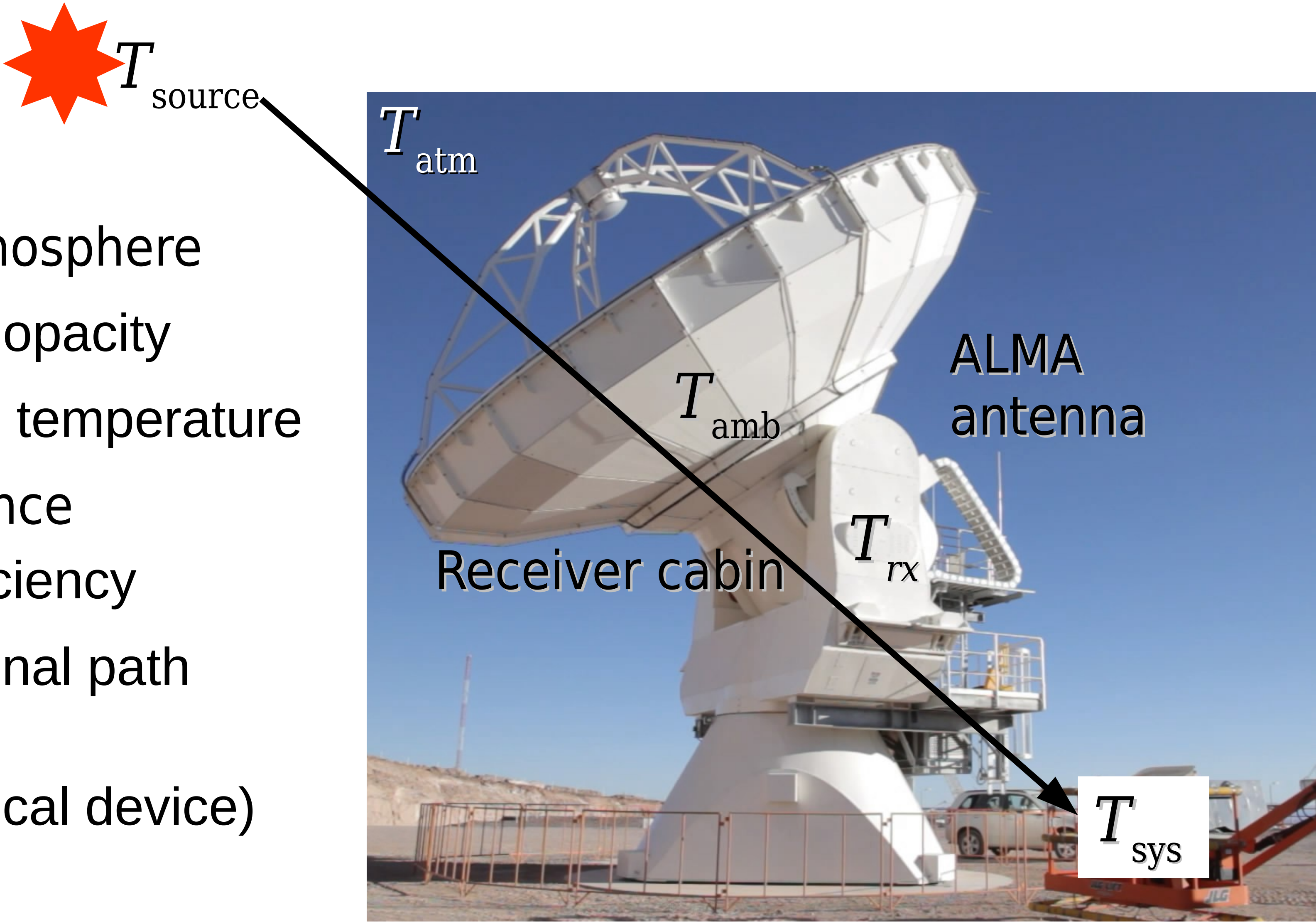
$$T_{sys} = \frac{1}{\eta_A e^{-\tau_{atm}}}\left[T_{Rx} + \eta_A T_{sky} + \left(1 - \eta_A\right) T_{amb}\right]$$

T_{sky}	incoming signal at antenna
T_{source}	above atmosphere
τ_{atm}	atmospheric opacity
T_{atm}	atmospheric temperature
z	zenith distance
η_{A}	antenna efficiency
T_{Rx}	noise of receiver & signal path
T_{amb}	ambient (or cal device) temp

- Measure T_{sys} using a standard signal ('cal device')
    - Long wavelengths: fire a noise diode
    - (sub-)mm: use a warm (thermal) 'load' (e.g. ALMA ACD)
  - Most used in VLBI and high-frequency interferometry

T_{sys} contributions

$$T_{sys} = \frac{1}{\eta_A e^{-\tau_{atm}}}\left[T_{Rx} + \eta_A\left(T_{source}\, e^{\tau_{atm}/\cos z} + T_{atm}\left(1 - e^{\tau_{atm}/\cos z}\right)\right) + \left(1 - \eta_A\right) T_{amb}\right]$$



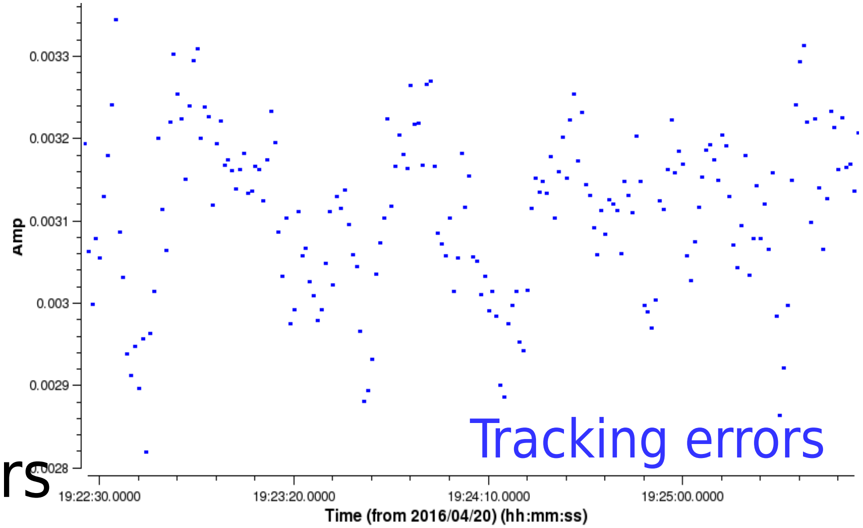
T_{source}	above atmosphere
τ_{atm}	atmospheric opacity
T_{atm}	atmospheric temperature
z	zenith distance
η_A	antenna efficiency
T_{Rx}	Receiver/signal path noise
T_{amb}	ambient (or cal device) temperature

System temperature measurement

- Typical T_{sys} 10 – 100 K @ 1 to ~200 GHz
  - Few 100 K at lower/higher frequencies
    - Bright (Jy) sources can raise T_{sys} significantly
    - Noise is increased for observing at low elevation (large *z*)
- Can provide time- and freq-dependent amp corrections for atmosphere, gain-el, bright souces
  - Use to estimate System Equivalent Flux Density
  - SEFD (Jy) = T_{sys}/K
    - where $K = \eta_{\mathrm{A}} A_{\mathrm{eff}} / 2\, k_{\mathrm{B}}$ (Kelvin per Jy)
      - Can be used to scale correlator units to Jy
- NB this sort of T_{sys} measurement not always needed
  - Can estimate SEFD ‘backwards’ from observed noise in images

More instrumental calibration

- Usually applied by observatory - can go wrong
- Bulk delay tracking
  - Calculated from elevation and atmospheric model
  - Phase tones (cm) can be used to align antenna signals
- Antennas: receiver/subreflector
  - Focus
  - Pointing and tracking
    - Error: 'scalloped' amps
  - Antenna position errors
    - Direction-dependent delay errors
    - Cannot transfer phase-ref corrections accurately to target
      - Can update positions in early processing stages
  - Mitigated near field centre by self-cal

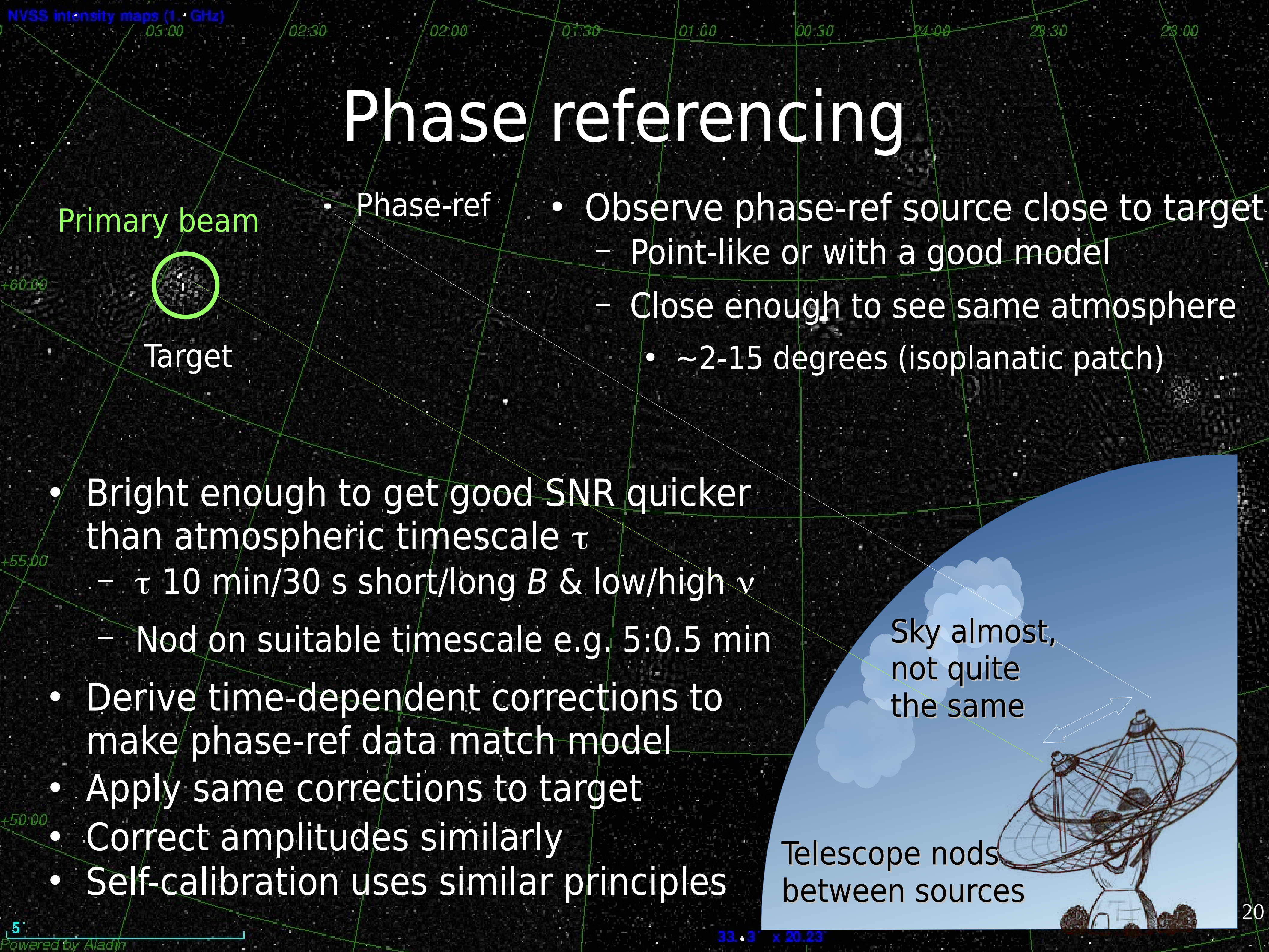


Tracking errors

Calibration using astrophysical sources

- A typical observation includes at least the following:
  - Science target source(s)
  - Bandpass calibration source
    - Strong enough to be seen in a single channel
  - Phase reference calibrator close on sky to target
    - Bright enough to give good S/N in each scan
  - Flux scale calibrator of known flux density
- A calibrator: may be used in more than one role
  - Needs accurate position, compact structure (or good model).
- **Calibration software compares the visibilities for a source with a model and calculates corrections to bring the observed visibilities closer to the model**

Phase referencing



Primary beam

Target

- Phase-ref

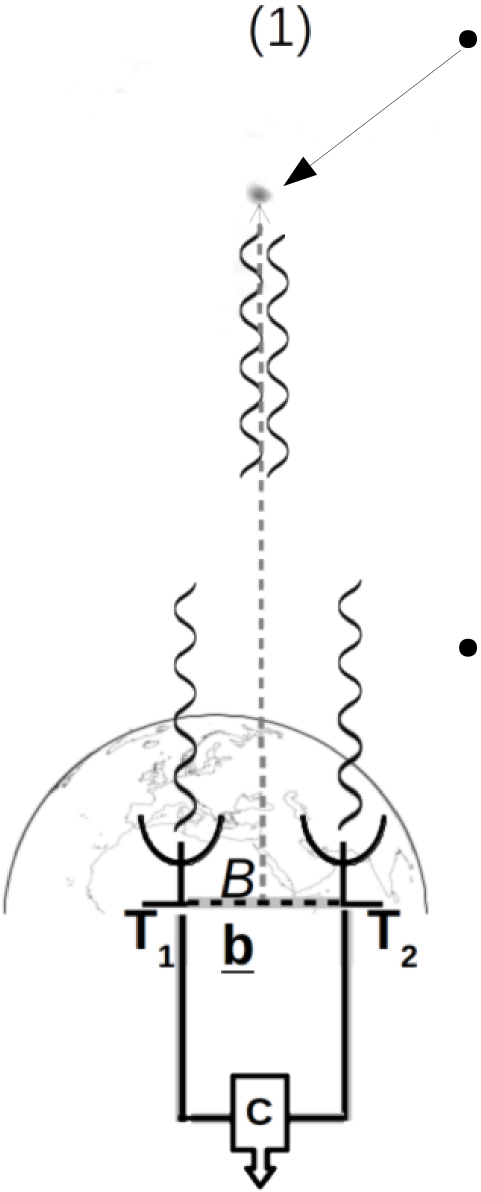
- Observe phase-ref source close to target
  - Point-like or with a good model
  - Close enough to see same atmosphere
    - ~2-15 degrees (isoplanatic patch)

- Bright enough to get good SNR quicker than atmospheric timescale τ
  - τ 10 min/30 s short/long *B* & low/high ν
  - Nod on suitable timescale e.g. 5:0.5 min
- Derive time-dependent corrections to make phase-ref data match model
- Apply same corrections to target
- Correct amplitudes similarly
- Self-calibration uses similar principles

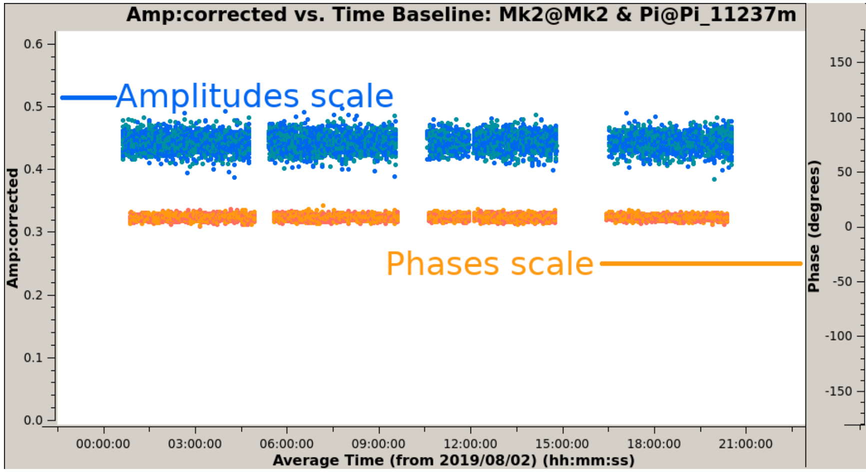
Sky almost,
not quite
the same

Telescope nods
between sources

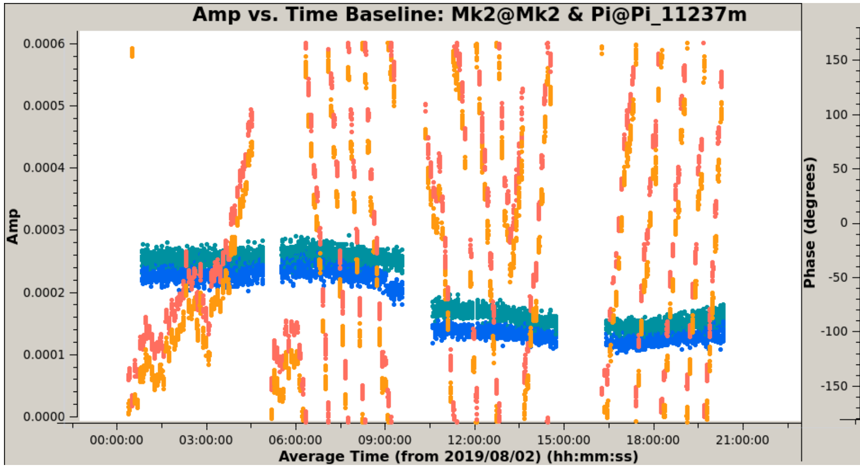
Response to point source



- Point source at phase centre, *no errors:*
  - Amplitudes constant with time at source flux density
  - Phases constant, 0°
- *Real raw data*:
  - Amplitudes scaled in correlator counts and corrupted by atmosphere & electronics
  - Phases also corrupted

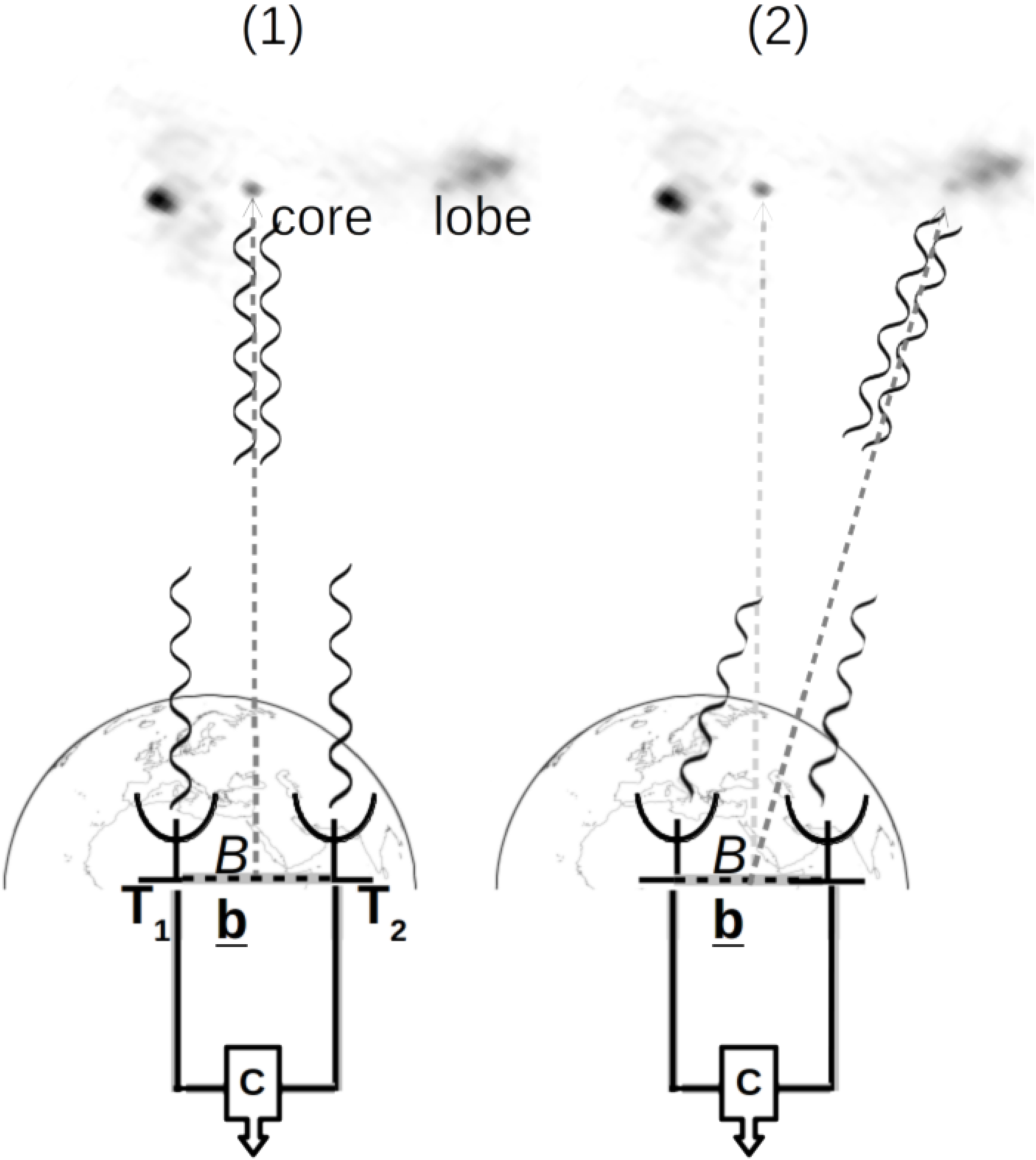


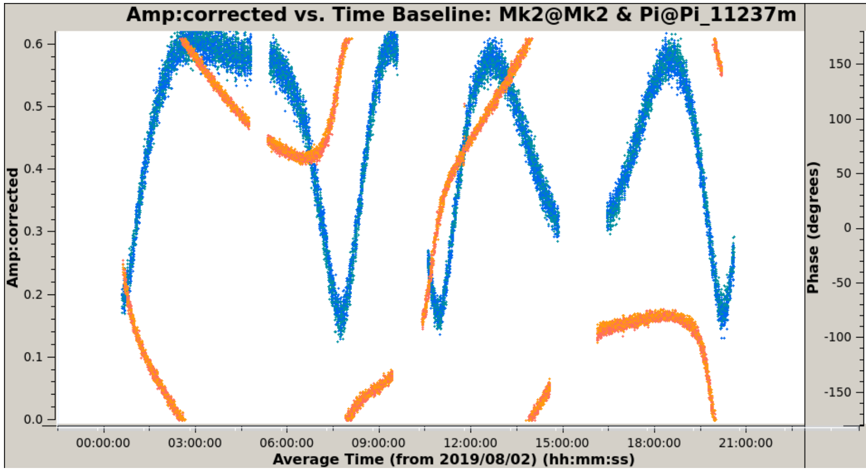
Single (short) baseline



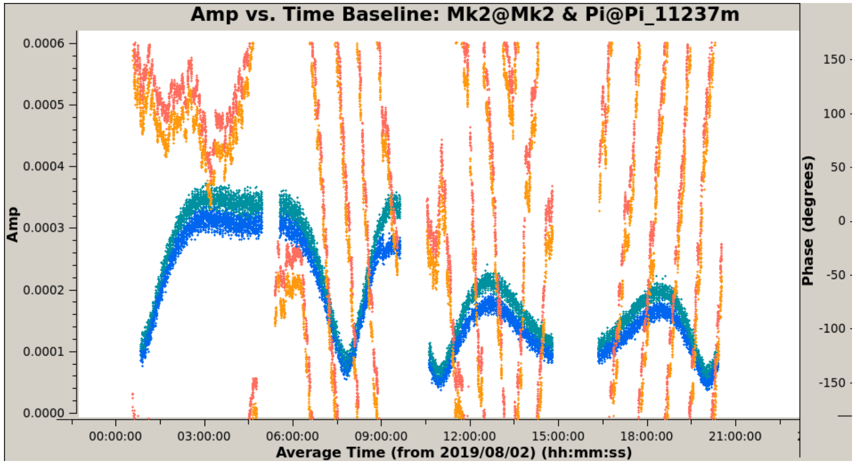
- *NB Assumptions*: single ν, some basic instrumental calibration applied.

Complex source





Corrected visibilities, complex source



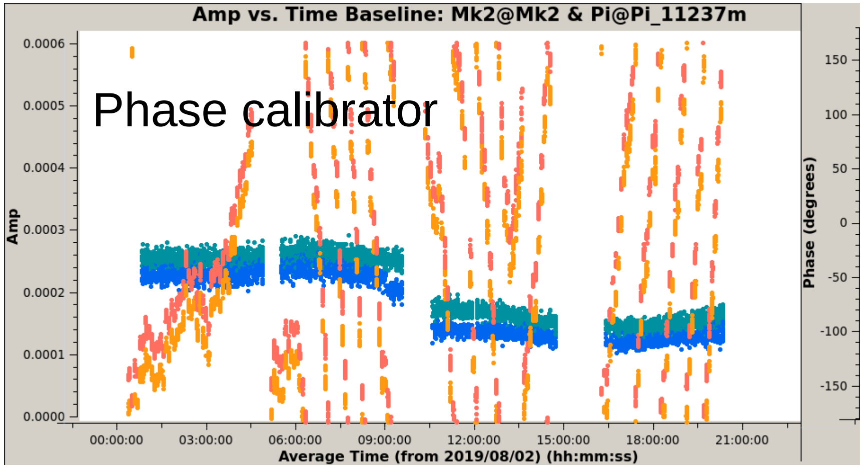
Uncorrected visibilities

- Extended source: emission from different parts arrives with different phases; correlated signal is sum of all.

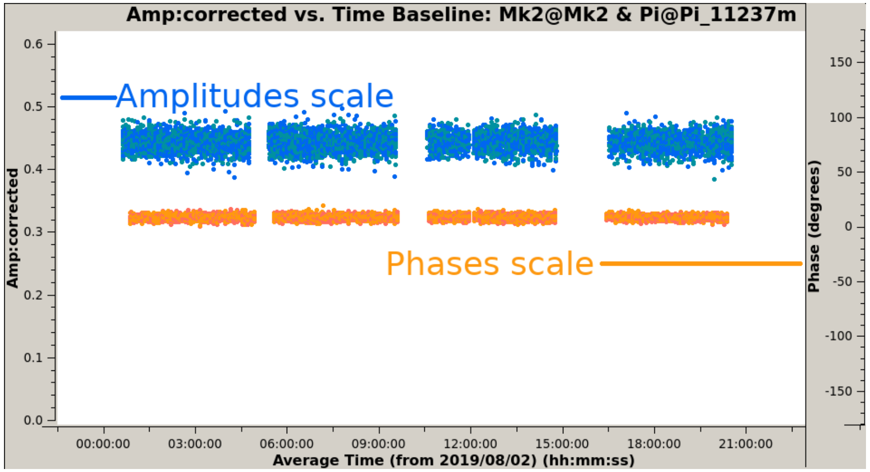
Phase correction

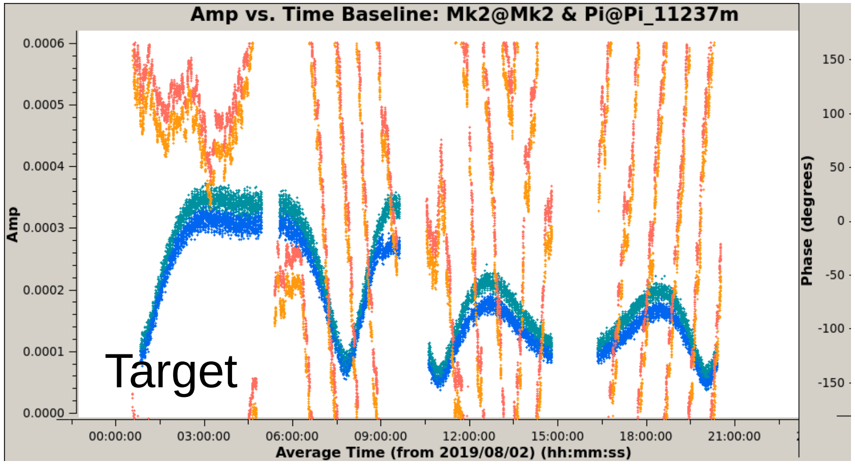
Raw visibilities

Corrected visibilities

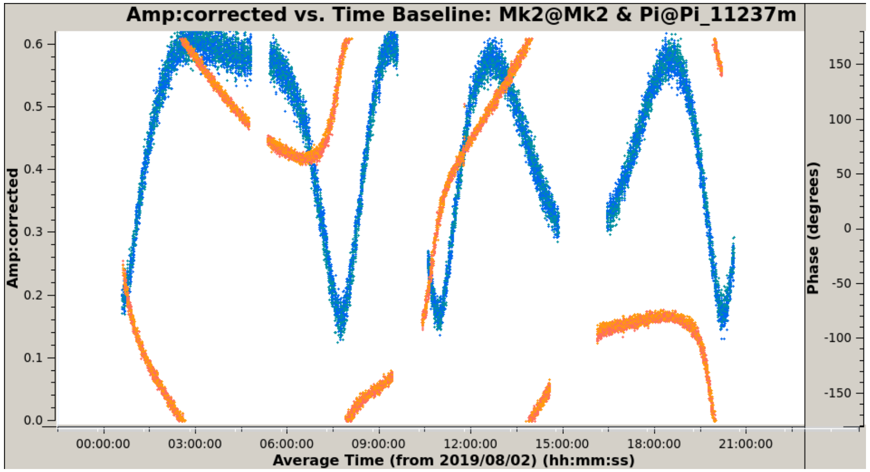


Derive corrections to make phase calibrator visibilities match point-source model



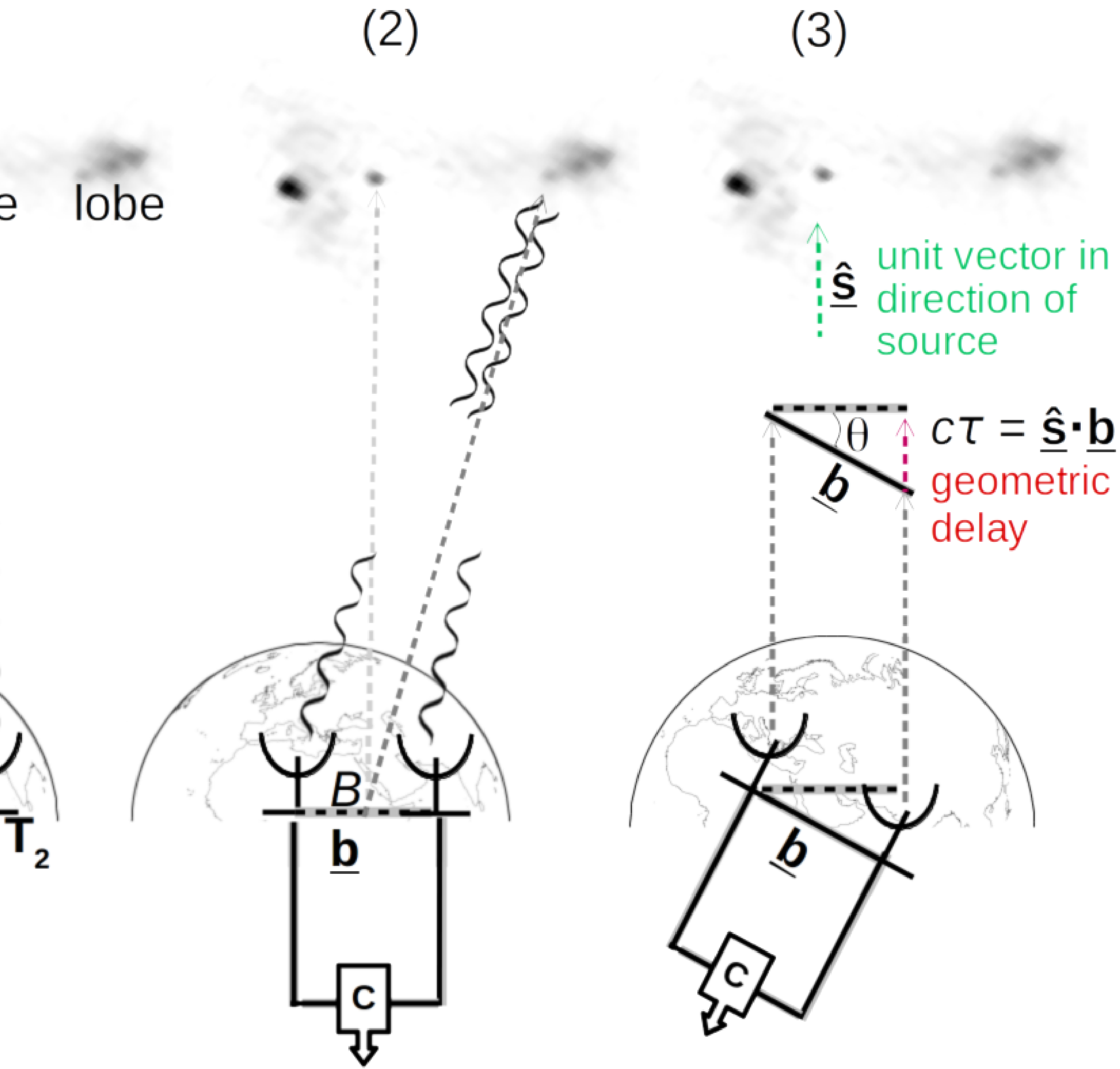


Apply corrections to target visibilities



- Raw visibility phases look similar – high phase rate
- Correct those, and true, distinct structures emerge

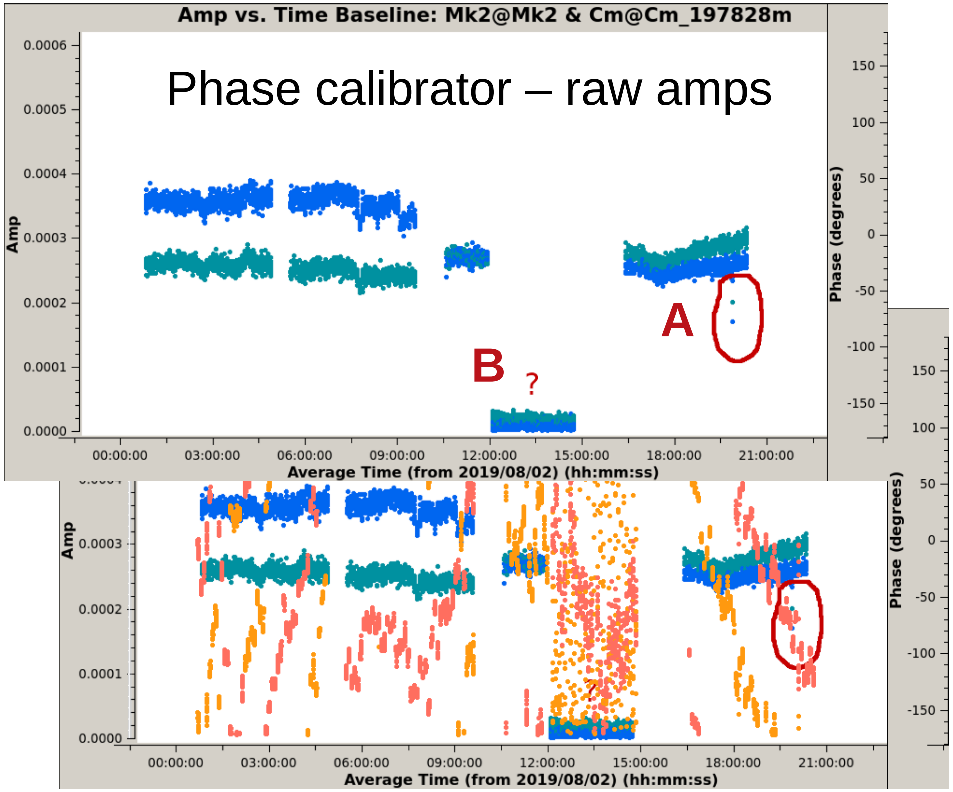
Aperture synthesis

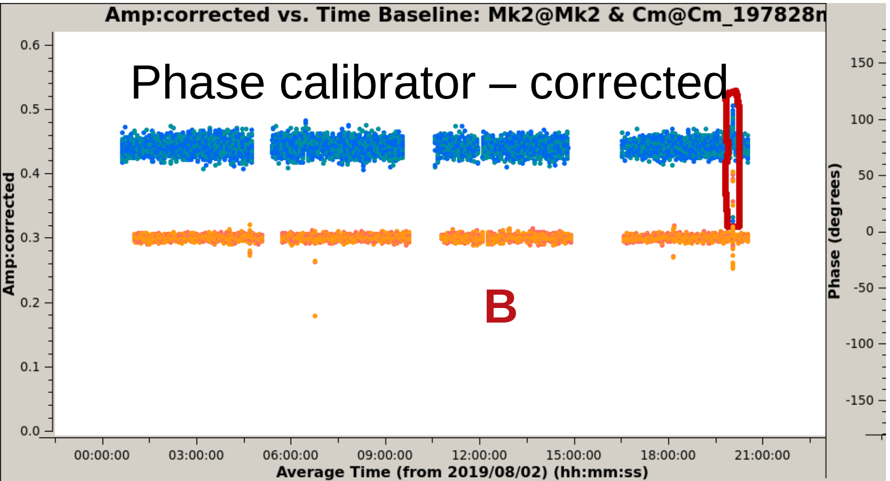


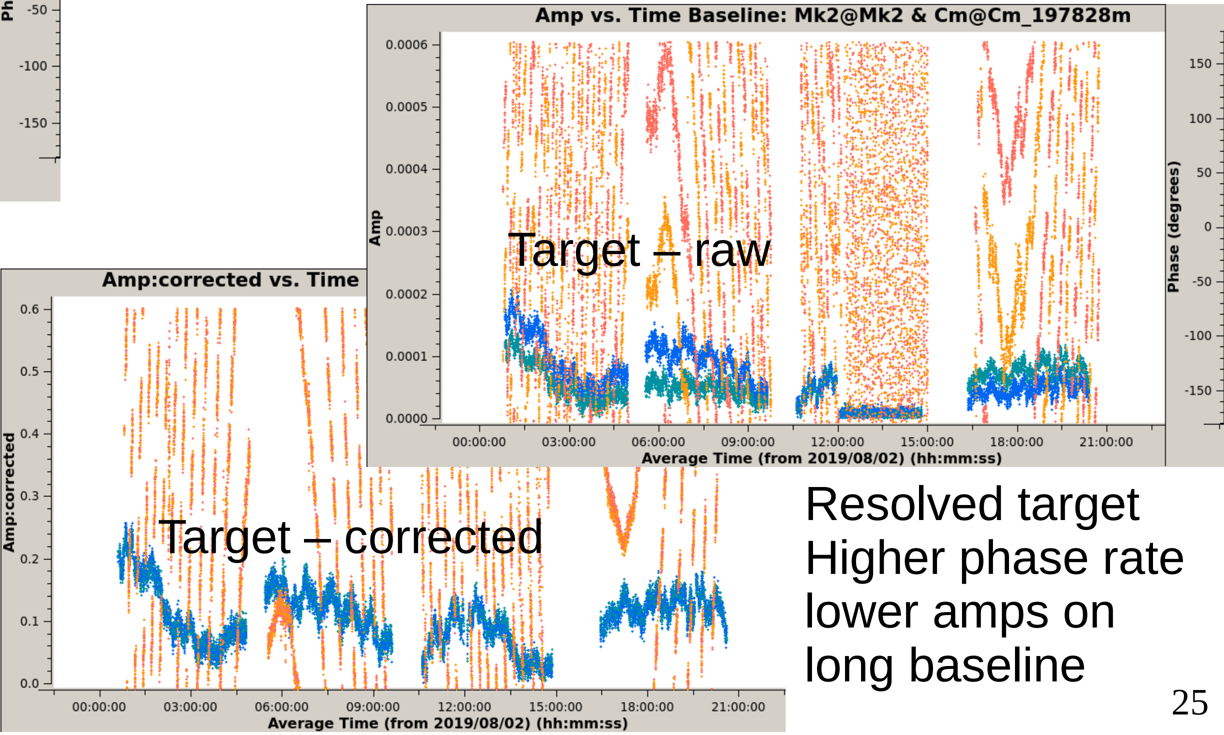
- Earth rotation samples target at different projections
- But signal path length also changes
  - “delayed” by τ over extra path length $\delta = c\tau$
- Must be corrected so phase is retained as if signals arrive “in time” at all telescopes

Quality of visibilities on long baseline

- Two doubtful regions
- A a few bad integrations
  - Still there after calibration
    - **Flag**
- B Noisy phase, very low amps
  - But well corrected!?!
  - Target similar



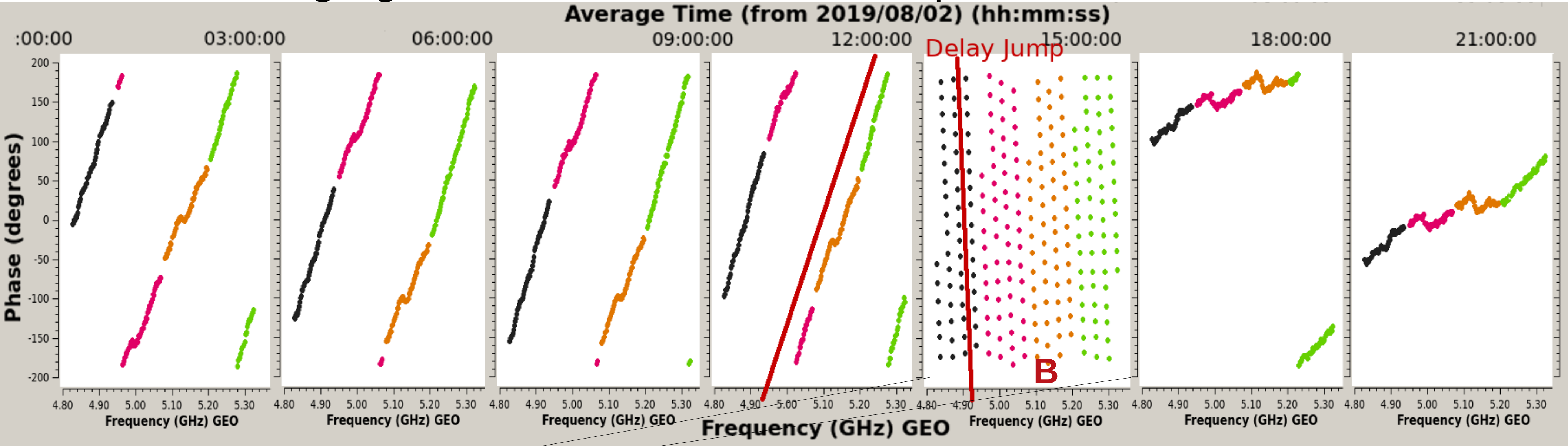




Resolved target
Higher phase rate
lower amps on
long baseline

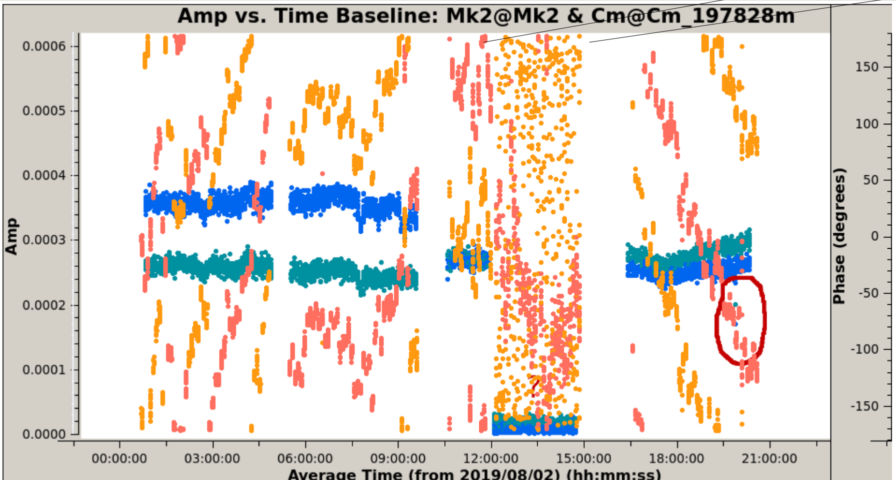
Delay errors

- Phase ref. plot phase against time, average every 3 hr
- Regular slopes, one chunk very high delay rate
    - Averaging channels decorrelates amplitudes

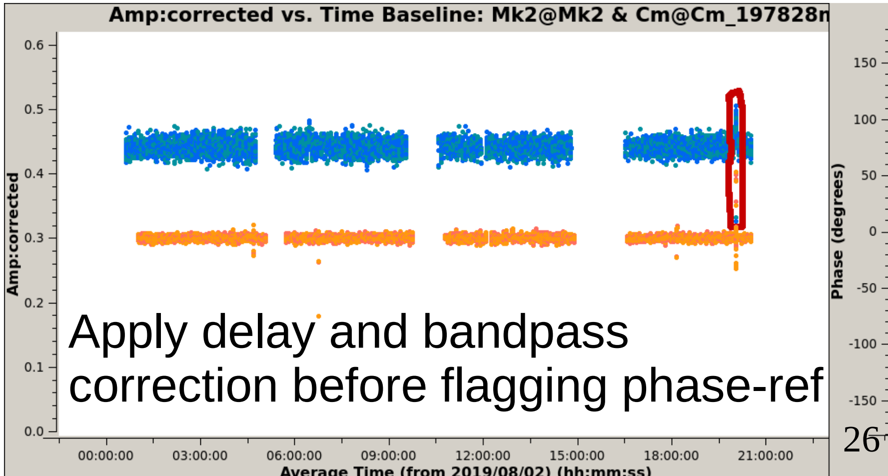


Delay Jump

B



Correct delay
Allows
averaging
across
spw without
supressing
amplitudes



Apply delay and bandpass
correction before flagging phase-ref

Hands-on

Now go to

Introduction to Interferometric Data

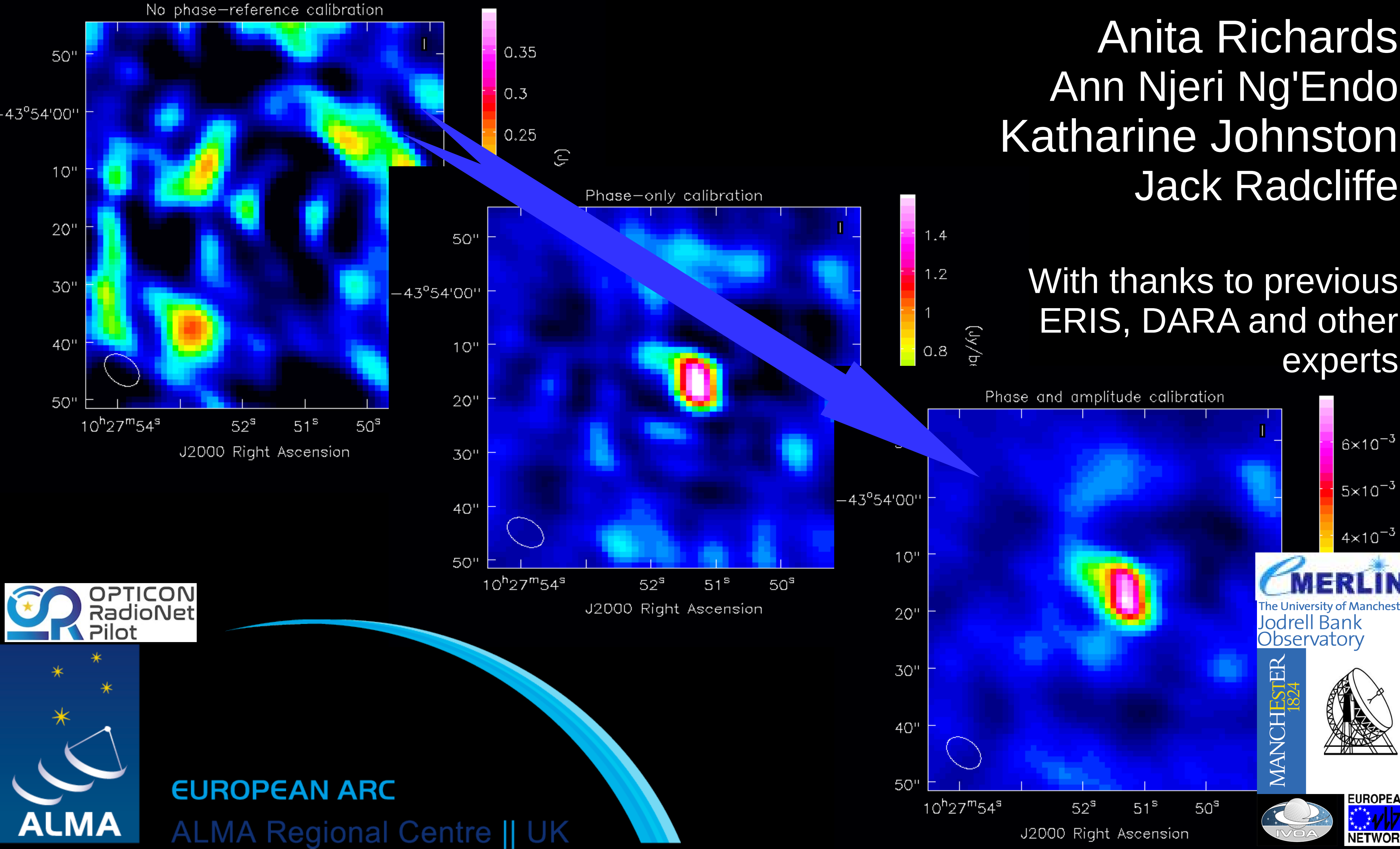
steps 2B,C,D,E

https://www.jb.man.ac.uk/DARA/ERIS22/intro_data.html

Introuction to Calibration

Anita Richards
Ann Njeri Ng'Endo
Katharine Johnston
Jack Radcliffe

With thanks to previous ERIS, DARA and other experts



OPTICON RadioNet Pilot

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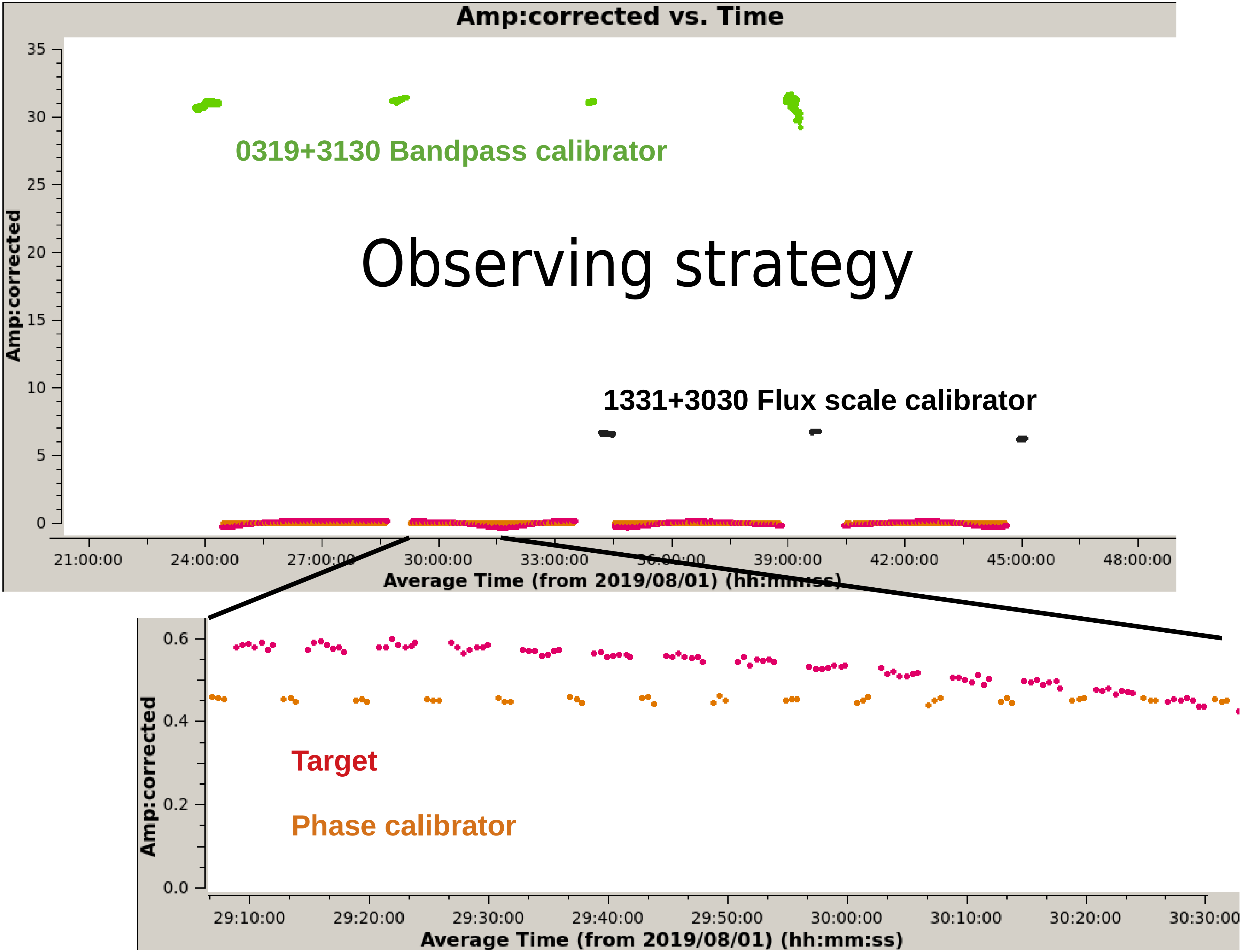
MANCHESTER 1824

IVOA

EUROPEAN VLBI NETWORK

Observing strategy



0319+3130 Bandpass calibrator

1331+3030 Flux scale calibrator

Target

Phase calibrator

Calibration strategy

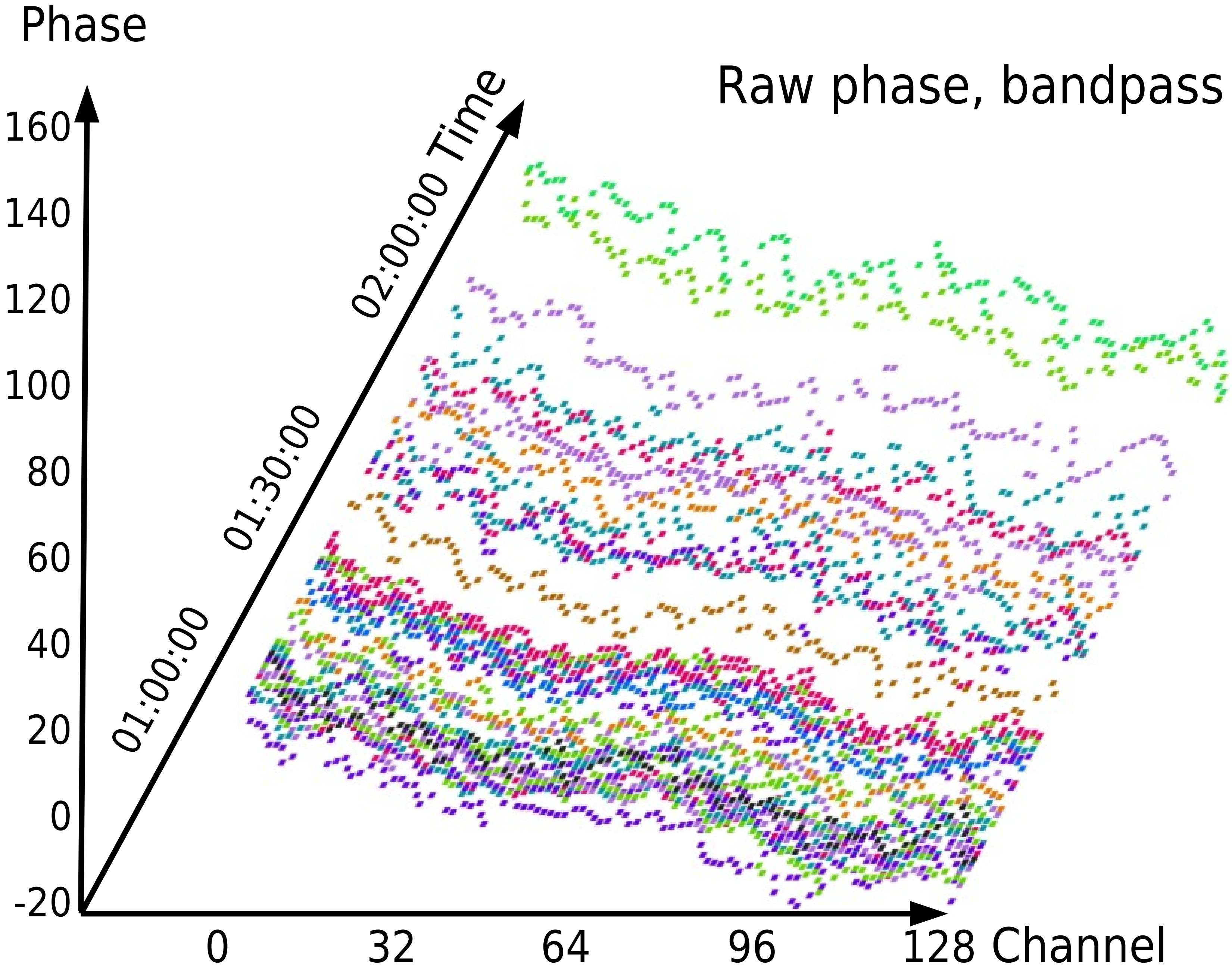
- Compare observed visibilities with model
  - Visibilities are per-baseline but most problems are per-antenna
    - Best fit solution derived using least-squares fit for each antenna
      - Better S/N as *n*(solints) are factored over *N* antennas, not *N(N-1)/2* baselines
- Good solutions need Signal to Noise ratio S/N $\sigma_{\mathrm{ant}}/S_{\mathrm{calsource}}>3$
  - per calibration interval δ*t,* per antenna, per frequency interval δ*ν* (typically spw), per Rx polarization

$$\sigma_{ant}(\delta t, \delta \nu) \approx \sigma_{array}\sqrt{\frac{N(N-1)/2}{N-3}}\sqrt{\frac{\Delta t}{\delta t}N_{spw}N_{pol}}$$

    - σ_{array} is noise rms estimated from T_{sys} or measured from image
    - for all N antennas, N_{pol} polarizations (e.g. *RR+LL*, total intensity), N_{spw} spectral windows (also called IFs)

**N-3* because there are 3 degrees of freedom for *N* antennas (*N*-1 baselines per antenna; origin of phase, refant).

Phase errors in 3D



Raw phase, bandpass calibrator

Each scan in a different colour

~30° phase slope across the band

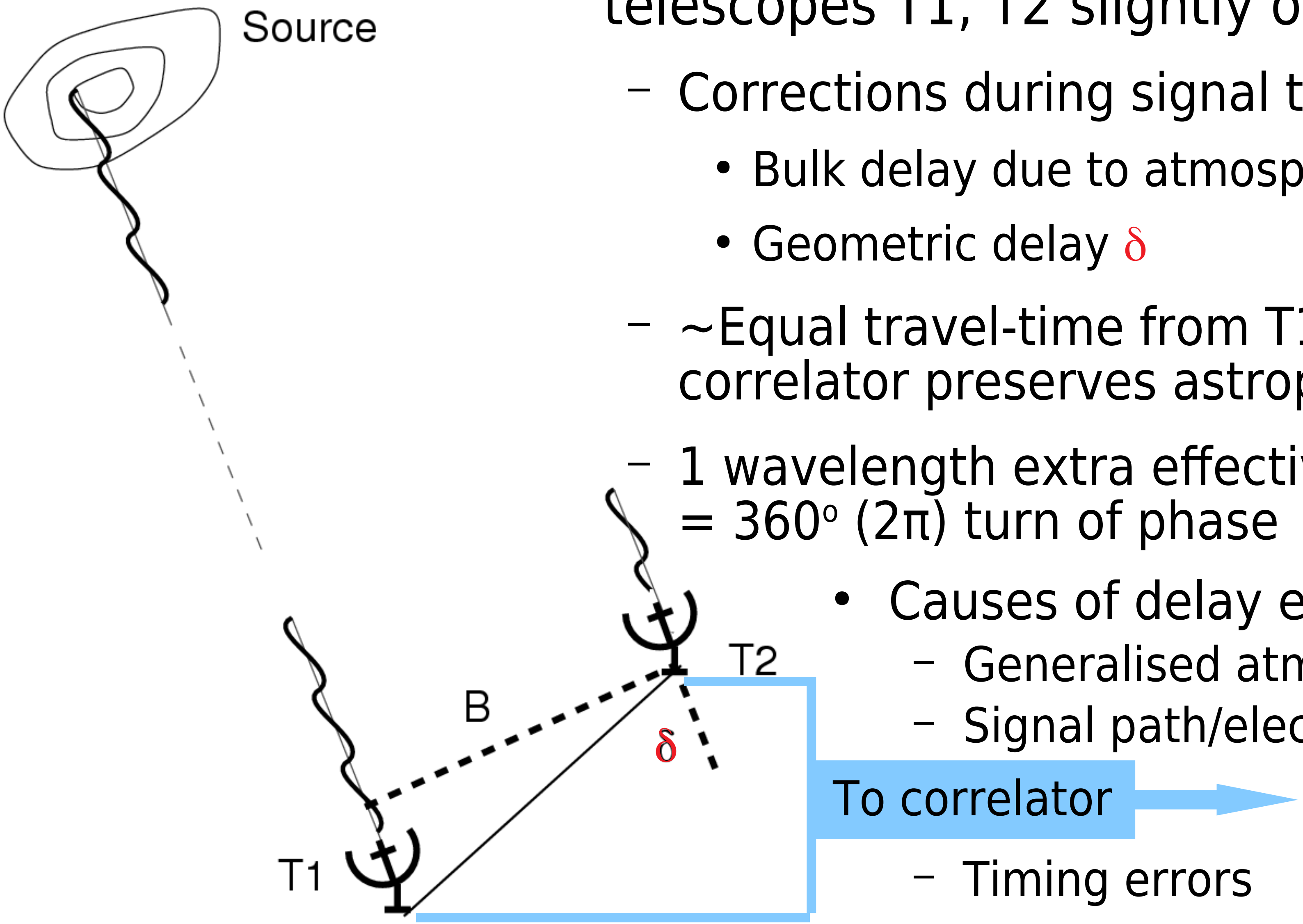
Phase as a whole drifts ~140° in 2 hr <2° per min

Calibration averaging

- Usually have to average data in time and/or frequency in order to get enough S/N per solution interval
  - Average all channels per spw for time-dependent calibration
  - Average all time on bandpass cal for bandpass calibration
- But.... averaging over large phase changes decorrelates
  - *Do not average over interval where phase change* $d\phi > \pi/4$
  - Keep polarizations and spectral windows separate if possible (until any phase offsets have been removed)
- Bandpass first or time-dependent calibration first?
  - Delay first! Fitting first derivative to phase v. freq. uses entire spw.
    - So a short time-average still gives enough S/N
      - If not possible, average central ~25% channels
- Applying the delay correction allows spw averaging for subsequent time-dependent calibration

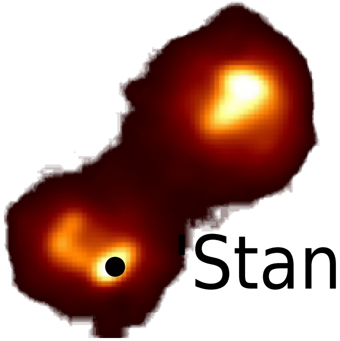
Delay errors

- Signals from off-centre source reach telescopes T1, T2 slightly out of phase
  - Corrections during signal transport:
    - Bulk delay due to atmosphere/elevation
    - Geometric delay δ
  - ~Equal travel-time from T1 and T2 to correlator preserves astrophysical phase
  - 1 wavelength extra effective path length = 360° (2π) turn of phase



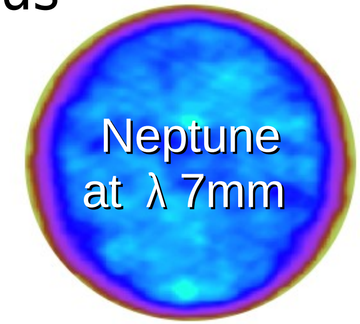
- Causes of delay error include:
  - Generalised atmospheric model
  - Signal path/electronics
  - Timing errors
  - Antenna position error

}constant in time


3C295
arcsec
resolution

Flux scale

- 'Standard' radio galaxies ~constant flux at longer λ
  - e.g. 3C286 e-MERLIN (cm), 3C295 LOFAR (m)
    - Originally defined wrt. Mars, NGC 7027, Cas A
  - These extended, stable sources are resolved-out by VLBI
    - Compact QSO variable; monitor wrt. standards
  - Extended, stable sources too faint at mm wavelengths
    - Use well-modelled planets/moons at low resolution
    - Monitor mm-bright compact QSO wrt Neptune/Uranus
- Set flux of standard as model.
  - Phase-ref has nominal model flux of 1 Jy
- Calibrate raw amps against model
  - Use standard scaling factor to derive phase-ref flux density
  - Set as model, re-derive amplitude calibration for phase-ref
  - This table contains correction to apply to target
    - Typical accuracy 5-10%



Hands-on

(after Introduction to Interferometry – inspection, flagging)

Prepare for calibration:

Set model for flux calibrator

Decide solution (averaging) interval for initial calibration

see web page

Introduction to Calibration

(sections 1, 2A; steps 11,12)

https://www.jb.man.ac.uk/DARA/ERIS22/calibration.html

Simple calibration with astrophysical sources

Example workflow assuming all calibrators are points

- **Delay** calibration K to allow channel-averaging of BP cal

  0. Usually stable in time, can be extrapolated if necessary

- **Bandpass** calibration – bright as possible source

  1. Time-dependent phase & amp calibration (applying K) G1

  2. Apply calibration (K, G1), average all time for freq. dependent phase and amplitude calibration, i.e. bandpass calibration B1

  - also residual delay correction for all calibrators

- **Phase-reference** – fairly bright source near target

  3. Apply K, B1 in time-dependent phase calibration G2a averaging all channels, shortest δ*t* for enough S/N

  - Also include flux scale calibrator
  - Table G2b is the same but per-scan solint for interpolation

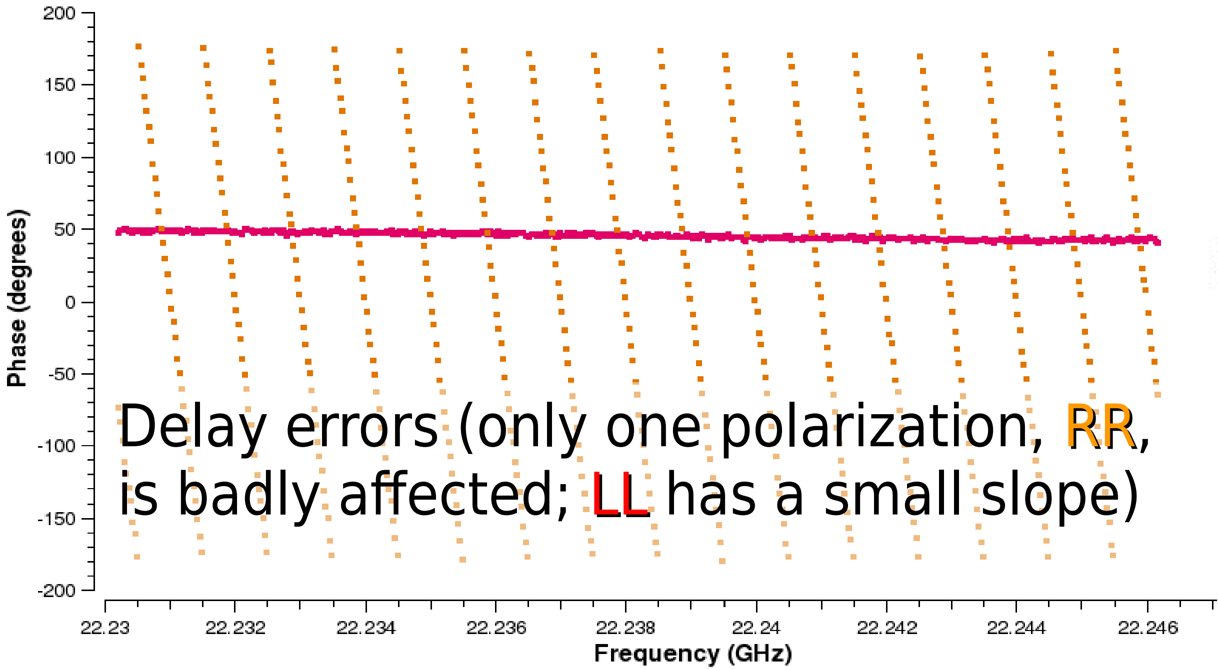
  4. Apply K, B1, G2a and perform time-dependent amp. cal. Gflux

  - Derive phase ref flux, set and repeat G3

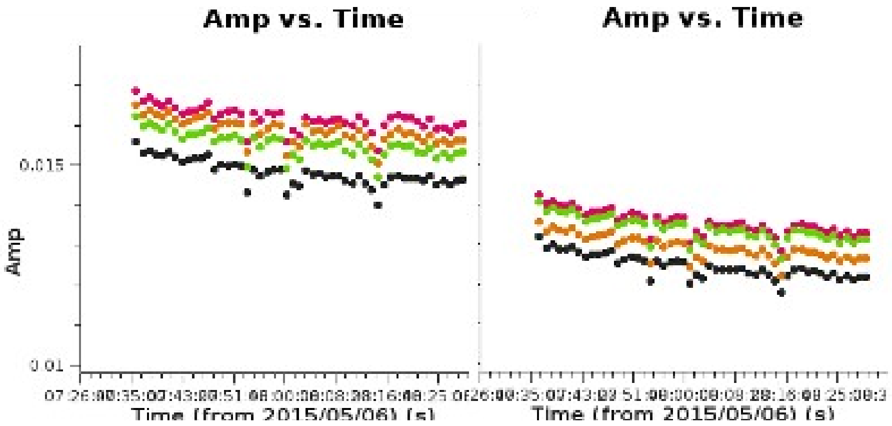
  5. Apply K, B1, G2b, G3 to target

Delay calibration

- Biggest errors due to instrumental timing errors
  - Usually stable for hours or more
    - Averaging across phase errors makes amps decorrelate



Delay errors (only one polarization, RR, is badly affected; LL has a small slope)



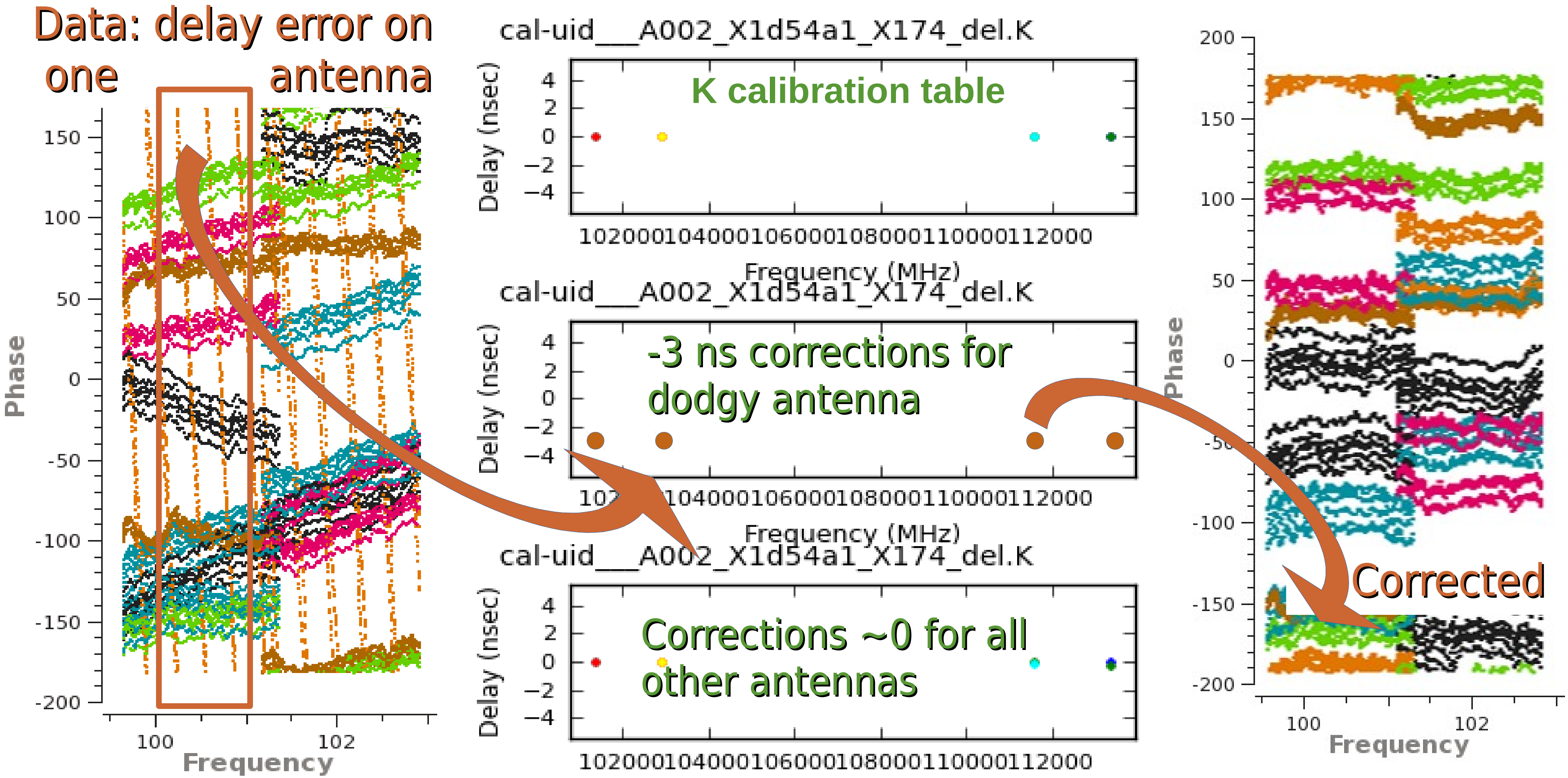
One channel (noisier but stronger)

All channels averaged (weaker)

- ~16 turns of phase in 16 MHz = 2π per MHz
  - 1/1MHz = 1μs delay correction needed

Delay correction

- Phase across 2 GHz undergoes ~3 full turns in 1 GHz
  - Delay error $3/10^9 = 3$ ns



Data: delay error on one antenna

K calibration table

-3 ns corrections for dodgy antenna

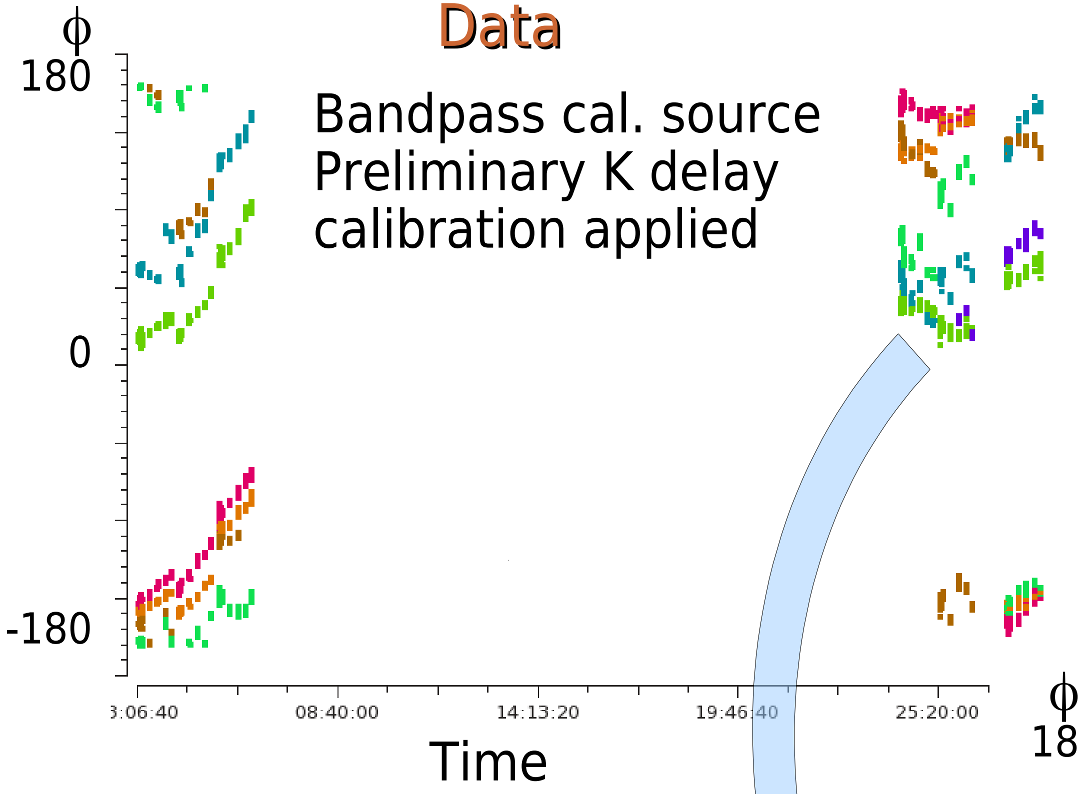
Corrections ~0 for all other antennas

Corrected

Two spw. Each baseline is shown in a different colour

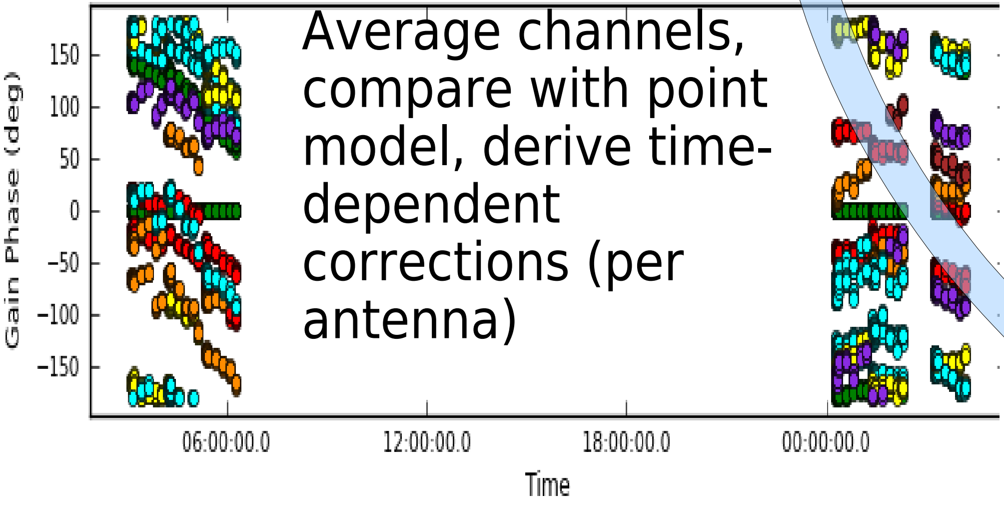
First time-dependent phase correction

Data



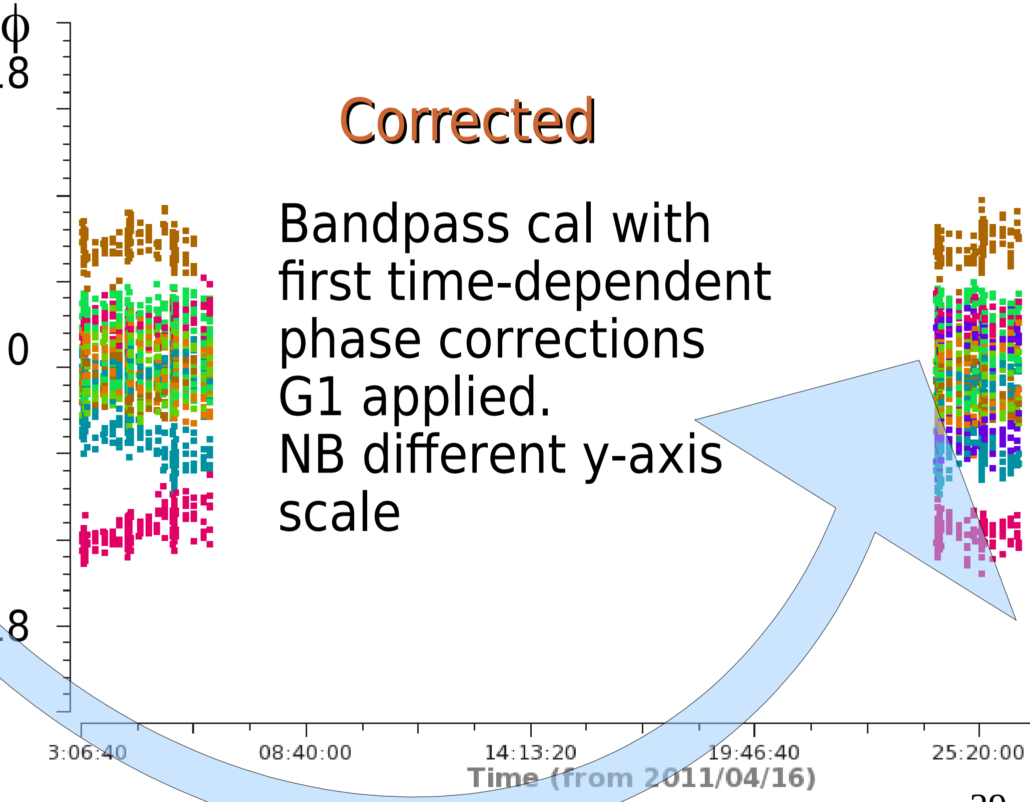
Bandpass cal. source
Preliminary K delay calibration applied

G1 calibration table



Average channels, compare with point model, derive time-dependent corrections (per antenna)

Corrected



Bandpass cal with first time-dependent phase corrections G1 applied.
NB different y-axis scale

Hands-on

(after Introduction to Interferometry, setting flux scale, inspection)

Bandpass calibrator pre-calibration

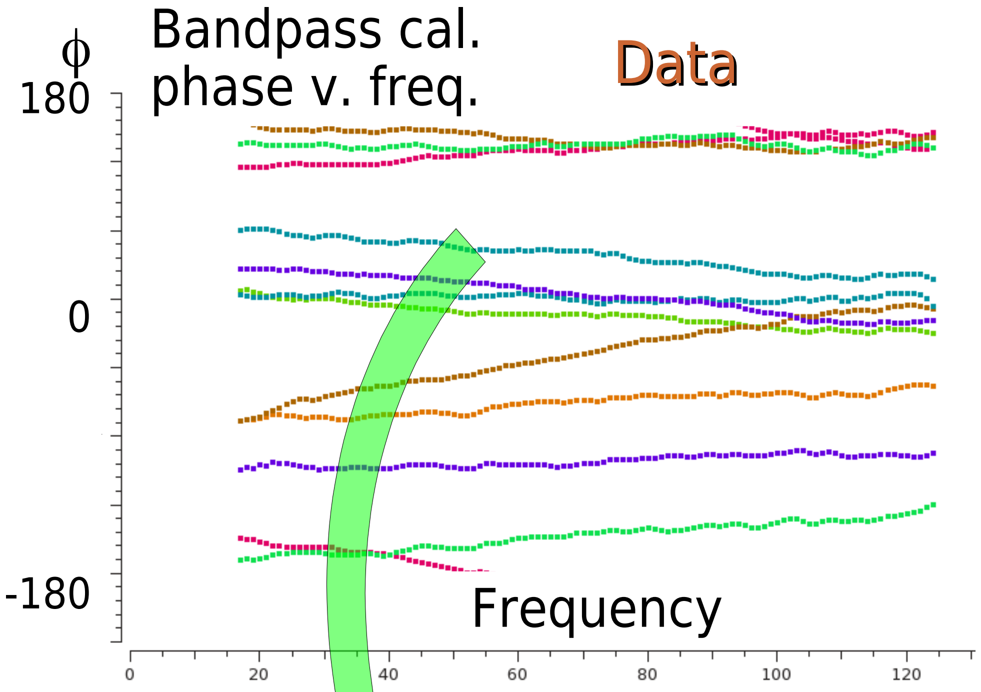
see web-page

Introduction to Calibration

(section 2B; step 13)

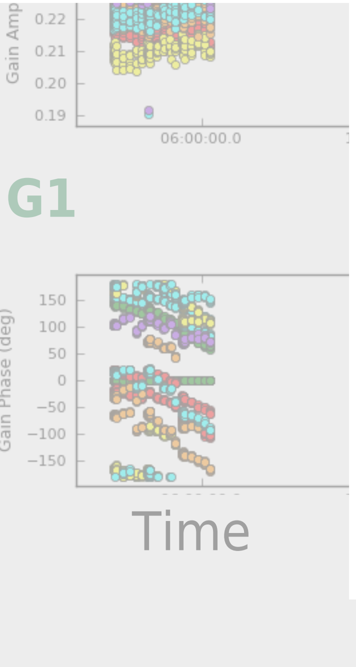
https://www.jb.man.ac.uk/DARA/ERIS22/calibration.html

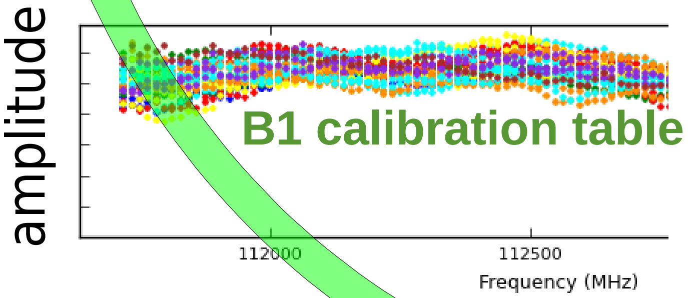
Bandpass calibration



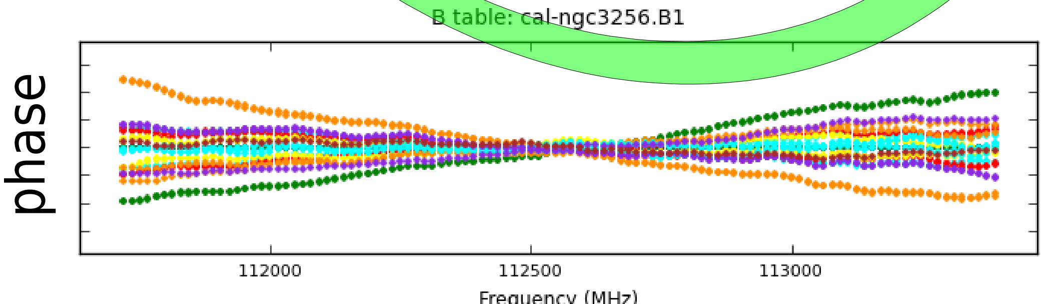
Bandpass cal. phase v. freq.

Apply 1st time-dependent corrections G1
Average all times, derive frequency-dependent calibration bandpass table B1

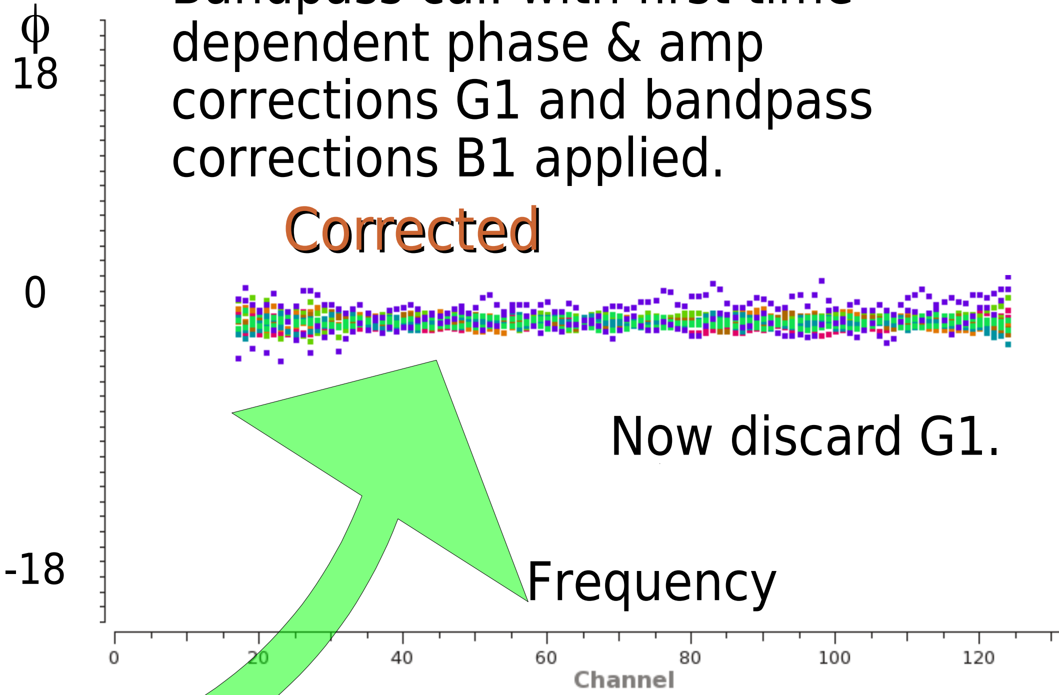




B1 calibration table



Bandpass cal. with first time-dependent phase & amp corrections G1 and bandpass corrections B1 applied.



Now discard G1.

NB1 Can't remove pure noise!

NB2 Continued application of K tables and other details omitted for simplicity

Hands-on

(after Introduction to Interferometry, setting flux scale,
inspection, BP cal pre-calibration)

Bandpass calibration

Residual delay calibration

Apply to check and decide phase-cal solution interval

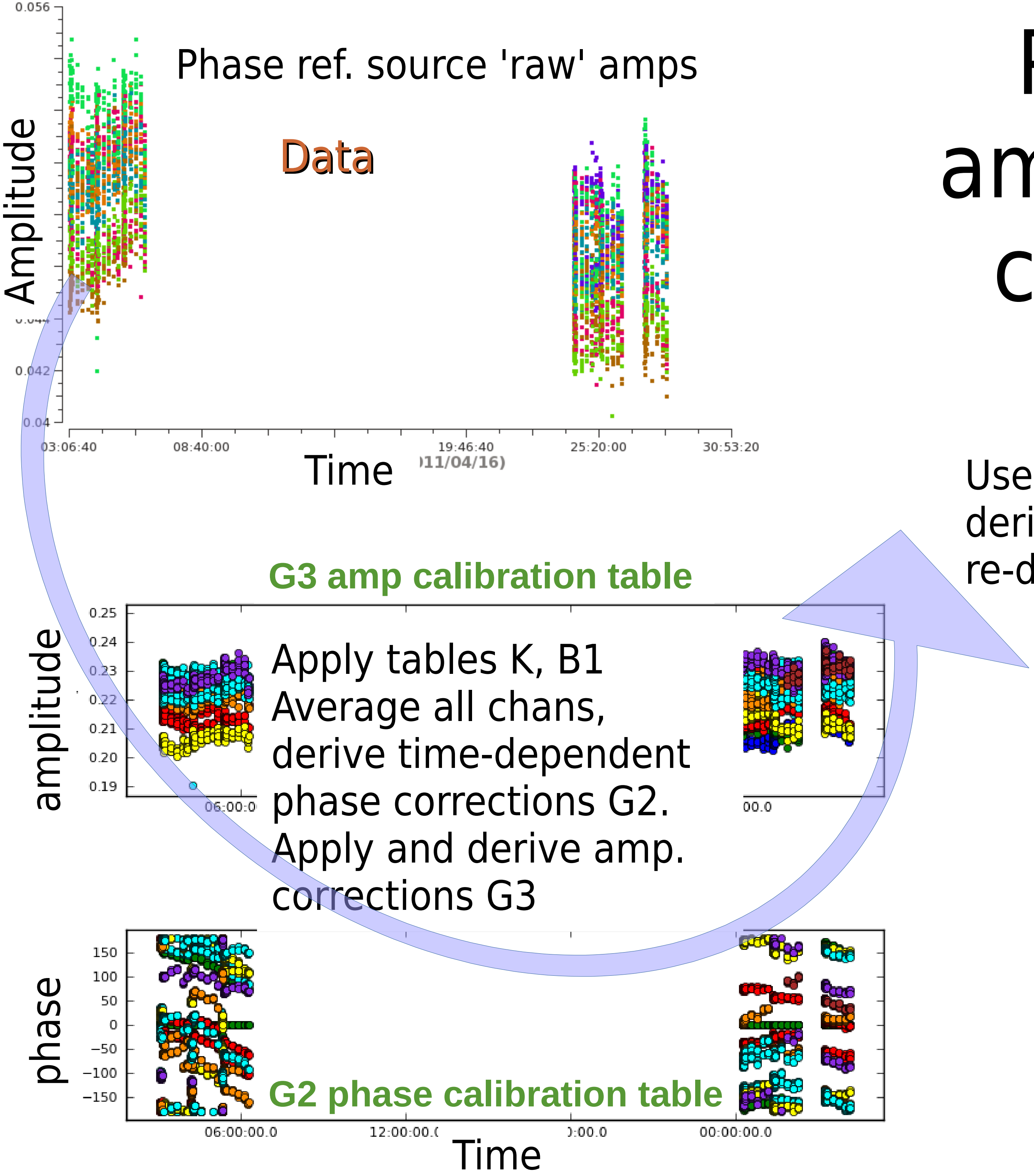
see web-page

Introduction to Calibration

(sections 2C, 3A; steps 14,15,16)

https://www.jb.man.ac.uk/DARA/ERIS22/calibration.html

Phase-ref amp & phase calibration



Phase ref. source 'raw' amps

Data

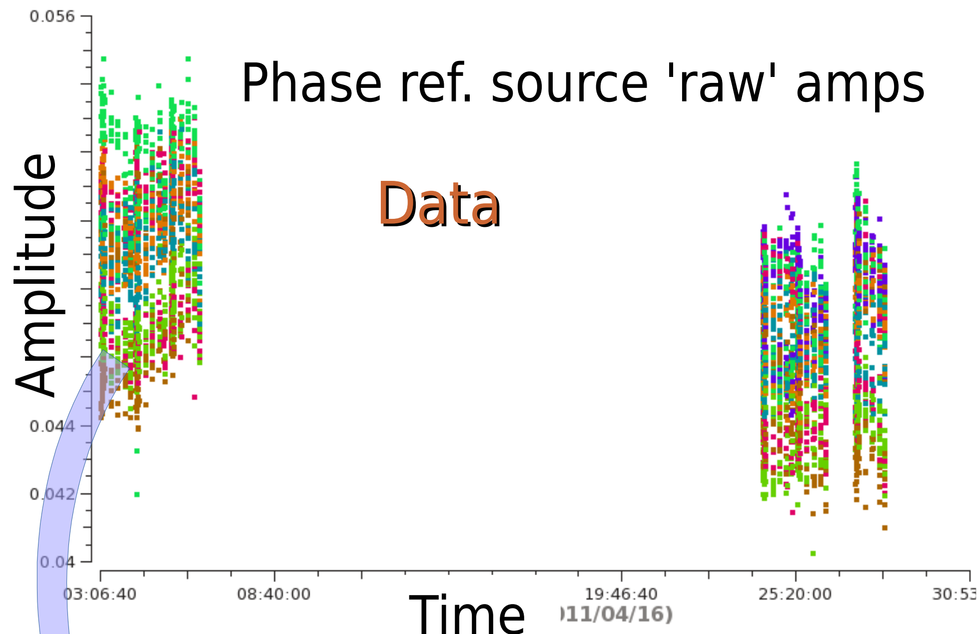
G3 amp calibration table

Apply tables K, B1
Average all chans,
derive time-dependent
phase corrections G2.
Apply and derive amp.
corrections G3

G2 phase calibration table

Use astrophysical standard to derive flux scale and use to re-derive amp corrections G3

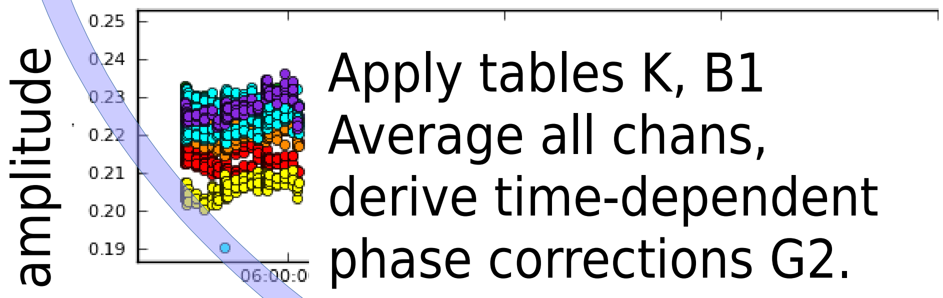
Phase-ref amp & phase calibration



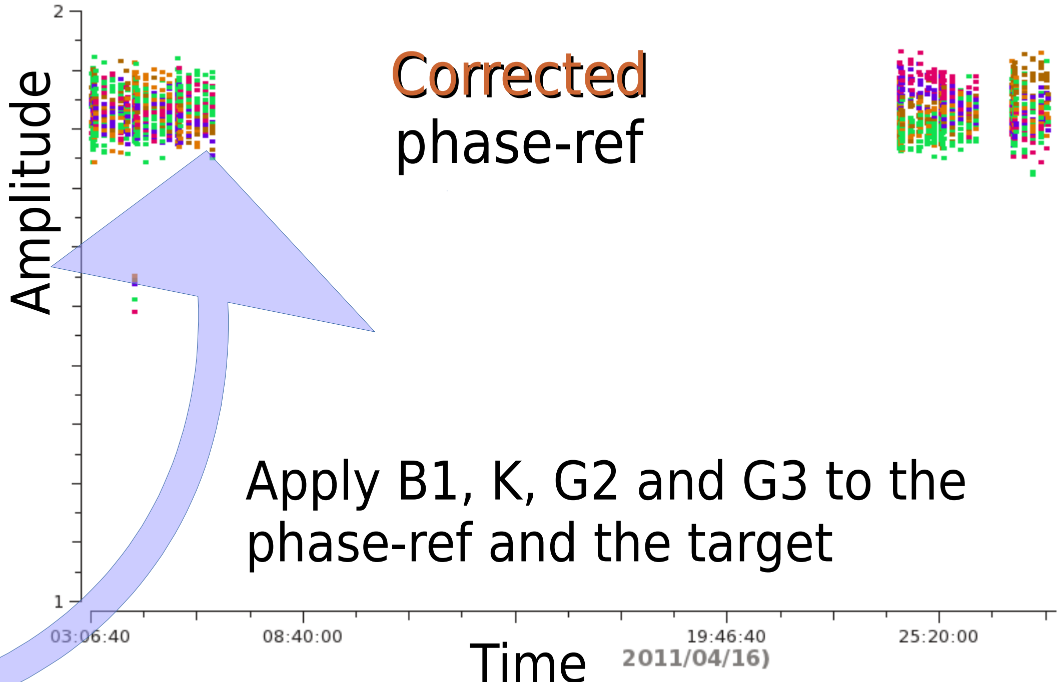
Phase ref. source 'raw' amps

Data

G3 amp calibration table

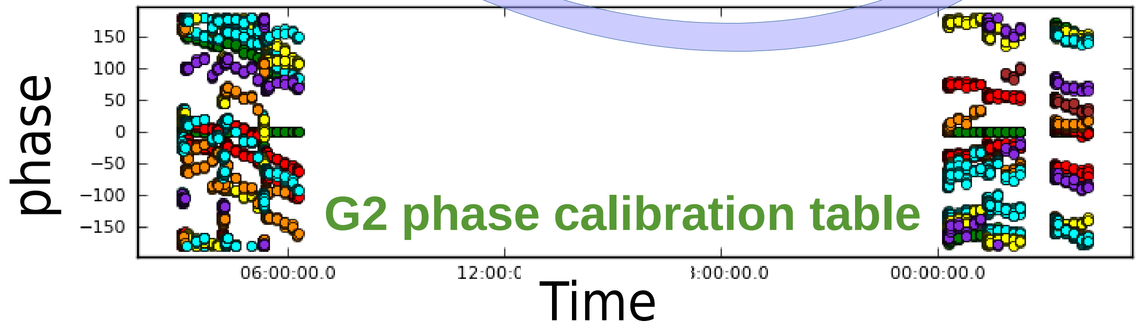


Apply tables K, B1
Average all chans,
derive time-dependent
phase corrections G2.
Apply and derive amp.
corrections G3



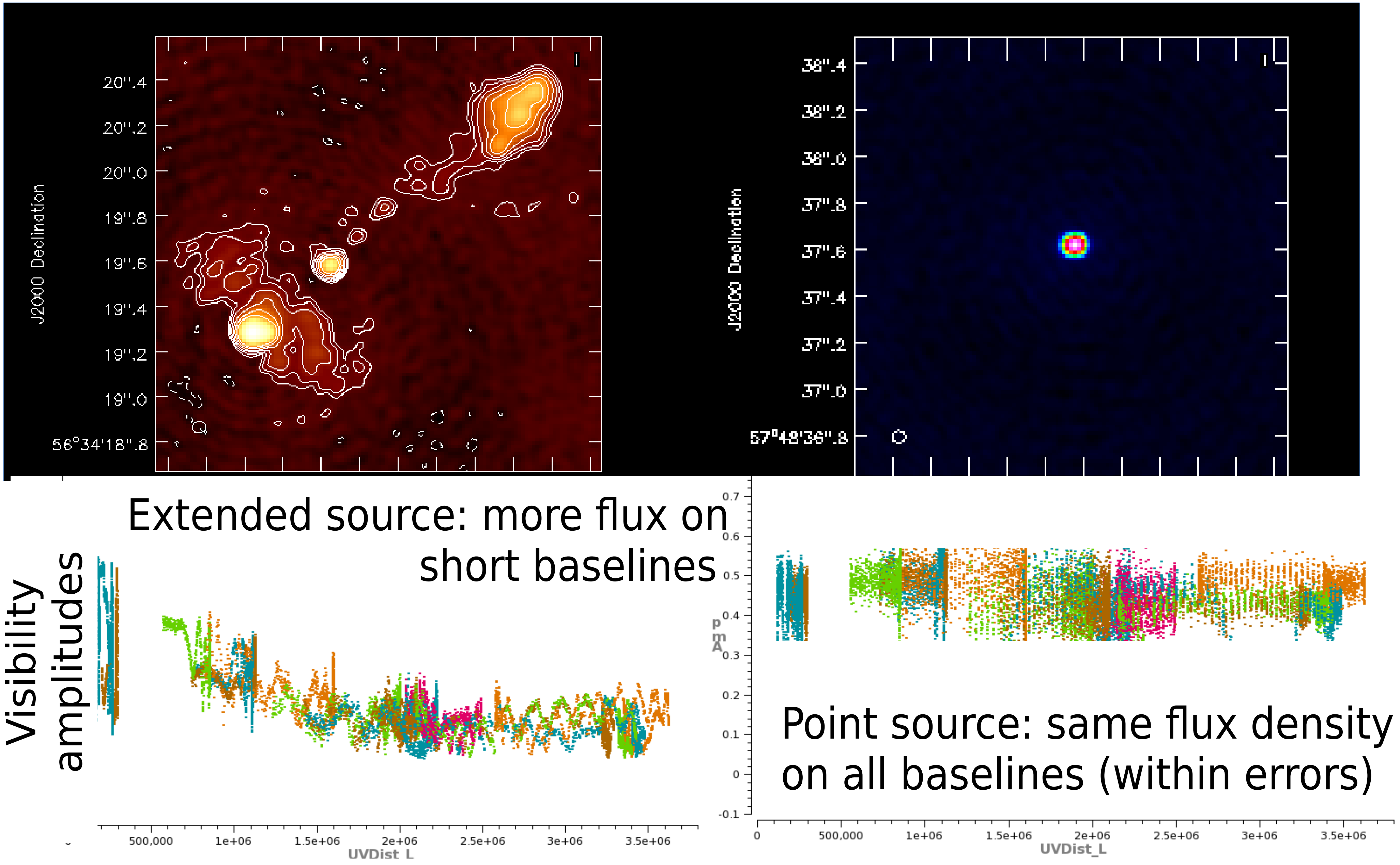
Corrected
phase-ref

Apply B1, K, G2 and G3 to the
phase-ref and the target



G2 phase calibration table

Source structure in *uv* plane



Extended source: more flux on short baselines

Point source: same flux density on all baselines (within errors)

Visibility amplitudes

Baseline length in wavelengths (*uv* distance)

Hands-on

(after Introduction to Interferometry, frequency-dependent calibration including bandpass and delay)

Derive phase calibrator calibration solutions

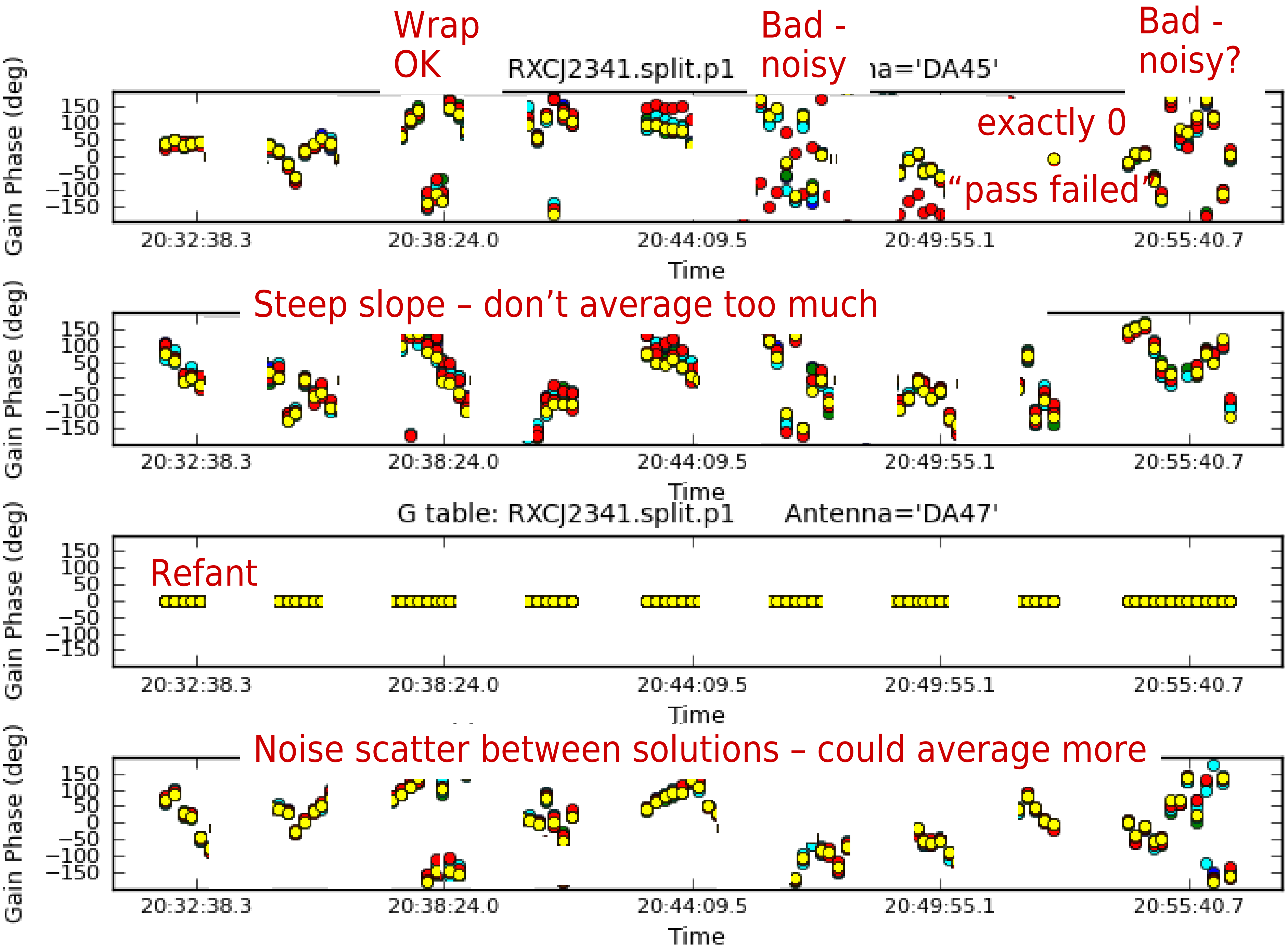
see web-page

Introduction to Calibration

(sections 4A, B, 5; steps 17,18,19)

https://www.jb.man.ac.uk/DARA/ERIS22/calibration.html

Short solint phase-ref phase solutions



Wrap OK

Bad - noisy

Bad - noisy?

exactly 0

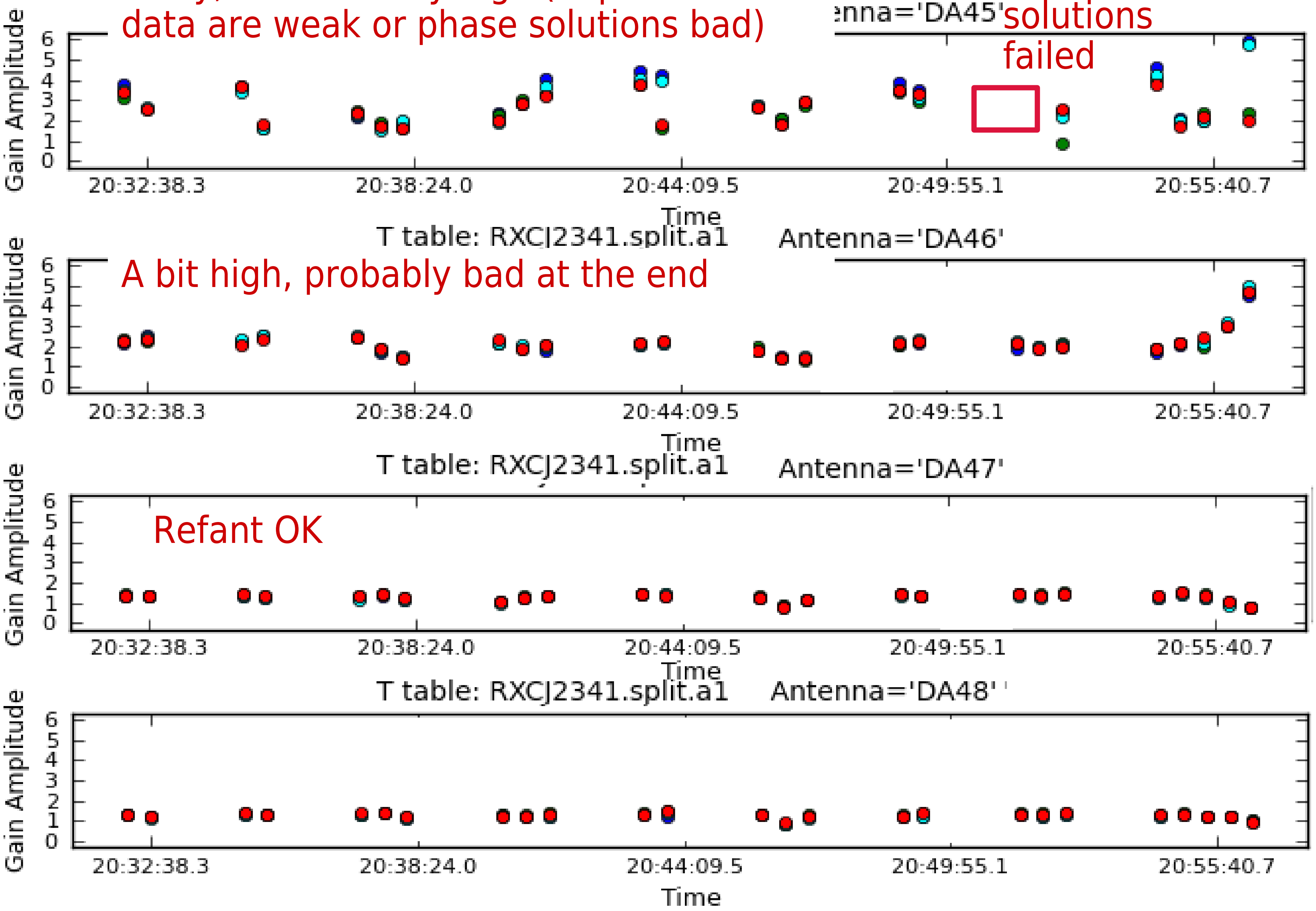
“pass failed”

Steep slope – don’t average too much

Refant

Noise scatter between solutions – could average more

Amp solutions



Noisy, anomalously high (implies data are weak or phase solutions bad)

Missing as solutions failed

A bit high, probably bad at the end

Refant OK

Calibration notes

- Always inspect the calibration solutions
  - If they look like random noise they won't do any good!
    - Look at the data – try a different averaging interval?
    - Have you applied necessary prior calibration?
    - Are there bad data?
      - You can always delete or clear calibration and try again
- Check for source resolution
  - Look at visibilities v. uv distance &/or image calibrators if you are not sure they are point-like
    - Build up model by cycles of imaging and calibration if a calibrator is resolved
- See later tutorials for advanced calibration

Hands-on

(after Introduction to Interferometry, frequency-dependent calibration including bandpass and delay, time dependent calibration of phase calibrator)

Apply calibration solutions

Split out target and inspect data

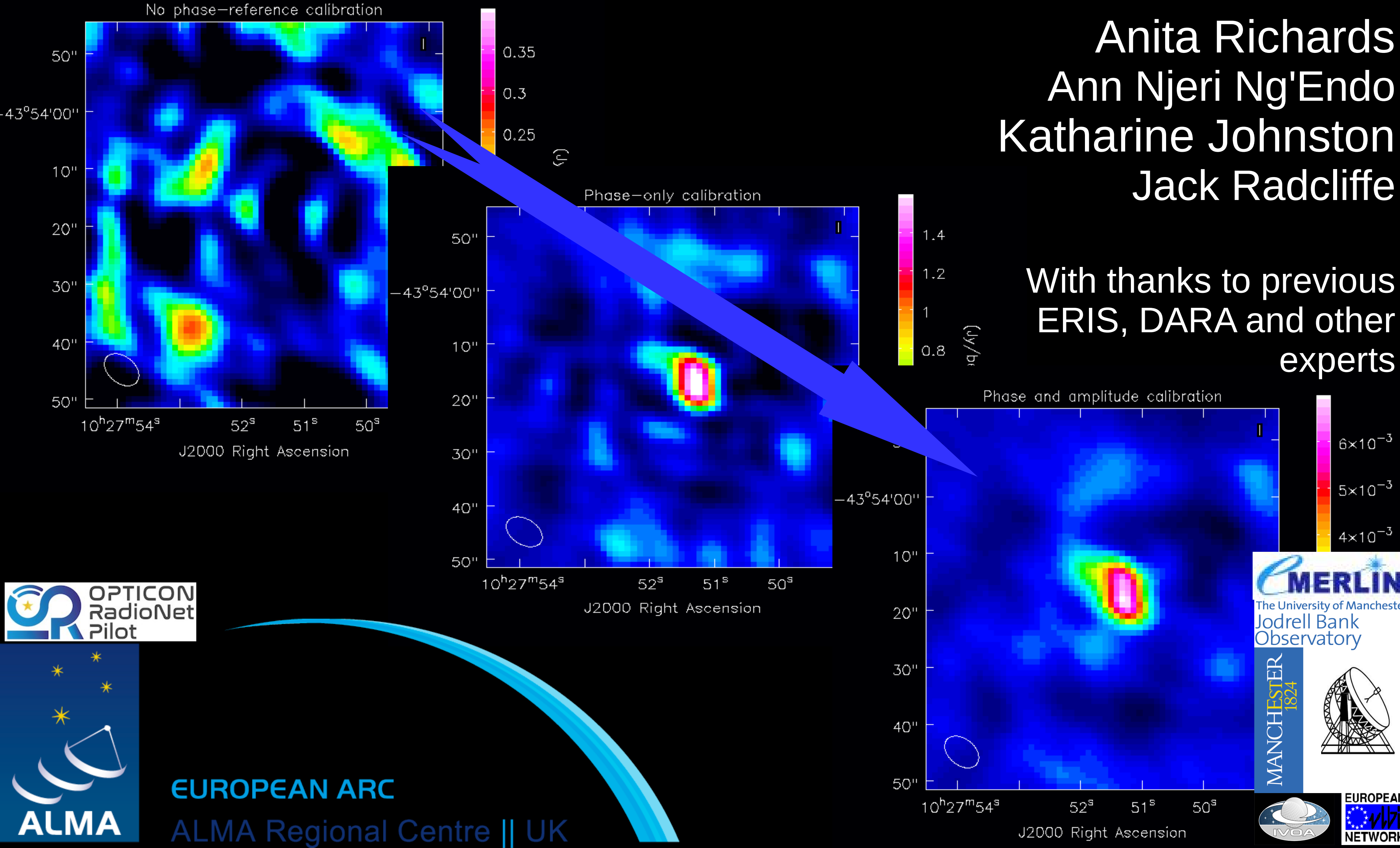
see web-page

Introduction to Calibration

(section 6; steps 20,21)

https://www.jb.man.ac.uk/DARA/ERIS22/calibration.html

Accuracy, references etc.



Anita Richards
Ann Njeri Ng'Endo
Katharine Johnston
Jack Radcliffe

With thanks to previous ERIS, DARA and other experts

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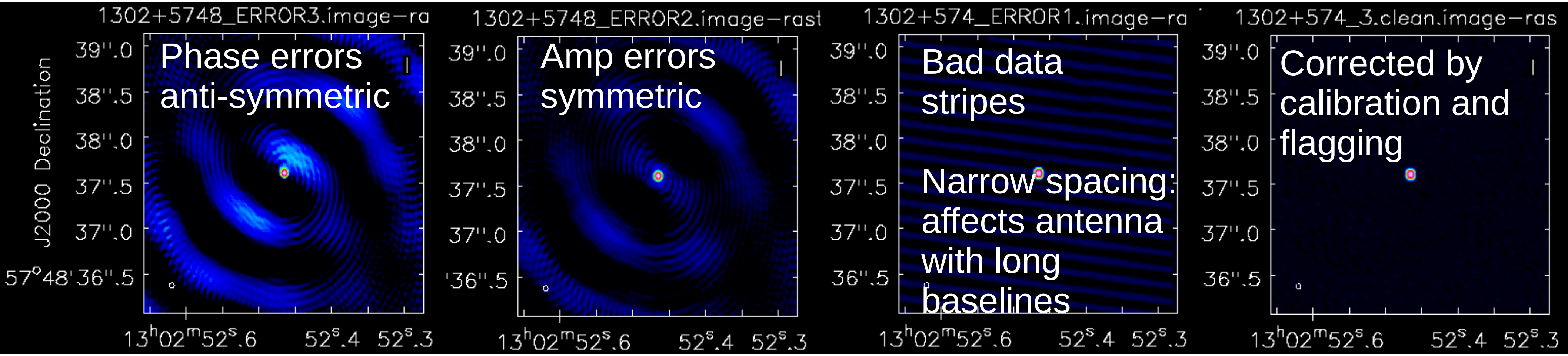
Error recognition from images

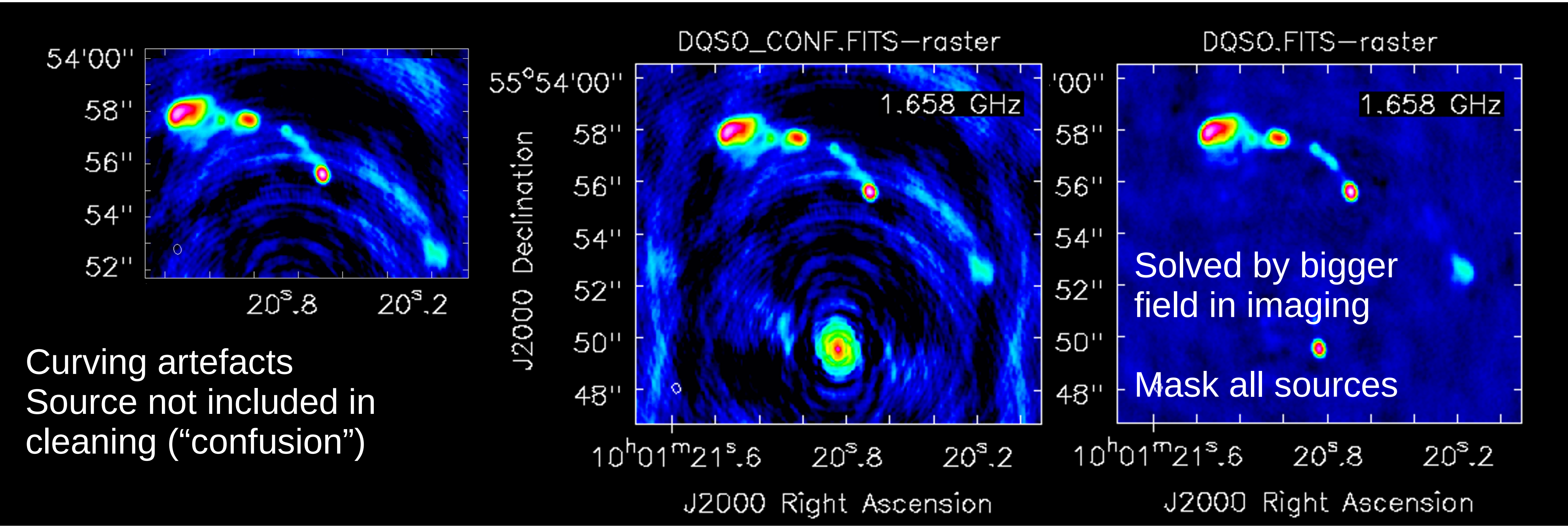

Phase errors anti-symmetric

Amp errors symmetric

Bad data stripes

Narrow spacing: affects antenna with long baselines

Corrected by calibration and flagging

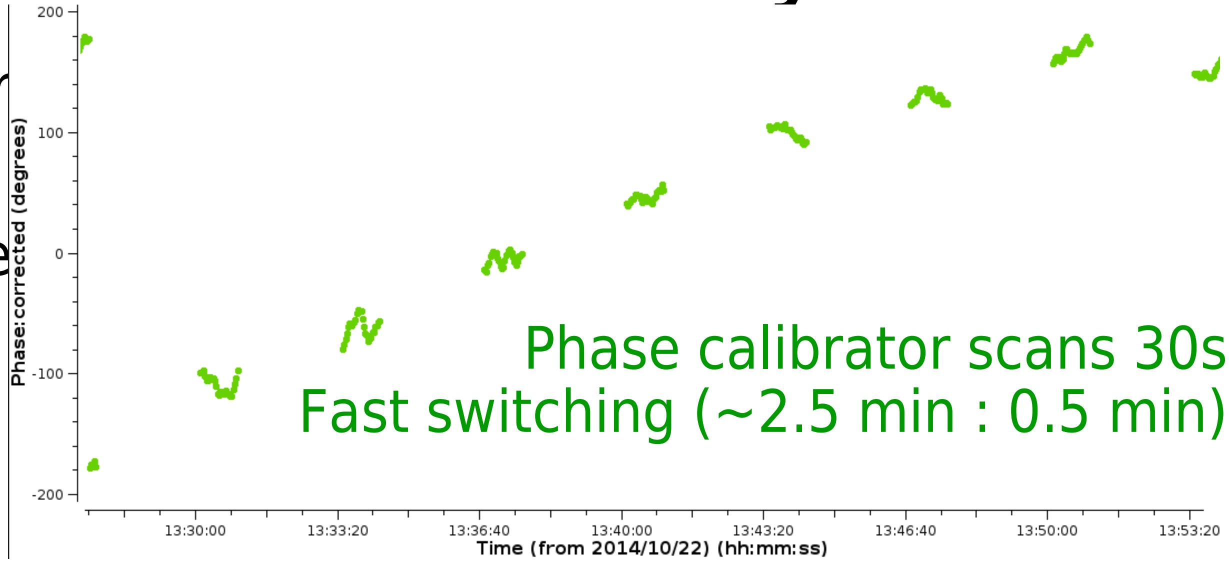

Curving artefacts
Source not included in cleaning ("confusion")

Solved by bigger field in imaging

Mask all sources

Phase transfer accuracy

- Phase-ref : target angular separation
  - Calibrator phase change $d\varphi_{atm} \sim \pi$ per ~30 min
  - $d\varphi_{atm/scan} = \pi/(30/2.5) = 15^{o}$ between scans



Phase calibrator scans 30s
Fast switching (~2.5 min : 0.5 min)

- Phase-ref: target separation, say $d\theta = 2^{o} = 120$ arcmin
  - Convert θ in degrees to 'R.A.-like' units of time
    - $(d\theta/360^{o})$ x cos(Dec.)x 24hr ~7.5^{min} of RA at Dec. 20^{o}
      - In 7.5 min, $d\varphi_{atm}$ gives $d\varphi_{atm/ang.sep.} = \pi/(30/7.5) = 45^{o}$ phase change
  - Atmospheric error $\sqrt{(d\varphi_{atm/ang.sep.}^{2} + d\varphi_{atm/scan}^{2})} \sim 47^{o}$
    - 25% target decorrelation, low dynamic range
  - Also noise, antenna pos. errors
    - But mitigated by independent baselines, many scans
      - And self-calibration of target

Astrometry

- Measure position of source before any self-calibration
- Relative accuracy of point source position fitting
  - $\sigma_{\mathrm{pos}} \sim k\theta_{\mathrm{beam}}$ /(S/N) (k ~0.5 to 1 for sparse - filled arrays)
    - σ_{pos} ~ 25 mas for 20σ source, 1 arcsec beam
- Position errors usually dominated by uncorrected phase errors due to phase-ref:target separation
  - e.g. phase corrections across target scan ~45°
    - Per antenna, so mean effect ~ 45° $/\sqrt{(N_{\mathrm{ant}} - 3)}$ ~7° 12-m array
    - Position error ~ θ_{beam} x 7/180 ~40 mas for 1 arcsec beam
- Limited by antenna position errors, residual delay etc.

Libraries use Measurement Equation

$$\underline{V}_{ij} = \mathbf{M}_{ij}\mathbf{B}_{ij}\mathbf{G}_{ij}\mathbf{D}_{ij}\int \mathbf{E}_{ij}\mathbf{P}_{ij}\mathbf{T}_{ij}\mathbf{F}_{ij} S \underline{I}_n (x,y)\, exp[i2\pi (u_{ij}x+v_{ij}y)]\, dxdy + \underline{A}_{ij}$$

Vectors

Starting point

$\underline{V}$ isibility = *f(u,v)*

Goal

$\underline{I}$ mage

$\underline{A}$dditive baseline error

Scalars

Methods

S (mapping $\underline{I}$ to observer polarization)

x,y image plane coords

u,v Fourier plane coords

i,j telescope pair

Jones Matrices

Hazards

Multiplicative baseline error

Bandpass response

Generalised electronic gain

Dterm (pol. leakage)

E (antenna voltage pattern)

Parallactic angle

Tropospheric effects

Faraday rotation

Using the Measurement Equation

- *Hamaker, Bregman & Sault 1996*
  - Decompose into relevant calibration components e.g.
- $\underline{V}_{ij}^{obs} = \mathbf{B}_{ij}\mathbf{G}_{ij}\mathbf{D}_{ij}\mathbf{P}_{ij}\mathbf{T}_{ij}\mathbf{F}_{ij}\underline{V}_{ij}^{ideal}$
  - Chose one (or a few) at a time
    - Usually solve fastest-varying first
      - (so averaging over slower-varying)
  - Compare data with model or idealisation
    - Linearise and solve by χ^2 (or other) minimization

The method behind solving the ME

- Express the correlator output as the coherency matrix of the signals from each pair of antennas *ij*.
  - Using a circular polarization basis, form outer product:

$$\mathbf{E}_{\mathrm{ij}}=\mathbf{e}_{\mathrm{i}}\,\mathbf{e}_{\mathrm{j}}^{\dagger}=\begin{pmatrix} R_i \\ L_i \end{pmatrix}\begin{pmatrix} R_j^{*} & L_j^{*} \end{pmatrix}=\begin{pmatrix} R_i R_j^{*} & R_i L_j^{*} \\ L_i R_j^{*} & L_i L_j^{*} \end{pmatrix}$$

    - Equivalent to $\boldsymbol{V}(u,v)_{\mathrm{ij}}=\begin{pmatrix} RR & RL \\ LR & LL \end{pmatrix}$

- Replace signal $\mathbf{e}$ from each antenna with corrupted signal $\mathbf{e}'_{\mathrm{i}} = \mathbf{J}_{\mathrm{i}}\,\mathbf{e}_{\mathrm{i}}$
  - $\mathbf{J}_{\mathrm{i}}$ is a (2 x 2) Jones matrix for antenna-based terms e.g., for the complex 'gain' errors affecting amplitude and phase:

$$\mathbf{J}_{\mathrm{G}}=\begin{pmatrix} g_R & 0 \\ 0 & g_L \end{pmatrix}$$

The method behind solving the ME

- The corruption of the 'true' visibilities $\mathbf{E}_{\mathrm{ij}}$ is written as

$$\mathbf{E}'_{\mathrm{ij}} = \mathbf{e}'_{\mathrm{i}} \mathbf{e}'^{\dagger}_{j} = \mathbf{J}_{\mathrm{i}} \mathbf{E}_{\mathrm{ij}} \mathbf{J}^{\dagger}_{\mathrm{j}}$$

  - Jones matrices known so expression can be inverted:

$$\mathbf{E}_{\mathrm{ij}} = \mathbf{J}_{\mathrm{i}}^{-1} \mathbf{E}'_{ij} \mathbf{J}_{\mathrm{j}}^{\dagger -1}$$

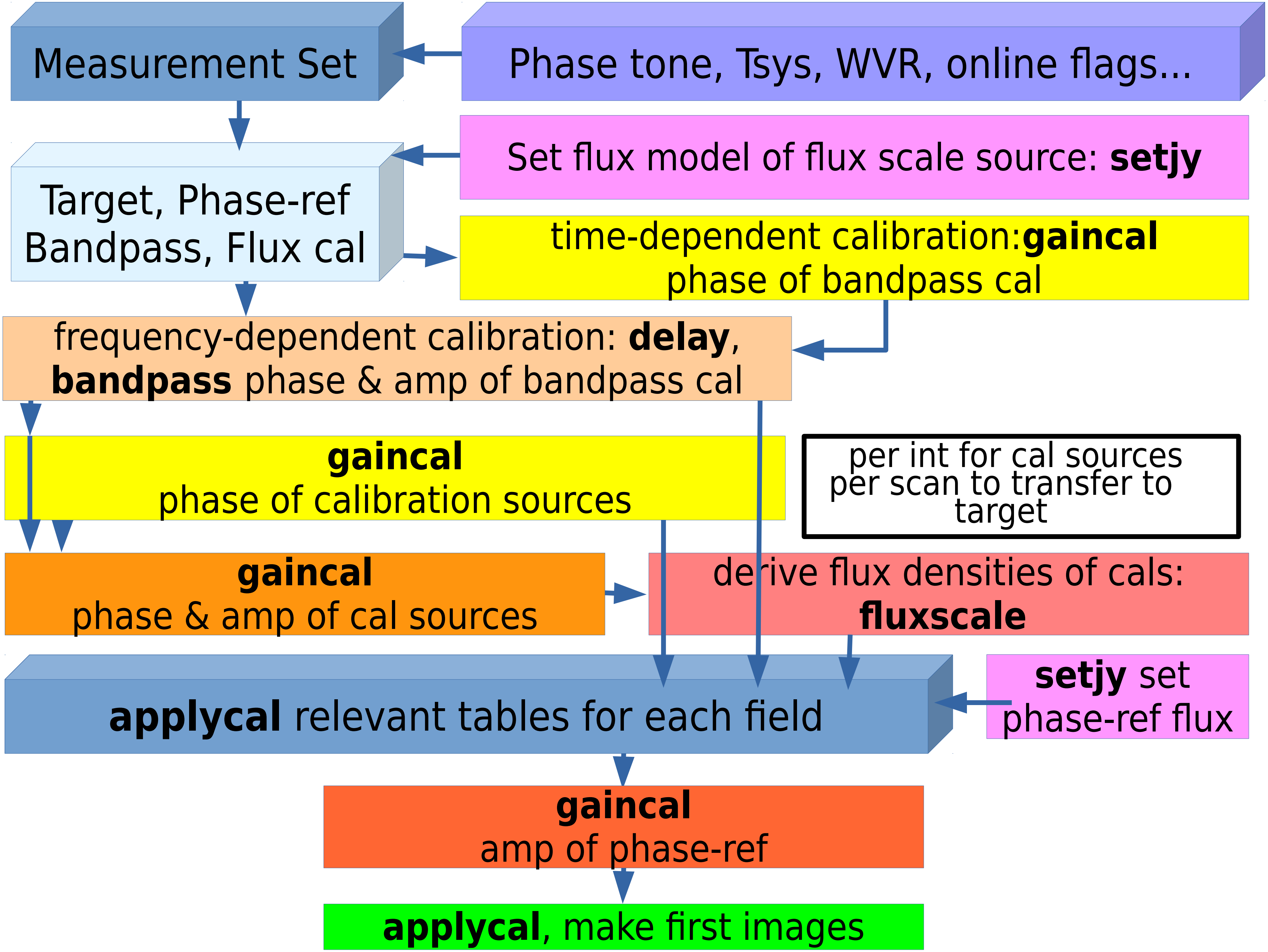
- If polarization is ignored and errors are constant across the (small) field of view, this can be linearised

$$V_{ij}^{obs} - J_i J_j^{*} V_{ij}^{mod}$$

  - V^{mod} are visibilities corrected for the errors represented by this Jones matrix, solved by to find corrections J_{i}, J_{j} to apply per antenna by minimising

$$\chi^2 = \sum \left| V_{ij}^{obs} - J_i J_j^{*} V_{ij}^{mod} \right|^2 W_{ij}$$

  - Weights (if any) $W_{\mathrm{ij}} = \mathrm{s}_{\mathrm{ij}}^{-2}$ are derived from previous noise estimates e.g. sample size, scatter in previous solutions



Measurement Set

Phase tone, Tsys, WVR, online flags...

Set flux model of flux scale source: **setjy**

Target, Phase-ref
Bandpass, Flux cal

time-dependent calibration:**gaincal**
phase of bandpass cal

frequency-dependent calibration: **delay**,
bandpass phase & amp of bandpass cal

gaincal
phase of calibration sources

per int for cal sources
per scan to transfer to target

gaincal
phase & amp of cal sources

derive flux densities of cals:
fluxscale

applycal relevant tables for each field

setjy set
phase-ref flux

gaincal
amp of phase-ref

applycal, make first images

References

- Thompson, Moran & Swenson, 2017, *Interferometry and Synthesis in Radio Astronomy* (theory) https://link.springer.com/book/10.1007/978-3-319-44431-4
- Taylor, Carilli & Perley, 1989, *Synthesis Imaging NRAO Summer School* (very practical although exact software dated) https://www.aspbooks.org/a/volumes/table_of_contents/?book_id=292
- Also see more recent NRAO Summer School lectures online
    - Some in arXiv e.g. Brogan et al. 2018 arXiv:1805.05266
- Previous ERIS lectures online
- ALMA memos (of general relevance for cm and VLBI also):
    - Maud et al. 2016 *Allegro Phase Metrics Workshop* https://library.nrao.edu/public/memos/alma/main/memo606.pdf
    - Richards et al. 2022 *Self-Calibration and Imaging Fidelity* https://library.nrao.edu/public/memos/alma/main/memo620.pdf