Polarization in interferometry

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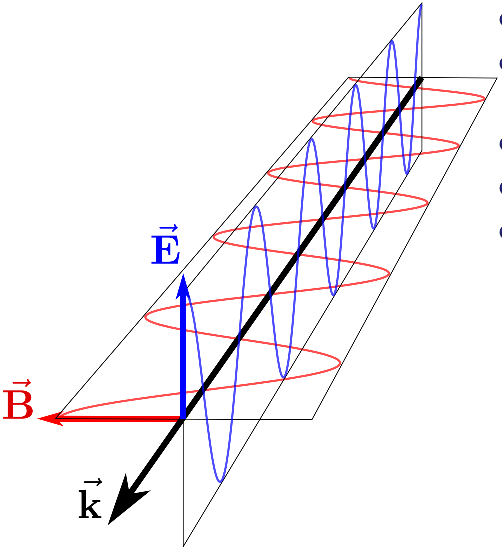
Reading list

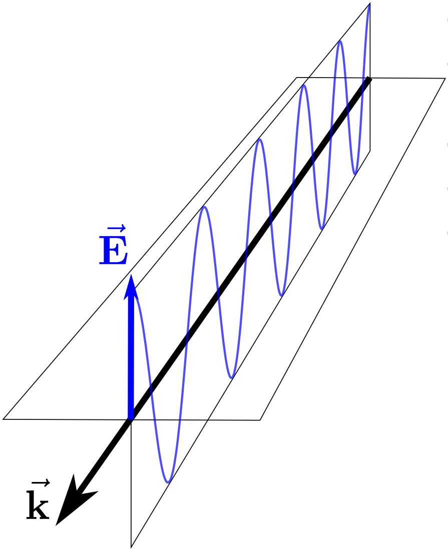
- Born & Wolf *Principles of optics*
- Thompson, Moran & Swenson *Interferometry and Synthesis in Radio Astronomy*
- Taylor, Carilli & Perley *Synthesis Imaging in Radio Astronomy II*
- Bracewell *The Fourier Transform & Its Applications*
- Hamaker, Bregman & Sault *Understanding radio polarimetry: paper I*(1996)
- Sault, Hamaker& Bregman *paper II*(1996)
- Hamaker & Bregman *paper III* (1996)
- Hamaker *paper IV* (2000)
- Hamaker *paper V* (2006)
- Brentjens & de Bruyn *Faraday rotation measure synthesis* (2005)

Outline

Electromagnetic (EM) wave



- Vector phenomenon
- From Maxwell's equations: $\hat{\mathbf{k}} = \hat{\mathbf{E}} \times \hat{\mathbf{B}}$
- We know **k**
- Measure either **E** or **B**
- **E** is easier



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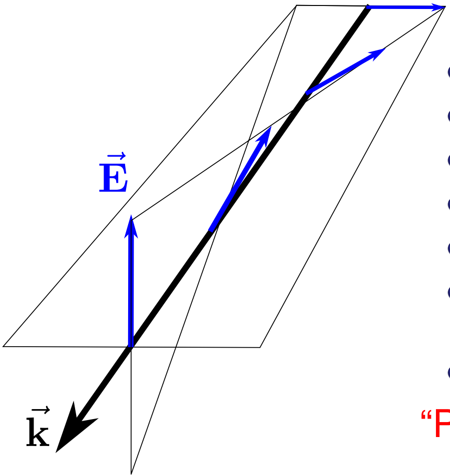
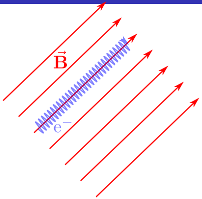
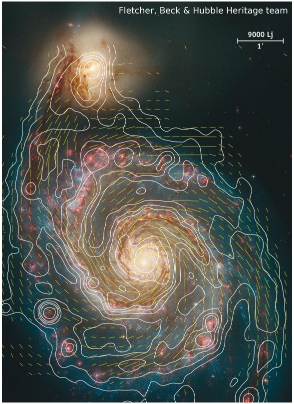


- Vector phenomenon
- From Maxwell's equations: $\hat{\mathbf{k}} = \hat{\mathbf{E}} \times \hat{\mathbf{B}}$
- We know **k**
- Measure either **E** or **B**
- **E** is easier
- But:
- E_x and E_y not equal
- **E** may rotate as function of **x** and t.
- **E** traces ellipse

"Polarization"

Outline





Process

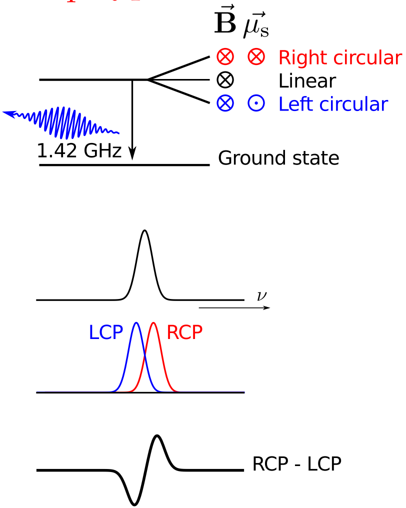
- Generates polarized emission
- Main emission mechanism at cm-m wavelength
- Up to 80% linearly polarized
- No circular
- $\langle \mathbf{E}_{\mathrm{source}} \rangle \perp \mathbf{B}_{\mathrm{source}}$

Polarimetry provides

- **B**-field direction
- Turbulence
- Indirectly: **B**-field strength

Zeeman splitting

$\mathrm{H_I}$ hyperfine transition



Process

- Generates polarized emission
- Only in spectral lines
- If magnetic moment: e.g. $\mathrm{H_I}$, OH, CN, H_2O
- **B**-field splits RCP and LCP
- Separation: 2.8 Hz mG^{-1}

Polarimetry provides

- **B**-field strength at source
- If detectable. . .

Zeeman effect *Vlemmings et al. (2001)*

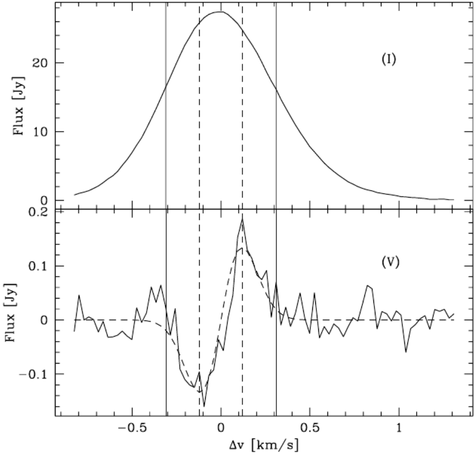
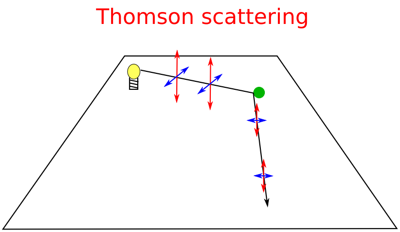
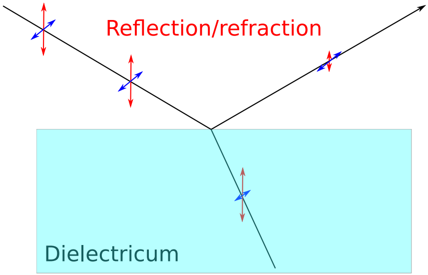


Fig. 4. Total power (I) and circular polarization (V) spectrum of the brightest H_2O maser feature around S Per. The dashed line is the fit of the synthetic V-spectrum to the observed spectrum. Also shown are the observed (dashed) and expected (solid) positions of the minimum and maximum of the V-spectrum.

Scattering/reflection



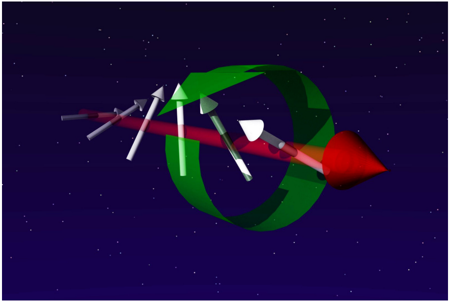


Process

- Modifies polarization state
- Thomson scattering: no T dependence
- Planets / Moon: dielectric transition

Polarimetry provides

- Electron densities in cool gas
- Dust properties
- Lunar dielectric constant

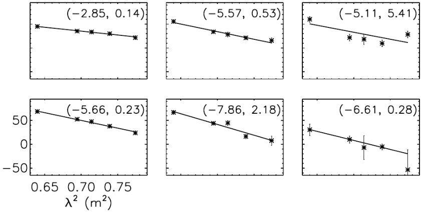


Process

- Modifies polarization state
- Delay between LCP and RCP
- Rotates linear pol angle
- $\Delta\chi = \chi_0 + \phi\lambda^2$

$$\phi = 0.812 \int_{\text{there}}^{\text{here}} n_e \mathbf{B} \cdot d\mathbf{l}$$

λ^2 law *Haverkorn et al. (2001)*

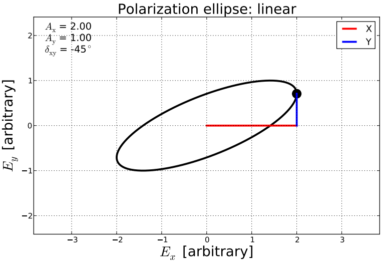


Polarimetry provides

- Source plasma properties
- Intervening plasma properties
- Rare cases: 3D tomography

Outline

Geometry

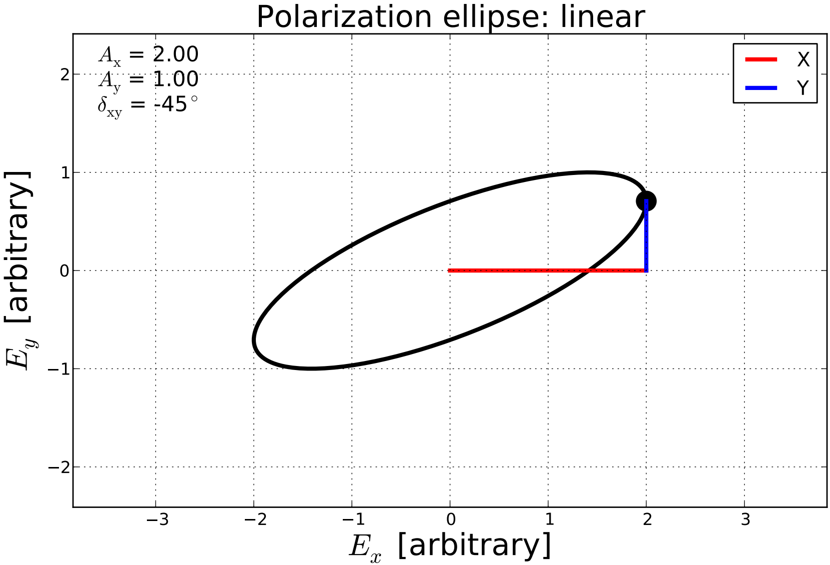


Viewing from antenna towards source, watching orientation and length of **E** vector on a plane at a fixed location in space.

$$\mathbf{E} = E_x\hat{\mathbf{e}}_x + E_y\hat{\mathbf{e}}_y$$
$$E_x = A_x\cos(2\pi\nu t + \delta_x)$$
$$E_y = A_y\cos(2\pi\nu t + \delta_y)$$

- A_x = x-amplitude
- A_y = y-amplitude
- $\delta_{xy} = \delta_y - \delta_x$
- δ_{xy} = measure of ellipticity
- $\delta_{xy} > 0$: CW rotation $\Rightarrow$ LEP
- $\delta_{xy} = 0$: linear polarization
- $\delta_{xy} < 0$: CCW rotation $\Rightarrow$ REP

Polarization ellipse: *xy* 000



Polarization ellipse: circular basis

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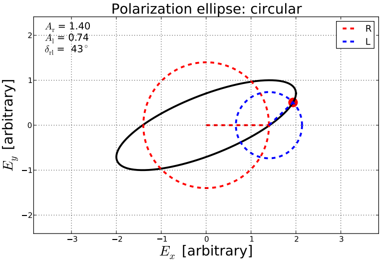
Geometry



Viewing from antenna towards source, watching orientation and length of **E** vector on a plane at a fixed location in space.

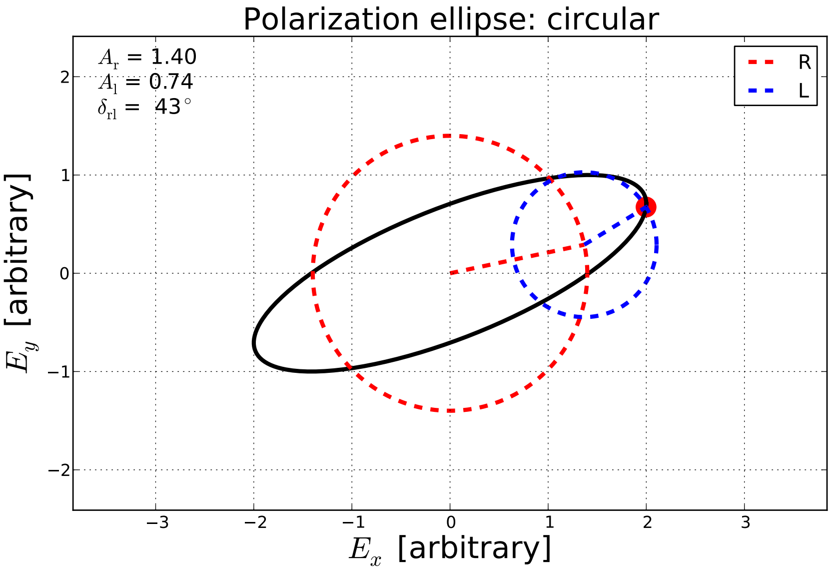
$$\mathbf{E} = A_\mathrm{r}\hat{\mathbf{e}}_\mathrm{r} + A_\mathrm{l}\hat{\mathbf{e}}_\mathrm{l}$$

$$\hat{\mathbf{e}}_\mathrm{r} = \begin{pmatrix} \cos(2\pi\nu t + \delta_\mathrm{r}) \\ \sin(2\pi\nu t + \delta_\mathrm{r}) \end{pmatrix}$$

$$\hat{\mathbf{e}}_\mathrm{l} = \begin{pmatrix} \cos(2\pi\nu t + \delta_\mathrm{l}) \\ -\sin(2\pi\nu t + \delta_\mathrm{l}) \end{pmatrix}$$

- $A_\mathrm{r} + A_\mathrm{l}$ = semi-major axis
- $\|A_\mathrm{r} - A_\mathrm{l}\|$ = semi-minor axis
- $\delta_\mathrm{rl} = \delta_\mathrm{l} - \delta_\mathrm{r}$
- $-\frac{1}{2}\delta_\mathrm{rl}$ = position angle of MA
- $\delta_\mathrm{rl} > 0$: MA rotated CW
- $\delta_\mathrm{rl} = 0$: MA along x-axis
- $\delta_\mathrm{rl} < 0$: MA rotated CCW

Polarization ellipse: circular 001



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$$A_{\mathrm{r}} = \frac{1}{2}\sqrt{A_{\mathrm{x}}^2 + A_{\mathrm{y}}^2 - 2A_{\mathrm{x}}A_{\mathrm{y}}\sin\delta_{\mathrm{xy}}}$$

$$A_{\mathrm{l}} = \frac{1}{2}\sqrt{A_{\mathrm{x}}^2 + A_{\mathrm{y}}^2 + 2A_{\mathrm{x}}A_{\mathrm{y}}\sin\delta_{\mathrm{xy}}}$$

$$\tan\delta_{\mathrm{rl}} = \frac{2A_{\mathrm{x}}A_{\mathrm{y}}\cos\delta_{\mathrm{xy}}}{A_{\mathrm{x}}^2 - A_{\mathrm{y}}^2}$$

Stokes parameters

- Three parameters enough
- Same units is convenient
- George Stokes defined four parameters (1852)
- Chandrasekhar introduced them to astronomy (1946)

$$I = A_\mathrm{x}^2 + A_\mathrm{y}^2 \qquad I = A_\mathrm{r}^2 + A_\mathrm{l}^2$$
$$Q = A_\mathrm{x}^2 - A_\mathrm{y}^2 \qquad Q = 2A_\mathrm{r}A_\mathrm{l}\cos\delta_\mathrm{rl}$$
$$U = 2A_\mathrm{x}A_\mathrm{y}\cos\delta_\mathrm{xy} \qquad U = -2A_\mathrm{r}A_\mathrm{l}\sin\delta_\mathrm{rl}$$
$$V = -2A_\mathrm{x}A_\mathrm{y}\sin\delta_\mathrm{xy} \qquad V = A_\mathrm{r}^2 - A_\mathrm{l}^2$$

- Monochromatic wave 100% polarized:

$$I^2 = Q^2 + U^2 + V^2$$

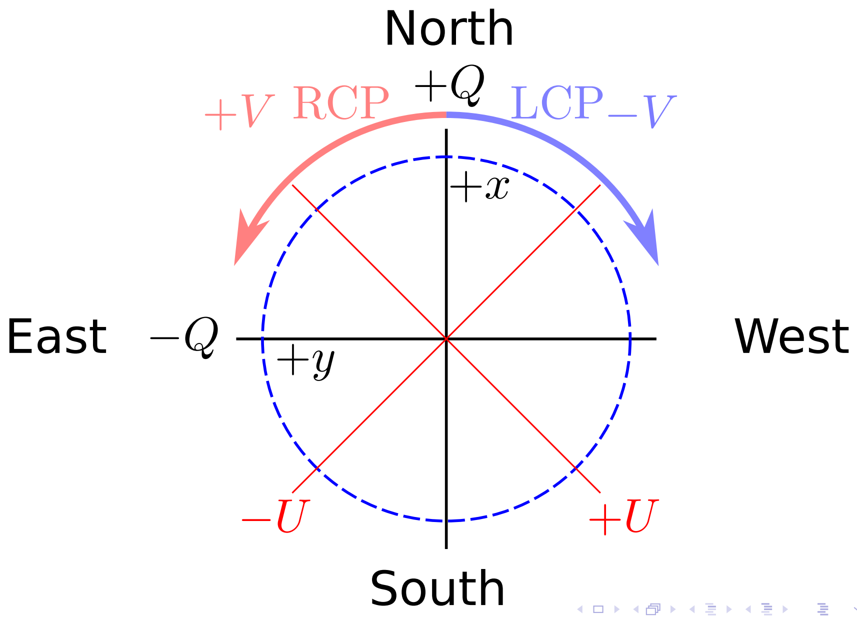
Stokes parameters

- Three parameters enough
- Same units is convenient
- George Stokes defined four parameters (1852) *ABCD*
- Chandrasekhar introduced them to astronomy (1946) $I_l I_r U V$

$$
\begin{aligned}
I &= A_{\mathrm{x}}^2 + A_{\mathrm{y}}^2 \\
Q &= A_{\mathrm{x}}^2 - A_{\mathrm{y}}^2 \\
U &= 2A_{\mathrm{x}}A_{\mathrm{y}} \cos\delta_{\mathrm{xy}} \\
V &= -2A_{\mathrm{x}}A_{\mathrm{y}} \sin\delta_{\mathrm{xy}}
\end{aligned}
\qquad
\begin{aligned}
I &= A_{\mathrm{r}}^2 + A_{\mathrm{l}}^2 \\
Q &= 2A_{\mathrm{r}}A_{\mathrm{l}} \cos\delta_{\mathrm{rl}} \\
U &= -2A_{\mathrm{r}}A_{\mathrm{l}} \sin\delta_{\mathrm{rl}} \\
V &= A_{\mathrm{r}}^2 - A_{\mathrm{l}}^2
\end{aligned}
$$

- Monochromatic wave 100% polarized:

$$I^2 = Q^2 + U^2 + V^2$$


North
+Q
+V RCP
LCP −V
+x
East
−Q
+y
West
−U
+U
South

Public angry letter from IAU (Dec 2015)

ASTRON

IAU

International Astronomical Union
Union Astronomique Internationale
98bis bd Arago, F - 75014 Paris, France ■ www.iau.org

Piero Benvenuti – General Secretary
e-mail: iau-general.secretary@iap.fr

To Whom It May Concern

The issue

Scientists working on the polarization of [redacted] use a convention for the polarization angle (PA) which is opposite to the IAU approved standard. This may cause confusion and misunderstandings.

Background

The convention astronomers follow for the PA (Polarization Angle) goes back to the 19th century and it has been in use for observations going from radio to gamma rays: the PA increases counter-clockwise when looking at the source. This convention is consistent with the one used for the Position Angle and it has been enforced by the IAU with a Resolution by Commissions 25 and 40 at the IAU XVth General Assembly in Sydney in 1973 (see Transactions of the IAU, Vol. XVB, pg. 166).

Recently, the scientists investigating the polarization of [redacted], have unfortunately adopted the opposite convention (PA increasing clockwise when looking at the source). This corresponds to a change of sign of the U Stokes parameter and is causing confusion and misunderstandings, in particular in the case of polarization data coming from experiments and satellites which are used by [redacted] and by other astronomers.

Recommendation

The IAU recommends that all astronomers, including those [redacted], follow the IAU Resolution for the Polarization Angle in all their publications.

Paris, December 8th, 2015

Piero Benvenuti	Pietro Ubertini	Saul J. Adelman
IAU General Secretary	President, Division B	President, Commission B6

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Quasi-monochromatic approximation

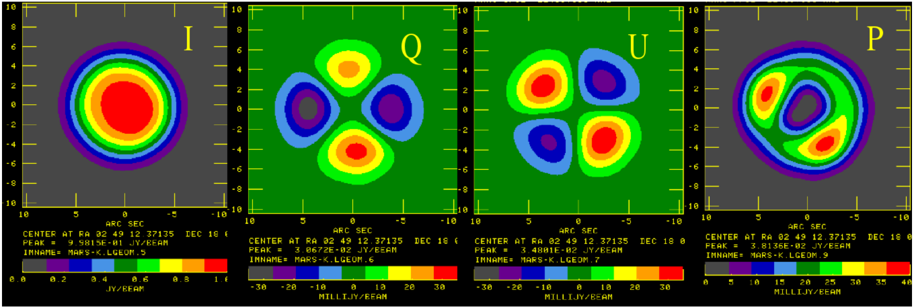
- Monochromatic radiation does not exist
- Finite bandwidth $\Delta\nu$; averaging time $\tau \gg \Delta\nu^{-1}$

$$
\begin{aligned}
I &= \langle A_\mathrm{x}^2 \rangle + \langle A_\mathrm{y}^2 \rangle \\
Q &= \langle A_\mathrm{x}^2 \rangle - \langle A_\mathrm{y}^2 \rangle \\
U &= \langle 2 A_\mathrm{x} A_\mathrm{y} \cos \delta_\mathrm{xy} \rangle \\
V &= \langle -2 A_\mathrm{x} A_\mathrm{y} \sin \delta_\mathrm{xy} \rangle
\end{aligned}
\qquad
\begin{aligned}
I &= \langle A_\mathrm{r}^2 \rangle + \langle A_\mathrm{l}^2 \rangle \\
Q &= \langle 2 A_\mathrm{r} A_\mathrm{l} \cos \delta_\mathrm{rl} \rangle \\
U &= \langle -2 A_\mathrm{r} A_\mathrm{l} \sin \delta_\mathrm{rl} \rangle \\
V &= \langle A_\mathrm{r}^2 \rangle - \langle A_\mathrm{l}^2 \rangle
\end{aligned}
$$

$$I^2 \geq Q^2 + U^2 + V^2$$

- Fractional linear pol: $p = \sqrt{Q^2 + U^2}/I \leq 1$
- Fractional circular pol: $v = \|V\|/I \leq 1$

TEST: Mars with VLA at 23.4 GHz *Perley*

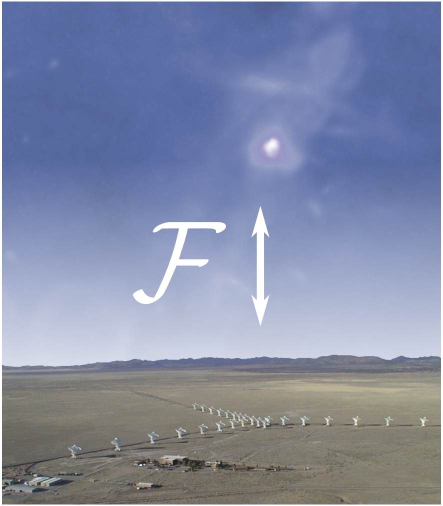


- This is thermal emission
- 1) Draw a map of the polarization vectors.
- 2) Why is it even polarized?

Outline

1. EM wave physics
2. Astrophysics
3. Polarized EM-waves
4. Interferometric polarimetry
5. Messy reality

Stokes visibilities



$$
\begin{aligned}
\mathcal{I}(u, v) &= \mathcal{F}^{+}(I(l, m)) \\
\mathcal{Q}(u, v) &= \mathcal{F}^{+}(Q(l, m)) \\
\mathcal{U}(u, v) &= \mathcal{F}^{+}(U(l, m)) \\
\mathcal{V}(u, v) &= \mathcal{F}^{+}(V(l, m)),
\end{aligned}
$$

where

$$
\mathcal{F}^{+}(f) = \int_{lm} f \mathrm{e}^{+2\pi \mathrm{i}\nu(ul+vm)/c} \mathrm{d}l\mathrm{d}m
$$

Cartesian

$$E_\mathrm{x} = \Re\left\{A_\mathrm{x}\mathrm{e}^{2\pi\mathrm{i}\nu t}\right\}$$

$$E_\mathrm{y} = \Re\left\{A_\mathrm{y}\mathrm{e}^{\mathrm{i}\delta_\mathrm{xy}}\mathrm{e}^{2\pi\mathrm{i}\nu t}\right\}$$

$$I = \langle A_\mathrm{x}^2\rangle + \langle A_\mathrm{y}^2\rangle = \langle E_\mathrm{x}E_\mathrm{x}^*\rangle + \langle E_\mathrm{y}E_\mathrm{y}^*\rangle$$

$$Q = \langle A_\mathrm{x}^2\rangle - \langle A_\mathrm{y}^2\rangle = \langle E_\mathrm{x}E_\mathrm{x}^*\rangle - \langle E_\mathrm{y}E_\mathrm{y}^*\rangle$$

$$U = \langle 2A_\mathrm{x}A_\mathrm{y}\cos\delta_\mathrm{xy}\rangle = \langle E_\mathrm{x}E_\mathrm{y}^*\rangle + \langle E_\mathrm{y}E_\mathrm{x}^*\rangle$$

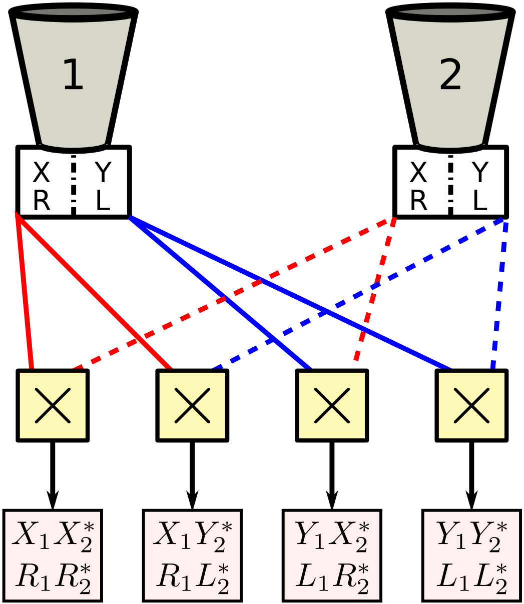
$$V = \langle -2A_\mathrm{x}A_\mathrm{y}\sin\delta_\mathrm{xy}\rangle = -\mathrm{i}\left(\langle E_\mathrm{x}E_\mathrm{y}^*\rangle - \langle E_\mathrm{y}E_\mathrm{x}^*\rangle\right)$$

Circular

$$E_\mathrm{r} = A_\mathrm{r}\mathrm{e}^{2\pi\mathrm{i}\nu t}$$
$$E_\mathrm{l} = A_\mathrm{l}\mathrm{e}^{-\mathrm{i}\delta_\mathrm{rl}}\mathrm{e}^{-2\pi\mathrm{i}\nu t}$$

$$I = \langle A_\mathrm{r}^2\rangle + \langle A_\mathrm{l}^2\rangle = \langle E_\mathrm{r}E_\mathrm{r}^*\rangle + \langle E_\mathrm{l}E_\mathrm{l}^*\rangle$$
$$Q = \langle 2A_\mathrm{r}A_\mathrm{l}\cos\delta_\mathrm{rl}\rangle = \langle E_\mathrm{r}E_\mathrm{l}^*\rangle + \langle E_\mathrm{l}E_\mathrm{r}^*\rangle$$
$$U = \langle -2A_\mathrm{r}A_\mathrm{l}\sin\delta_\mathrm{rl}\rangle = \mathrm{i}\left(\langle E_\mathrm{r}E_\mathrm{l}^*\rangle - \langle E_\mathrm{l}E_\mathrm{r}^*\rangle\right)$$
$$V = \langle A_\mathrm{r}^2\rangle - \langle A_\mathrm{l}^2\rangle = \langle E_\mathrm{r}E_\mathrm{r}^*\rangle - \langle E_\mathrm{l}E_\mathrm{l}^*\rangle$$

Correlating interferometer



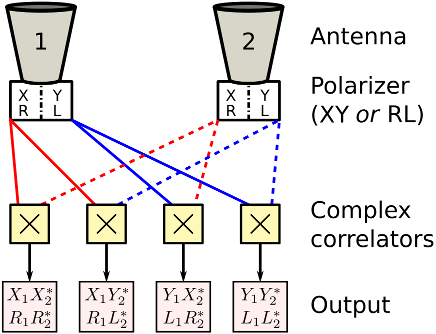
Antenna

Polarizer (XY *or* RL)

Complex correlators

Output

Stokes visibilities from correlator outputs



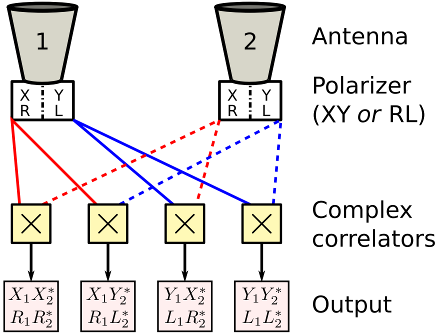
- From here on, $\langle\cdot\rangle$ is implied for correlator outputs.

Cartesian

$$\mathcal{I} = x_1 x_2^* + y_1 y_2^*$$
$$\mathcal{Q} = x_1 x_2^* - y_1 y_2^*$$
$$\mathcal{U} = x_1 y_2^* + y_1 x_2^*$$
$$\mathcal{V} = -\mathrm{i}\,(x_1 y_2^* - y_1 x_2^*)$$

Circular

$$\mathcal{I} = r_1 r_2^* + l_1 l_2^*$$
$$\mathcal{Q} = r_1 l_2^* + l_1 r_2^*$$
$$\mathcal{U} = \mathrm{i}\,(r_1 l_2^* - l_1 r_2^*)$$
$$\mathcal{V} = r_1 r_2^* - l_1 l_2^*$$



From here on, *p* and *q* designate either *x* and *y*, or *r* and *l*.

- Polarizers produce vector:

$$\mathbf{e}_i = \begin{pmatrix} p_i \\ q_i \end{pmatrix}$$

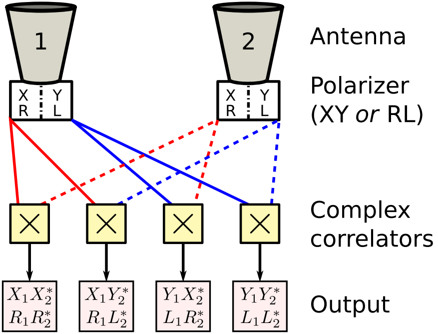
- Correlator multiplies:

$$\mathbf{E}_{ij} = \mathbf{e}_i\mathbf{e}_j^\dagger = \begin{pmatrix} p_i \\ q_i \end{pmatrix} \begin{pmatrix} p_j^* & q_j^* \end{pmatrix}$$

$$\mathbf{E}_{ij} = \begin{pmatrix} p_ip_j^* & p_iq_j^* \\ q_ip_j^* & q_iq_j^* \end{pmatrix}$$

- $\mathbf{E}_{ij}$ is the coherency matrix

Outline



Until now...

- Assumed all systems perfect

From now...

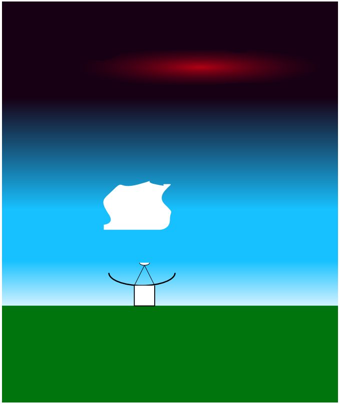
- Assume all systems linear:

$$\mathbf{e}'_i = \mathbf{J}_i\mathbf{e}_i$$

- $\mathbf{J}_i$ (2×2) is Jones matrix
- Cross correlation:

$$\begin{aligned}
\mathbf{E}'_{ij} &= \mathbf{e}'_i\mathbf{e}'^{\dagger}_j \\
\mathbf{E}'_{ij} &= \mathbf{J}_i\mathbf{e}_i\left(\mathbf{J}_j\mathbf{e}_j\right)^{\dagger} \\
\mathbf{E}'_{ij} &= \mathbf{J}_i\mathbf{e}_i\mathbf{e}^{\dagger}_j\mathbf{J}^{\dagger}_j \\
\mathbf{E}'_{ij} &= \mathbf{J}_i\mathbf{E}_{ij}\mathbf{J}^{\dagger}_j
\end{aligned}$$

Measurement equation



- The measurement equation:

$$\mathbf{E}'_{ij} = \mathbf{J}_i \mathbf{E}_{ij} \mathbf{J}_j^{\dagger}$$

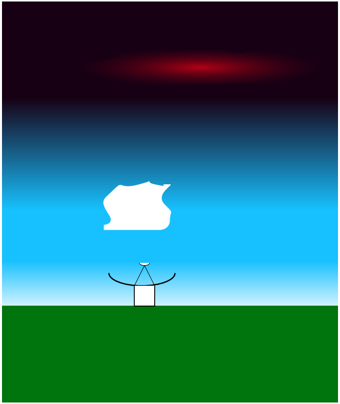
- Invertible!

$$\mathbf{E}_{ij} = \mathbf{J}_i^{-1} \mathbf{E}'_{ij} \mathbf{J}_j^{\dagger -1},$$

- where

$$\mathbf{J} = \mathbf{RPDOWFT}$$

- ...riiiiight...



- Perfect instrument:

$$\mathbf{J} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

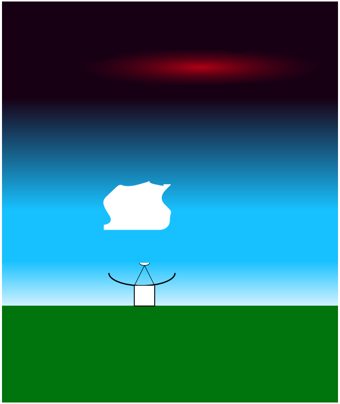
- Time delay:

$$\mathbf{J} = \begin{pmatrix} \mathrm{e}^{2\pi\mathrm{i}\nu\tau_p} & 0 \\ 0 & \mathrm{e}^{2\pi\mathrm{i}\nu\tau_q} \end{pmatrix}$$

- Receiver gain:

$$\mathbf{J} = \begin{pmatrix} g_\mathrm{p} & 0 \\ 0 & g_\mathrm{q} \end{pmatrix}$$

Measurement equation: examples



- Polarization leakage:

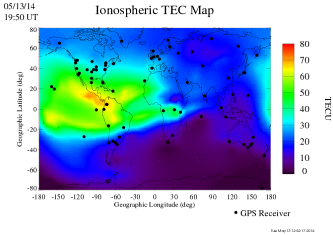
$$\mathbf{J} = \begin{pmatrix} g_{\mathrm{p}} & d_{\mathrm{q}\rightarrow\mathrm{p}} \\ d_{\mathrm{p}\rightarrow\mathrm{q}} & g_{\mathrm{q}} \end{pmatrix}$$

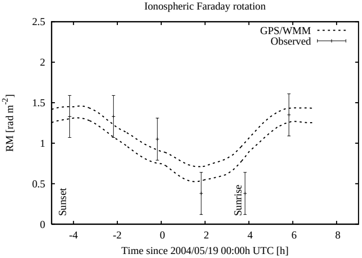
- Parallactic angle or feed rotation XY:

$$\mathbf{J} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

- Parallactic angle or feed rotation RL:

$$\mathbf{J} = \begin{pmatrix} \mathrm{e}^{+\mathrm{i}\theta} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\theta} \end{pmatrix}$$



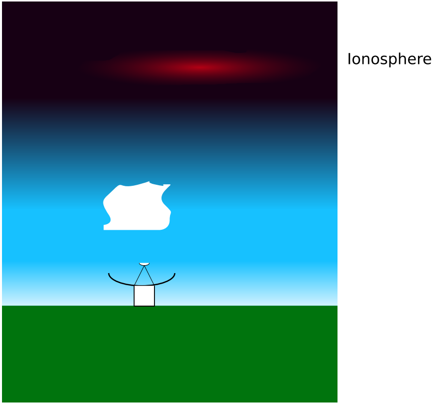


- Remember: $\Delta\chi = \chi_0 + \phi\lambda^2$
- Faraday depth

$$\phi = 0.812 \int_{\text{there}}^{\text{here}} n_e \mathbf{B} \cdot d\mathbf{l}$$

- ionosphere: plasma within Earth's magnetic field
- $\phi \approx$ -10 – +10 rad m^{-2}
- *Very* significant below 1 GHz
- Use TEC/IGRF models for correction, check with pulsar.

Differential Ionospheric Faraday rotation



- $\Delta\chi = \chi_0 + \phi\lambda^2$
- Rotation of linear pol = delay between RCP and LCP
- Antennas see different ionosphere
- Leakage from LL to RR or v.v. during cross correlation
- Leaks $\mathcal{I}$ into $\mathcal{V}$ and v.v.
- Important below 300 MHz at baselines $\geq$ 20 km

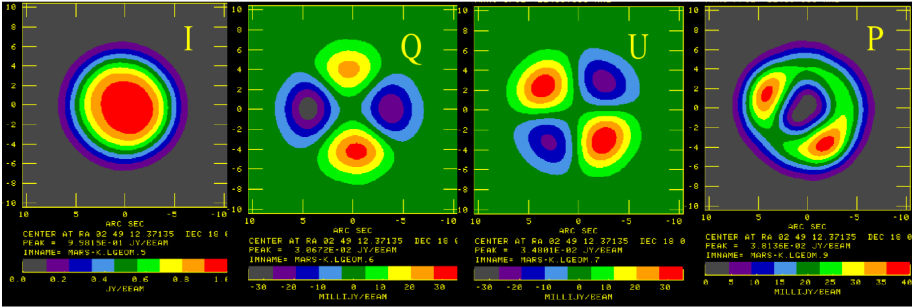
Take away this...

- Radio antennas are fundamentally polarized
- Polarimetry required for certain astrophysical observations
- Linear systems make for fairly straightforward calibration
- Understanding polarimetry improves your unpolarized calibration and imaging

Reading list

- Born & Wolf *Principles of optics*
- Thompson, Moran & Swenson *Interferometry and Synthesis in Radio Astronomy*
- Taylor, Carilli & Perley *Synthesis Imaging in Radio Astronomy II*
- Bracewell *The Fourier Transform & Its Applications*
- Hamaker, Bregman & Sault *Understanding radio polarimetry: paper I*(1996)
- Sault, Hamaker& Bregman *paper II*(1996)
- Hamaker & Bregman *paper III* (1996)
- Hamaker *paper IV* (2000)
- Hamaker *paper V* (2006)
- Brentjens & de Bruyn *Faraday rotation measure synthesis* (2005)

TEST: Mars with VLA at 23.4 GHz *Perley*



- This is thermal emission
- 1) Draw a map of the polarization vectors.
- 2) Why is it even polarized?

Answer: Mars polarization vectors *Perley*