Some History behind the Measurement Equation

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Calibration and Self-calibration

- Need rigorous mathematical framework.
- Relate observed quantities to wanted quantities including all distortions by instrument and observing procedures.
- In our case a 2-step process
  - From sky brightness to Radio Interferometer visbility power
  - From visibilty power to sky brightness image
- This presentation is about the first step, i.e. the RIME with focus on ASTRON contributions.
- Following historical milestones and 150 years of progress in understanding distorted wave propagation.
- Key issue is the polarization state of the propagating wave.

Milestones in Polarization Calibration

- 1852 Stokes polarized –optical- power parameters
- 1862 Maxwell EM-field

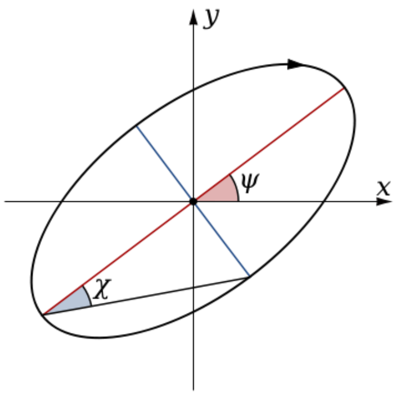
- 1941 Jones 2x2 matrices for –optical- field vector propagation
- 1943 Mueller 4x4 matrices for Stokes power vector propagation

- 1964 Morris –radio- interferometer power response
- 1973 Weiler crossed dipoles allow polarization calibration with unpol source
  Blessing and curse of the WSRT

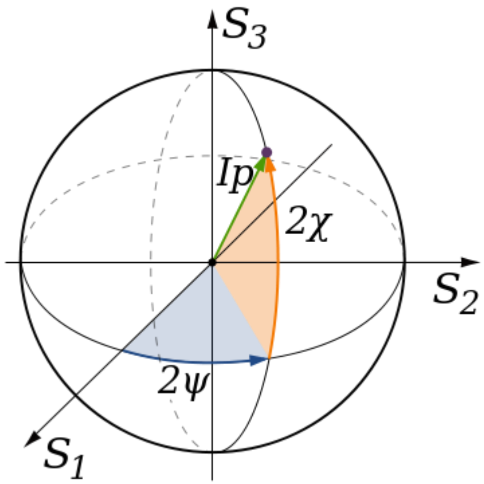
- 1980 Receiver based self-calibration
- 1982 From antenna voltage beam to interferometer power beam
- 1993 Discovery of 17 MHz polarization structure in 90 cm beam

- 1995 AIPS++ endorses Jones matrix formalism
- 1996 HBS relate Kronecker product of Jones matrices to Muller matrix
- 2000 Hamaker matrix self-cal theory and Unitarian Pol-rotation ambiguity
- 2006 Polarization self-cal demonstration for heterogeneous array

Stokes polarized power parameters



- Polarization ellipse



- Orientation and axis ratio of polarization ellipse defines state of relative polarization part of light.
- Reformulated in 1852 by Stokes as a Quaternion.
  - Invented in 1843 by Hamilton.
- i.e. Scalar & 3-D vector describe total intensity & relative polarization:

$$S_0 = I$$

$$S_1 = Ip\cos 2\psi \cos 2\chi$$

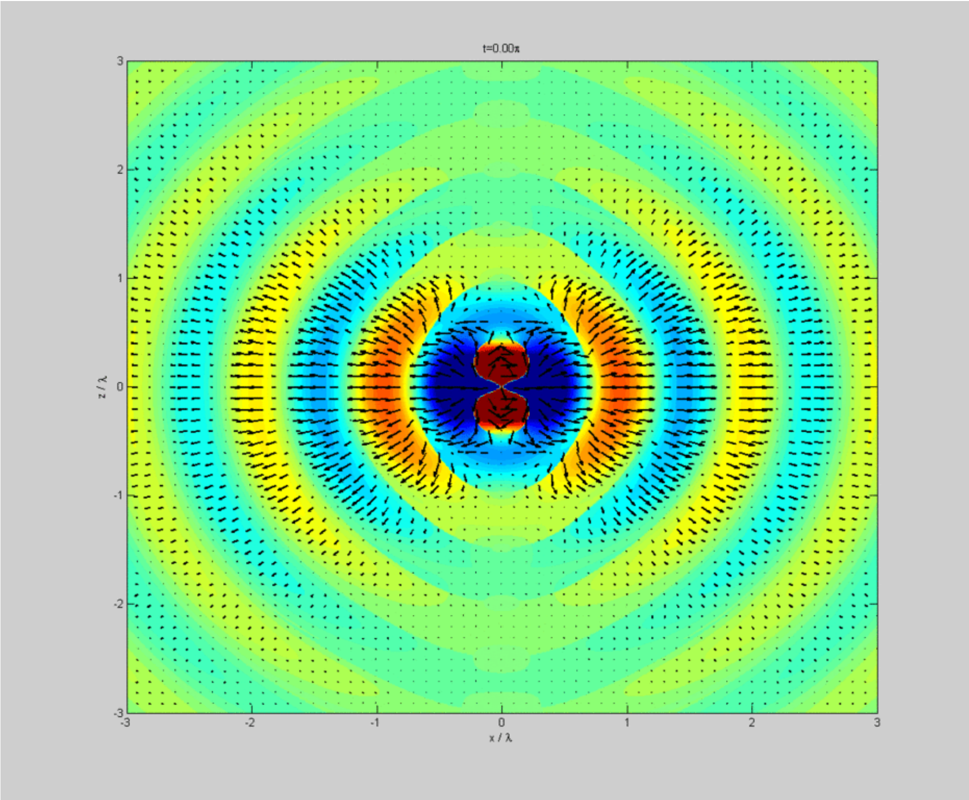
$$S_2 = Ip\sin 2\psi \cos 2\chi$$

$$S_3 = Ip\sin 2\chi$$

- Relative polarization as 3-D vector on abstract (Poincaré) sphere

Maxwell EM-field theory (1862)

- Energy flux from radiating dipole



- Originally derived with 24 equations relating electrical quantities using quaternion formalism.
- Allowed waves propagating at about speed of light suggesting that light is indeed an EM-wave.
- Reformulated later by 4 vector equations.
- Energy flux by Poynting (1884) propagation vector:

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

- Conversely, EM wave induces electric current in an antenna structure.
- 2 orthogonal antennas needed to observe arbitrarily polarized wave incident along third orthogonal direction.
- Projection effects for non-orthogonal incidence, i.e. beam polarization

Jones field transforming matrices

- Matrix equation chain
- $\mathbf{E}' = J_1 \; J_2 \; J_n \; \mathbf{E}$
- For field vector $\mathbf{E} = (E_x \; E_y)^T$
- $J = \begin{bmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{bmatrix}$
- 4 complex numbers define change of polarization state
- Including propagation phase
- Jones valid for instantaneous fields and therefore narrow band

- R.C. Jones working at Polaroid and Harvard developed a matrix formalism for systems analysis in 1941.
- For a plane wave traversing a chain of optical elements such as rotators, polarizers, wave plates, etc.
- 2x2 matrices transform polarization of a fully polarized plane EM wave in the field domain.
- Cannot handle depolarization.
- Could describe a single arm of an interferometer.

Mueller polarization transforming

- Matrix equation

  $$\mathbf{S}_{out} = M_T \ \mathbf{S}_{in}$$

  $$M_T = M_3 M_2 M_1$$

- Mathematical description method
- Simple matrices for wave plates, polarizers, etc.

- Hans Mueller worked at MIT and developed in 1943 a systems analysis tool for a chain of optical elements.
- Chain of 4x4 matrices transform the Stokes vector of partially polarized light power.
- Includes depolarization
- Dominating approach in Astronomy and Radio astronomy
- Not in Antenna Engineering
  - Complicates specification

Morris, Radhakrishnan & Seielstad

- Stokes by coherences

$$I = \langle (E_l^0)^2 \rangle + \langle (E_r^0)^2 \rangle \,,$$

$$Q = \langle (E_l^0)^2 \rangle - \langle (E_r^0)^2 \rangle \,,$$

$$U = 2\langle E_l^0 E_r^0 \rangle \cos \delta \,,$$

$$V = 2\langle E_l^0 E_r^0 \rangle \sin \delta \,,$$

- Morris provided (without derivation) the power response of an interferometer in 1963
- using the polarization ellipse of each antenna, referring to:
- Chandrasekhar who introduced in 1950 coherency parameters related to Stokes parameters .

$$\begin{aligned} R(t) = \tfrac{1}{2}k\{ & I[\cos \phi_1 - \phi_2) \cos (\theta_1 - \theta_2) + i \sin (\phi_1 - \phi_2) \sin (\theta_1 + \theta_2)] \\ & + Q[\cos (\phi_1 + \phi_2) \cos (\theta_1 + \theta_2) + i \sin (\phi_1 + \phi_2) \sin (\theta_1 - \theta_2)] \\ & + U[\sin (\phi_1 + \phi_2) \cos (\theta_1 + \theta_2) - i \cos (\phi_1 + \phi_2) \sin (\theta_1 - \theta_2)] \\ & + V[\cos (\phi_1 - \phi_2) \sin (\theta_1 + \theta_2) + i \sin (\phi_1 - \phi_2) \cos (\theta_1 - \theta_2)]\} \end{aligned}$$

Crossed dipole mode for WSRT

- Effectively a chain of 4x4 matrices

  S = P C G **R**

  G diagonal gains
  C from coherence to Stokes
  P instrument polarization (Mueller)

- C configuration matrix has only 0 and 1 to obtain the calibrated Stokes visibilities

- P has 1 on the diagonal and off-axis errors < 0.02 after nominal setting of each dipole pair per antenna

- In general Q, U < 0.1 and V < 0.01 so polarization errors < 0.002

- Kurt W. Weiler described in 1973 a convenient method for full WSRT polarization calibration.

- Relate Stokes **S** vector to Visibility vector **R**

- Derived and applied Morris formula to linear dipoles
  - ϕ nominal at 0, 90, 45, 135 with small errors $\Delta\phi$
  - Small ellipticity $\Delta\theta$

- Interferometer based gain factors using an un-polarized source include combinations of $\Delta\phi$ and i $\Delta\theta$

- No proper decomposition in station based gain factors possible

Self-calibration

Issues at WSRT

- DCB and DXB allowed now also correlation between fixed antennas.
- Mixture of ++ and +x dipoles.
- Standard crossed dipole interferometer calibration destroys receiver gain decomposition.
- New standard calibration package needed i.e. NEWSTAR

Round 1980 self-calibration was developed.

- Cooled receivers at VLA provided sufficient sensitivity.
- Use receiver based amplitude and phase calibration factors.
- Couple X with Y receiver for ++ by external means
  - By noise source
  - By polarized source
  - By change of parallactic angle

Beam polarization by Jones matrices

- Observed coherence matrix

$$R_o = (J_1 \mathbf{E})(J_2 \mathbf{E})^{*T} = J_1 \mathbf{E}\,\mathbf{E}^{*T} J_2^{*T} = J_1 R_S J_2^{*T}$$

- with

$$R_S = \begin{bmatrix} I+Q & U+iV \\ U-iV & I-Q \end{bmatrix}$$

- For un-polarized source flux I_S

$$R_S = I_S \, \mathrm{I}$$

- If J_2 tracks the source and J_1 scans we measure the voltage beam components of J_1

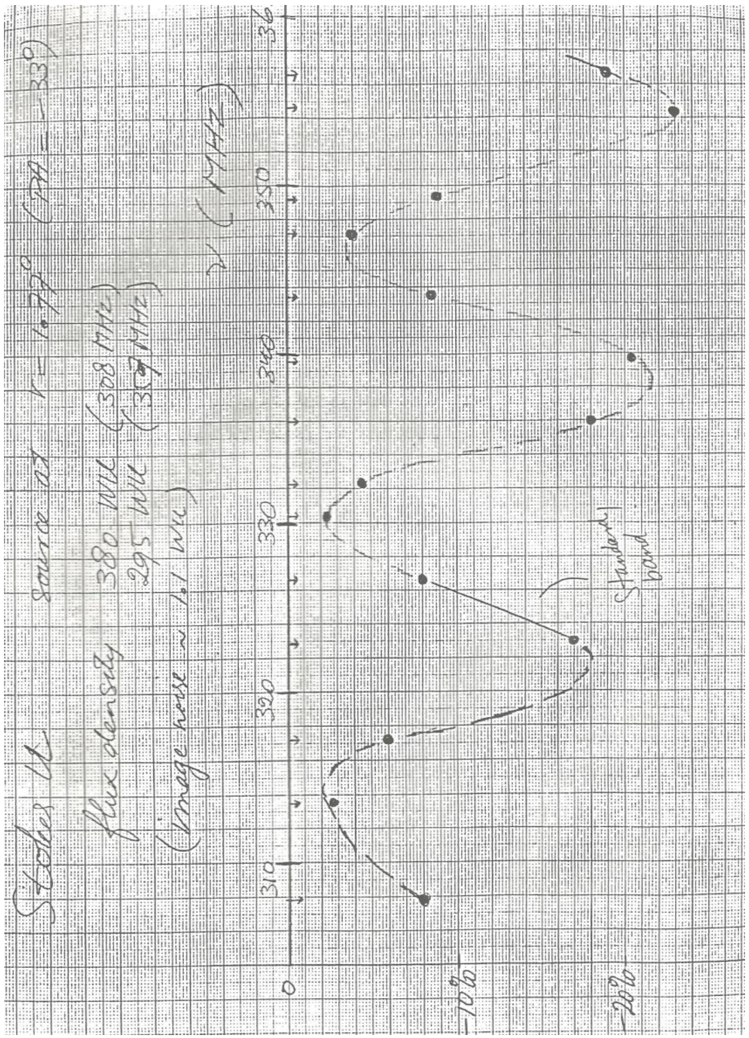
- Bregman, Hoekstra & de Waard used in 1982 measured voltage beams to describe power beam for crossed and parallel modes.
- Product of field vectors provides 2x2 coherency matrix.
- Jones matrix per telescope.
- Complication since rotation of feed between feed legs creates a different antenna pattern.
- Power beam given by

$$J_1 J_2^{*T} \neq J_1 R(\phi) J_1^{*T}$$

+x and ++ beam polarization

- 1973 Weiler, van Someren Greve & Pierson give +x beam results at 21 cm.
- 1978 QMC report also shows < 1% relative polarization within half power area of +x 6 cm beam.
- Strong relative polarization in beam area below 20% level at 6 and 21 cm.
- 1982 Bregman, Hoekstra & de Waard provide ++ end +x power beams from 6 cm voltage pattern.
- 1989 Henneken & Robijn verified ++ power beam at 92 cm.
- 1992 Blok & Woudt verified ++ power beam at 21 cm.
- 1993 de Bruyn discovered with wide band system 17 MHz ripple in 90 cm polarization beam.
- 1993 beam summary in new Observers Handbook

Beam with frequency fine structure



- 1993 Wide band 90 cm system available
- Evaluation of 90 cm part of MFFE triple feed
- De Bruyn discovers 17 MHz polarization ripple in U
- Earlier found in varying off-axis baseline of line observations
- Standing wave between feed and apex identified as culprit
- Important argument for off-axis SKA telescopes

AIPS++ endorses the Jones ME in 1995

- 1996 Noordam
  - AIPS++ note 185
  - Basis of full Jones chain
  - Allows full polarization decomposition for self-calibration.
- 1996 Hamaker, Bregman & Sault
  - Kronecker product of 2x2 Jones matrices gives 4x4 chereny transfer matrix
  - Averaging narrow band Jones allows wideband depolarization
  - Relates 2x2 Jones to 4x4 Mueller
  - Connecting known polarization knowledge to the new formalism
- 2000 Hamaker
  - matrix self-cal theory and Unitarian Pol-rotation ambiguity
- 2000 de Bruyn
  - Demonstrates 2 source peeling
  - Approach proposed by Noordam
- 2003 AIPS++ consortium ends
  - Provided basic toolkit for calibration and imaging
  - Based on the Jones ME

3C343 / 3C343.1   a suitable pair to test peeling

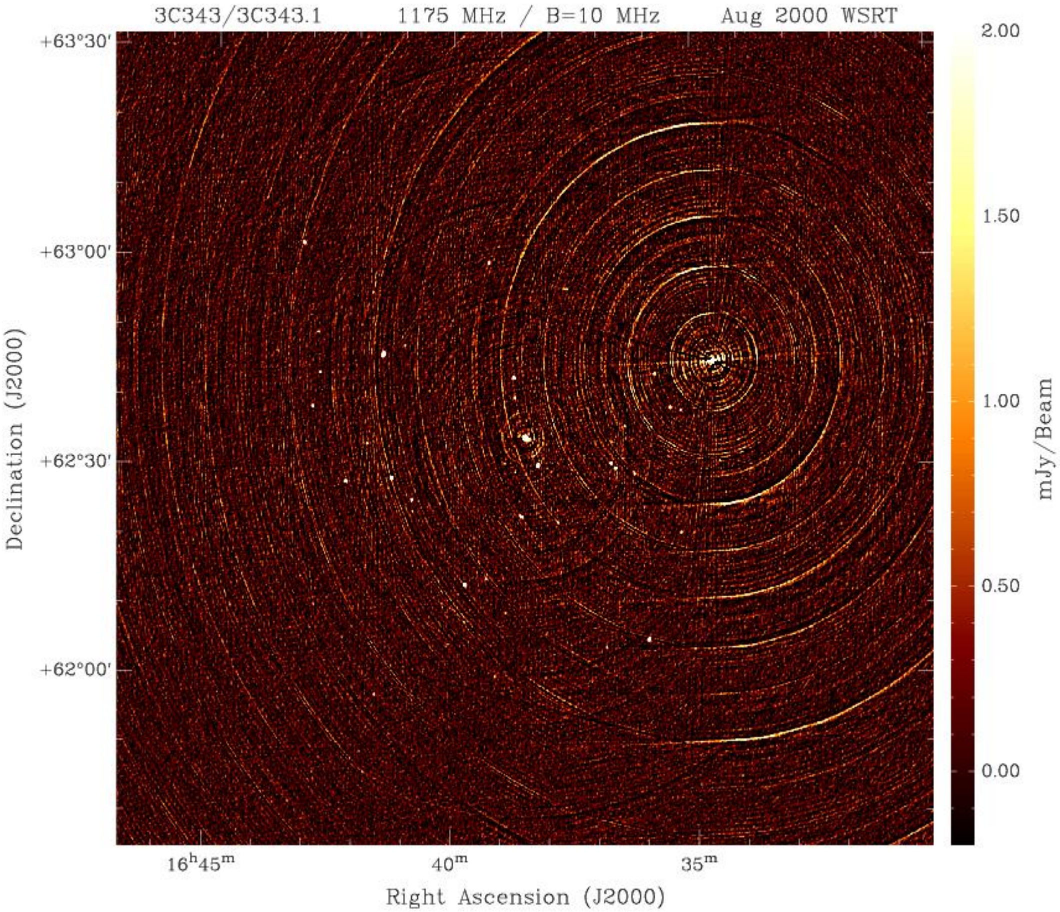
3C343.1   5 Jy
3C343   1.5 Jy (apparent, 3x attenuated)

Thermal noise ~ 30 μJy   (~ 100,000 : 1)

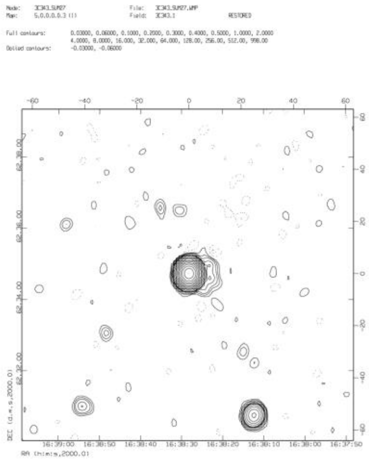
1x12h   4 Aug 2000
1175 MHz   B=10 MHz (64 ch)

NEWSTAR processing:
(~40 job script using programs NCALIB, NMODEL, NMAP, NFLAG, NCOPY, NCLEAN)

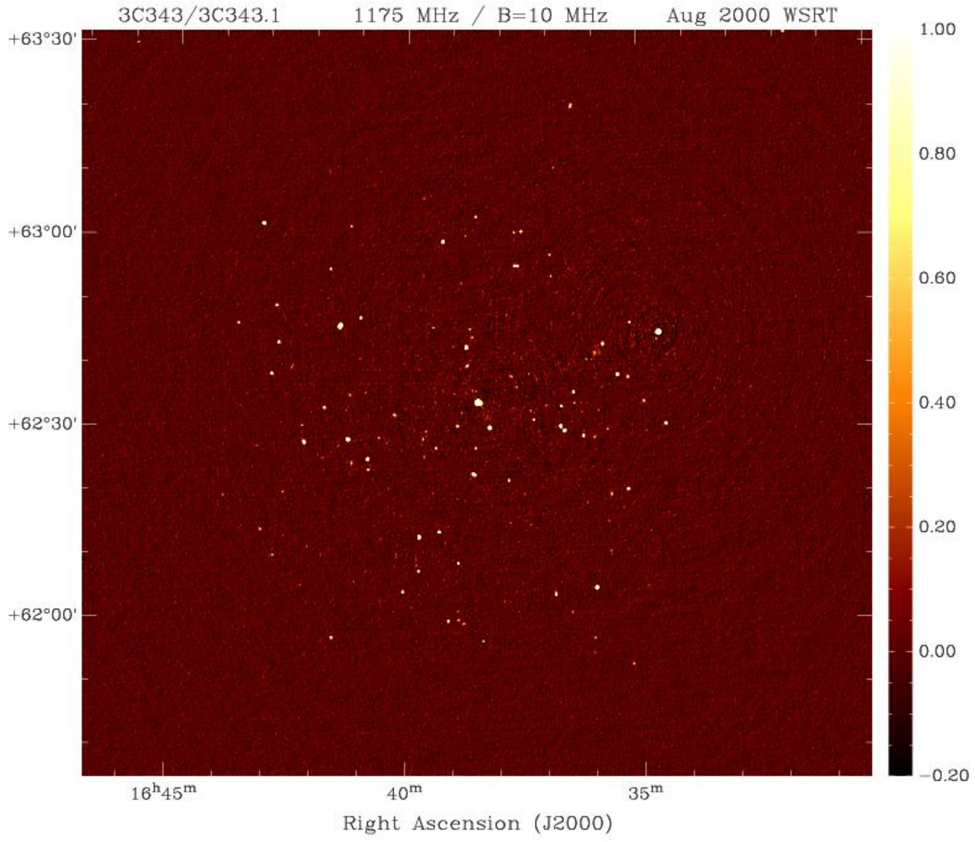
Small closure errors on each source location.

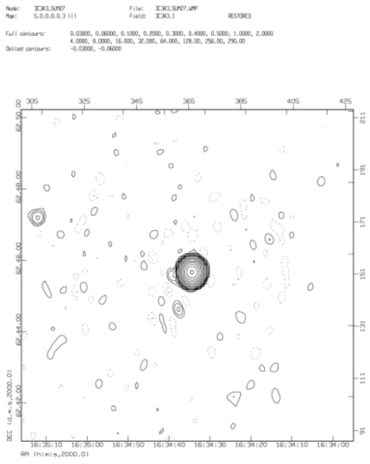


after ~1h and three selfcals....



3C343.1





3C343

Towards LOFAR and APERTIF

- 2006 Hamaker
  - Polarization self-cal demonstration for heterogeneous array
- 2006 v.d. Tol & Jeffs
  - Multi source peeling
  - demo for 6 sources
- 2006 Wijnholds & Bregman
  - SNR > 3 per source per baseline
  - Simplified derivation
- 2007 Smirnov & Noordam
  - MeqTrees
  - user implementation
- 2008 Pandey et al
  - BlackBoard Selfcal for LOFAR
  - Calibration and Subtraction of few strongest sources in field
- 2009 Yatawatta
  - Nominal polarized beamshape corrections in LOFAR
  - Multi source SAGE-cal for tens of sources
- 2012 Wijholds, Ivashina, Maaskant, Warnick
  - Phased array polarization calibration, i.e APERTIF
  - Only sensitivity loss by projection