Tomography of the ionosphere at LOFAR

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The Earth’s ionosphere

(from S. J. Bauer, Physics of Planetary Ionospheres, 1973)
Ionospheric Corruptions
Ionospheric corruptions
Medical Computed Tomography

www.siemens.com
The LOFAR Core

CORE

Central Core
Beams overlap
Power-spectrum Tomography

\[
\delta I^{(s)}(s_u, s_v) = \frac{1}{s_w^2} \int \int I^{(i)}(s_{0,u}, s_{0,v}) |\tilde{\Phi}(s - s_0)|^2 ds_{0,u} ds_{0,v}
\]

\[s_w^2 = 1 - s_u^2 + s_v^2\]

Directional Cosines

Geometric Term due to Curve Sky and Planar Array

Rescaled Intensity

\[
\delta J^{(s)}(s_u, s_v) \equiv s_w^2 \cdot \delta I^{(s)}(s_u, s_v)
\]

Koopmans 2010
Estimating the dTEC

Fitting Tec to Sagecal Solutions

Fit for frequency dependence of the phase due to the ionosphere

115 - 189 MHz
0.2 MHz Bandwidth Channel
379 subbands
Box splines Tomography

Ray-trace and estimate all the weights at the voxels

\[ d = fs + n \]

d is data points
s is the model parameters, in this case the electron density at each pixel.
f is a constant linear transformation matrix, in this case the weights at each voxels
n is noise
Regularised Maximum Likelihood Inversion

consider $f$ to be a constant linear transformation matrix of dimensions $N_d$-by-$N_s$ such that

$$d = fs + n$$  \hspace{1cm} (1)

where $n$ is the noise in the data characterised by the covariance matrix $C_D$ (here and below, subscript D indicates “data”).

Modelling the noise as Gaussian, the probability of the data given the model parameters $s$ is

$$P(d|s,f) = \frac{\exp(-E_D(d|s,f))}{Z_D}$$  \hspace{1cm} (2)

where

$$E_D(d|s,f) = \frac{1}{2} (fs - d)^T C_D^{-1} (fs - d)$$
$$= \frac{1}{2} \chi^2$$  \hspace{1cm} (3)

and $Z_D = (2\pi)^{Nd/2} (\det C_D)^{1/2}$ is the normalisation for the probability. The probability $P(d|s,f)$ is called the likelihood.

Regularisation parameter since $Ed$ is ill-posed.

$$P(s|g,\lambda) = \frac{\exp(-\lambda E_S(s|g))}{Z_S(\lambda)}$$  \hspace{1cm} (4)

Bayes’ rule tells us that the posterior probability of the parameters $s$ given the data, response function and prior is

$$P(s|d,\lambda, f, g) = \frac{P(d|s,f)P(s|g,\lambda)}{P(d|\lambda, f, g)}$$  \hspace{1cm} (5)

Suyu et al. 2006
Building a 3D model

Optimise Bayesian evidence based on:

- Regularisation parameter
- Grid Size
- Height of the ionosphere layers

2D model
Height of the ionosphere

Changing of the height of the ionosphere in a 2D model leads to rearranging of the pierce points. This could lead to conflicting constraints on the model. If this persists then the ionosphere is 3D.
Building a 3D model

![Graph showing time in hours UTC vs. dTEC with RECONSTRUCTION: BLUE - measured, RED - 2D, Green - 3D.](image-url)
Tomography
To be done

- Multiple grids for the 3D model to take into account the different station density of the LOFAR core and remote stations.

- Prediction for ionospheric correction for arbitrary directions in the field of view.

- Application of ionospheric correction to the uv data.
Thank you