Tomography of the ionosphere at LOFAR

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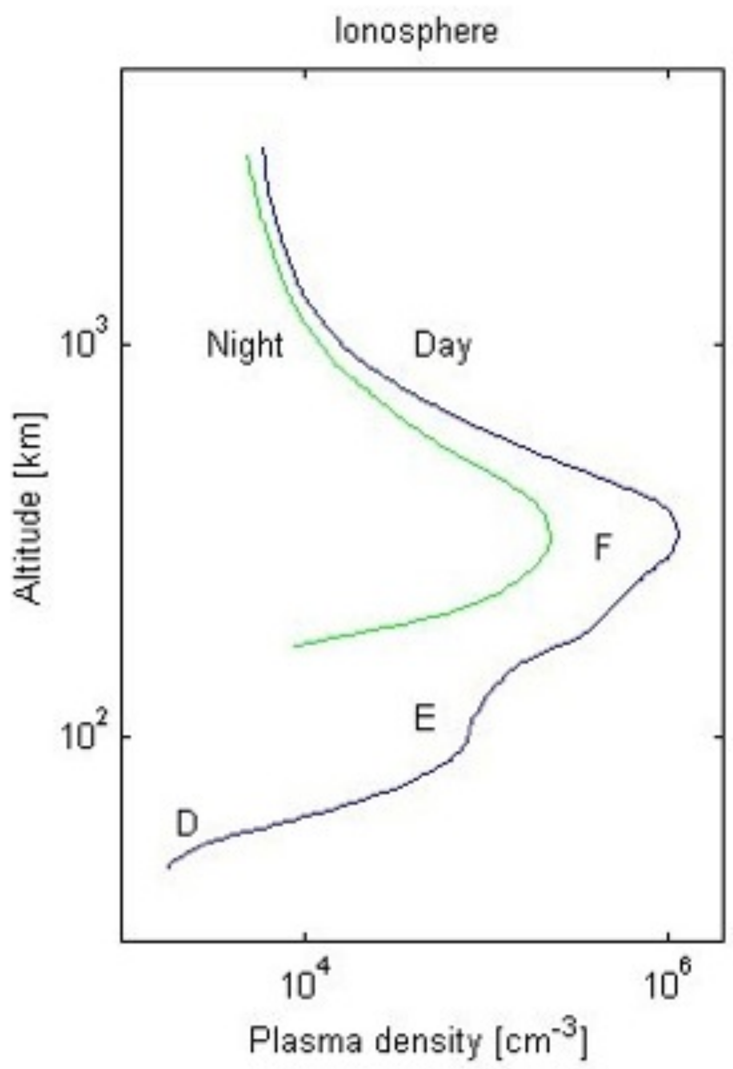
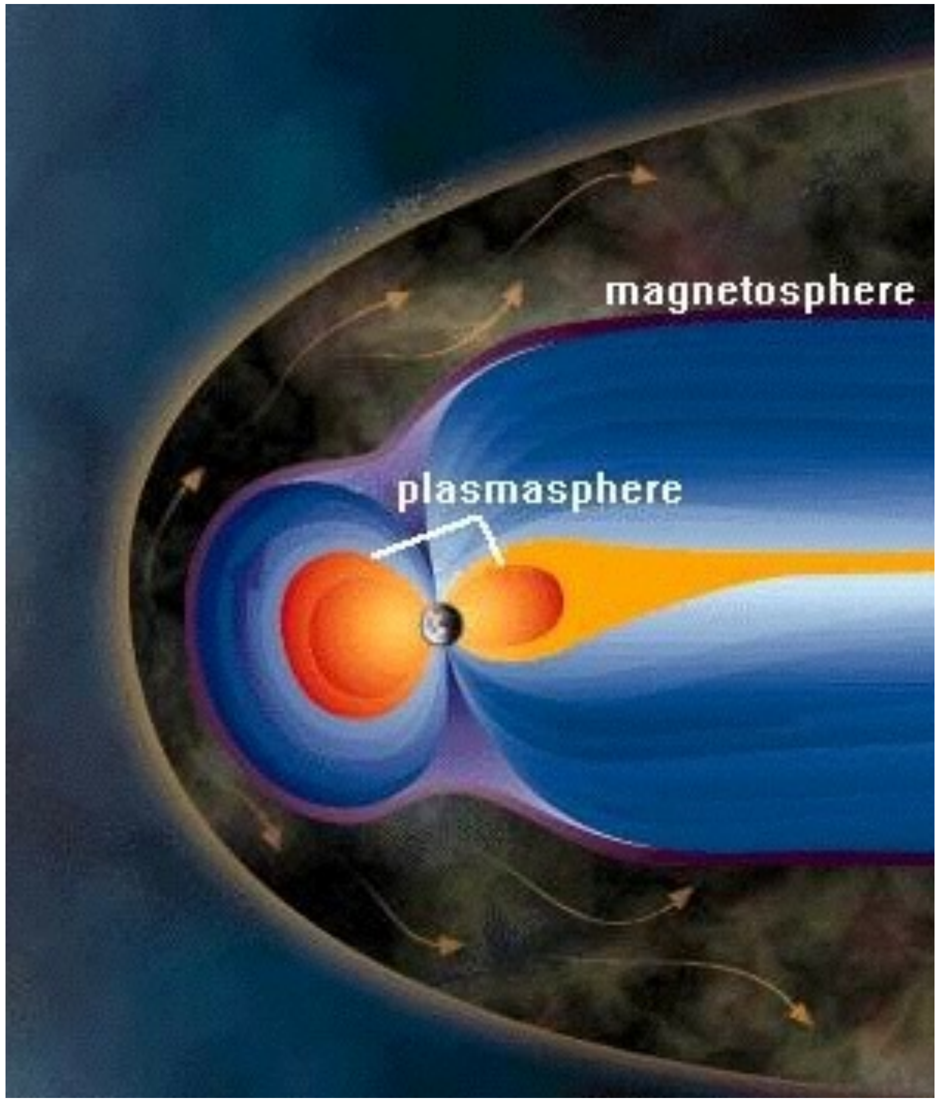
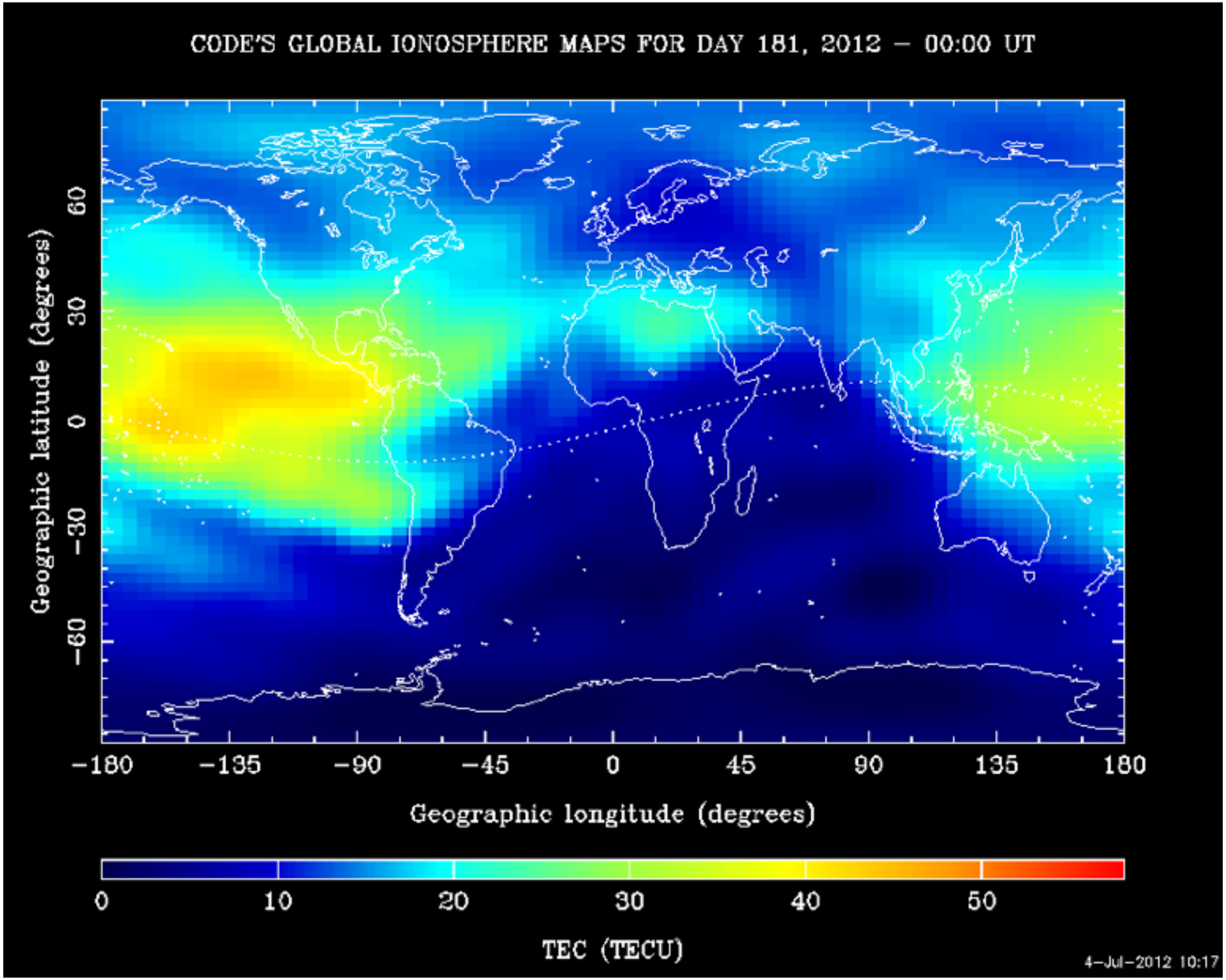
The Earth's ionosphere

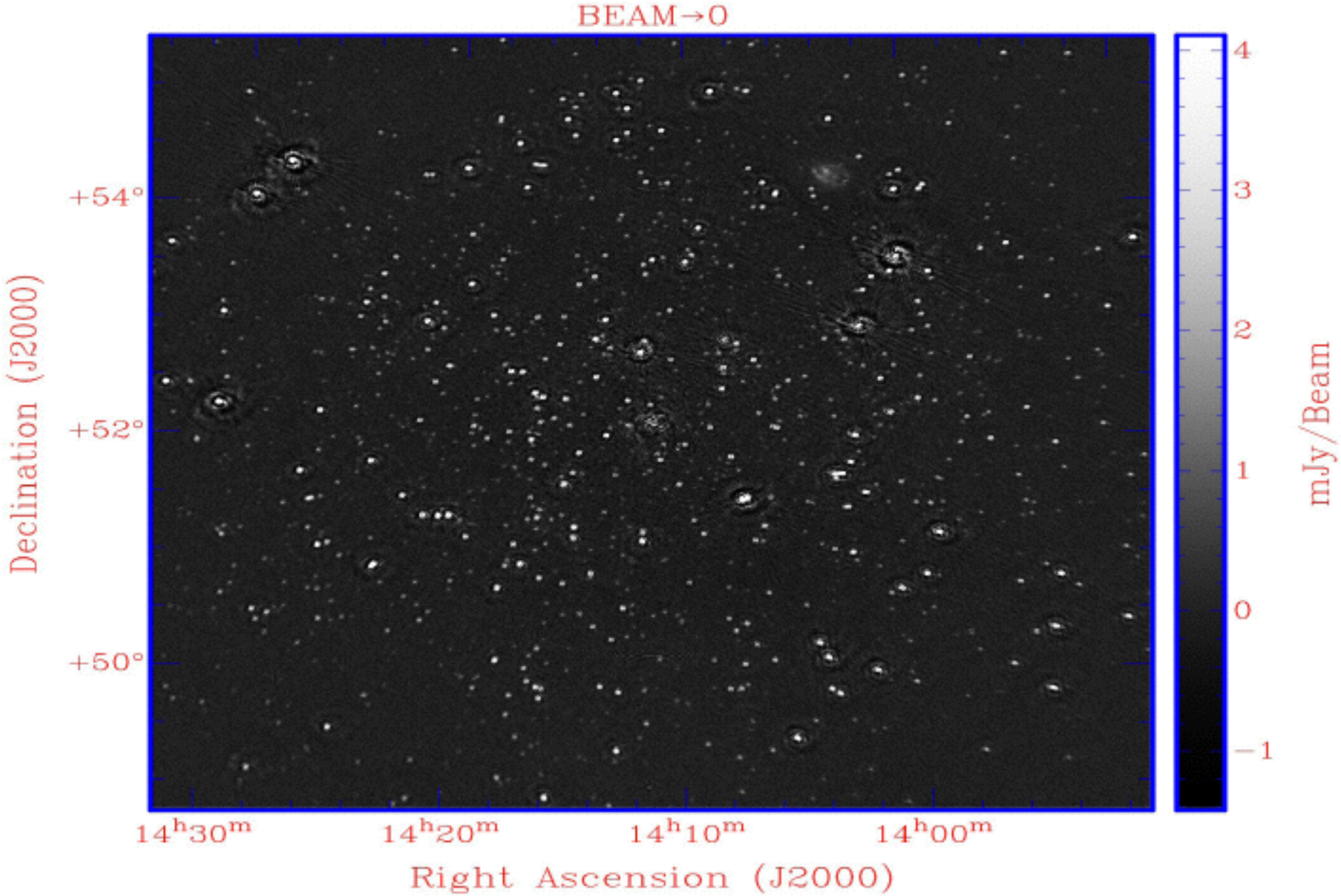




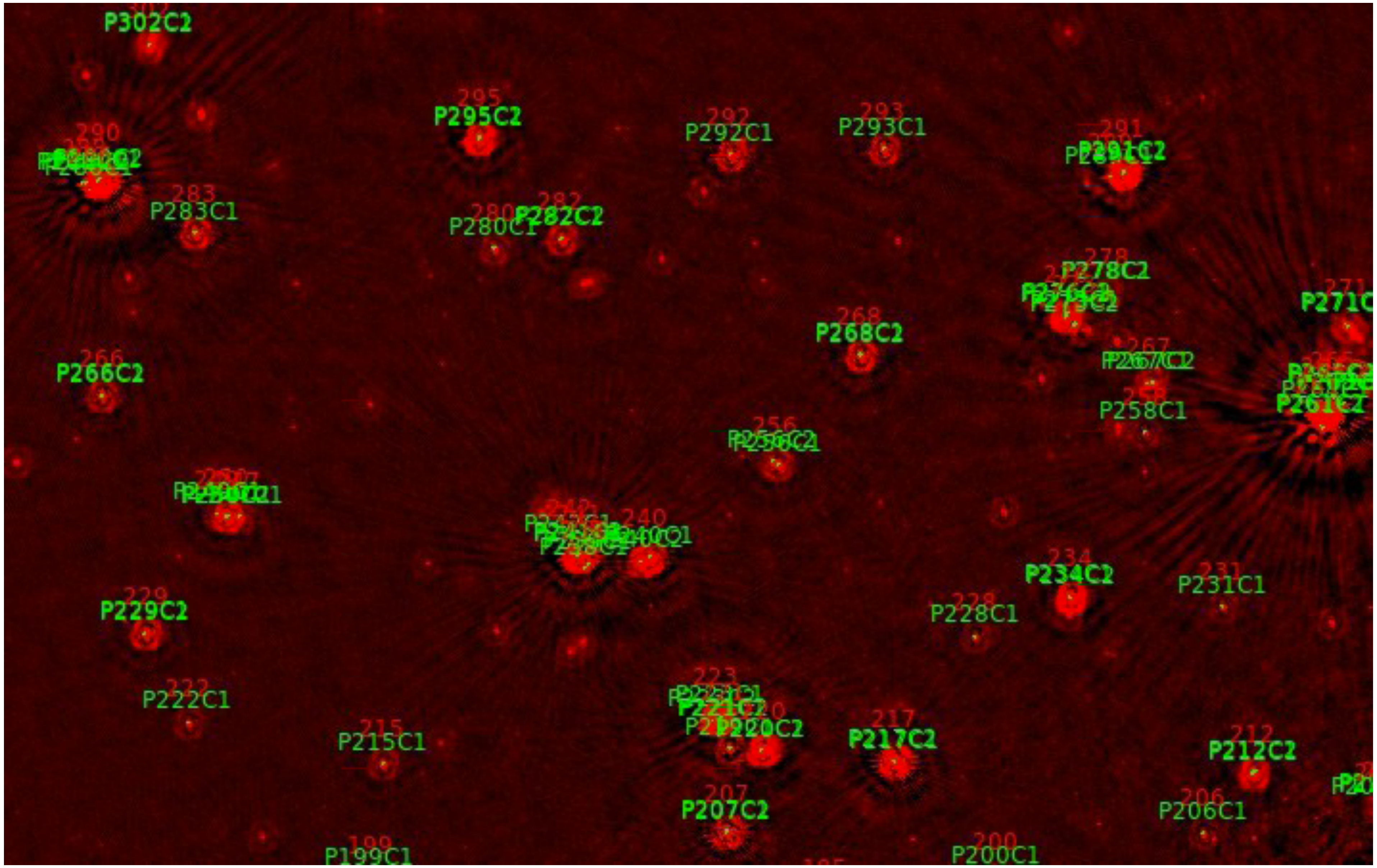
(from S. J. Bauer, Physics of Planetary Ionospheres, 1973)


CODE'S GLOBAL IONOSPHERE MAPS FOR DAY 181, 2012 – 00:00 UT
Geographic latitude (degrees)
60
30
0
–30
–60
–180
–135
–90
–45
0
45
90
135
180
Geographic longitude (degrees)
0
10
20
30
40
50
TEC (TECU)
4–Jul–2012 10:17

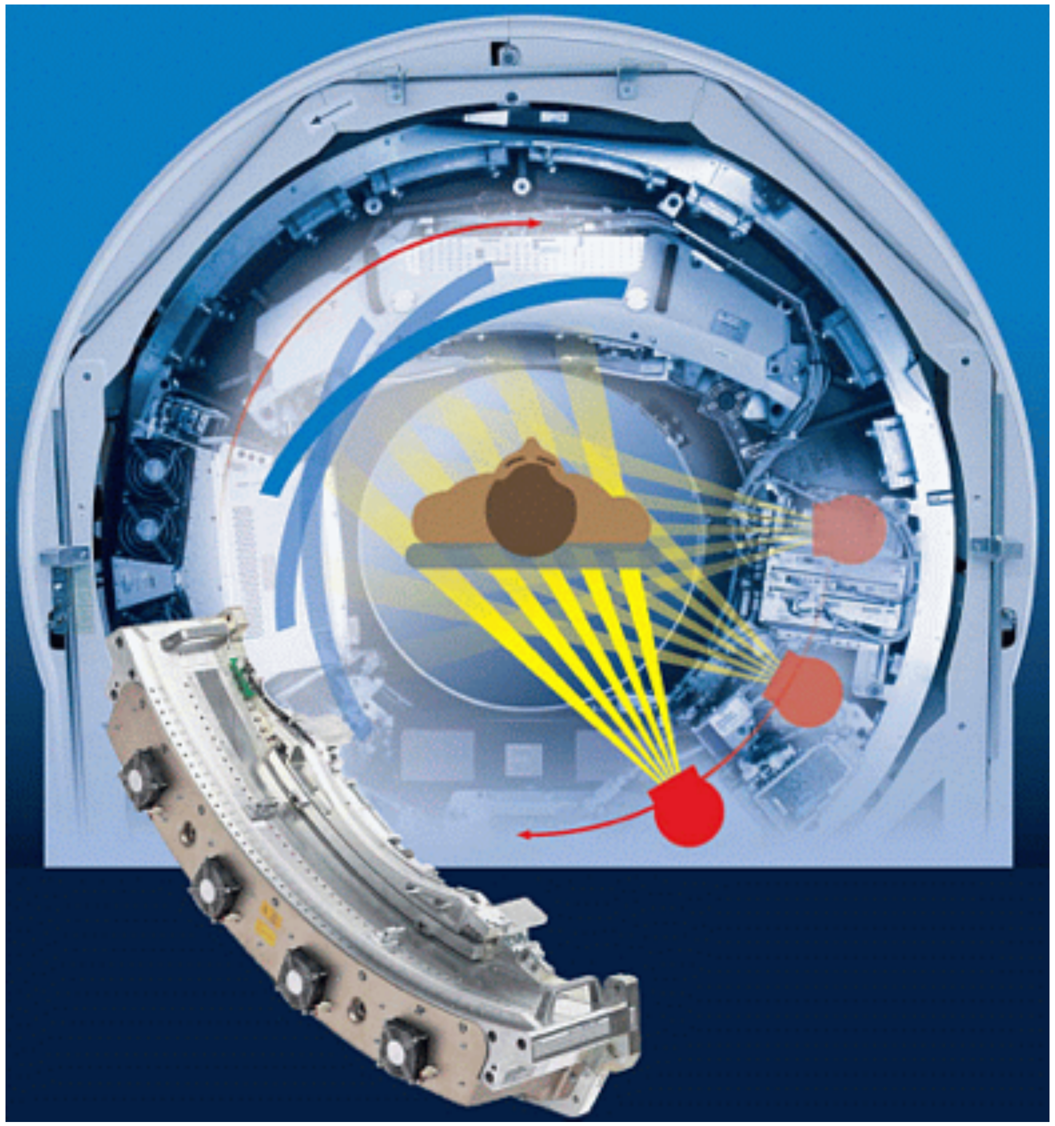
Ionospheric Corruptions

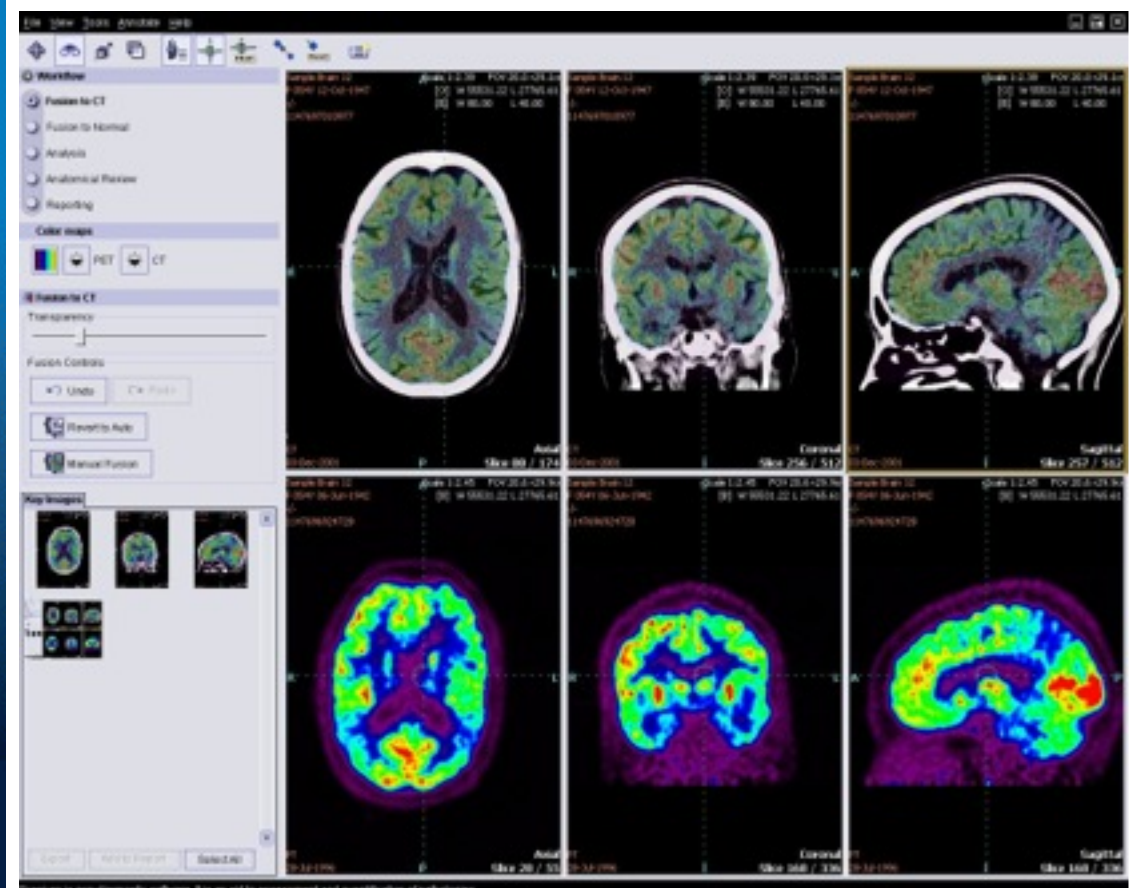


Ionospheric corruptions



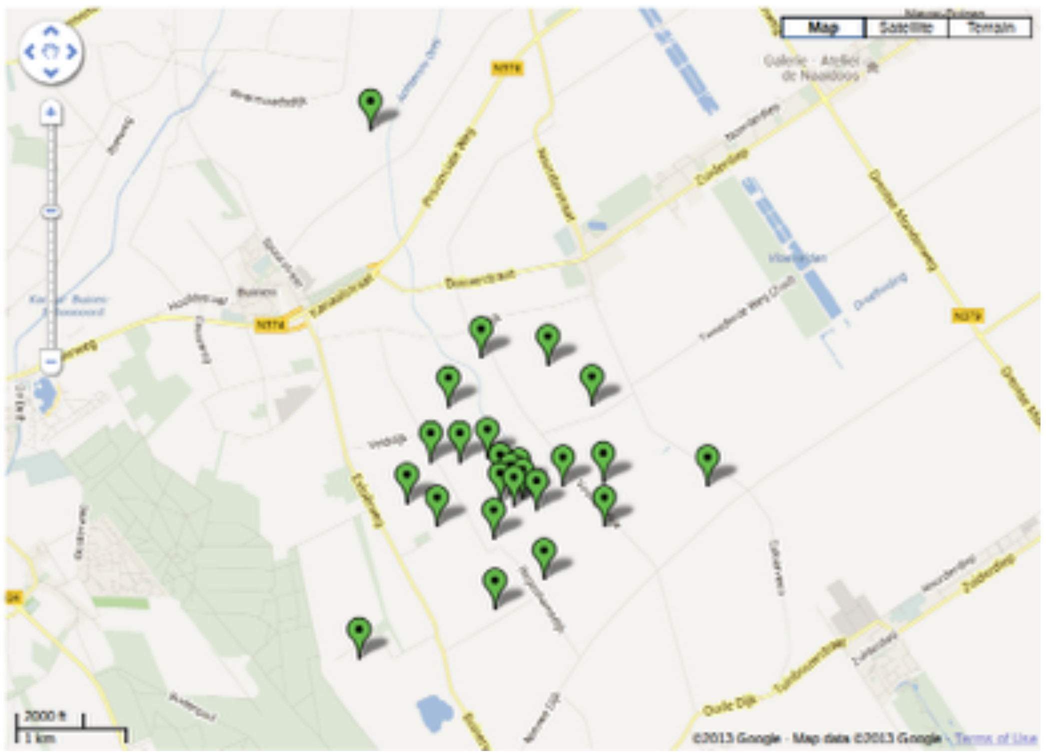
Medical Computed Tomography





www.siemens.com

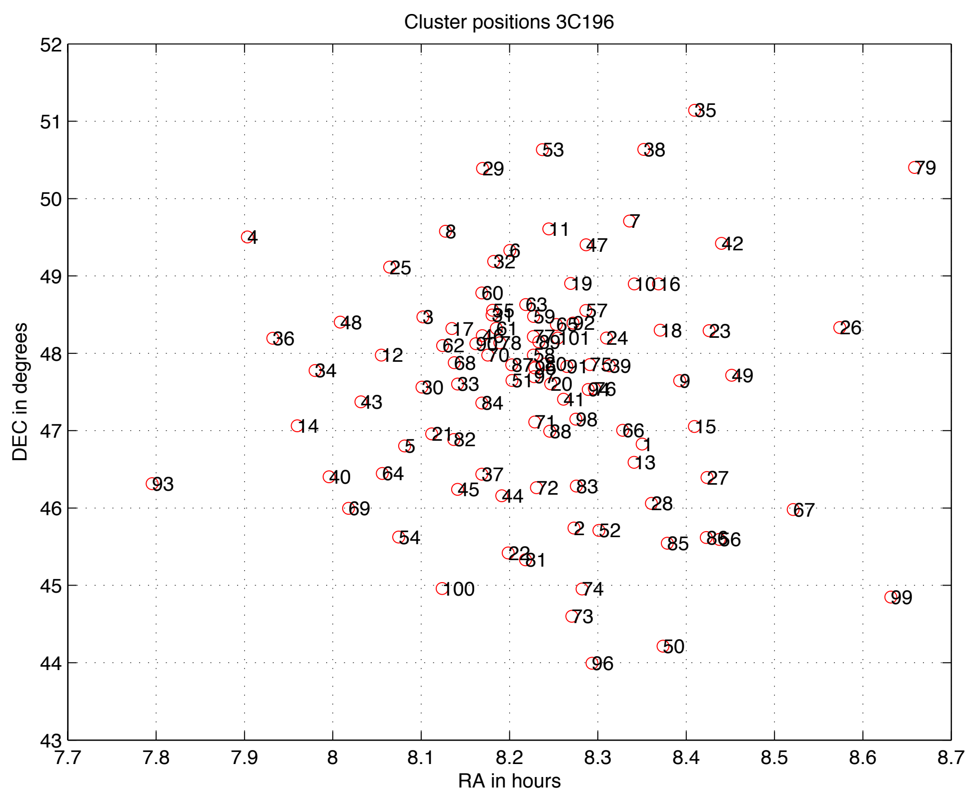
The LOFAR Core

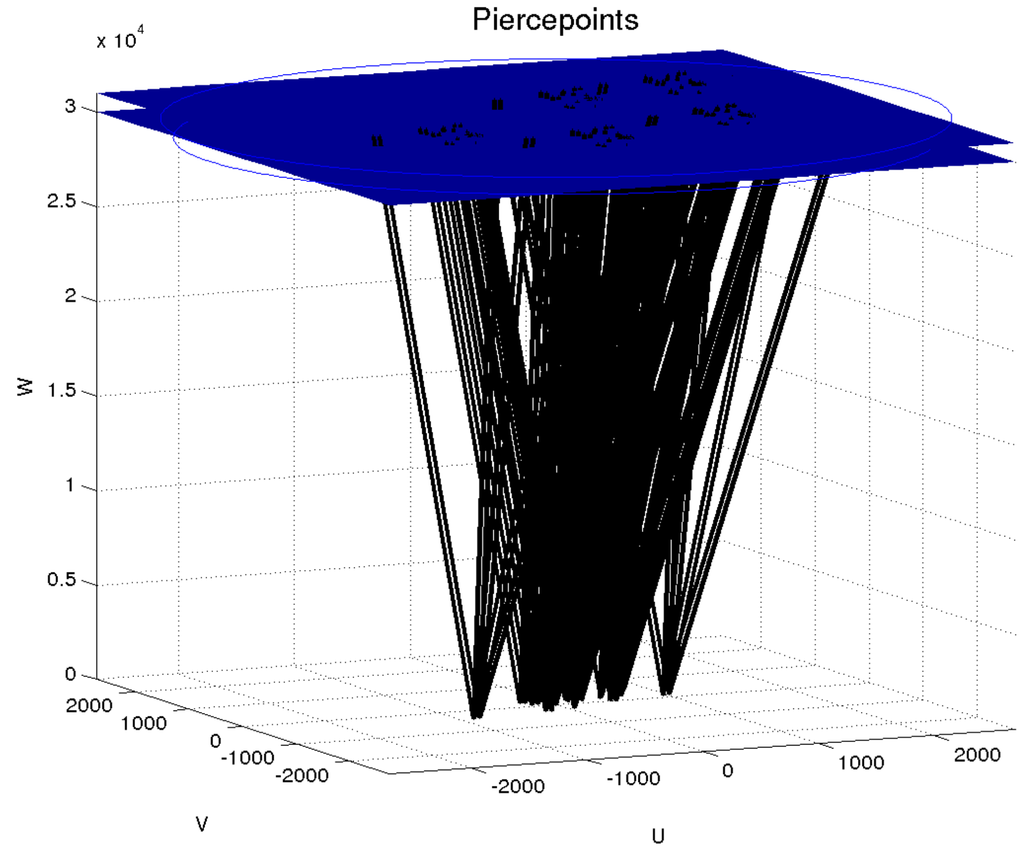


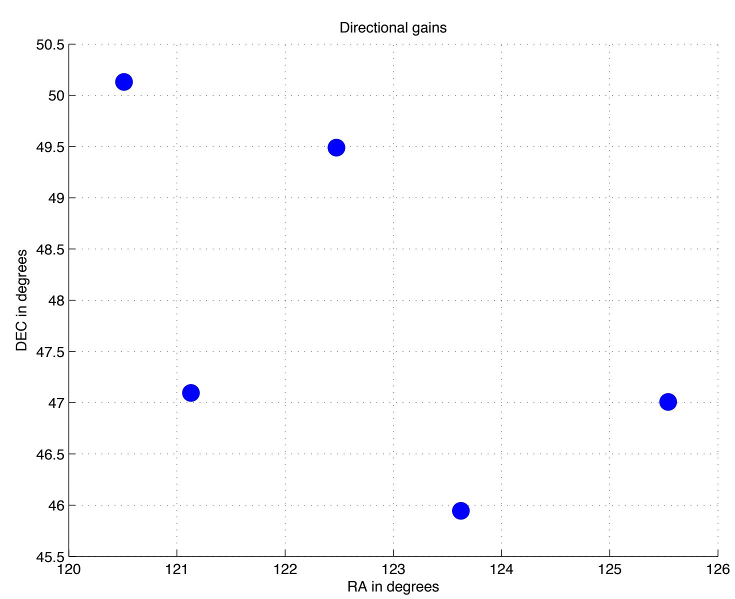
CORE

Central Core

Beams overlap







Power-spectrum Tomography

Scattered Intensity — 2D Sky Intensity — 3D power-spectrum

$$\delta I^{(s)}(s_u, s_v) = \frac{1}{s_w^2} \iint I^{(i)}(s_{0,u}, s_{0,v})\, |\tilde{\Phi}(\mathbf{s} - \mathbf{s}_0)|^2 ds_{0,u} ds_{0,v}$$

Directional Cosines

$$s_w^2 = 1 - s_u^2 + s_v^2$$

Geometric Term due to
Curve Sky and Planar Array

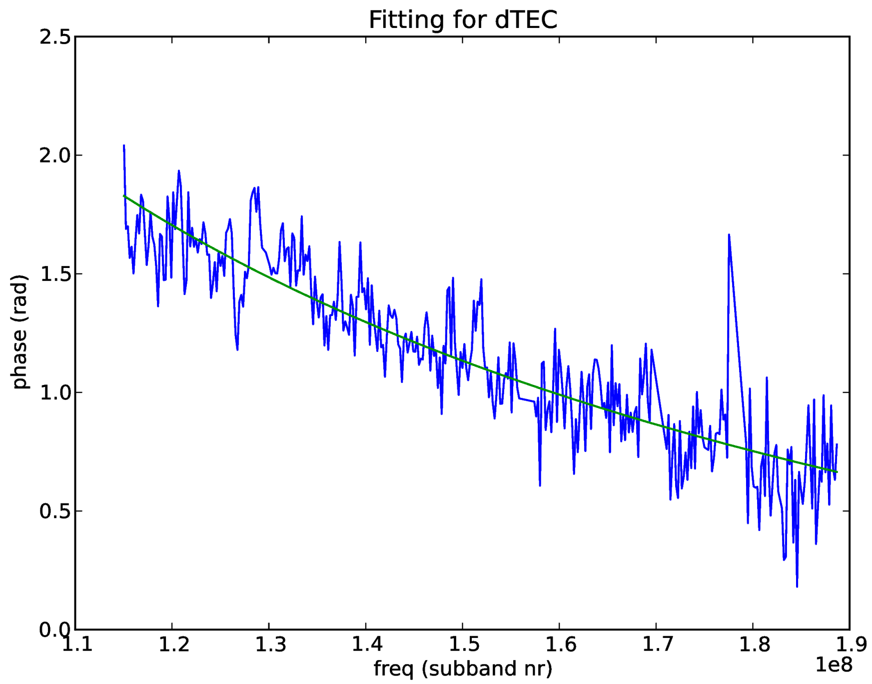
Rescaled Intensity

$$\delta J^{(s)}(s_u, s_v) \equiv s_w^2 \cdot \delta I^{(s)}(s_u, s_v)$$

Koopmans 2010

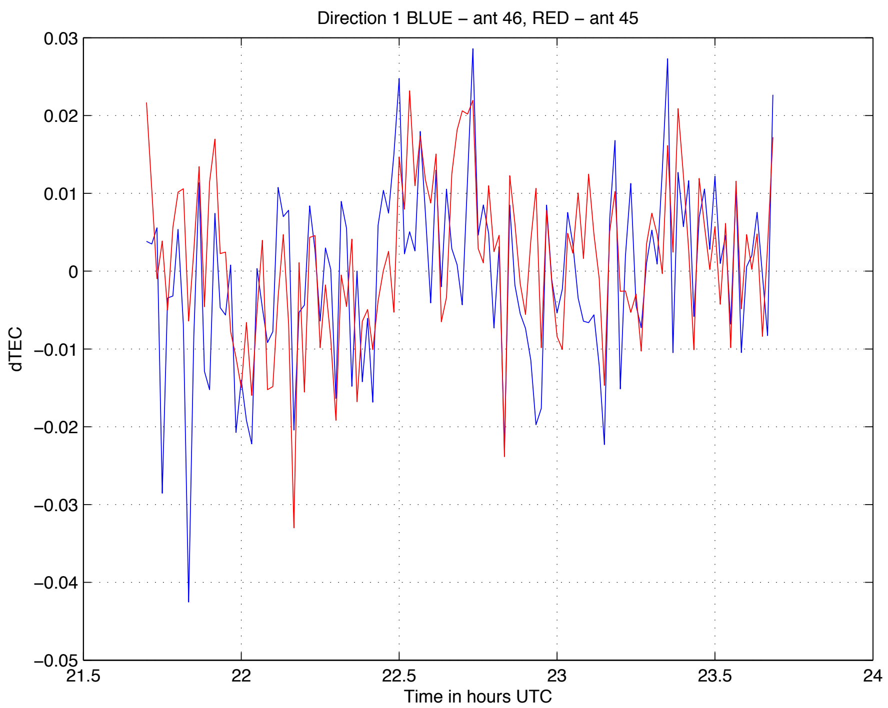
Estimating the dTEC

Fitting Tec to Sagecal Solutions



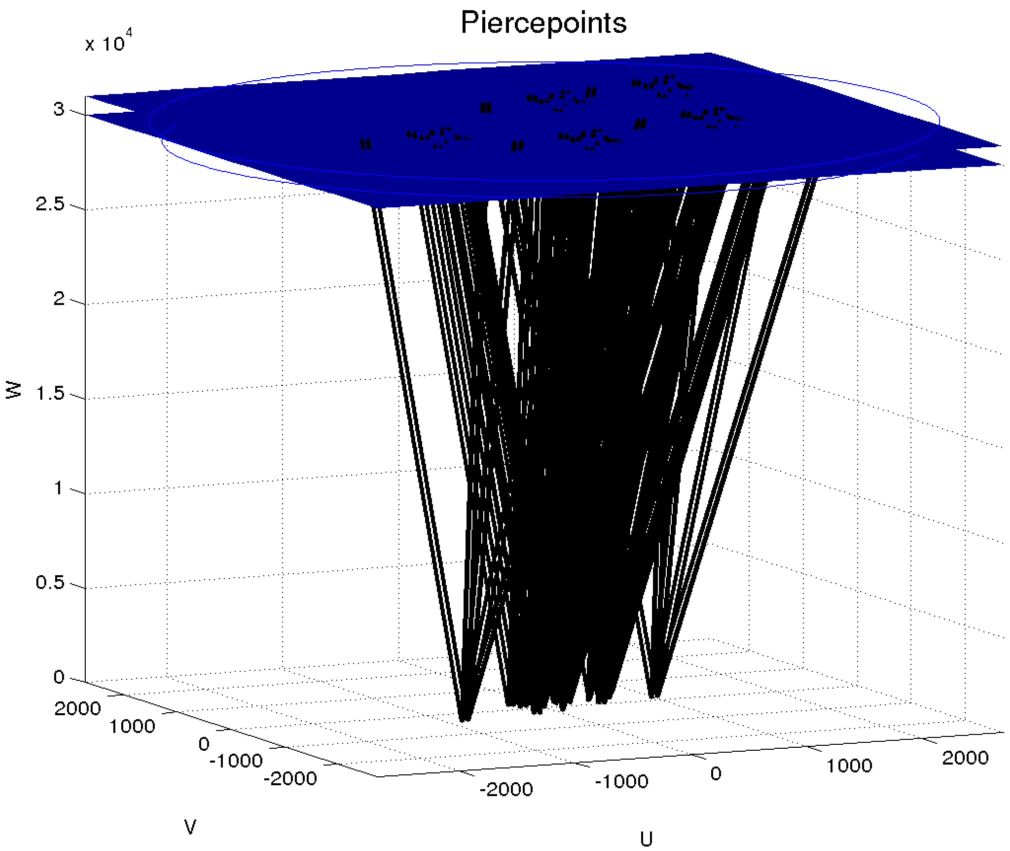
Fit for frequency dependence of the phase due to the ionosphere

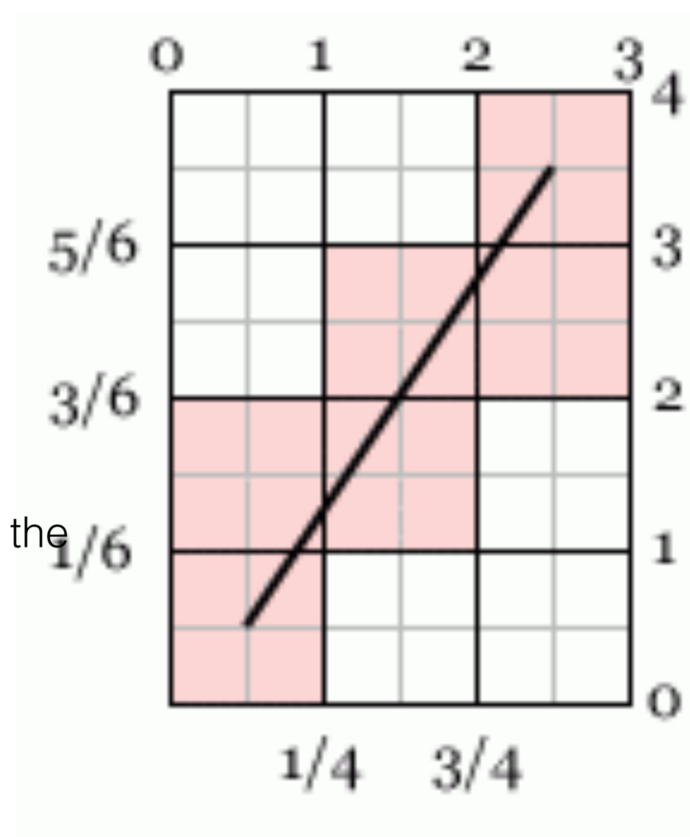
115 - 189 MHz
0.2 MHz Bandwidth Channel
379 subbands



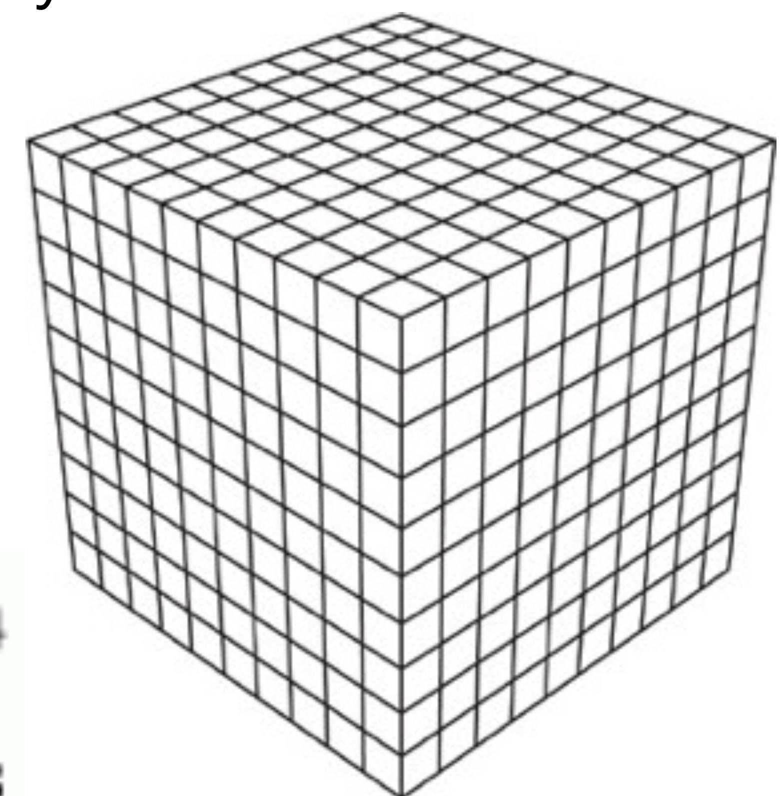
2 antennas a few metres apart

Box splines Tomography





Ray-trace and estimate all the weights at the voxels



$$d = fs + n$$

d is data points

s is the model parameters, in this case the electron density at each pixel.

f is a constant linear transformation matrix, in this case the weights at each voxels

n is noise

Regularised Maximum Likelihood Inversion

consider f to be a constant linear transformation matrix of dimensions N_d-by-N_s such that

$$\boldsymbol{d} = \mathbf{f}\boldsymbol{s} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{n}$ is the noise in the data characterised by the covariance matrix $\mathbf{C}_D$ (here and below, subscript D indicates "data").

Modelling the noise as Gaussian,[2] the probability of the data given the model parameters $\boldsymbol{s}$ is

$$P(\boldsymbol{d}|\boldsymbol{s},\mathbf{f}) = \frac{\exp(-E_D(\boldsymbol{d}|\boldsymbol{s},\mathbf{f}))}{Z_D}, \tag{2}$$

where

$$\begin{aligned} E_D(\boldsymbol{d}|\boldsymbol{s},\mathbf{f}) &= \frac{1}{2}(\mathbf{f}\boldsymbol{s}-\boldsymbol{d})^T \mathbf{C}_D^{-1} (\mathbf{f}\boldsymbol{s}-\boldsymbol{d}) \\ &= \frac{1}{2}\chi^2 \end{aligned} \tag{3}$$

and $Z_D = (2\pi)^{N_d/2}(\det \mathbf{C}_D)^{1/2}$ is the normalisation for the probability. The probability $P(\boldsymbol{d}|\boldsymbol{s},\mathbf{f})$ is called the *likelihood*,

Regularisation parameter since Ed is ill-posed.

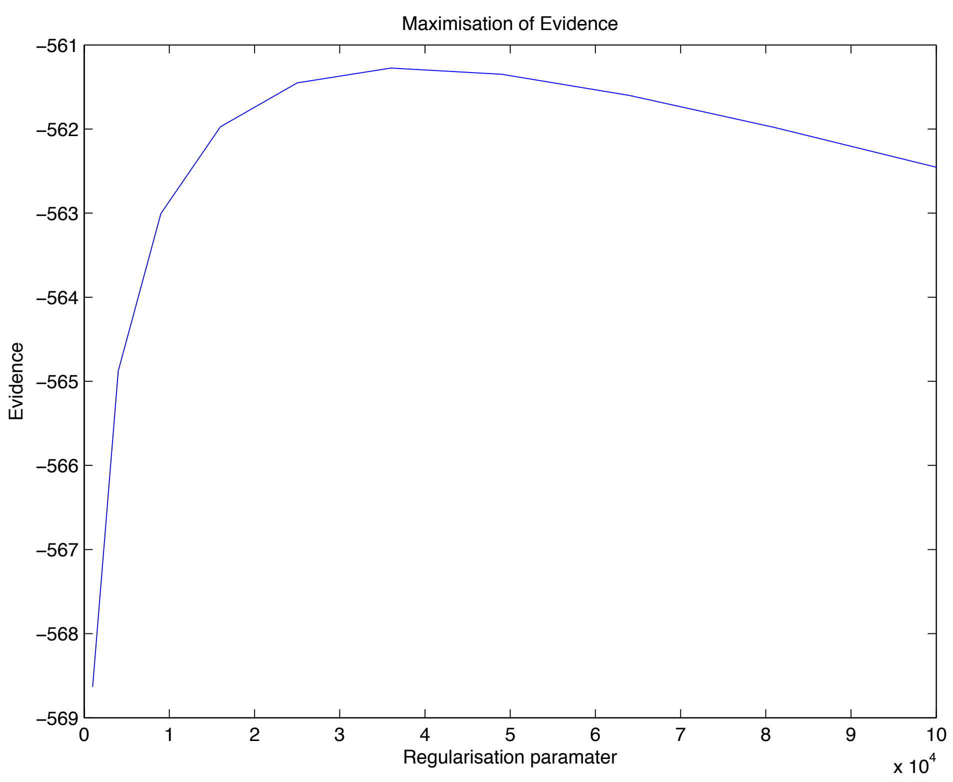
$$P(\boldsymbol{s}|\mathbf{g},\lambda) = \frac{\exp(-\lambda E_S(\boldsymbol{s}|\mathbf{g}))}{Z_S(\lambda)}, \tag{4}$$

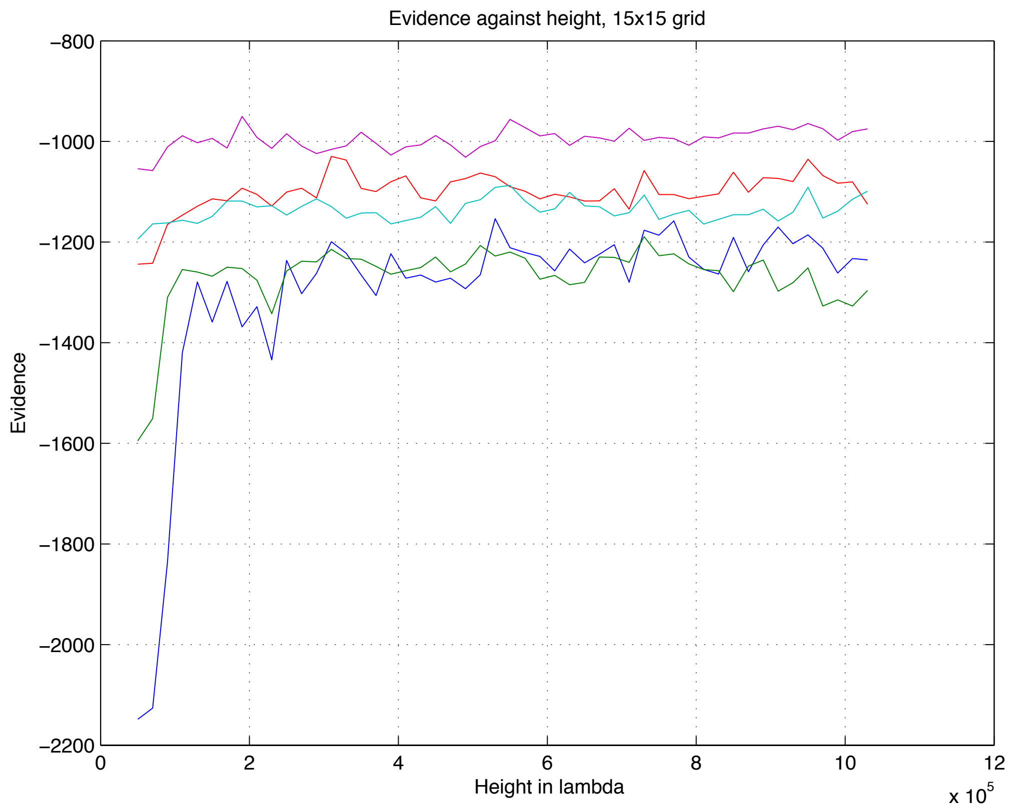
Bayes' rule tells us that the *posterior probability* of the parameters $\boldsymbol{s}$ given the data, response function and prior is

$$P(\boldsymbol{s}|\boldsymbol{d},\lambda,\mathbf{f},\mathbf{g}) = \frac{P(\boldsymbol{d}|\boldsymbol{s},\mathbf{f})P(\boldsymbol{s}|\mathbf{g},\lambda)}{P(\boldsymbol{d}|\lambda,\mathbf{f},\mathbf{g})}, \tag{5}$$

Suyu et al. 2006

Building a 3D model



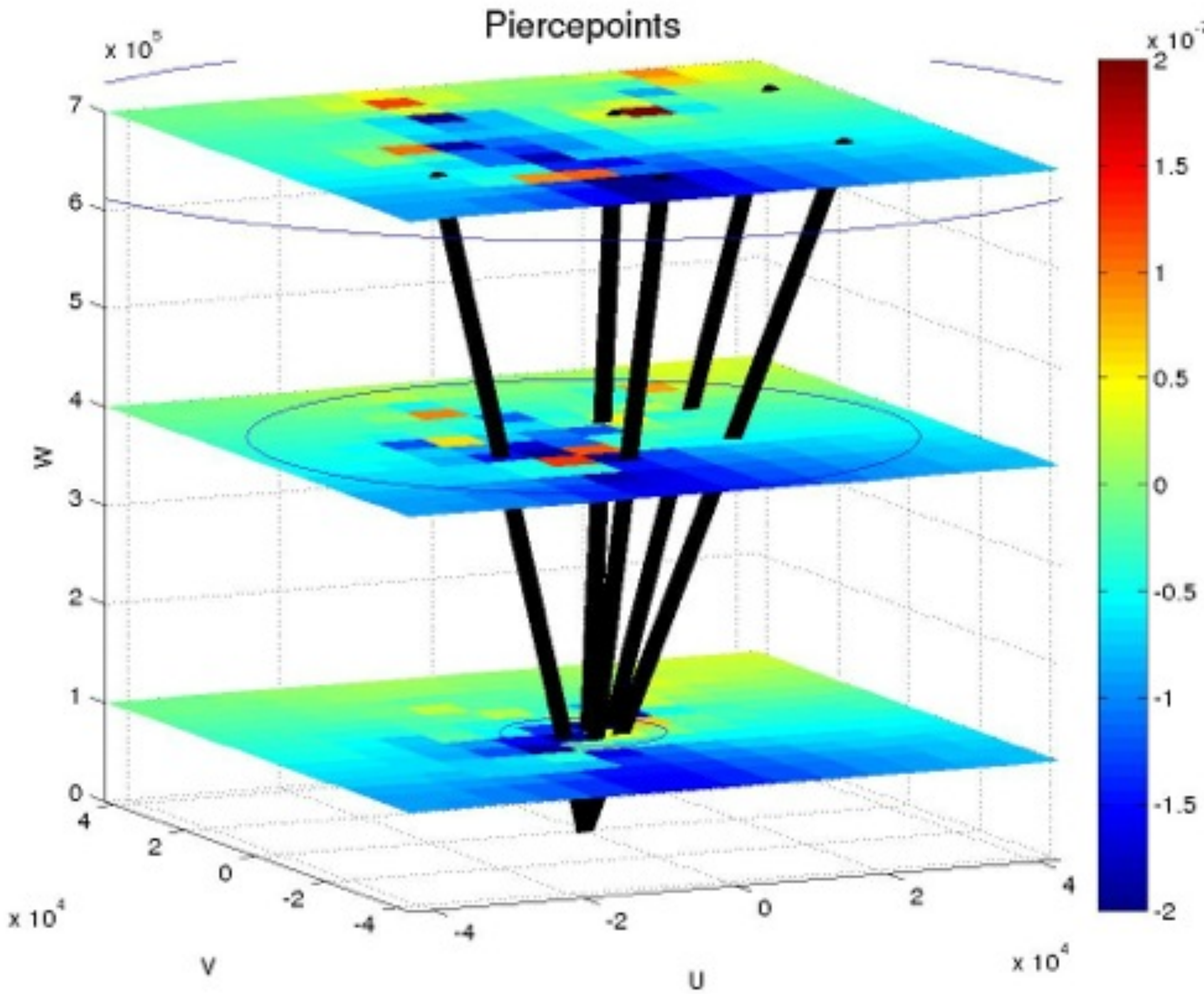


2D model

Optimise Bayesian evidence based on:

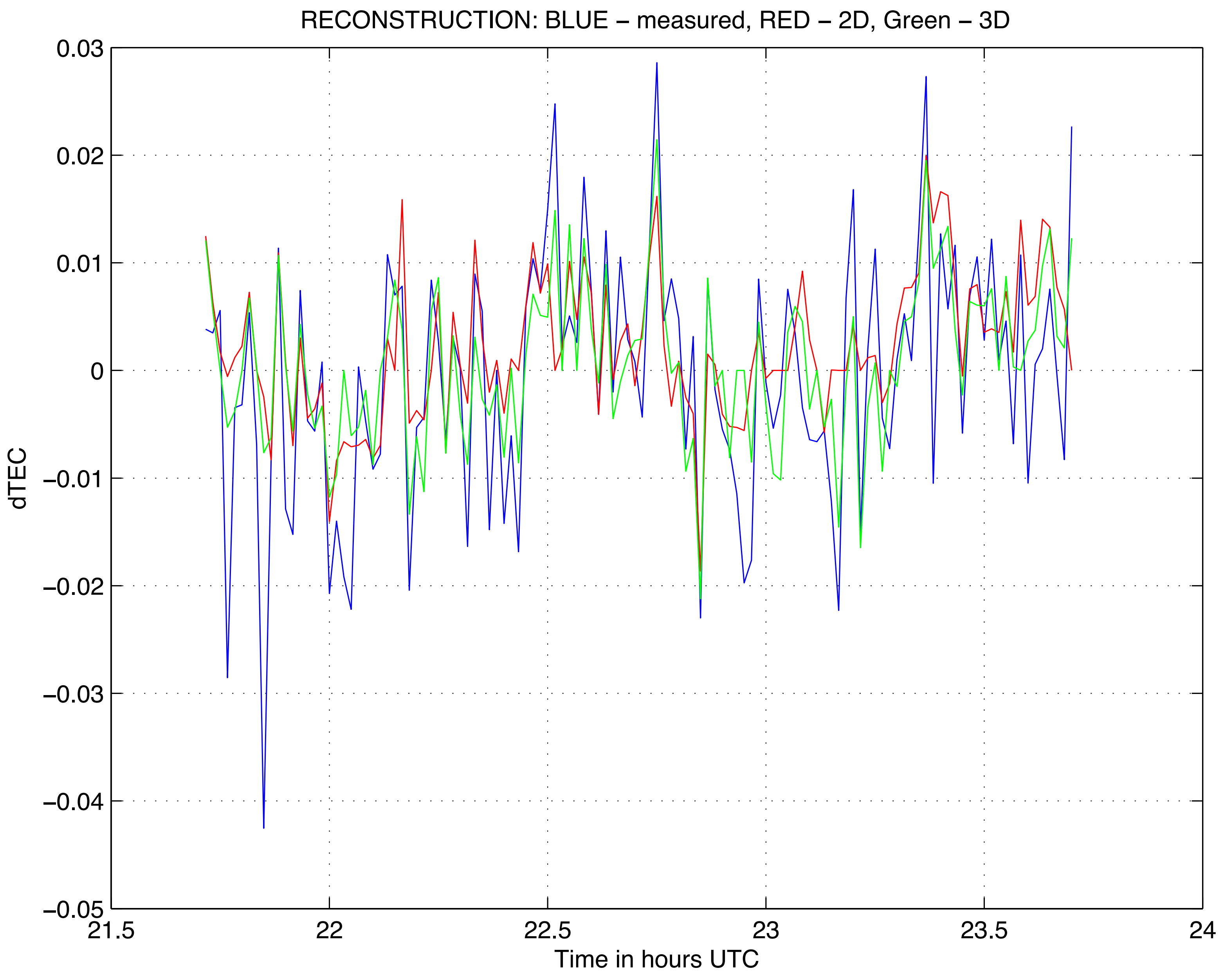
- Regularisation parameter
- Grid Size
- Height of the ionosphere layers

Height of the ionosphere

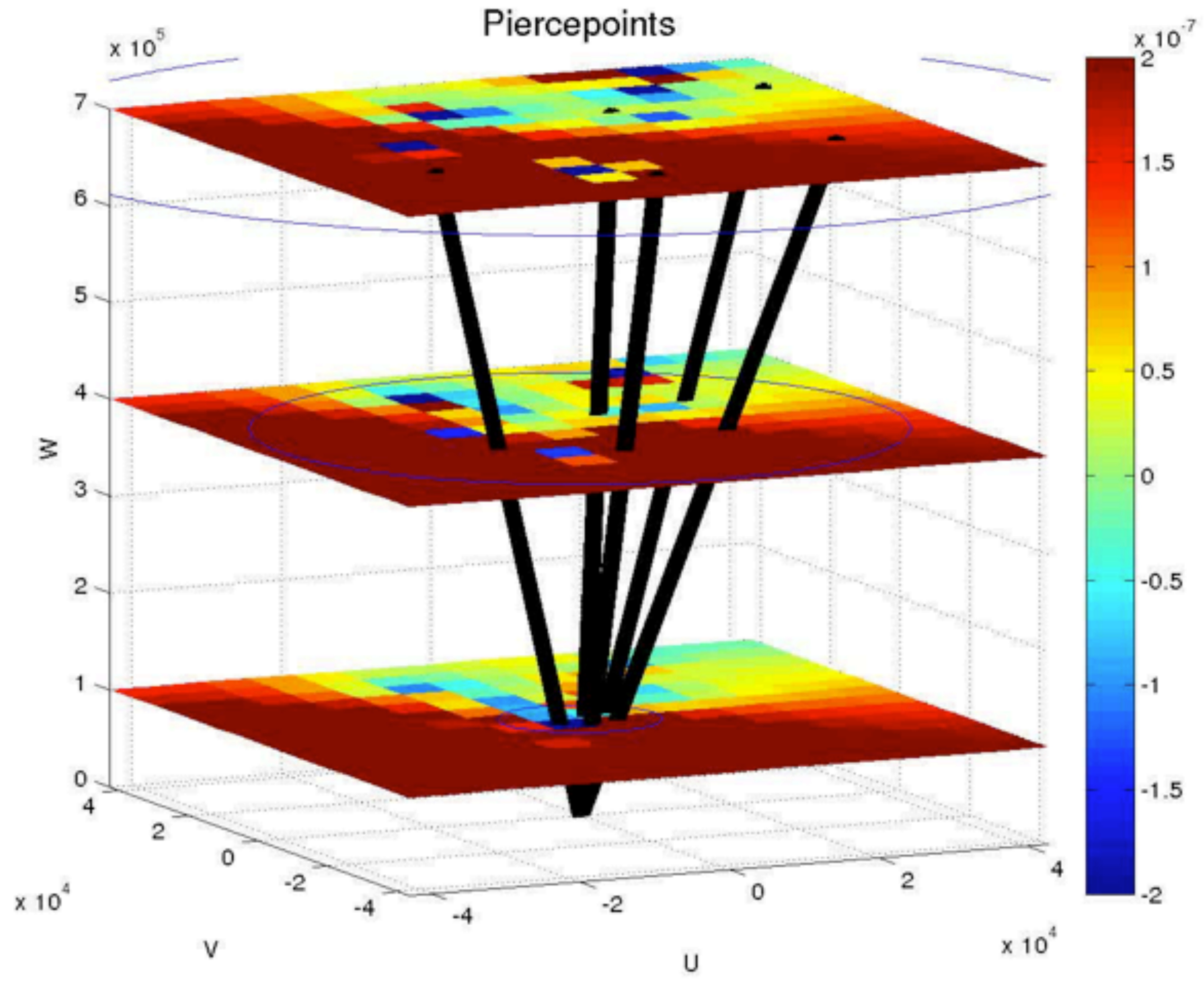


Changing of the height of the ionosphere in a 2D model leads to re-arranging of the pierce points. This could lead to conflicting constraints on the model . If this persists then the ionosphere is 3D.

Building a 3D model



Tomography



Piercepoints
x 10^5
7
6
5
4
3
2
1
0
W
4
2
0
-2
-4
x 10^4
V
-4
-2
0
2
4
U
x 10^4
x 10^-7
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

To be done

- Multiple grids for the 3D model to take into account the different station density of the LOFAR core and remote stations.
- Prediction for ionospheric correction for arbitrary directions in the field of view.
- Application of ionospheric correction to the uv data.

Thank you