Impact of observing strategy on EoR statistical estimation: drift and drift 'n' shift

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Motivation

- ✦How we observe the sky determines the information available for science
- ✦Zenith drift versus tracked scans have different benefits and costs
- ✦How does observing strategy impact EoR power spectrum estimation?

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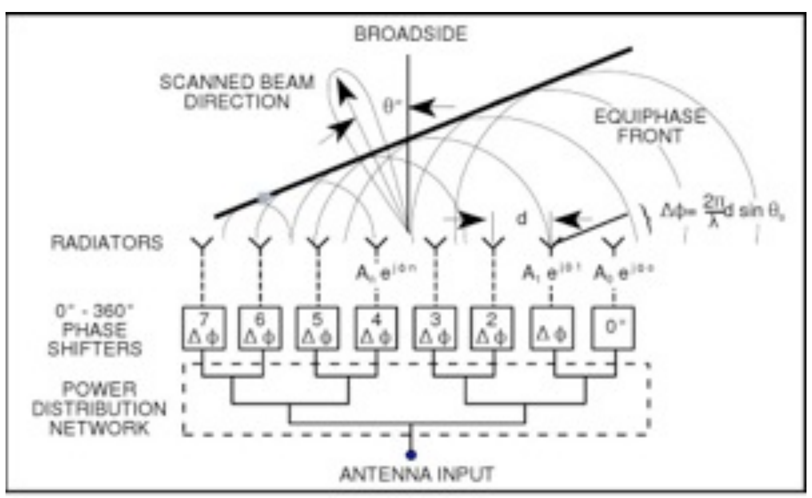
Observing with phased arrays

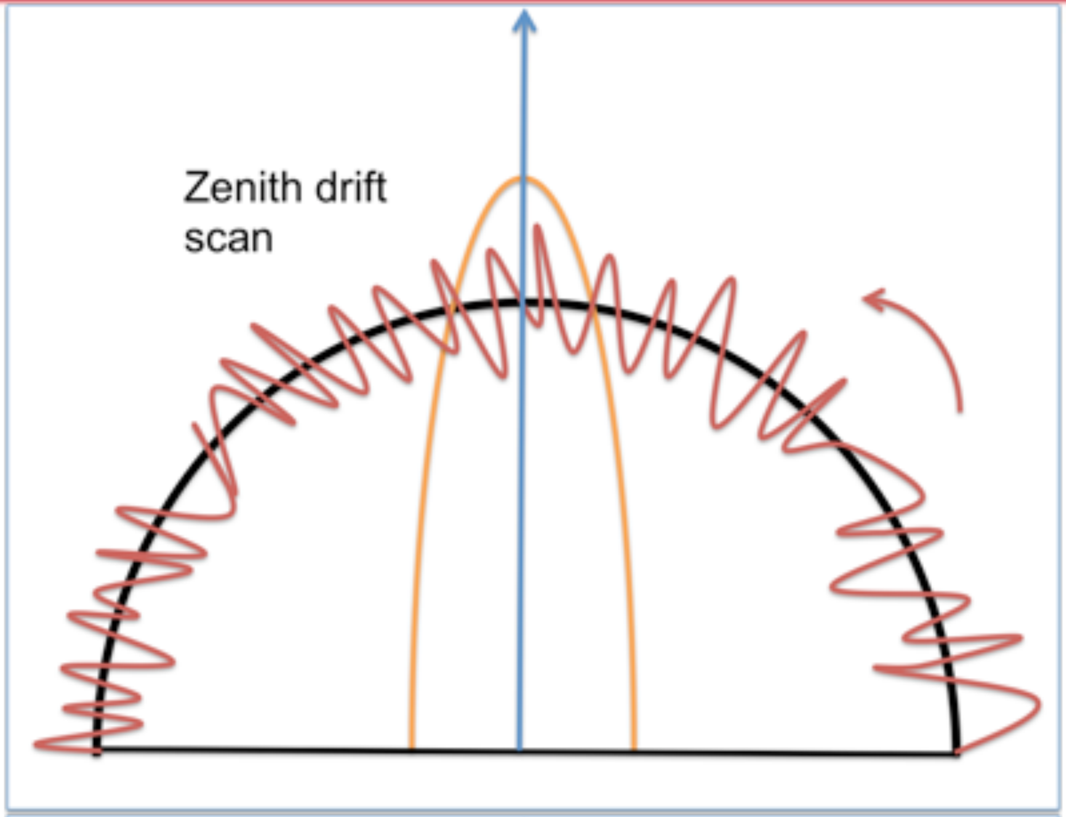
- Beam shape and sensitivity independent of pointing
- Mechanical steering

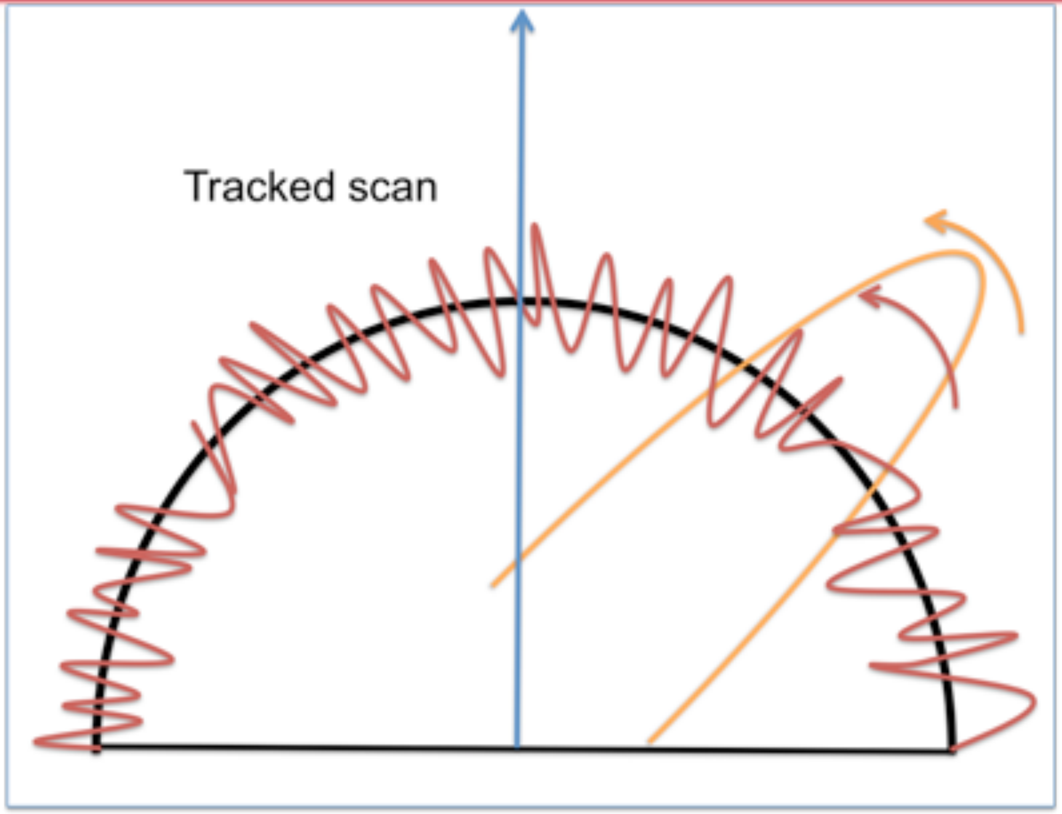


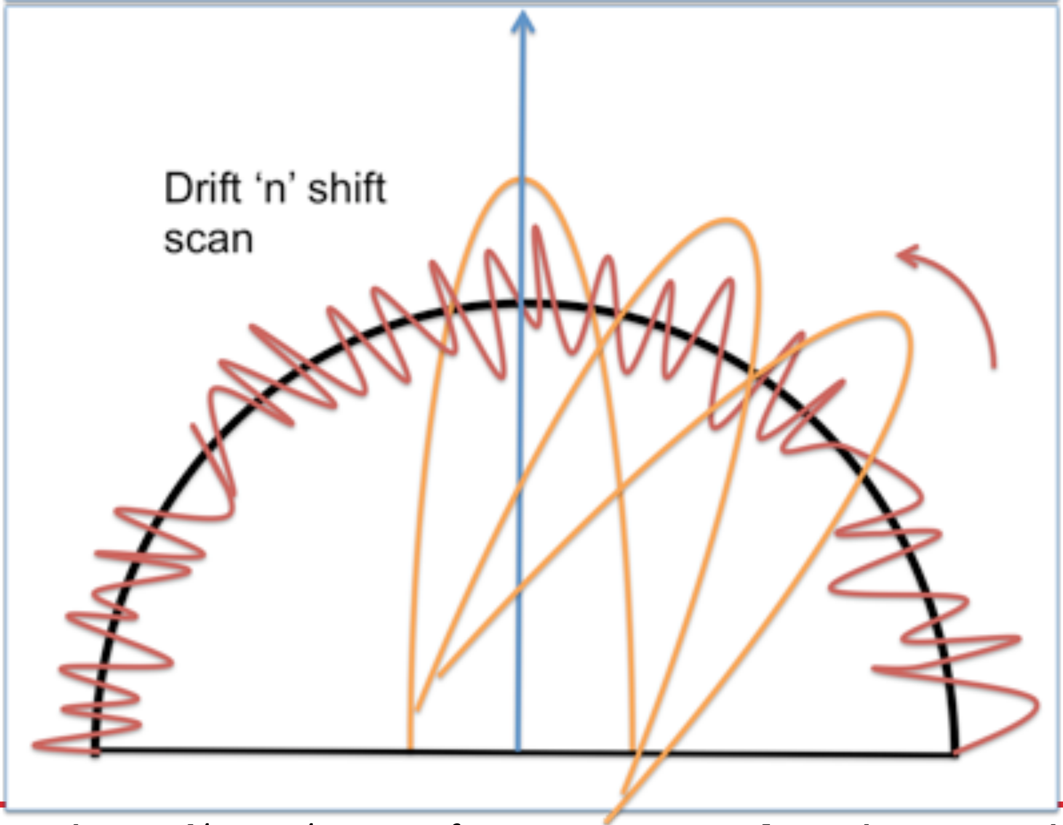
- Beam shape and sensitivity dependent on pointing
- Zenith sensitivity optimal
- Electronic steering

Fixed beamformer settings; zenith sensitivity highest

Observing modes







Drift 'n' shift scan:
11 pointings over 54°
(~30 mins per pointing)

EoR power spectrum

$$P(\vec{u})\delta_{uu'} = \left\langle S^\dagger(\vec{u})S(\vec{u}')\right\rangle = \left\langle \tilde{T}^\dagger(\vec{u})\tilde{T}(\vec{u}')\right\rangle$$

[S(u)]: K sr $\longrightarrow$ temp. fluc. integrated over obs volume

[P(u)]: K^2 sr $\longrightarrow$ temp. variance integrated over obs volume

Sky signal: samples of a Gaussian random field

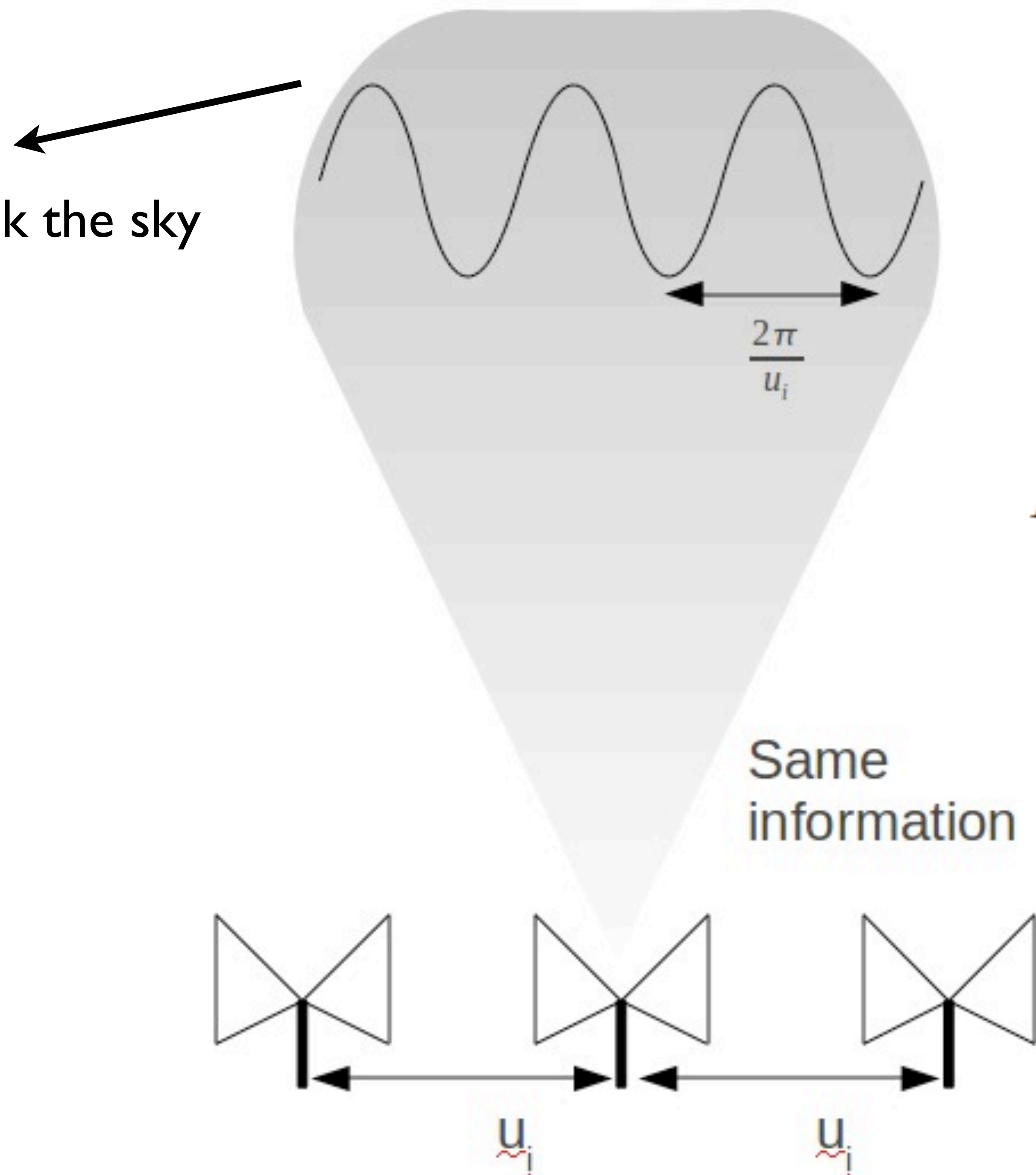
Radiometric noise: reduced effectively by coherent addition of information (summing visibilities)

Visibility:

$$\begin{aligned} V(\vec{u}) &= B_w(\vec{u}) * S(\vec{u}) + N \\ &= \sum_{\vec{u}'} B_w(u - u')S(\vec{u}') + N. \end{aligned}$$

Coherence versus incoherence

Track the sky



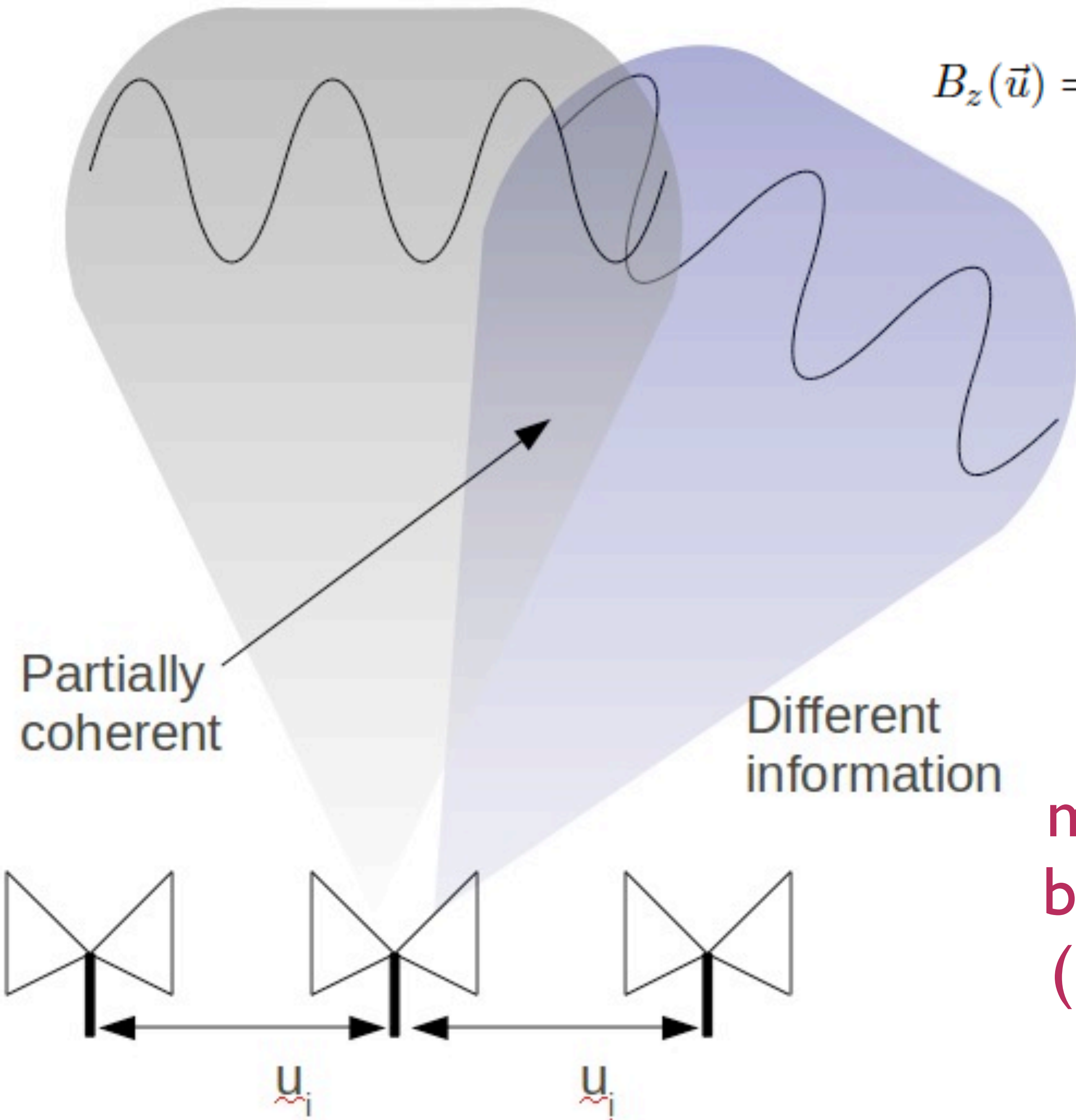
A tracked scan provides a single sample of the sky power, but repeated observation of the same sky (coherent addition - phase intact).

$$B_{tr}(\vec{u}) = \mathcal{FT}(B(\vec{l})) = \int B(\vec{l}) \exp{(-2\pi i \vec{u} \cdot \vec{l})}.$$

21cm signal can be described as a Gaussian random field: **each patch of sky yields a sample of this random field**

Coherence versus incoherence

Incoherent samples of the sky signal



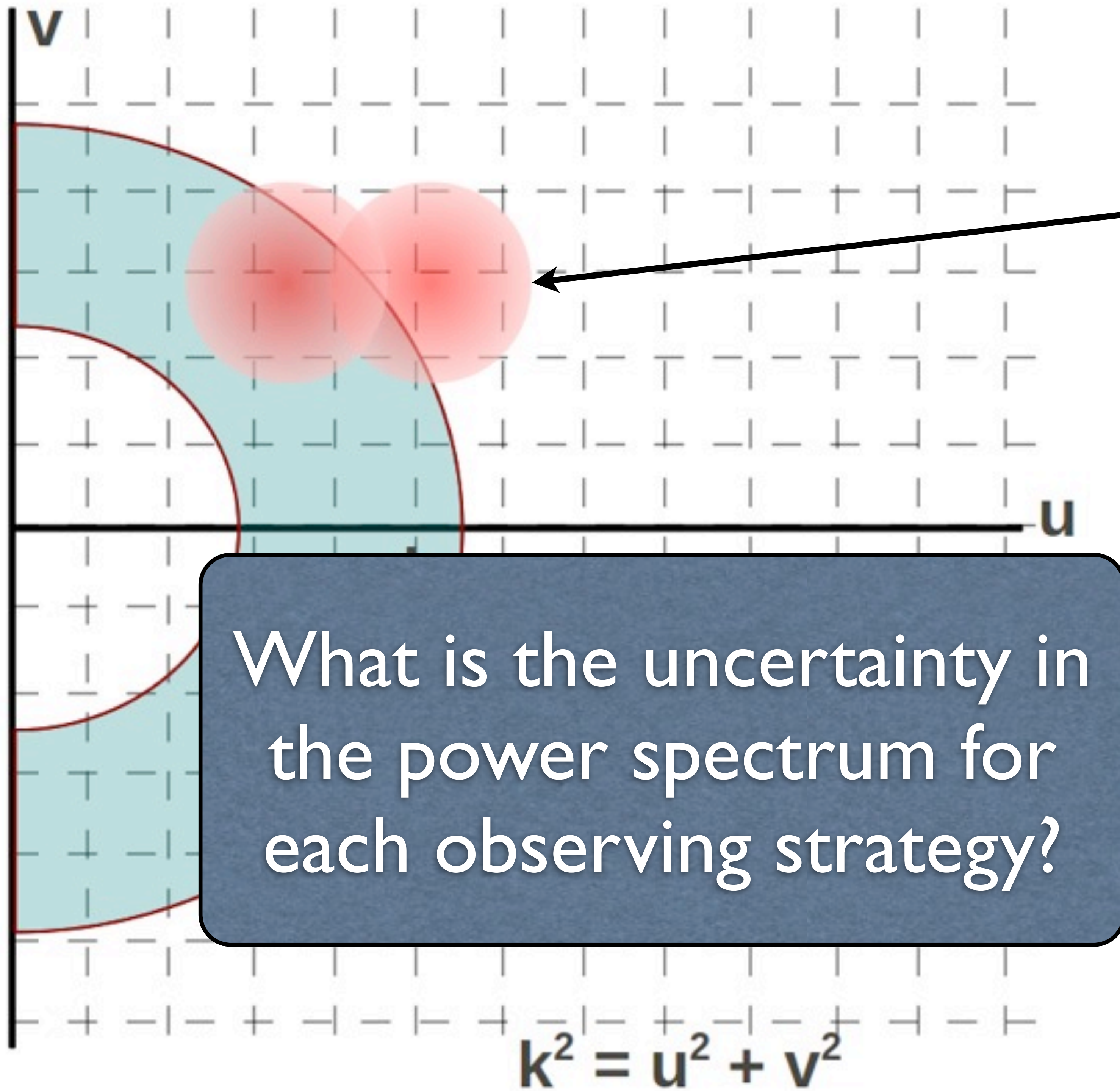
$$B_z(\vec{u}) = \mathcal{FT}(B(\vec{l} - \Delta\vec{\theta})) = \int B(\vec{l} - \Delta\vec{\theta}) \exp\left(-2\pi i \vec{u} \cdot \vec{l}\right)$$

$$= \exp\left(-2\pi i \vec{u} \cdot \Delta\vec{\theta}\right) B_{tr}(\vec{u}),$$

u-dependent phase shift

A zenith drift scan provides multiple samples of the sky power, but limited mutual sky information (incoherent addition - phase lost).

Coherence versus incoherence



Beam footprint on *uv* plane

$B_w^\dagger B_w$ encodes the correlation between uv cells and the amount of information obtained about signal in each uv cell

Noise considerations

	Radiometric Noise	Sample variance
Coherent addition	Reduces ~1/N	Unchanged
Incoherent addition	Reduces $1/\sqrt{N}$	Reduces $1/\sqrt{N}$

Observing strategy pros and cons

	Zenith drift	Drift 'n' shift
Coherence	< 1	~1
Sensitivity (zenith=1)	1	< 1
Power samples	> 1	~1

Ratio of radiometric noise to sample variance determines balance of coherent and incoherent combination.

Building a power spectrum

1. Form sky signal

$$\vec{S}(\vec{u}_\alpha) = ({B_w}^\dagger B_w)^{-1} {B_w}^\dagger \vec{V}$$

1b. Fourier transform along line-of-sight (frequency)

$$\vec{S}(\vec{k}) = F({B_w}^\dagger B_w)^{-1} {B_w}^\dagger \vec{V}$$

2. Form power *sample* (coherent)

$$P(\vec{u_i}) = S^\dagger(\vec{u_i}) S(\vec{u_j}) V_u$$

3. Form ML binned power (incoherent)

$$P(\vec{k}) = \frac{\sum_{\vec{u_i} \in k} N(\vec{u_i}) P(\vec{u_i})}{\sum_{\vec{u_i} \in k} N(\vec{u_i})},$$

Propagate power spectrum uncertainties

Coherent power

Radiometric noise

Sample variance

$$
\begin{aligned}
C_p &= C_s^\dagger C_s V_u^2 \\
&= \left(\sigma^2 V_u \left(B^\dagger B\right)^{-1}_{ij} + P\right)^\dagger \left(\sigma^2 V_u \left(B^\dagger B\right)^{-1}_{ij} + P\right).
\end{aligned}
$$

$$\left(B_w^\dagger B_w\right)^{-1}$$

encodes coherence of measurements and information in data about an angular mode

$$V_u = \int d^3\vec{u}|B(\vec{u})|^2$$

(tracked mode with a single sample of sky)

$$V_u = \int d^3\vec{u}|B(\vec{u})\mathrm{sinc}(2\pi\vec{\theta}_{\mathrm{max}} \cdot \vec{u})|^2$$

$2\theta_{max}$ is angular coverage of drift

(drift mode with >1 samples of sky: beam is tapered by sinc function --> larger *physical* sky volume)

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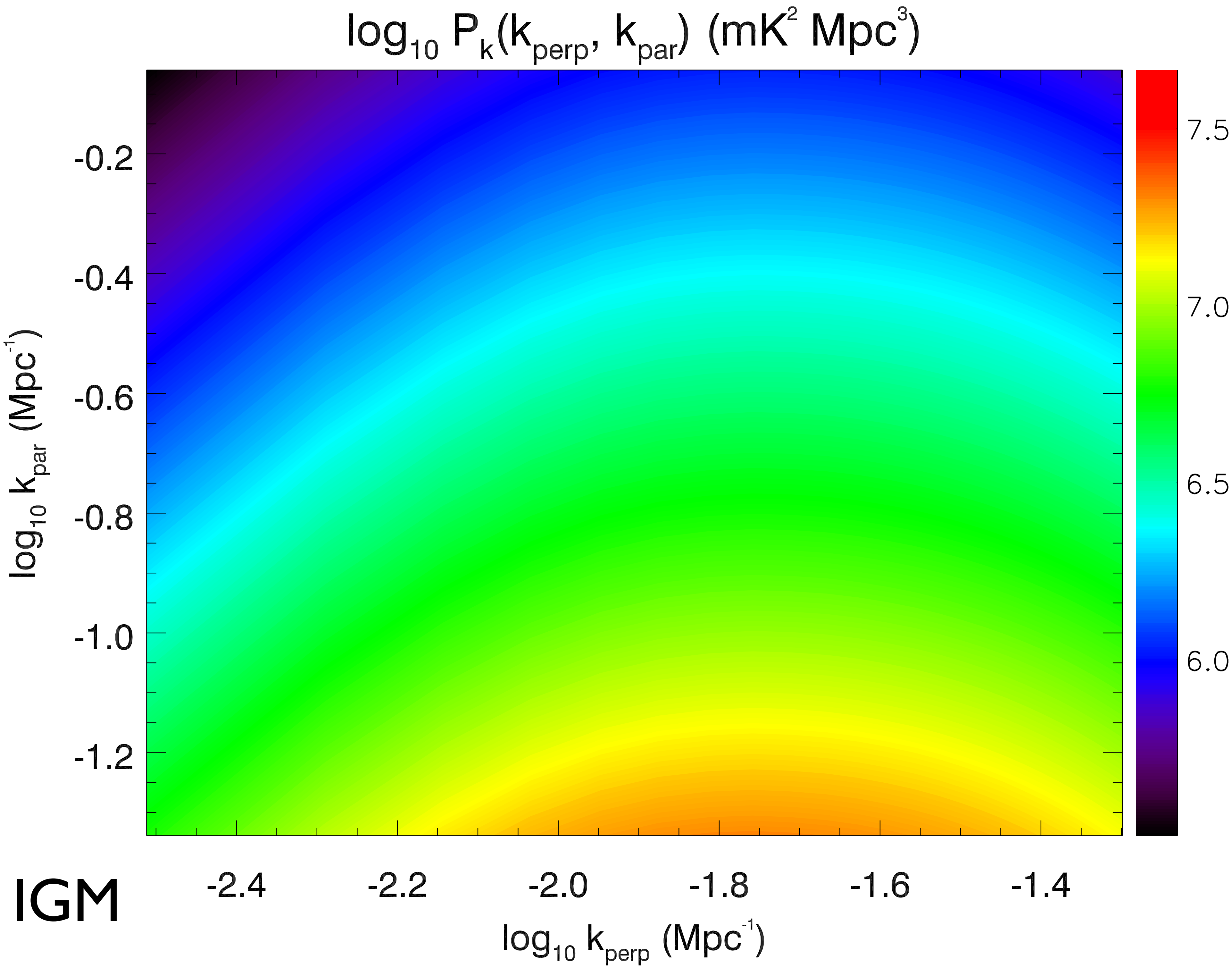
Preliminary MWA simulation

- 900 hour observation
- HA range: [-1.8, 1.8]
- 11 beamformer positions
- w-terms, direction-dependent beams included
- Frequency: 150 MHz
- BW: 8 MHz
- $\Delta u = 0.5$ ($\Delta u = 2$ typical for "independence")
- $T_{sys} = 440K$
- $u_{max} = 80$ ($l = 500$, $\log_{10} k_{\perp} = -1.3$)

- Coarse binning in $k_{\perp}$, $k_{||}$

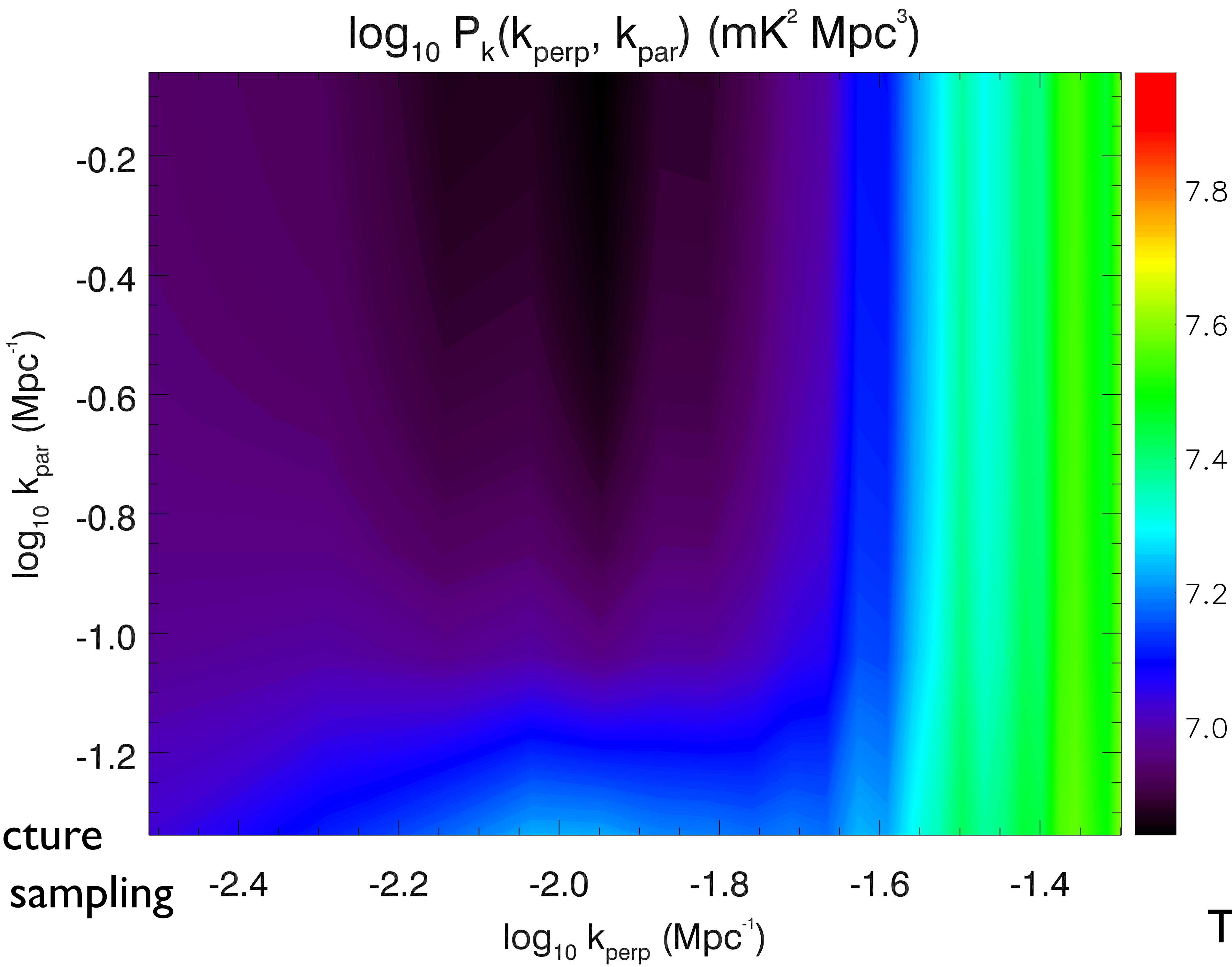
2D signal power spectrum



z=8

Neutral IGM

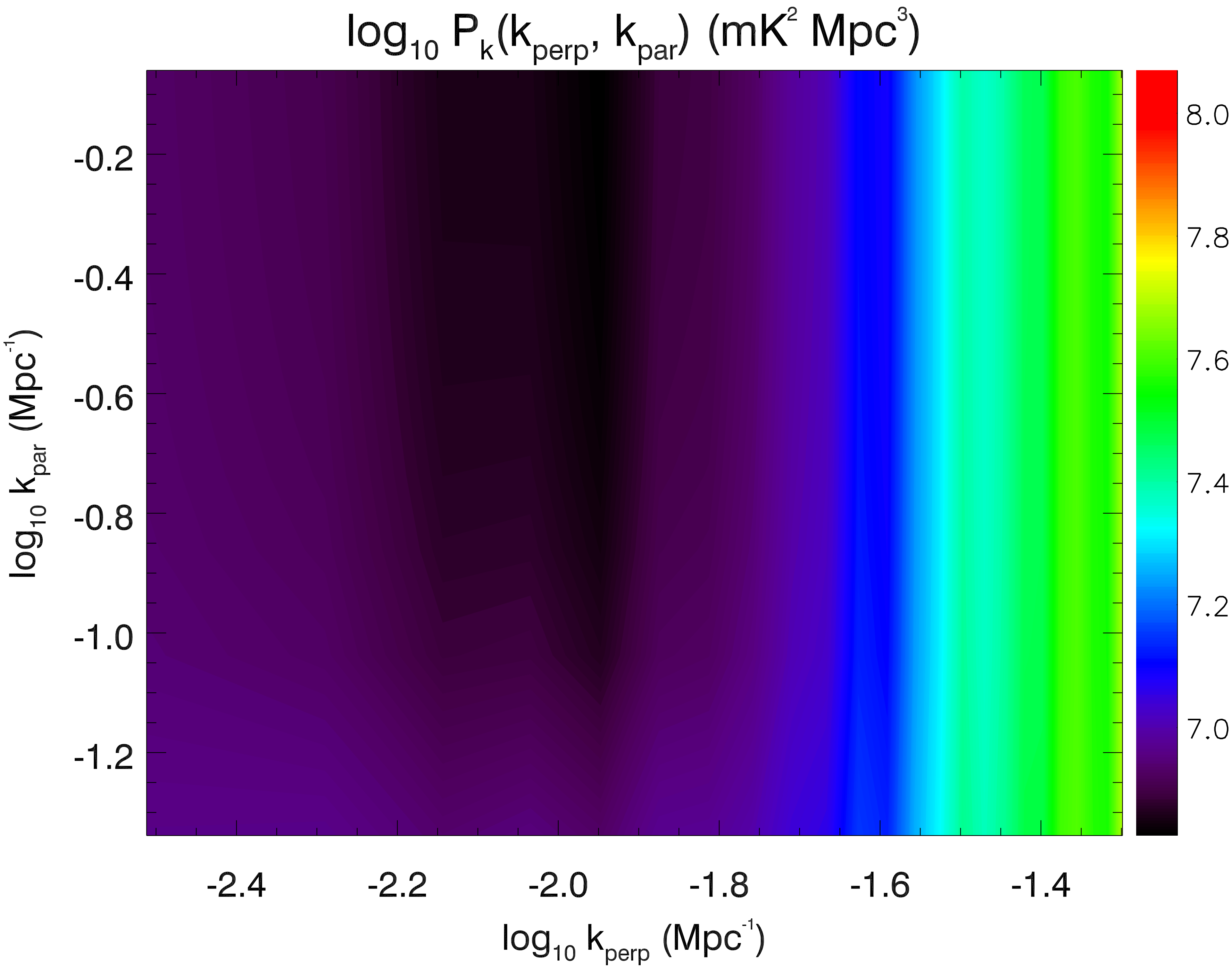
2D power spectrum uncertainty: drift 'n' shift



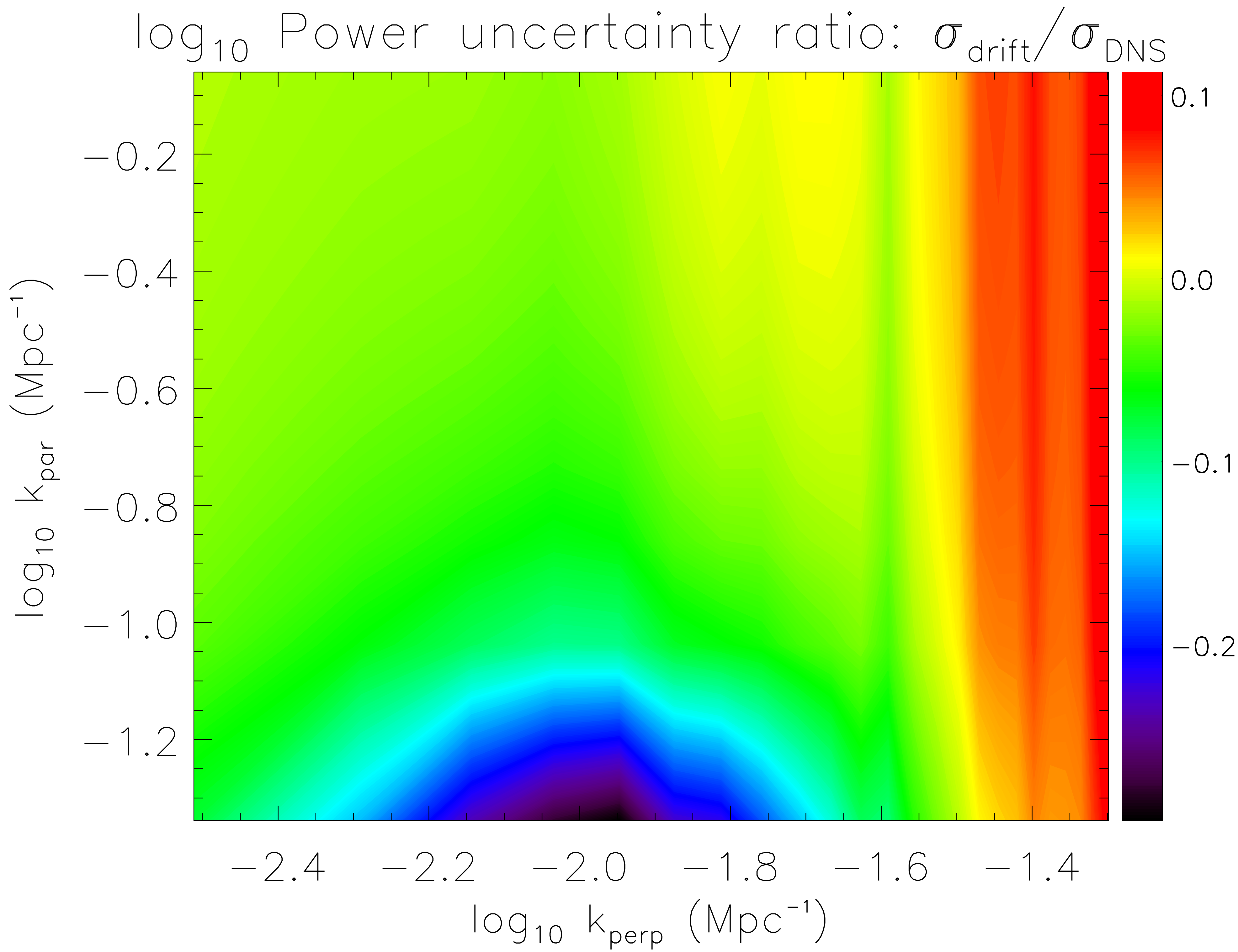
$k_\perp{}^2$ structure encodes uv sampling

Trott (2013)

2D power spectrum uncertainty: drift

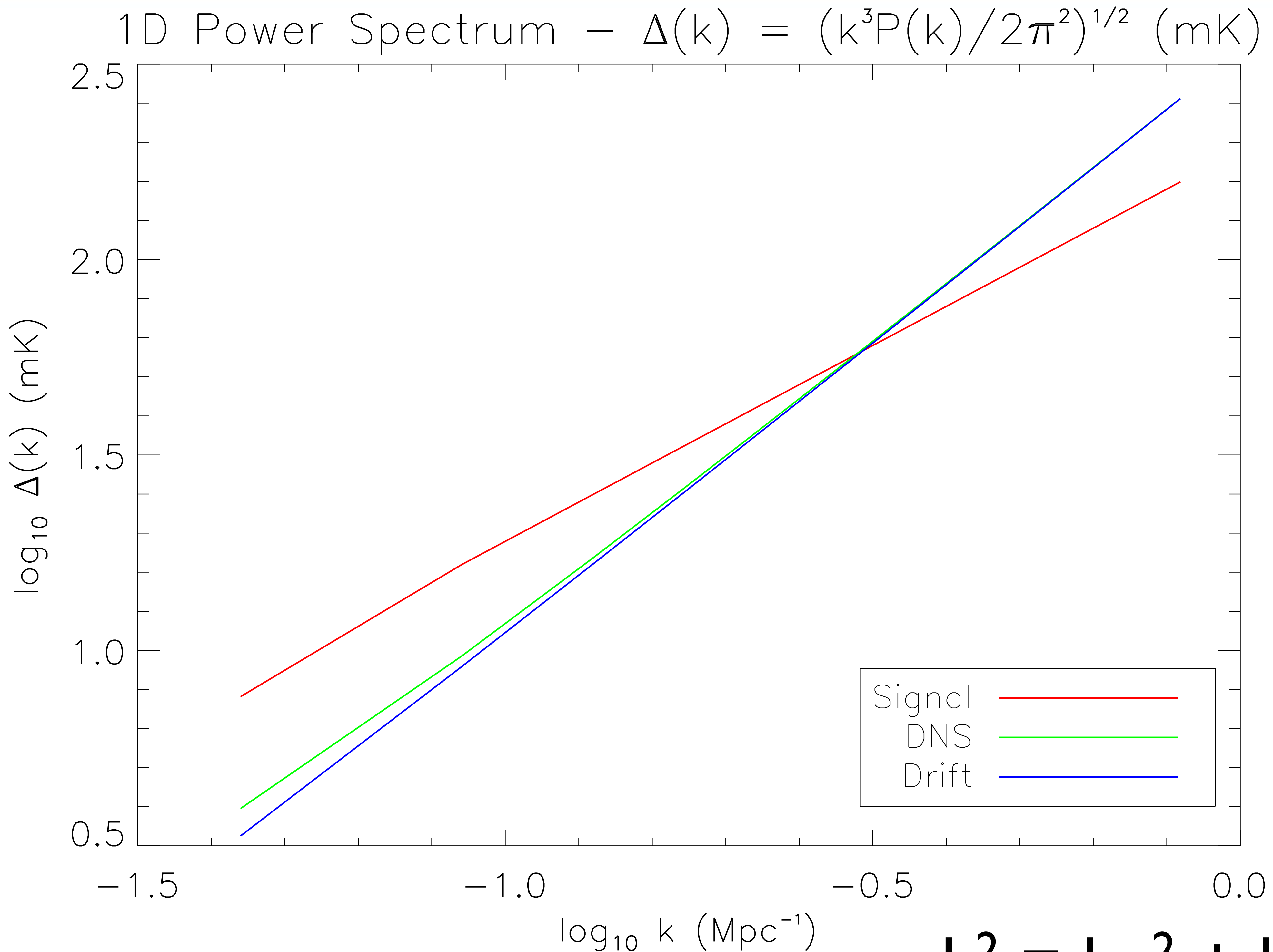


2D power spectrum uncertainty ratio: drift / drift 'n' shift



1D spherically-averaged "power"

Dimensionless (mK)



$$k^2 = k_\perp^2 + k_{||}^2$$

Preliminary conclusions

✦An understanding of the instrument beam, and baseline distribution yields the balance of coherent and incoherent information
✦Optimal observational strategies can be computed for a particular instrument
✦Zenith drift scans competitive with tracked scans for thermal-noise limited observations.

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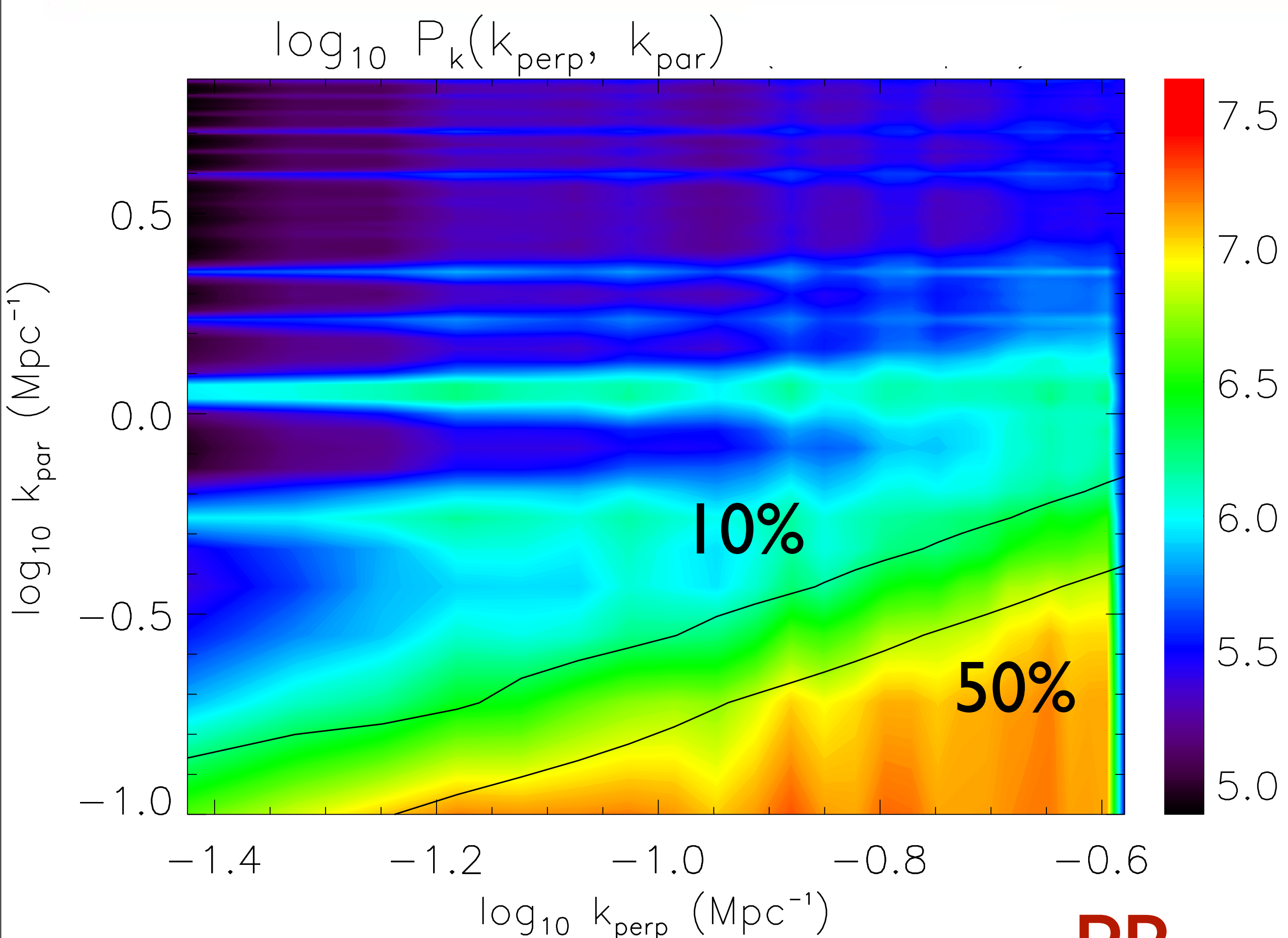
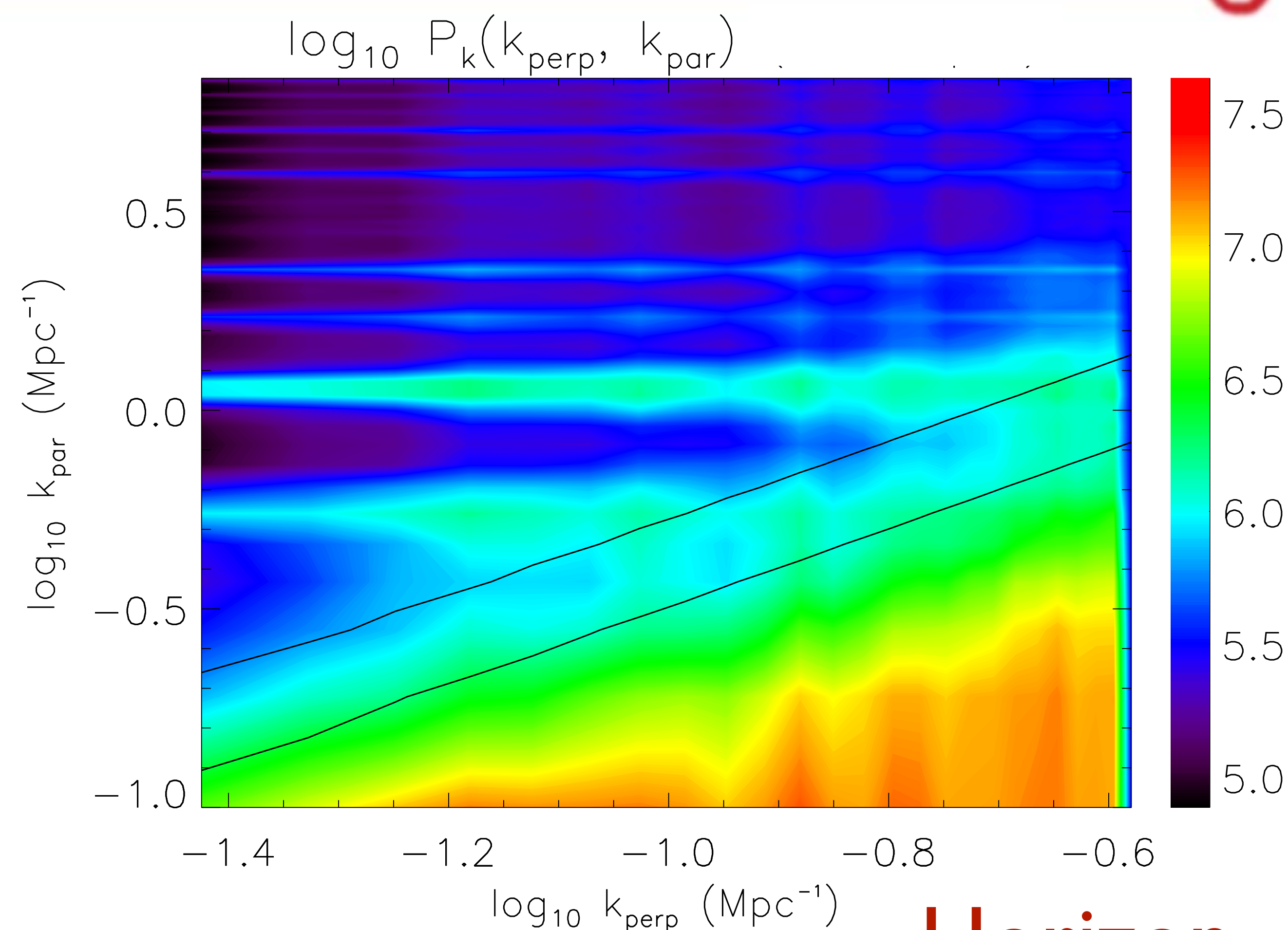
MWA **Test** Power Spectrum ML Estimate



PB



Horizon

$\mathrm{sinc}^2(\alpha k_{||}/k_{\perp})$ structure to wedge: $\alpha \propto 1/\ell_{max}$ (Trott+ 12, Vedantham+ 11, Datta+ 10...)

- ✦8 MHz bandwidth (167 - 175 MHz)
- ✦224s data, XX pol
- ✦Simple calibration, no source subtraction, no CLEANing
- ✦Simple frequency DFT
- ✦No imaging -> produced directly from visibility data

Wedge feature: signature of smooth-spectrum foregrounds in PS space

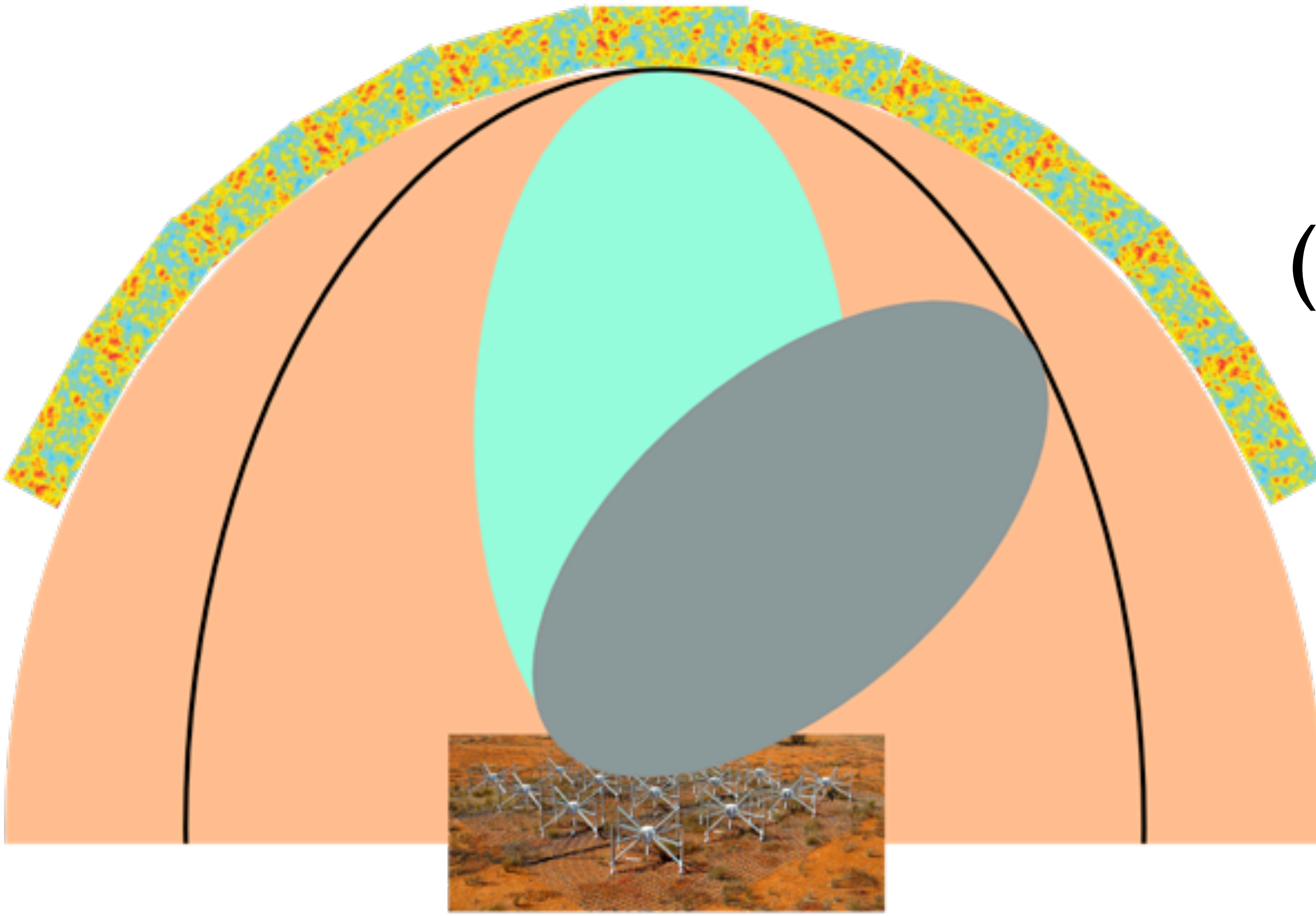
Units *approximately* log K^2 Mpc^3

Next-generation instruments

Considerations for SKA-Low, HERA...

- ✦Excellent instantaneous uv coverage --> high coherence
- ✦Small FOV yields a broad Fourier beam --> large spectral leakage
- ✦High sensitivity reduces radiometric noise component --> sample variance fraction of noise budget is larger
- ✦Multiple independent FOV versus drift scanning --> drifts sample small k information

Observing modes



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Propagate power spectrum uncertainties

Visibilities

$$S(\vec{u}) = \left(B_w^\dagger B_w\right)^{-1} B_w^\dagger V(\vec{u}')$$

Sky modes

$$\boldsymbol{C}_s \equiv \left\langle S(\vec{u_i}) S^\dagger(\vec{u_j}) \right\rangle$$

Coherent power (uv cells)

Radiometric noise

Sample variance

$$\begin{aligned} \boldsymbol{C}_p V_u^2 &= \boldsymbol{C}_s^\dagger \boldsymbol{C}_s \\ &= \left(\sigma^2 \left(B^\dagger B\right)_{ij}^{-1} + P'\right)^\dagger \left(\sigma^2 \left(B^\dagger B\right)_{ij}^{-1} + P'\right) \end{aligned}$$

Incoherent power (k)

$$\sigma_k^2 V_u^2 = \frac{1}{\sum\limits_{i \in k} \boldsymbol{C}_p^{-1}} = \frac{1}{\sum\limits_{i \in k} \boldsymbol{C}_s^{-1} \boldsymbol{C}_s^{-1}}$$