SPATIAL FILTERING OF RFI USING A REFERENCE ANTENNA ARRAY

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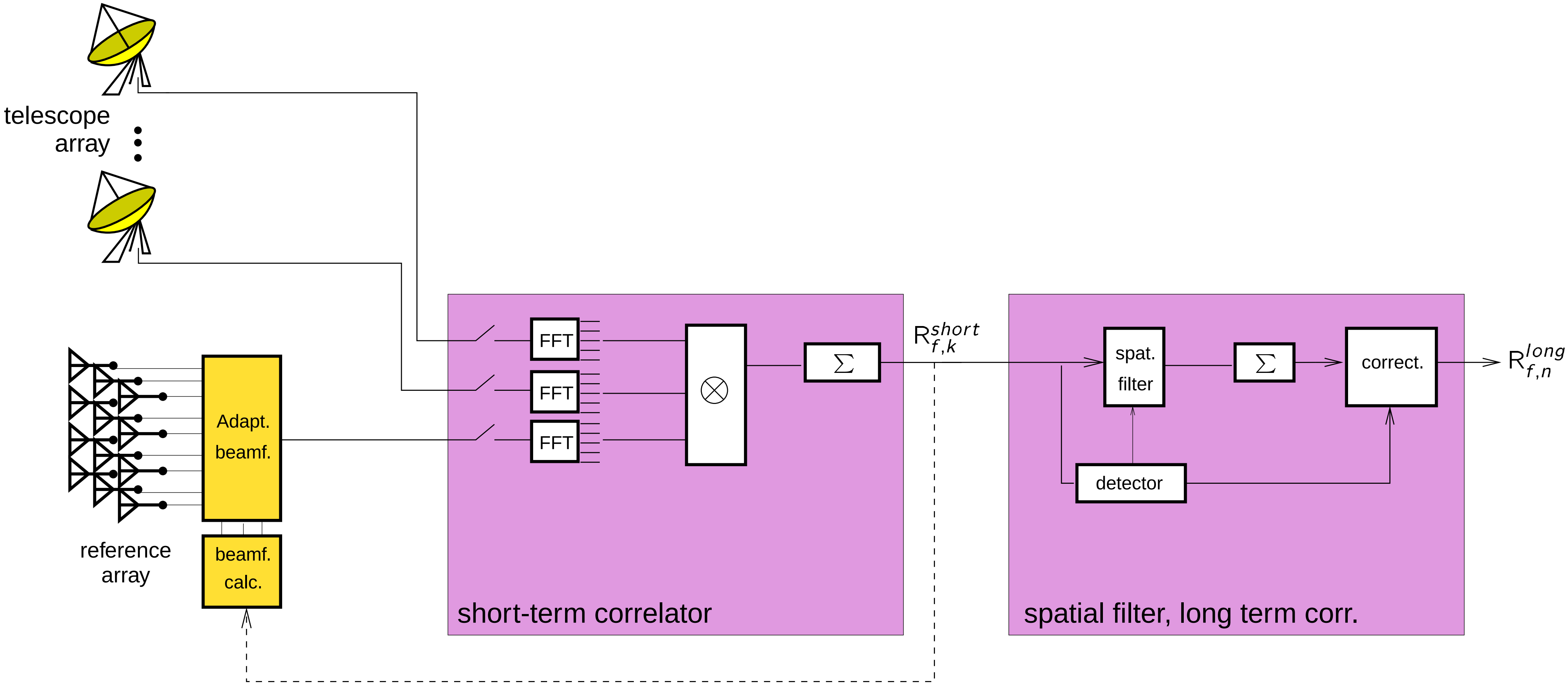
- We investigate **multichannel spatial filtering** techniques for removing continually present interference such as TV signals, radio broadcasts, or the GPS satellite system.
- The techniques are based on **subspace projections** on short-term spatial covariance matrices.
- Tested on (1) WSRT/focal plane array (3C48 contaminated by Afristar)
  (2) RS409 LOFAR station (TV station interference)

Setup

- The outputs of the reference array are processed as additional telescopes
- Correlation and short-time integration (e.g. 10 ms), followed by offline filtering



Data model

- **Interference free case:** the received data vector on primary array (p_0 elements) is

$$\tilde{\mathbf{x}}_0(t) = \mathbf{v}_0(t) + \mathbf{n}_0(t)$$

where $\mathbf{v}_0$ is the astronomical signal, $\mathbf{n}_0$ the noise.

- **With interference** $s(t)$, we receive

$$\mathbf{x}_0(t) = \mathbf{v}_0(t) + \mathbf{a}_0(t)s(t) + \mathbf{n}_0(t)$$

- **With a reference antenna** (p_1 elements):

$$\mathbf{x}_1(t) = \mathbf{a}_1(t)s(t) + \mathbf{n}_1(t)$$

- Stacking all antenna signals in a single vector $\mathbf{x}(t) = \begin{bmatrix} \mathbf{x}_0(t) \\ \mathbf{x}_1(t) \end{bmatrix}$, we obtain

$$\mathbf{x}(t) = \mathbf{v}(t) + \mathbf{a}(t)s(t) + \mathbf{n}(t)$$

Covariance model

From the observed data, construct short-term covariance estimates

$$\hat{\mathbf{R}}_k = \frac{1}{M}\sum_{n=kM}^{(k+1)M} \mathbf{x}_n \mathbf{x}_n^{\mathrm{H}}$$

with expected value

$$\mathbf{R}_k := \begin{bmatrix} \mathbf{R}_{00,k} & \mathbf{R}_{01,k} \\ \mathbf{R}_{10,k} & \mathbf{R}_{11,k} \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{R}_{v,0} + \mathbf{a}_{0,k}\mathbf{a}_{0,k}^{\mathrm{H}} + \mathbf{\Sigma}_0 & \mathbf{a}_{0,k}\mathbf{a}_{1,k}^{\mathrm{H}} \\ \hline \mathbf{a}_{1,k}\mathbf{a}_{0,k}^{\mathrm{H}} & \mathbf{a}_{1,k}\mathbf{a}_{1,k}^{\mathrm{H}} + \mathbf{\Sigma}_1 \end{array}\right]$$

where $\mathbf{R}_{v,0}$ are the visibilities on the primary array, $\mathbf{\Sigma}_0$ is the noise at the primary array, $\mathbf{\Sigma}_1$ is the noise at the reference array.

Objective: estimate interference-free visibilities, $\mathbf{\Psi}_{00} := \mathbf{R}_{v,0} + \mathbf{\Sigma}_0$.

We assume $\mathbf{R}_{v,0} \ll \mathbf{\Sigma}_0$, and $\mathbf{a}_k$ stationary over short processing times ($\sim$ 10 ms).

We also assumed no astronomical signal on the reference antennas.

First technique: subtraction (cf. Briggs e.a., Jeffs e.a.)

With a single reference antenna without noise, the expected value is

$$\mathbf{R}_k = \left[\begin{array}{c|c} \mathbf{R}_{v,0} + \mathbf{a}_{0,k}\mathbf{a}_{0,k}^{\mathsf{H}} + \mathbf{\Sigma}_0 & \mathbf{a}_{0,k}\bar{\alpha} \\ \hline \mathbf{a}_{0,k}^{\mathsf{H}}\alpha & \alpha^2 \end{array} \right]$$

Thus, form the ‘clean’ instantaneous estimates

$$\hat{\mathbf{\Psi}}_{00,k} = \hat{\mathbf{R}}_{00,k} - \hat{\mathbf{R}}_{01,k}\hat{\mathbf{R}}_{11,k}^{-1}\hat{\mathbf{R}}_{10,k} \quad \sim \quad \mathbf{R}_{v,0} + \mathbf{\Sigma}_0$$

and average them to obtain an estimate of $\mathbf{\Psi}_{00}$.

Disadvantage:

- Not general: assumes no noise on reference; can cancel at most p_1 interferers. With noise, subtraction introduces a bias
- Bias can be avoided, but for poor INR of the reference antenna, subtraction can become unstable

Second technique: spatial filtering with projections

- Estimate $\mathbf{a}_k$ from eigenvalue or Factor Analysis computations, and form a projection matrix:

$$\mathbf{P}_k := \mathbf{I} - \mathbf{a}_k(\mathbf{a}_k^{\mathrm{H}}\mathbf{a}_k)^{-1}\mathbf{a}_k^{\mathrm{H}} \qquad \text{Note: } \mathbf{P}_k\mathbf{a}_k = 0$$

- Apply the projection: $\hat{\mathbf{Q}}_k := \mathbf{P}_k\hat{\mathbf{R}}_k\mathbf{P}_k$.

  Ideally, the interferer is gone and $\hat{\mathbf{Q}}_k$ is equal to $\mathbf{P}_k\hat{\boldsymbol{\Psi}}_k\mathbf{P}_k$ where $\hat{\boldsymbol{\Psi}}_k$ is the interference-free data covariance.

- Average the results:

$$\hat{\mathbf{Q}} := \frac{1}{N}\sum_{k=1}^{N}\mathbf{P}_k\hat{\mathbf{R}}_k\mathbf{P}_k \;=\; \frac{1}{N}\sum_{k=1}^{N}\mathbf{P}_k\hat{\boldsymbol{\Psi}}_k\mathbf{P}_k$$

  The astronomical data is modified by the projections as well.

Second technique: spatial filtering with projections

- Let $\mathbf{C}_k = \mathbf{P}_k^\top \otimes \mathbf{P}_k$, then

$$\text{vec}(\hat{\mathbf{Q}}) \sim \underbrace{\left(\frac{1}{N}\sum_{k=1}^{N}\mathbf{C}_k\right)}_{\mathbf{C}} \text{vec}(\boldsymbol{\Psi})$$

- Apply correction: $\hat{\boldsymbol{\Psi}} := \text{unvec}(\ \mathbf{C}^{-1}\text{vec}(\hat{\mathbf{Q}})\ )$.

  $\hat{\boldsymbol{\Psi}}_{00}$ is the $p_0 \times p_0$ submatrix corresponding to the primary array.

Disadvantage:

- $\mathbf{C}$ can be ill-conditioned, e.g., for a stationary interferer ($\mathbf{a}_k$ constant), or interference entering only on a single telescope.
- $\mathbf{C}$ is quite large ($p^2 \times p^2$)

Third algorithm: spatial filter with reduced-size correction

- Previously, we solved

$$\hat{\boldsymbol{\Psi}} = \arg\min_{\boldsymbol{\Psi}} \| \operatorname{vec}(\hat{\mathbf{Q}}) - \mathbf{C}\operatorname{vec}(\boldsymbol{\Psi}) \|^2$$

  and then reduced $\hat{\boldsymbol{\Psi}}$ to size $p_0 \times p_0$ to obtain the estimate $\hat{\boldsymbol{\Psi}}_{00}$.

- Instead, we can use the known structure of $\boldsymbol{\Psi}$ and solve

$$\hat{\boldsymbol{\Psi}}_{00} = \arg\min_{\boldsymbol{\Psi}_{00}} \|\operatorname{vec}(\hat{\mathbf{Q}}) - \mathbf{C}\operatorname{vec}\left(\left[\begin{array}{c|c} \boldsymbol{\Psi}_{00} & \mathbf{0} \\ \hline \mathbf{0} & \boldsymbol{\Sigma}_1 \end{array}\right]\right)\|^2$$

- This is a standard Least Squares problem after separating the knowns from the unknowns:

$$\begin{aligned} \operatorname{vec}(\hat{\boldsymbol{\Psi}}_{00}) &= \arg\min_{\boldsymbol{\Psi}_{00}} \| \operatorname{vec}(\hat{\mathbf{Q}}) - [\mathbf{C}_1 \ \mathbf{C}_2] \begin{bmatrix} \operatorname{vec}(\boldsymbol{\Psi}_{00}) \\ \sigma_1 \end{bmatrix} \|^2 \\ &= \mathbf{C}_1^{\dagger}(\operatorname{vec}(\hat{\mathbf{Q}}) - \mathbf{C}_2\sigma_1) \end{aligned}$$

Third algorithm: spatial filter with reduced-size correction

Advantages:

- Can still work for stationary interferers ($\mathbf{a}_k$ constant): $\mathbf{C}_1$ is tall and expected to have full column rank
- Same advantage in case only one of the primary antennas is contaminated ($\mathbf{a}_{k,0}$ has only one nonzero entry).

  Without reference antenna, the projection is always the same and cannot be corrected.
- Unlike the subtraction technique, can project more interferers than number of reference antennas (subject to a non-stationary $\mathbf{a}_k$).
- With Factor Analysis, can work on uncalibrated arrays.

Disvantage:

- $\mathbf{C}_1$ is still a quite large matrix to invert.

Fourth algorithm: Extended Factor Analysis

Factor Analysis is a numerical technique that generalizes the Eigenvalue Decomposition: given $\hat{\mathbf{R}}$ with model

$$\mathbf{R} = \mathbf{A}\mathbf{A}^H + \mathbf{\Sigma}$$

(low rank plus diagonal), find estimates for $\mathbf{A}$ (low-rank factor) and $\mathbf{\Sigma}$ (diagonal).

We extended this to low-rank plus block-diagonal, or even more general, here

$$\mathbf{R} = \mathbf{A}\mathbf{A}^H + \left[\begin{array}{c|c} \mathbf{\Psi}_{00} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{\Sigma}_1 \end{array}\right]$$

where $\mathbf{\Psi}_{00}$ is a full matrix and $\mathbf{\Sigma}_1$ is a diagonal. The EFA algorithm gives directly $\mathbf{A}$ and $\mathbf{\Psi}_{00}$ (no calibration needed). The estimate for $\mathbf{\Psi}_{00}$ is the RFI-free visibility estimate. Compute the decomposition for every $\hat{\mathbf{R}}_k$, then average.

Properties:

- No projections, no prior calibration needed
- No assumptions on stationarity of interferers
- Need more reference antennas than interferers

Simulations

Scenario

- $p_0 = 5$ primary antennas (telescopes) and $p_1 = 2$ reference antennas
- Source: $\mathrm{SNR}_0 = -20$ dB with respect to each primary array element, and $\mathrm{SNR}_1 = -40$ dB towards the reference antenna.
- Interferer: various INRs towards the primary and reference array (INR_0 and $\mathrm{INRdiff} = \mathrm{INR}_1 - \mathrm{INR}_0$), varying $\mathbf{a}_k$ for each data block.

Simulations

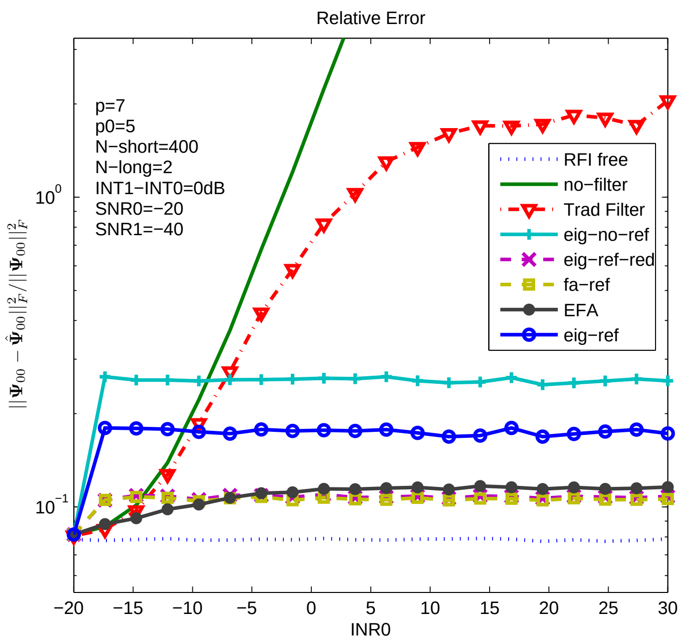
Algorithms

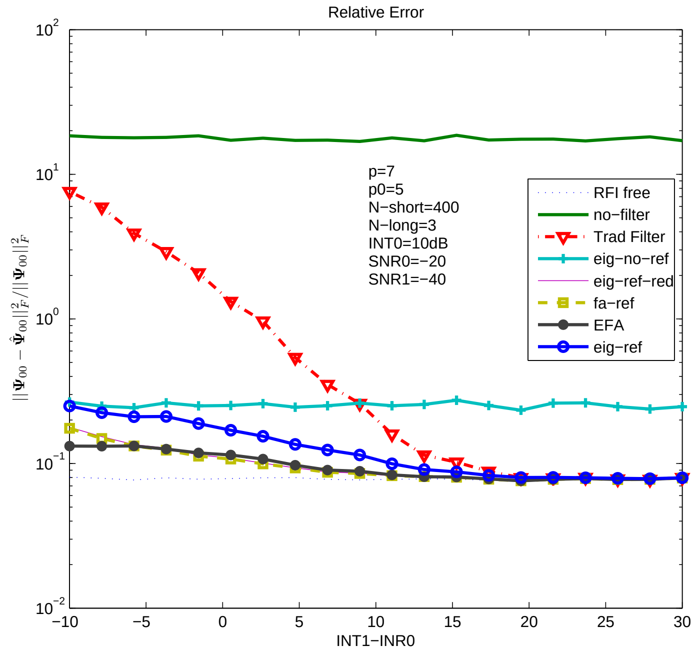
- **First method:** "subtraction"; no rank detection
- **Second method:** spatial filtering ("eig ref"); rank detection
- **Third method:** improved spatial filtering ("eig ref red" and "fa ref"); rank detection
- **Fourth method:** extended factor analysis ("EFA"); rank detection
- For comparison, spatial filtering without reference antenna ("eig-no-ref"), interference-free ("RFI free"), no filtering ("no-filter")

Shown is the relative Mean Squared Error of the estimated filtered covariance compared to the theoretical value $\boldsymbol{\Psi}_{00} = \mathbf{R}_{v,0} + \boldsymbol{\Sigma}_0$, normalized by $\boldsymbol{\Psi}_{00}$.

Simulations

Very short long-term integration (=2)





(*a*) relative MSE as function of interferer power at the reference antenna,

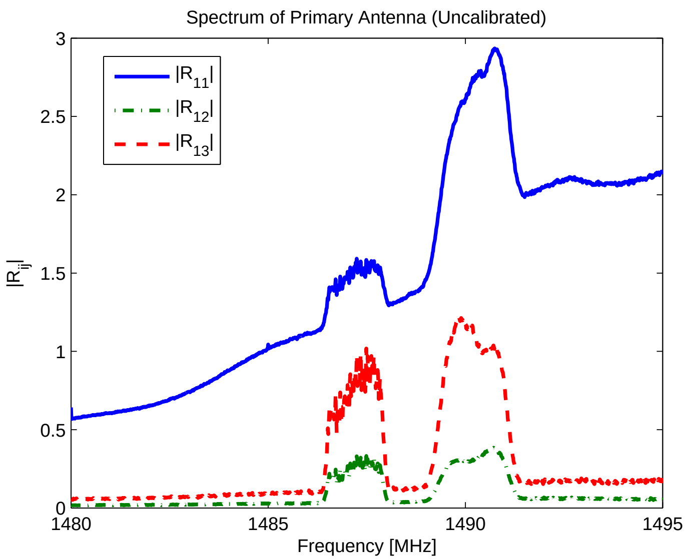
(*b*) relative MSE as function of the interferer power difference

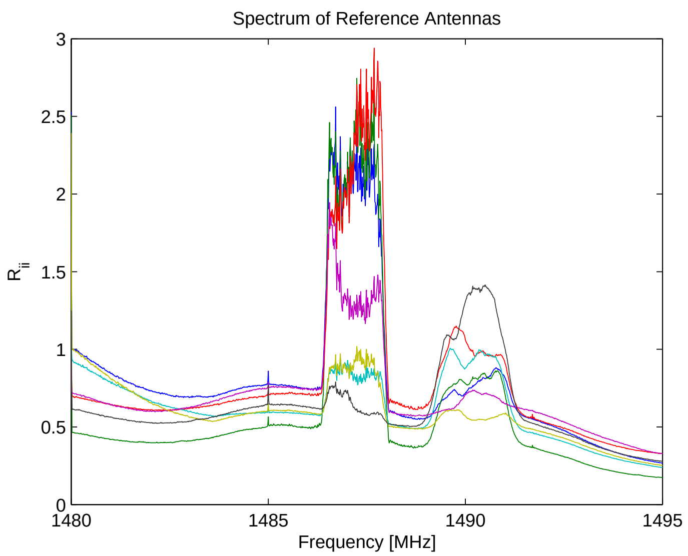
- Flat MSE for varying INRs (desirable)
- "Eig ref red", "fa ref" and "EFA" are best (similar performance)

Experiment: Afristar

- WSRT is the primary array ($p_0 = 3$ telescopes used).
- Reference antenna array is focal plane array on one telescope pointed to zenith ($p_1 = 27$ elements used).

Experiment: Afristar



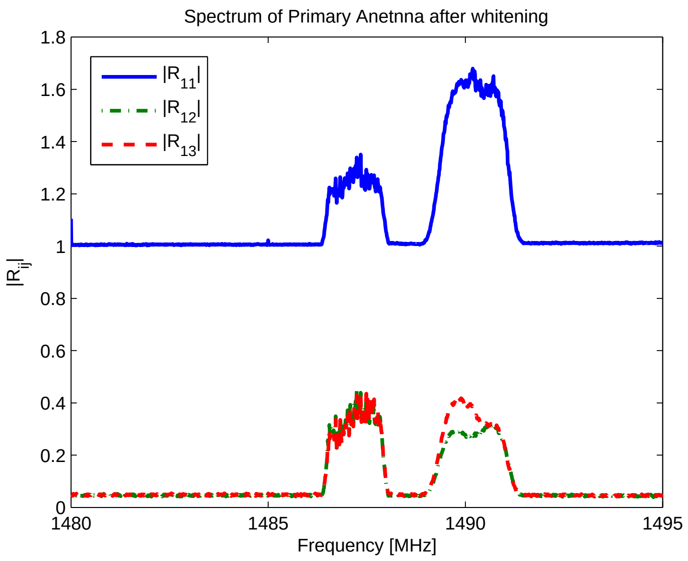


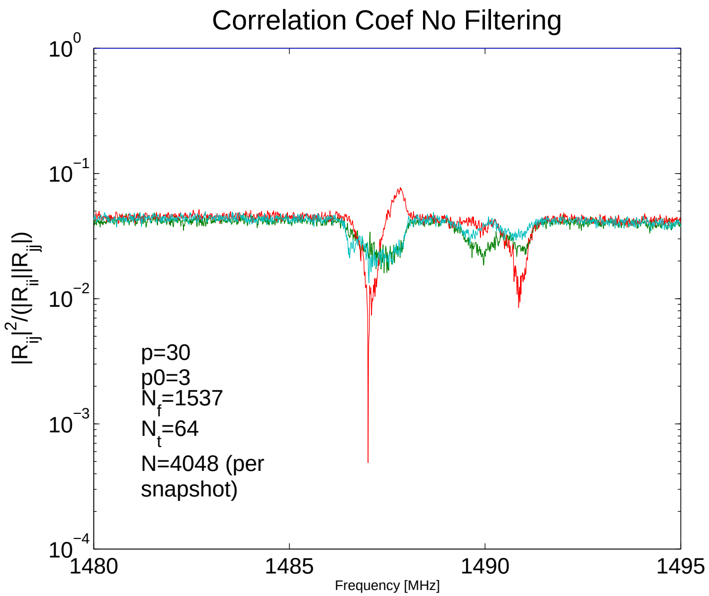
Observed spectrum from (*a*) the primary telescopes (*b*) 6 of the reference antennas

Telescopes are uncalibrated

TUDelft

Experiment: Afristar

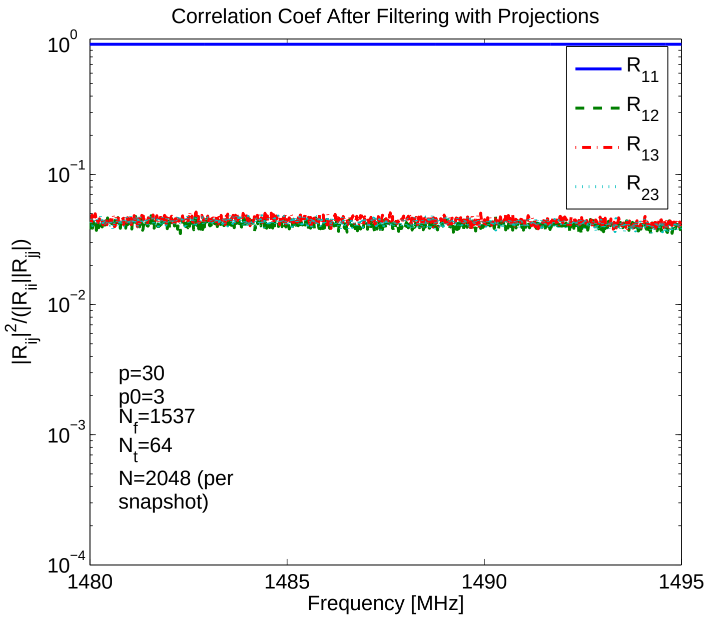


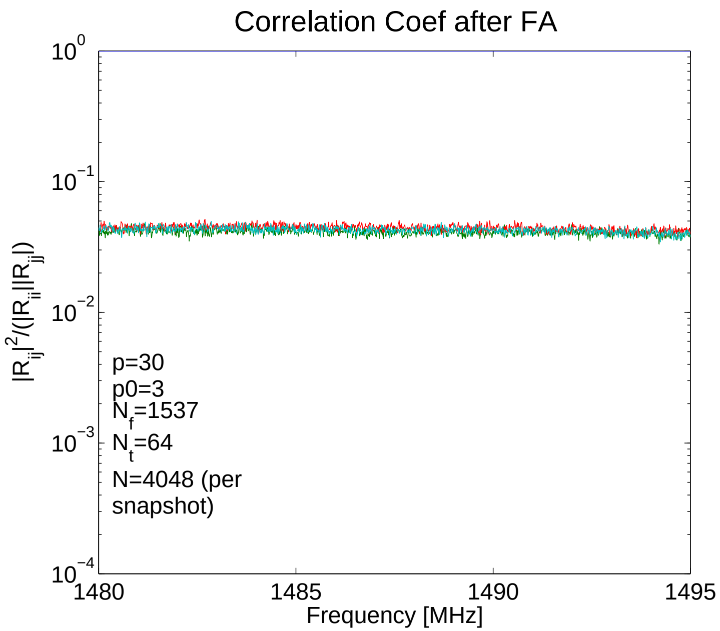


(a) Spectrum of primary antenna after whitening (calibrated using Factor Analysis),

(b) Average normalized Correlation Coefficients

Experiment: Afristar





Averaged Normalized Correlation Coefficients

(a) after improved spatial filtering using Factor Analysis, (b) after using EFA.

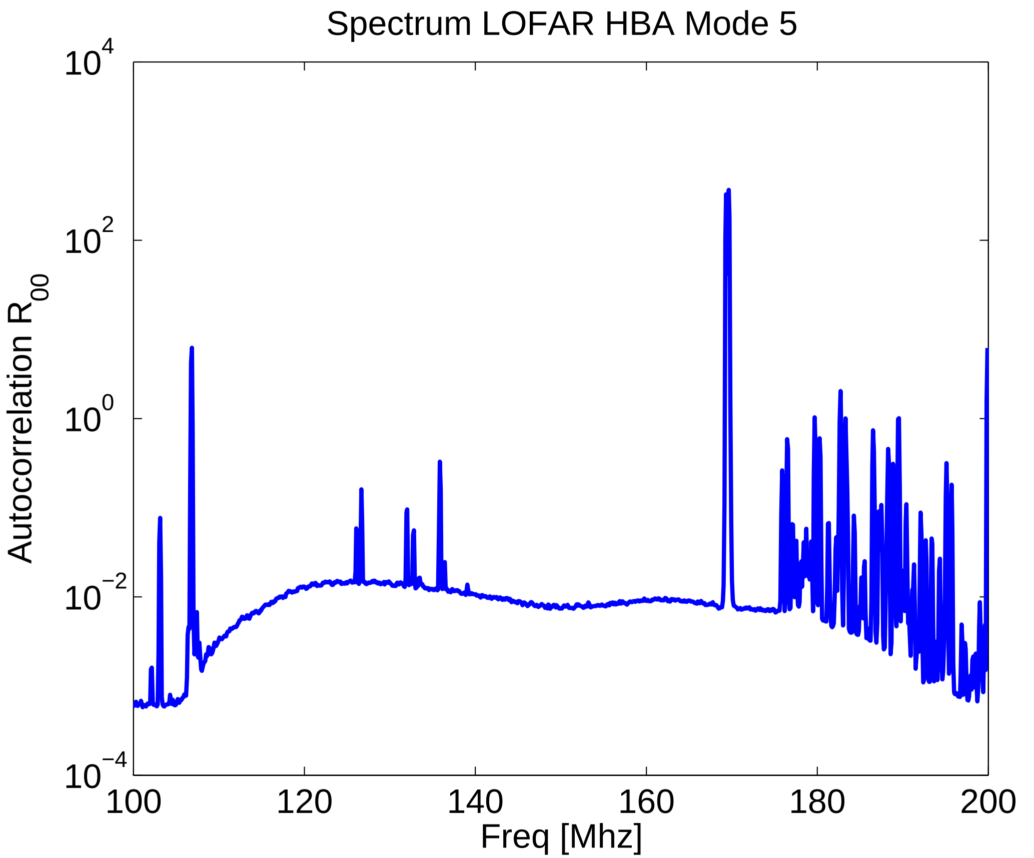
Experiment: LOFAR RS409

RS409 is a LOFAR station close to TV transmitters. In particular it receives interference from DVB-T.

Experiment

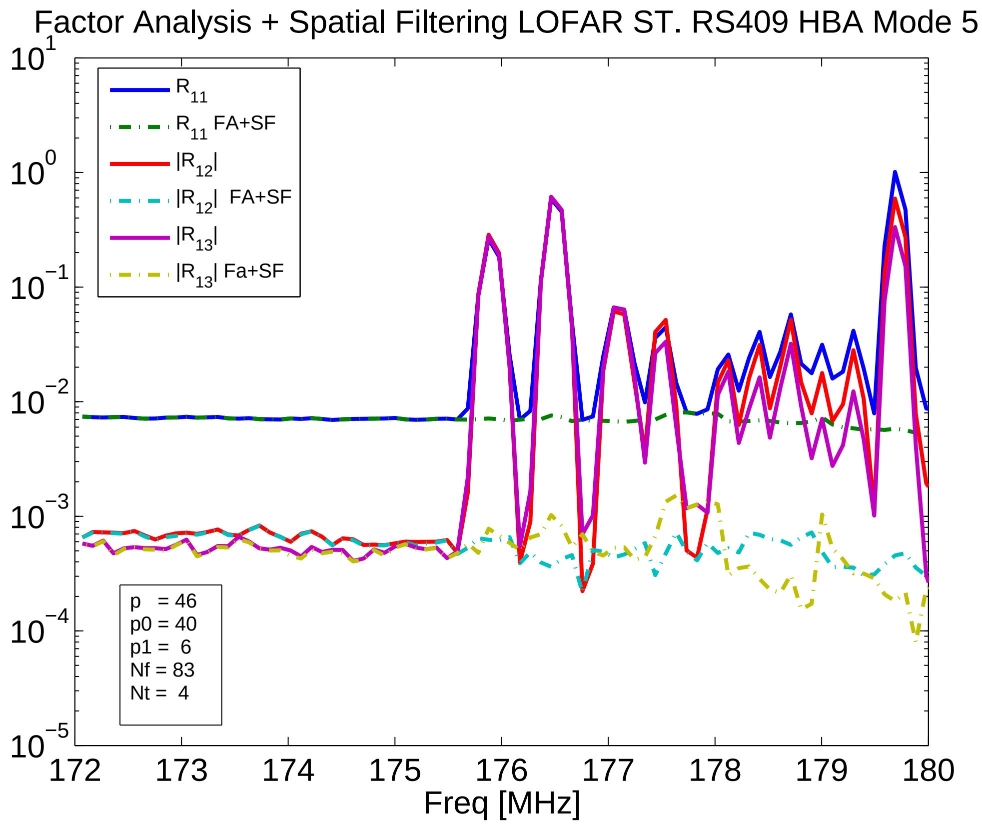
- all HBA tiles are tracking Cygnus A. $p_0 = 40$ tiles are "primary", and $p_1 = 6$ tiles are "reference".
- Data in HBA mode 5 (110-190 MHz); sampling at 200 MHz, direct dump of 100 sec of time-domain data from TBB boards.
- Preprocessing: split into 1024 frequency channels, short-time avaraging to 19 ms (available: 4 long-term covariance estimates)

Experiment: LOFAR RS409



Autocorrelation spectrum

Experiment: LOFAR RS409

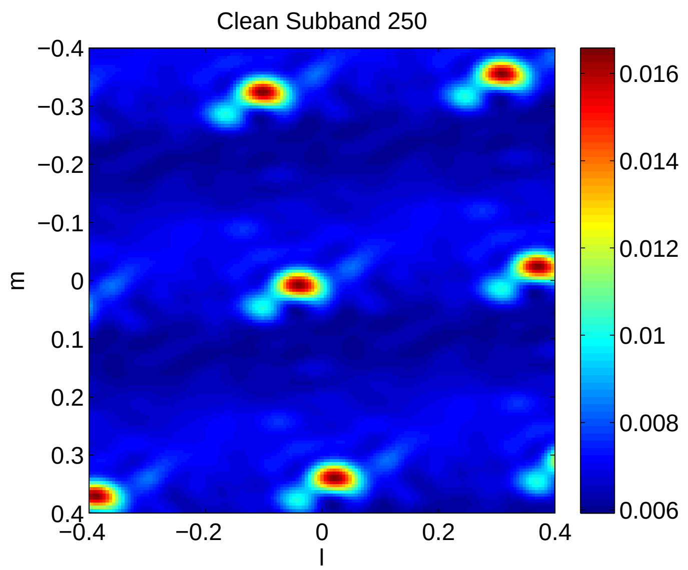


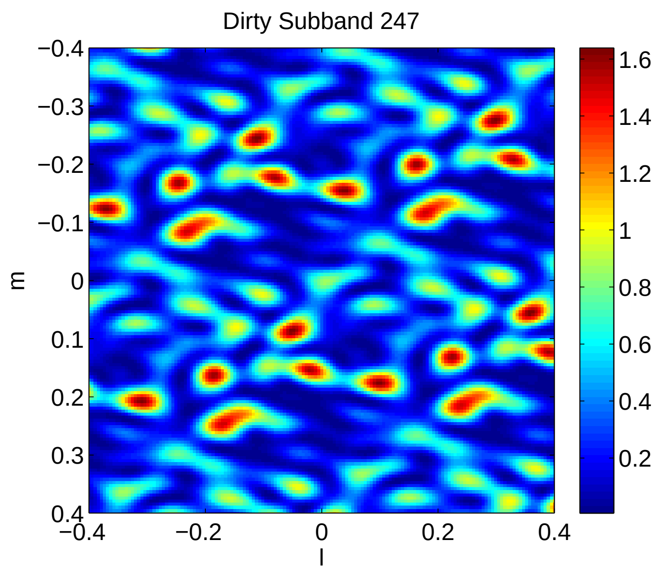
Filtered spectrum: (solid): unfiltered; (dashed) improved spatial filter with Factor Analysis

Performance of filtering somewhat limited due to violated assumption: reference antennas were also pointed at astronomical signal.

TUDelft

Experiment: LOFAR RS409

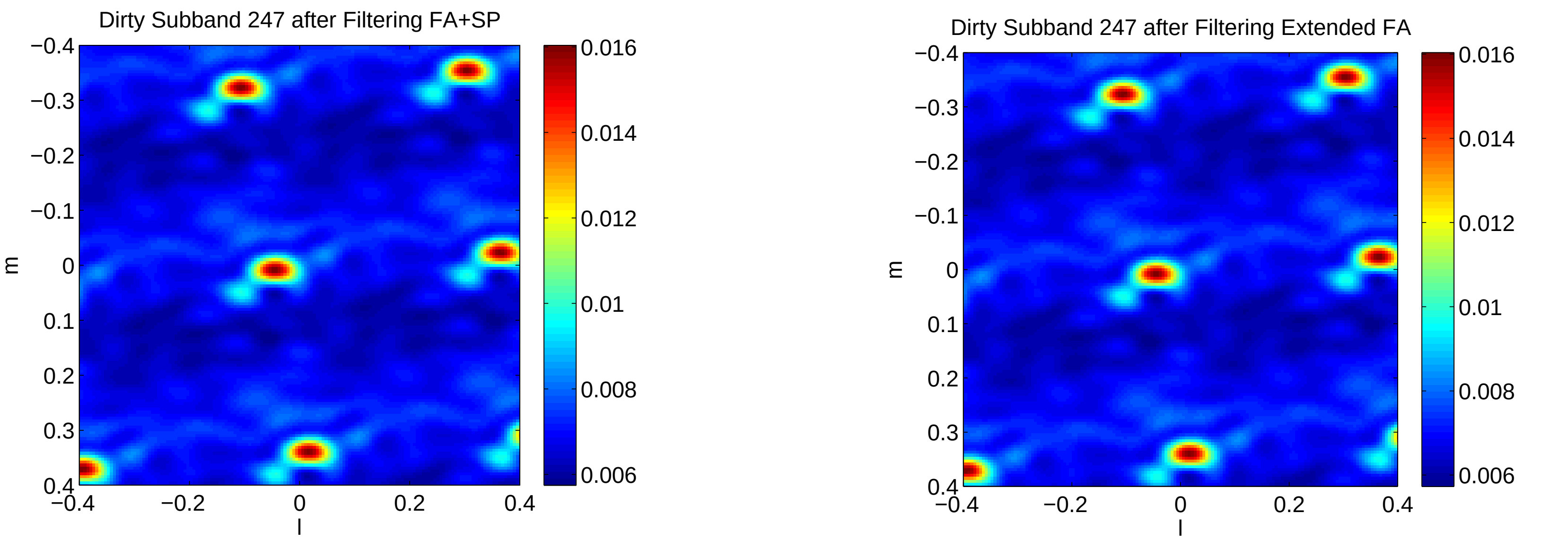




(a) Clean (at 175.59 MHz) (b) Contaminated (at 175.88 MHz)

TUDelft

Experiment: LOFAR RS409



(a) Improved spatial filter with Factor Analysis (b) Extended Factor Analysis

Conclusions

- Reference antennas give powerful additional information
- Factor Analysis and Extended Factor Analysis appear to be robust over a range of INRs and INR differences
- For LOFAR, need extensions to work on interference cancellation at station level (adaptive beamforming) whereas covariance data is needed at instrument level

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Thanks to Albert-Jan Boonstra, Stefan Wijnholds, Menno Norden (ASTRON) for suggestions and providing experimental data.