New Definition of Resolution Based on Correlation Coefficient ${\tt occocccco}$

Resolution in Deconvolution

Statistical Evaluation of Two Image Deconvolution Algorithms

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New Definition of Resolution Based on Correlation Coefficient ${\tt occocccco}$

Resolution in Deconvolution

Outline

The Resolution of an Image

- Two Classical Criteria
- The Problems in Classical Definition
- Deconvolution and Super Resolution

New Defintion of Resolution Based on Correlation Coefficient

- Correlation Coefficient and Coherence
- The Relationship Between Correlation and PSF
- New Definition of Resolution

3 Resolution in Deconvolution

- CLEAN
- Wiener Deconvolution

New Definition of Resolution Based on Correlation Coefficient

Resolution in Deconvolution

The Higher the Resolution, the Better the Image

M100 Galactic Nucleus

Hubble Space Telescope Wide Field Planetary Camera 2



Wide Field Planetary Camera 1

Wide Field Planetary Camera 2

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Airy Pattern

Airy Pattern : The PSF of a circular aperture telescope.



The Airy pattern could be described by the following equation :

$$I(\theta) = I_0 \left(\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right)^2 = I_0 \left(\frac{2J_1(x)}{x}\right)^2$$

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Two Classical Criteria

The Rayleigh Criterion

Two point sources are regarded as just resolved when the principal diffraction maximum of one image coincides with the first minimum of the other.



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Two Classical Criteria

Full Width at Half Maximum, FWHM

For those PSFs which don't have the similar structure with Airy Pattern, the FWHM is usually used to define their resolution.



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Two Classical Criteria

Full Width at Half Maximum, FWHM

In VLBI, a Gaussian beam is used to fit the dirty beam, and the FWHM area is usually displayed in a map to demonstrate the resolution. Clean map. Array: EHMNNSTGNFLPKBOMG 2215+020 at 1.634 GHz 1997 Sep 14



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The Problems in Classical Definition

Imaging and Convolution

The Mathematical Description of Imaging

 $D = I \otimes P + N$

where D is an observed image, I is an original image, P is the PSF, N is the noise.

In the classical definition of resolution, only PSF is used, whereas a real observed image is not only influenced by PSF, but also by original image and noise.

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The Problems in Classical Definition

The Stochastic Property of a Physical Image



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The Problems in Classical Definition

Two Sources under Different Contrasts

The PSF is a normal Gaussian function, with FWHM equals 2.3. The distance between two sources is 3. In left figure, the total photon number for both sources are 10^5 . In right figure, the total photon numbers are 10^5 and 10^3 respectively.



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The Problems in Classical Definition

A Real Astronomical Image



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Deconvolution and Super Resolution

An Example of Maximum Entropy Method



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Resolution in Deconvolution

Correlation Coefficient and Coherence

The Correlation in Real World



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Correlation Coefficient and Coherence

The Physical Meanings of Correlation Coefficient





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Resolution in Deconvolution

Correlation Coefficient and Coherence

The Physical Meanings of Correlation Coefficient





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Correlation Coefficient and Coherence

The Pearson's Defintion of Cross-Correlation Coefficient

$$\rho_{X,Y} = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y} \\ = \frac{E[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y}$$

Where *E* is the operator of expection, *cov* is the operator of covariance.



Karl Pearson

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The Relationship Between Correlation and PSF

An Original Image in One Dimension



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The Relationship Between Correlation and PSF

An Original Image in One Dimension



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The Relationship Between Correlation and PSF

Correlation Coefficient Curves and Matrix



Resolution in Deconvolution

The Relationship Between Correlation and PSF

Correlation Coefficient Curves and Matrix



Correlation Coefficient Matrix

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The Relationship Between Correlation and PSF

The Influence by a Telescope



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Correlation Coefficient Matrix



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Resolution in Deconvolution

New Definition of Resolution

New Definiton of Resolution Based on Correlation Coefficient

Definition

A correlation coefficient matrix (tensor) can be derived from a serier of observed images targetting the same area. Estimate an standard variance σ in an region without any sources. In a Convidence Level(For example 3σ), find the resolvable boudary for a fixed point.



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CLEAN

A Sample



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Correlation Coefficient Matrix

Correlation Coefficient Matrix



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Correlation Coefficient Lines



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Correlation Coefficient Lines

Correlation Coefficient Matrix of Residual Images



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Wiener Deconvolution

A Sample with SNR=1.3



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Wiener Deconvolution

Correlation Coefficient Matrix of Observed Images

Cross-Correlation Coefficient Matrix of Observed Images



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Wiener Deconvolution

Correlation Coefficient Matrix of Deconvolved Images



Cross-Correlation Coefficient Matrix of Deconvolved Images

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Wiener Deconvolution

Correlation Coefficient Lines



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Wiener Deconvolution

A Sample with SNR=9300



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Wiener Deconvolution

Correlation Coefficient Matrix of Observed Images

Cross-Correlation Coefficient Matrix of Observed Images



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Wiener Deconvolution

Correlation Coefficient Matrix of Deconvolved Images





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Wiener Deconvolution

Correlation Coefficient Lines



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New Definition of Resolution Based on Correlation Coefficient

Resolution in Deconvolution

Wiener Deconvolution



- We propose a statistical defition of resolution which is based cross-correlation coefficient. It is not only suitable for real observed images, but also for deconvolved images.;
- With new definition of resolution, we evaluate the CLEAN and Wiener deconvolution algorithms.

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