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Density structures caused by supersonic turbulent motions

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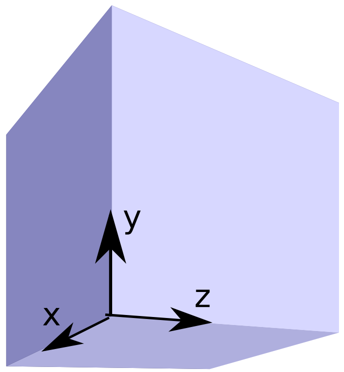
Motivation

- Understanding turbulence
- Numerical simulations: isolate turbulence
- Statistical properties of turbulence:

  One-point: e.g. PDF

  Two-point : e.g. power spectrum

Turbulence in a box :



- continuity equation
- Euler equation
- isothermal $p = c_S^2 \varrho$
- FLASH 4 with HLLR solver
- homogeneous grid
- 3D, periodic boundary conditions
- initial conditions: $v(r, t=0) = 0, \varrho(r, t=0) = \varrho_0$

Flash Center
for computational science

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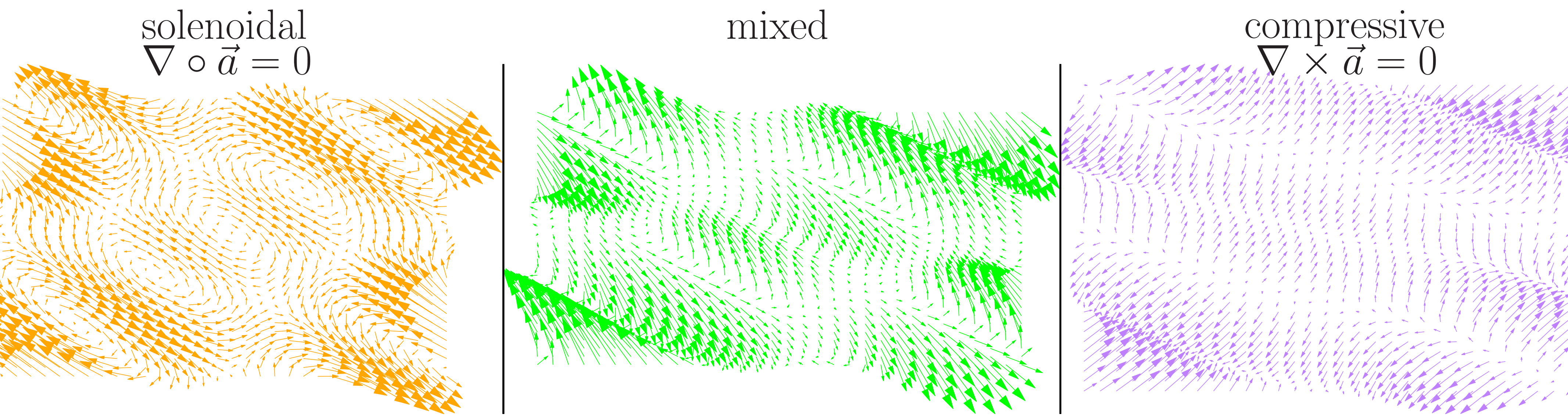
Forcing routine

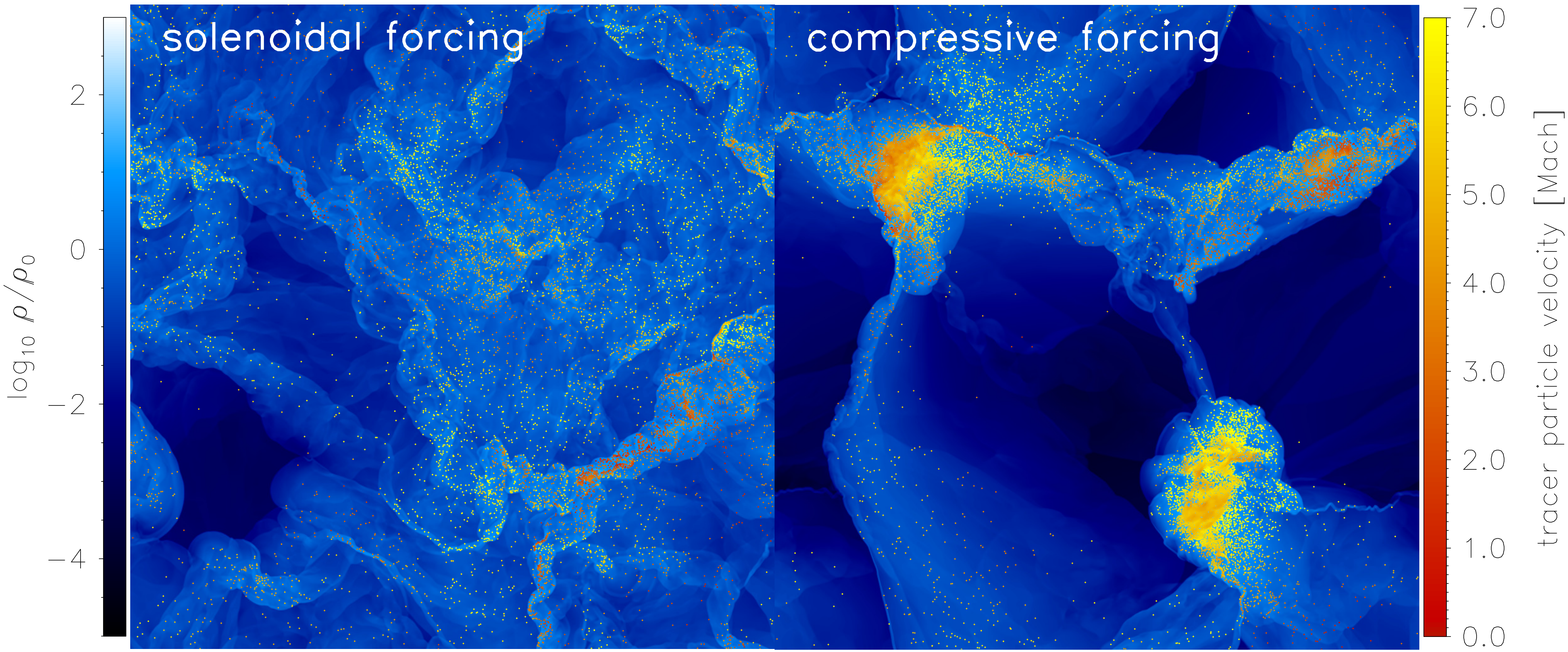


Ornstein-Uhlenbeck process

$$\mathrm{d}\mathbf{F}(\mathbf{k},\, t) = g\left[-\mathbf{F}(\mathbf{k},\, t)\frac{\mathrm{d}t}{T_{ac}} + \mathbf{F}_0\left(\frac{2\sigma^2(\mathbf{k})}{T_{ac}}\right)^{1/2}\boldsymbol{\mathcal{P}}_\zeta \mathrm{d}\boldsymbol{\mathcal{W}}\right]$$

→ forcing field varies smoothly in space and time

(Schmidt W., et al., 2009, A&A, 494, 127)



(Konstandin L., et al. 2012)

- *solenoidal* forcing is more space filling, whereas *compressive* yields larger voids and stronger shocks

One point statistic: density PDF

- $\rho(t_{i+1}) = \rho(t_i)\mathcal{M}^2 \longrightarrow p(s)ds = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(s-\mu_s)^2}{2\sigma^2}\right)$

PDFs are log-normal distributed

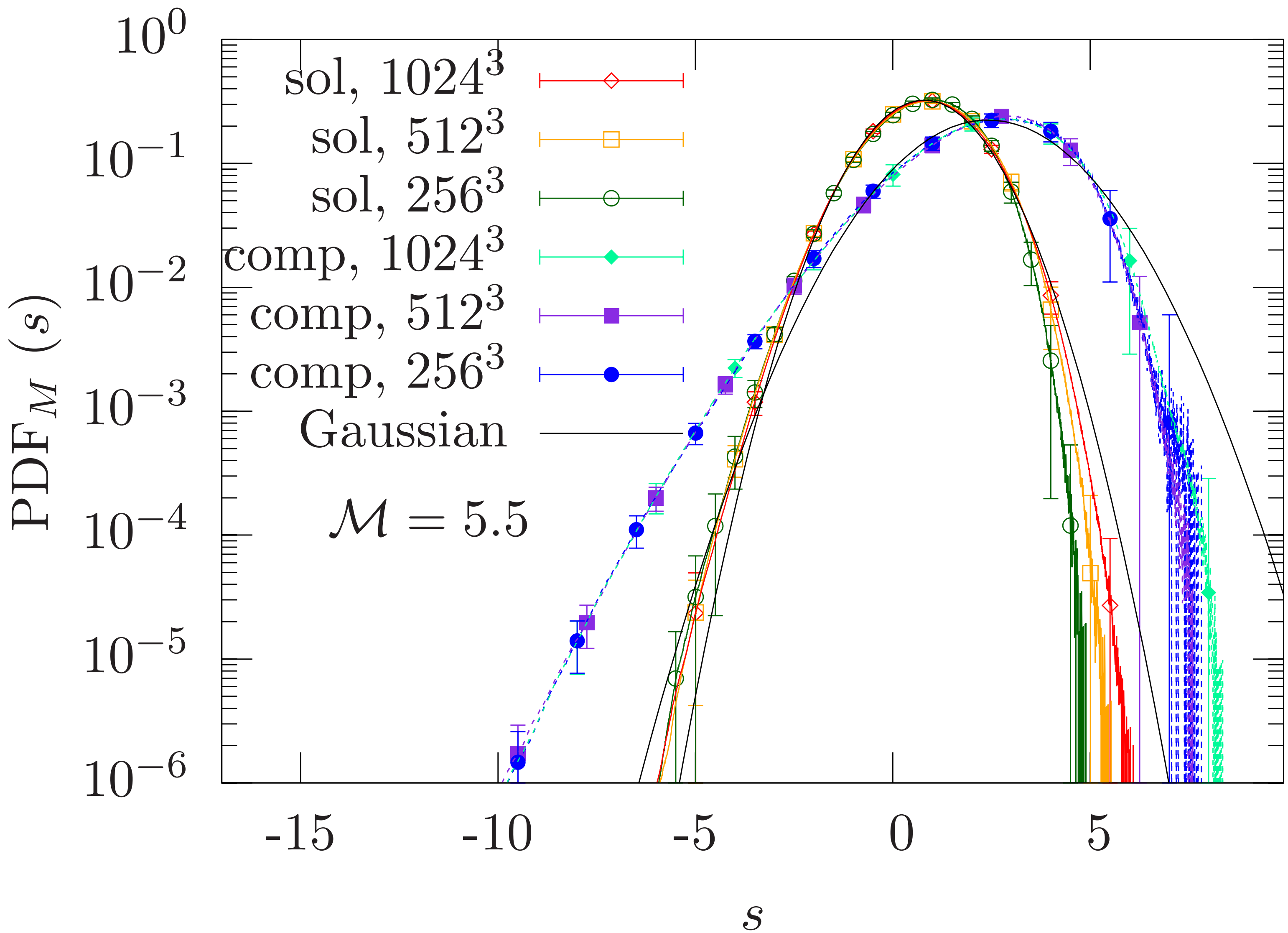
(Passot T.,Vasquez-Semadeni E., 1994, Phys. Rev. E, 58, 4)

- Log normal $\longrightarrow \sigma_\rho^2 = \mu_\rho^2(\exp(\sigma_s^2) - 1)$

with $s = \log(\rho)$, $\mu_\rho = 1$

- Passot 1994 : $\sigma_\rho \approx \sigma_s \approx \mathrm{b}\mathcal{M}$

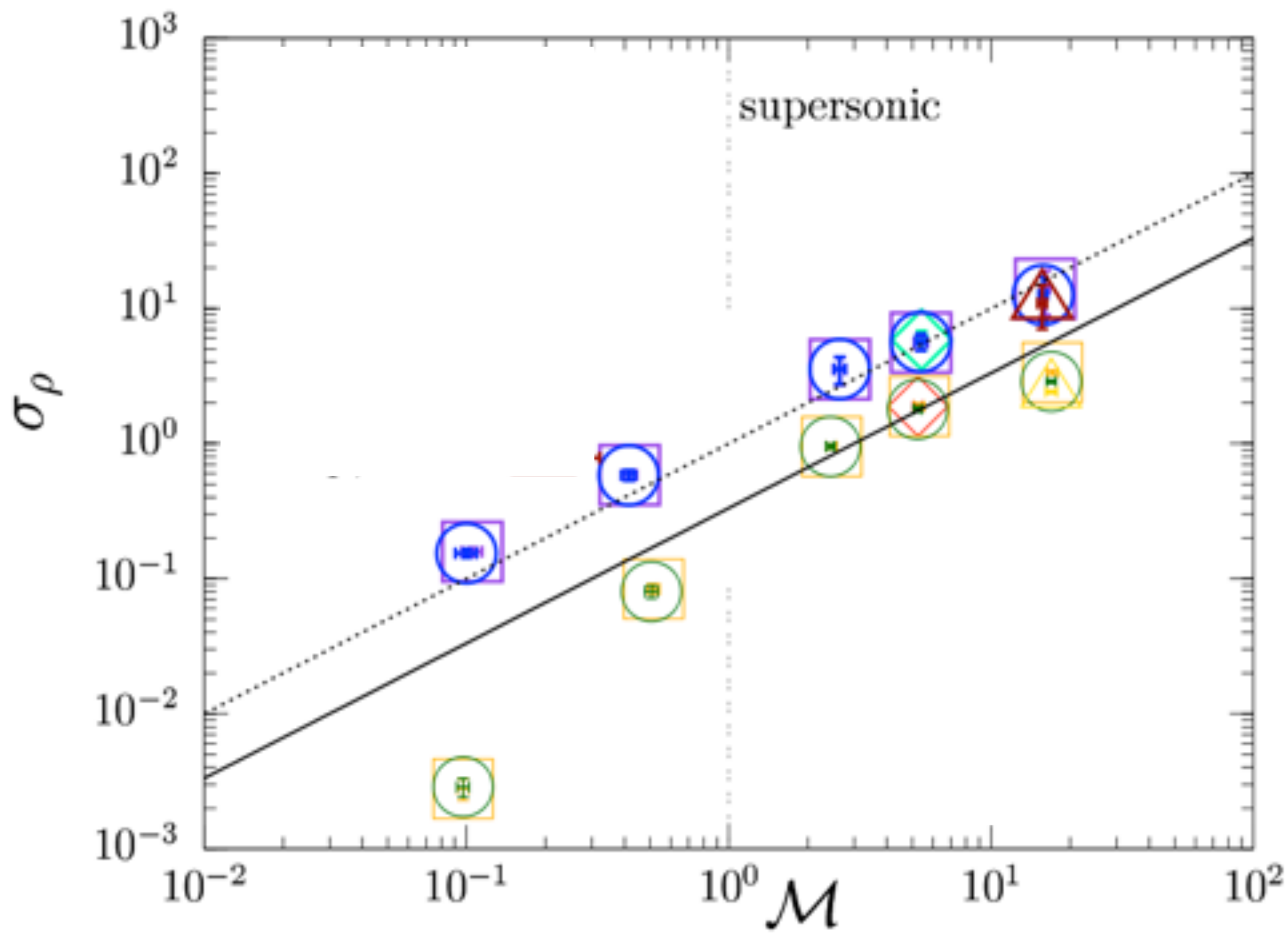
One point statistic: density PDF



- deviations from log-normal are caused by mass-conservation.

(Hopkins 2013)

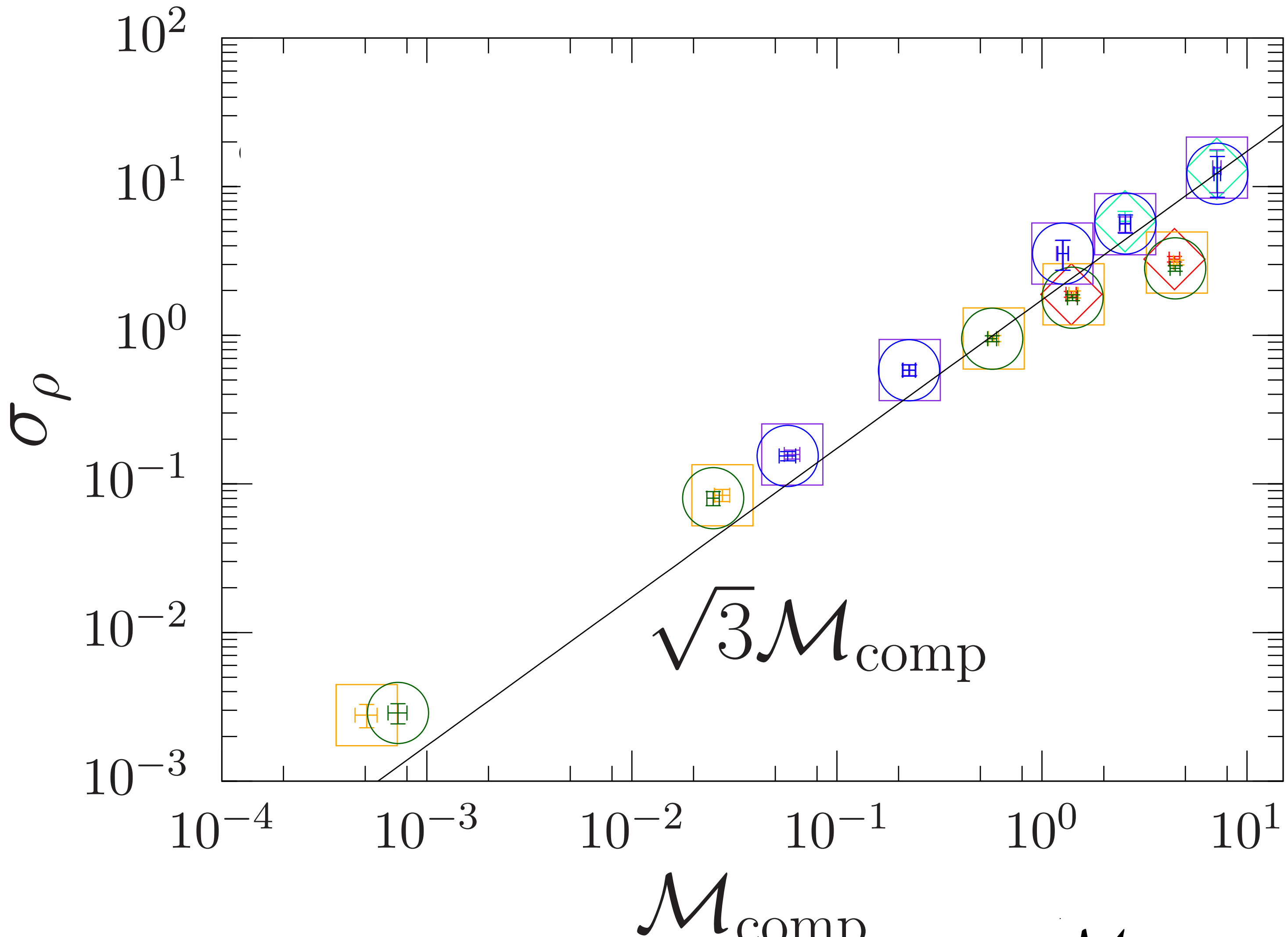
The variance of density PDF



$$\sigma_\rho \approx b\mathcal{M}$$

- The b-parameter depends on the forcing mechanism

The variance of density PDF



- The b-parameter is a measure of $\frac{\mathcal{M}_{\mathrm{comp}}}{\mathcal{M}}$

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Two point statistics:
The density power spectrum

Richardson-Obukhov: Energy-cascade

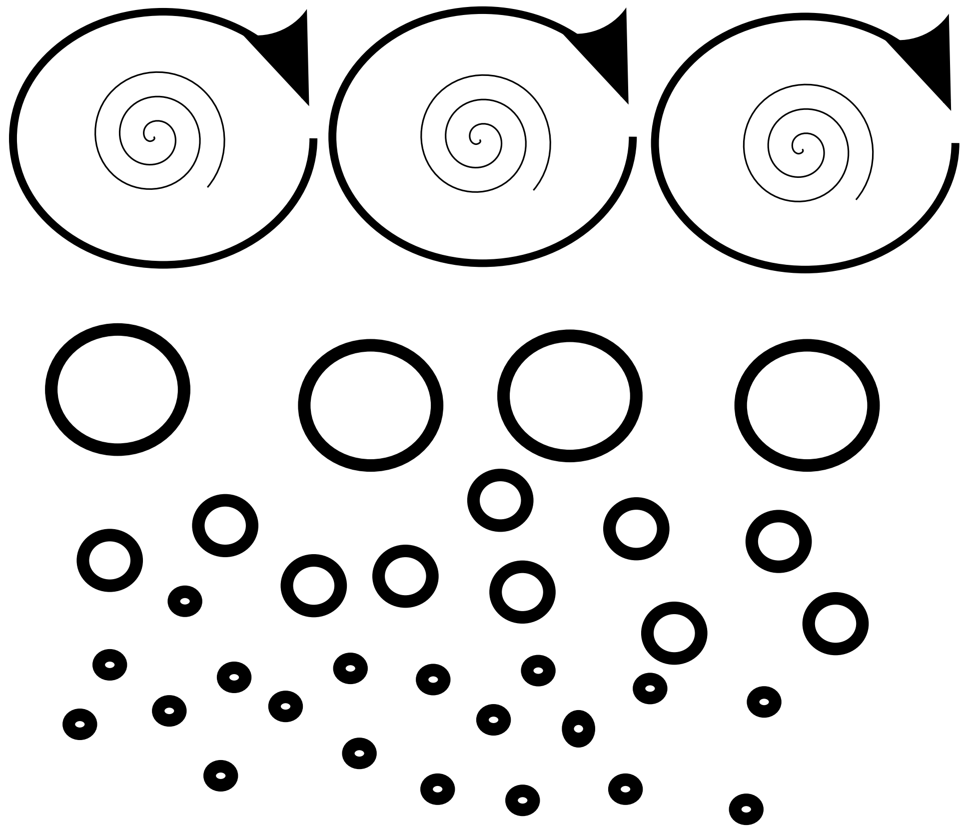
large scales | F | energy injection

↓

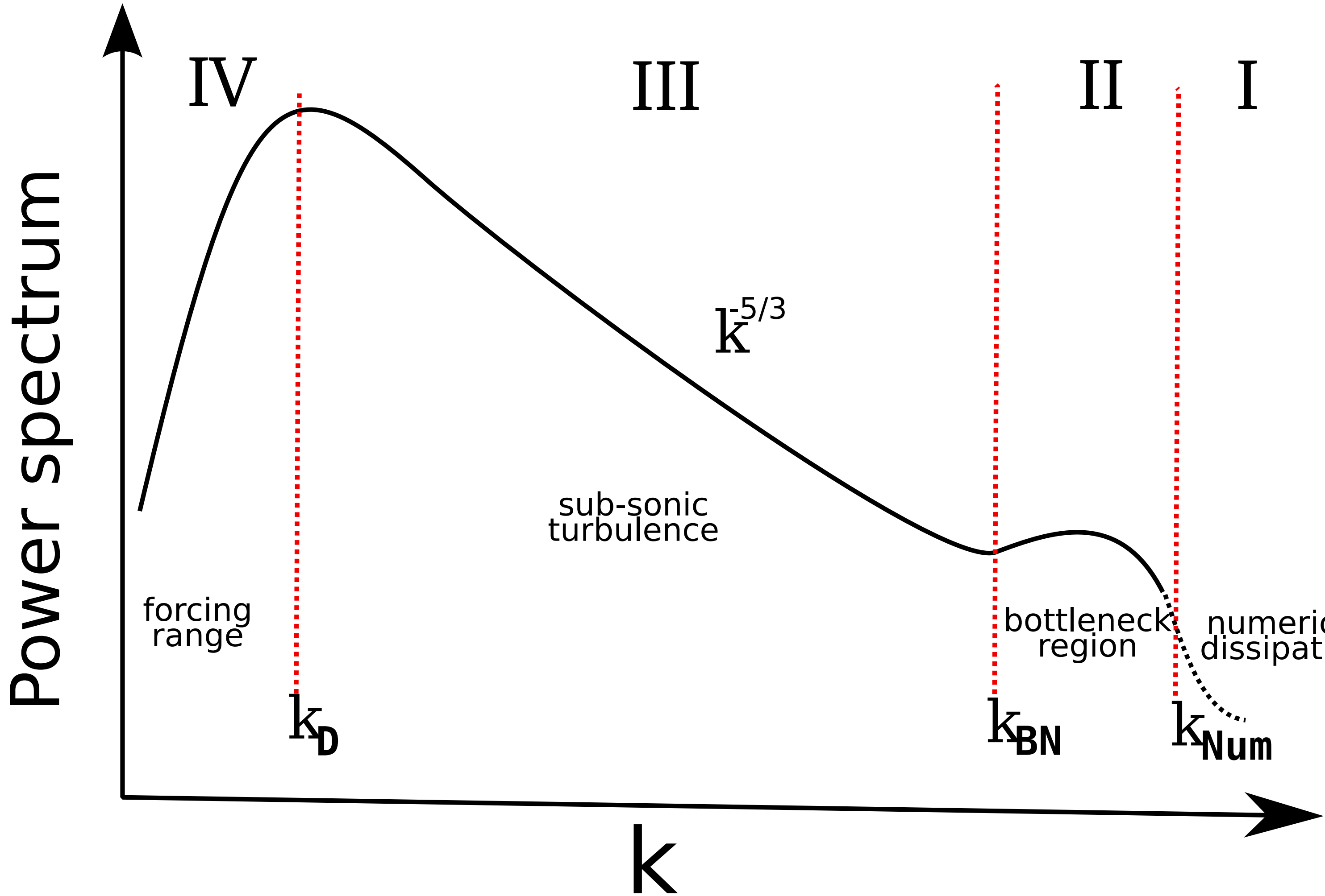
| E | kinetic

↓

small scales | E | internal



The density power spectrum



$$\mathcal{D}(k,\, t_i,\, \mathcal{M})\mathrm{d}k = 4\pi k^2 \hat{\rho}(k,\, t_i,\, \mathcal{M}) \cdot \hat{\rho}^*(k,\, t_i,\, \mathcal{M})\, \mathrm{d}k$$

Why the density power spectrum?

- Weizsaecker1951

$$\rightarrow \quad \rho(\ell) \propto \Theta(\ell)\ell^{-3\gamma} \propto \Theta(\ell)\ell^{D-3}$$

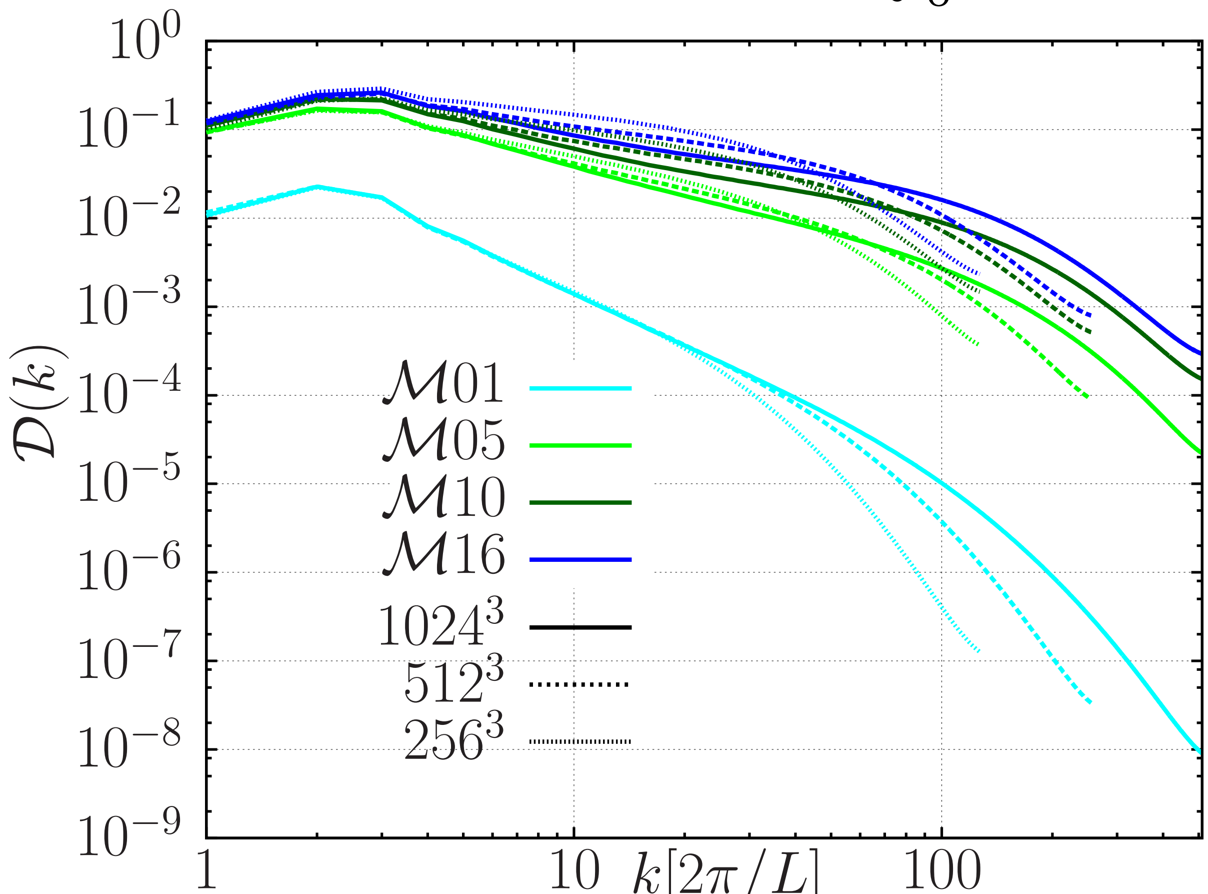
FT

γ , degree of compression
D , fractal dimension
f , volume filling factor

$$\rightarrow \quad \rho(k) = \Gamma(1-3\gamma)k^{-(1-3\gamma)}$$

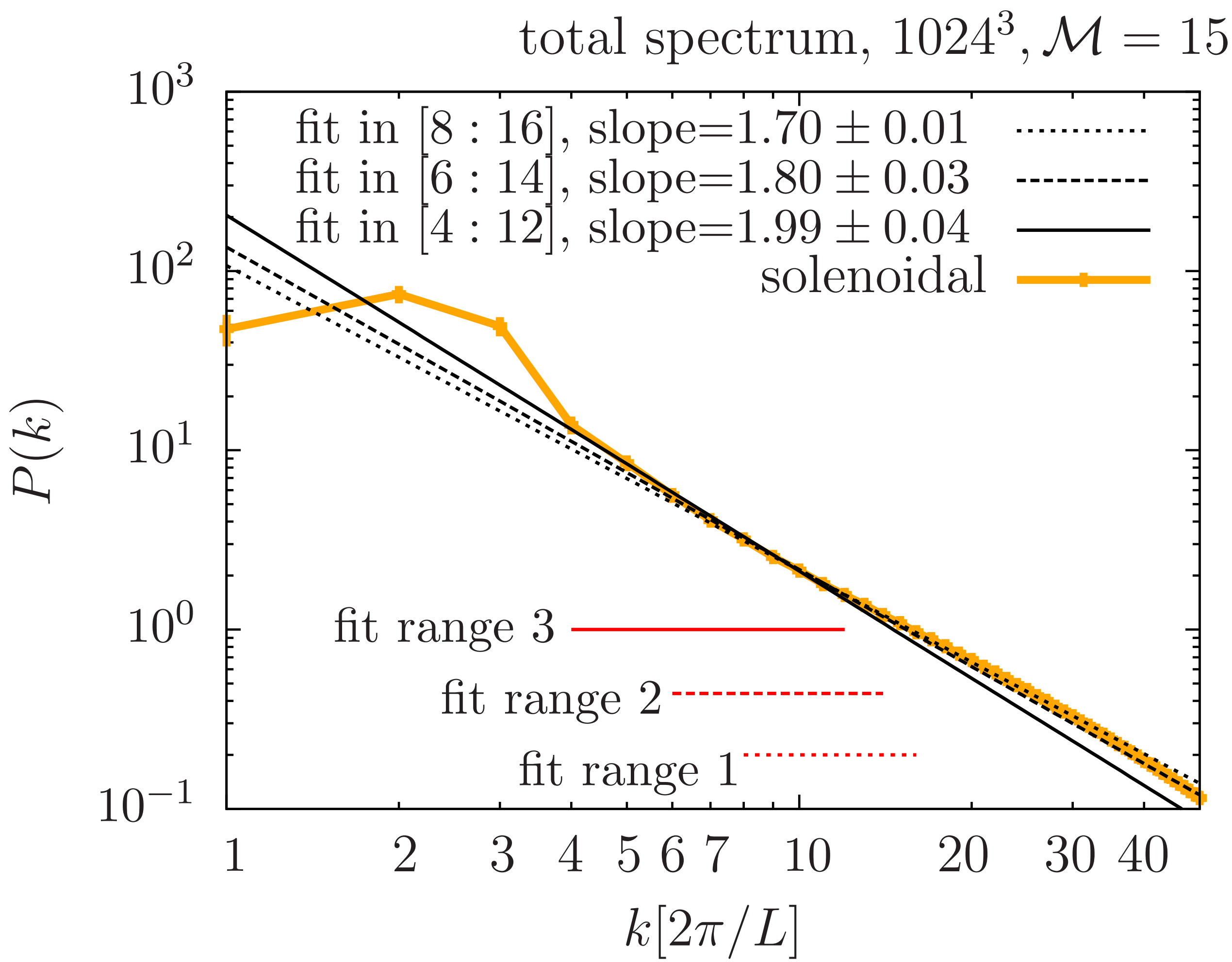
Two point statistic: density power spectrum

The Parseval theorem $\sigma_\rho^2 + \mu_\rho^2 = \int_0^\infty \mathcal{D}(k)\,\mathrm{d}k$



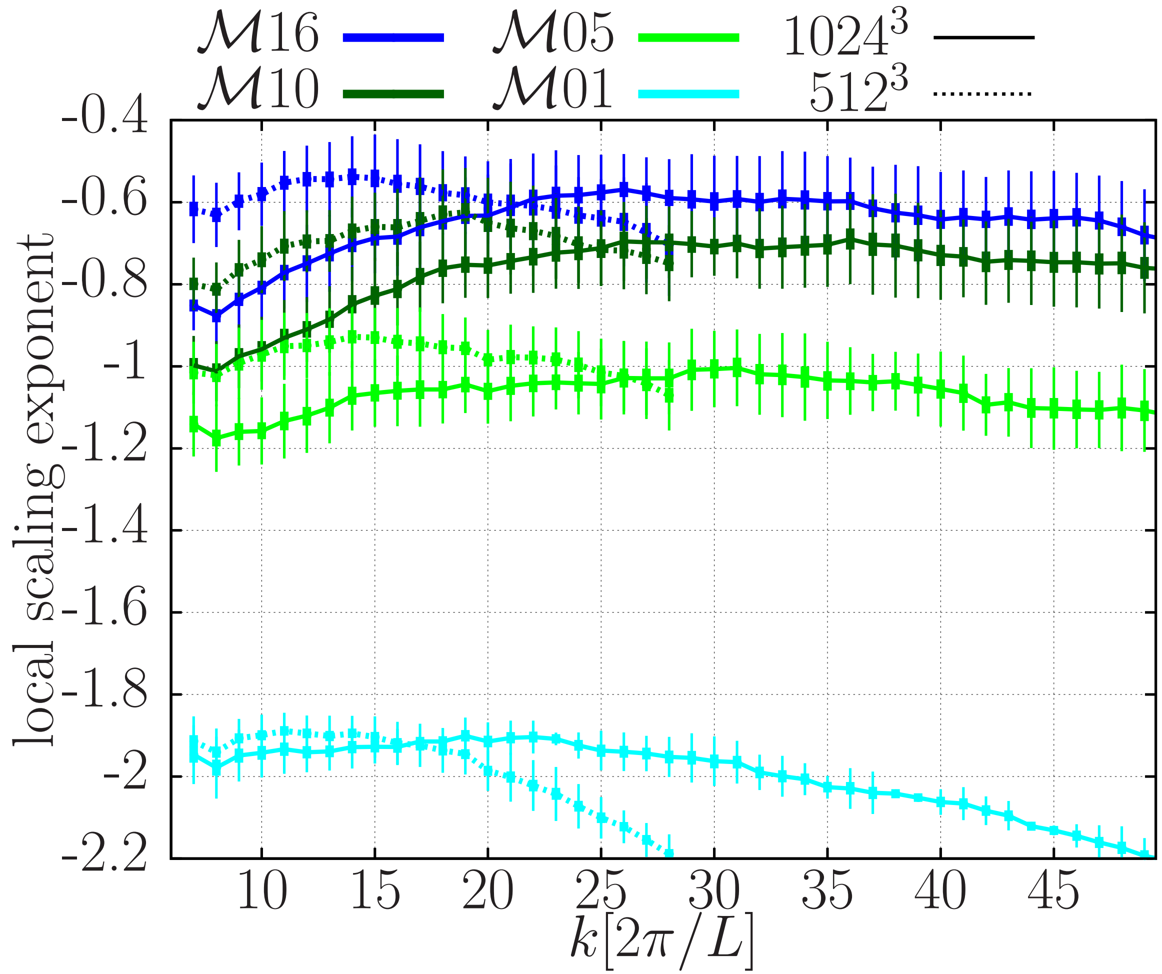
- increasing $\mathcal{M} \longrightarrow$ spectra get shallower

Caveats measuring the scaling exponent



- The slope depends on the fitting range
- Develop a hierarchical fitting technic

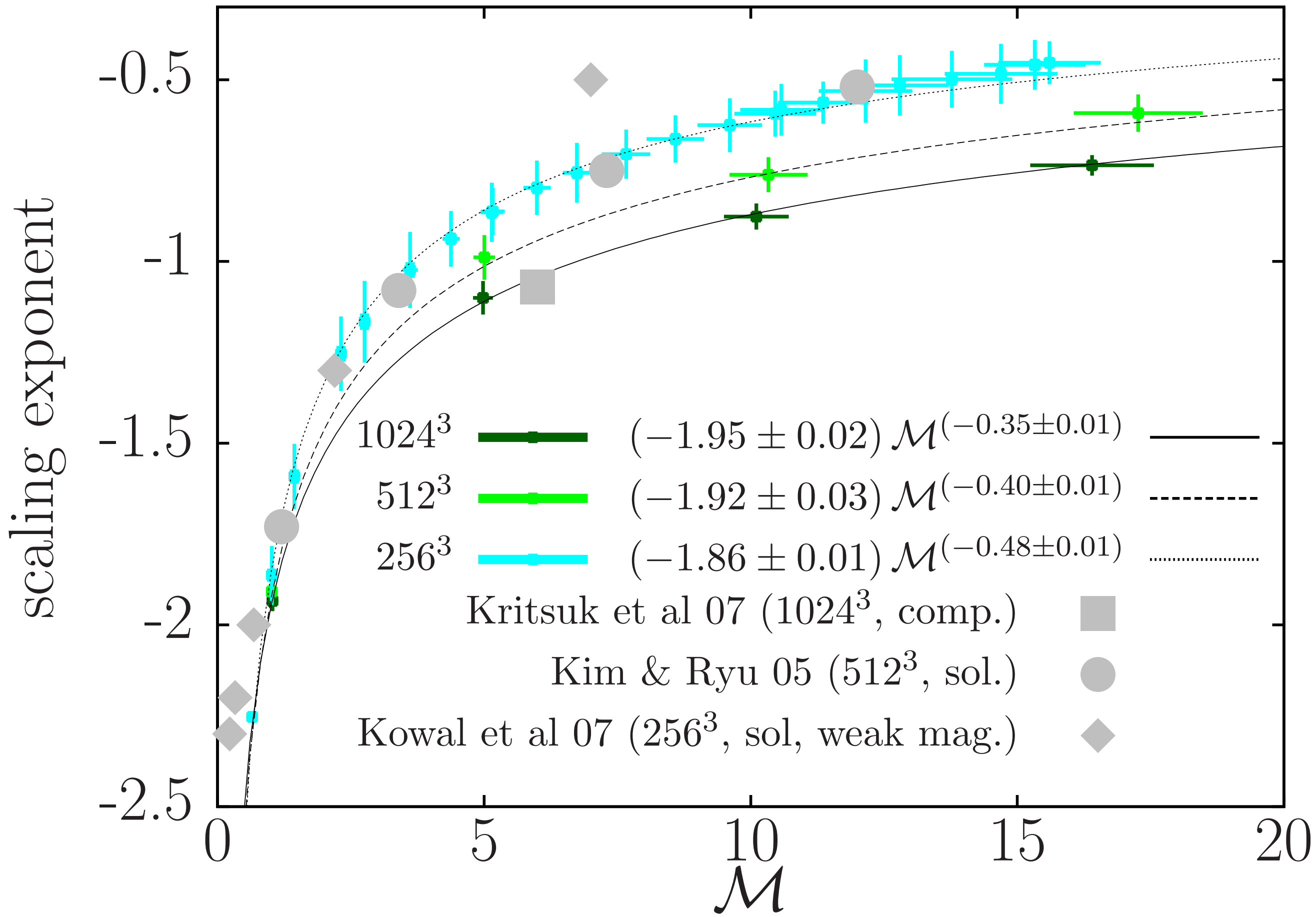
Local slope of the density power spectrum



- spectra get steeper with resolution
- spectra get shallower with $\mathcal{M}$

Influence of the Mach number and resolution

fitting range: $k \in [4:10]_{256},\ [4:17]_{512},\ [4:31]_{1024}$

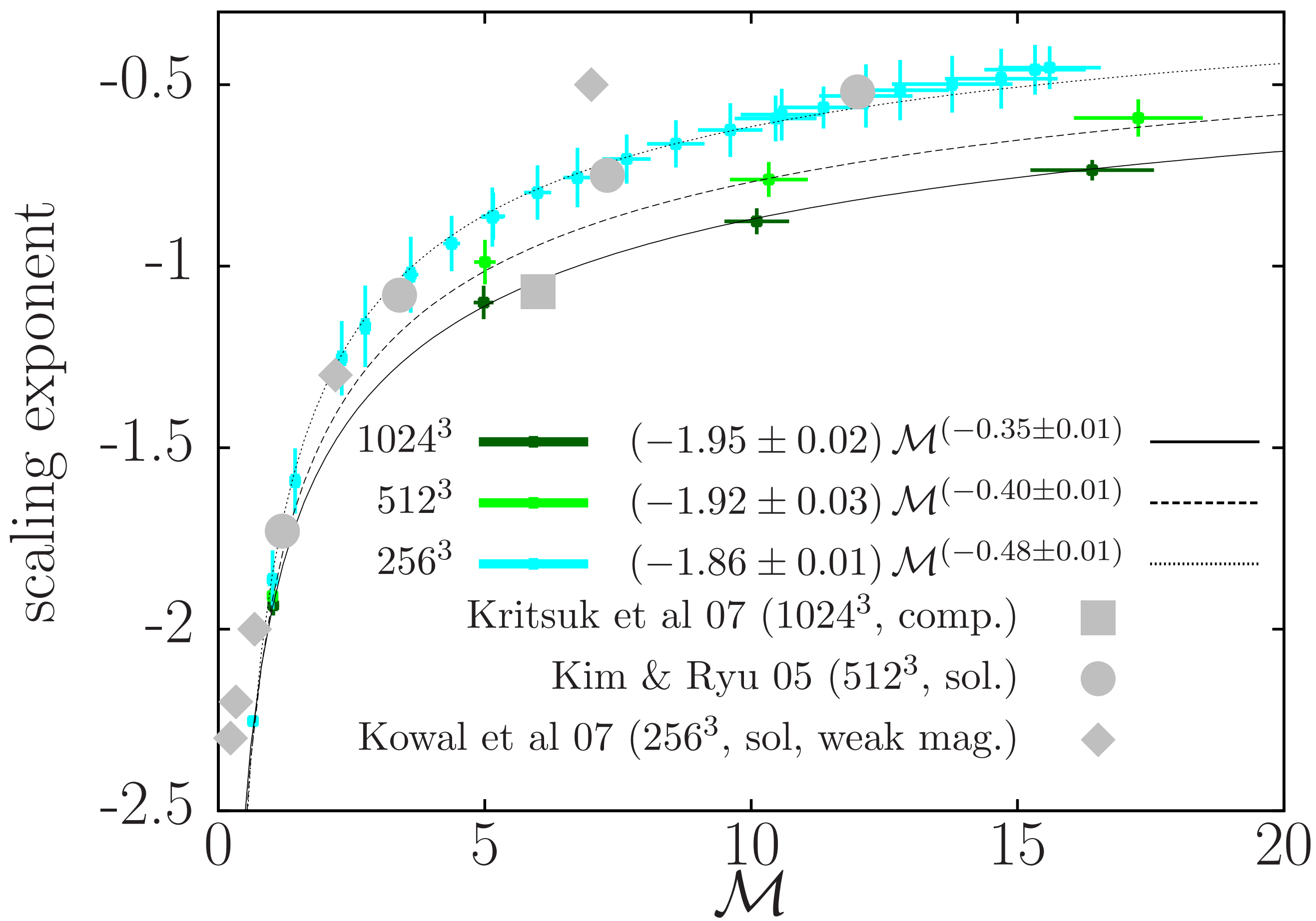


$\mathcal{M} \to 0$, constant density
spectrum = delta function

$\mathcal{M} \to \infty$
strongest shock contains all mass
peaky density → constant spectrum

Influence of the Mach number and resolution

fitting range: $k \in [4:10]_{256},\ [4:17]_{512},\ [4:31]_{1024}$



$$\zeta(\mathcal{M},\, n) = \alpha \mathcal{M}^{\tilde{\beta} + \omega \sum_{i=0}^{n} (1/2)^i}$$

$$\zeta(\mathcal{M}) = (-1.91 \pm 0.01)\,\mathcal{M}^{-0.30\pm0.03}$$

Conclusion

- The width of the density PDF is $\sigma_\rho \approx \mathrm{b}\mathcal{M}$
- The slope of the density power spectrum is:

$$\zeta(\mathcal{M}) = (-1.91 \pm 0.01)\,\mathcal{M}^{-0.30\pm 0.03}$$

- This influences e.g. fractal dimension, space filling factor, and degree of compression.

$$D = 2 - 1/2\,\zeta(\mathcal{M}) = 2 + 0.96\mathcal{M}^{-0.30}.$$

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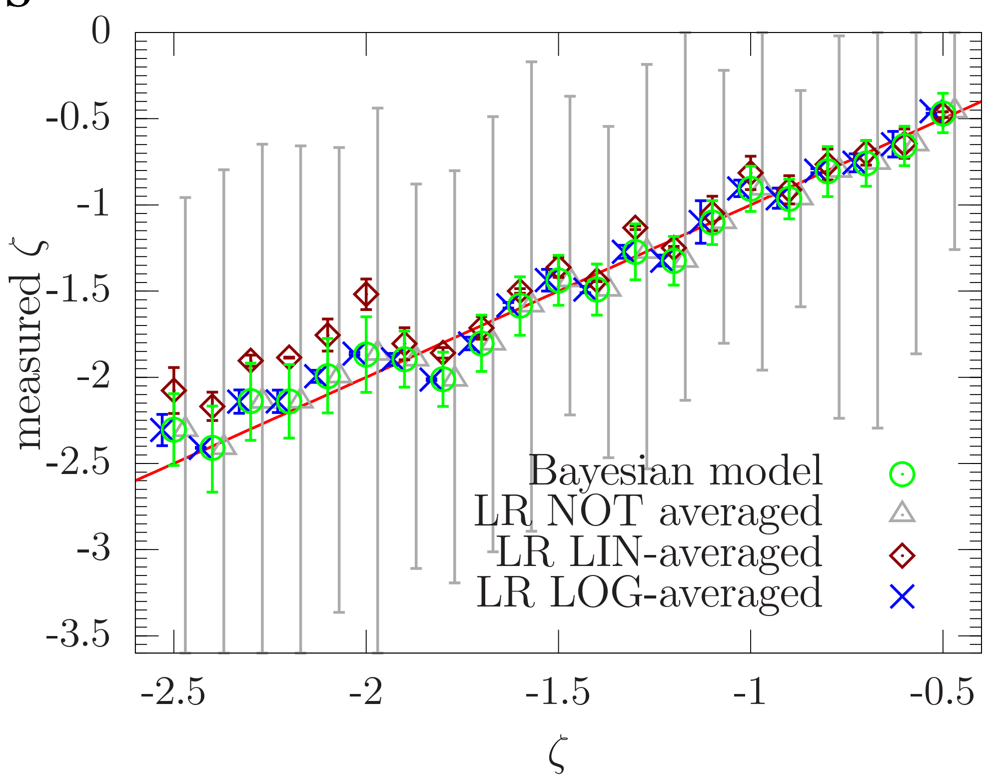
Thank you for your attention!

Hierarchical Bayesian model of the power spectrum

- Linear model with scatter
  $\log(P(k)) = A + \zeta \cdot \log(k) + \delta s$
- Fitting range are just 5 points for this test
- 30% time variation and scatter on all quantities

Test on synthetic data



(Konstandin et al.,2015)

One point statistic: density PDF

- Log normal $\longrightarrow$ $\sigma_\rho^2 = \mu_\rho^2(\exp(\sigma_s^2) - 1)$

  with $s = \log(\rho)$, $\mu_\rho = 1$

- Passot 1994 : $\sigma_\rho \approx \sigma_s \approx \mathrm{b}\mathcal{M}$

- Konstandin et. al. 2015 :

$$\mathrm{d}s = B_1(s,\, t)\mathrm{d}t + B_2(s,\, t)\mathrm{d}W$$

$$B_1(s(t)) = -1/\tau_\alpha(s - \mu_s) \quad B_2^2 = 1/\tau_\beta \log(1 + \mathrm{b}^2\mathcal{M}^2)$$

pressure redistributes the compressed gas

$$\rho^{-1}\nabla p = c_s^2 \nabla s \approx c_s^2(s - \mu_s)$$

$$\longrightarrow \sigma_s^2 = \tau_\alpha/2\tau_\beta \log(1 + \mathrm{b}^2\mathcal{M}^2)$$