The W-F Mechanism Revisited or The Temperature of Light

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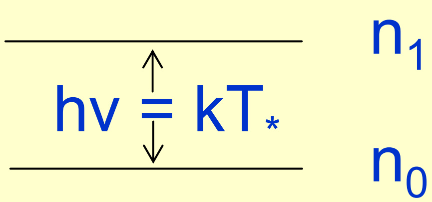
The spin temperature

The ratio of triplet state (upper) to singlet state (lower) is specified by the spin temperature T_S



$$\frac{n_1}{n_0} = 3\exp[-T_*/T_S]$$

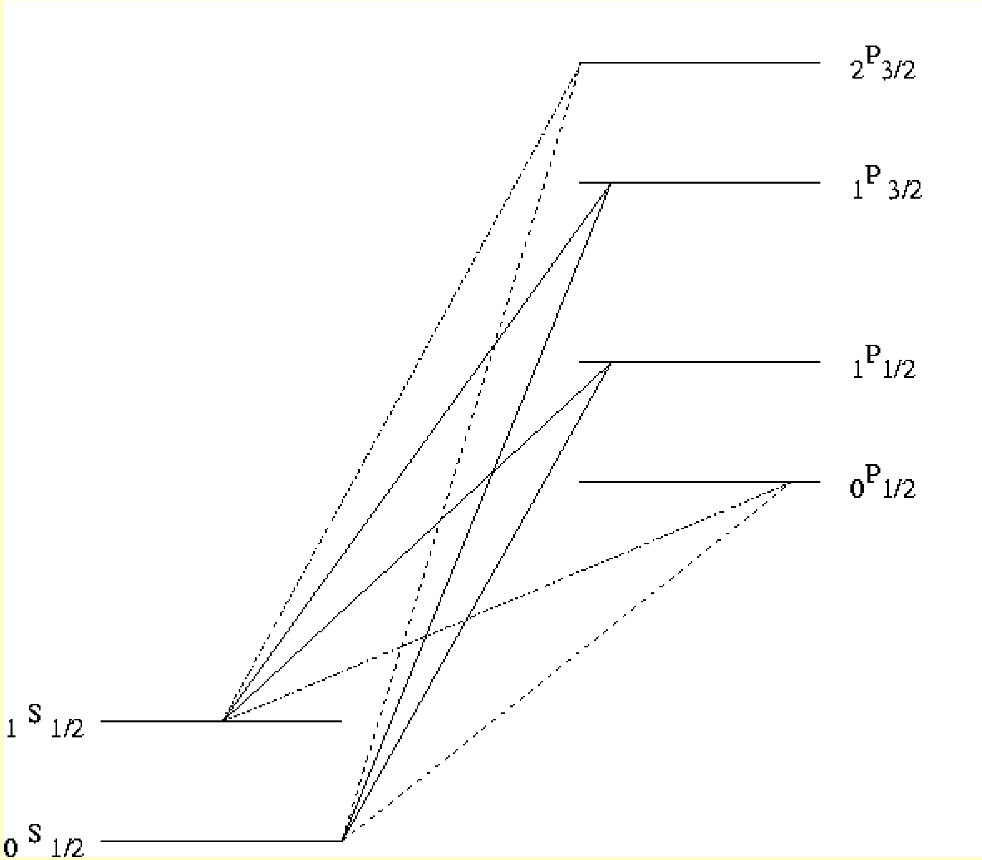
Coupling to the Cosmic Background Radiation ensures

$$T_S = T_{CBR}.$$

In this case, there is no net signal: the HI absorbs and re-emits the CBR photons at the same rate and the IGM is *invisible*.

How is the 21cm line excited?

The Wouthuysen-Field mechanism: reset the spin temperature by scattering Lyα photons



$$T_S = \frac{T_{\rm CBR} + y_\alpha T_\alpha}{1 + y_\alpha}$$

$$y_\alpha \equiv \frac{P_{10}}{A_{10}} \frac{T_*}{T_\alpha}$$

T_α is the *light temperature*

P_{10} is the Lyα scattering rate due to sources

What sets the light temperature?

Field (1958) showed:

$$\frac{P_{01}^{L}}{P_{10}^{L}} = 3\frac{\langle u_{\nu_0}\rangle + \langle u_{\nu_0'}\rangle}{\langle u_{\nu_1}\rangle + \langle u_{\nu_1'}\rangle} = 3\exp(-T_*/ T_\alpha)$$

The averages are carried over the line absorption profile centred on each frequency. Field (1959) showed for pure Doppler scattering and a Doppler source:

u_ν is proportional to $\exp[-h(\nu-\nu_0)/kT]$ for ν near ν_0 and $T_\alpha = T$, the matter temperature,

…but the proof was valid only for $h(\nu-\nu_0) << kT$.

Lyα photon scattering

Since the full Doppler core must be included to evaluate the averages $<u_\nu>$, the complete radiative transfer equation must be solved:

$$\frac{Du_\nu(\mathbf{r},t)}{Dt} = \frac{\chi_0(\mathbf{r},t)c}{h\nu_0}h\nu\left[\int_{-\infty}^{\infty} dQ' W(\nu',Q')u_{\nu'} - \int_{-\infty}^{\infty} dQ W(\nu,Q)u_\nu\right] + h\nu S(\nu),$$

W(ν,Q) describes the scattering of a photon of frequency $\nu \rightarrow \nu' = \nu - Q$.

Diffusion approximation

Rybicki & Dell'Antonio (1994) and Rybicki (2006) derive the scattering equation in the diffusion approximation:

$$\frac{1}{c\chi}\frac{\partial J}{\partial t} = \frac{\Delta\nu_{\rm D}^2}{2}\frac{\partial}{\partial \nu}\left[D(\nu)\left(\frac{\partial J(\nu)}{\partial \nu} + \frac{hJ}{kT}\right)\right]$$

where J (= n) is the photon number density.

In a steady-state ($\partial J/ \partial t = 0$), $J(v) = J(v_0)\exp[-h(v-v_0)/ kT]$, as found by Field (1959) near line centre. This is the expectation (for $hv_0 >> kT$) as the radiation approaches statistical equilibrium, and establishes a Bose-Einstein distribution about the line centre:

$$n = \frac{1}{e^{(h\nu-\mu)/kT} - 1}$$

Fokker-Planck approximation

But the diffusion approximation neglects non-particle conserving terms, and so leaves D(v) ambiguous:

$$\frac{1}{c\chi}\frac{\partial J}{\partial t} = \frac{\Delta\nu_{\rm D}^2}{2}\frac{\partial}{\partial\nu}\left[D(\nu)\left(\frac{\partial J(\nu)}{\partial\nu} + \frac{hJ}{kT}\right)\right]$$

+ non-particle conserving terms

The Fokker-Planck approximation exactly conserves particle number:

$$\frac{Dn_\nu(\mathbf{r},t)}{Dt} = \frac{\chi_0(\mathbf{r},t)c}{h\nu_0}\frac{\partial}{\partial\nu}\left\{\langle Q\rangle\varphi(\nu)u_\nu + \frac{1}{2}\frac{\partial}{\partial\nu}\left[\langle Q^2\rangle\varphi(\nu)u_\nu\right]\right\} + S(\nu)$$

AM (2006)

Kramers-Moyal expansion

The Fokker-Planck approximation may be extended by Taylor-expanding beyond second order

$$W(\nu', Q')u_{\nu'} \approx W(\nu, Q')u_\nu + Q' \frac{\partial}{\partial \nu}\left[W(\nu, Q')u_\nu\right] + \frac{1}{2}Q'^2 \frac{\partial^2}{\partial \nu^2}\left[W(\nu, Q')u_\nu\right].$$

with generally defined moments of the frequency redistribution function:

$$\langle Q^n \rangle \varphi(\nu) \equiv \left[\int_{-\infty}^{\infty} dQ Q^n W(\nu, Q)\right]$$

Light thermodynamics

But the Fokker-Planck approximation doesn't give $n(\nu) = n(\nu_0)\exp[-h(\nu-\nu_0)/kT]$. How then can it describe the approach to statistical equilibrium and provide the correct light temperature $T_\alpha \rightarrow T$?

Use a light temperature defined *thermodynamically*. Equilibrium is reached when heat transfer between the radiation and matter ceases. The rate of heat exchange is:

$$G = P_l n_l \frac{h\nu_0}{m_a c^2} h\nu_0 \left(1 - \frac{T}{\langle T_u \rangle_H}\right)$$

where $\langle T_u \rangle_H$ is the thermodynamic light temperature.

Thermodynamic light temperature

The thermodynamic light temperature is defined through the photon statistical distribution temperature:

$$T_u(\nu) = -\frac{h}{k}\left(\frac{d\log u_\nu}{d\nu}\right)^{-1}$$ (u = hv*n)

This recovers $T_u(v) = T$ for $n(v) = n(v_0)\exp[-h(v-v_0)/kT]$.

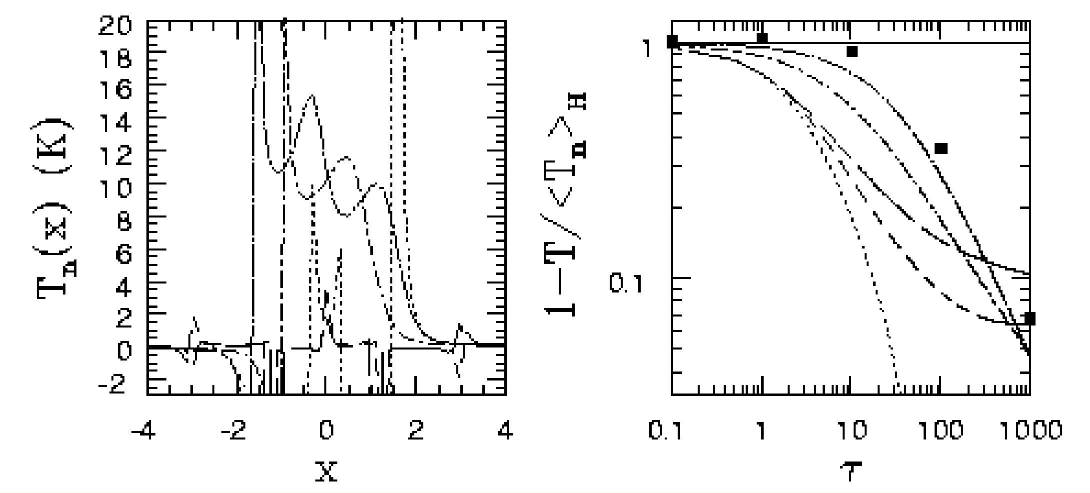
The frequency-averaged thermodynamic temperature is:

$$\langle T_u\rangle_H = \int_0^\infty d\nu u_\nu \varphi_V(\nu) \Bigg/ \int_0^\infty d\nu u_\nu \varphi_V(\nu) \frac{1}{T_u(\nu)}$$

Relaxation of the temperature

The full-frequency time-dependent solution for a unit Doppler profile source for T = 10K gives:

Statistical distribution temperature definition



Thermodynamic temperature definition

This is a surprise: it is the thermodynamically-motivated definition of light temperature which converges to the matter temperature, not the frequency-dependent definition.

This is good news for the Wouthuysen-Field mechanism, since it can be shown: $T_\alpha = \langle T_u \rangle_H$.

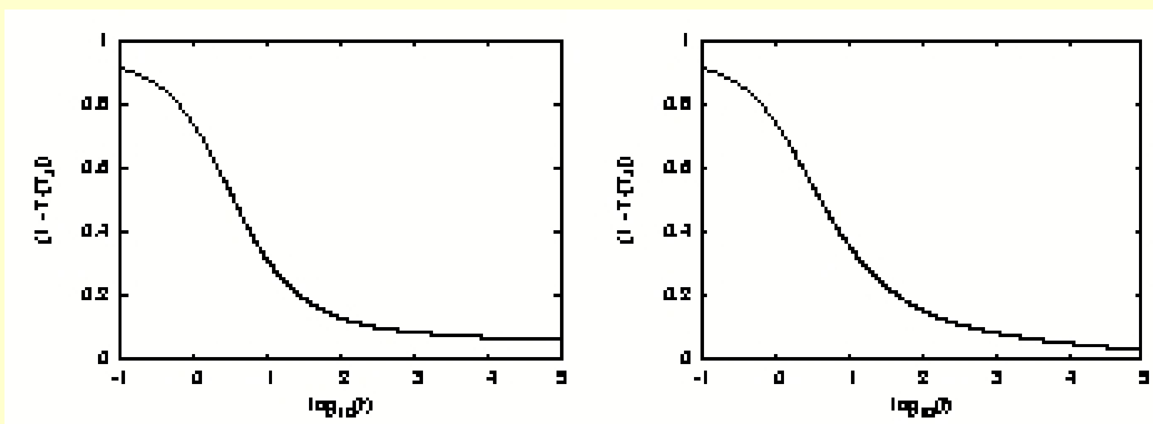
How fast is equilibrium reached?

Two cases:

- Static medium
- Expanding medium

Static medium:

2^{nd} order F-P $\quad$ 3^{rd} order K-M

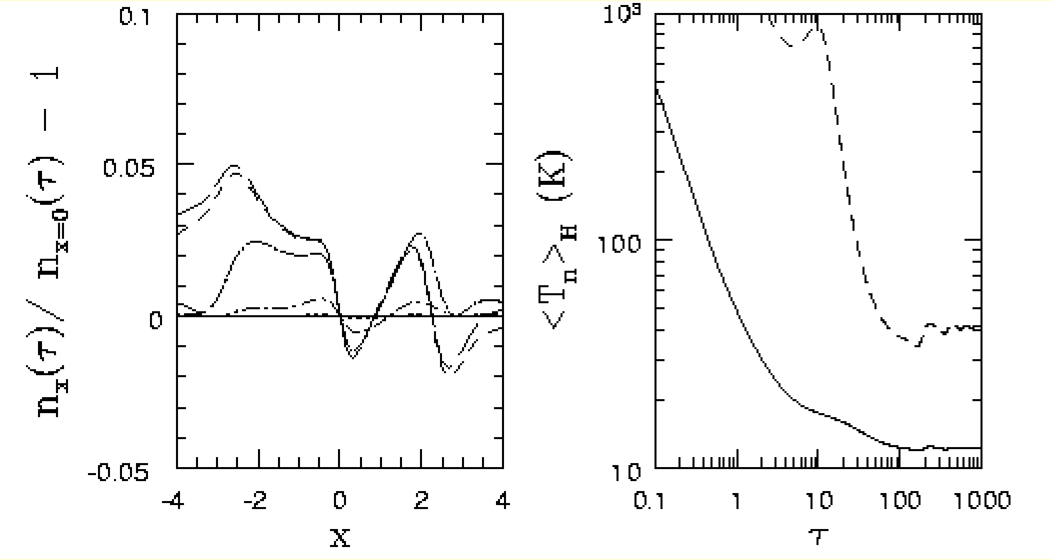


Higgins & AM

Including expansion

Allowing for cosmological expansion has only a small effect once steady-state equilibrium is achieved (Chen & Miralda-Escude 1994).

In more extreme situations (eg, an AGN jet or wind), the expansion may lead to a freezing out of the light temperature with $T_\alpha \neq T$:



T = 10K
a = 0.015
γ = 0.1

Summary

- The definition of the **light temperature** may be physically motivated through thermodynamics considerations.
- The light temperature approaches the kinetic temperature within about 1000 scattering times at line centre.
- Convergence of the light temperature to the kinetic temperature to better than a few percent may take more than 10^5 scattering times.

  This means Lyα heating could be cosmologically significant for bright sources (10 x W-F coupling rate).
- The light temperature can freeze out in extreme flows, never converging to the kinetic temperature.

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