Polarization imaging with LOFAR

Radio Observatory
ASTRON, Dwingeloo, The Netherlands

LOFAR data school

Reading list

- Born & Wolf *Principles of optics*
- Thompson, Moran & Swenson *Interferometry and Synthesis in Radio Astronomy*
- Taylor, Carilli & Perley *Synthesis Imaging in Radio Astronomy II*
- Bracewell *The Fourier Transform & Its Applications*
- Hamaker, Bregman & Sault *Understanding radio polarimetry: paper I*(1996)
- Sault, Hamaker& Bregman *paper II*(1996)
- Hamaker & Bregman *paper III* (1996)
- Hamaker *paper IV* (2000)
- Hamaker *paper V* (2006)
- Brentjens & de Bruyn *Faraday rotation measure synthesis* (2005)

Outline

1. Polarized EM waves
2. Stokes parameters
3. Interferometric polarimetry
4. Messy reality

Electromagnetic (EM) wave

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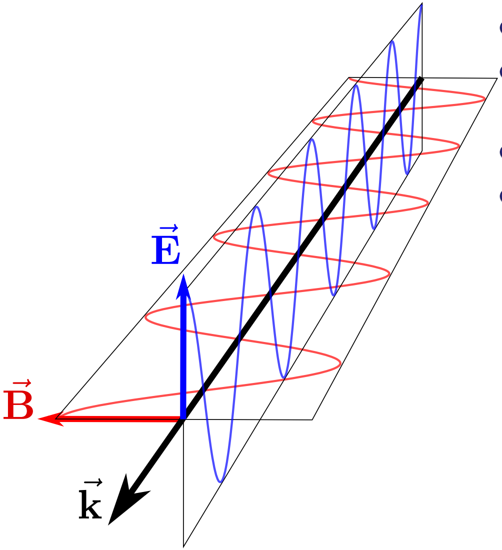
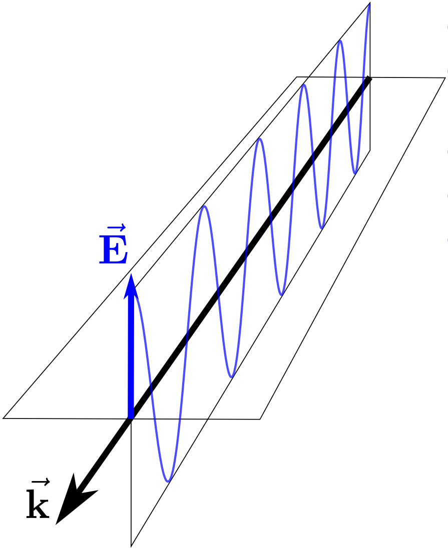


- Vector phenomenon
- From Maxwell's equations: $\hat{\mathbf{k}} = \hat{\mathbf{E}} \times \hat{\mathbf{B}}$
- We know **k**
- Measure either **E** or **B**



- Vector phenomenon
- From Maxwell's equations: $\hat{\mathbf{k}} = \hat{\mathbf{E}} \times \hat{\mathbf{B}}$
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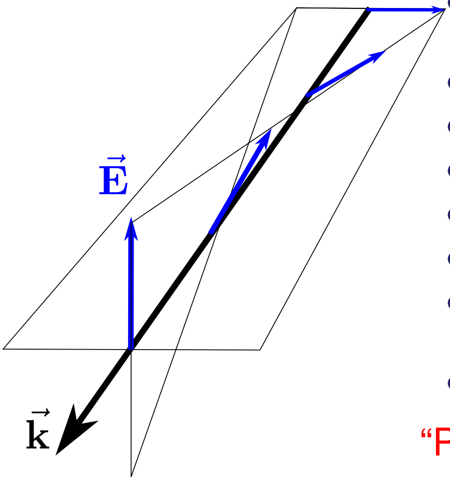
- Measure either **E** or **B**
- **E** is easier

Electromagnetic (EM) wave

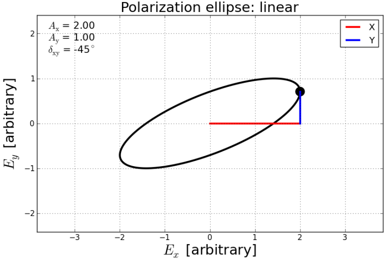


- Vector phenomenon
- From Maxwell's equations: $\hat{\mathbf{k}} = \hat{\mathbf{E}} \times \hat{\mathbf{B}}$
- We know **k**
- Measure either **E** or **B**
- **E** is easier
- But:
- E_x and E_y not equal
- **E** may rotate as function of **x** and t.
- **E** traces ellipse

"Polarization"

ASTRON

Geometry

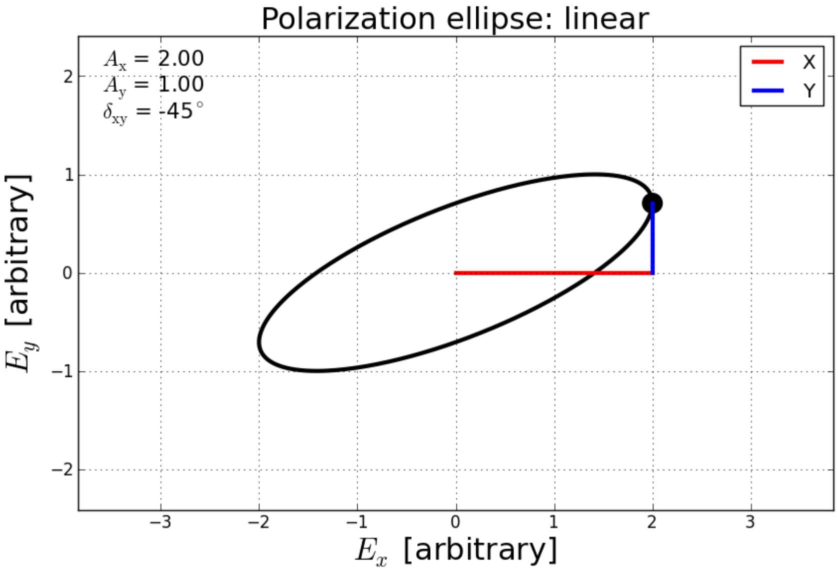


Viewing from antenna towards source, watching orientation and length of **E** vector on a plane at a fixed location in space.

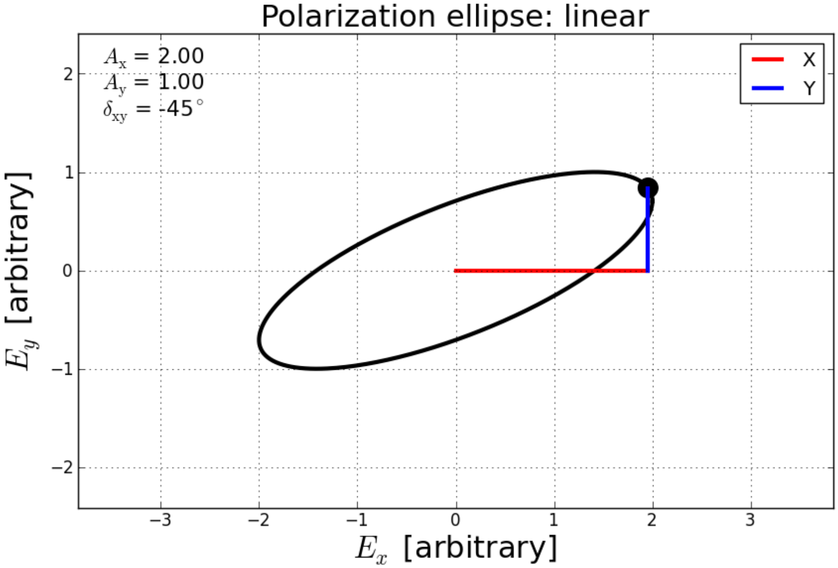
$$\mathbf{E} = E_x\hat{\mathbf{e}}_x + E_y\hat{\mathbf{e}}_y$$
$$E_x = A_x\cos(2\pi\nu t + \delta_x)$$
$$E_y = A_y\cos(2\pi\nu t + \delta_y)$$

- A_x = x-amplitude
- A_y = y-amplitude
- $\delta_{xy} = \delta_y - \delta_x$
- δ_{xy} = measure of ellipticity

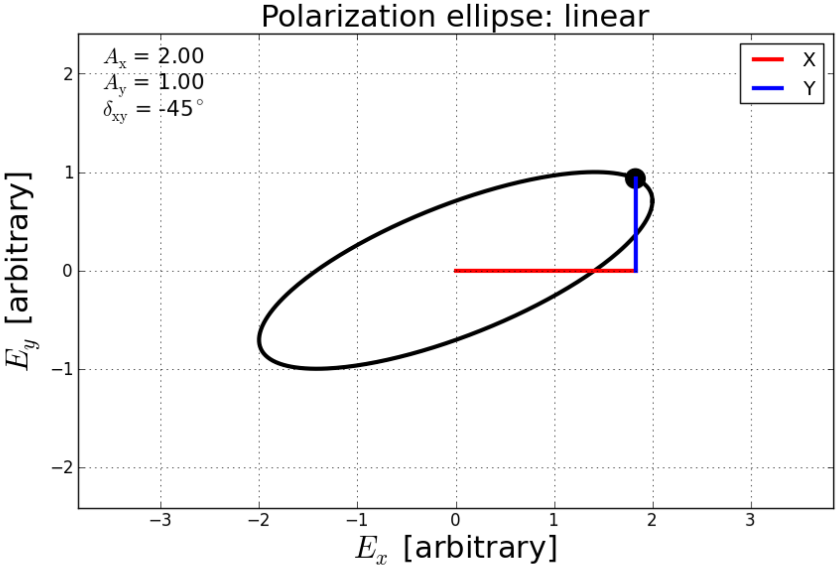
- $\delta_{xy} > 0$: CW rotation $\Rightarrow$ LEP
- $\delta_{xy} = 0$: linear polarization
- $\delta_{xy} < 0$: CCW rotation $\Rightarrow$ REP


Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_x [arbitrary]
E_y [arbitrary]

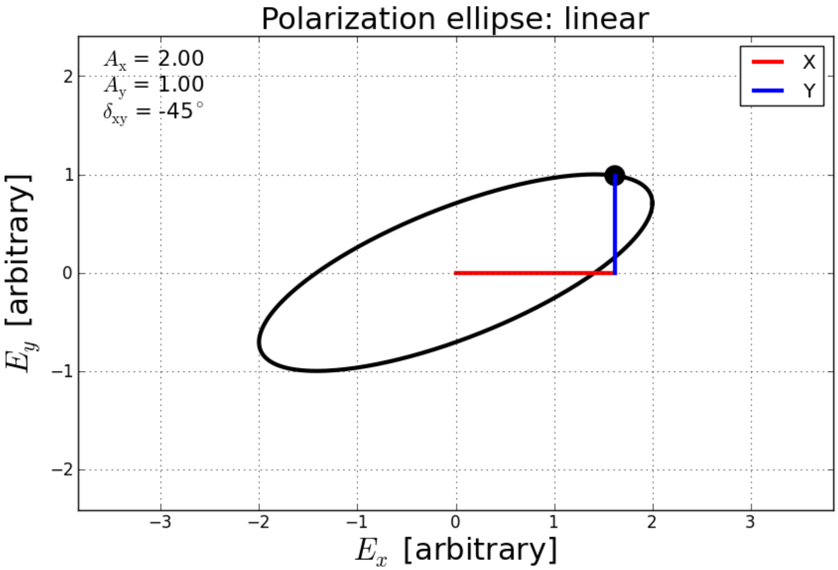
Polarization ellipse: *xy* 001



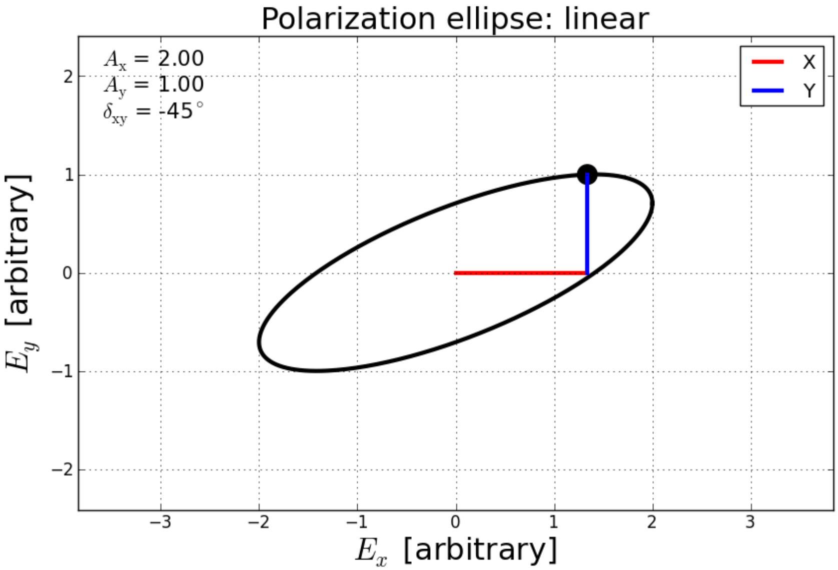
Polarization ellipse: *xy* 002



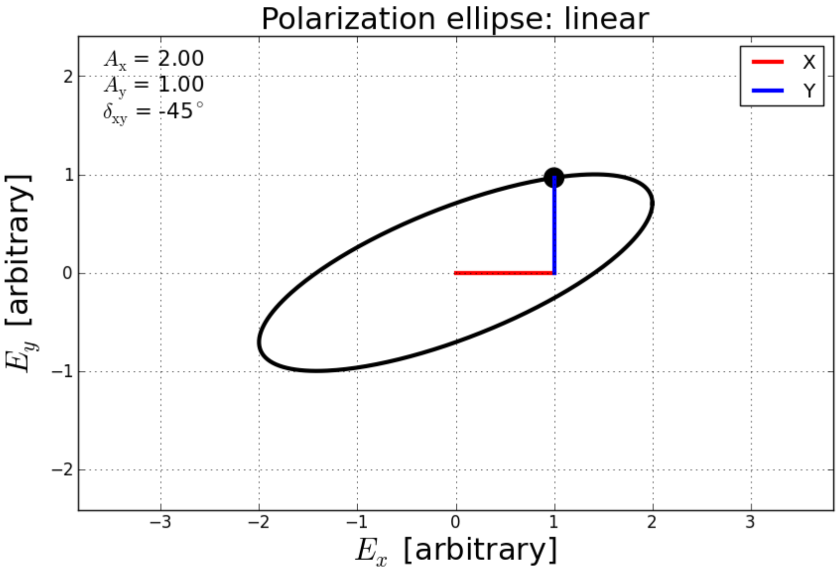
Polarization ellipse: *xy* 003

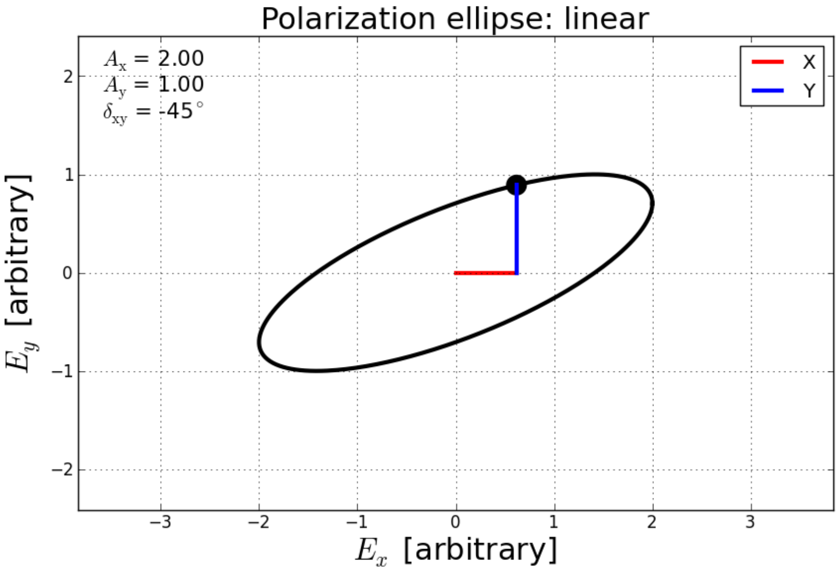


Polarization ellipse: *xy* 004



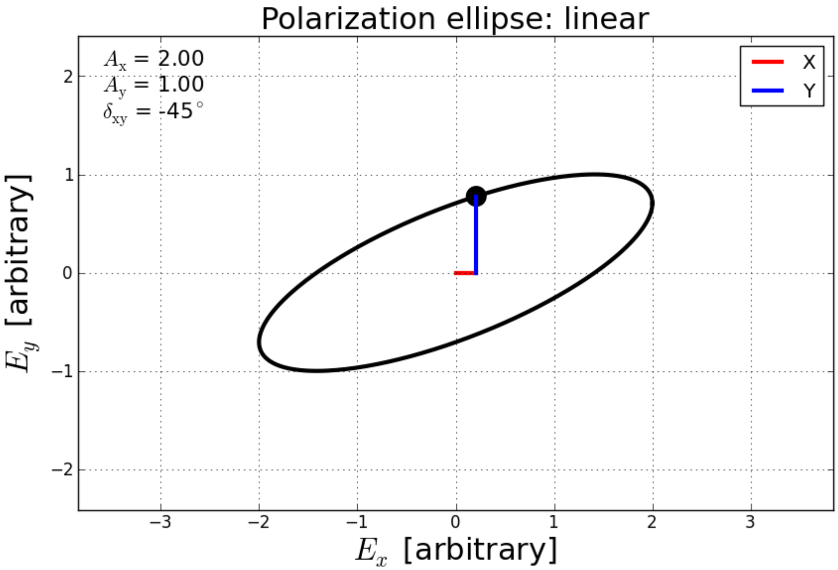
Polarization ellipse: *xy* 005



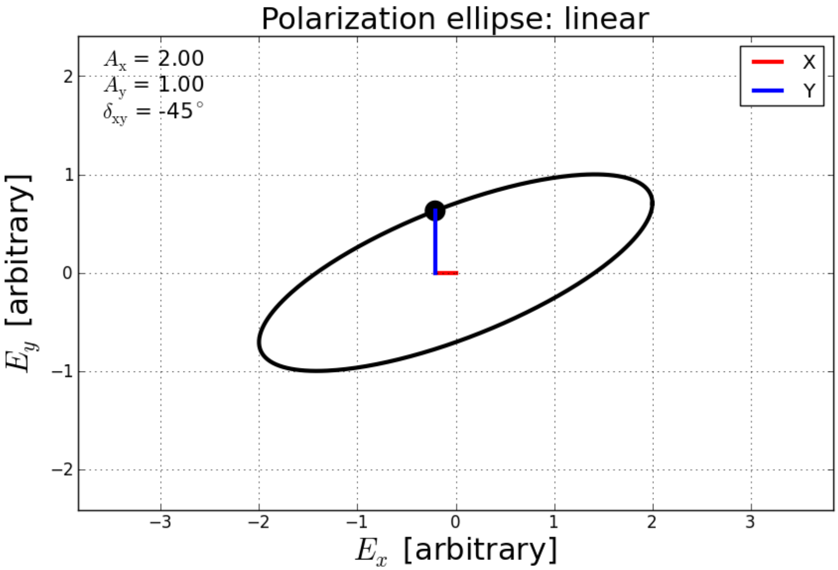


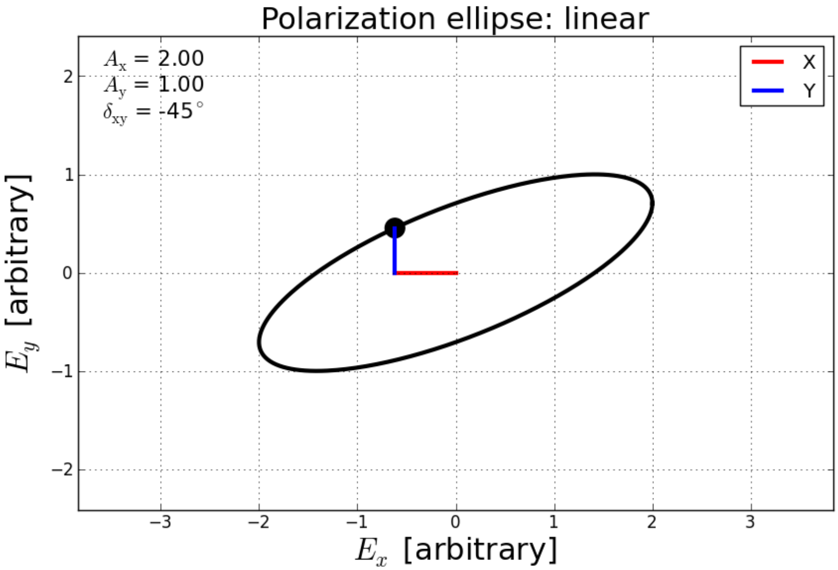
Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
E_y [arbitrary]
E_x [arbitrary]
−3
−2
−1
0
1
2
3

Polarization ellipse: *xy* 007



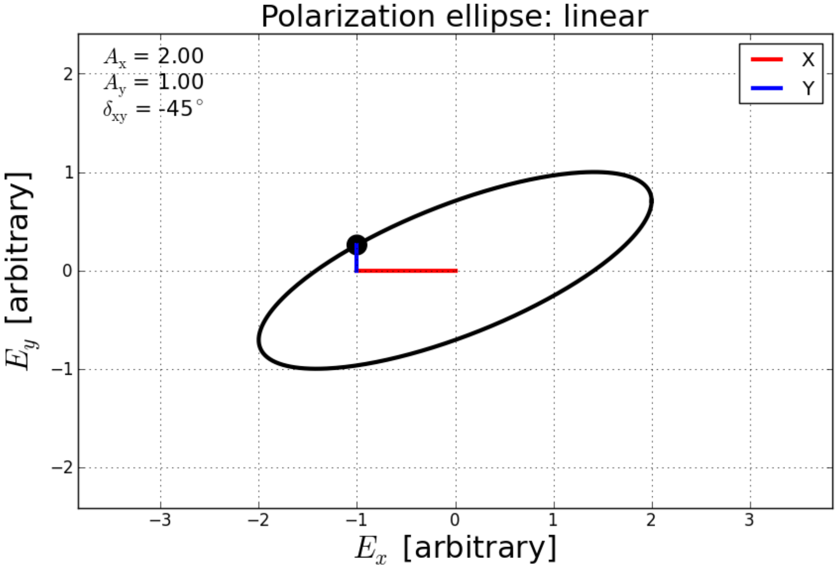
Polarization ellipse: *xy* 008

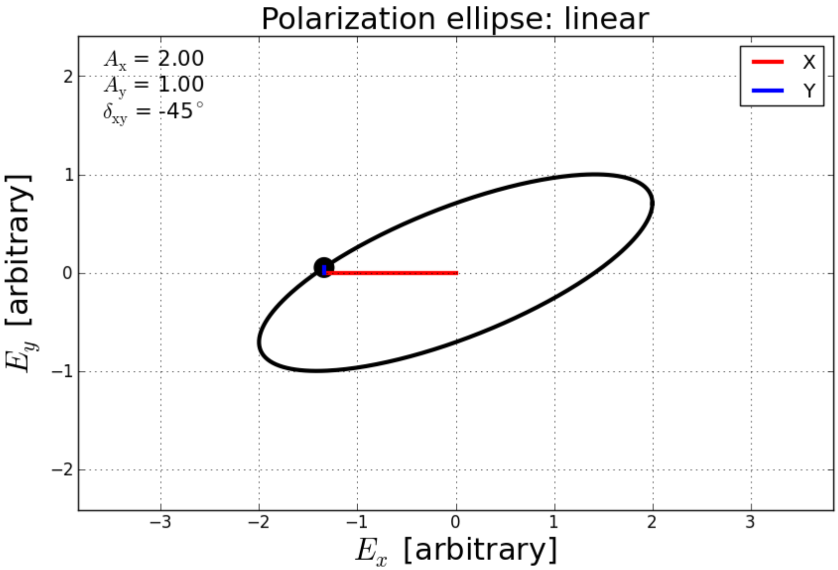




Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

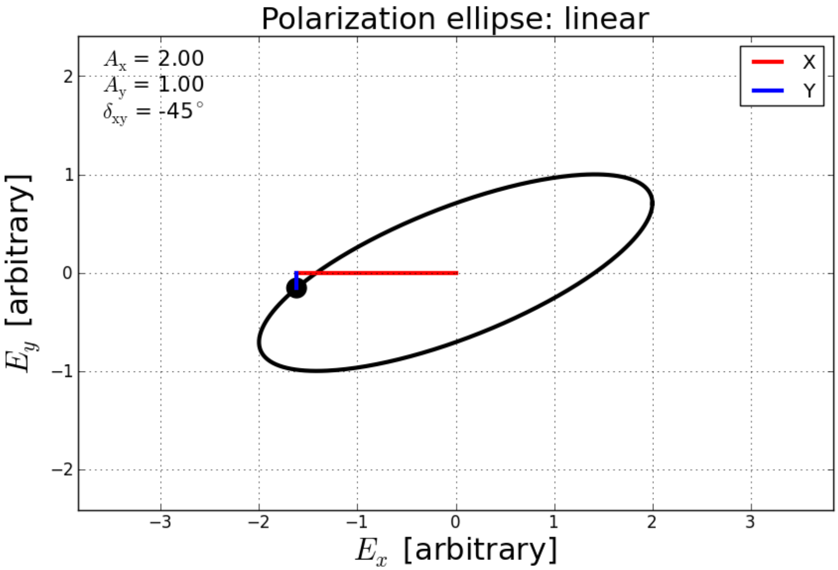
Polarization ellipse: *xy* 010



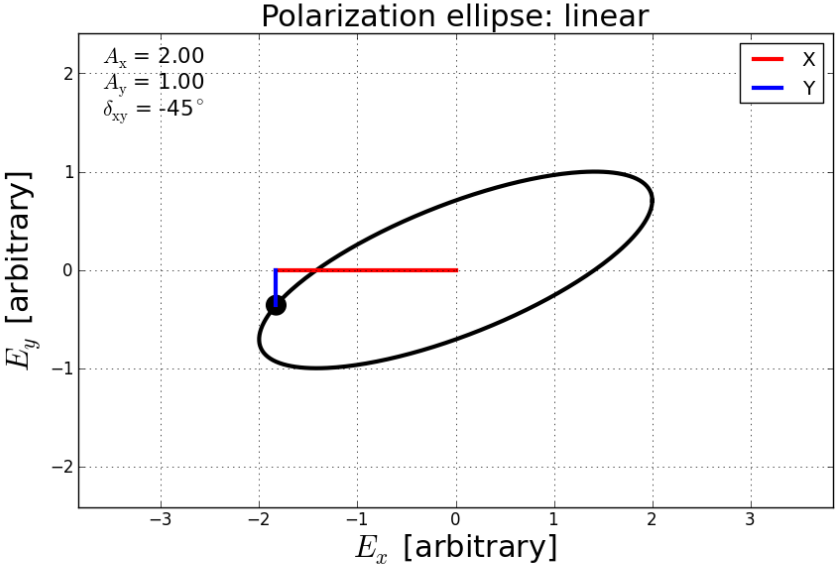


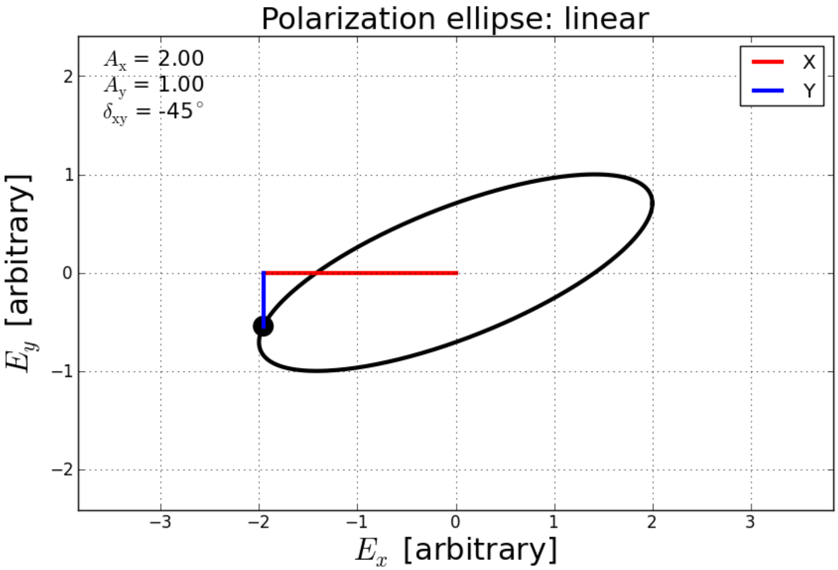
Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
2
1
0
−1
−2
−3
3
E_y [arbitrary]
E_x [arbitrary]

Polarization ellipse: *xy* 012



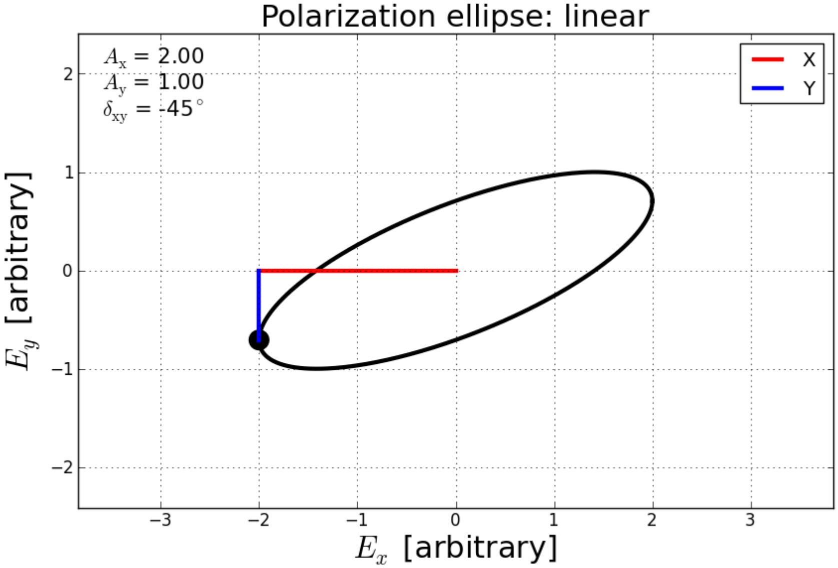
Polarization ellipse: *xy* 013



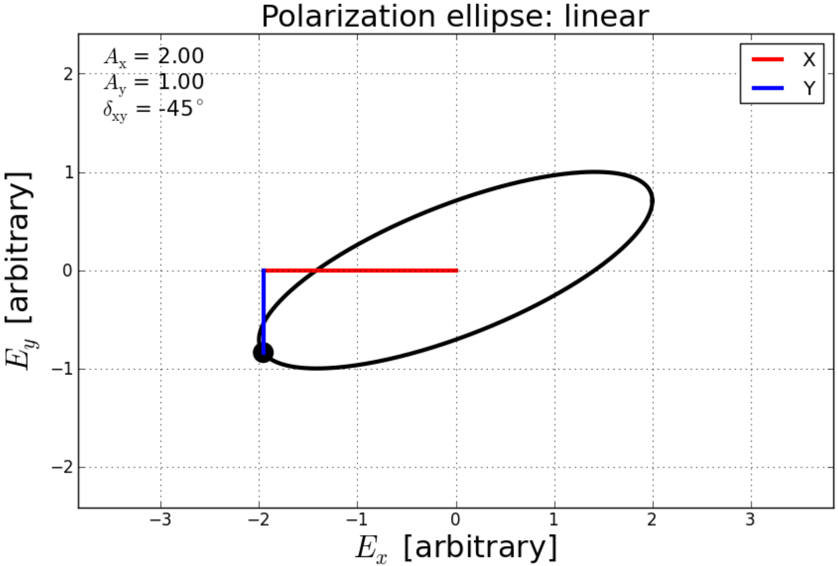


Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

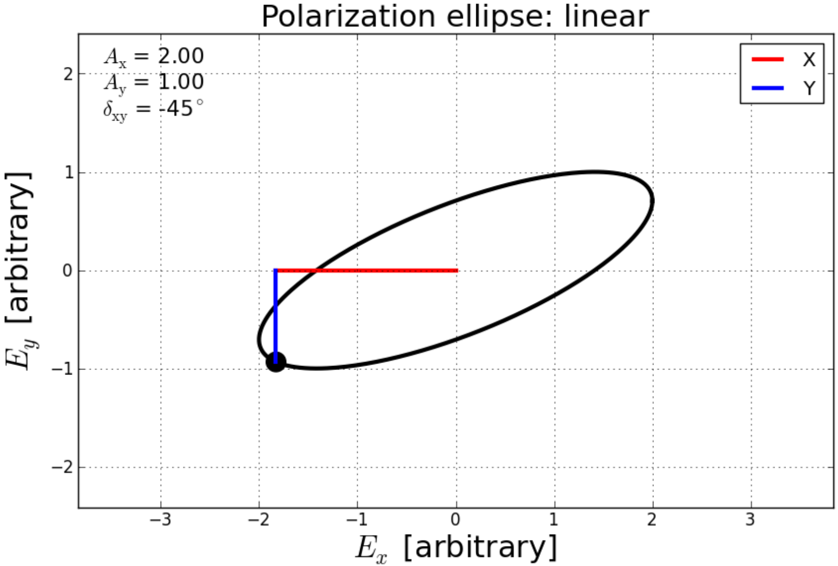
Polarization ellipse: *xy* 015

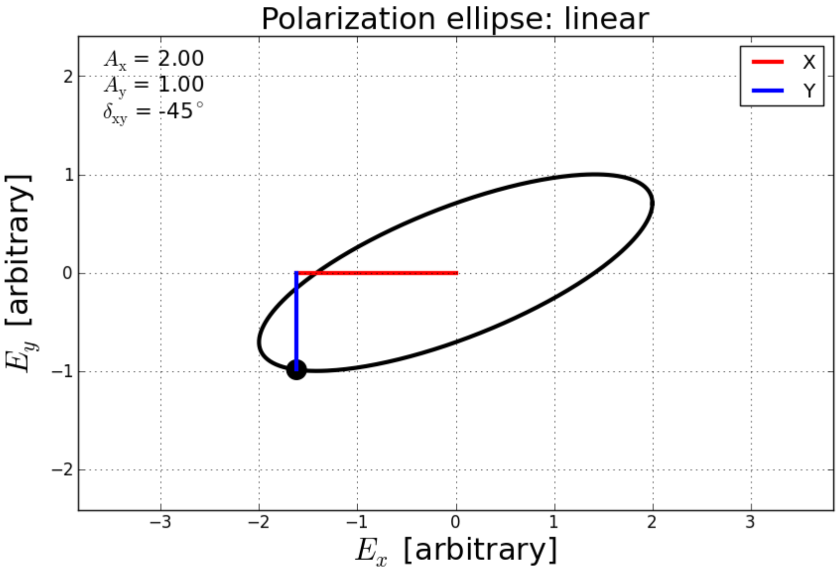


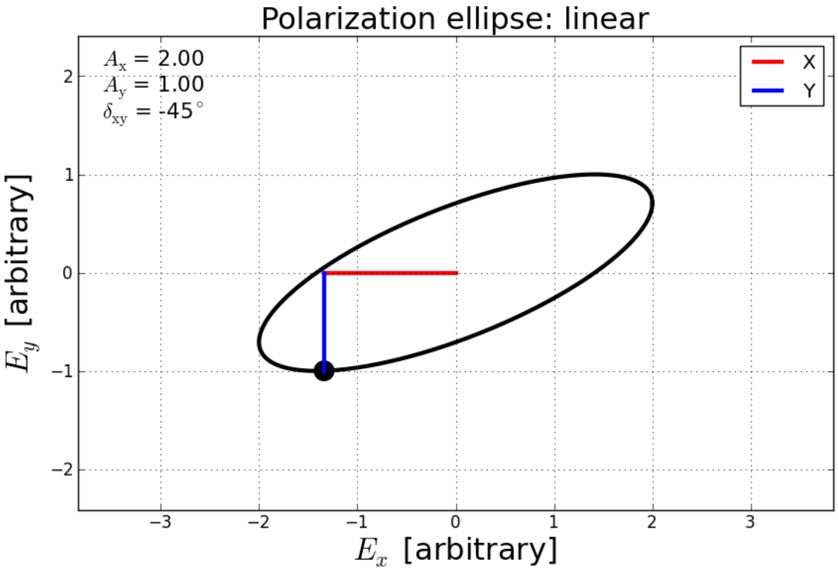
Polarization ellipse: *xy* 016

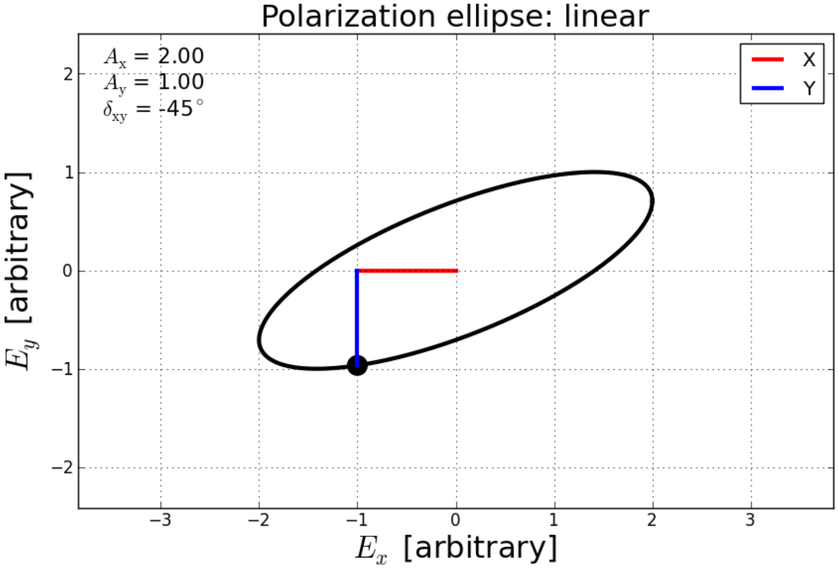


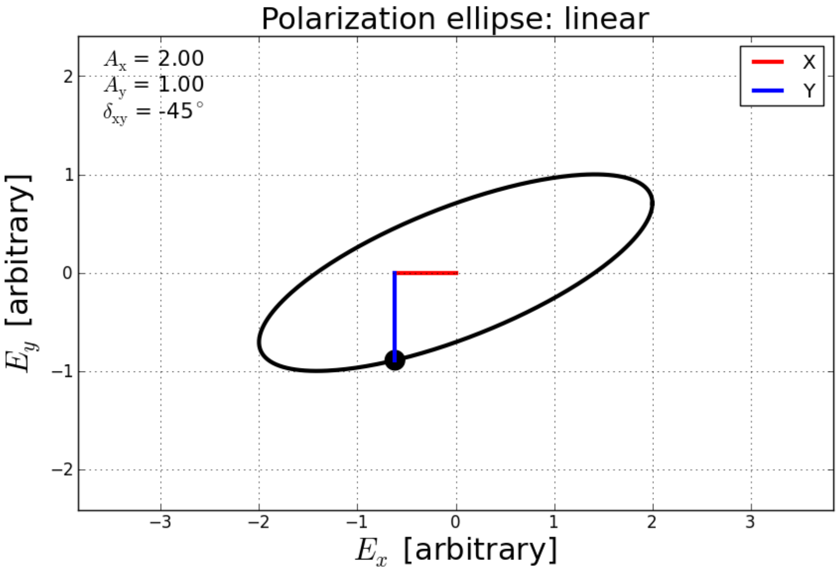
Polarization ellipse: *xy* 017




Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

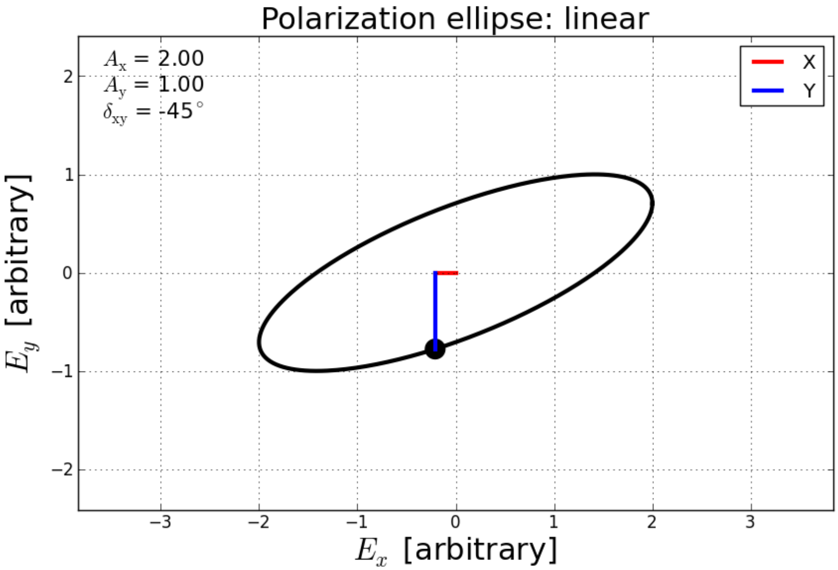

Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]


Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
E_y [arbitrary]
E_x [arbitrary]
−3
−2
−1
0
1
2
3

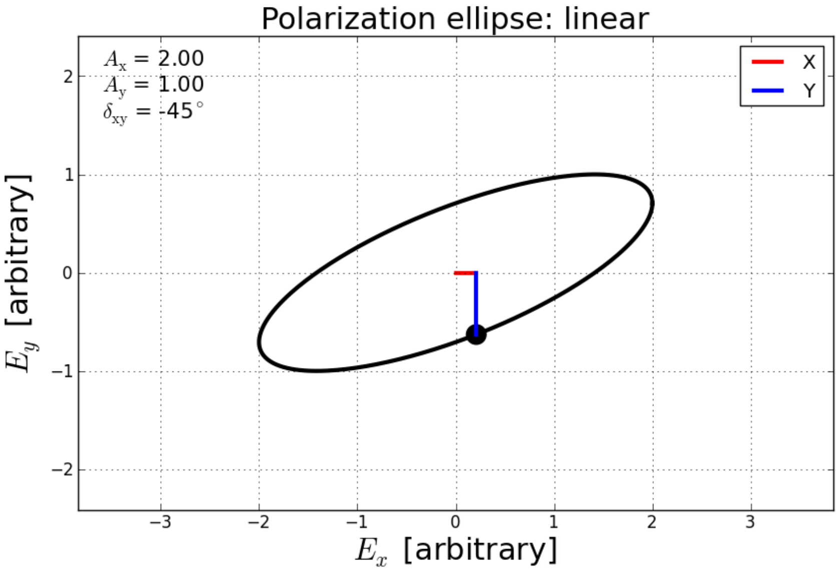


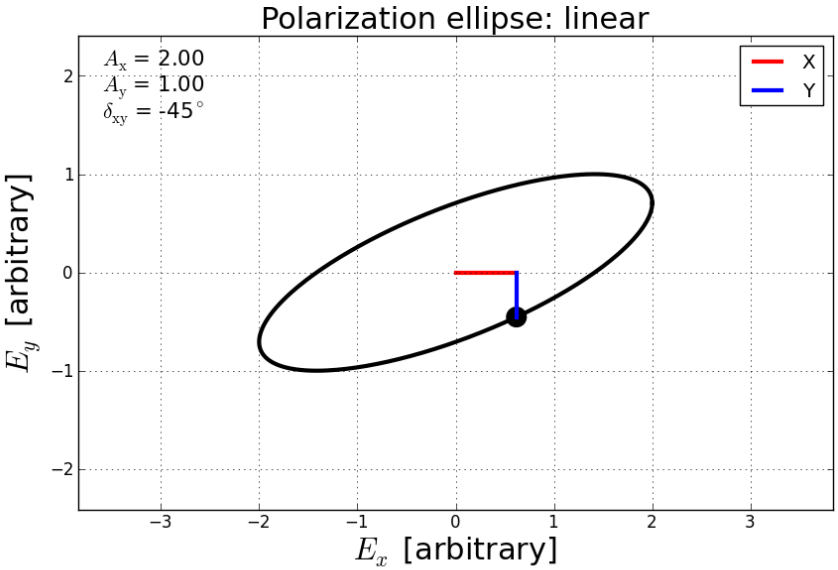
Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

Polarization ellipse: *xy* 022

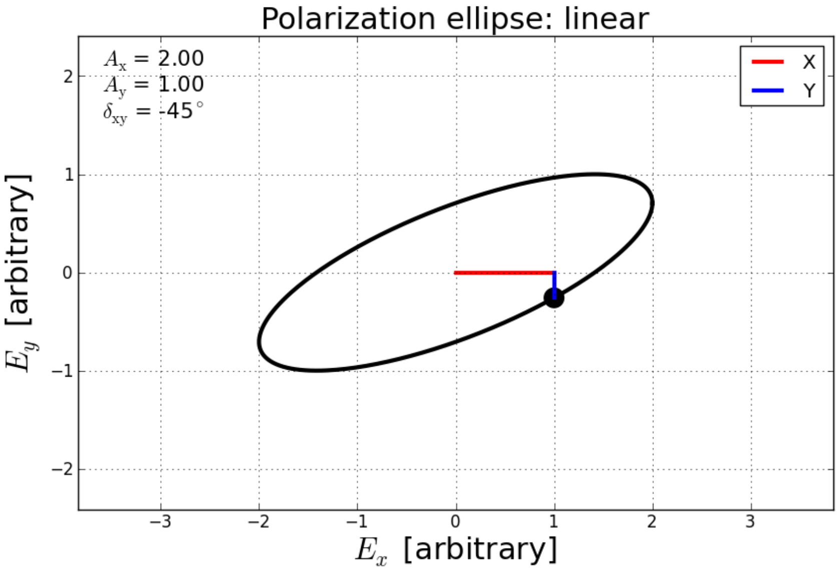


Polarization ellipse: *xy* 023

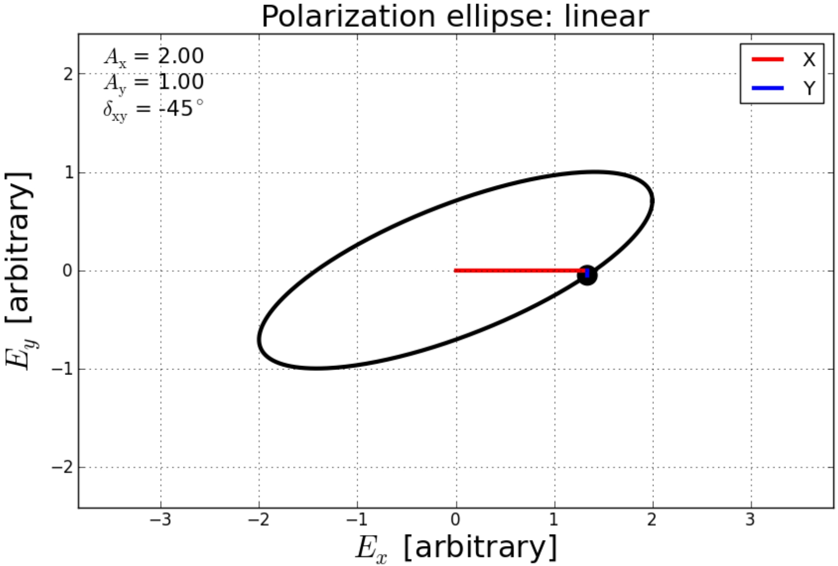




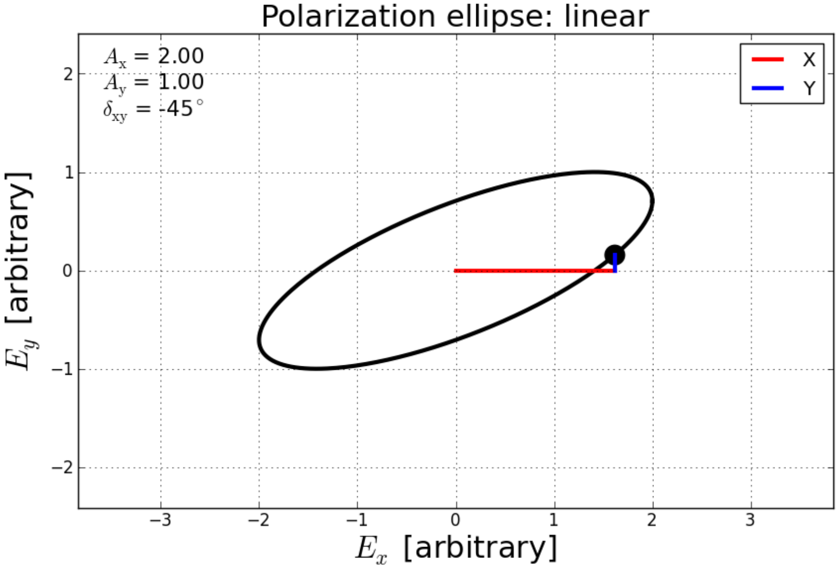
Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

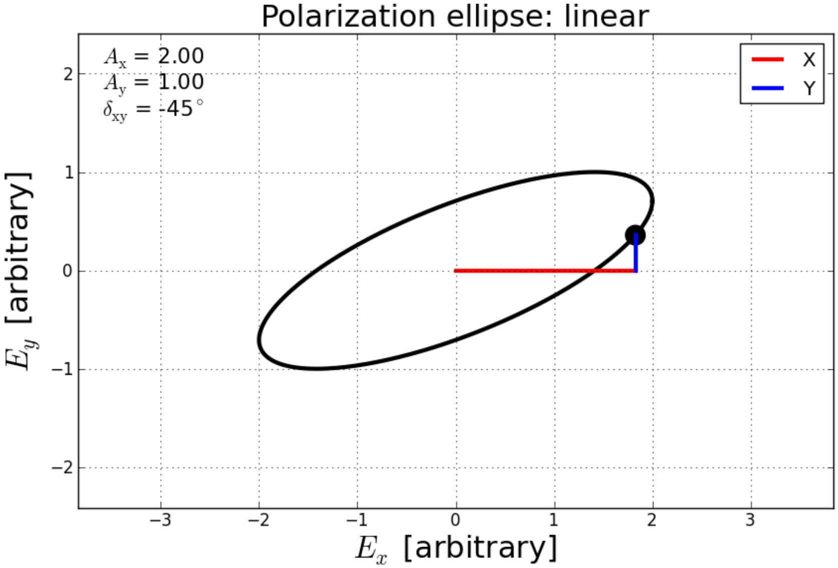

Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
2
1
0
−1
−2
E_y [arbitrary]
−3
−2
−1
0
1
2
3
E_x [arbitrary]

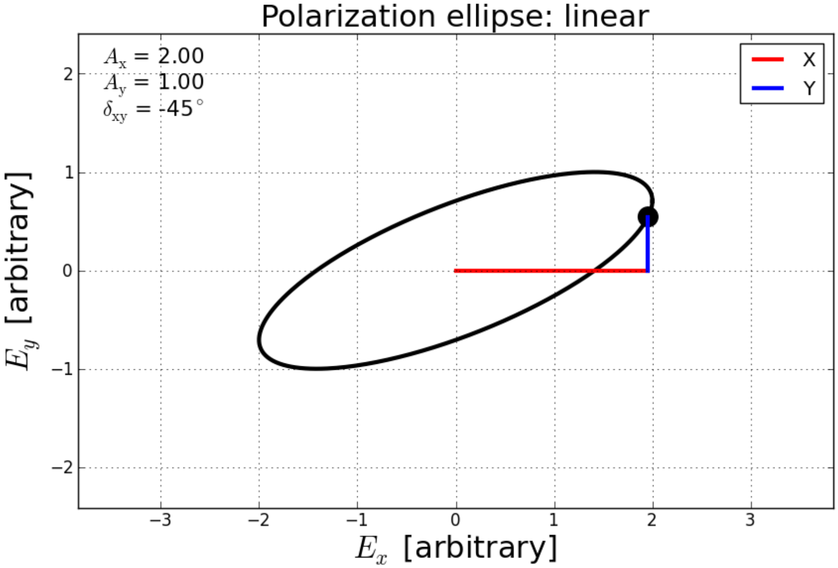
Polarization ellipse: *xy* 026



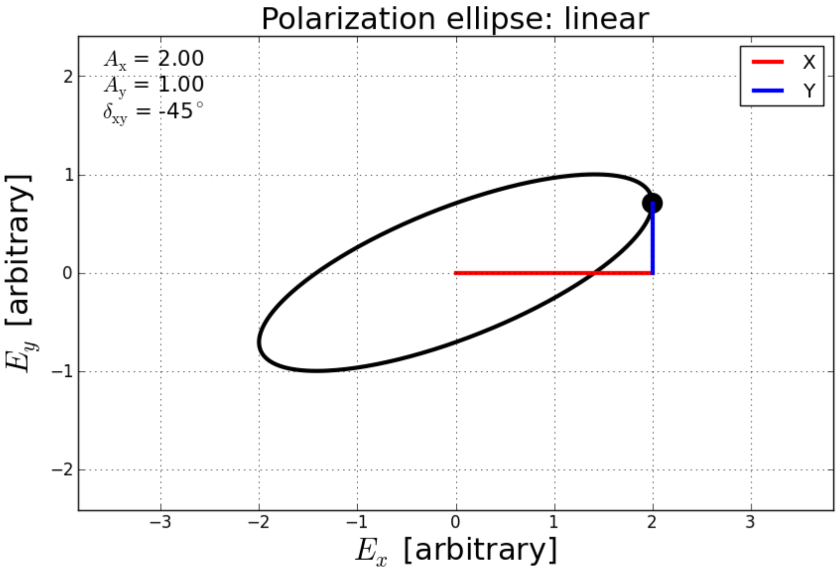
Polarization ellipse: *xy* 027



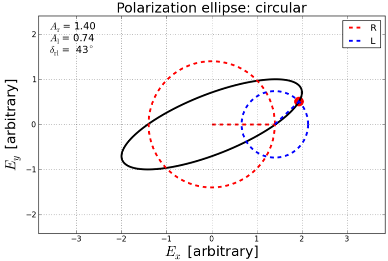

Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]


Polarization ellipse: linear
A_x = 2.00
A_y = 1.00
δ_{xy} = -45°
X
Y
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
3

Polarization ellipse: *xy* 030



Geometry



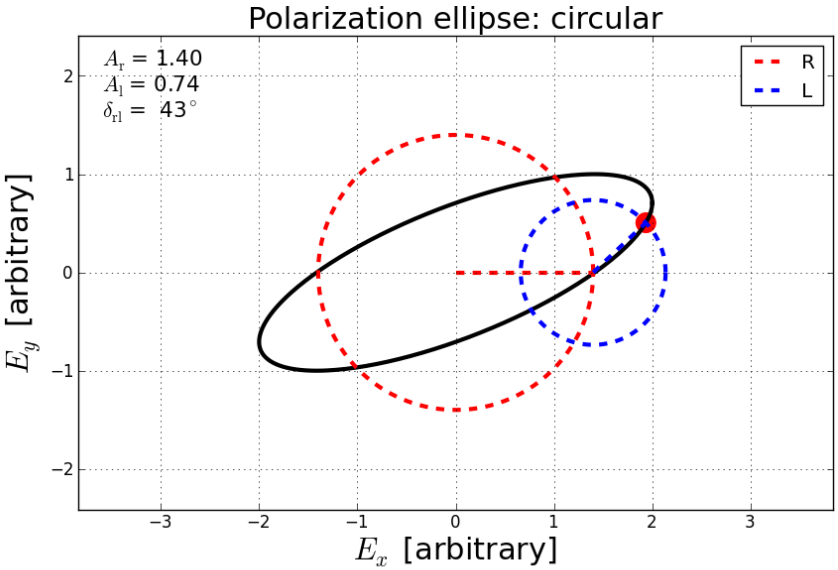
Viewing from antenna towards source, watching orientation and length of **E** vector on a plane at a fixed location in space.

$$\mathbf{E} = A_r \hat{\mathbf{e}}_r + A_l \hat{\mathbf{e}}_l$$

$$\hat{\mathbf{e}}_r = \begin{pmatrix} \cos(2\pi\nu t + \delta_r) \\ \sin(2\pi\nu t + \delta_r) \end{pmatrix}$$

$$\hat{\mathbf{e}}_l = \begin{pmatrix} \cos(2\pi\nu t + \delta_l) \\ -\sin(2\pi\nu t + \delta_l) \end{pmatrix}$$

- $A_r + A_l$ = semi-major axis
- $\|A_r - A_l\|$ = semi-minor axis
- $\delta_{rl} = \delta_l - \delta_r$
- $-\frac{1}{2}\delta_{rl}$ = position angle of MA
- $\delta_{rl} > 0$: MA rotated CW
- $\delta_{rl} = 0$: MA along x-axis
- $\delta_{rl} < 0$: MA rotated CCW


Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_x [arbitrary]
E_y [arbitrary]

Polarization ellipse: circular 001

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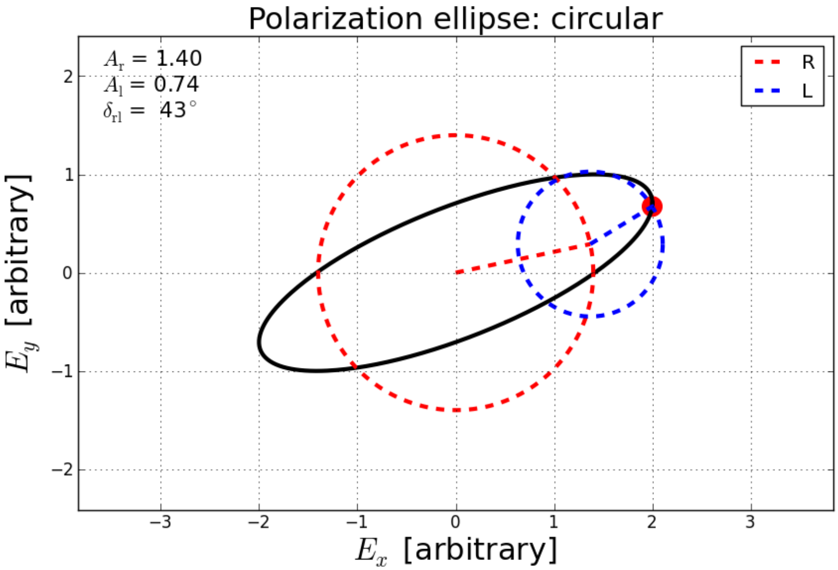
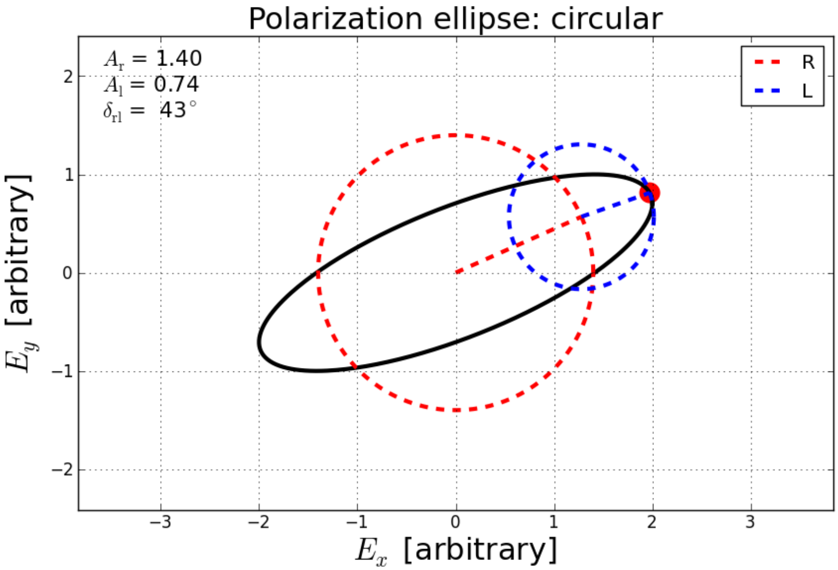
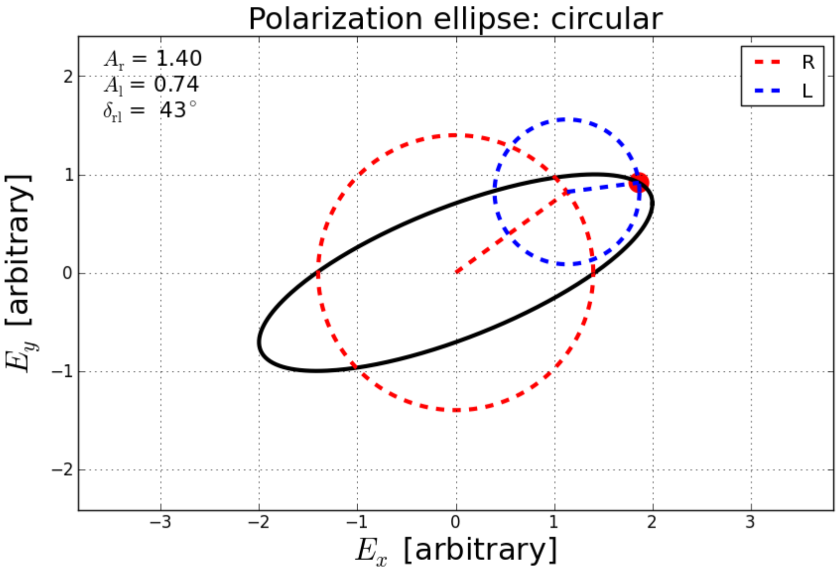
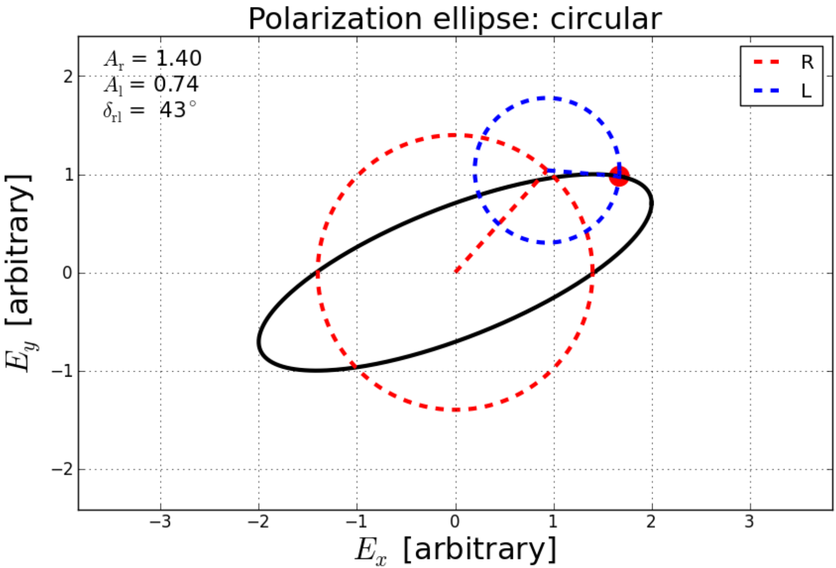



Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
E_y [arbitrary]
E_x [arbitrary]
−3
−2
−1
0
1
2
3
−2
−1
0
1
2

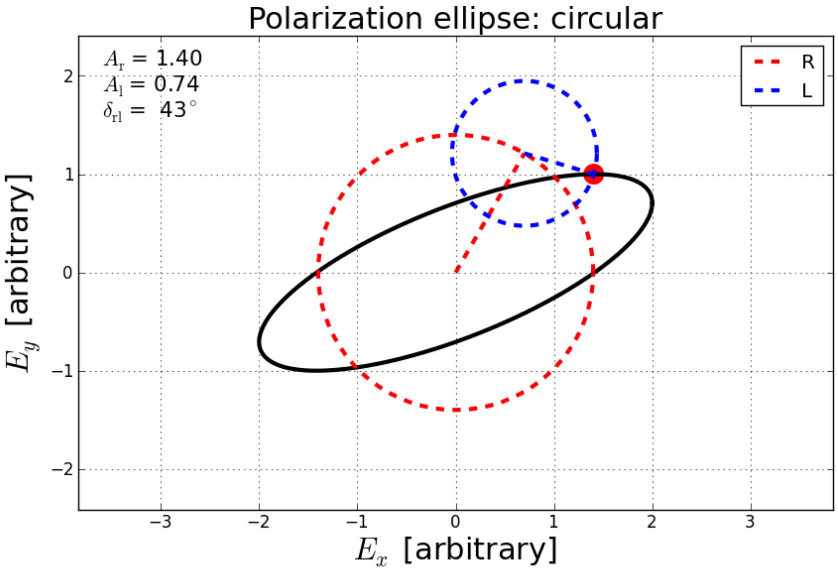
Polarization ellipse: circular 003



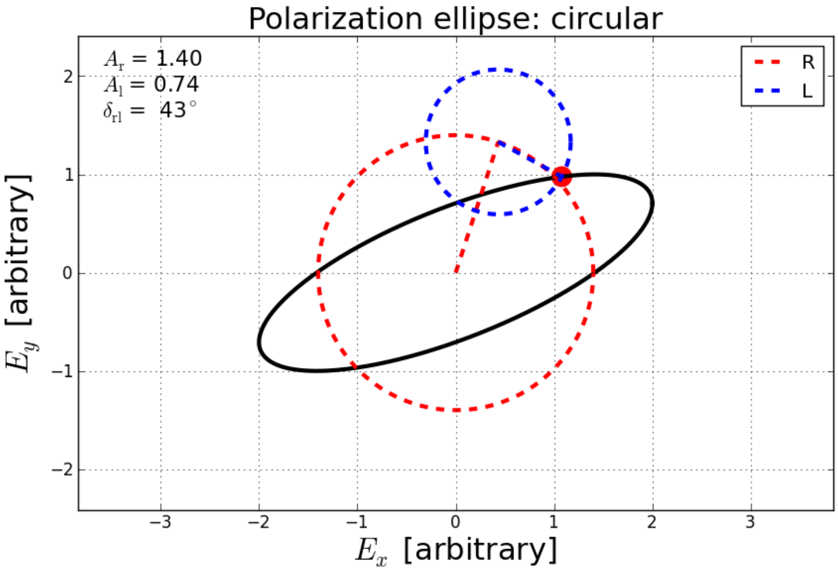
Polarization ellipse: circular 004

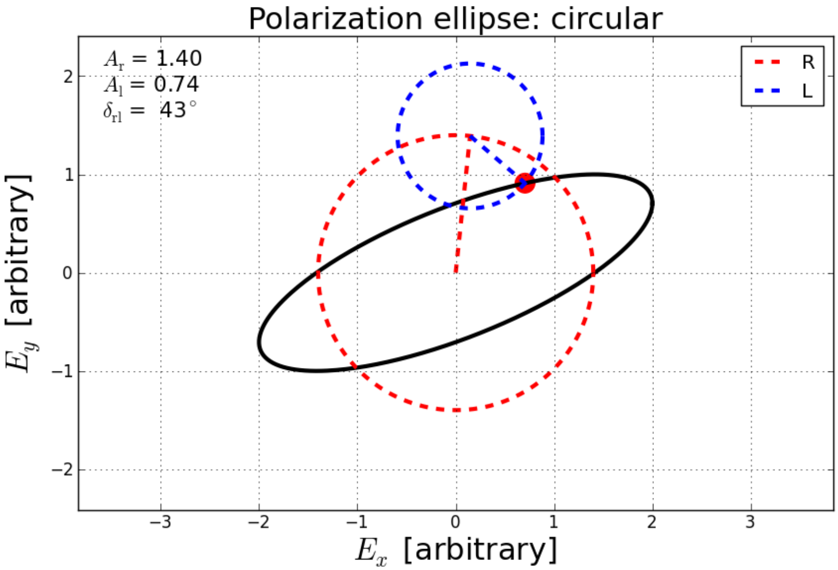


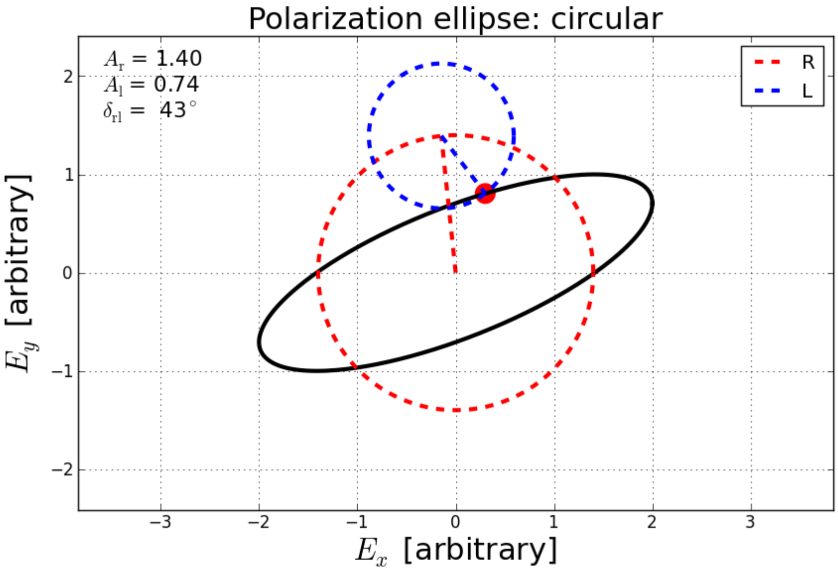
Polarization ellipse: circular 005



Polarization ellipse: circular 006




Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
E_y [arbitrary]
E_x [arbitrary]
2
1
0
−1
−2
−3
−2
−1
0
1
2
3


Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
2
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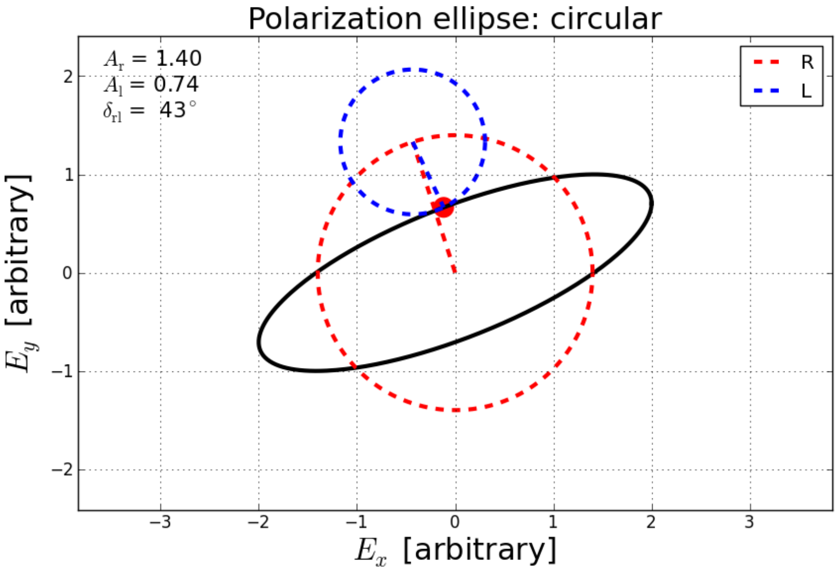
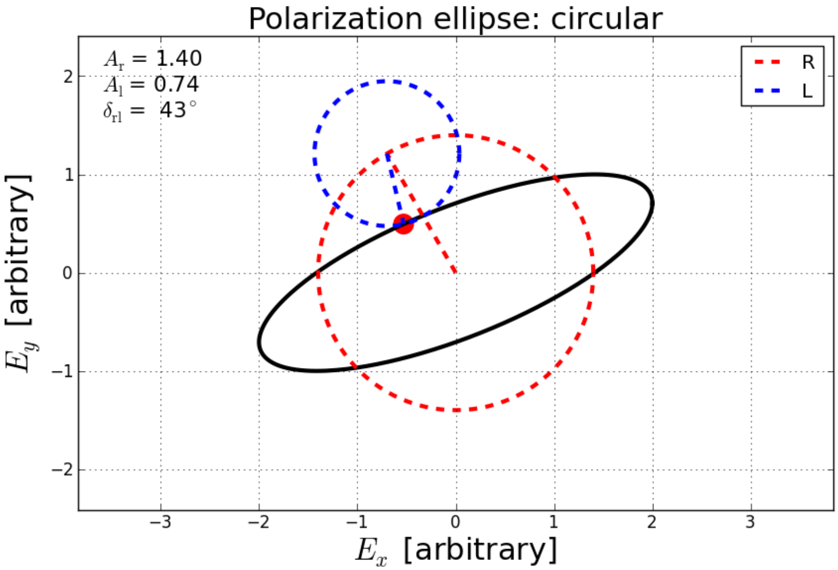
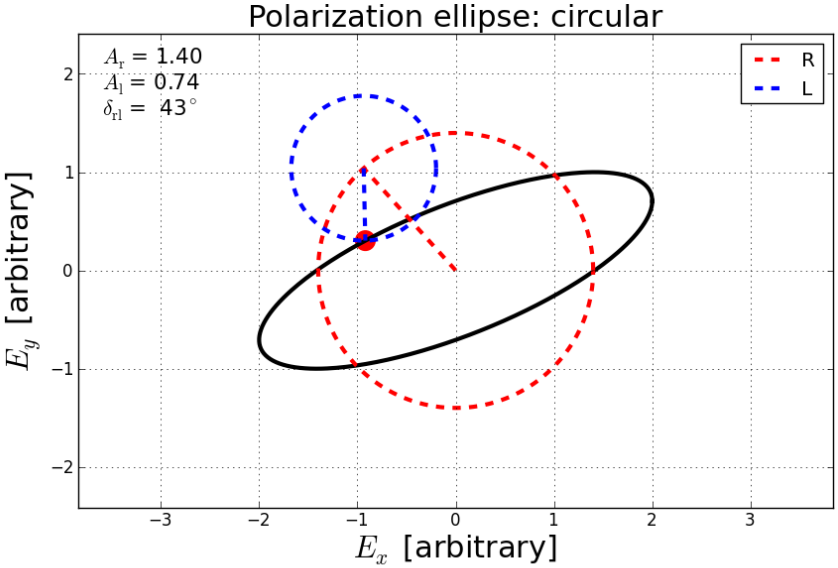
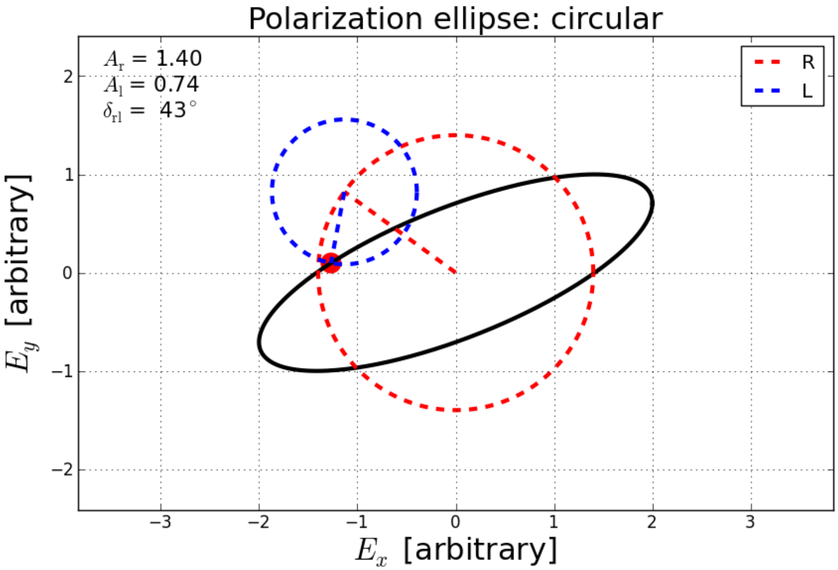
0
−1
−2
−3
−2
−1
0
1
2
3
E_x [arbitrary]
E_y [arbitrary]


Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
2
1
0
−1
−2
−3
3
E_y [arbitrary]
E_x [arbitrary]

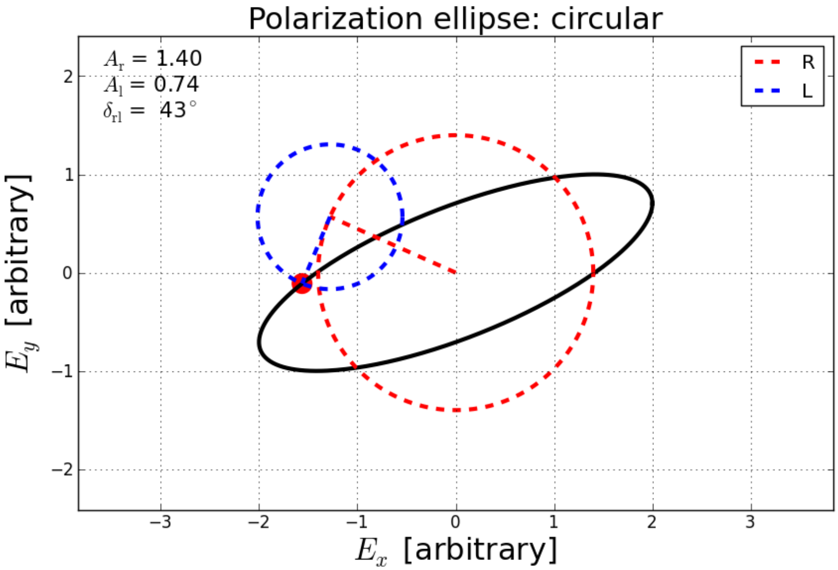
Polarization ellipse: circular 010




Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]


Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_x [arbitrary]
E_y [arbitrary]

ASTRON


Polarization ellipse: circular
A_r = 1.40
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δ_{rl} = 43°
R
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−2
−1
0
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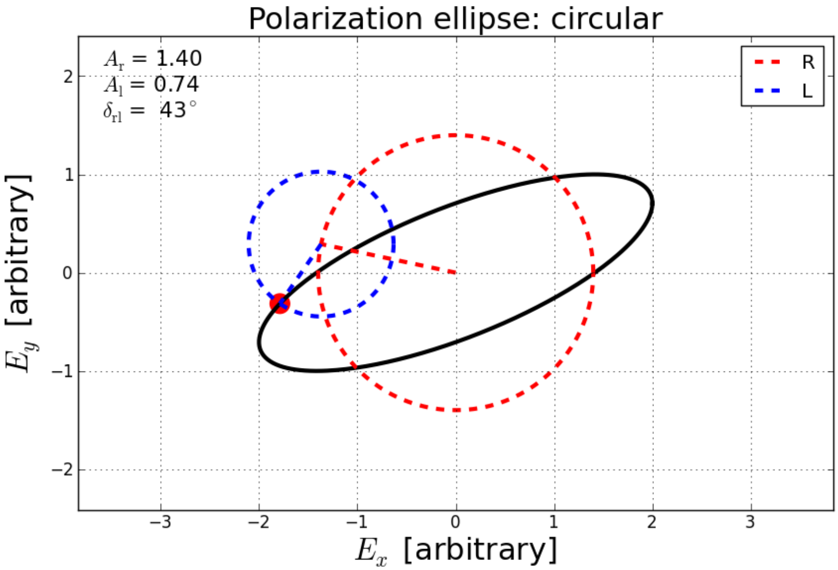
2
3
E_y [arbitrary]
E_x [arbitrary]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A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
2
1
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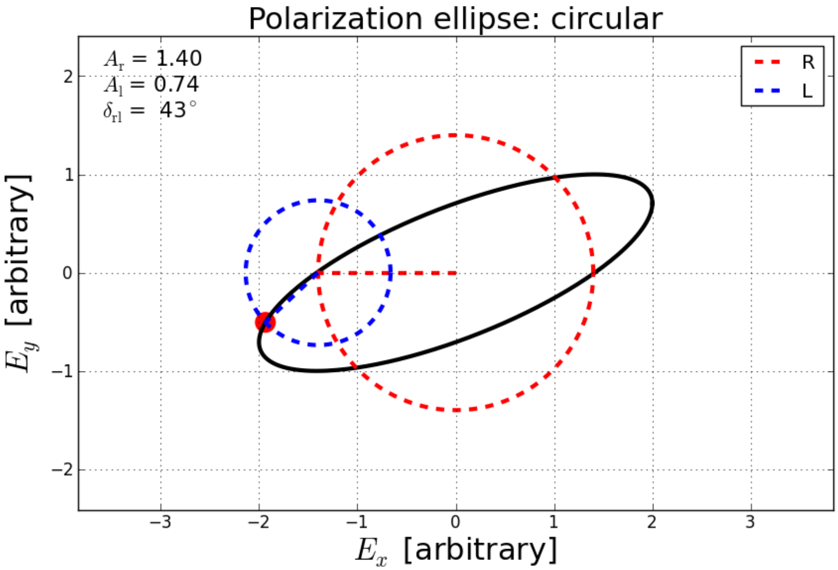
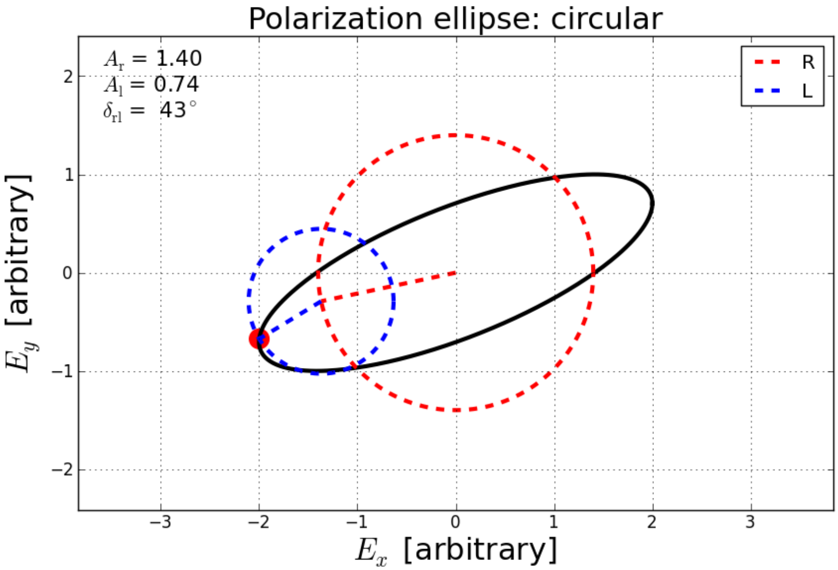
−2
−3
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

Polarization ellipse: circular 015




Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
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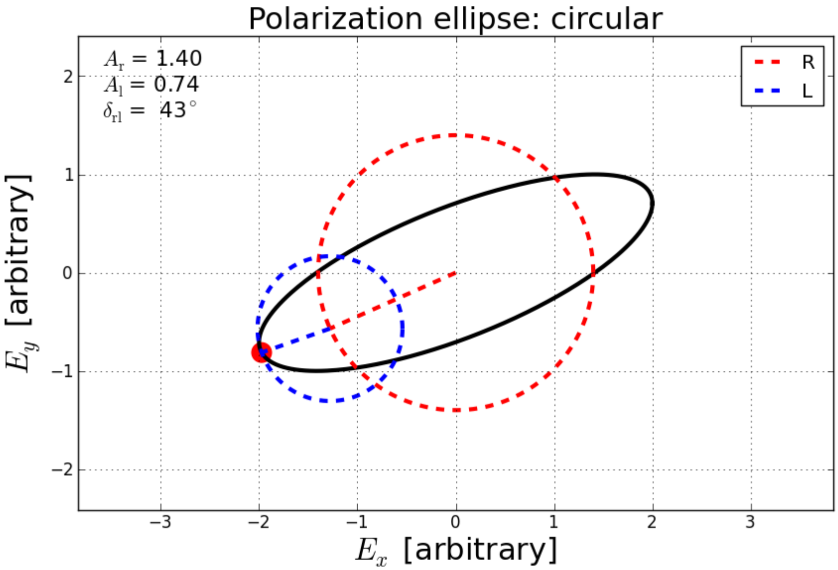
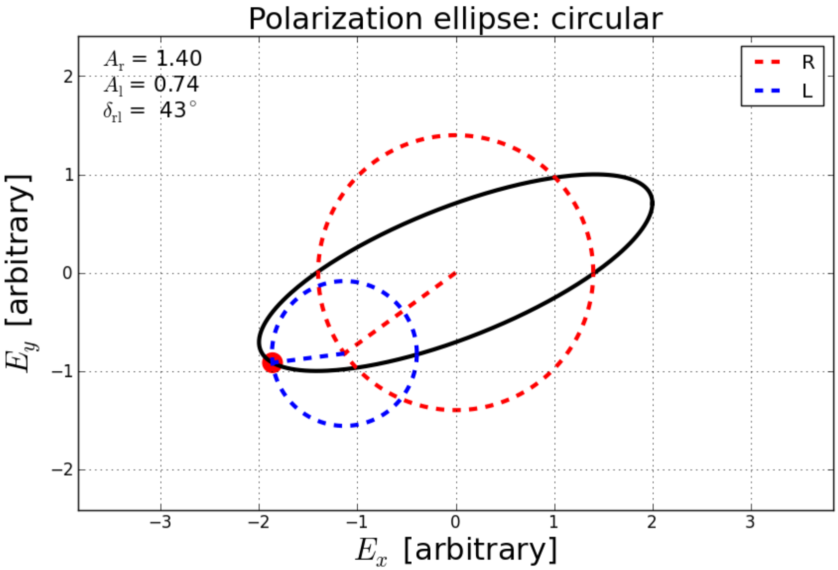
1
0
−1
−2
−3
−2
−1
0
1
2
3
E_x [arbitrary]
E_y [arbitrary]

Polarization ellipse: circular 017




Polarization ellipse: circular
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A_l = 0.74
δ_{rl} = 43°
R
L
2
1
0
−1
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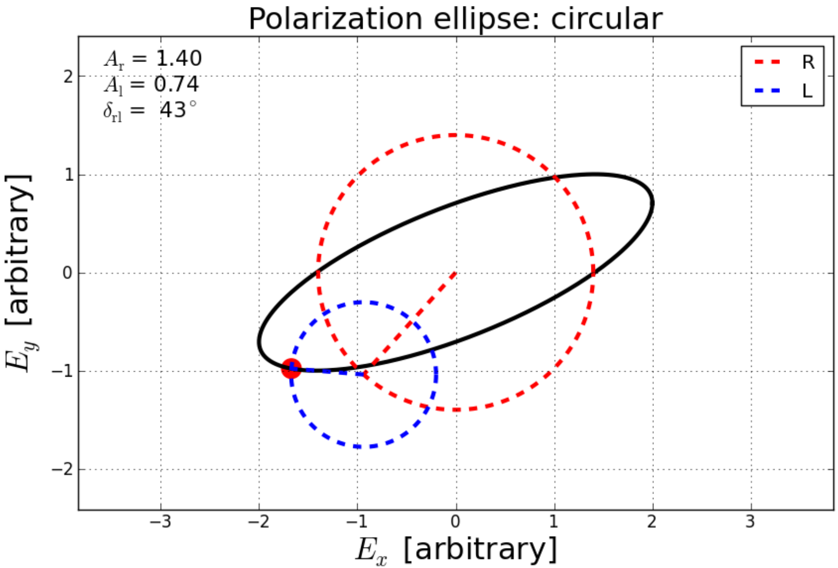
−3
3
E_y [arbitrary]
E_x [arbitrary]


Polarization ellipse: circular
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A_l = 0.74
δ_{rl} = 43°
R
L
E_y [arbitrary]
E_x [arbitrary]
−3
−2
−1
0
1
2
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2
1
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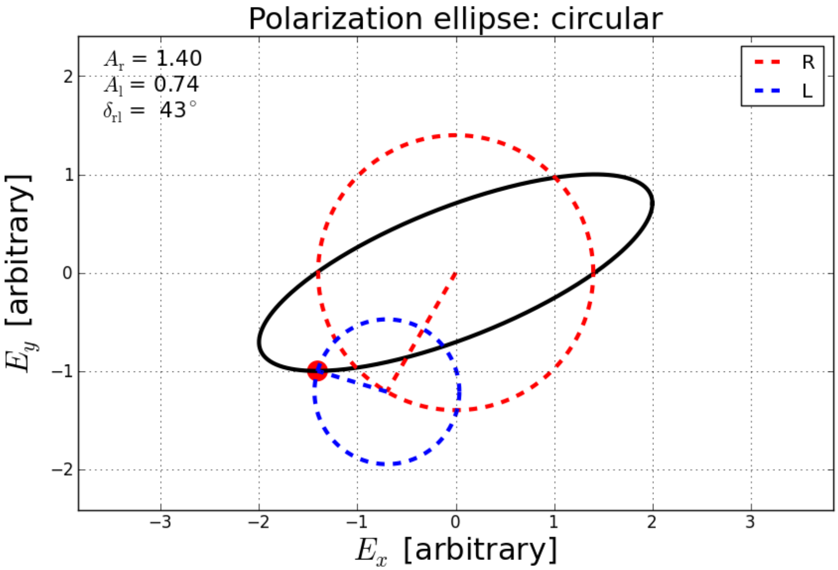
−2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A_l = 0.74
δ_{rl} = 43°
R
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0
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2
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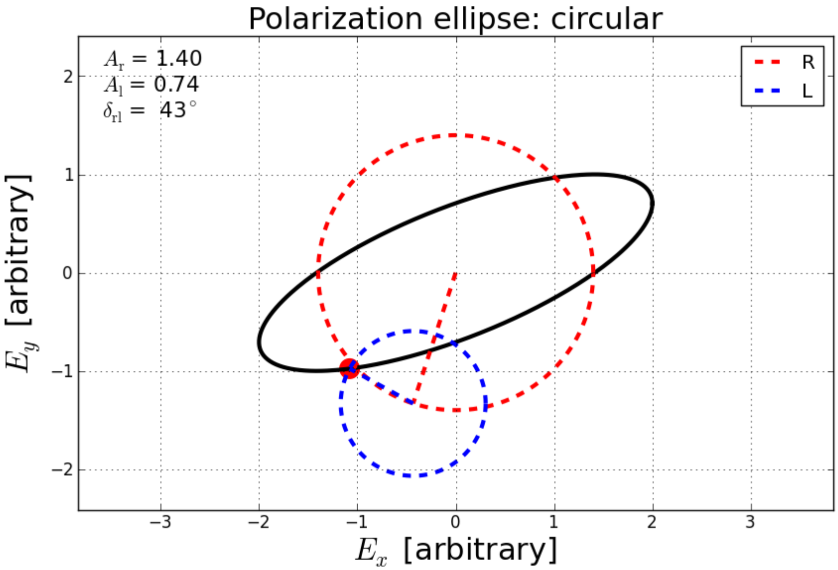
E_x [arbitrary]
E_y [arbitrary]


Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
L
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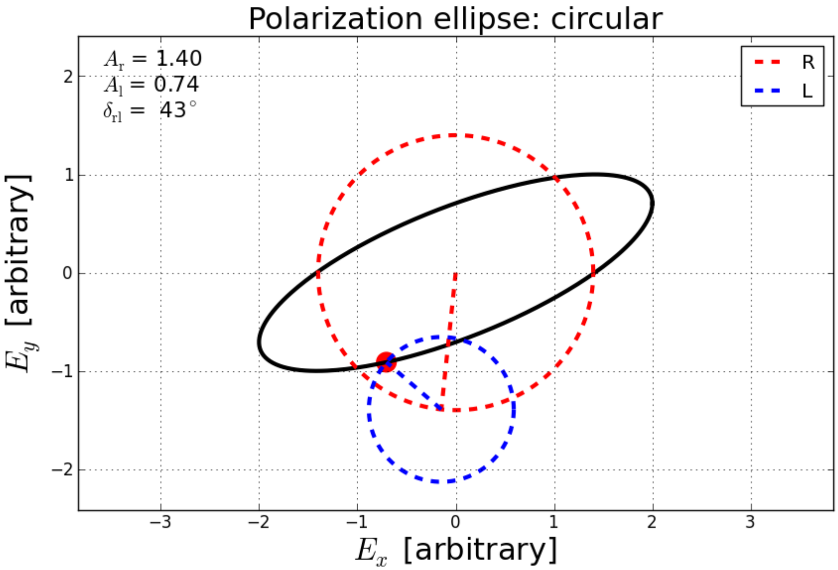
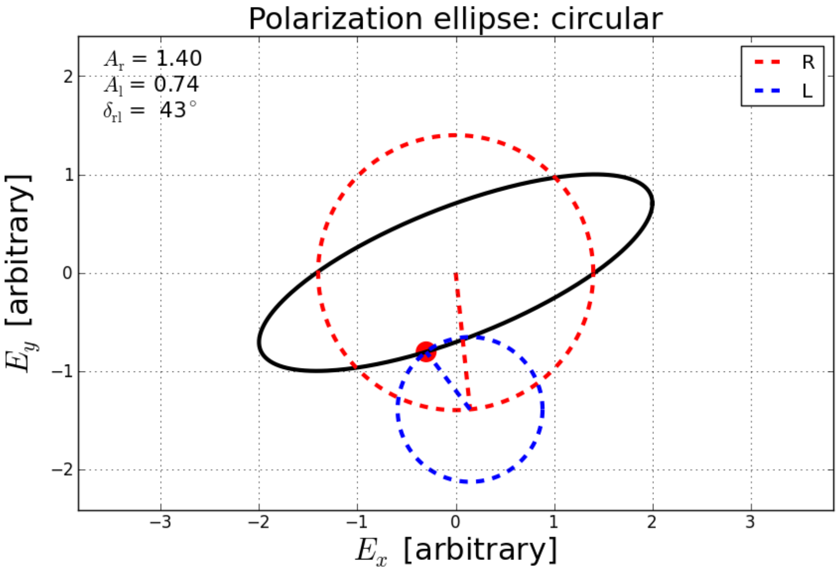
−2
−1
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

Polarization ellipse: circular 022




Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
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−1
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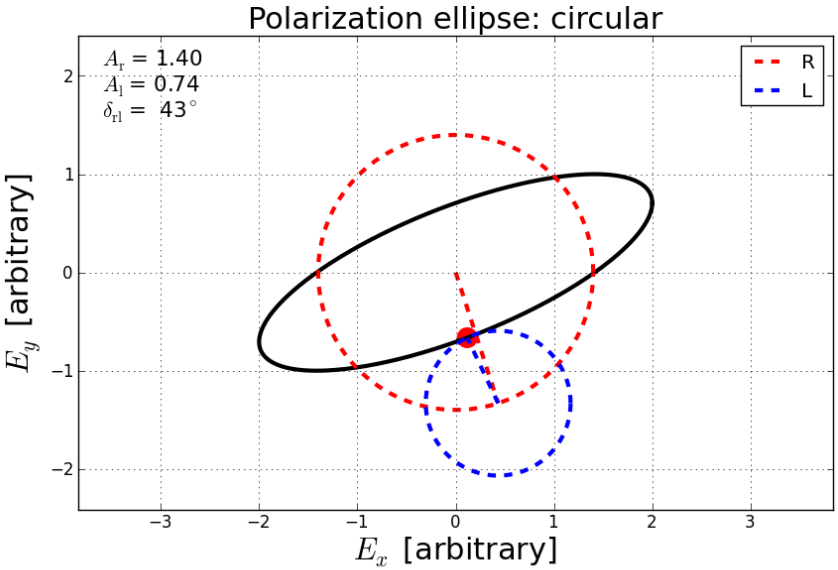
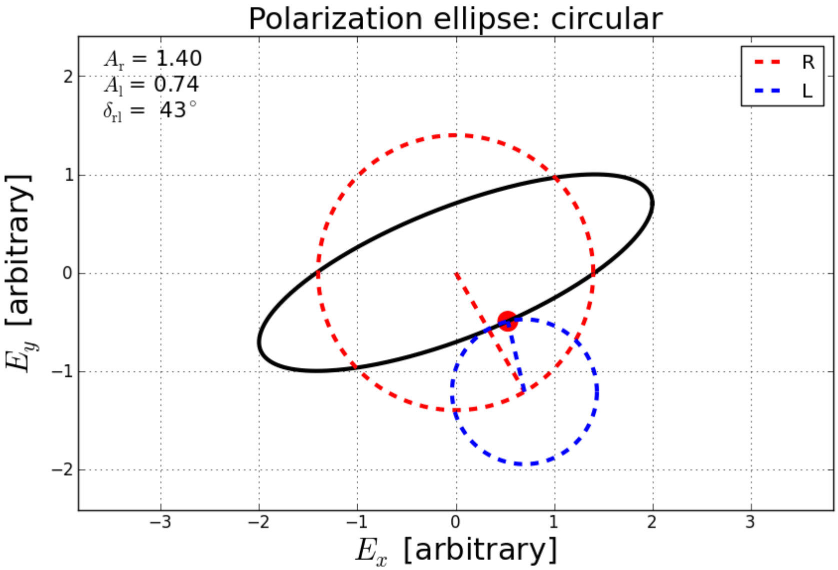
1
2
3
E_y [arbitrary]
E_x [arbitrary]

Polarization ellipse: circular 024




Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
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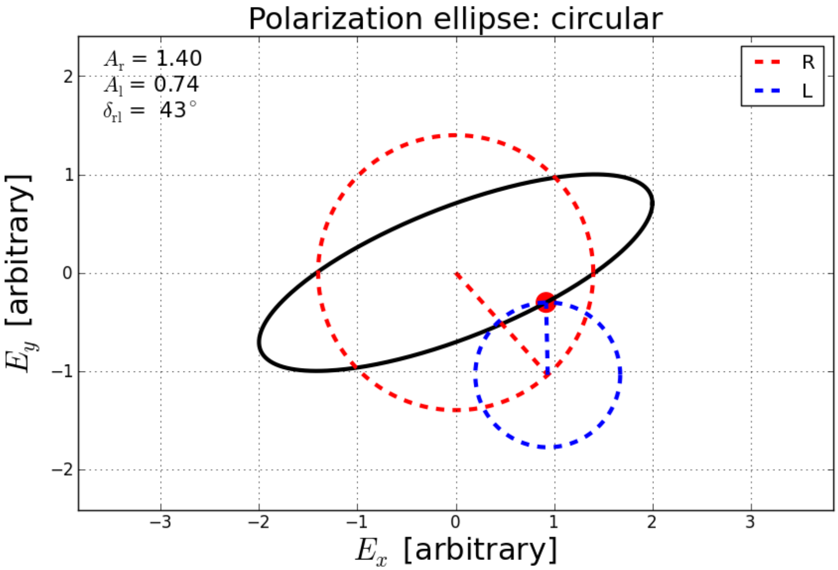
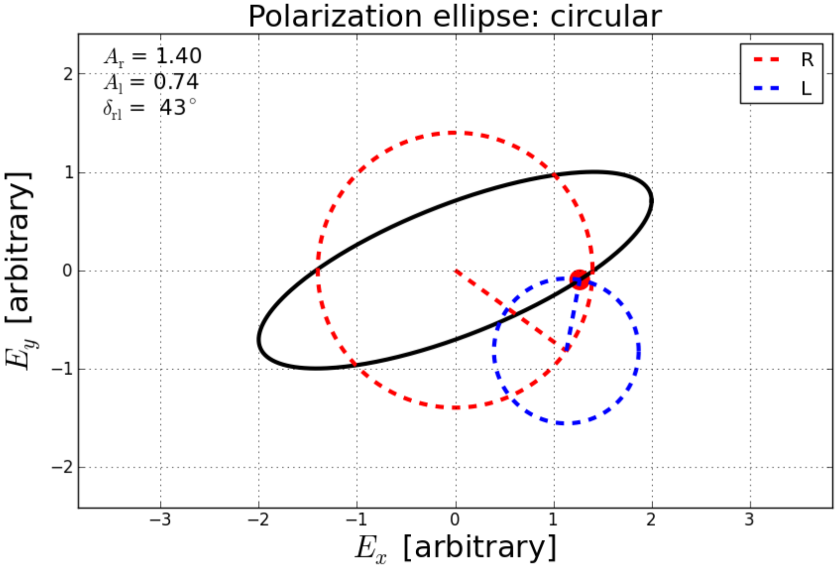
0
1
2
3
E_y [arbitrary]
E_x [arbitrary]

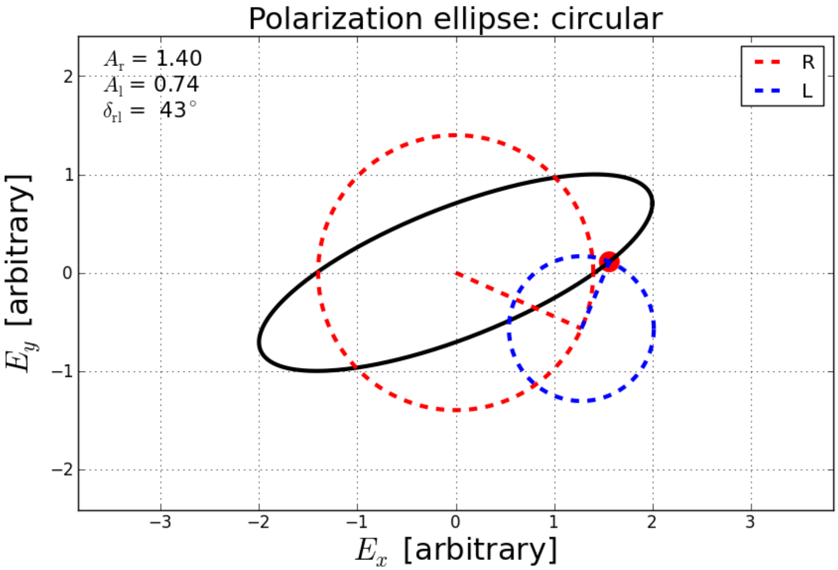
Polarization ellipse: circular 026

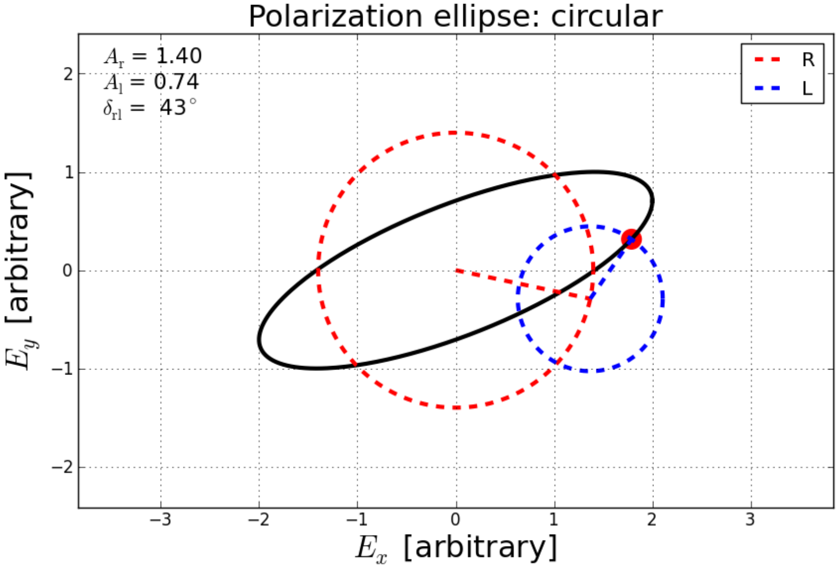


Polarization ellipse: circular 027



Polarization ellipse: circular 028




Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
R
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0
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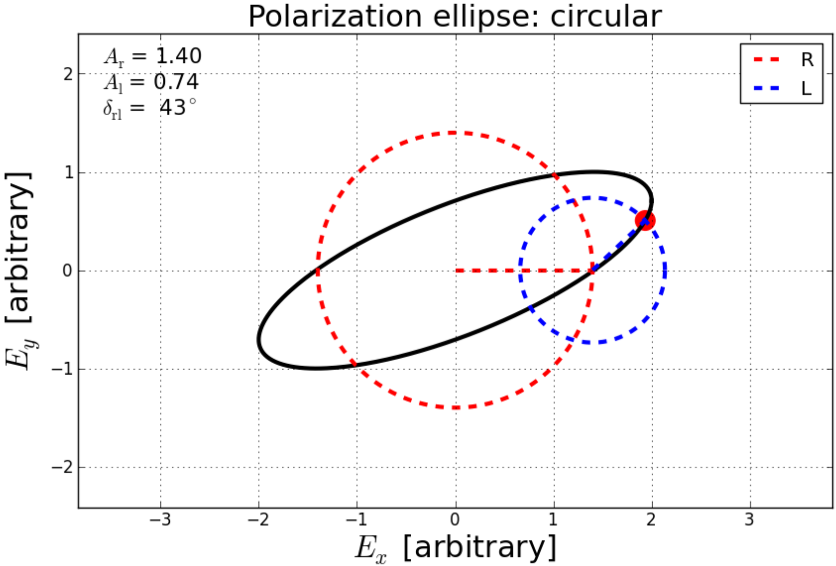
2
3
E_x [arbitrary]
E_y [arbitrary]


Polarization ellipse: circular
A_r = 1.40
A_l = 0.74
δ_{rl} = 43°
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−2
−1
0
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2
3
E_x [arbitrary]
E_y [arbitrary]

Outline

1. Polarized EM waves
2. Stokes parameters
3. Interferometric polarimetry
4. Messy reality

Stokes parameters

- Three parameters enough
- Same units is convenient
- George Stokes defined four parameters (1852)
- Chandrasekhar introduced them to astronomy (1946)

$$\begin{aligned} I &= A_{\mathrm{x}}^2 + A_{\mathrm{y}}^2 \\ Q &= A_{\mathrm{x}}^2 - A_{\mathrm{y}}^2 \\ U &= 2A_{\mathrm{x}}A_{\mathrm{y}} \cos\delta_{\mathrm{xy}} \\ V &= -2A_{\mathrm{x}}A_{\mathrm{y}} \sin\delta_{\mathrm{xy}} \end{aligned}$$

$$\begin{aligned} I &= A_{\mathrm{r}}^2 + A_{\mathrm{l}}^2 \\ Q &= 2A_{\mathrm{r}}A_{\mathrm{l}} \cos\delta_{\mathrm{rl}} \\ U &= -2A_{\mathrm{r}}A_{\mathrm{l}} \sin\delta_{\mathrm{rl}} \\ V &= A_{\mathrm{r}}^2 - A_{\mathrm{l}}^2 \end{aligned}$$

- Monochromatic wave 100% polarized:

$$I^2 = Q^2 + U^2 + V^2$$

Stokes parameters

- Three parameters enough
- Same units is convenient
- George Stokes defined four parameters (1852) *ABCD*
- Chandrasekhar introduced them to astronomy (1946) $I_l I_r U V$

$$I = A_x^2 + A_y^2 \qquad I = A_r^2 + A_l^2$$
$$Q = A_x^2 - A_y^2 \qquad Q = 2A_r A_l \cos\delta_{rl}$$
$$U = 2A_x A_y \cos\delta_{xy} \qquad U = -2A_r A_l \sin\delta_{rl}$$
$$V = -2A_x A_y \sin\delta_{xy} \qquad V = A_r^2 - A_l^2$$

- Monochromatic wave 100% polarized:

$$I^2 = Q^2 + U^2 + V^2$$

Quasi-monochromatic approximation

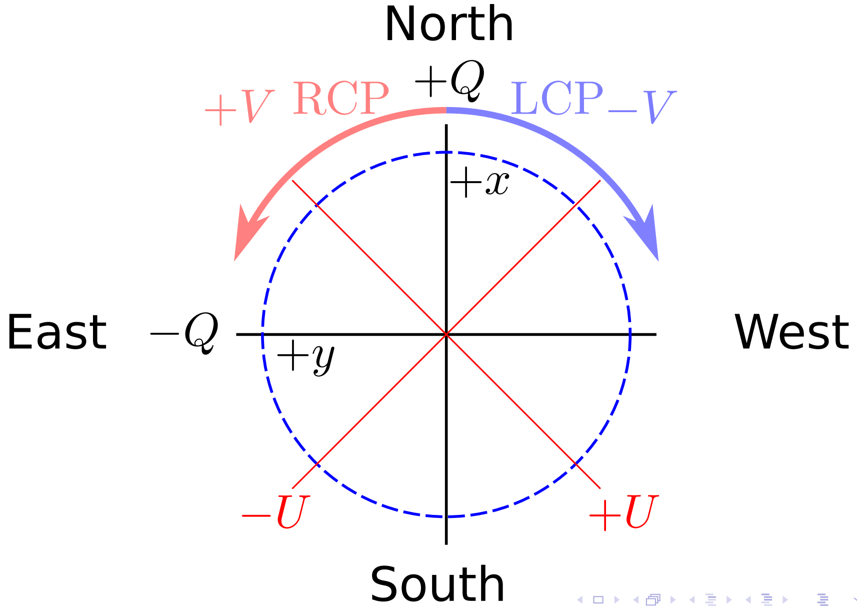
- Monochromatic radiation does not exist
- Finite bandwidth $\Delta\nu$; averaging time $\tau \gg \Delta\nu^{-1}$

$$\begin{aligned} I &= \langle A_{\rm x}^2\rangle + \langle A_{\rm y}^2\rangle \\ Q &= \langle A_{\rm x}^2\rangle - \langle A_{\rm y}^2\rangle \\ U &= \langle 2A_{\rm x}A_{\rm y}\cos\delta_{\rm xy}\rangle \\ V &= \langle -2A_{\rm x}A_{\rm y}\sin\delta_{\rm xy}\rangle \end{aligned}$$

$$\begin{aligned} I &= \langle A_{\rm r}^2\rangle + \langle A_{\rm l}^2\rangle \\ Q &= \langle 2A_{\rm r}A_{\rm l}\cos\delta_{\rm rl}\rangle \\ U &= \langle -2A_{\rm r}A_{\rm l}\sin\delta_{\rm rl}\rangle \\ V &= \langle A_{\rm r}^2\rangle - \langle A_{\rm l}^2\rangle \end{aligned}$$

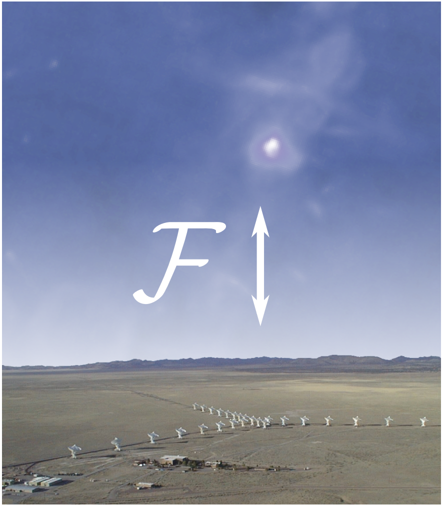
$$I^2 \geq Q^2 + U^2 + V^2$$

- Fractional linear pol: $p = \sqrt{Q^2 + U^2}/I \leq 1$
- Fractional circular pol: $v = \|V\|/I \leq 1$


North
+Q
+V RCP
LCP −V
+x
East
−Q
+y
West
−U
+U
South

Outline

Stokes visibilities



$$
\begin{aligned}
\mathcal{I}(u, v) &= \mathcal{F}^{+}(I(l, m)) \\
\mathcal{Q}(u, v) &= \mathcal{F}^{+}(Q(l, m)) \\
\mathcal{U}(u, v) &= \mathcal{F}^{+}(U(l, m)) \\
\mathcal{V}(u, v) &= \mathcal{F}^{+}(V(l, m)),
\end{aligned}
$$

where

$$
\mathcal{F}^{+}(f) = \int_{lm} f \mathrm{e}^{+2\pi \mathrm{i} \nu (ul + vm)/c} \mathrm{d}l \mathrm{d}m
$$

Cartesian

$$E_x = \Re\left\{A_x e^{2\pi i \nu t}\right\}$$
$$E_y = \Re\left\{A_y e^{i\delta_{xy}} e^{2\pi i \nu t}\right\}$$

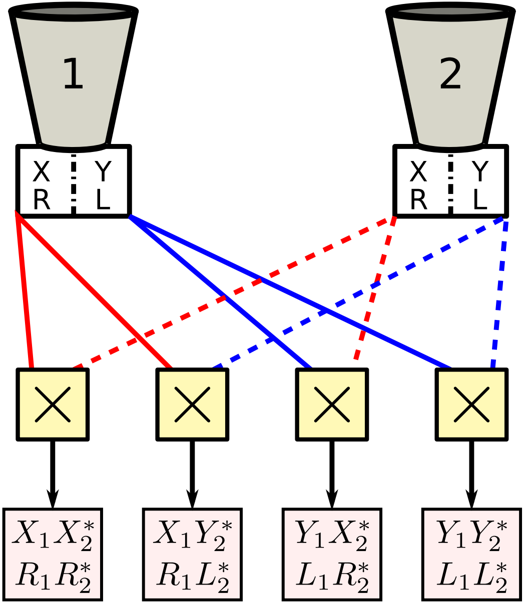
$$I = \langle A_x^2\rangle + \langle A_y^2\rangle = \langle E_x E_x^*\rangle + \langle E_y E_y^*\rangle$$
$$Q = \langle A_x^2\rangle - \langle A_y^2\rangle = \langle E_x E_x^*\rangle - \langle E_y E_y^*\rangle$$
$$U = \langle 2A_x A_y \cos\delta_{xy}\rangle = \langle E_x E_y^*\rangle + \langle E_y E_x^*\rangle$$
$$V = \langle -2A_x A_y \sin\delta_{xy}\rangle = -i\left(\langle E_x E_y^*\rangle - \langle E_y E_x^*\rangle\right)$$

Circular

$$E_\mathrm{r} = \Re\left\{A_\mathrm{r} \mathrm{e}^{2\pi\mathrm{i}\nu t}\right\}$$
$$E_\mathrm{l} = \Re\left\{A_\mathrm{l} \mathrm{e}^{\mathrm{i}\delta_\mathrm{rl}} \mathrm{e}^{2\pi\mathrm{i}\nu t}\right\}$$

$$I = \langle A_\mathrm{r}^2\rangle + \langle A_\mathrm{l}^2\rangle \qquad = \langle E_\mathrm{r}E_\mathrm{r}^*\rangle + \langle E_\mathrm{l}E_\mathrm{l}^*\rangle$$
$$Q = \langle 2A_\mathrm{r}A_\mathrm{l}\cos\delta_\mathrm{rl}\rangle \qquad = \langle E_\mathrm{r}E_\mathrm{l}^*\rangle + \langle E_\mathrm{l}E_\mathrm{r}^*\rangle$$
$$U = \langle -2A_\mathrm{r}A_\mathrm{l}\sin\delta_\mathrm{rl}\rangle \qquad = -\mathrm{i}\left(\langle E_\mathrm{r}E_\mathrm{l}^*\rangle - \langle E_\mathrm{l}E_\mathrm{r}^*\rangle\right)$$
$$V = \langle A_\mathrm{r}^2\rangle - \langle A_\mathrm{l}^2\rangle \qquad = \langle E_\mathrm{r}E_\mathrm{r}^*\rangle - \langle E_\mathrm{l}E_\mathrm{l}^*\rangle$$

Correlating interferometer

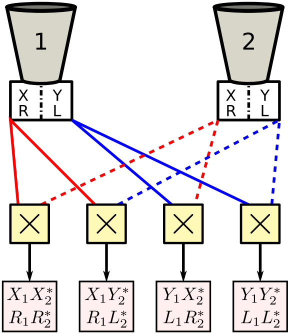


Antenna

Polarizer (XY *or* RL)

Complex correlators

Output



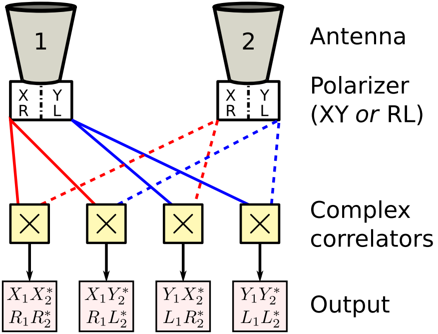
- From here on, $\langle\cdot\rangle$ is implied for correlator outputs.

Cartesian

$$\mathcal{I} = x_1 x_2^* + y_1 y_2^*$$
$$\mathcal{Q} = x_1 x_2^* - y_1 y_2^*$$
$$\mathcal{U} = x_1 y_2^* + y_1 x_2^*$$
$$\mathcal{V} = -\mathrm{i}\,(x_1 y_2^* - y_1 x_2^*)$$

Circular

$$\mathcal{I} = r_1 r_2^* + l_1 l_2^*$$
$$\mathcal{Q} = r_1 l_2^* + l_1 r_2^*$$
$$\mathcal{U} = -\mathrm{i}\,(r_1 l_2^* - l_1 r_2^*)$$
$$\mathcal{V} = r_1 r_2^* - l_1 l_2^*$$



From here on, *p* and *q* designate either *x* and *y*, or *r* and *l*.

- Polarizers produce vector:

$$\mathbf{e}_i = \begin{pmatrix} p_i \\ q_i \end{pmatrix}$$

- Correlator multiplies:

$$\mathbf{E}_{ij} = \mathbf{e}_i \mathbf{e}_j^\dagger = \begin{pmatrix} p_i \\ q_i \end{pmatrix} \begin{pmatrix} p_j^*, q_j^* \end{pmatrix}$$

$$\mathbf{E}_{ij} = \begin{pmatrix} p_i p_j^* & p_i q_j^* \\ q_i p_j^* & q_i q_j^* \end{pmatrix}$$

- $\mathbf{E}_{ij}$ is the coherency matrix

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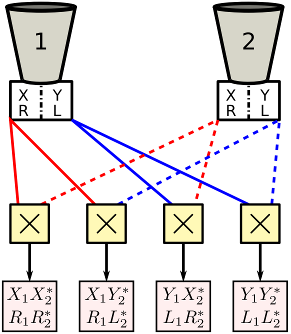
Jones calculus



Until now...

- Assumed all systems perfect

From now...

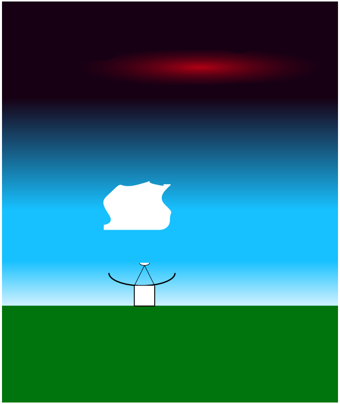
- Assume all systems linear:

$$\mathbf{e}'_i = \mathbf{J}_i \mathbf{e}_i$$

- $\mathbf{J}_i$ (2×2) is Jones matrix
- Cross correlation:

$$\begin{aligned}
\mathbf{E}'_{ij} &= \mathbf{e}'_i \mathbf{e}'^{\dagger}_j \\
\mathbf{E}'_{ij} &= \mathbf{J}_i \mathbf{e}_i \left(\mathbf{J}_j \mathbf{e}_j\right)^{\dagger} \\
\mathbf{E}'_{ij} &= \mathbf{J}_i \mathbf{e}_i \mathbf{e}^{\dagger}_j \mathbf{J}^{\dagger}_j \\
\mathbf{E}'_{ij} &= \mathbf{J}_i \mathbf{E}_{ij} \mathbf{J}^{\dagger}_j
\end{aligned}$$

Measurement equation



- The measurement equation:

$$\mathbf{E}'_{ij} = \mathbf{J}_i \mathbf{E}_{ij} \mathbf{J}_j^\dagger$$

- Invertible!

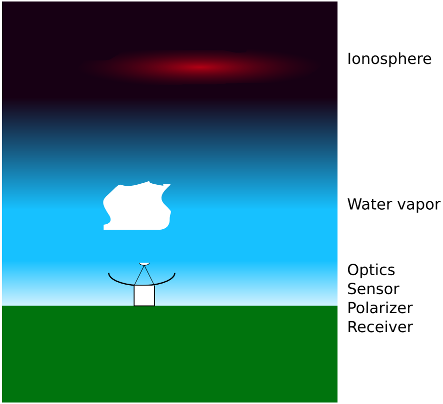
$$\mathbf{E}_{ij} = \mathbf{J}_i^{-1} \mathbf{E}'_{ij} \mathbf{J}_j^{\dagger -1},$$

- where

$$\mathbf{J} = \mathbf{RPDOWFT}$$

- ...riiiiight...

Jones matrices: examples



- Perfect instrument:

$$\mathbf{J} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

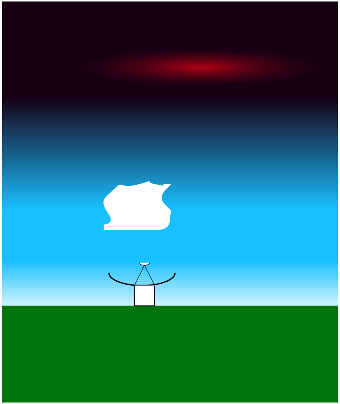
- Time delay:

$$\mathbf{J} = \begin{pmatrix} \mathrm{e}^{2\pi\mathrm{i}\nu\tau_p} & 0 \\ 0 & \mathrm{e}^{2\pi\mathrm{i}\nu\tau_q} \end{pmatrix}$$

- Receiver gain:

$$\mathbf{J} = \begin{pmatrix} g_\mathrm{p} & 0 \\ 0 & g_\mathrm{q} \end{pmatrix}$$

Jones matrices: examples



Ionosphere

Water vapor

Optics
Sensor
Polarizer
Receiver

- Polarization leakage:

$$\mathbf{J} = \begin{pmatrix} g_{\mathrm{p}} & d_{\mathrm{q}\rightarrow\mathrm{p}} \\ d_{\mathrm{p}\rightarrow\mathrm{q}} & g_{\mathrm{q}} \end{pmatrix}$$

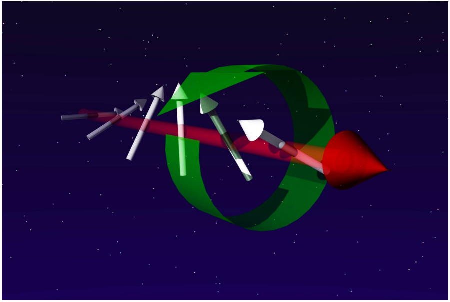
- Parallactic angle or feed rotation XY:

$$\mathbf{J} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

- Parallactic angle or feed rotation RL:

$$\mathbf{J} = \begin{pmatrix} 1 & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\theta} \end{pmatrix}$$

Faraday rotation

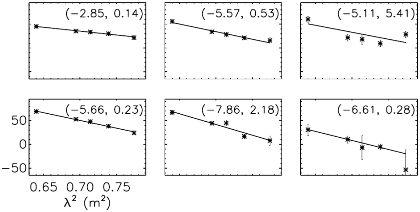


Process

- Modifies polarization state
- Delay between LCP and RCP
- Rotates linear pol angle
- $\Delta\chi = \chi_0 + \phi\lambda^2$

$$\phi = 0.812 \int_{\mathrm{there}}^{\mathrm{here}} n_e \mathbf{B} \cdot \mathrm{d}\mathbf{l}$$

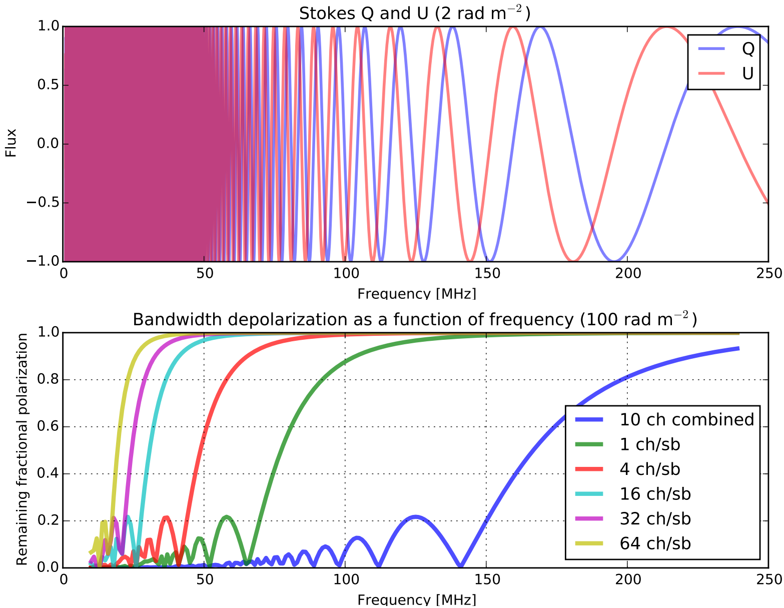
λ^2 law *Haverkorn et al. (2001)*



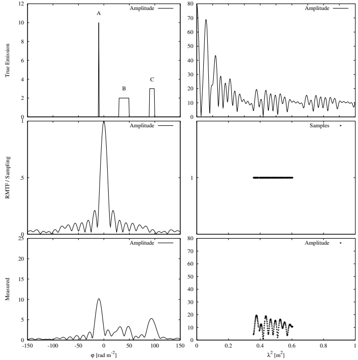
Polarimetry provides

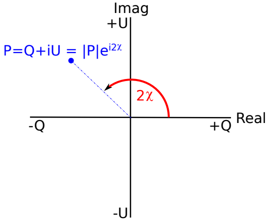
- Source plasma properties
- Intervening plasma properties
- Rare cases: 3D tomography

Faraday rotation in practice



Faraday rotation measure synthesis

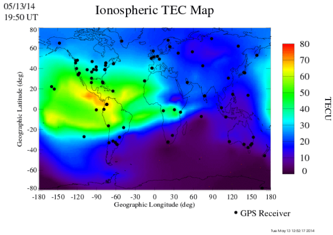


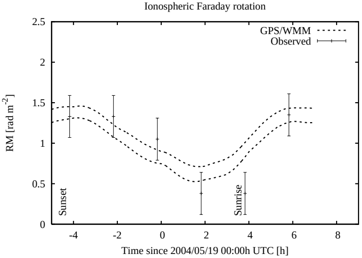


$$P(\lambda_c^2) = \int_{\lambda^{2\prime}} \int_{\Omega} \int_{-\infty}^{\infty} W(\Omega, \lambda_c^2 + \lambda^{2\prime}) f(\Omega, \phi, \lambda_c^2 + \lambda^{2\prime}) e^{2i\phi(\lambda_c^2 + \lambda^{2\prime})} \, d\phi \, d\Omega \, d\lambda^{2\prime}$$

$$P(\lambda^2) = W(\lambda^2) \int_{-\infty}^{\infty} f(\phi) e^{2i\phi\lambda^2} \, d\phi$$

$$f(\phi) * R(\phi) = \int_{-\infty}^{\infty} P(\lambda^2) e^{-2i\phi\lambda^2} \, d\lambda^2$$





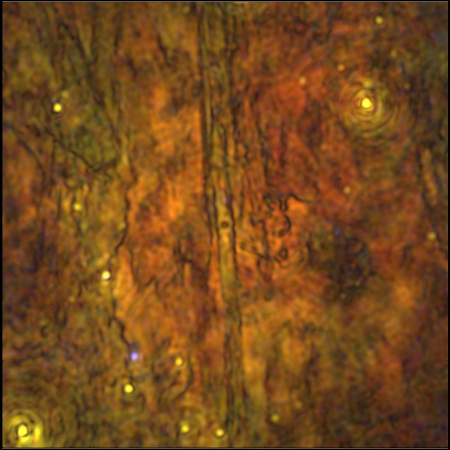
- Remember: $\Delta\chi = \chi_0 + \phi\lambda^2$
- Faraday depth

$$\phi = 0.812 \int_{\text{there}}^{\text{here}} n_e \mathbf{B} \cdot d\mathbf{l}$$

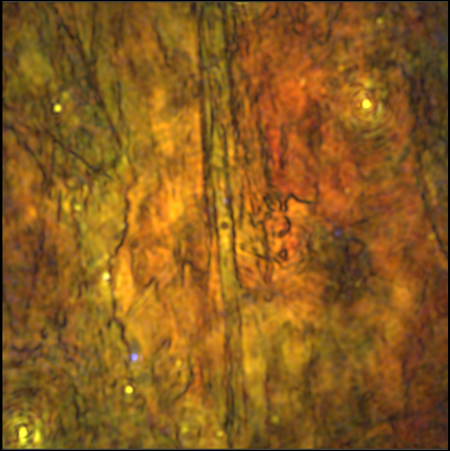
- ionosphere: plasma within Earth's magnetic field
- $\phi \approx$ -10 – +10 rad m^{-2}
- *Very* significant below 1 GHz
- Use TEC/WMM models for correction, check with pulsar.

Applied to maps of 3C 196

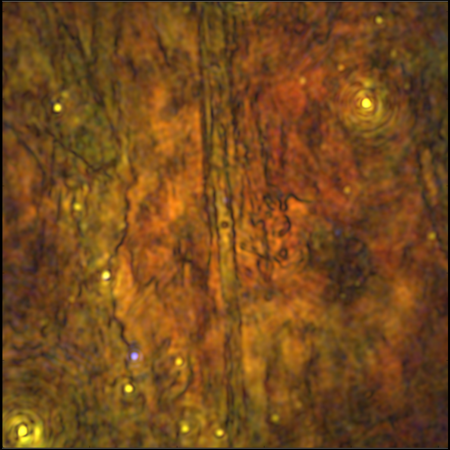
No correction



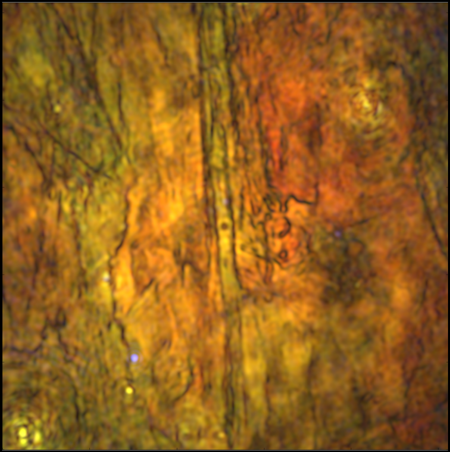
GPS TEC Corrected



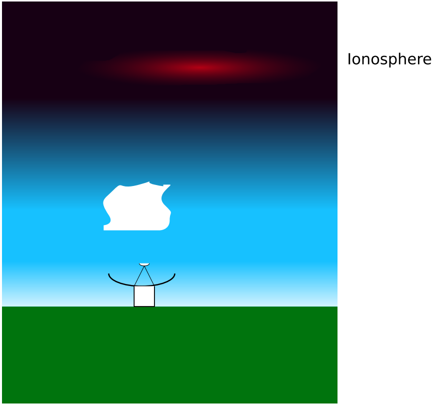
No correction



Corrected RM self cal

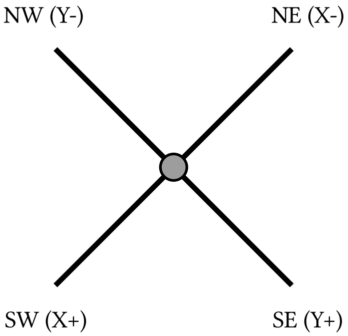


Differential Ionospheric Faraday rotation

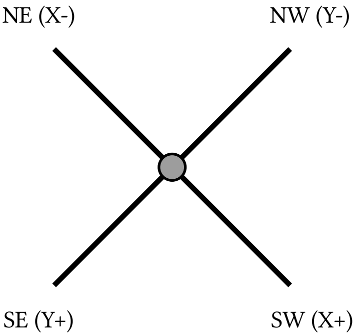


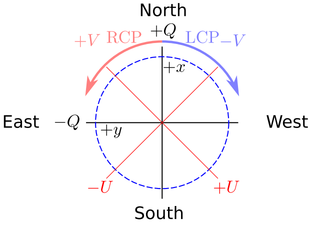
- $\Delta\chi = \chi_0 + \phi\lambda^2$
- Rotation of linear pol = delay between RCP and LCP
- Antennas see different ionosphere / Faraday rotation
- Changes $\mathcal{I}$ into $\mathcal{V}$ during cross correlation
- PROVE IT! (hint: use Jones / coherency matrices)
- Important below 300 MHz at baselines $\geq$ 20 km

Top-down zenith view

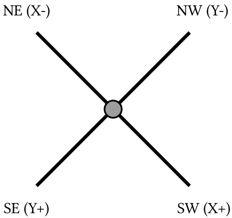


Bottom-up zenith view





Bottom-up zenith view



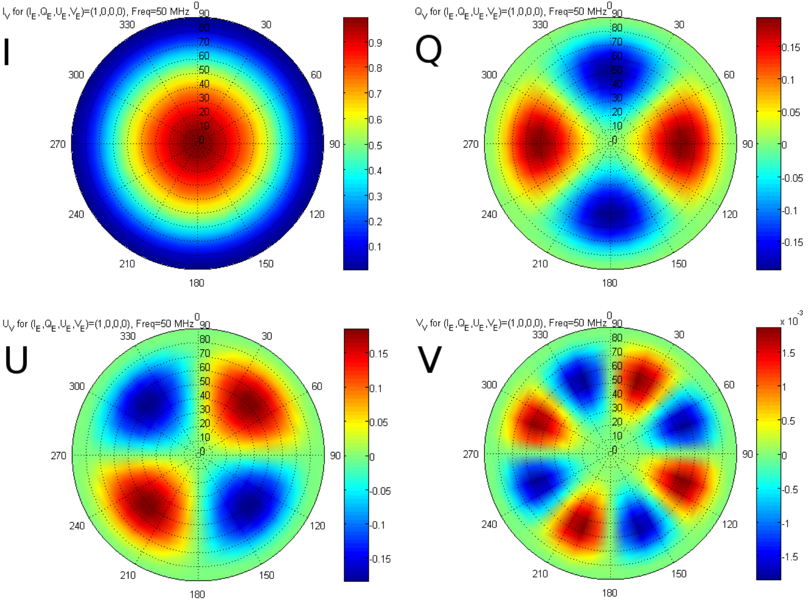
Raw visibilities

- Left-hand vs right-hand coordinate systems
- Rotated by 45 degrees
- $I_{\mathrm{raw}} \sim I_{\mathrm{true}}$
- $Q_{\mathrm{raw}} \sim U_{\mathrm{true}}$
- $U_{\mathrm{raw}} \sim -Q_{\mathrm{true}}$
- $V_{\mathrm{raw}} \sim -V_{\mathrm{true}}$
- $\phi_{\mathrm{raw}} = -\phi_{\mathrm{true}}$

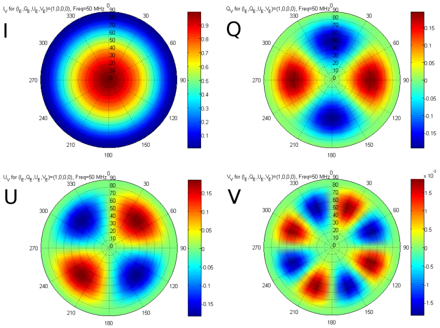
REMEMBER:

- Apply element beam model!

LBA element beam model (50 MHz)



LBA element beam model (50 MHz)



- Axes are *not* azimuth elevation
- Plots are looking *down* to antenna
- “Azimuth” is measured with respect to x dipole, not bisector between x and y dipoles
- Not actually IAU Stokes parameters
- Instead, just $xx + yy$, $xx - yy$, $xy + yx$, $-\mathrm{i}(xy - yx)$
- Fortunately, BBS beam model does take all these things into account

Take away this...

- Radio antennas are fundamentally polarized
- Polarimetry required for certain astrophysical observations
- Linear systems make for fairly straightforward calibration
- Understanding polarimetry essential for unpolarized calibration and imaging
- Although LOFAR polarization leakage in any random direction is likely horrific, it hardly changes across the field of view in the HBA
- Compensation for ionospheric (differential) Faraday rotation required
- *Listen and read very carefully, and believe nobody, including me and yourself*

Reading list

- Born & Wolf *Principles of optics*
- Thompson, Moran & Swenson *Interferometry and Synthesis in Radio Astronomy*
- Taylor, Carilli & Perley *Synthesis Imaging in Radio Astronomy II*
- Bracewell *The Fourier Transform & Its Applications*
- Hamaker, Bregman & Sault *Understanding radio polarimetry: paper I*(1996)
- Sault, Hamaker& Bregman *paper II*(1996)
- Hamaker & Bregman *paper III* (1996)
- Hamaker *paper IV* (2000)
- Hamaker *paper V* (2006)
- Brentjens & de Bruyn *Faraday rotation measure synthesis* (2005)