Wide Field Imaging

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Introduction

Imaging LOFAR data poses additional challenges

- Very wide fields of view
- Stations are fixed on the ground. Beam shape varies with time

AWImager

- Stand-alone version of CASA Imager
- Support for LOFAR Beam and ionosphere
- W-Stack: Faster gridding for very wide fields

Imaging as Inversion Problem

The Measurement Equation

$$v = \int\int \exp 2\pi(ul + vm + nw) A(l,m) I(l,m)$$

The Measurement Equation can be written as a system of linear equations:

$$\mathbf{v} = \mathbf{A}\mathbf{i}$$

where $\mathbf{v}$ are the visibilities, $\mathbf{i}$ the source fluxes or pixel values, $\mathbf{A}$ describes the relation between the two. Problem: what is $\mathbf{i}$ given $\mathbf{v}$? Generic solution:

$$\mathbf{i} = \left(\mathbf{A}^{\mathbf{H}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathbf{H}}\mathbf{v}$$

Direct inversion of $\mathbf{A}^{\mathbf{H}}\mathbf{A}$ is not feasible.

Iterative Solution

Minimize sum of squared errors

$$\mathbf{i} = \arg\min_{\mathbf{i}} \|\mathbf{v} - \mathbf{A}\mathbf{i}\|^2$$

Find the best matching image, starting from an empty image, updating the image using the derivative

$$\frac{\partial}{\partial \mathbf{i}} \|\mathbf{v} - \mathbf{A}\mathbf{i}\|^2 = \mathbf{A}^{\mathrm{H}} (\mathbf{v} - \mathbf{A}\mathbf{i})$$

The residual image is the derivative of the cost function.

CLEAN major cycle

1. start with an empty model image
2. make a dirty (residual) image
3. if threshold has been reached then stop
4. find clean components and update the model image
5. compute model data
6. compute residual data and go to step 2

Convolution Theorem

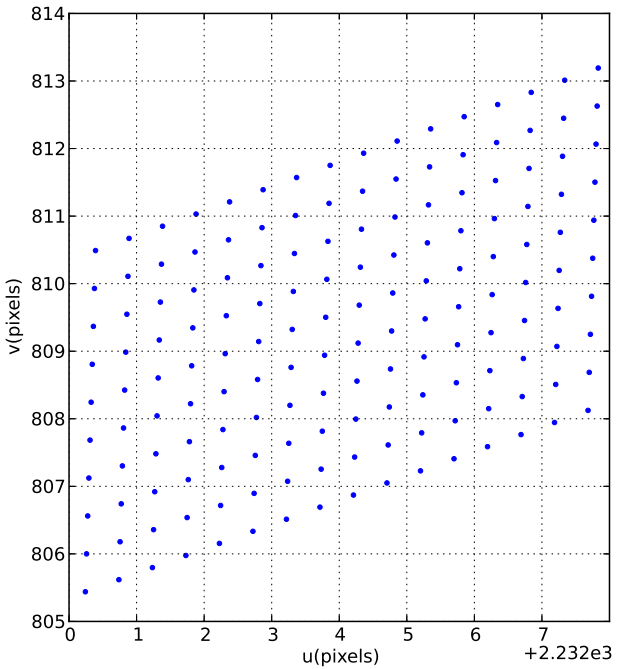
Measurement Equation resembles a Fourier transform.

An important property of the Fourier transform is the convolution theorem.

$$\mathcal{F}(f * g) = \mathcal{F}(f) \cdot \mathcal{F}(g)$$

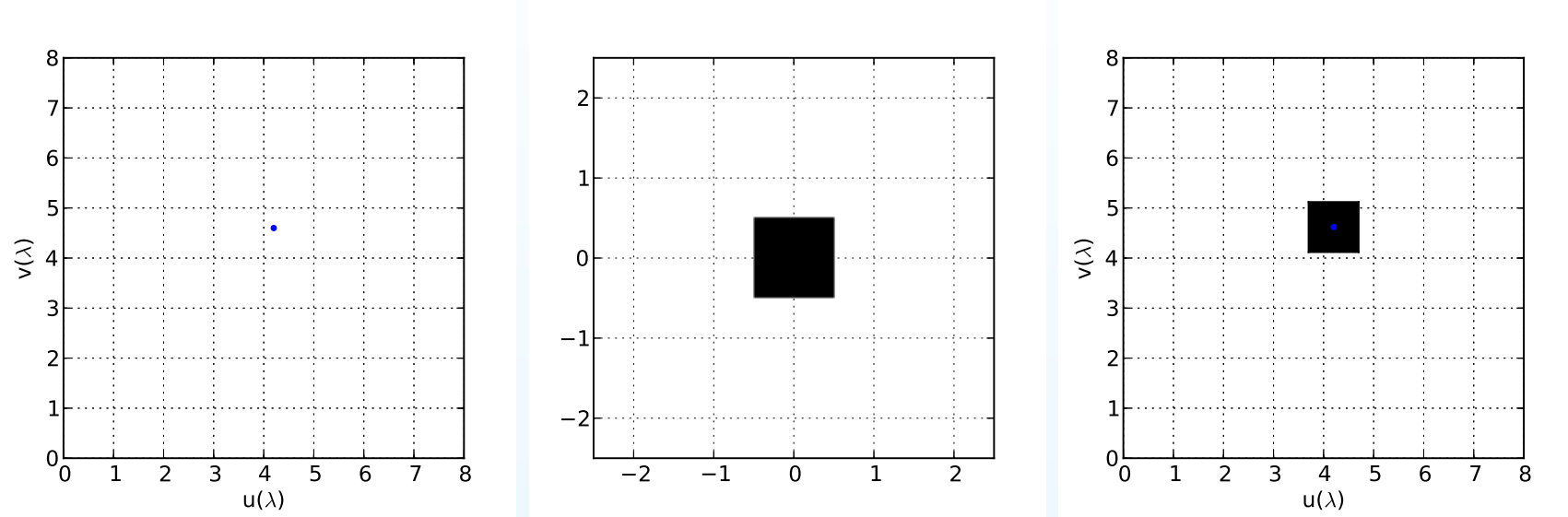
$$\mathcal{F}(f \cdot g) = \mathcal{F}(f) * \mathcal{F}(g)$$

Track of a baseline in uv grid



Gridding

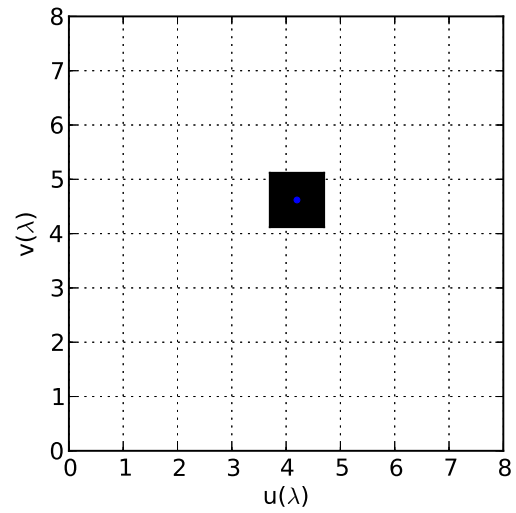
Convolution with 2D box function

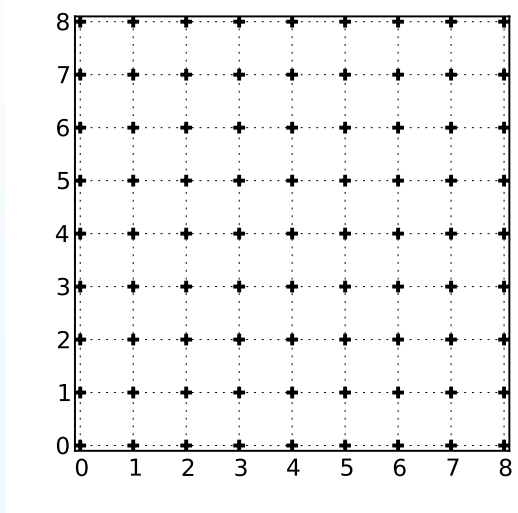


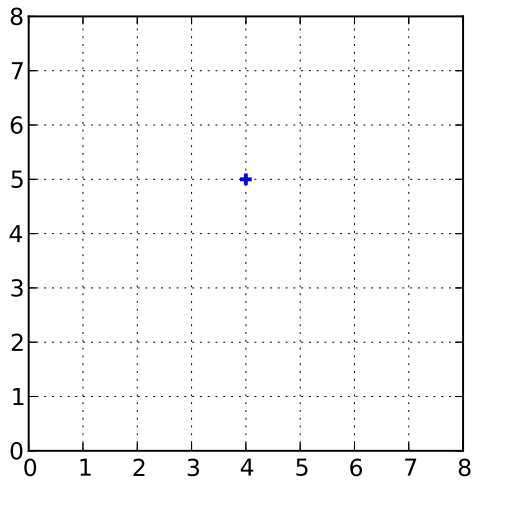
The result overlaps with only a single (integer) grid point

Gridding

Multiplication by 2D Dirac comb function



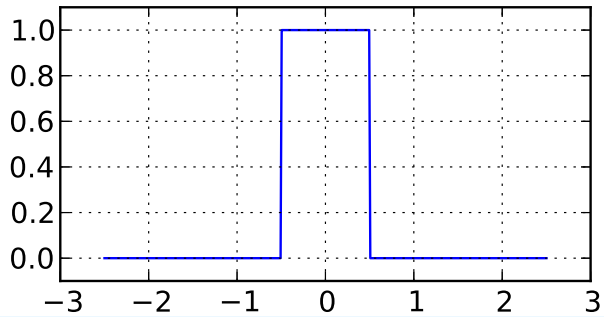


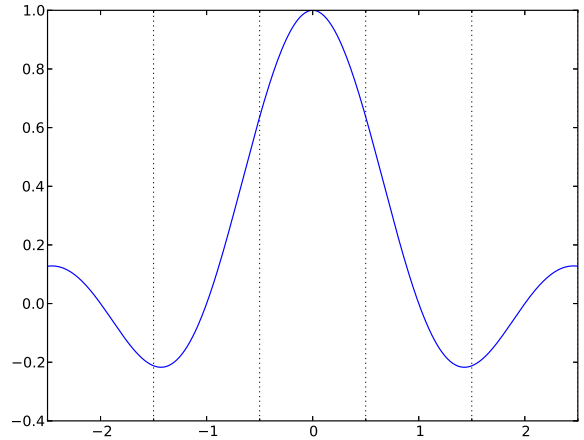


Selects only the value on integer grid points. The result is gridding on the nearest grid point.

What does gridding do in the image domain?

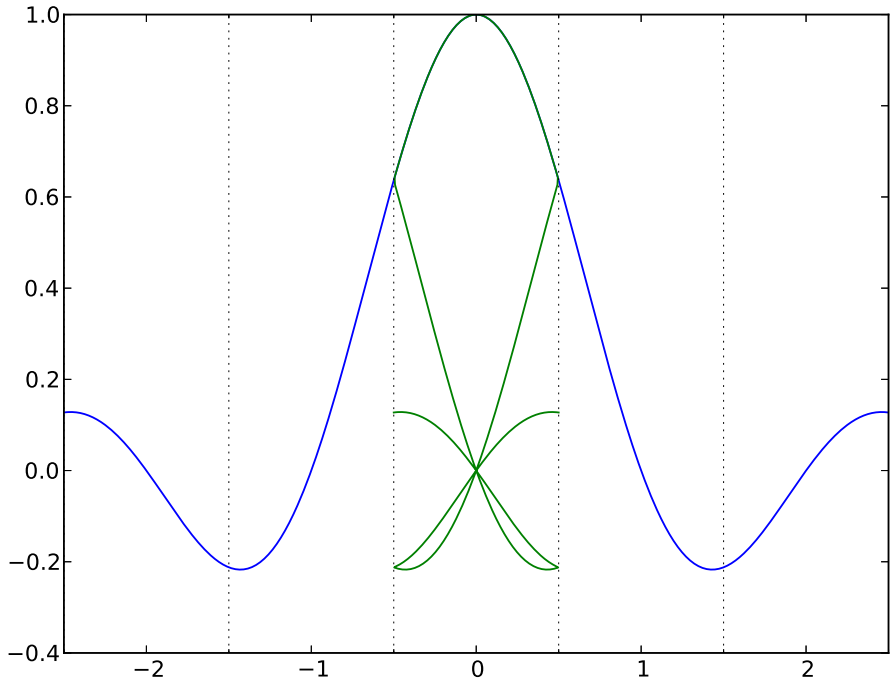
Remember: convolution in the uv domain is a multiplication in the image domain





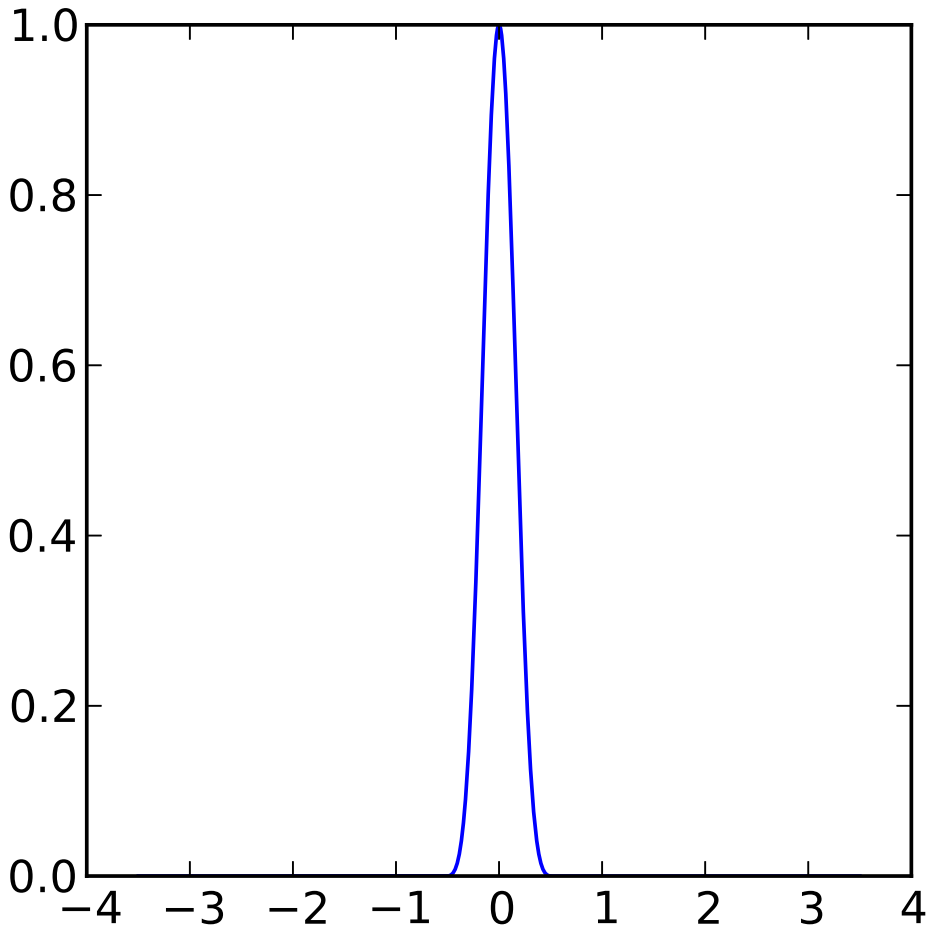
What does gridding do in the image domain?

And multiplication in the uv domain is a convolution in the image domain



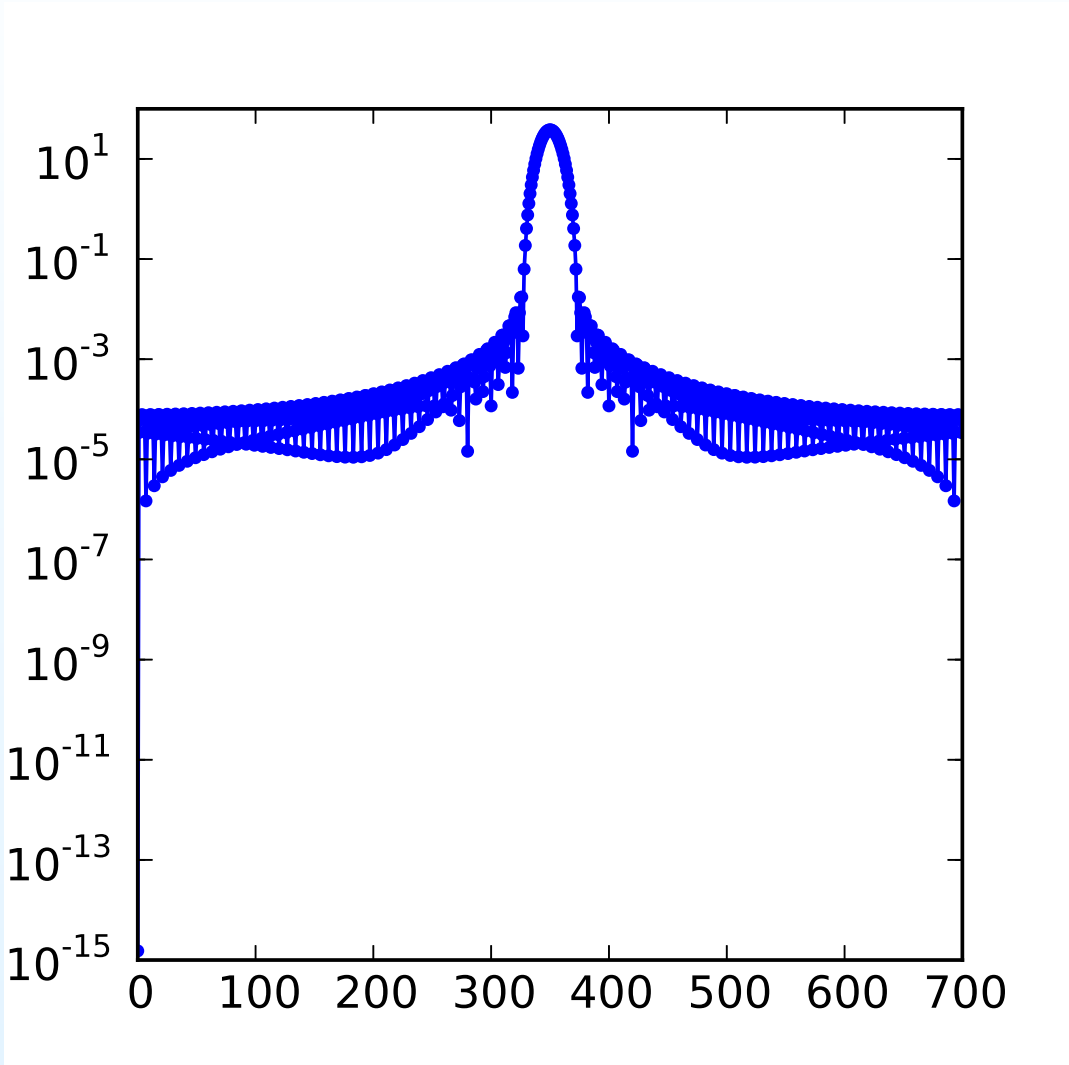
The Fourier transform of a comb function is again a comb. A convolution with a comb is a sum of shifted versions of the original. Aliases of sources outside the image will appear in the image.

Spheroidal I

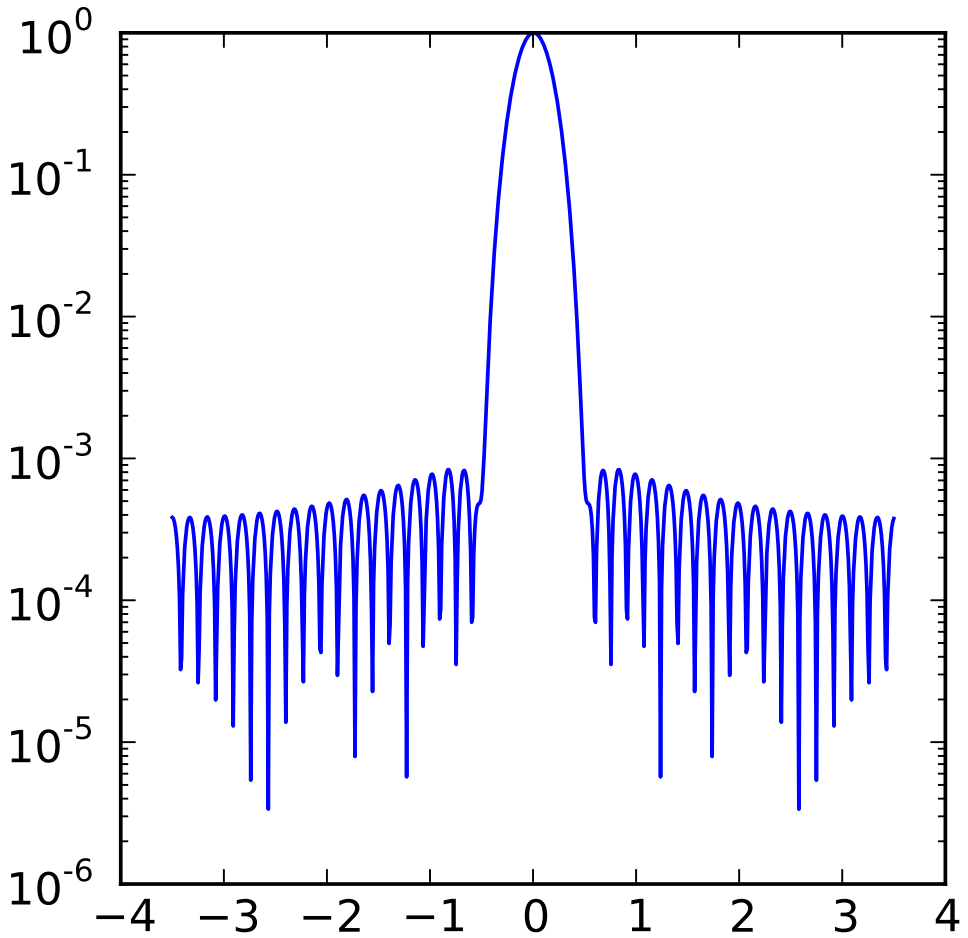


Parameters: gridding.padding

Spheroidal II

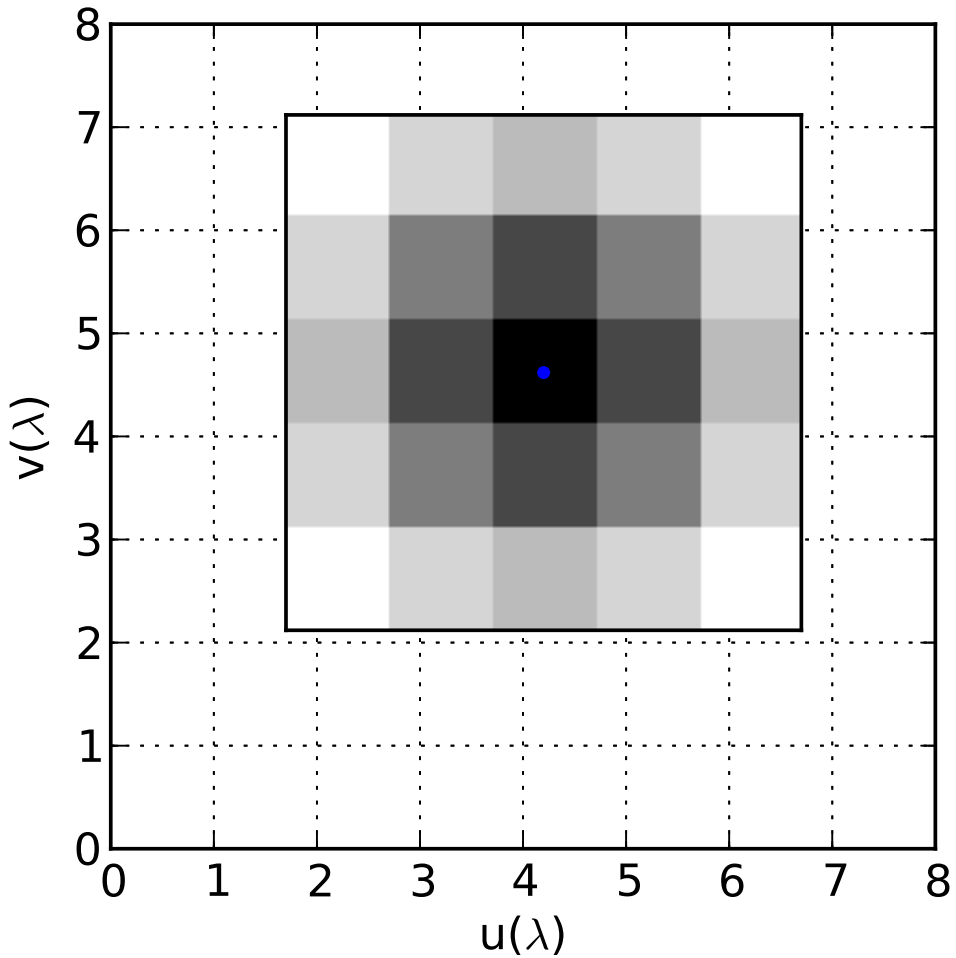


Spheroidal III

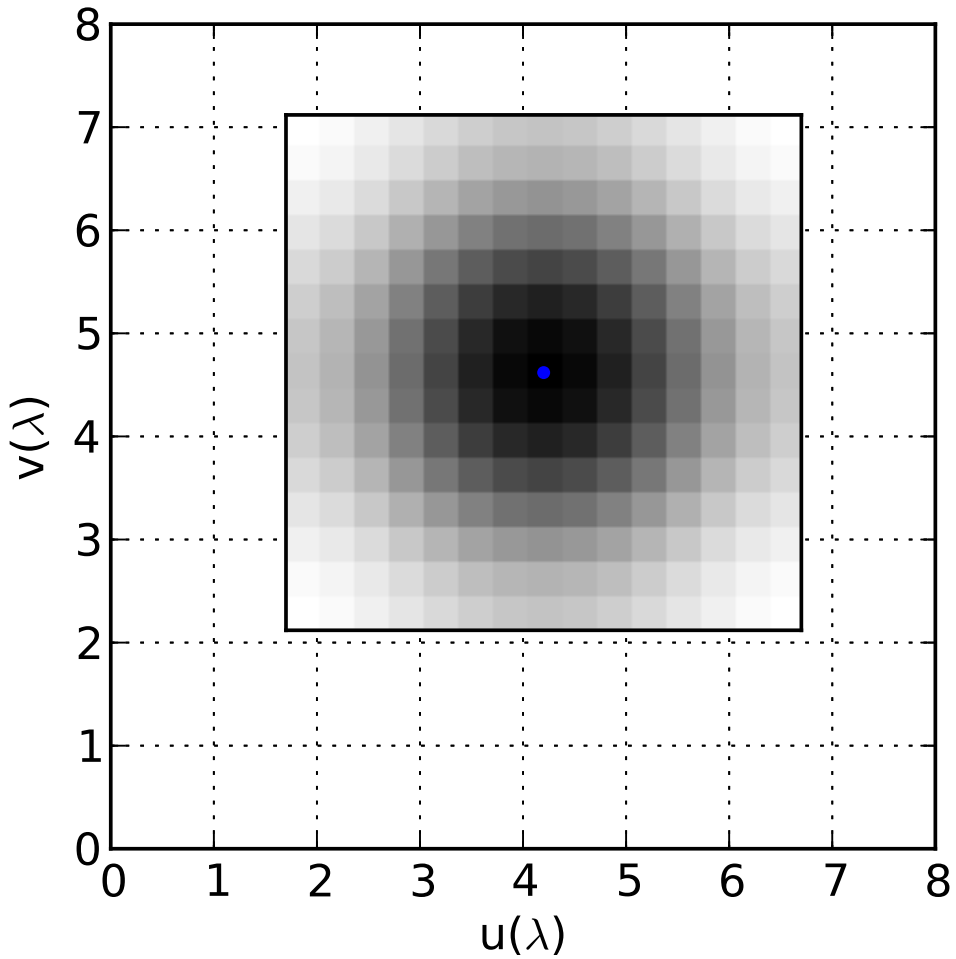


Spheroidal truncated in uv domain is still nice in image domain.

Gridding with Spheroidal

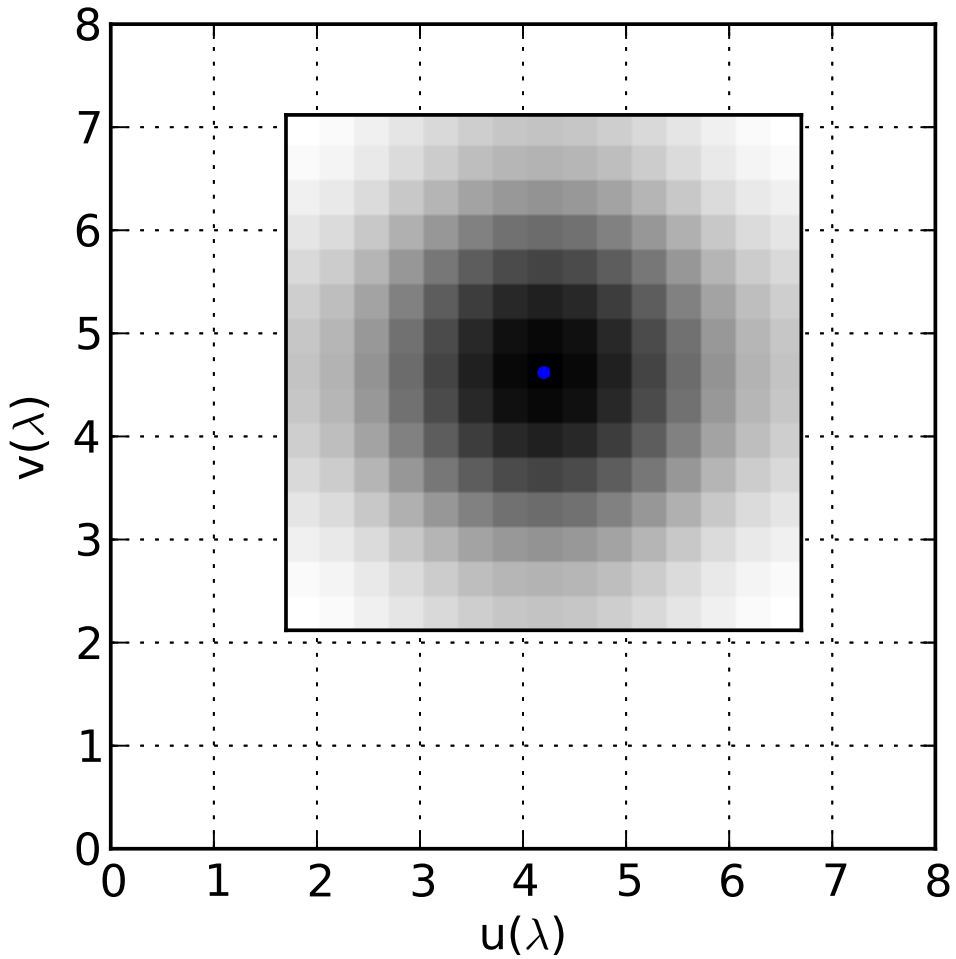


Oversampling



Parameters: gridding.oversampling

Degridding



Parameters: gridding.oversampling

Image plane effects

Gridding the data with a convolution function applies the Fourier transform of that function in the image domain. We can make use of that to apply other corrections in the image domain.

- Tapering (Spheroidal)
- W term (Curvature of the sky)
- A term (Beam effects)

Consequence: need to compute many Convolution Functions

W projection

Correction for the W-term

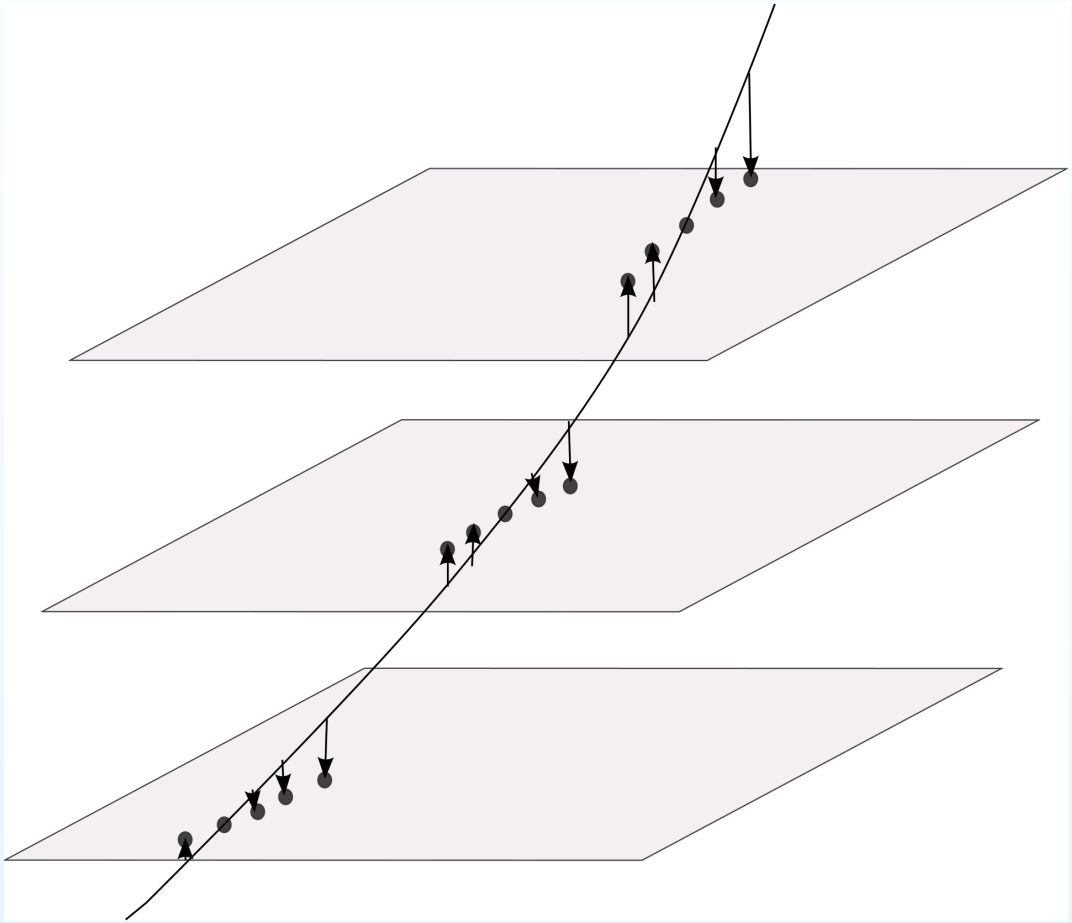
$$\exp 2\pi n w$$

where

$$n = 1 - \sqrt{l^2 + m^2}$$

Support of the convolution function grows with the square of the angular size of the image.

W Stack algorithm



Parameter: chunksize

A projection

The LOFAR beams are time varying, so the convolution functions need to be recomputed for each time window. The length of the window is set with parameter gridding.timewindow

How to inspect the effect of parameters

The most critical step in the CLEAN cycle is the computation of the model data (degridding). Other errors can be corrected in the next iteration. Errors in degridding will not be corrected. To inspect the performance of the degridding step look at

- model/residual data
- residual image

Compare BBS and awimager predict

BBS and awimager can both predict data, BBS from a catalog file and awimager from a model image.

```
Step.predict.Operation                  = PREDICT
Step.predict.Output.Column              = DATA
```

```
awimager operation=predict
```

Now compare the DATA and the MODEL_DATA column.