

## Radio polarimetry with LOFAR

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## Radio polarimetry at Low Frequency (LF):

### ➤ physics/instrumental

*Radio antennas are fundamentally polarized  
Understanding polarimetry improves your unpolarized  
calibration and imaging*

### ➤ astrophysics

*Polarimetry required for certain astrophysical observations*

- \* *synchrotron emission → B-field direction (indirectly strength) / Turbulence*
- \* *Zeeman splitting → B-field strength at source*
- \* *Scattering/reflection → electron densities in cool gas / Dust properties*
- \* *Faraday rotation → source / intervening plasma properties*

# Radio Polarimetry at low frequency (LF)



- LF Instruments: Dishes and Dipoles → different polarimetric (beam) properties
- LF challenges: Confusion + Ionosphere + Radio Frequency Interference + Large Field of View + Wide Bandwidth
- Large Field of View + Wide Bandwidth → RM Synthesis

## ➤ *LF Instruments: Dishes vs Dipoles* → *different polarimetric (beam) properties*

Electronic beamforming of dipole arrays:

- \* A single dipole sees the entire sky (element pattern)
- \* Station of dipoles can be combined to create station beams
- \* Multiple stations combined to create synthesized beams

Dipole array beams:

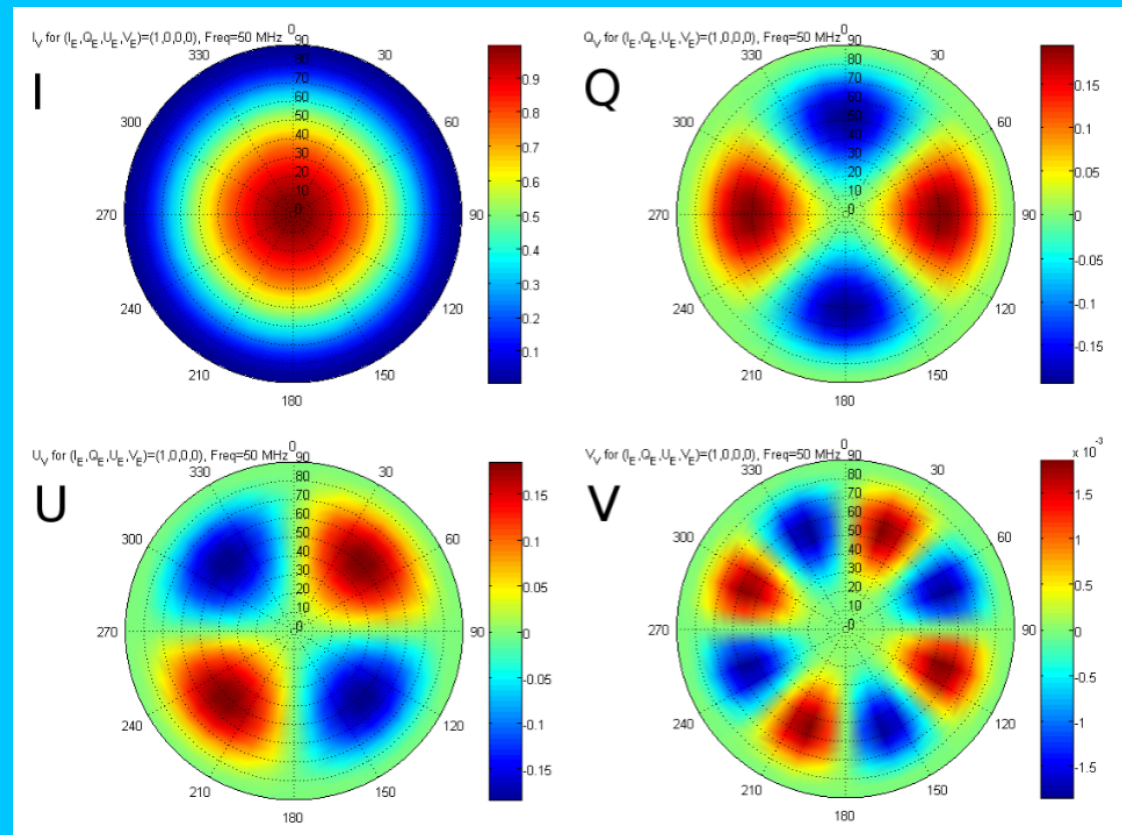
- \* Changing Gain with Time → complicated beam pattern
- \* Projection effects of dipoles viewed out of the zenith → Change of polarimetric response on scales of the entire sky, instead of on scales of the station's beam

# Polarimetry at LF & the LOFAR case

- **LF Instruments: Dishes vs Dipoles** → **different polarimetric (beam) properties**

Because the digital beam is small compared to the element beam, the polarization response does not vary significantly across a station beam.

The beam is polarization dependent.

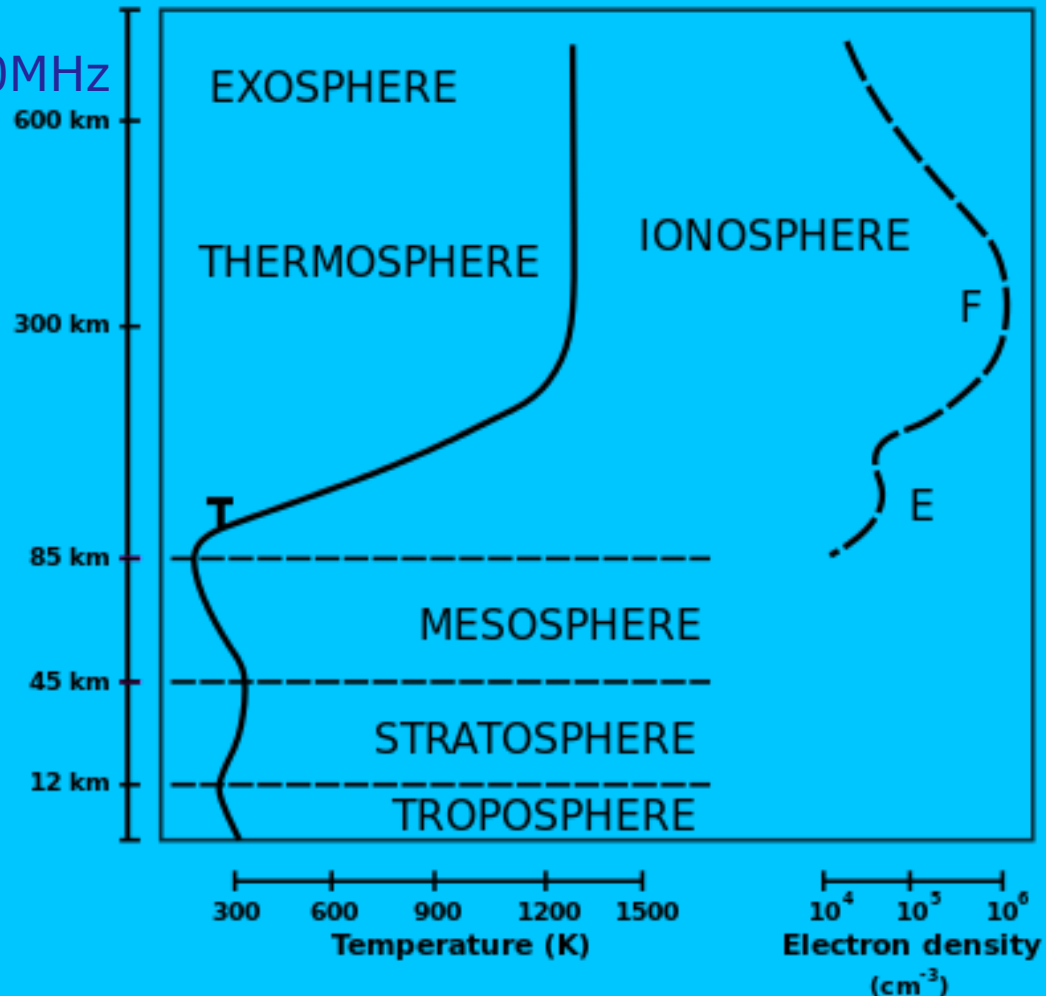


Calibrating the polarization response towards the pointing center is enough to obtain low leakage wide field maps

# Polarimetry at low frequency & the LOFAR case

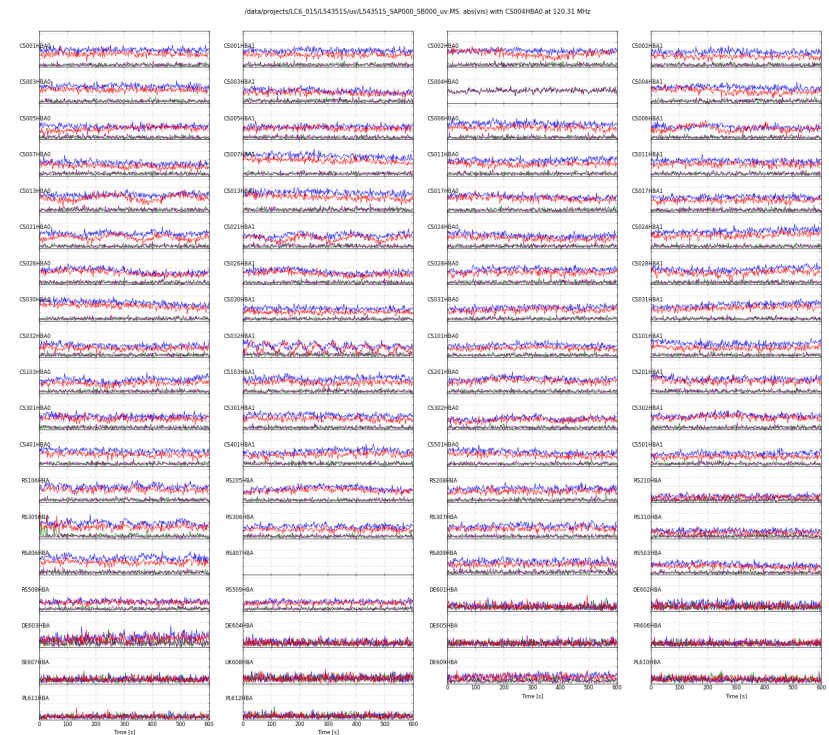
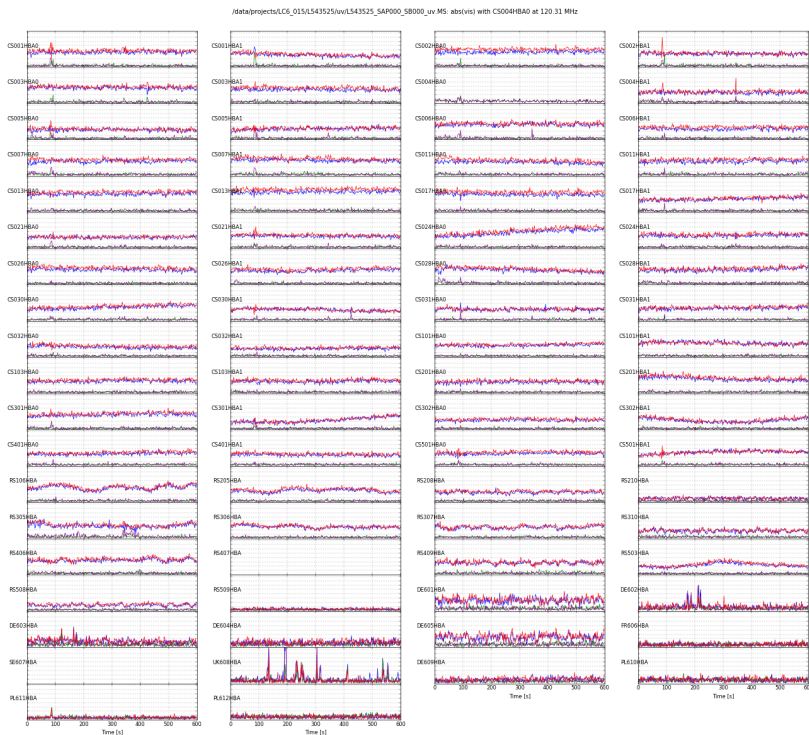
## ➤ *Ionosphere and ionospheric effects:*

- Ionospheric Cutoff:  
Plasma opacity below  $\sim 10\text{MHz}$



# Polarimetry at low frequency & the LOFAR case

- ***Ionosphere and ionospheric effects:***
  - **Disturbed Ionosphere:**
    - Scintillation
    - Image distortion / Rapid position shift → on image plane



# Polarimetry at low frequency & the LOFAR case

## ➤ *Ionosphere and ionospheric effects:*

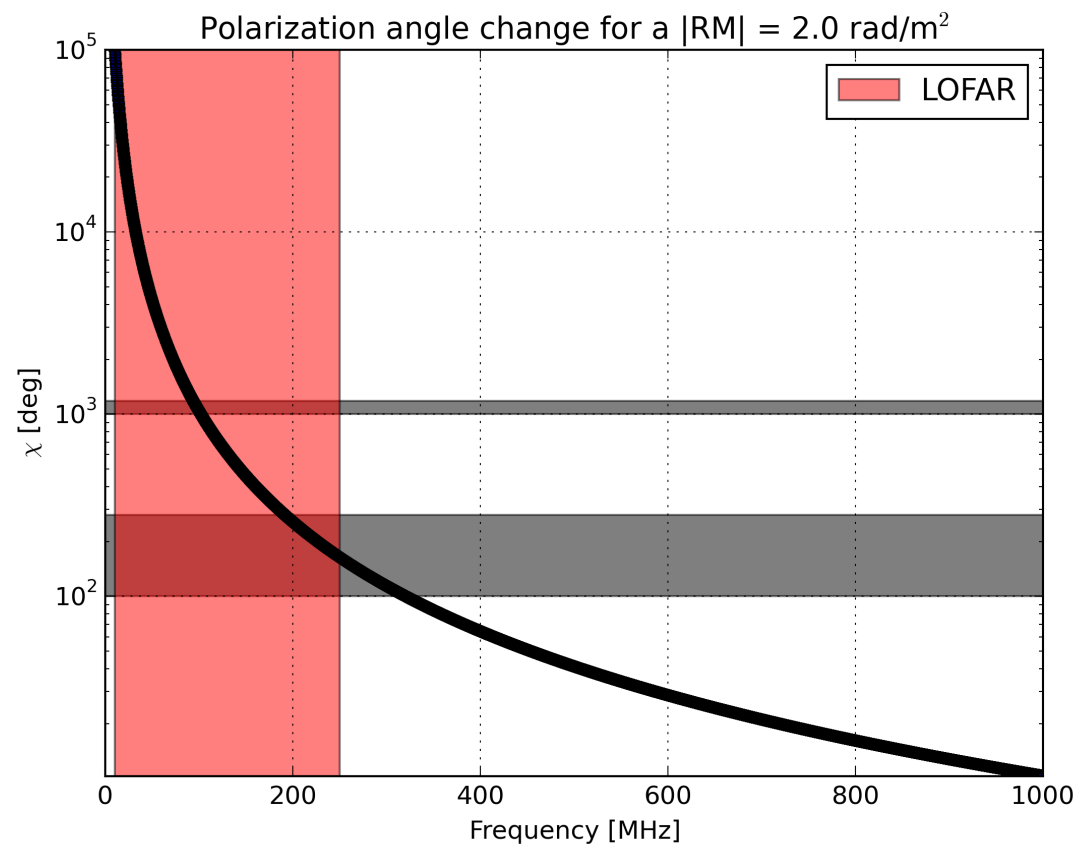
### ➤ Ionospheric Cutoff:

Plasma opacity below  $\sim 10\text{MHz}$

### ➤ Quiescent Ionosphere:

Refraction

Faraday Rotation





# Polarimetry at low frequency & the LOFAR case

- ***Ionosphere and ionospheric effects:***
  - Ionospheric Cutoff:
    - Plasma opacity below  $\sim 10\text{MHz}$
  - Quiescent Ionosphere:
    - Refraction
    - Faraday Rotation
  - Disturbed Ionosphere:
    - Scintillation
    - Image distortion / Rapid position shift

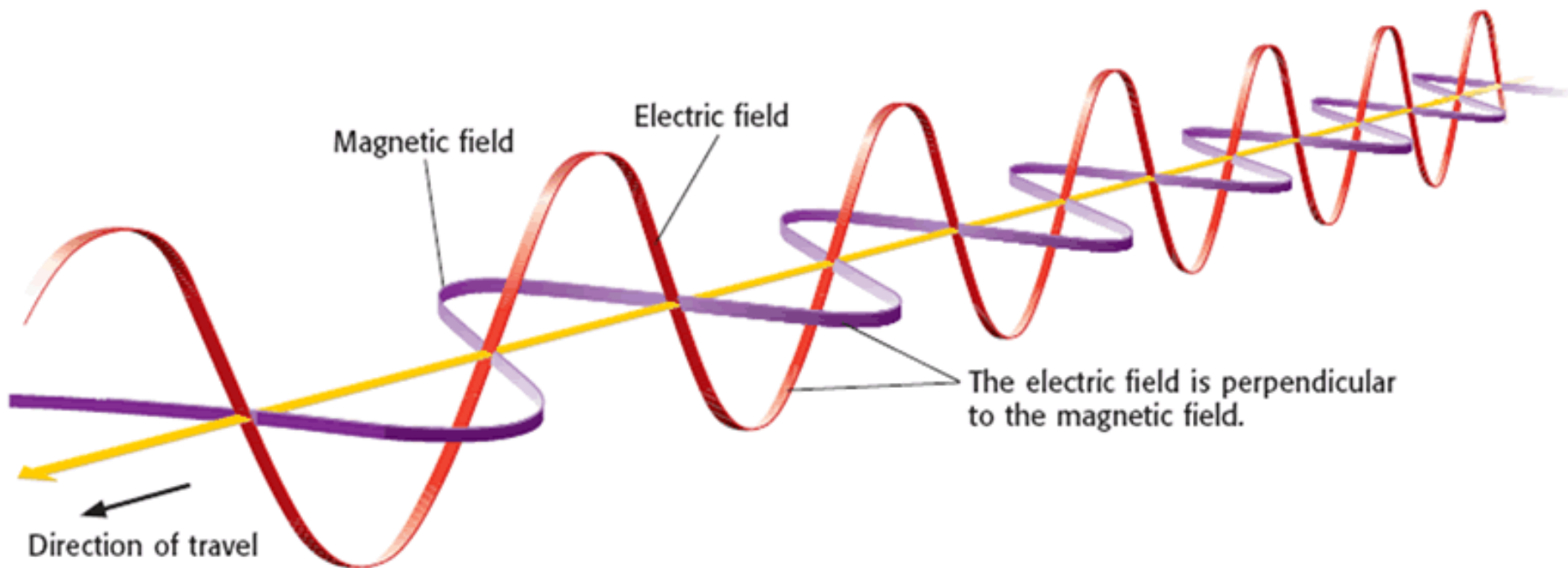
- ***Polarized EM waves***
  - different polarization states
- ***Stokes parameters***
  - an experimentally convenient description of the polarization state
- ***Radio polarimetry in practice***
  - Stokes visibilities
  - Jones matrices / antenna beam polarization
  - Faraday rotation / Ionospheric (differential) Faraday rotation / RM-synthesis

# Polarized EM waves: a vector nature

$\mathbf{k}$  = direction of propagation,  $\mathbf{B}$  = magnetic field,  $\mathbf{E}$  = Electric field

Antennas & detectors give  $\mathbf{k}$  and  $\mathbf{E}$

But  $\mathbf{E}$  may rotate as function of  $x$  and  $t$  tracing an ellipse → Polarization

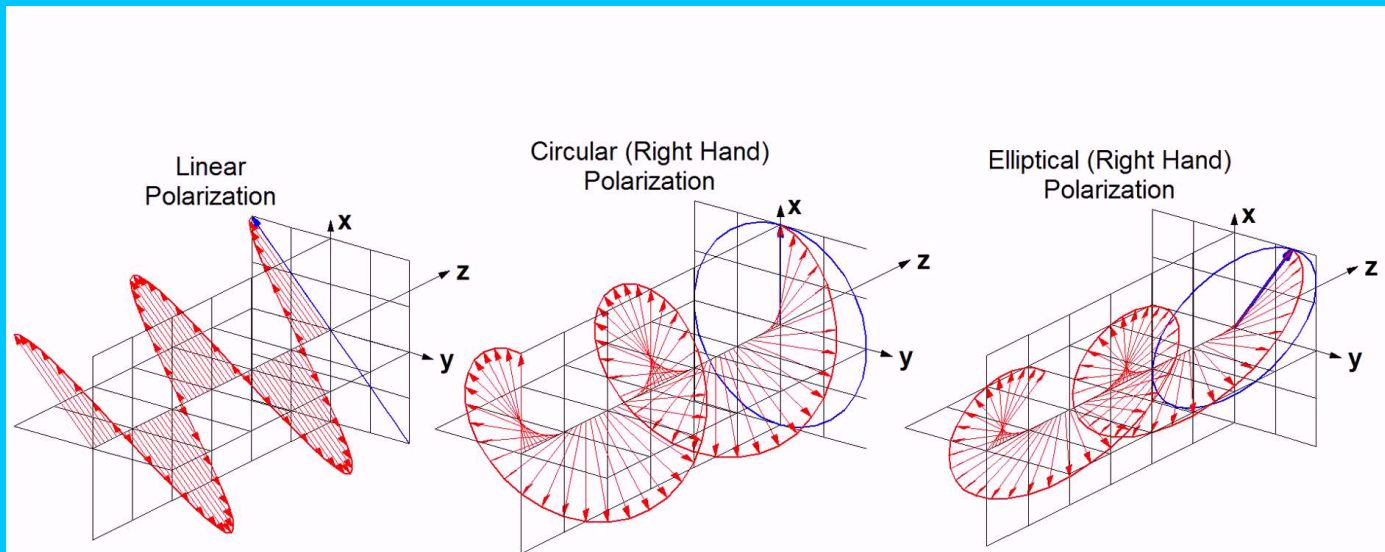


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But  $\mathbf{E}$  may rotate as function of  $x$  and  $t$  tracing an ellipse → Polarization



# Polarized EM waves: a vector nature

$$\mathbf{E} = E_x \mathbf{e}_x + E_y \mathbf{e}_y$$

$$E_x = A_x \cos(2\pi\nu t + \delta_x)$$

$$E_y = A_y \cos(2\pi\nu t + \delta_y)$$

$$\delta_{xy} = \delta_y - \delta_x \text{ measure of ellipticity}$$

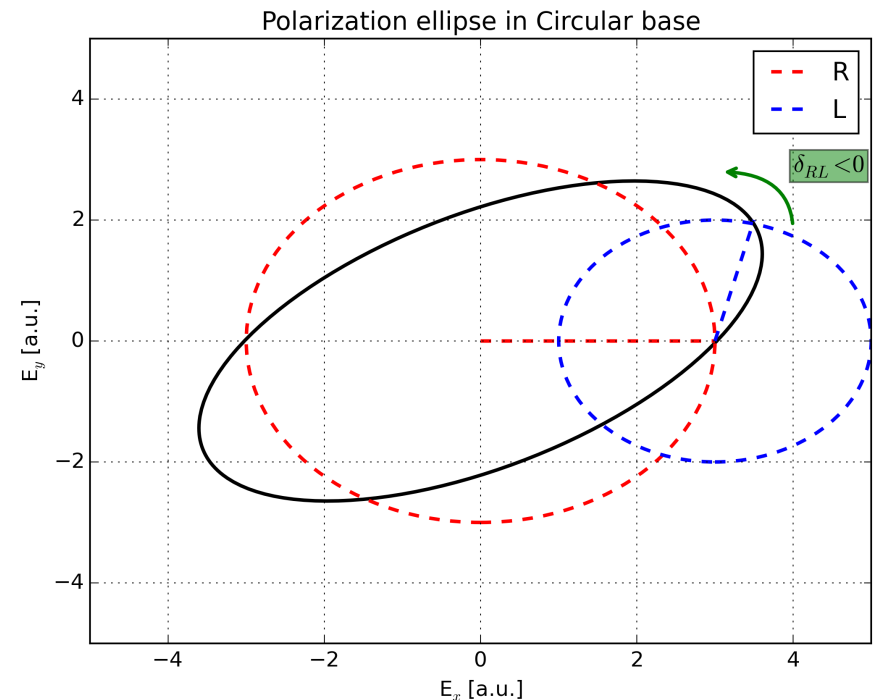
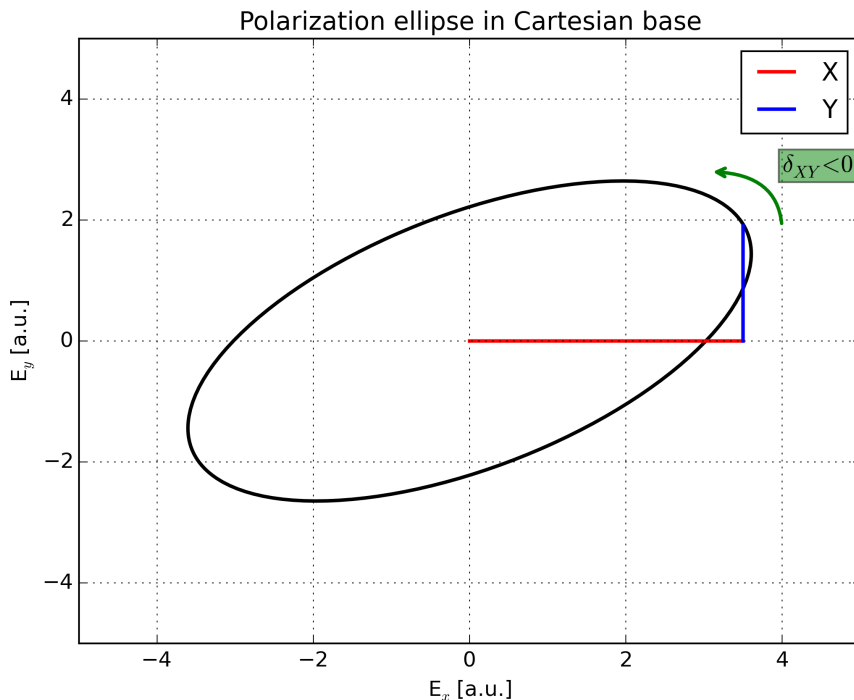
$$\mathbf{E} = A_r \mathbf{e}_r + A_l \mathbf{e}_l$$

$$\mathbf{e}_r = [\cos(2\pi\nu t + \delta_r), \sin(2\pi\nu t + \delta_r)]$$

$$\mathbf{e}_l = [\cos(2\pi\nu t + \delta_l), -\sin(2\pi\nu t + \delta_l)]$$

$$\delta_{rl} = \delta_r - \delta_l \text{ measure of ellipticity}$$

$$-1/2 \delta_{rl} \rightarrow \text{major axis position angle}$$



# Stokes parameters: a set of values to describe the polarization state

Polarization state described by a set of 4 parameters, e.g.: the semi-major and semi-minor axes of the polarization ellipse, its orientation and the sense of rotation.

The Stokes parameters → an alternative description of the polarization state experimentally convenient because each parameter corresponds to a sum or difference of measurable intensities.

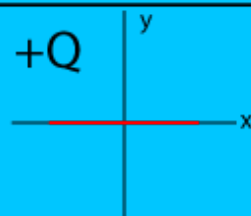
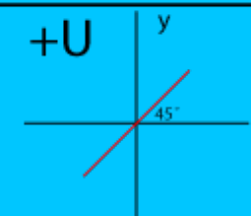
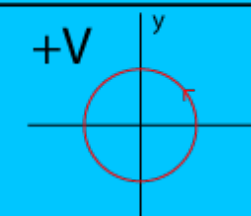
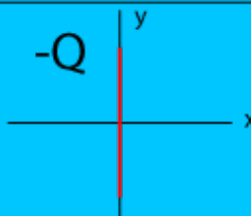
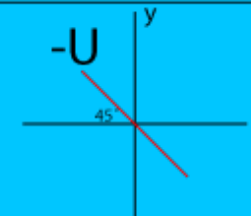
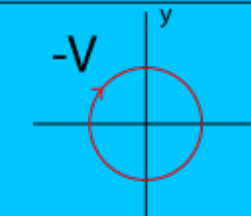
Polarization states:

1. Elliptical (general case)
2. Linear (degenerate case of 1.)
3. circular (degenerate case of 1.)

$I$  → mean flux density

$(Q, U)$  → linear polarization

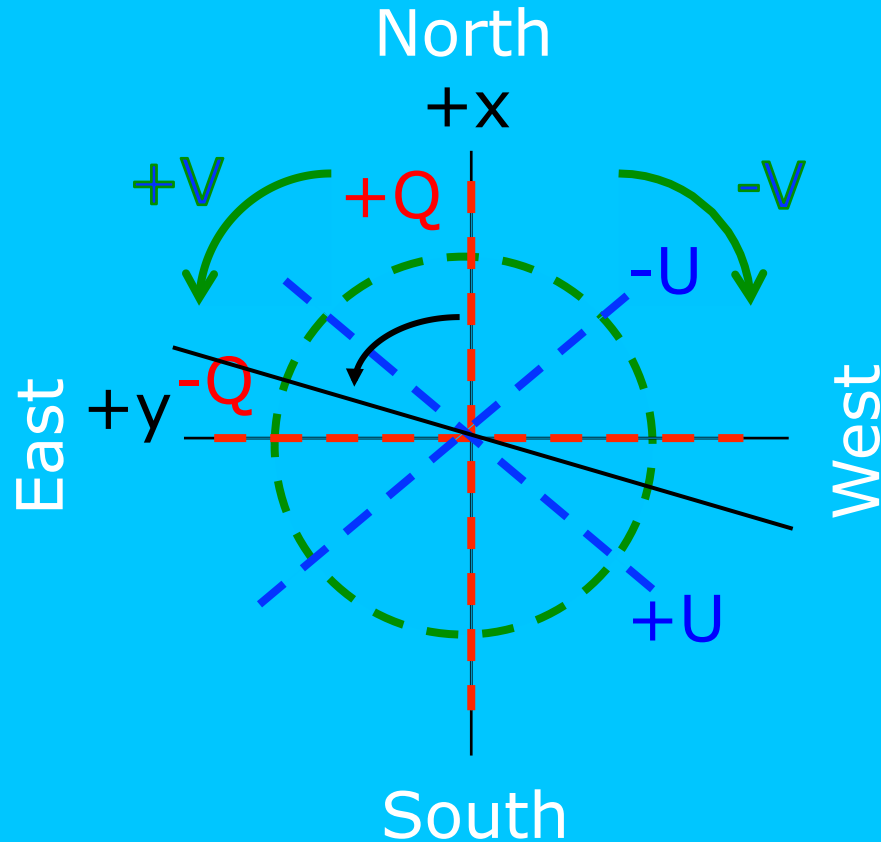
$V$  → circular polarization

100% Q	100% U	100% V
<p><b>+Q</b></p>  <p><math>Q &gt; 0; U = 0; V = 0</math></p> <p>(a)</p>	<p><b>+U</b></p>  <p><math>Q = 0; U &gt; 0; V = 0</math></p> <p>(c)</p>	<p><b>+V</b></p>  <p><math>Q = 0; U = 0; V &gt; 0</math></p> <p>(e)</p>
<p><b>-Q</b></p>  <p><math>Q &lt; 0; U = 0; V = 0</math></p> <p>(b)</p>	<p><b>-U</b></p>  <p><math>Q = 0; U &lt; 0; V = 0</math></p> <p>(d)</p>	<p><b>-V</b></p>  <p><math>Q = 0; U = 0; V &lt; 0</math></p> <p>(f)</p>

# Stokes parameters: IAU convention

Polarization angle measurements are done North through East

North  $\rightarrow$   $+x$     East  $\rightarrow$   $+y$



# Stokes parameters: a set of values to describe the polarization state

Polarization state described by e.g.: the semi-major and semi-minor axes of the polarization ellipse, its orientation, and the sense of rotation.

The Stokes parameters → an alternative description of the polarization state experimentally convenient because each parameter corresponds to a sum or difference of measurable intensities.

$I$  → total intensity,  $(Q,U)$  → linear polarization,  $V$  → circular polarization

Monochromatic wave fully polarized:  $I^2 = Q^2 + U^2 + V^2$

$$\begin{aligned} I &= A_x^2 + A_y^2 \\ Q &= A_x^2 - A_y^2 \\ U &= 2A_x A_y \cos\delta_{xy} \\ V &= -2A_x A_y \sin\delta_{xy} \end{aligned}$$

$$\Psi = \frac{1}{2} \arctan(U, Q)$$

$$P = \sqrt{Q^2 + U^2}$$

$$p = \frac{\sqrt{Q^2 + U^2}}{I}$$

$$\begin{aligned} I &= A_r^2 + A_l^2 \\ Q &= 2A_r A_l \cos\delta_{rl} \\ U &= -2A_r A_l \sin\delta_{rl} \\ V &= A_r^2 - A_l^2 \end{aligned}$$



# Radio polarimetry in practice: Stokes visibilities vs sky images

Sky brightness



FT(Sky brightness):  
the visibilities

$$I(u,v) = \text{FT}(I(l,m))$$

$$Q(u,v) = \text{FT}(Q(l,m))$$

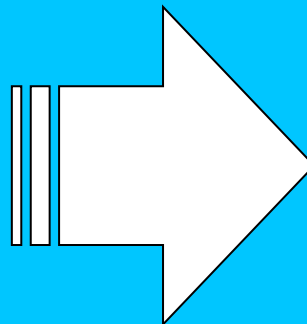
$$U(u,v) = \text{FT}(U(l,m))$$

$$V(u,v) = \text{FT}(V(l,m))$$

No mono-chromatic radiation approximation  
Stokes parameters sampled in {time} / frequency

$$E_x = \text{Re}(A_x e^{2\pi i \nu t}) \quad E_y = \text{Re}(A_y e^{2\pi i \nu t} e^{i\delta_{xy}})$$

$$\begin{aligned} I &= A_x^2 + A_y^2 \\ Q &= A_x^2 - A_y^2 \\ U &= 2A_x A_y \cos \delta_{xy} \\ V &= -2A_x A_y \sin \delta_{xy} \end{aligned}$$



$$\begin{aligned} I &= \{E_x E_x^*\} + \{E_y E_y^*\} \\ Q &= \{E_x E_x^*\} - \{E_y E_y^*\} \\ U &= \{E_x E_y^*\} + \{E_y E_x^*\} \\ V &= -i(\{E_x E_y^*\} - \{E_y E_x^*\}) \end{aligned}$$

# Radio polarimetry in practice: Stokes visibilities vs sky images

Sky brightness



FT(Sky brightness):  
the visibilities

$$I(u,v) = \text{FT}(I(l,m))$$

$$Q(u,v) = \text{FT}(Q(l,m))$$

$$U(u,v) = \text{FT}(U(l,m))$$

$$V(u,v) = \text{FT}(V(l,m))$$

No mono-chromatic radiation approximation  
Stokes parameters sampled in {time} / frequency

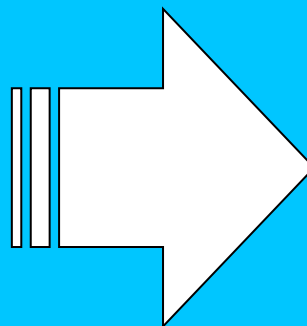
$$E_r = \text{Re}(A_r e^{2\pi i \nu t}) \quad E_l = \text{Re}(A_l e^{2\pi i \nu t} e^{i\delta_{rl}})$$

$$I = A_r^2 + A_l^2$$

$$Q = 2A_r A_l \cos \delta_{rl}$$

$$U = -2A_r A_l \sin \delta_{rl}$$

$$V = A_r^2 - A_l^2$$



$$I = \{E_r E_r^*\} + \{E_l E_l^*\}$$

$$Q = \{E_r E_l^*\} + \{E_l E_r^*\}$$

$$U = -i(\{E_r E_l^*\} - \{E_l E_r^*\})$$

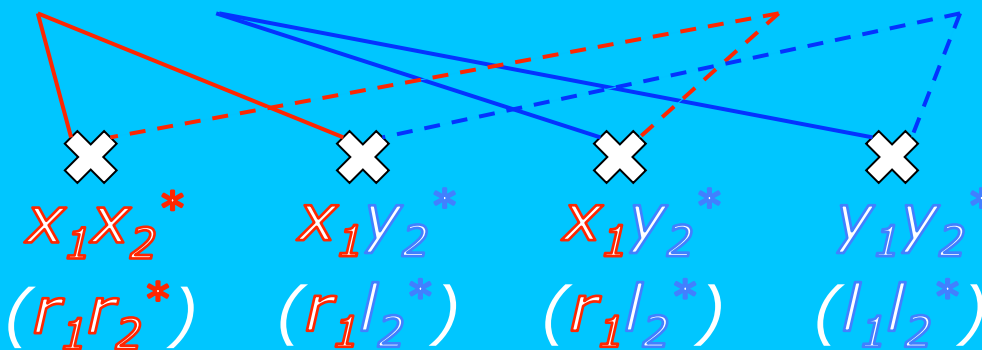
$$V = \{E_r E_r^*\} - \{E_l E_l^*\}$$

# Radio polarimetry in practice: Stokes visibilities & correlators I



$$x_1(r_1) - y_1(l_1)$$

$$x_2(r_2) - y_2(l_2)$$



$$I(u,v) = \{x_1 x_2^*\} + \{y_1 y_2^*\}$$

$$Q(u,v) = \{x_1 x_2^*\} - \{y_1 y_2^*\}$$

$$U(u,v) = \{x_1 y_2^*\} + \{y_1 x_2^*\}$$

$$V(u,v) = -i(\{x_1 y_2^*\} - \{y_1 x_2^*\})$$

$$I(u,v) = \{r_1 r_2^*\} + \{l_1 l_2^*\}$$

$$Q(u,v) = \{r_1 l_2^*\} + \{l_1 r_2^*\}$$

$$U(u,v) = -i(\{r_1 l_2^*\} - \{l_1 r_2^*\})$$

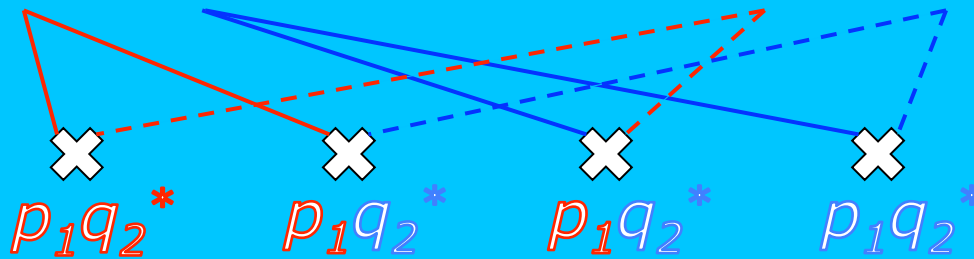
$$V(u,v) = \{r_1 r_2^*\} - \{l_1 l_2^*\}$$

# Radio polarimetry in practice: Stokes visibilities & correlators II



$p_1 - q_1$

$p_2 - q_2$



Signal editing by linear operators: Jones matrix and coherence matrix

$$\mathbf{e}_i = \begin{pmatrix} p_i \\ q_i \end{pmatrix}$$

$$\mathbf{E}_{ij} = \mathbf{e}_i \mathbf{e}_j^\dagger = \begin{pmatrix} p_i \\ q_i \end{pmatrix} (p_i^*, q_i^*)$$

$$\mathbf{E}_{ij} = \begin{pmatrix} p_i p_j^*, p_i q_j^* \\ q_i p_j^*, q_i q_j^* \end{pmatrix}$$

# Radio polarimetry: signal editing & Jones matrices I

Polarization state changes assumed to be linear → corruptions described by linear operators as 2x2 complex valued matrices: the Jones matrices ( $\mathbf{J}$ )

$$\mathbf{e}'_i = \mathbf{J}_i \mathbf{e}_i$$

Signal perturbation at correlator input  $i$

$$\mathbf{E}'_{ij} = \mathbf{e}'_i \mathbf{e}'_j{}^\dagger = \mathbf{J}_i \mathbf{E}_{ij} \mathbf{J}_j{}^\dagger$$

Cross correlation of the corrupted signal: the measurement equation (baseline  $i$ - $j$ )

$$\mathbf{E}_{ij} = \mathbf{J}_i^{-1} \mathbf{E}'_{ij} \mathbf{J}_j{}^{\dagger -1}$$

Inversion of the equation → Calibration

$$\mathbf{J} = \mathbf{G} \mathbf{P} \mathbf{I}$$

Matrix product is not commutative: right to left!

# Radio polarimetry: signal editing & Jones matrices II

Polarization state changes assumed to be linear → corruptions described by linear operators as 2x2 complex valued matrices: the Jones matrices ( $\mathbf{J}$ )

A relevant case at LF: (Faraday or feed) rotation or Parallactic angle

Cartesian base

$$\mathbf{J} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

Circular base

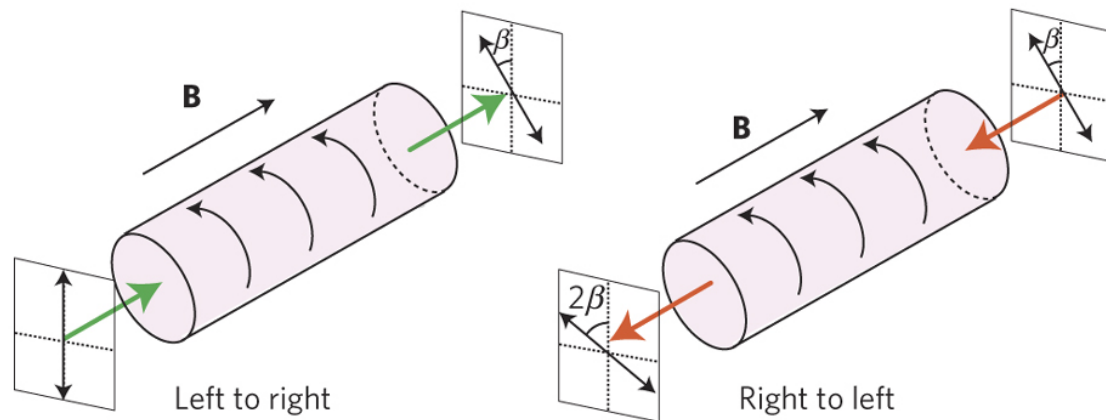
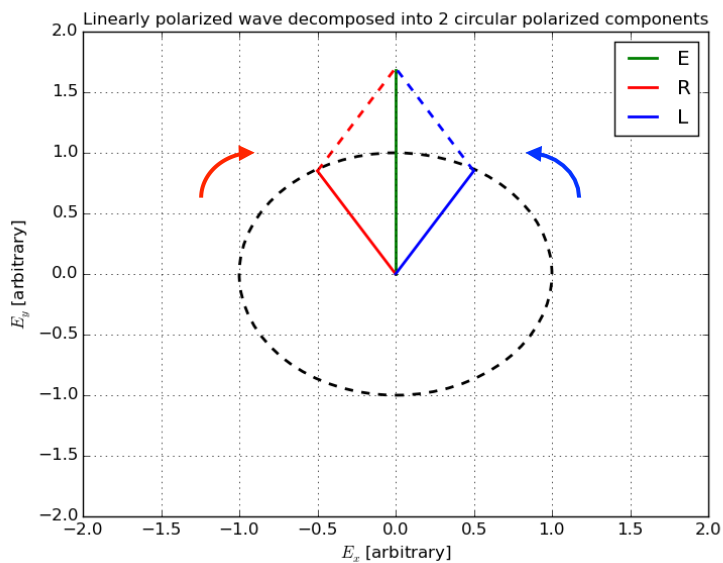
$$\mathbf{J} = \begin{pmatrix} e^{+i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix}$$

Choose the proper base!

To correct your LOFAR data for rotations / Parallactic angle use BBS software package

# Radio polarimetry: Faraday rotation

Faraday rotation is a magneto-optical phenomenon → in a magnetized plasma, L and R circularly polarized waves propagate at slightly different speeds at different wavelengths (circular birefringence).



Frequency dependent delay:

IF:

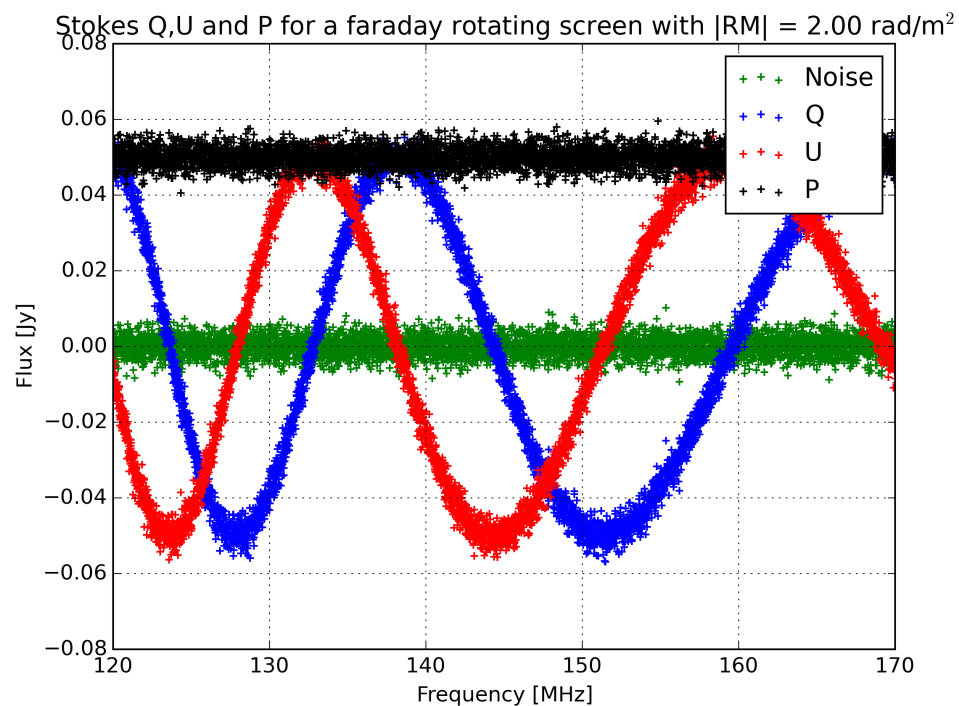
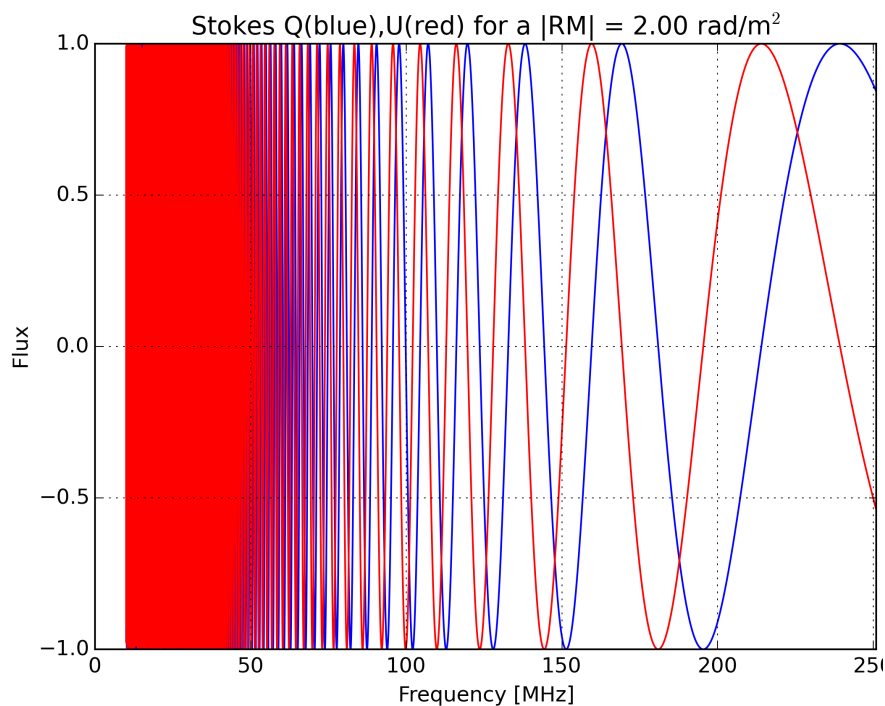
✓ Linear dependence on  $\lambda^2$   $\Psi(\lambda) = \Psi_0 + RM\lambda^2$  and  $RM = \frac{\partial \Psi}{\partial \lambda^2}$

ELSE:

✓ Non linear dependence on  $\lambda^2$  → Faraday depth  $\phi = 0.81 \int_{source}^{observer} n_e \vec{B} \cdot d\vec{l}$

# Radio polarimetry: Faraday rotation & Stokes Q,U parameters

Faraday rotation imprints on Stokes Q,U → fast rotation / band depolarization

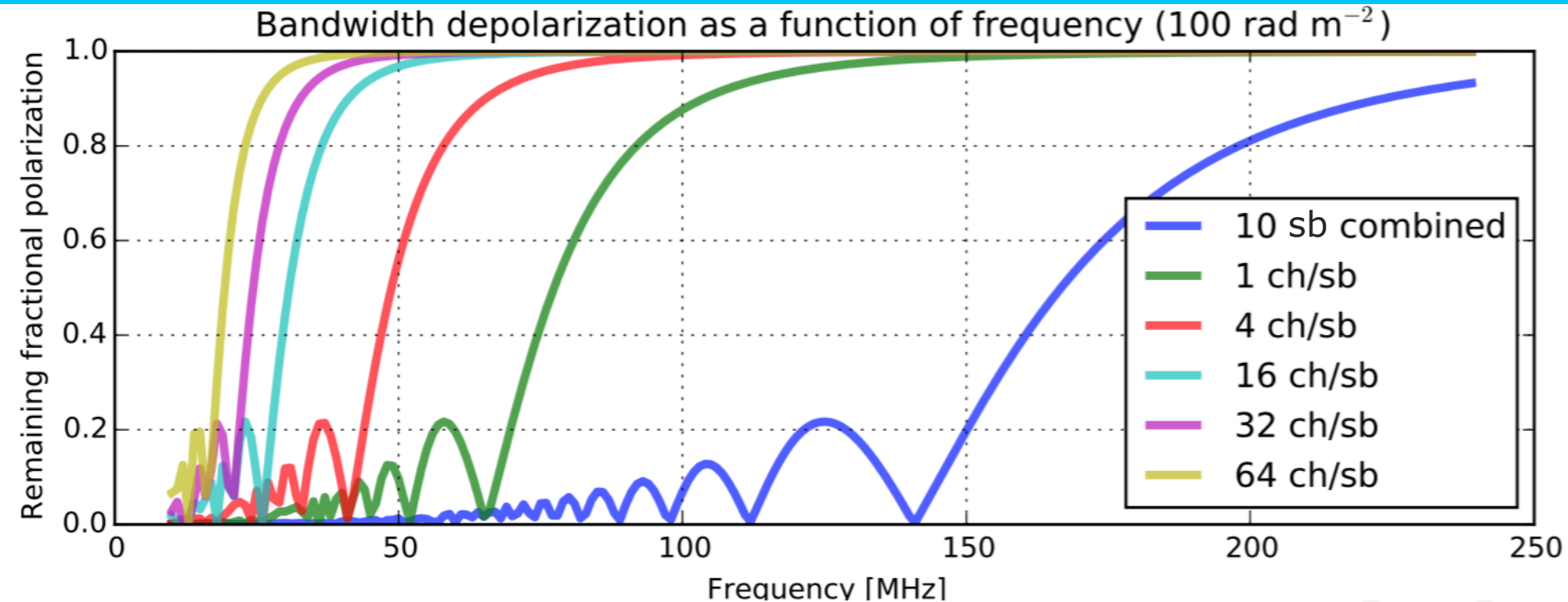


When frequency averaging your LOFAR data the signal can completely depolarize. Carefully choose averaging factors for LBA and HBA polarimetry!



# Radio polarimetry: Faraday rotation & Stokes parameters

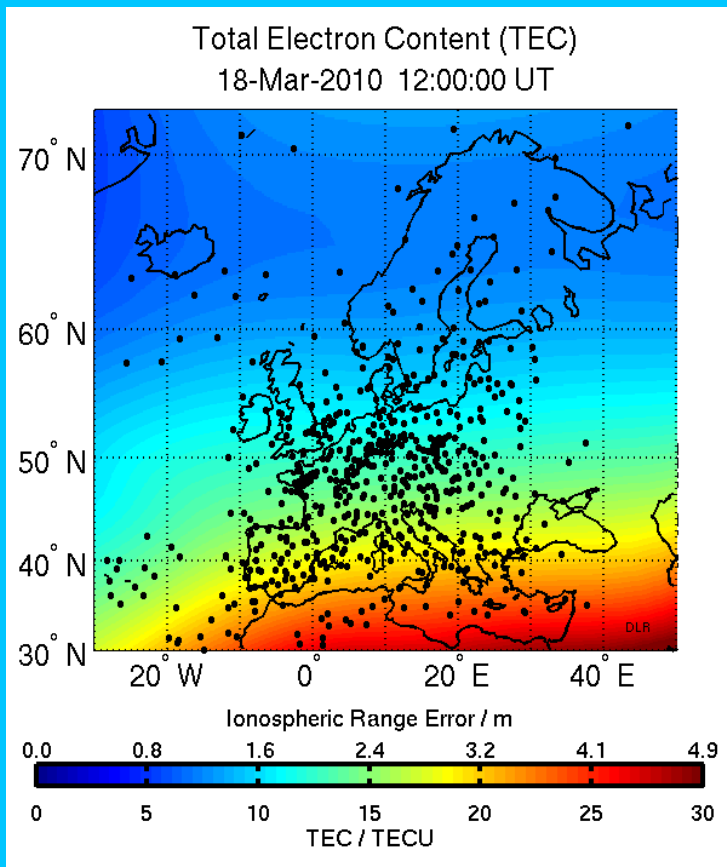
Faraday rotation imprints on Stokes Q,U → fast rotation / band depolarization



When frequency averaging your LOFAR data the signal can completely depolarize. Carefully choose averaging factors for LBA and HBA polarimetry!

# Radio polarimetry: Ionospheric Faraday rotation

Ionization (day) vs recombination (night): 1 TEC Unit =  $10^{12}$  el/cm<sup>2</sup>



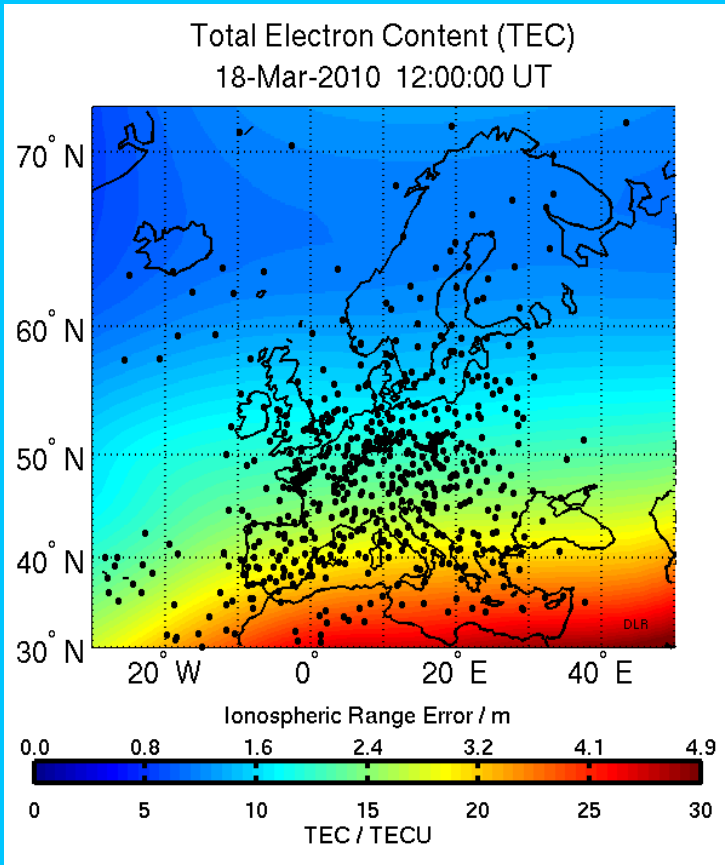
Typically daily variations up to  $\pm 5$  rad m<sup>-2</sup>  
If not corrected, variations cause depolarization when combining data from long day/night runs.

Typical accuracies are of the order of 0.2 rad m<sup>-2</sup>, which is good enough for the HBA but not for the LBA.

To correct your LOFAR data for ionospheric Faraday rotation use RExtract package → createRMParmdb script

# Radio polarimetry: Ionospheric differential Faraday rotation

Ionization (day) vs recombination (night): 1 TEC Unit =  $10^{12}$  el/cm<sup>2</sup>



Stations apart >10 km see different ionosphere, i.e. different amounts of ionospheric Faraday rotation.

During cross correlation Stokes I leakages into Stokes V and viceversa.

For LOFAR, differential Faraday rotation becomes important at baselines of only a few tens of km in the LBA and at the longer in the HBA.

To correct your LOFAR data for differential ionospheric RM use BBS package → solve for diagonal gains and a common rotation angle

# Radio polarimetry: Faraday rotation Measure synthesis

A Fourier transform between  $\Phi$  and  $\lambda^2$  space:  $P(\lambda^2) \rightarrow F(\Phi)$

$$P = Q + iU = pIe^{2i\Psi} + \Psi(\lambda) = \Psi_0 + RM\lambda^2 + \phi \equiv RM$$

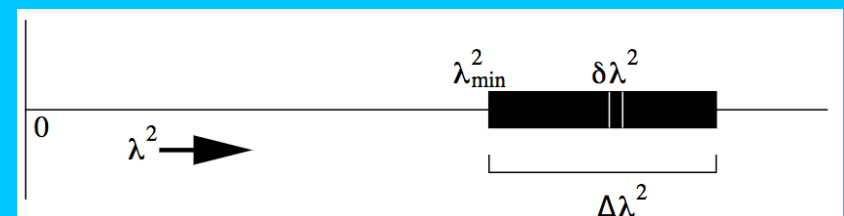
$$P(\lambda^2) = W(\lambda^2) \int_{-\infty}^{+\infty} F(\phi) e^{2i\phi\lambda^2} d\phi$$

$W(\lambda^2)$ : the **sampling (window) function**

$F(\Phi)$ : the **Faraday dispersion function**

By inverting, a finite point spread function (the **Rotation Measure Spread Function**) is obtained!

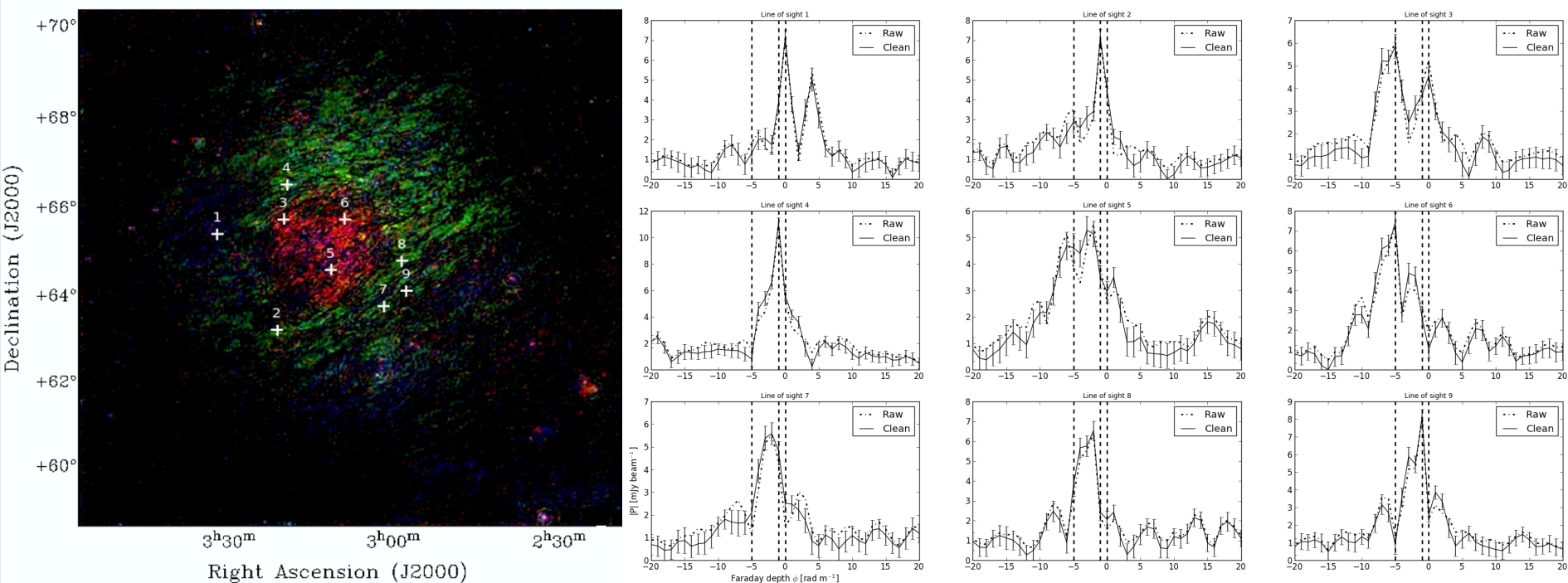
**3 instrumental parameters fix the output of RM synthesis**



# Radio polarimetry: Faraday rotation Measure synthesis

A Fourier transform between  $\Phi$  and  $\lambda^2$  space:  $P(\lambda^2) \rightarrow F(\Phi)$

In practice ... the reconstructed **Faraday dispersion function**



To apply RM synthesis to your LOFAR data use PYRMSINTH or RM-SYNTHESES packages (github)