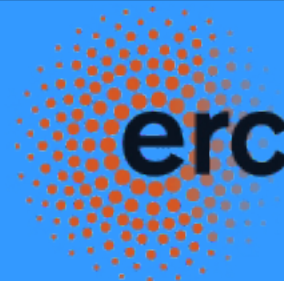


# Wide-field radio interferometric imaging

*LOFAR data school 2016*

2016-09-07

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**European Research Council**

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**Supporting top researchers  
from anywhere in the world**

- Basics of widefield imaging
- Imaging LOFAR data
  - Software
  - Methods

# Contents

- Output of an interferometer after calibration:

$$V(u, v, w) = \iint \frac{I(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-2\pi i (ul + vm + w(\sqrt{1 - l^2 - m^2} - 1))} dl dm$$

- $(u, v, w)$  : interferometer's geometrical vector
- $(l, m)$  : position on the sky
- $I$  : sky brightness (“image”)

**Imaging : Calculating  $I(l, m)$  from  $V(u, v, w)$**

# Visibility function

- Full visibility function:

$$V(u, v, w) = \iint \frac{I(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-2\pi i (ul + vm + w(\sqrt{1 - l^2 - m^2} - 1))} dl dm$$

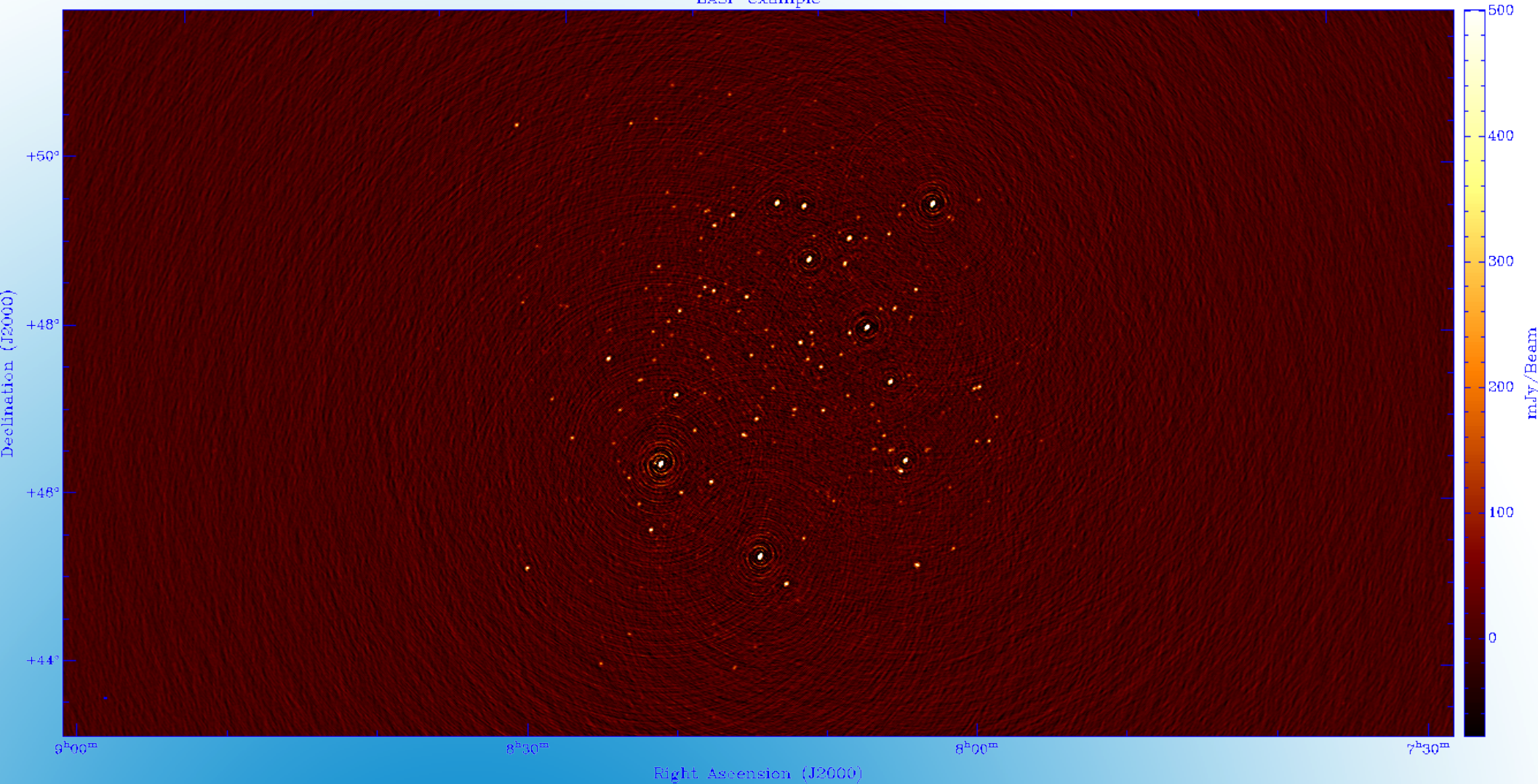
- For small field of view ( $l \sim 0$ ,  $m \sim 0$ ) or  $w \sim 0$  :

$$V(u, v, w) \approx \iint I(l, m) e^{-2\pi i (ul + vm)} dl dm$$

- $(u, v, w)$  : interferometer's geometrical vector
- $(l, m)$  : position on the sky
- $I$  : sky brightness (“image”)

## Fourier relation

BASP example

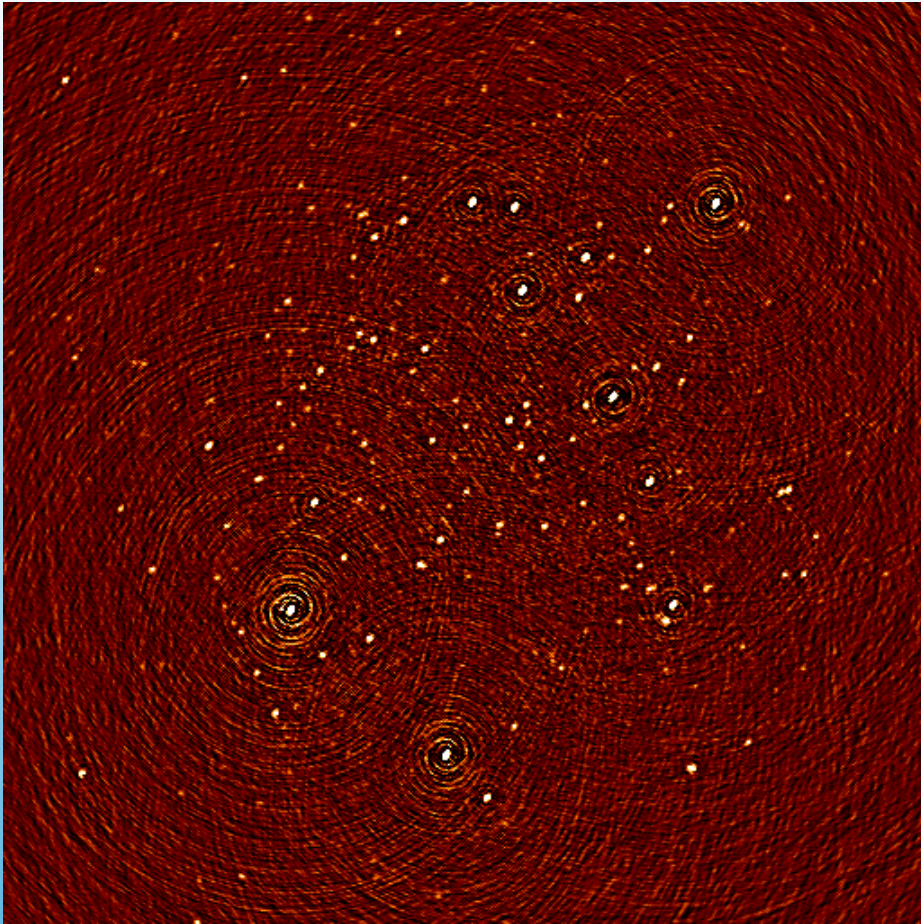


LOFAR dirty image (3c196)

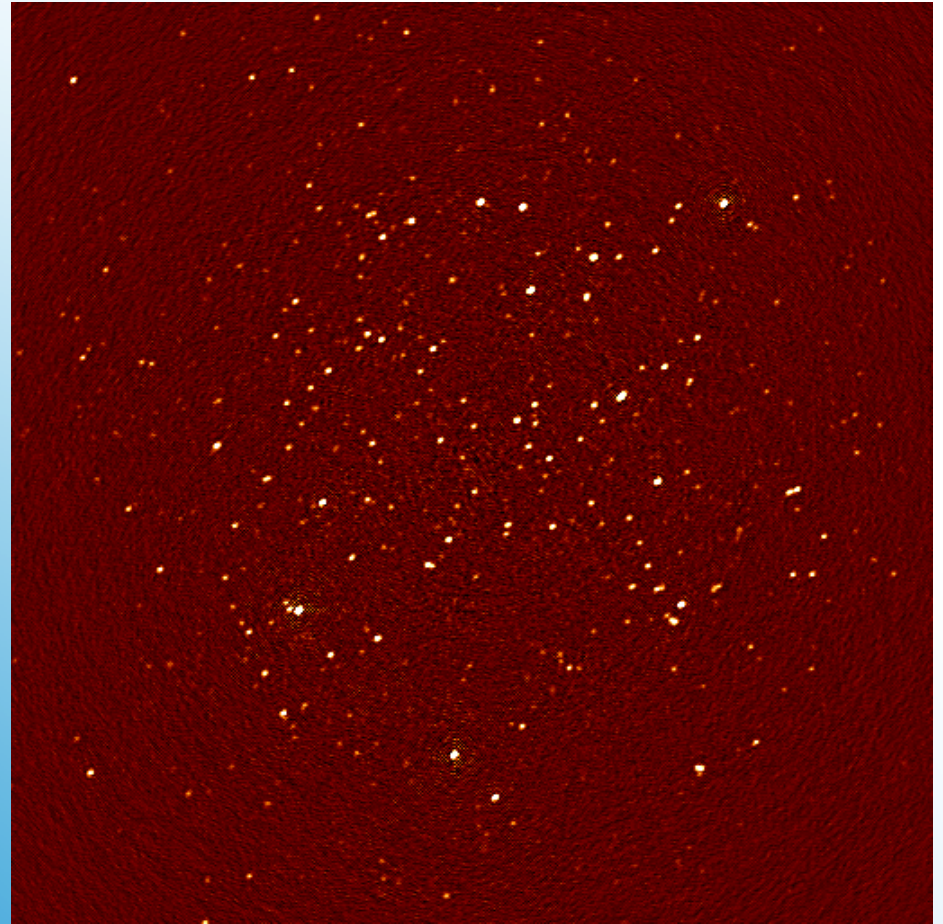
# The dirty image

- Högbom CLEAN algorithm (1974):
  - Find largest peak in image
  - Scale PSF to fraction of peak and subtract
  - Repeat until peak  $<$  threshold or niter  $>$  limit
  - Finally: restore subtracted components

**Högbom CLEAN**

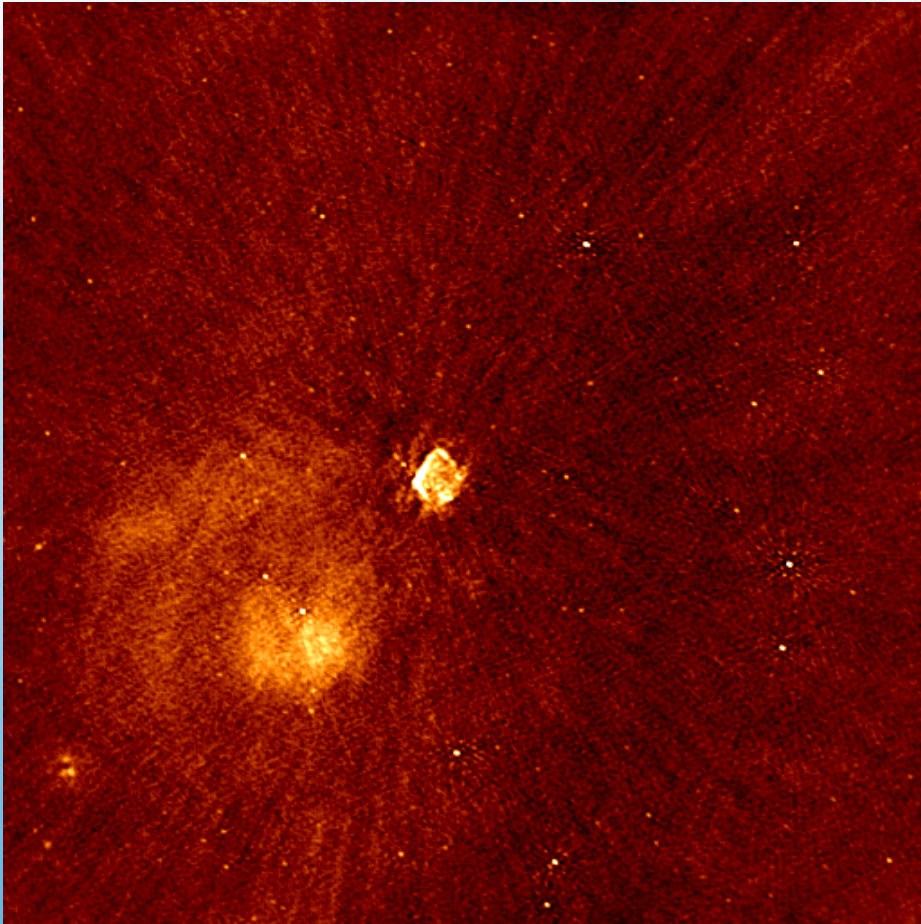


LOFAR undeconvolved ("dirty") image

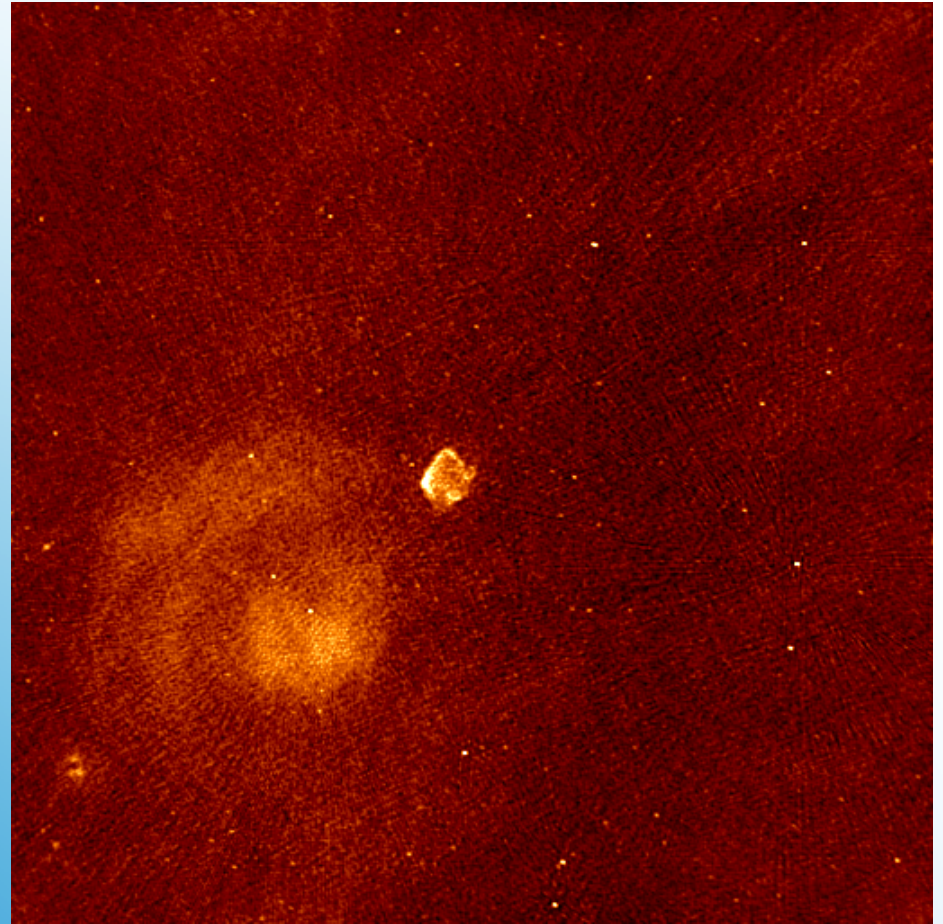


Deconvolved with Högbom CLEAN

# Högbom CLEAN



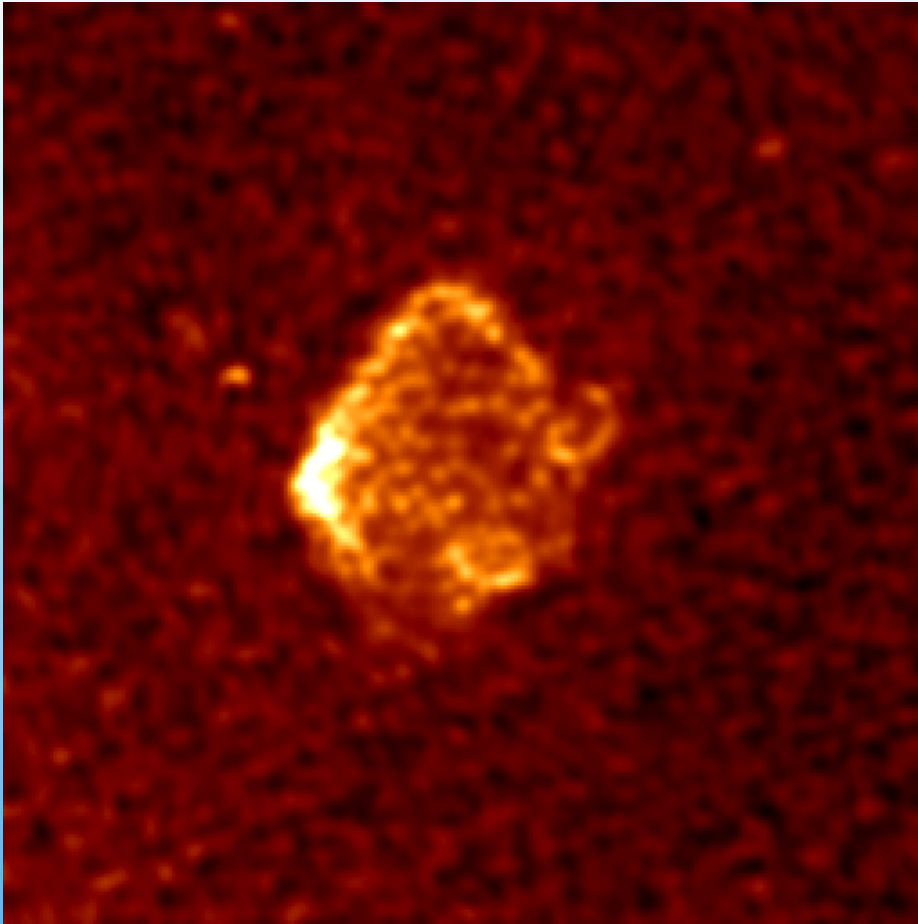
Undeconvolved "dirty" image



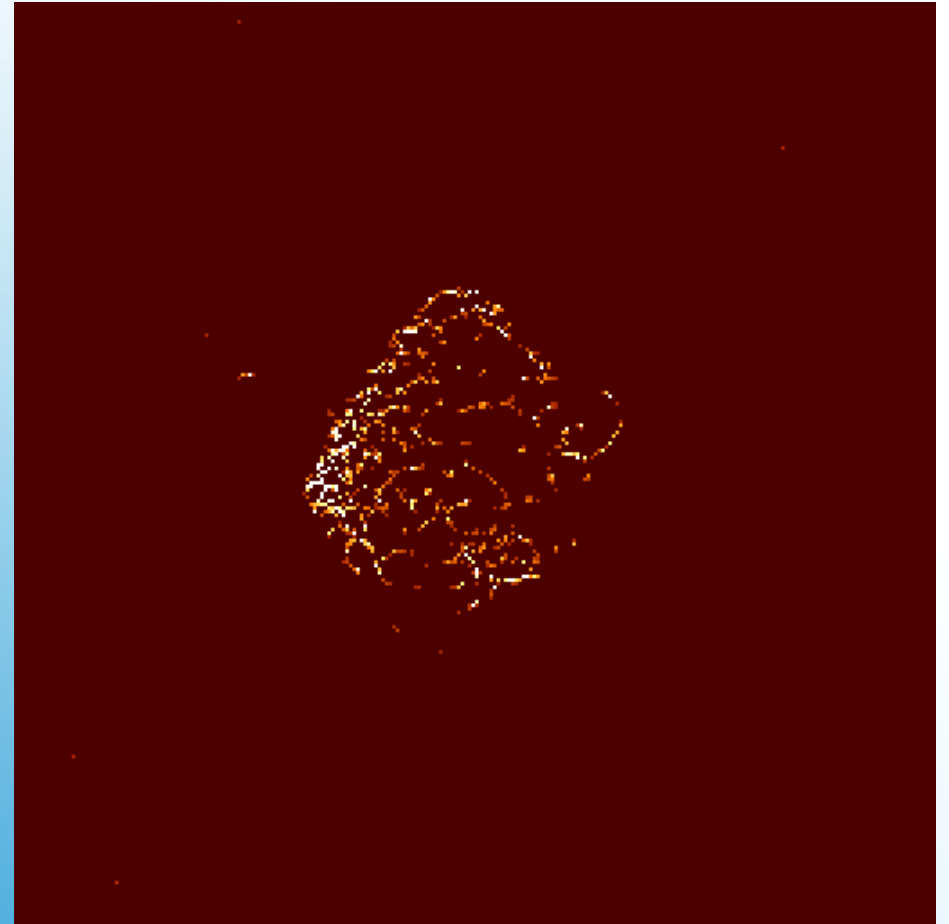
Deconvolved image with Högbom CLEAN

# Deconvolving diffuse structures





Deconvolved image (Högbom CLEAN)



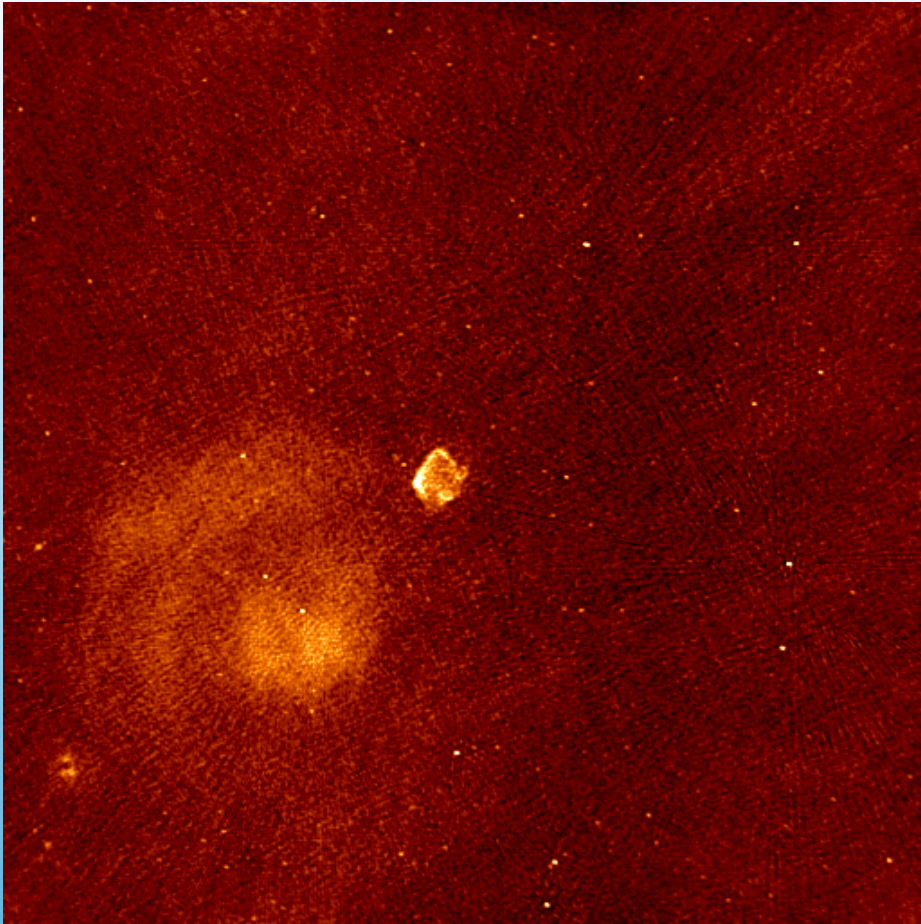
Actual model

# Deconvolving diffuse structures

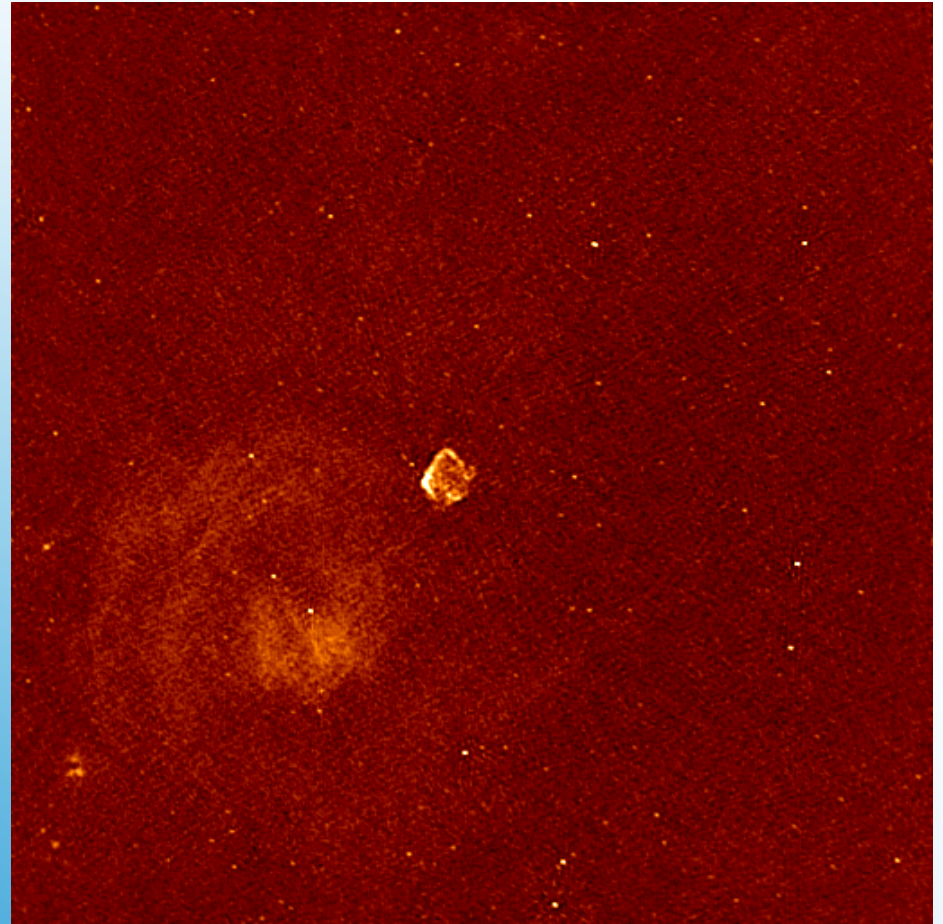
Improved algorithm by Cornwell (2008) :

- “Multi-scale clean”
- Fits small smooth kernels (and delta functions) during a Högbom CLEAN iteration

**Multi-scale CLEAN**

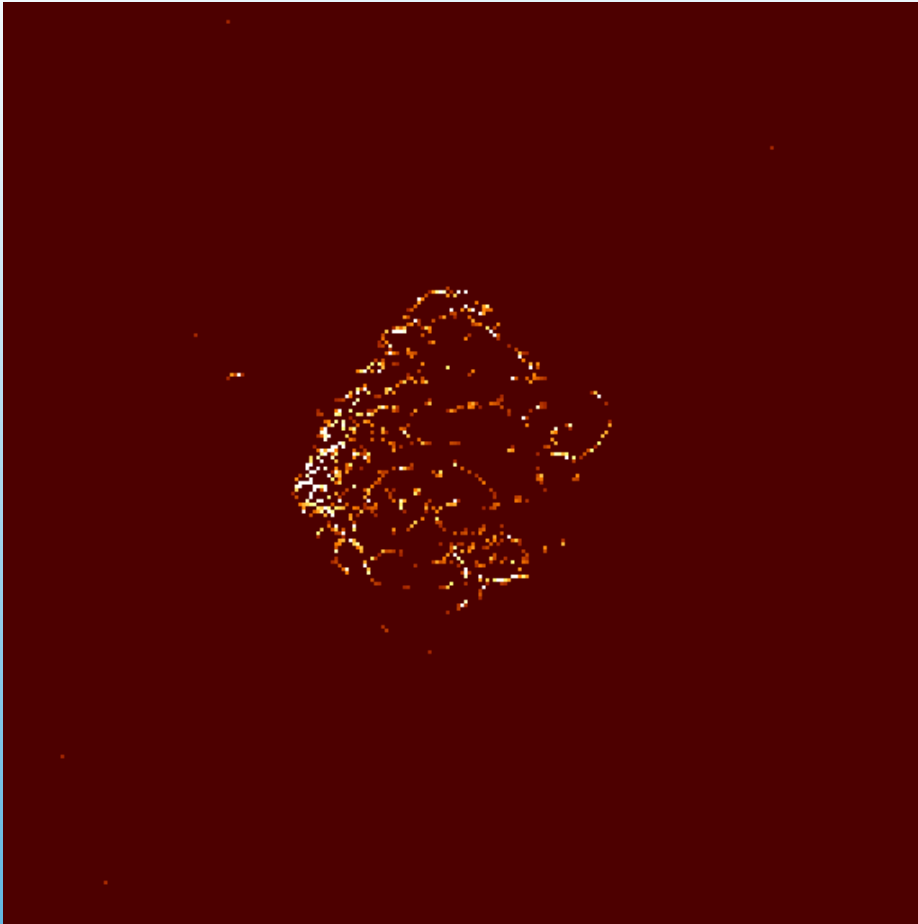


Normal Högbom CLEAN

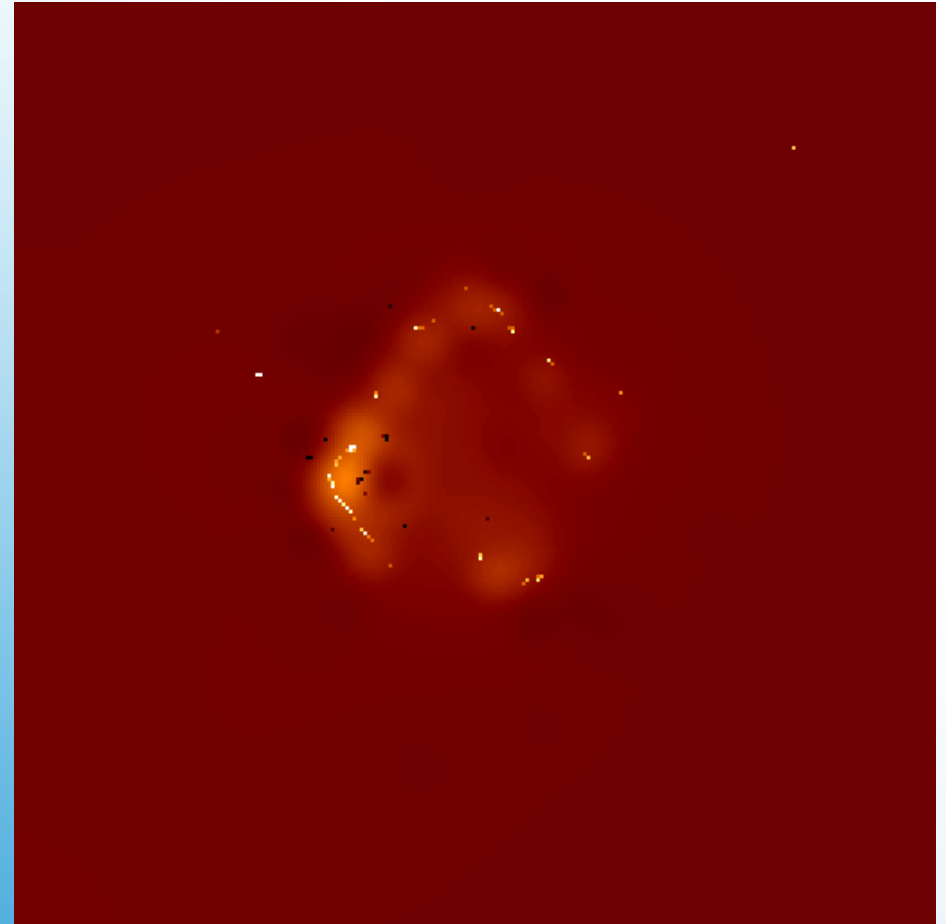


Multi-scale CLEAN  
(implementation in WSClean)

# Multi-scale CLEAN



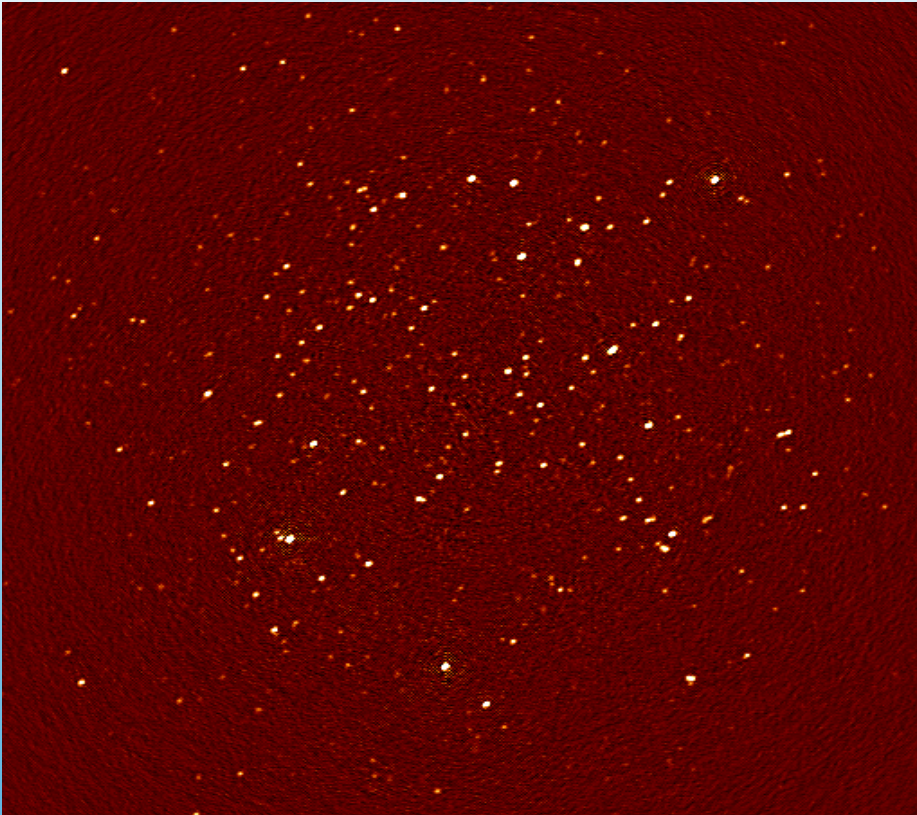
Normal Högbom CLEAN  
Output model



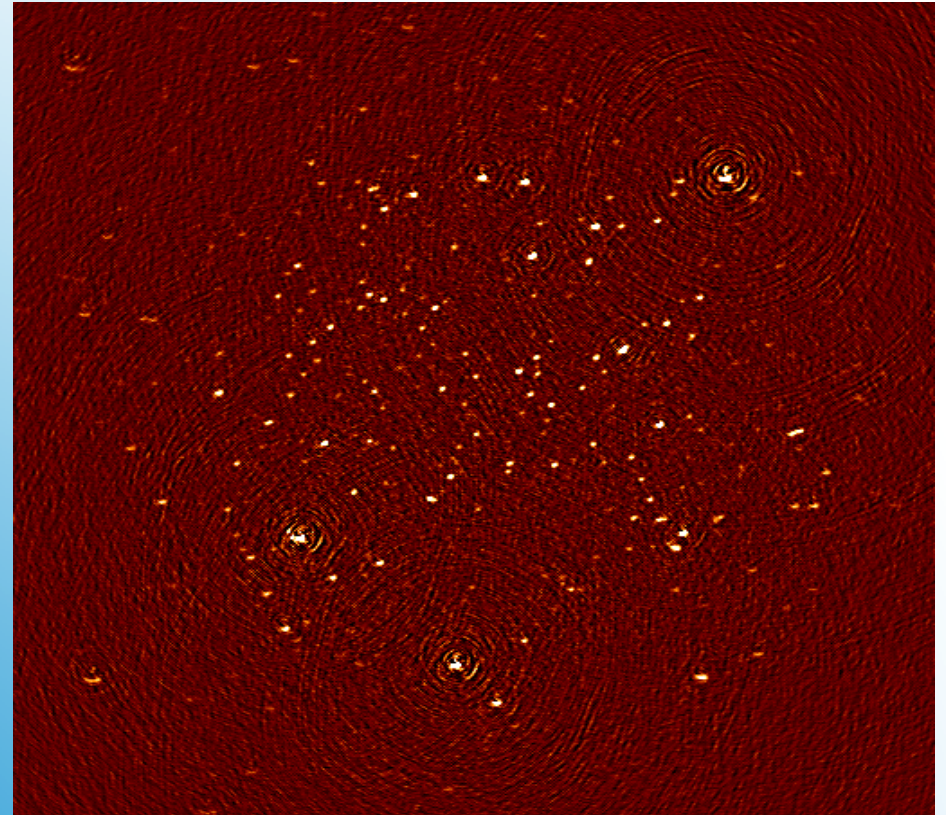
Multi-scale CLEAN  
(as implementation in WSClean)

# Multi-scale CLEAN

- 2D FT does not hold for new arrays:  $l, m, w \gg 0$



Correcting w-terms



Without correcting w-terms

# The w-term

- 2D FT relationship does not hold for low-frequency/widefield arrays:  $l, m, w \gg 0$
- Have to use full function:

$$V(u, v, w) = \iint \frac{I(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-2\pi i (ul + vm + w(\sqrt{1 - l^2 - m^2} - 1))} dl dm$$

- Easy solution: facetting
  - But: slow, stitching artefacts
- Better & commonly used solution: 'w-projection'


## The w-term

- Visibility function:

$$V(u, v, w) = \iint \frac{I(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-2\pi i (ul + vm + w(\sqrt{1 - l^2 - m^2} - 1))} dl dm$$

- W-projection: (Cornwell et al, 2008)

$$V(u, v, w) * \mathcal{F}(e^{-2\pi i w(\sqrt{1 - l^2 - m^2} - 1)}) = \iint \frac{I(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-2\pi i (ul + vm)} dl dm$$

  
 This convolution turns out  
 to have a “limited” support

- Performance very dependent on zenith angle, coplanarity of array, field of view and resolution.

# w-projection

- Another problem; convolution theorem no longer works when w-terms present in

$$V(u, v, w) = \iint \frac{I(l, m)}{\sqrt{1 - l^2 - m^2}} e^{-2\pi i (ul + vm + w(\sqrt{1 - l^2 - m^2} - 1))} dl dm$$

- Högbom CLEAN assumes constant PSF
- But PSF changes (slightly) over the image
- Solved with Cotton-Schwab algorithm (schwab 1984)
- Normal CASA/awimager imaging mode will automatically use CS, WSClean with '-mgain' param

# w-projection



- The Cotton-Schwab + w-projection algorithm:
  - Make initial dirty image & central PSF
  - Perform minor iterations:
    - Find peak
    - Subtract scaled PSF at peak with small gain
    - Repeat until highest peak ~ 80-90% decreased
  - Major iteration: “Correct” residual
    - Predict visibility for current model
    - Subtract predicted contribution and re-image

**w-projection**

- W-projection is the standard way to solve w-terms in radio astronomy
- W-term convolution can be *slow*
- New imager with new algorithm implemented: WSClean<sup>1</sup> (“w-stacking clean”).
  - Offringa et al, 2014

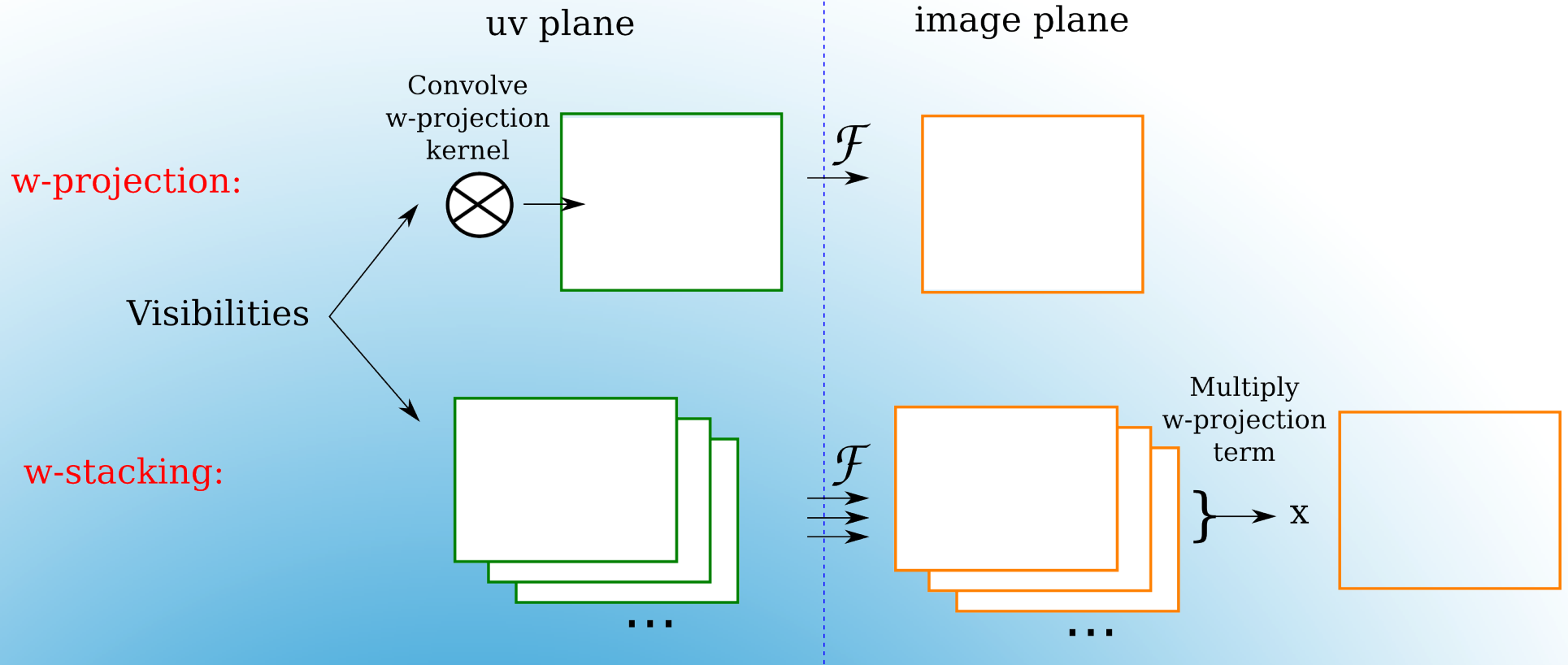
<sup>1</sup>docs can be found at <http://wsclean.sourceforge.net/>

# w-projection

There are three packages suitable for imaging of LOFAR data:

- CASA (task “clean” with `gridmode='widefield'`)
  - Not optimized for LOFAR
  - But has many options and is well known
- AWImager
  - Slow, but currently required for full polarimetric imaging
- WSClean
  - Fast & many LOFAR specific features
  - Becoming the “de facto” LOFAR imager (?)
  - (+awesome author)

# Imaging software for LOFAR



# w-stacking

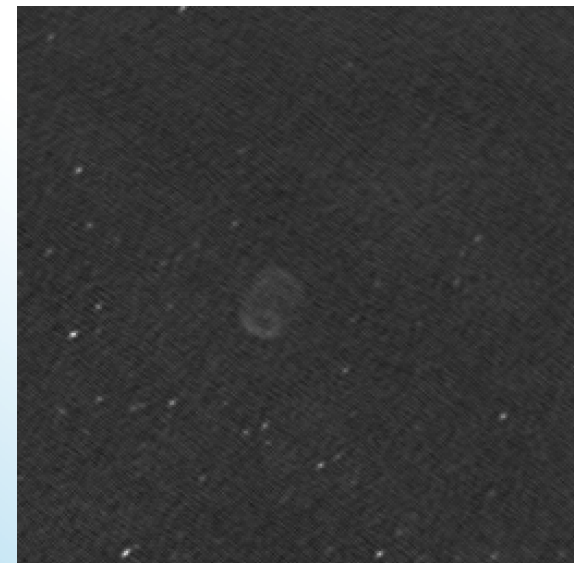
- LOFAR has Core, Remote and International stations.
- With only core stations, a resolution of  $\sim$ arcmin can be reached.
    - Ionosphere hardly relevant, easy to image
    - Most sources unresolved
  - With remote stations, the resolution increases to  $\sim$ 5 arcsec
    - Ionosphere very relevant, harder to image
    - Many sources resolved
  - International stations increase the resolution to  $\sim$ 200 masec.
    - Might require specialized calibration

## LOFAR's resolution

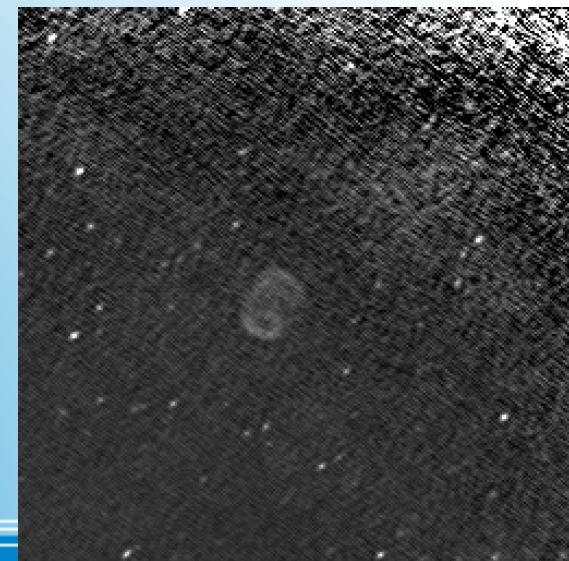
- Because of LOFAR's dense core, using natural or Briggs' weighting highlights large-scale structure
- More natural weighting (or positive Briggs' weighting) does however not change the sensitivity of LOFAR much.
- Hence, uniform weighting or Briggs' weighting with robustness value (e.g. -0.5) is used most commonly.

# Weighting

- To get proper Jy values, images need to be **corrected for the LOFAR beam**.
- Both **WSClean** and **awimager** can do this.

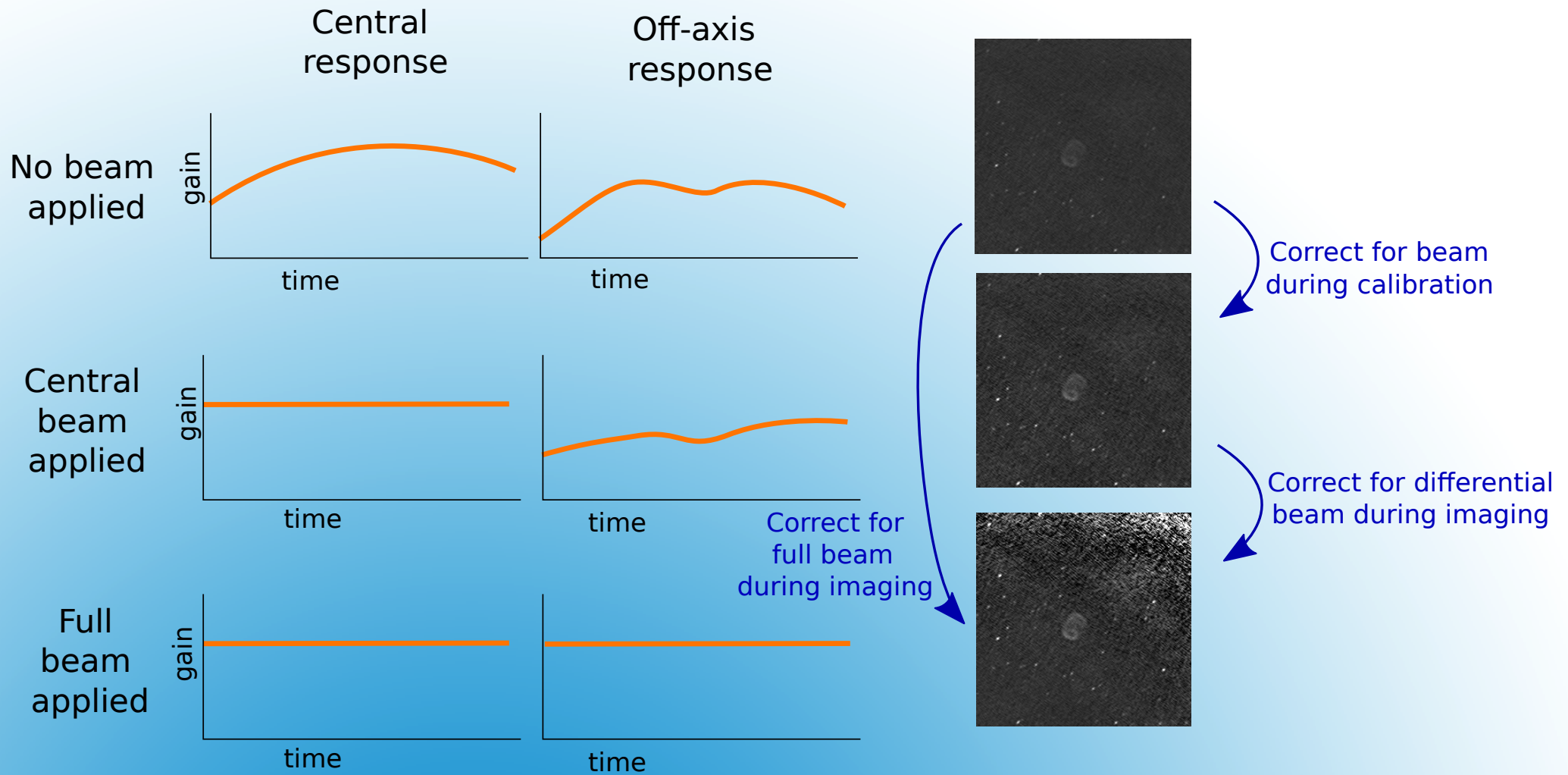


Beam not applied



Beam applied

# Applying the LOFAR beam

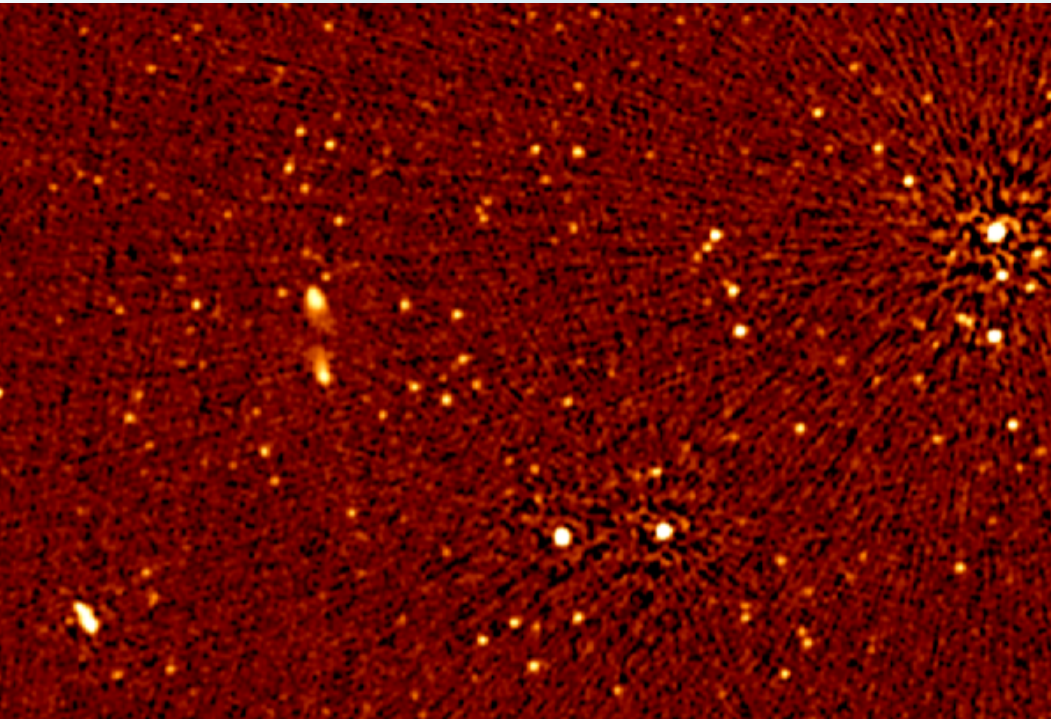


# LOFAR beam correction

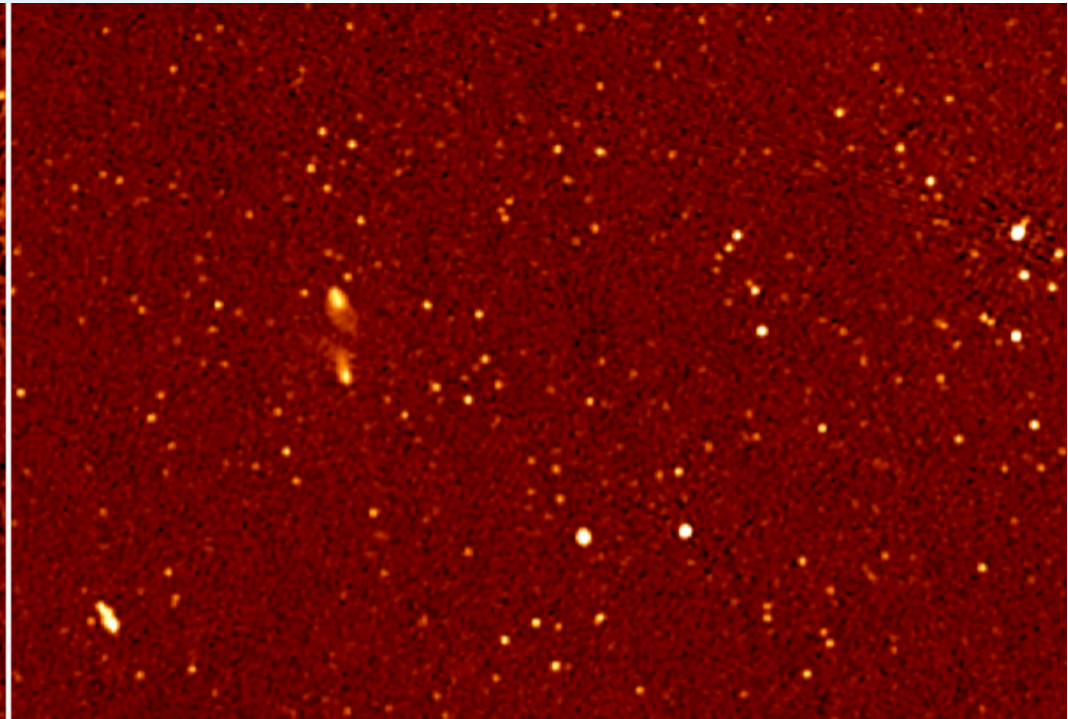


- Deconvolution (cleaning) creates a model of the field
- These clean components can be used as calibration model (“selfcal”)
- WSClean will store the predicted model visibilities in the MODEL\_DATA column
- NDPPP can calibrate the data using this column

**Self-cal & CLEAN**



After initial calibration

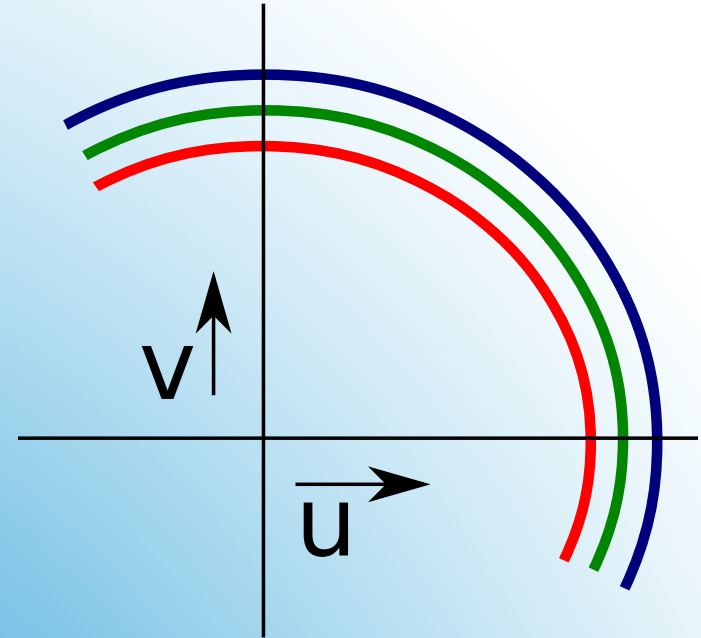


After self-cal on clean components

*Image credit: N. Hurley walker*

# Self-calibration using CLEAN

- Multi-frequency synthesis (MFS) means gridding different frequencies on the same uv grid:
- This is the standard for modern telescopes
- MFS is often confused with multi-frequency *deconvolution*



# Multi-frequency synthesis

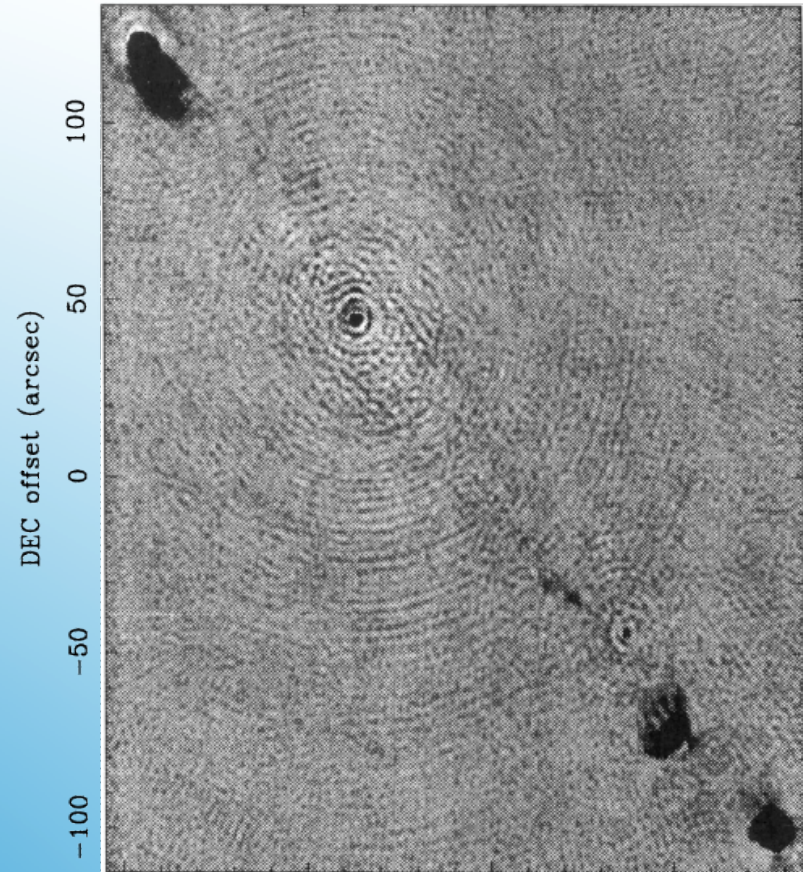
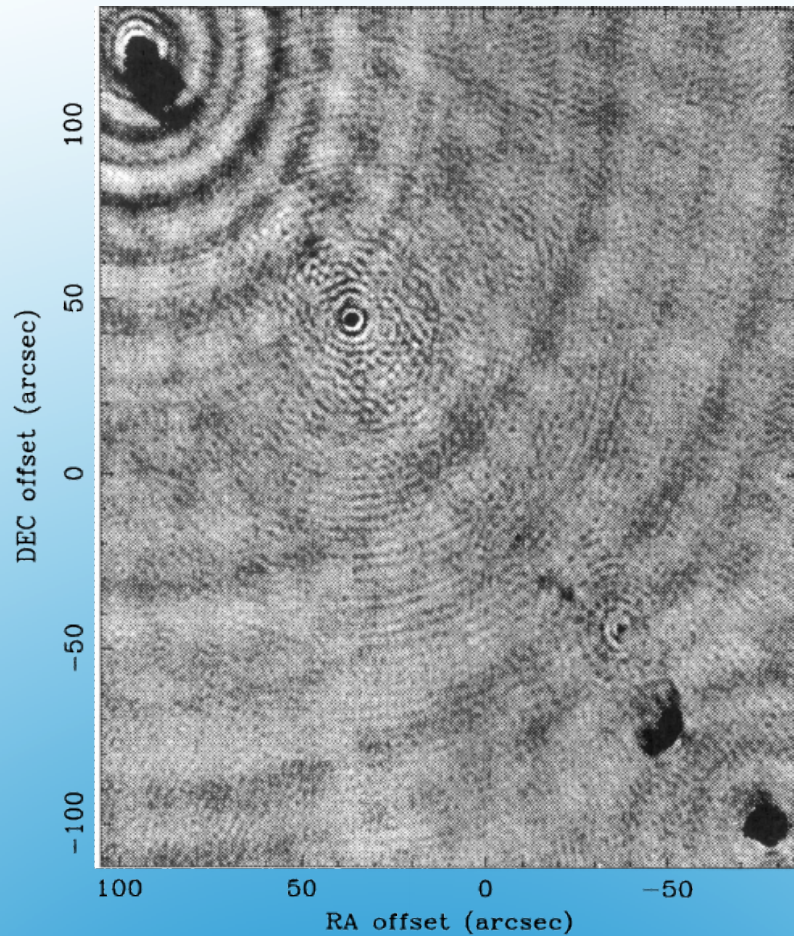
Related, but not the same:

- **Multi-frequency deconvolution** (see Rau and Cornwell, 2011)  
sometimes called  
**multi-term deconvolution**

*Selected by setting `nterms` in CASA's clean task*

- Takes spectral variation into account during deconvolution
- Useful for wide-band, sensitive imaging
  - Useful for LOFAR!

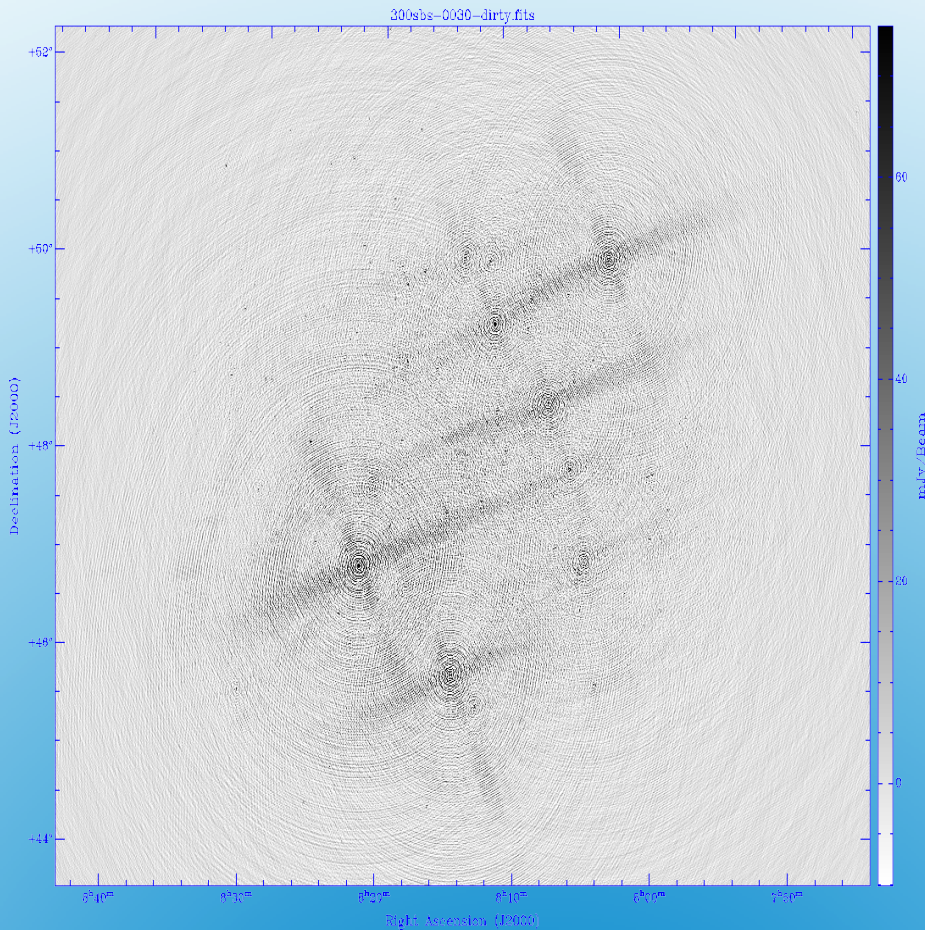
# Multi-frequency deconvolution



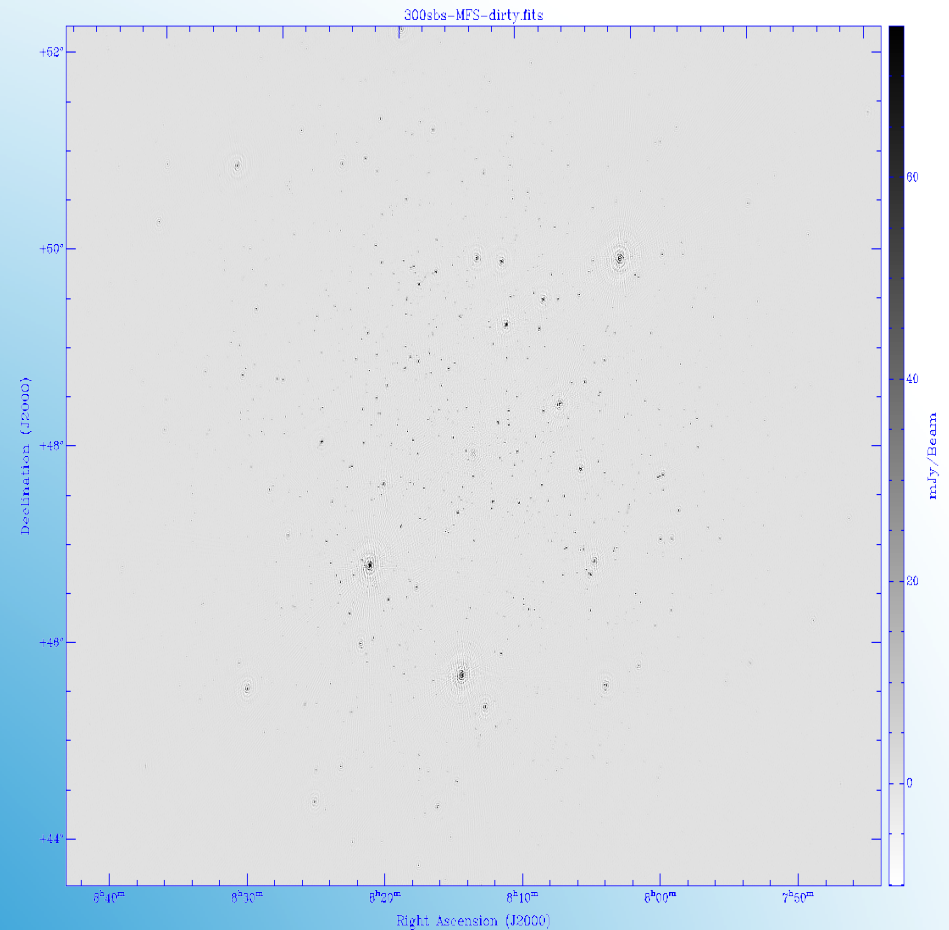
- Right image: fit for flux over frequency to improve deconvolution (Sault & Wieringa, 1994)

# Frequency-dependent deconvolution

- How to (efficiently) include spectral information in the deconvolution?



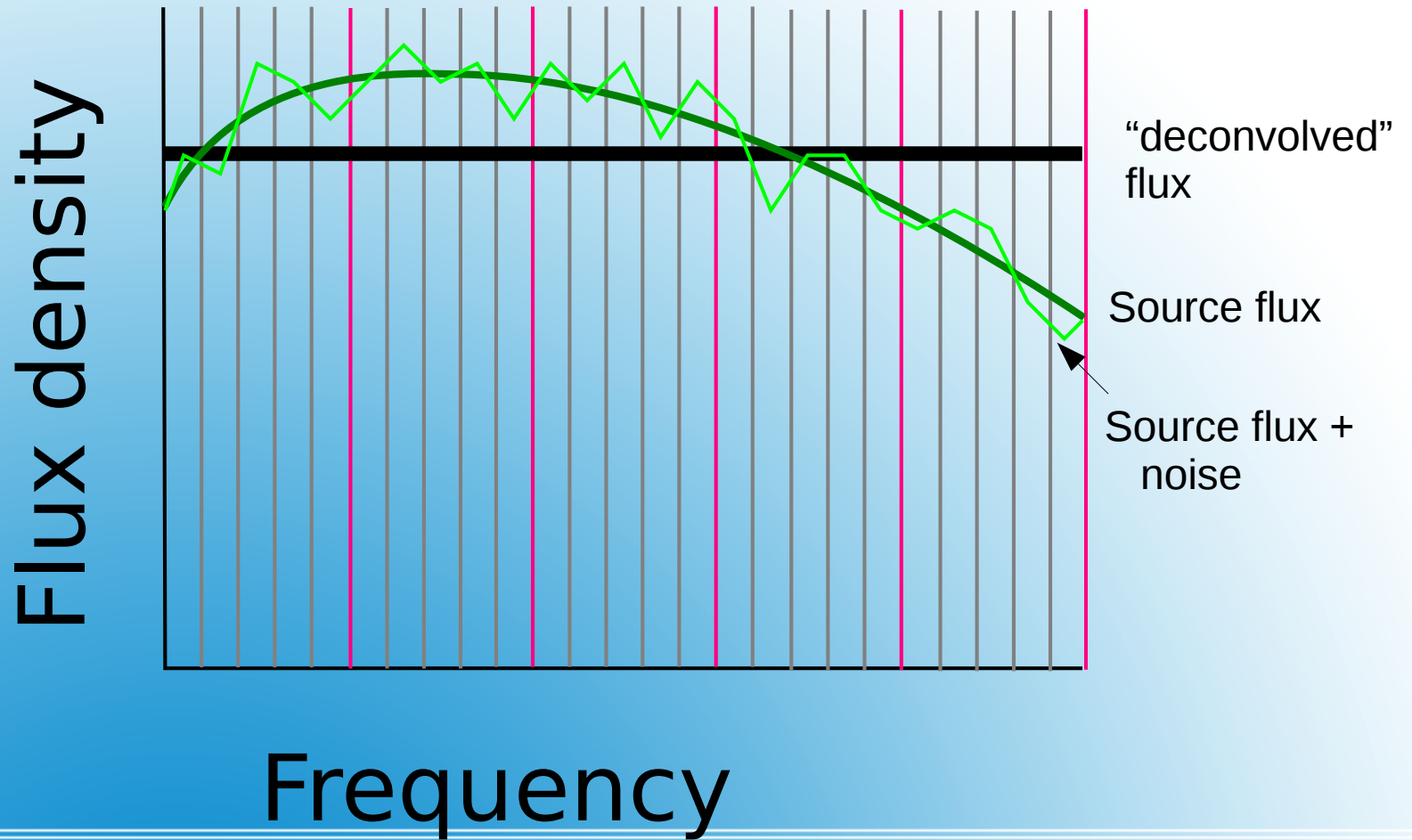
Dirty image of 0.4 kHz (two subbands)



Dirty image of 60 MHz

# Imaging with large bandwidth

# Single deconvolution over full integrated bandwidth

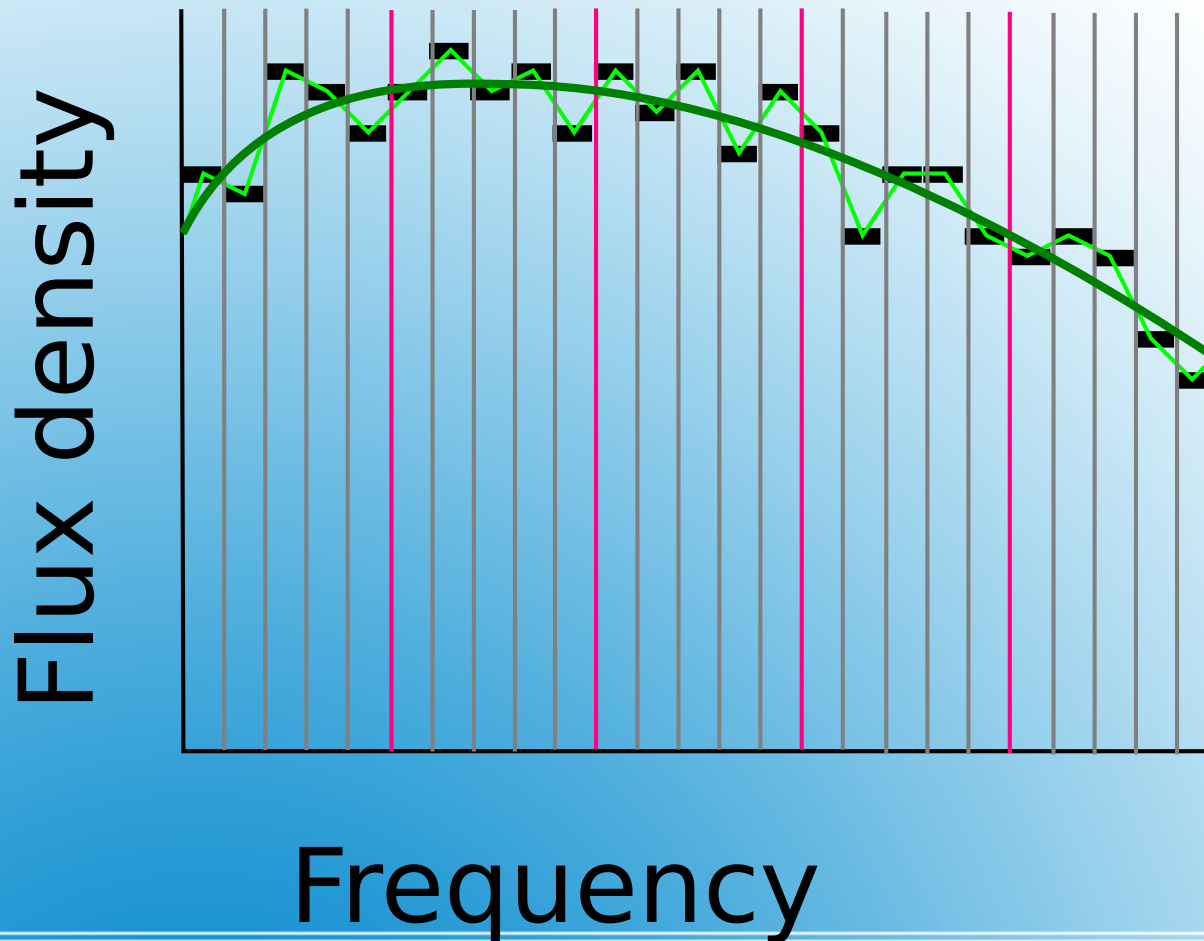


## Spectral fitting

# “Joined channel” deconvolution

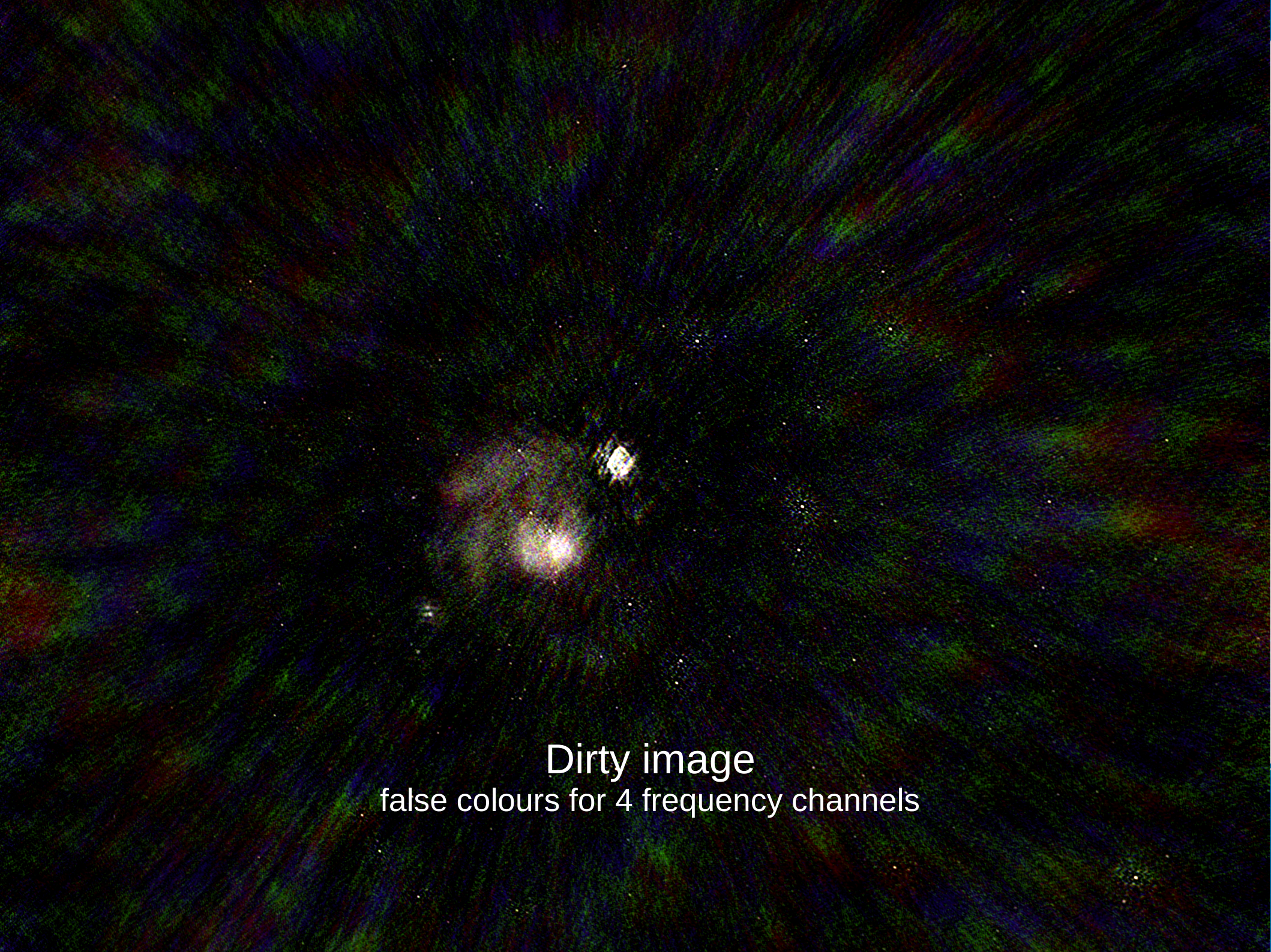
Affected by noise

Expensive: all channels are in memory/cleaned during deconvolution



## Spectral fitting





Dirty image  
false colours for 4 frequency channels



Restored image  
false colours for 4 frequency channels



Model image

false colours for 4 frequency channels

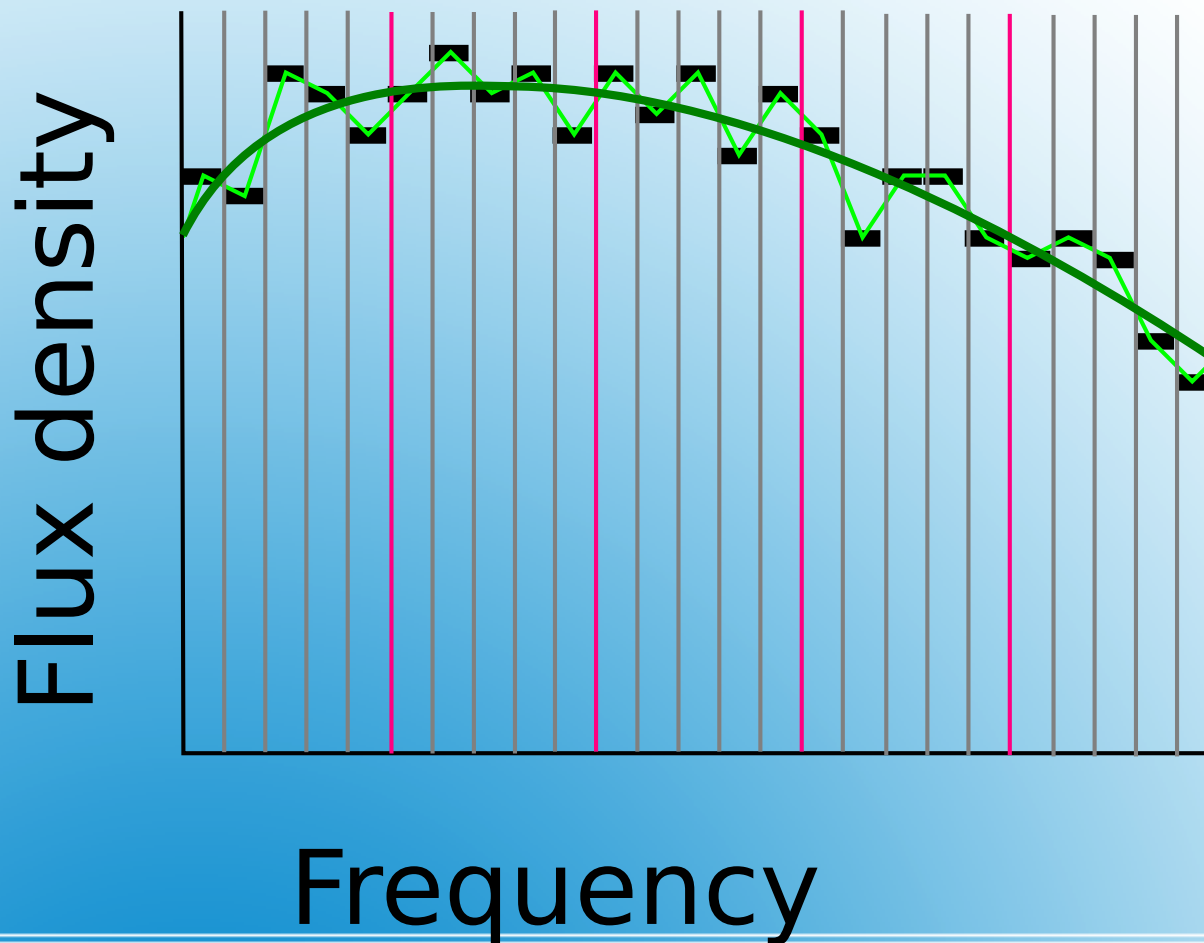
(patchy contours are artefact of increasing contrast)

Residual image  
false colours for 4 frequency channels

# “Joined channel” deconvolution

Affected by noise

Expensive: all channels are in memory/cleaned during deconvolution

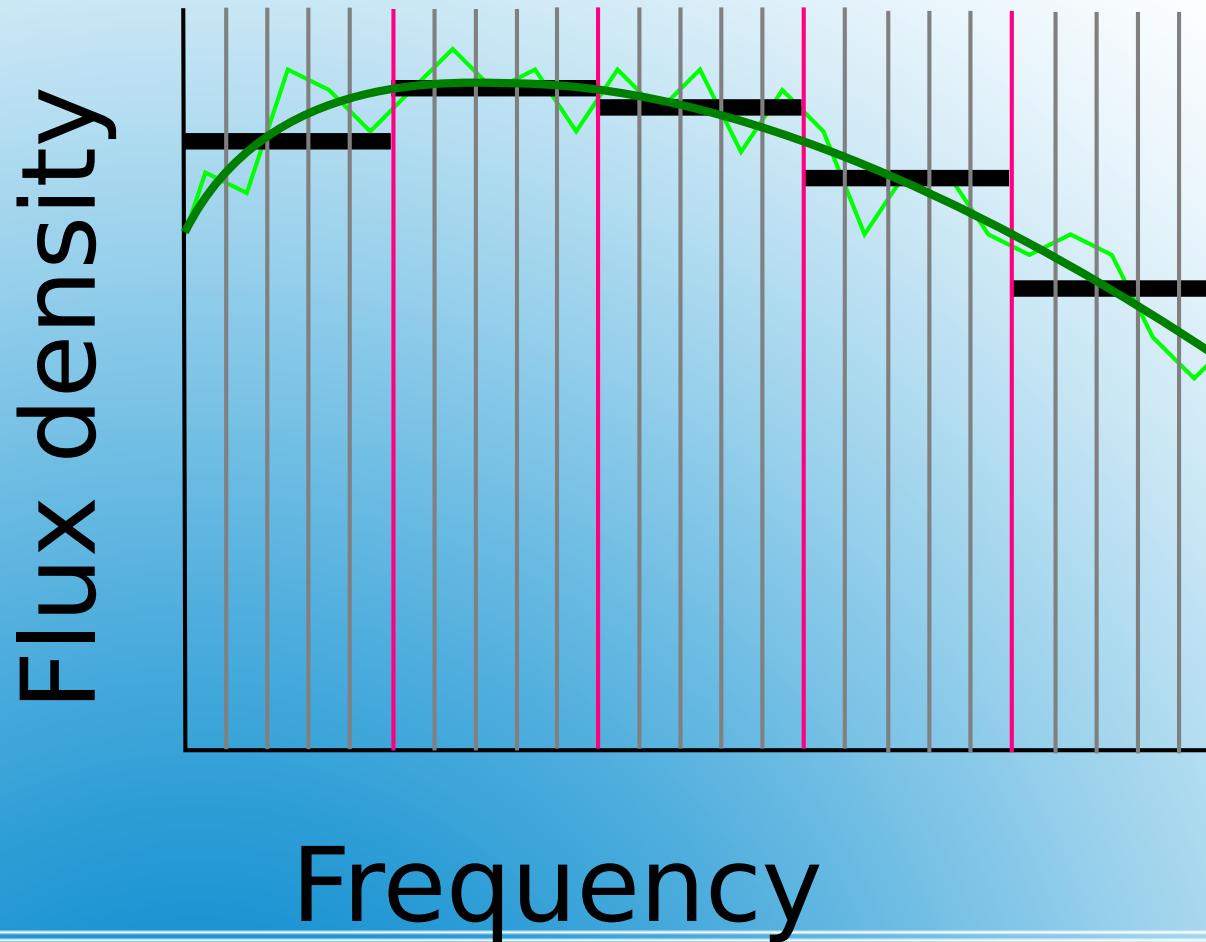


## Spectral fitting

# “Joined channel” deconvolution on fewer channels

Less expensive, less affected by noise

Not as accurate

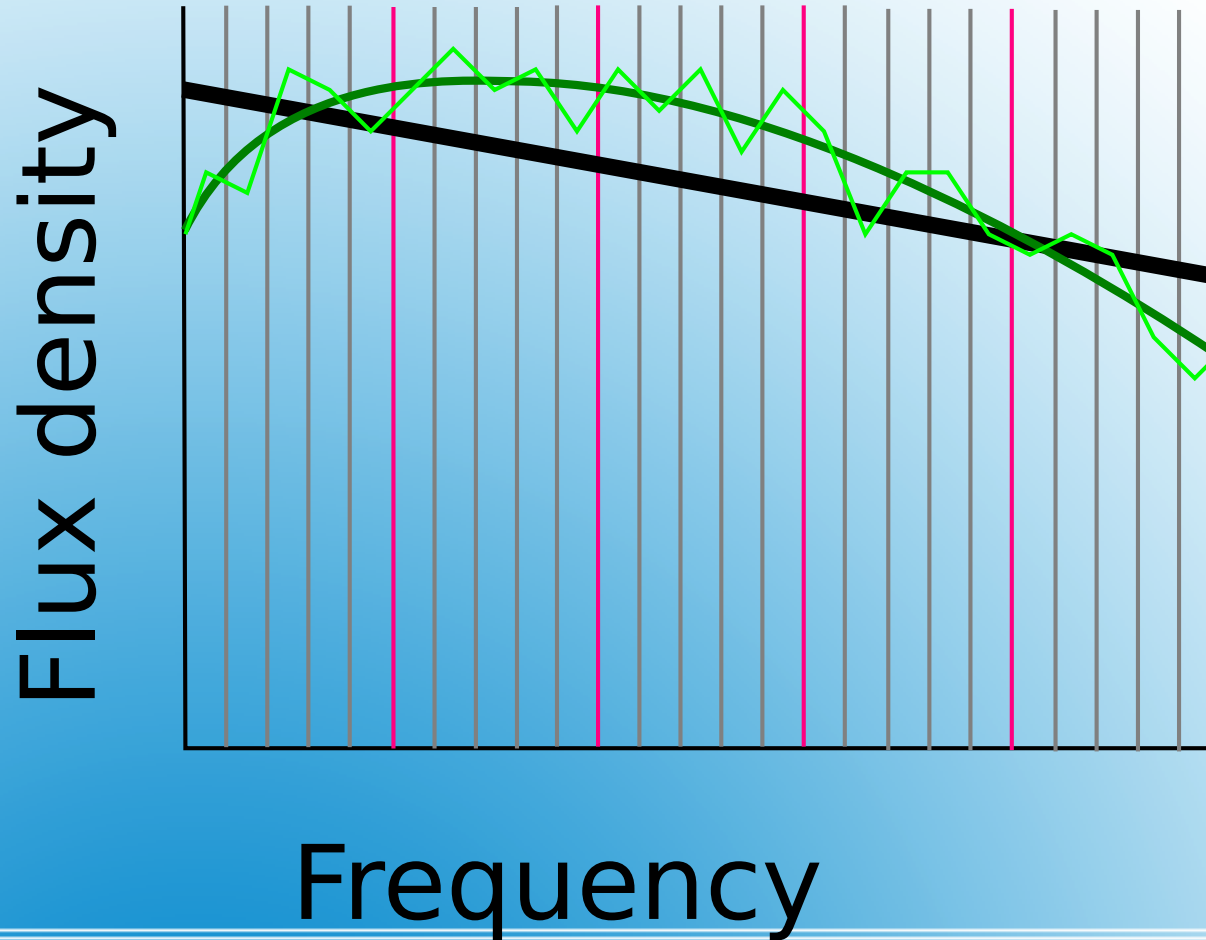


## Spectral fitting

# Joined deconvolution & fit 2-term function

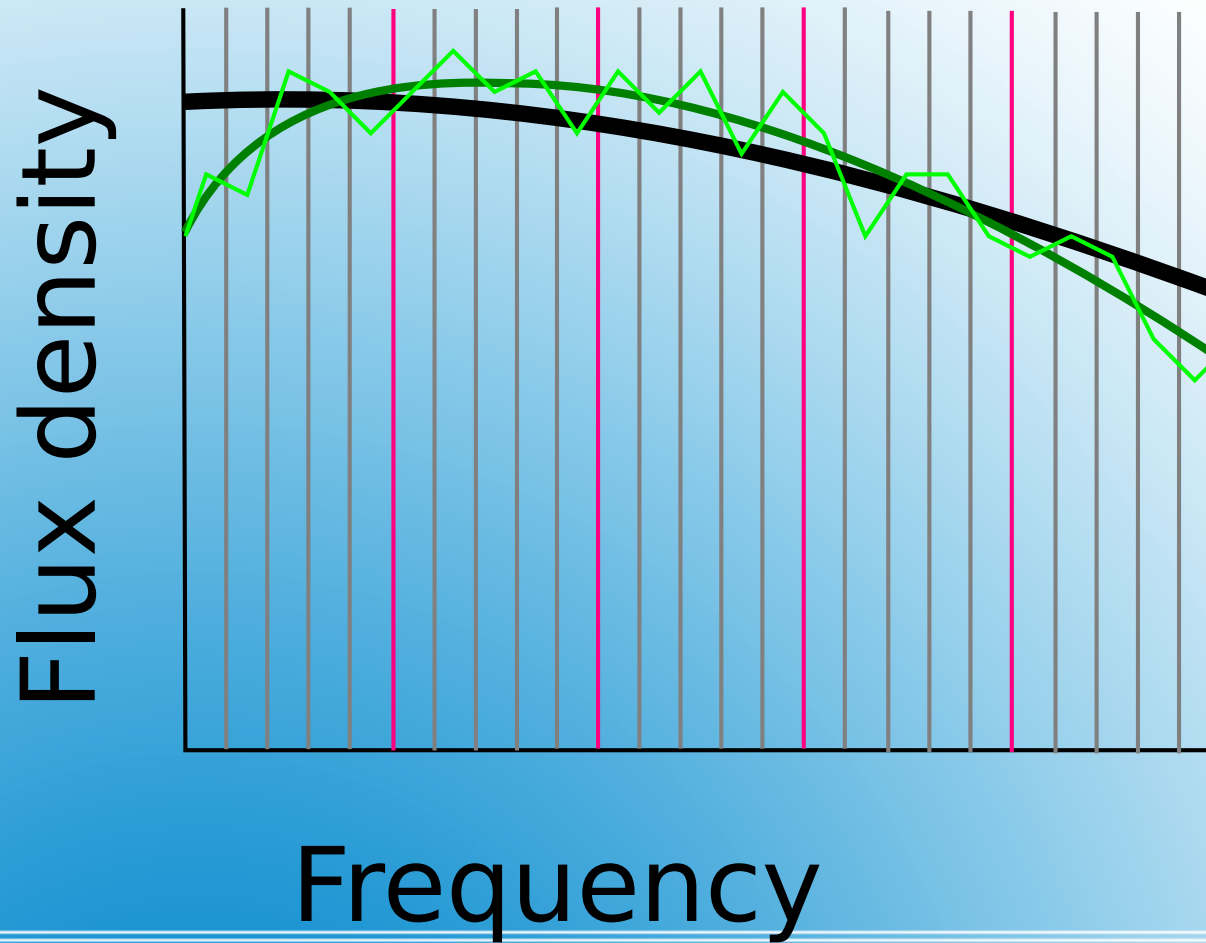
Either linear or power-law fit

Can be performed on averaged channels and interpolated for prediction



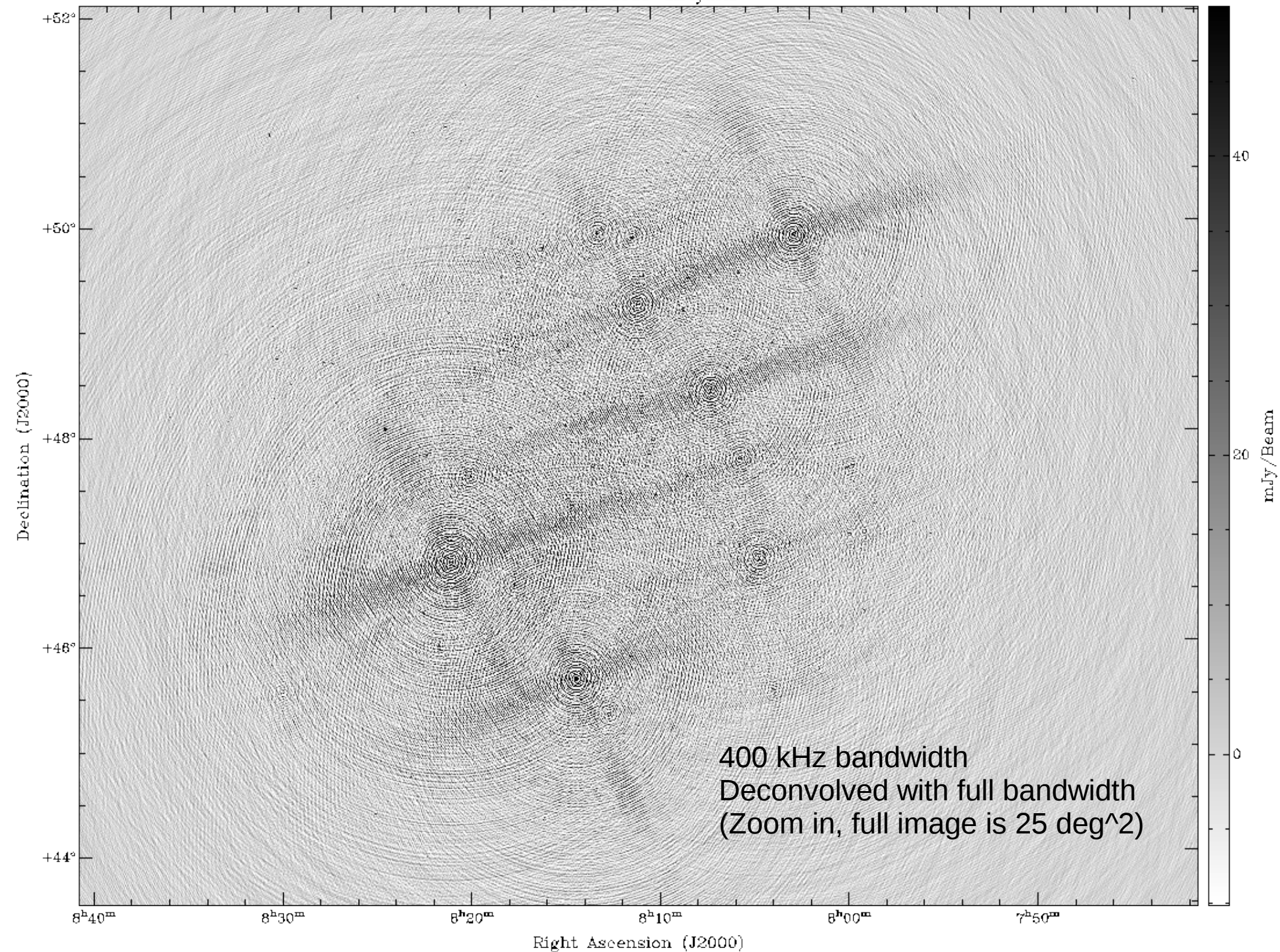
## Spectral fitting

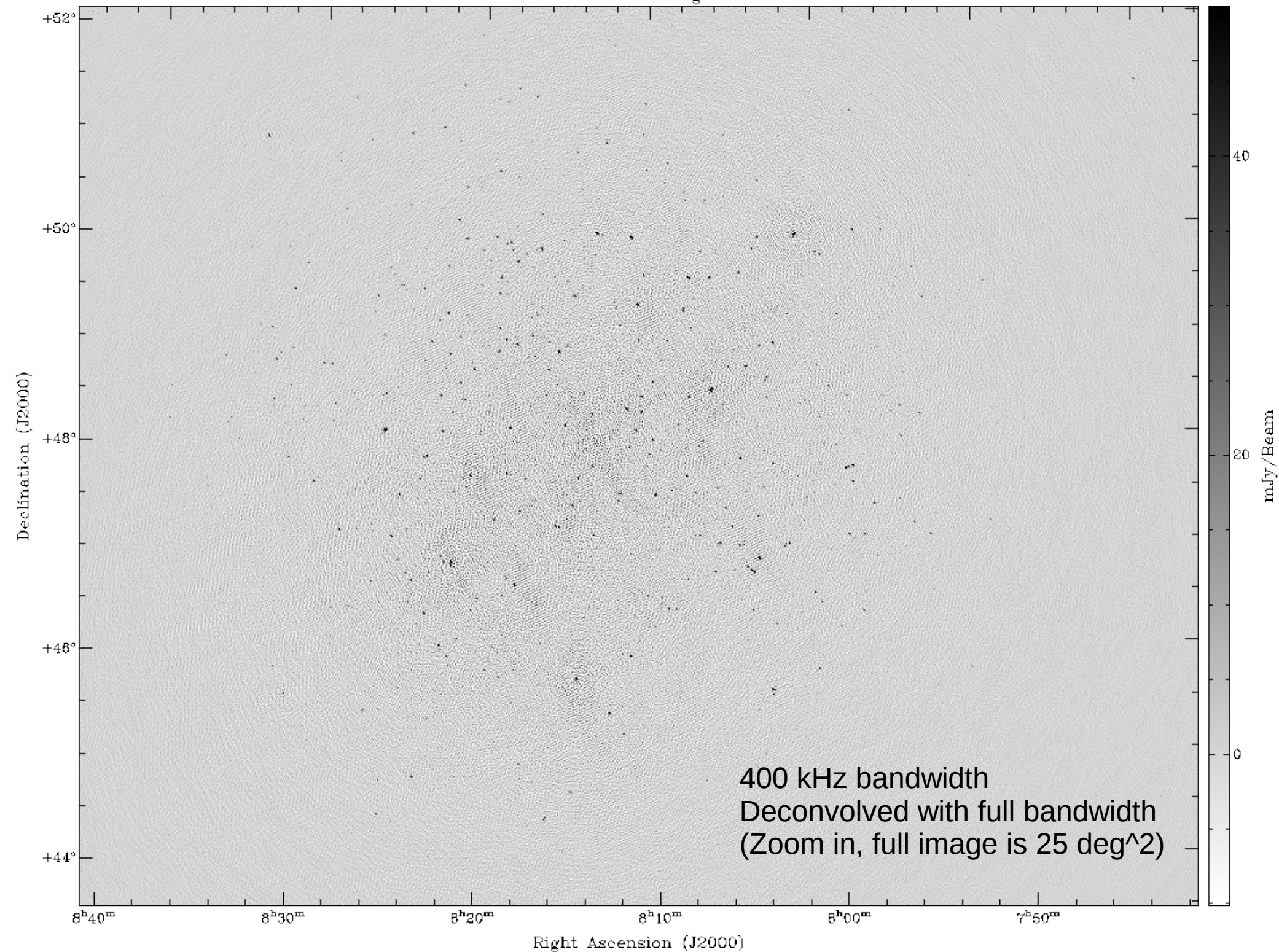
# Joined deconvolution & fit 3-term function (either in linear space or in log-log space)

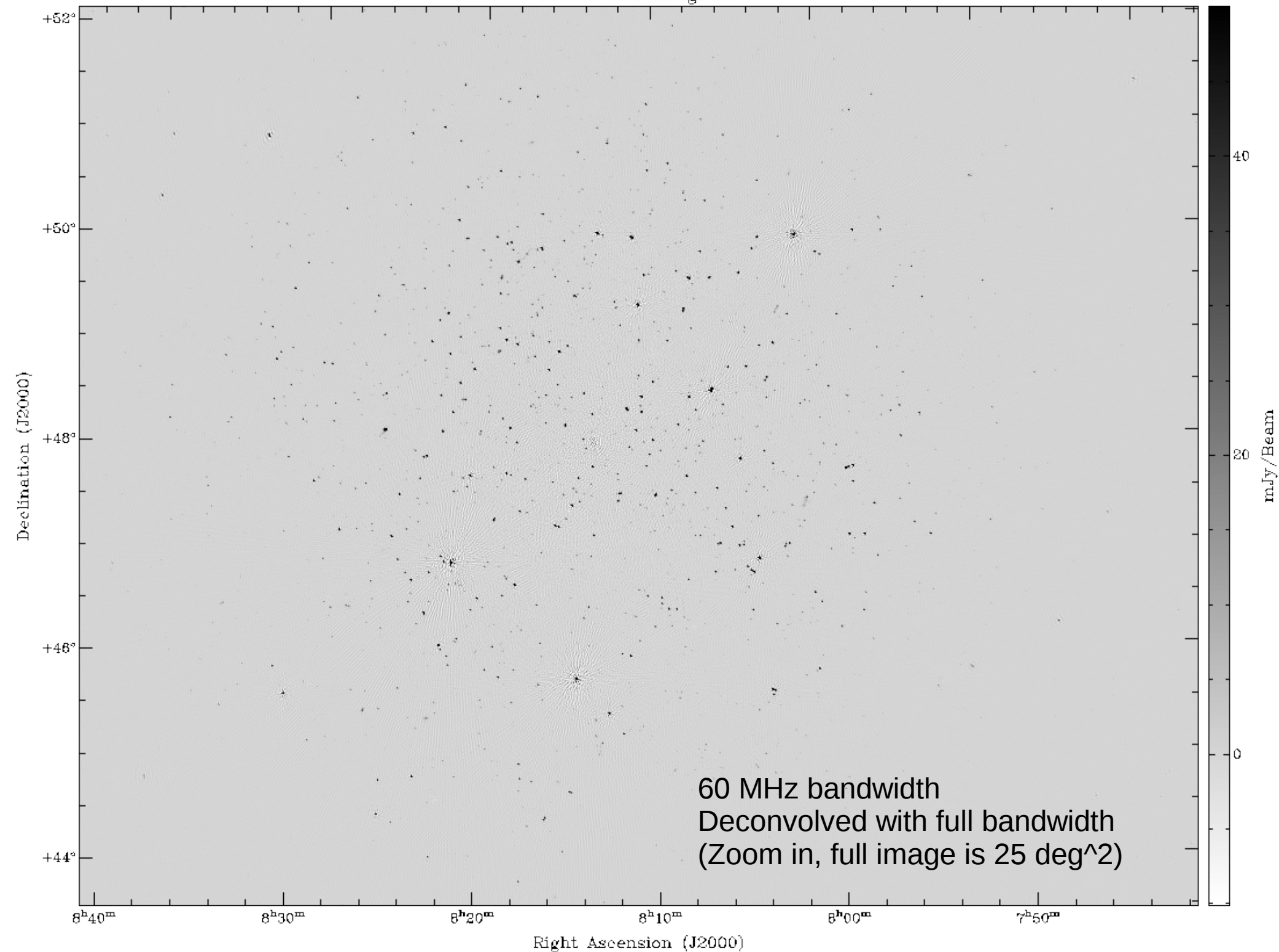


## Spectral fitting



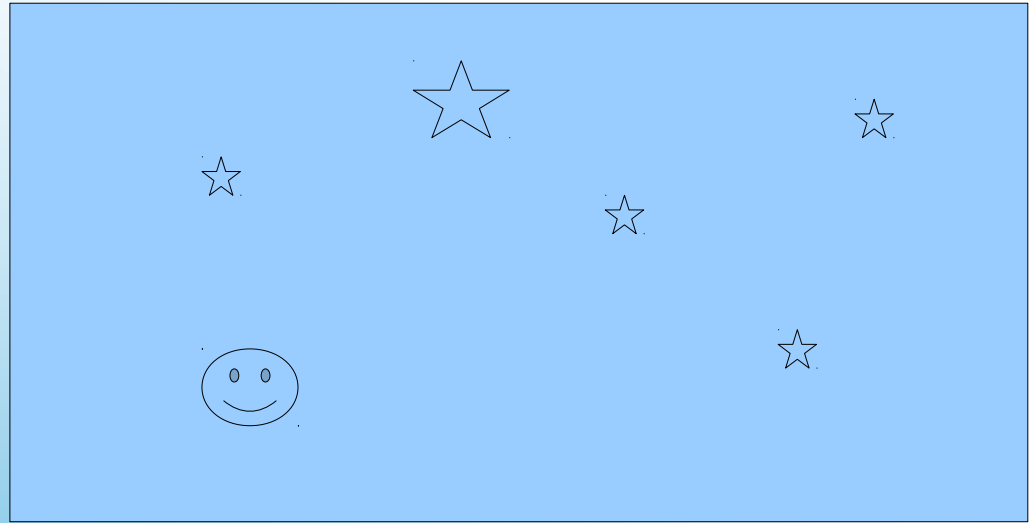






What if...

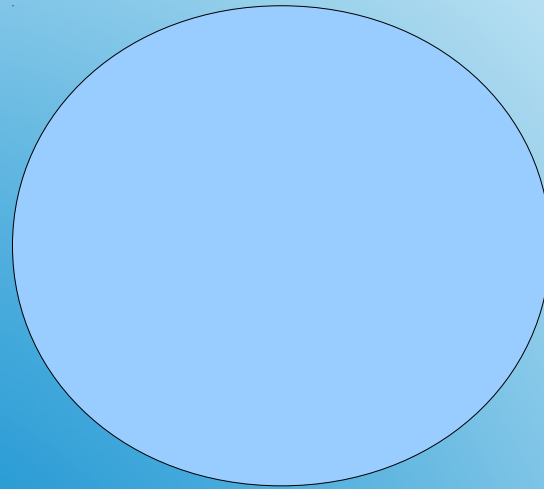
This is our field of interest →



(In practice, actual galaxies look different)

... and

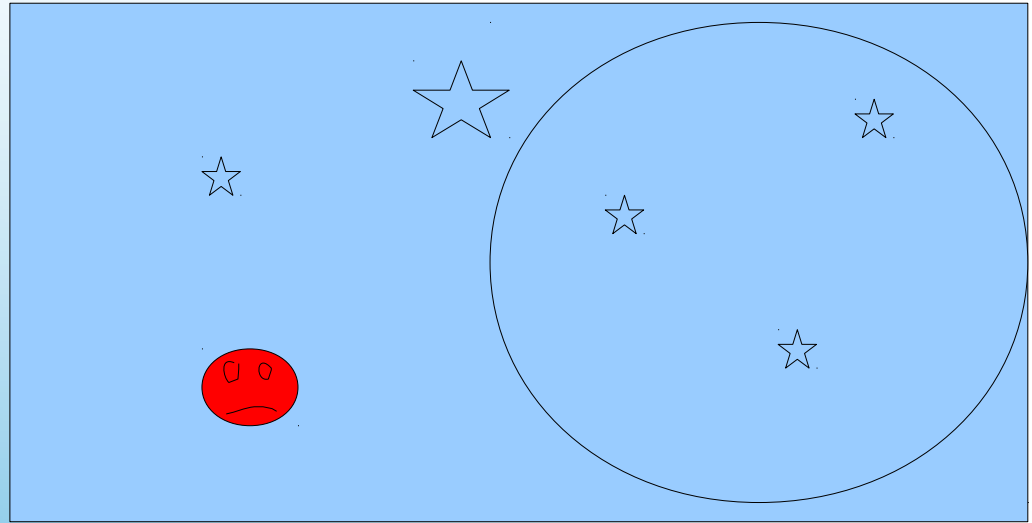
this is our primary beam →



# Mosaicing

What if...

This is our field of interest →

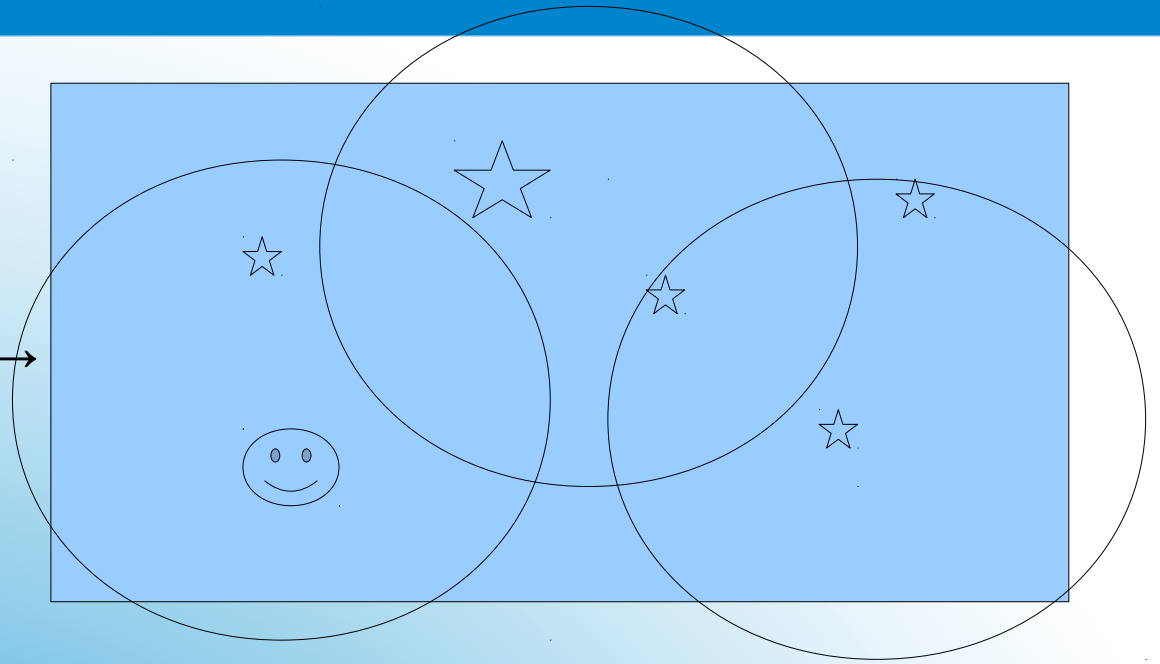


(In practice, actual galaxies look different)

**Mosaicing**

What if...

This is our field of interest →



- This is called mosaicing
- Perfect combination with LOFAR multi-beaming
- How to average the images together?

# Mosaicing

Inverse-variance  
weighting

Primary-beam-corrected image

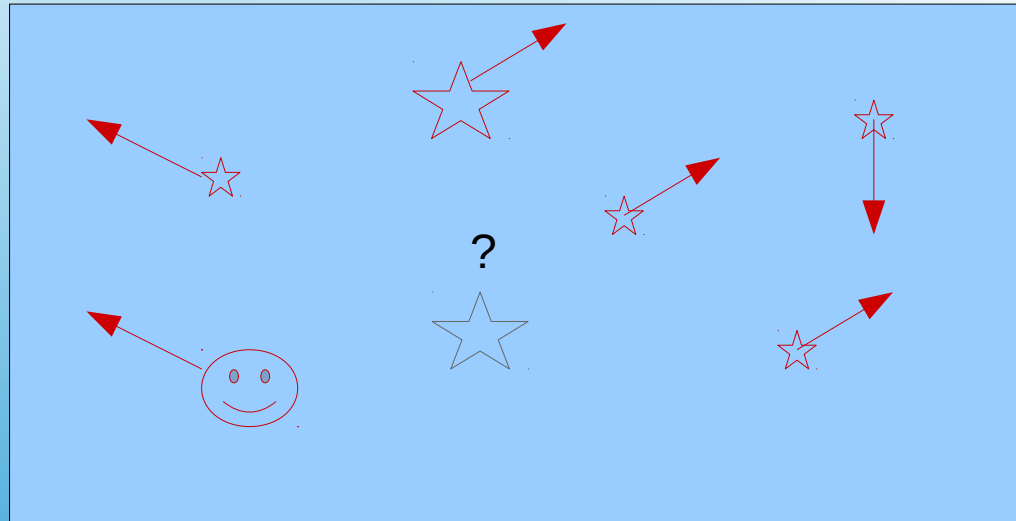
$$M(l, m) = \frac{\sum_i B_i^2(l, m) \overbrace{(I_i(l, m) / B_i(l, m))}^{\text{Primary-beam-corrected image}}}{\sum_i B_i^2(l, m)}$$
$$= \frac{\sum_i B_i(l, m) I_i(l, m)}{\sum_i B_i^2(l, m)}$$

- This is called mosaicing
- Perfect combination with LOFAR multi-beaming
- How to average the images together?

Weight with  $1/\sigma^2 = (\text{primary beam})^2$

# Mosaicing

- **Direction-dependent effects** might require further correction during imaging:



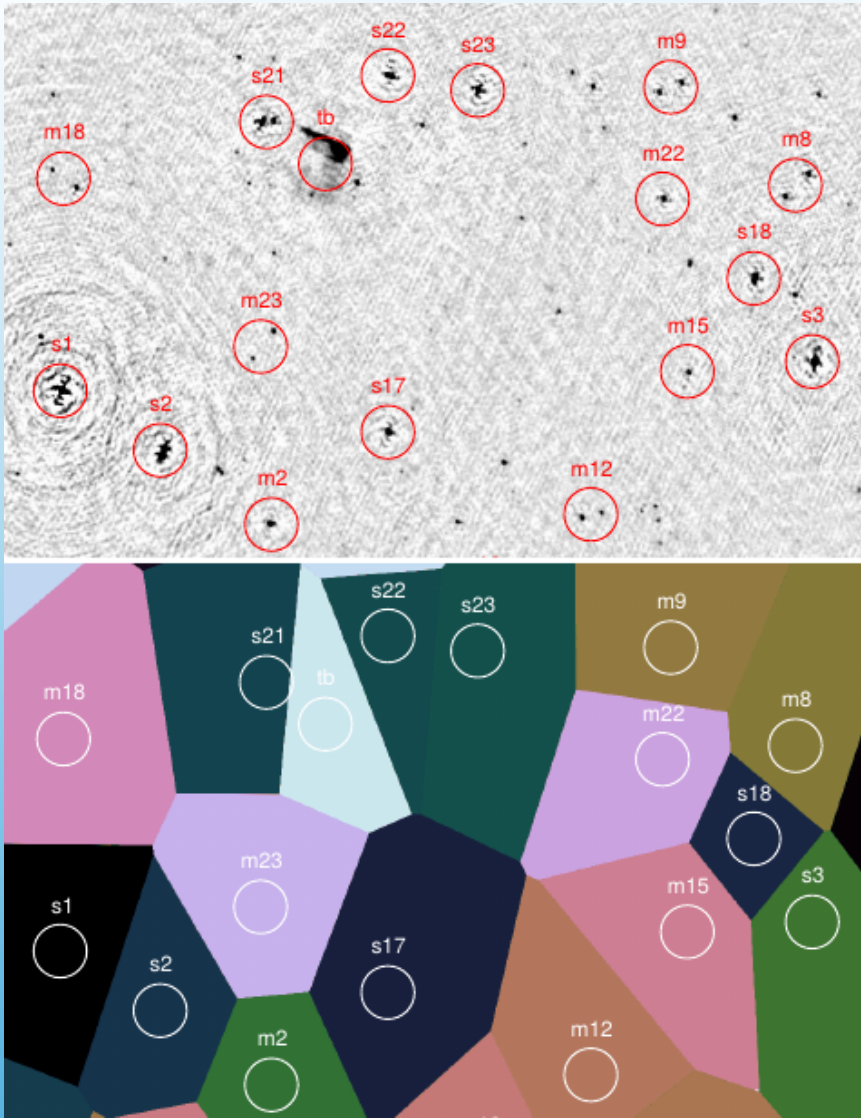
- Positions of 'calibrators' (red) are known
- Apparent position has moved due to ionosphere

**More variable effects...**



- **Direction-dependent effects** might be time-variable (e.g. ionosphere)
- Besides position, **DD effects** can also affect polarization angle and brightness
- Not a fully solved problem, but possible solutions:
  - image in small **facets** where **DDE**'s are constant
  - or interpolation – AWImager can do this.
  - Peeling

## Direction-dependent effects



Factor is currently the best pipeline to produce high-resolution high-dynamic-range images.

- Works by facetting the sky
- Each facet is independently self-calibrated

# Factor

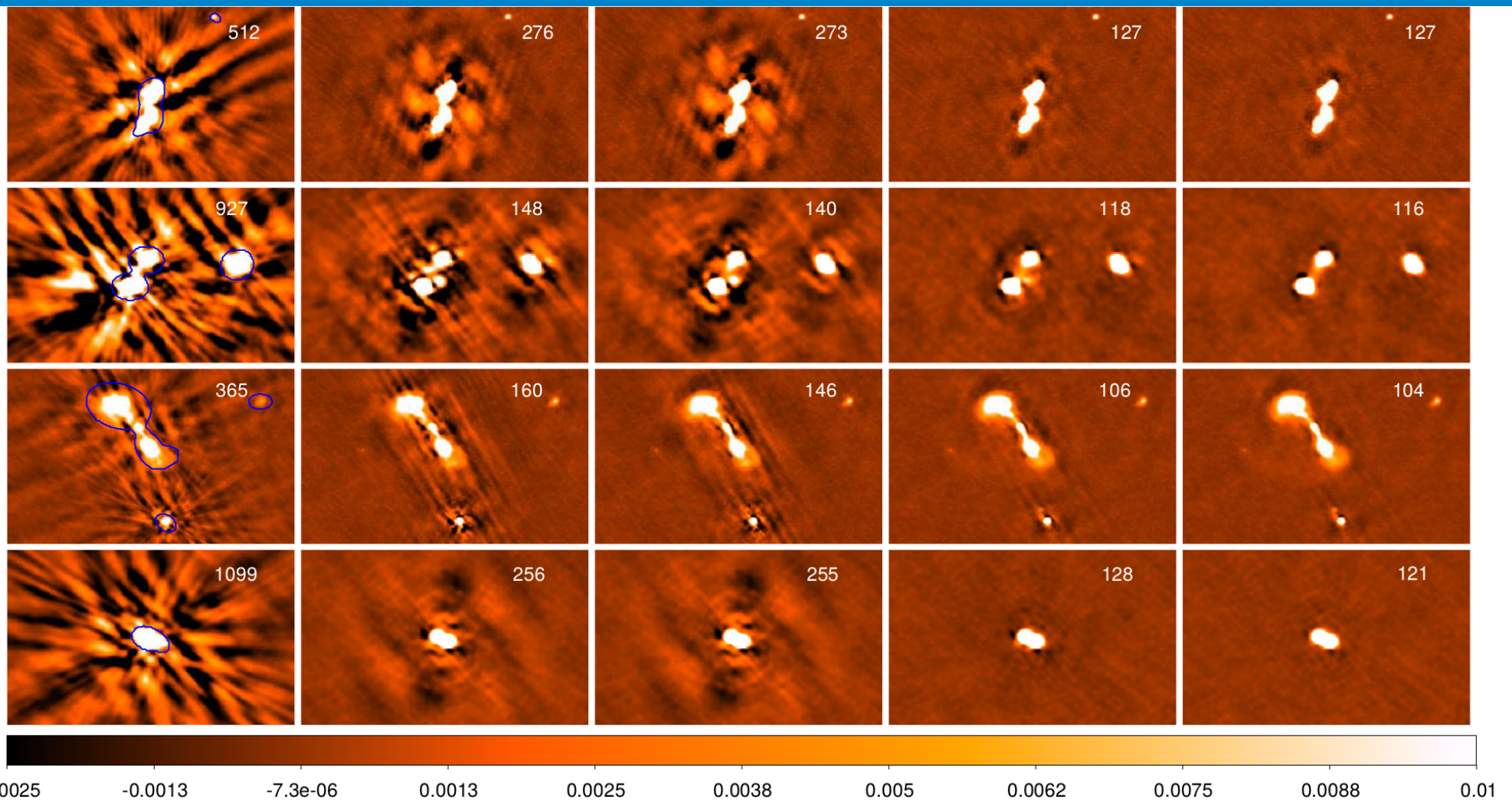


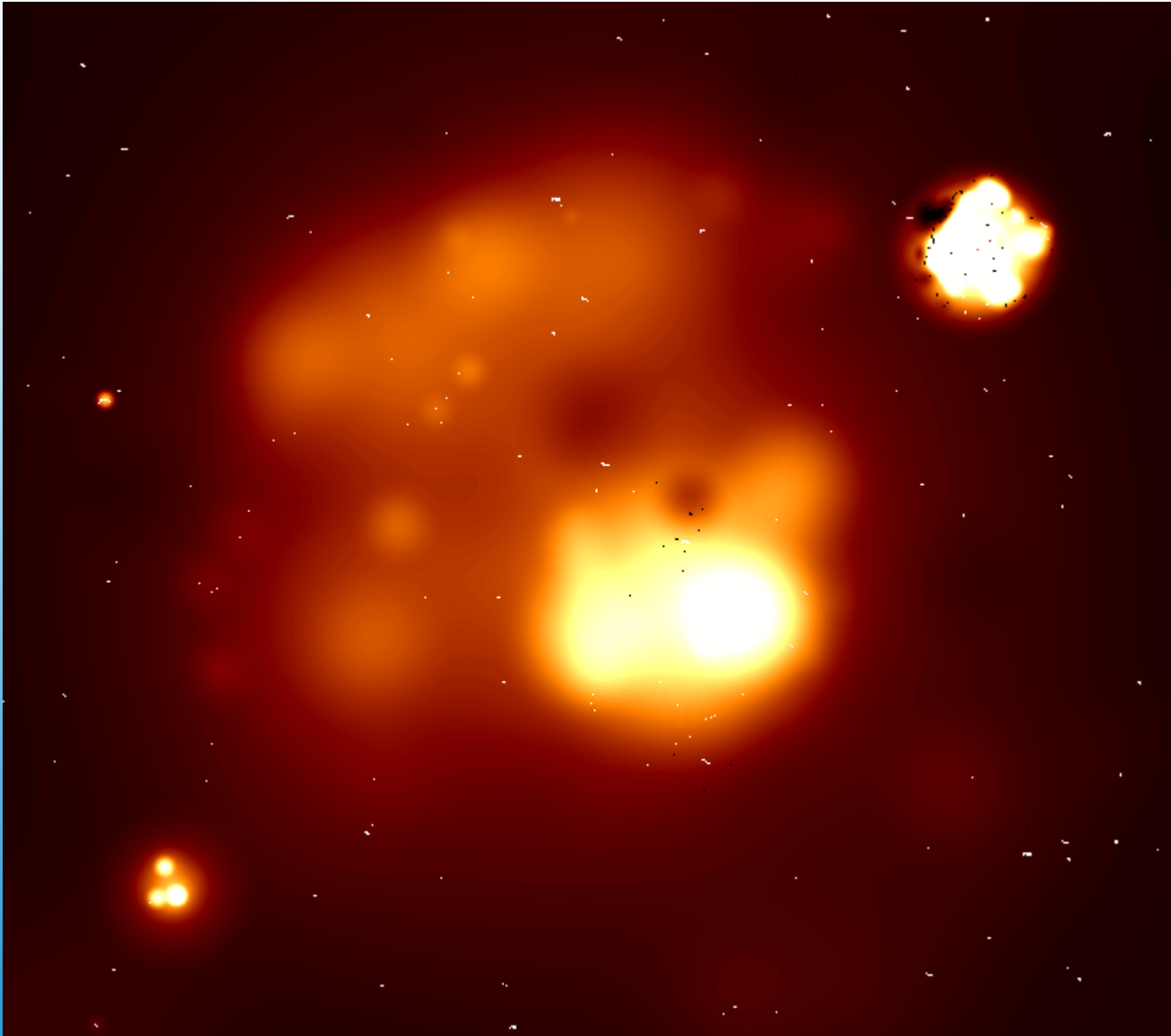
FIG. 10.— Images showing the incremental improvements during the DDE calibration, see Sect. 5.3. For reference, the first and second row of images show direction s2 and s21, respectively (Figure 6). All images are made using the full dataset (120–181 MHz,  $\text{robust}=-0.25$ ) and have a resolution of  $8'' \times 6.5''$ . Note that at this resolution many of the bright DDE calibrator sources are resolved. The first column displays the images made with the (direction independent) self-calibration solutions, see Sect 4.6. The blue contours show the clean mask that was created with PyBDSM for the imaging. The clean mask is updated at each imaging step during the DDE calibration (not shown). The next columns display improvements during the DDE calibration step (see also Figure 9). Second column: first DDE TEC+phase iteration. Third column: second DDE TEC+phase iteration. Fourth column: third DDE TEC+phase iteration and first DDE XX and YY gain (amplitude and phase) iteration. Fifth column: fourth DDE TEC+phase iteration and second DDE XX and YY gain (amplitude and phase) iteration. For all four directions the TEC+phases were solved for on a 10 s timescale. The XX and YY gains were solved for on a 10 min timescale, except for the source in the top row for which this was 5 min. The scale bar at the bottom is in units of  $\text{Jy beam}^{-1}$ . The images in the first and third row were cleaned with multi-scale clean because of extended emission. The r.m.s. noise level in each of the images is indicated in the top right corner in units of  $\mu\text{Jy beam}^{-1}$ .

- Recent focus on deconvolution using '**compressed sensing**' (abbrev. **CS** – but **CS** can mean “**Cotton-Schwab**” too)
- **CS** methods assume the sky is 'sparse' (“solution matrix is sparse in some basis”)
- Minimizes “L1-norm” (= abs sum of CLEAN components)
- Högbom clean is actually (almost) a compressed sensing method called “**Matching Pursuit**”
- **CS** considers **MP** to be non-ideal... but radio data is not the perfect **CS** case: **Calibration errors, w-terms**

# Compressed sensing



**Model created by a CS method  
("non-linear conjugate gradient using IUWT")**



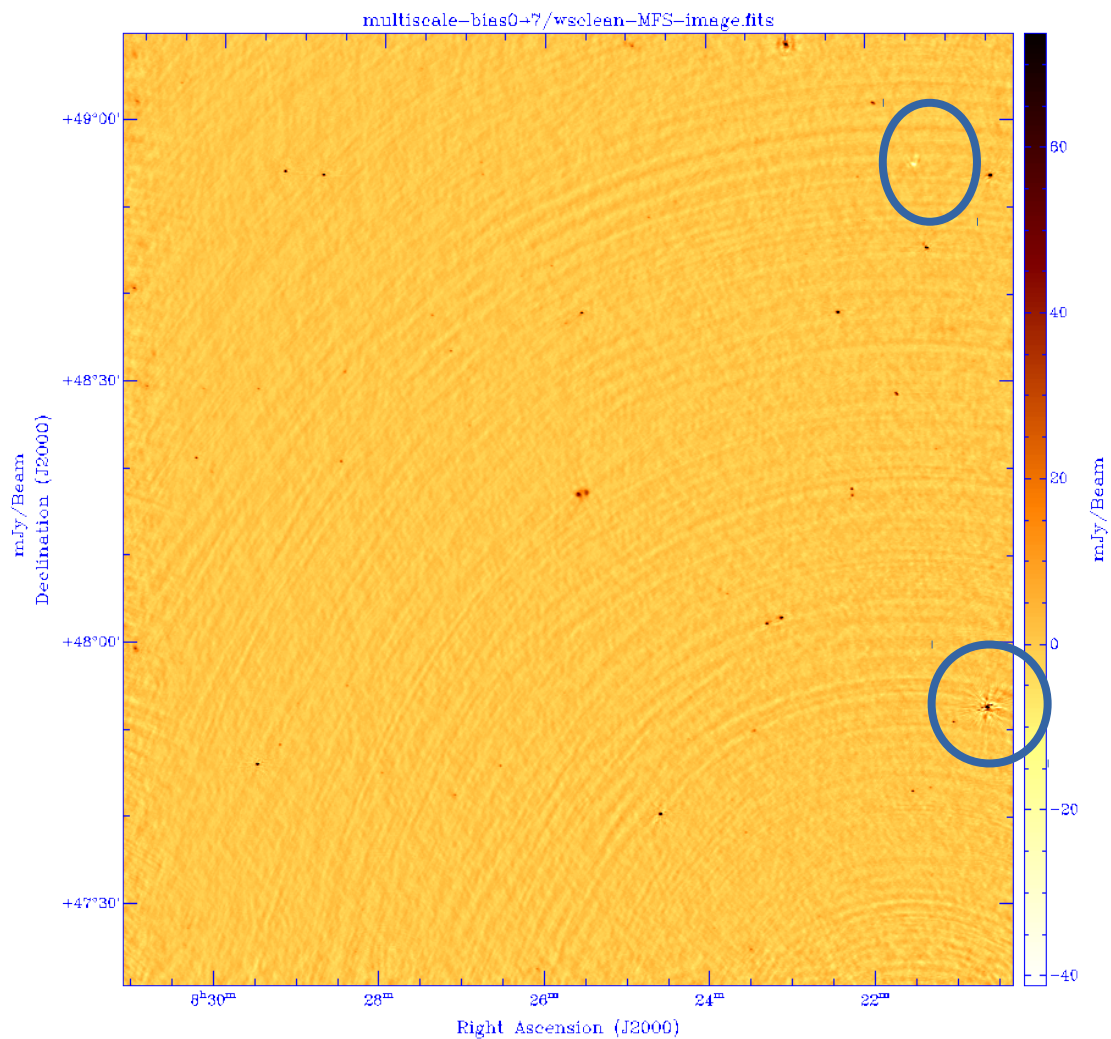
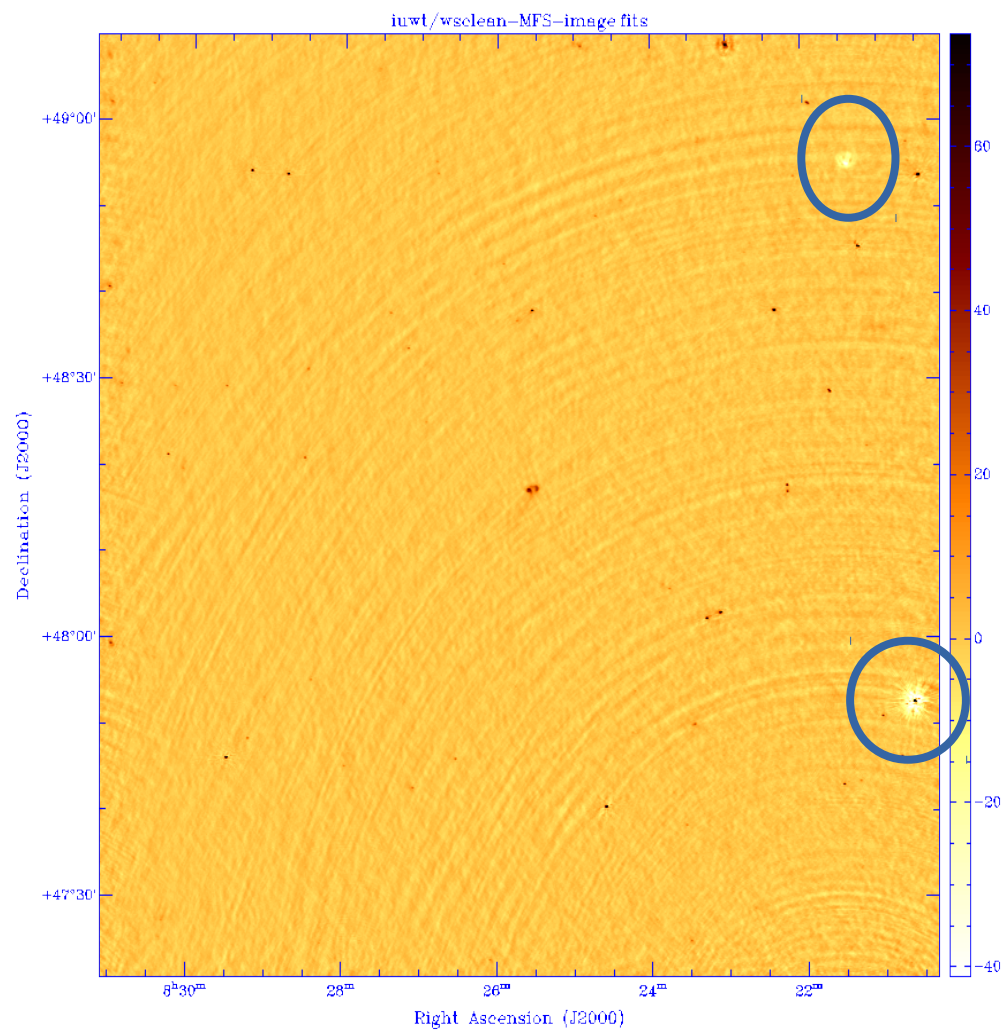
**Model created by multi-scale clean**

Both WSClean's IUWT and Moresane diverge on sources with calibration errors

Multi-scale clean is more robust to calibration errors

WSClean MF IUWT:

MF Multi-scale:



Thank you for your attention!