Wide Field Imaging

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Introduction

- Why Wide Field Imaging
- LOFAR imaging challenges
- Imaging fundamentals
- Algorithms

Why is Wide Field Imaging Important?

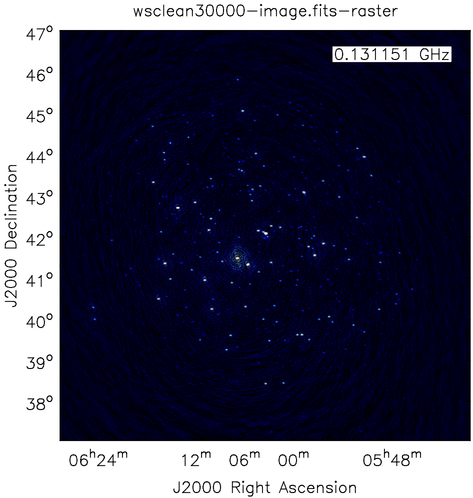
- Nice for surveys of course...,
- but what if you are interested in a single target?
- visibilities are the sum over the entire beam
- need WF imaging to achieve dynamic range

LOFAR imaging challenges

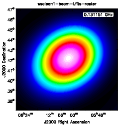
Imaging LOFAR data poses additional challenges

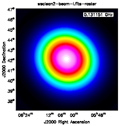
- Very wide fields of view
- Stations are fixed on the ground. Beam shape varies with time
- Ionosphere
- Wide frequency band
- Diffuse sources

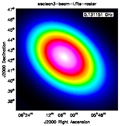
Wide Fields of View



Varying beam shape

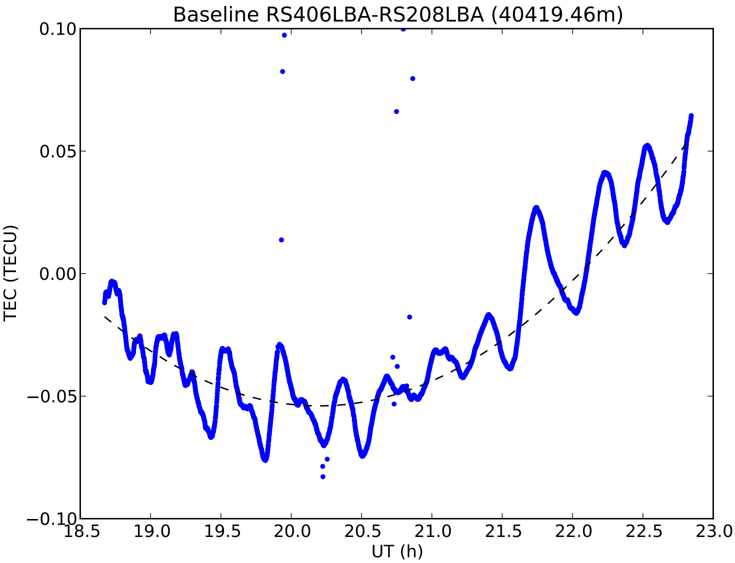






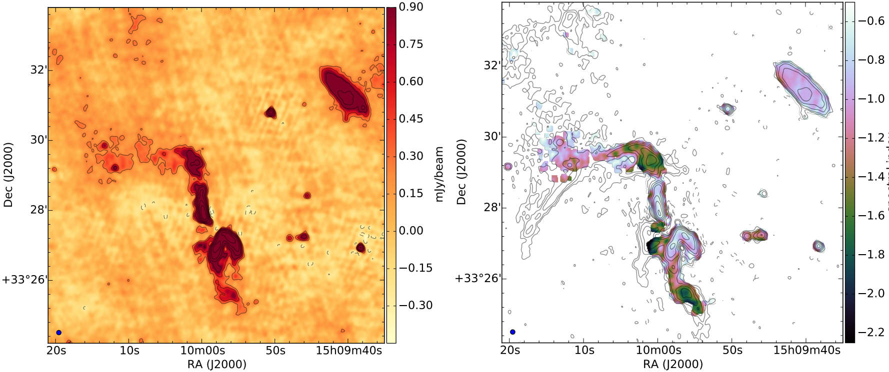

```
wsclean -interval 3000 3100 -size 3000 3000 -scale
12asec -use-idg -idg-mode hybrid -grid-with-beam
-weight briggs 0 -make-psf-only
/var/scratch/bvdtol/RX42_SB100-109.2ch10s.ms
```

Ionosphere



Baseline RS406LBA-RS208LBA (40419.46m)
TEC (TECU)
UT (h)
0.10
0.05
0.00
−0.05
−0.10
18.5
19.0
19.5
20.0
20.5
21.0
21.5
22.0
22.5
23.0

Wide Frequency Band & Diffuse sources



"A plethora of diffuse steep spectrum radio sources in Abell 2034 revealed by LOFAR" by T. W. Shimwell et.al.

frame

- Imaging as inversion problem
- Iterative solutions
- Gridding and degridding

Imaging as Inversion Problem

The Measurement Equation

$$v = \int\int \exp 2\pi(ul + vm + nw)A(l, m)I(l, m)$$

The Measurement Equation can be written as a system of linear equations:

$$\mathbf{v} = \mathbf{A}\mathbf{i}$$

where **v** are the visibilities, **i** the source fluxes or pixel values, **A** describes the relation between the two. Problem: what is **i** given **v**? Generic solution:

$$\mathbf{i} = \left(\mathbf{A}^{\mathbf{H}}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathbf{H}}\mathbf{v}$$

Direct inversion of $\mathbf{A}^{\mathbf{H}}\mathbf{A}$ is not feasible.

Iterative Solution

Minimize sum of squared errors

$$\mathbf{i} = \arg\min_{\mathbf{i}} \|\mathbf{v} - \mathbf{A}\mathbf{i}\|^2$$

Find the best matching image, starting from an empty image, updating the image using the derivative

$$\frac{\partial}{\partial \mathbf{i}} \|\mathbf{v} - \mathbf{A}\mathbf{i}\|^2 = \mathbf{A}^{\mathrm{H}} (\mathbf{v} - \mathbf{A}\mathbf{i})$$

The residual image is the derivative of the cost function.

CS Clean

1. start with an empty model image
2. make a dirty (residual) image
3. if threshold has been reached then stop
4. find clean components and update the model image (psf -¿ Hessian)
5. compute model data
6. compute residual data and go to step 2

FFT, gridding and degridding

- Evaluation of products $\mathbf{Ai}$ and $\mathbf{A}^{\mathrm{H}}\mathbf{v}$ is expensive.
- Equations resemble a Fourier transform.
- Fast Fourier Transform is efficient
- Need visibilities on a regular grid, but u, v, w coordinates are continuous.
- need to resample visibilities

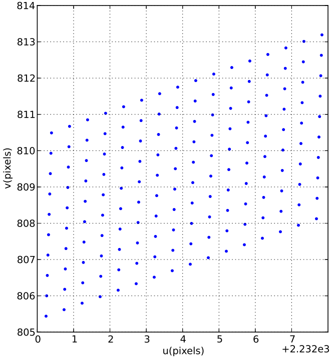
Convolution Theorem

Measurement Equation resembles a Fourier transform.
An important property of the Fourier transform is the convolution theorem.

$$\mathcal{F}(f * g) = \mathcal{F}(f) \cdot \mathcal{F}(g)$$

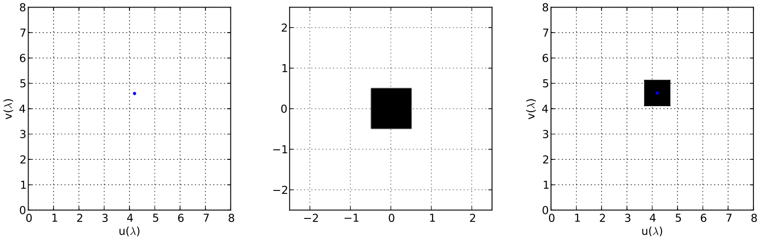
$$\mathcal{F}(f \cdot g) = \mathcal{F}(f) * \mathcal{F}(g)$$

Track of a baseline in uv grid



Gridding (I)

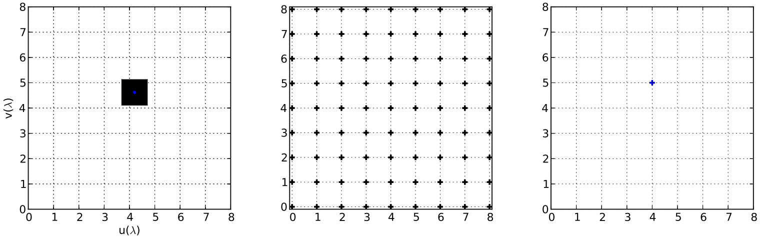
Convolution with 2D box function



The result overlaps with only a single (integer) grid point

Gridding (II)

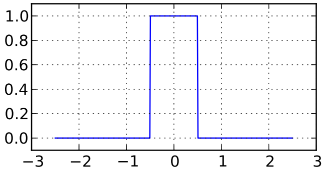
Multiplication by 2D Dirac comb function

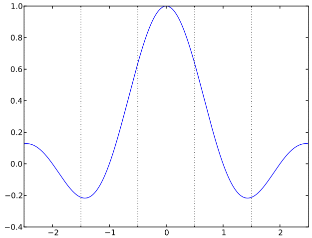


Selects only the value on integer grid points. The result is gridding on the nearest grid point.

What effect has gridding in the image domain? (I)

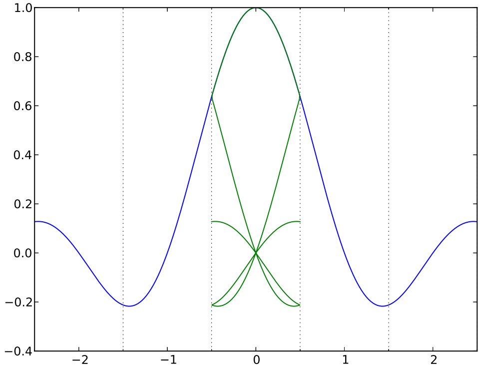
Remember: convolution in the uv domain is a multiplication in the image domain





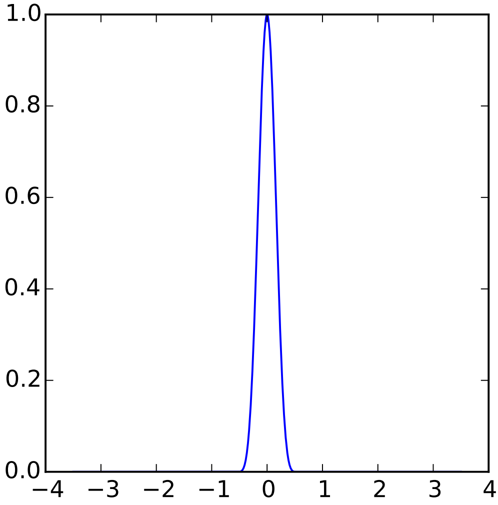
What effect has gridding in the image domain? (II)

And multiplication in the uv domain is a convolution in the image domain

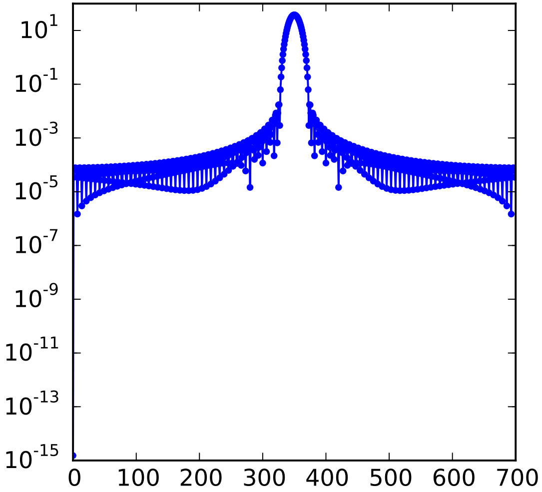


The Fourier transform of a comb function is again a comb. A convolution with a comb is a sum of shifted versions of the original. Aliases of sources outside the image will appear in the image.

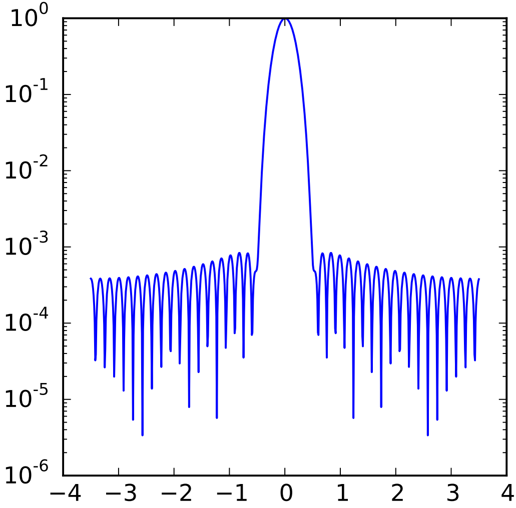
Spheroidal (I)



frametitleSpheroidal (II)

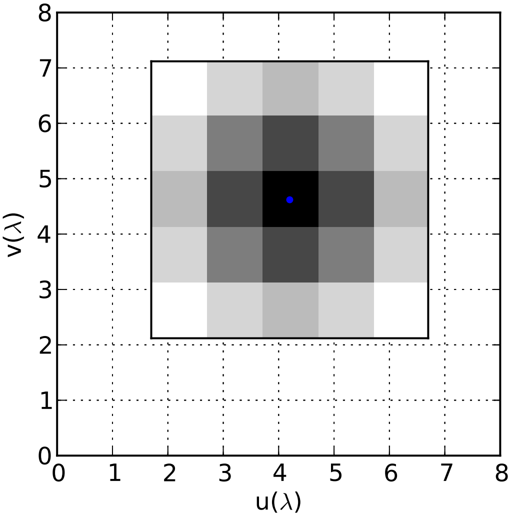


Spheroidal (III)



Spheroidal truncated in uv domain is still nice in image domain.

Gridding with Spheroidal



Including Image plane effects

Assumptions that allow us to use 2D FFT no longer hold for large images

W - term A - term

Solutions

- divide image in to smaller sub images or facets
  assumptions do hold for the facets
  pro
  - straight forward applications of existing tools
  - further averaging of visibilities

  con
  - edge effects
  - piece wise constant, facets need to be small enough to approximate
  - facets are not independent, need to cycle multiple times through facets
- Projection methods - include correction in the gridding kernel (explained in the following slides)
  pro

Projection methods

Gridding the data with a convolution function applies the Fourier transform of that function in the image domain. We can make use of that to apply other corrections in the image domain.

- Tapering (Spheroidal)
- W term (Curvature of the sky)
- A term (Beam effects)

Consequence: need to compute many Convolution Functions

W projection

Correction for the W-term

$$\exp 2\pi n w$$

where

$$n = 1 - \sqrt{l^2 + m^2}$$

Support of the convolution function grows with the square of the angular size of the image.

Image Domain Gridding (IDG)

(warning: shameless promotion of own work)

- Eliminates need to compute convolution kernels
- At the expense of doing more computations, sine-cosine evaluations
- Algorithm maps very well on Graphical Processing Units (GPU)
- Especially GPU with special function units to evaluate sine-cosine (Nvidia) (CEP4, dawn eor cluster)
- On GPU faster than classical gridding

Wide Band Deconvolution

Multi Scale Deconvolution

Summary

- Wide Field & Wide Band Imaging is essential to make deep images
- Very important to have good direction dependent calibration (Factor)
- The tools are there, different approaches, not everything is settled (faceting ¡-¿ projection, casa nterm ¡-¿ wsclean join-channels, deconvolution methods (multiscale, compresssed sensing)
- Active development, expect improvements in the near future