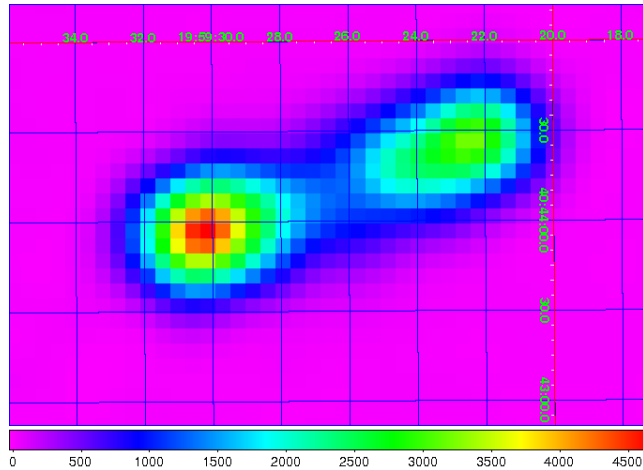


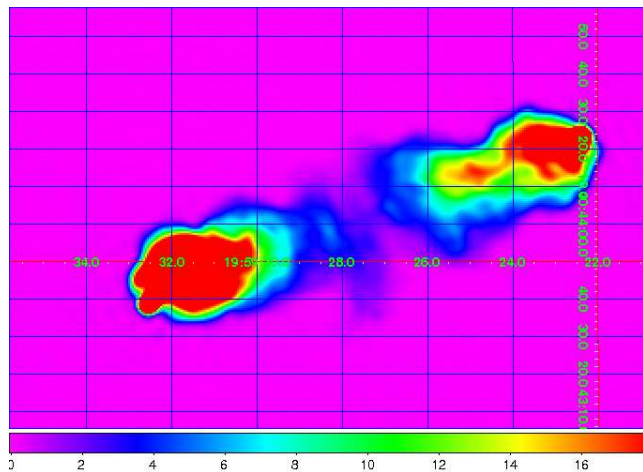
# WSRT LFFE CygA 5000 to 150000

Sarod Yatawatta, Ger de Bruyn and Michiel Brentjens

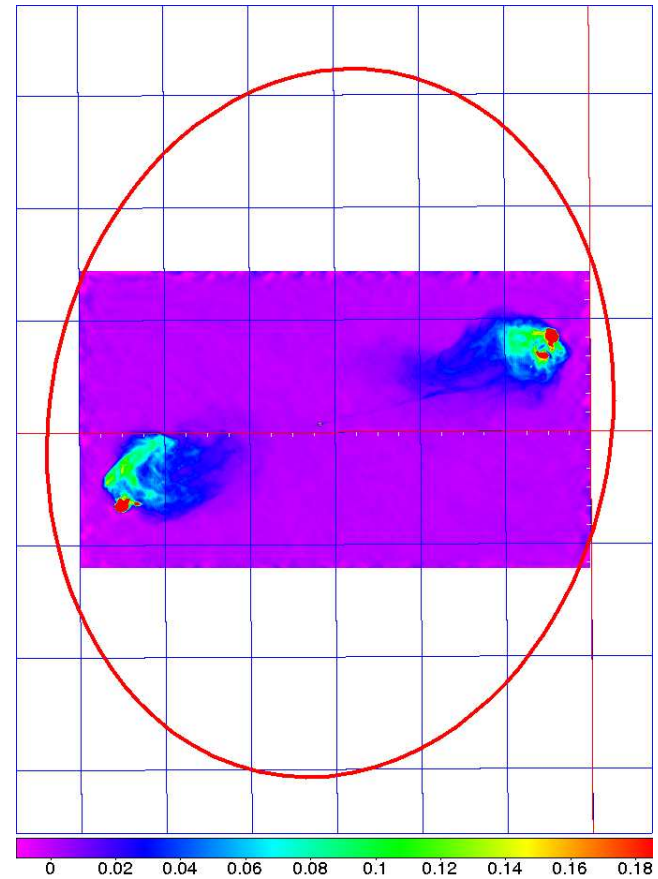
# Cygnus A



VLA 74 MHz



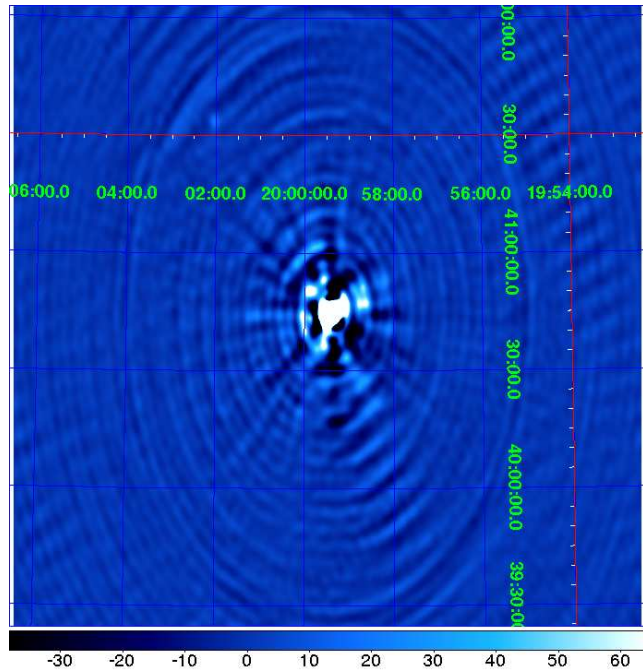
VLA 327 MHz



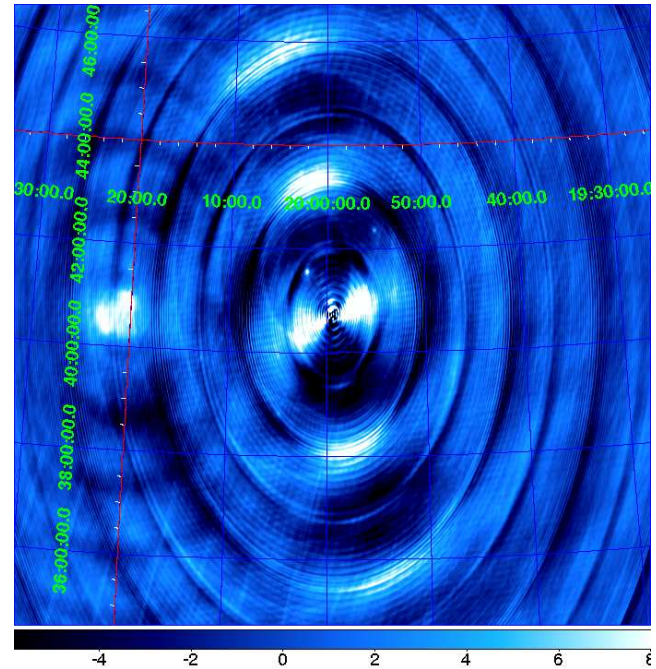
VLA 4.5 GHz, WSRT PSF at 140 MHz

Credits: R.A. Perley, J.W. Dreher, J.J. Cowan, J. Lazio

# Cygnus A



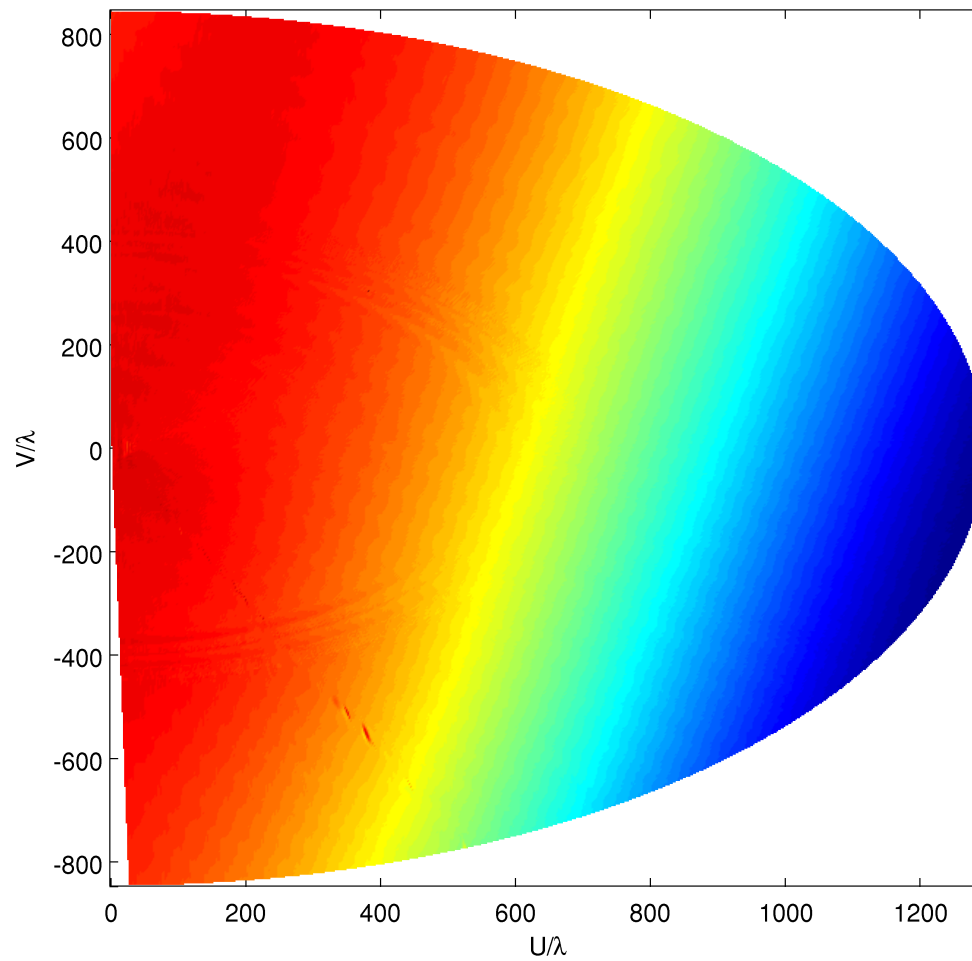
WSRT 138.84 MHz image



WSRT 138.84 MHz image, HB20 on left

□ Clean dynamic range  $\approx 5000$

# Visibilities



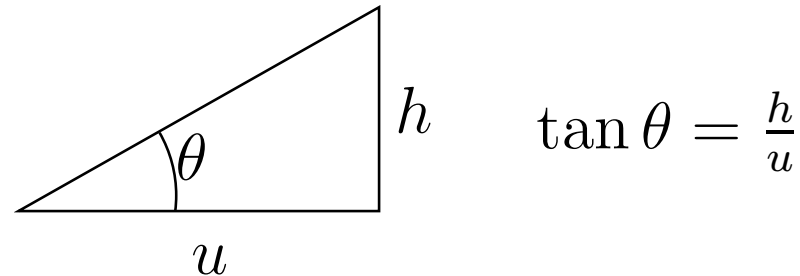
Calibrated  $|I|$

# Clean fails because...

Elementary Fourier transform:

$$f(l - a, m - b) \Leftrightarrow \exp(-j2\pi(au + bv))F(u, v)$$

Estimation of phase (position) of a clean component:



$$\Delta\theta = \frac{u}{u^2 + h^2} \Delta h - \frac{h}{u^2 + h^2} \Delta u$$

$$E\{\Delta\theta^2\} = \frac{u^2}{(u^2 + h^2)^2} \sigma^2$$

# Shapelets

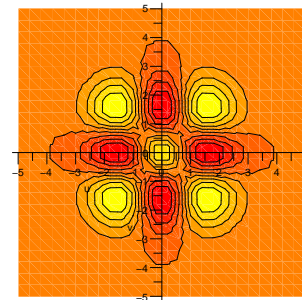
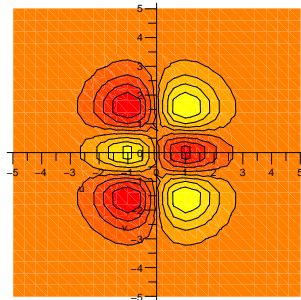
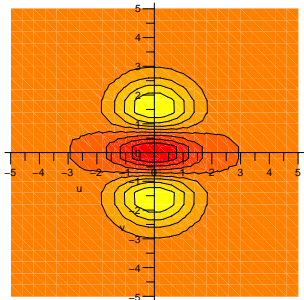
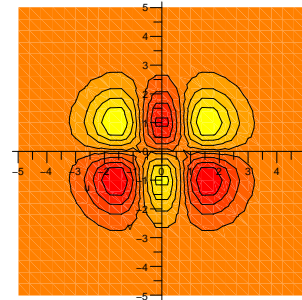
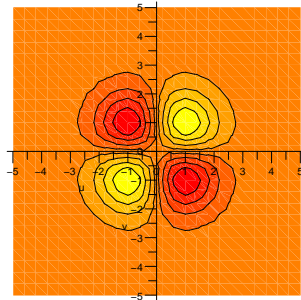
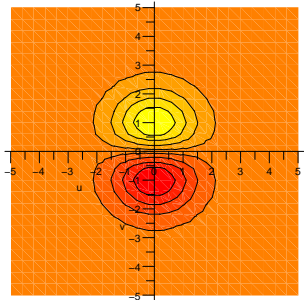
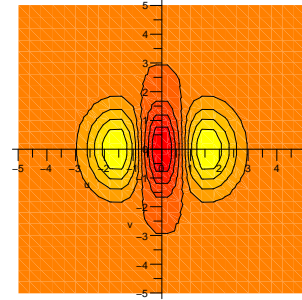
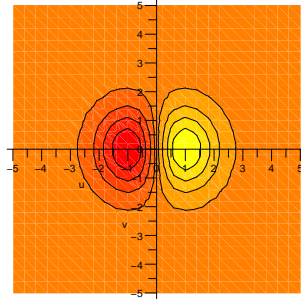
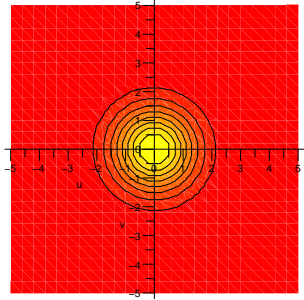
- Quite popular and well studied: [Refregier 2001] Paper I, [Refregier and Bacon 2001] Paper II, [Massey and Refregier 2003] Paper III
- Hermite Gaussian Orthonormal basis functions, (also polar shapelets)

$$\phi_{m,n}(x, y) = \frac{H_m(x/\beta)H_n(y/\beta)}{\beta\sqrt{m!n!2^{(m+n)}\pi}} e\left(-\frac{(x/\beta)^2}{2} - \frac{(y/\beta)^2}{2}\right), \quad x, y \in \mathbb{R}, \quad m, n \in [0, K-1]$$

where  $H_n(x)$ :  $n$ -th order Hermite function,  $K$ : order,  $\beta$ : scale

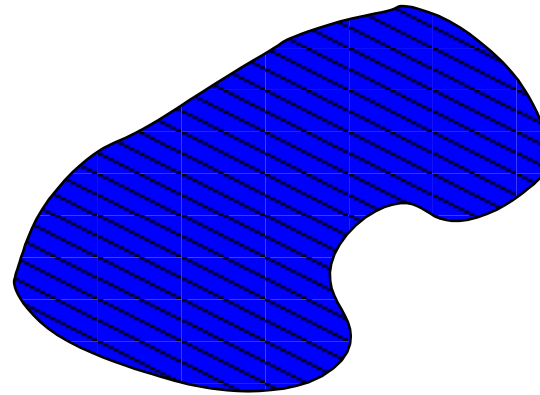
- Analytic relationships for Fourier transform, Convolution, Integration, Linear transform etc.
- Simple to evaluate using recursive relationships.
- Problems: hard to find optimal linear transform, order and scale for Shapelet Decomposition (mixed mode optimization problem). Shapelet evaluation can become negative or even complex.

# Some basis functions



# Shapelet Decomposition

Represent an arbitrary image



$N$  pixels

$= \mathbf{b}$

$N \times 1$  vector

$$\mathbf{A}\mathbf{c} = \mathbf{b}$$

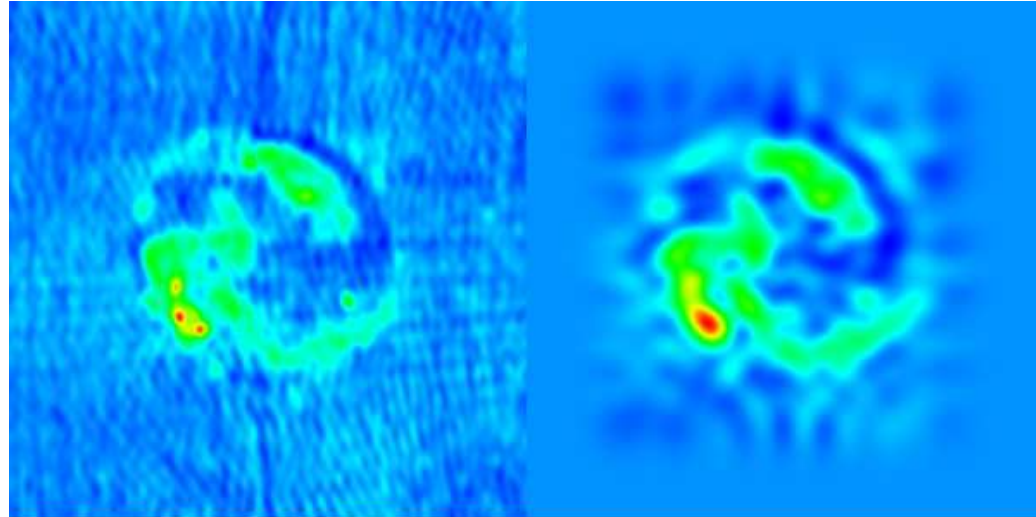
where columns of  $\mathbf{A}$ : shapelet basis functions ( $N \times M$ ),  
 $\mathbf{c}$  coefficients of shapelet decomposition ( $M \times 1$ ),  
 $\mathbf{b}$  image vector ( $N \times 1$ ). ( $N \gg M$ )

$\mathbf{c} = \mathbf{A}^\dagger \mathbf{b}$ .

$\mathbf{A}^\dagger$  found using SVD, can also use Tikhonov regularization.

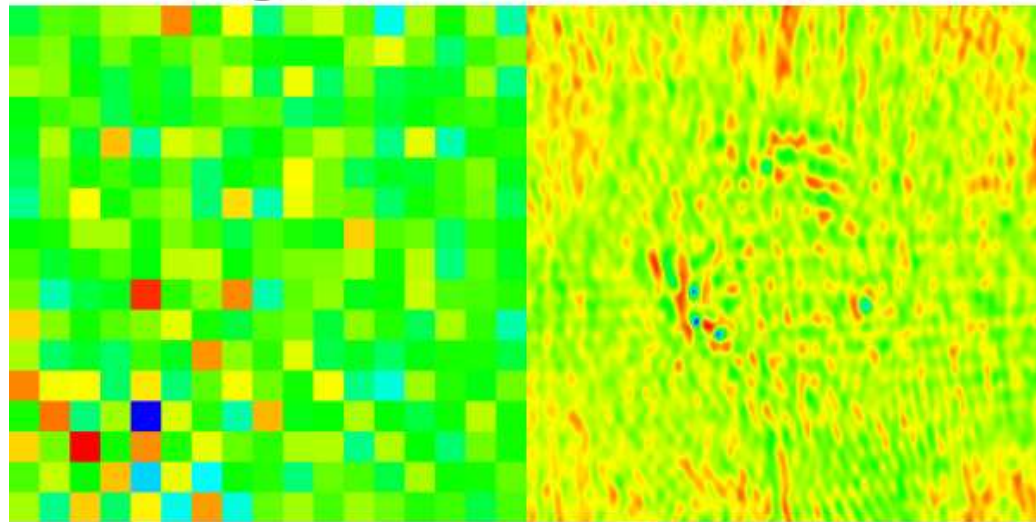


# Example



Original

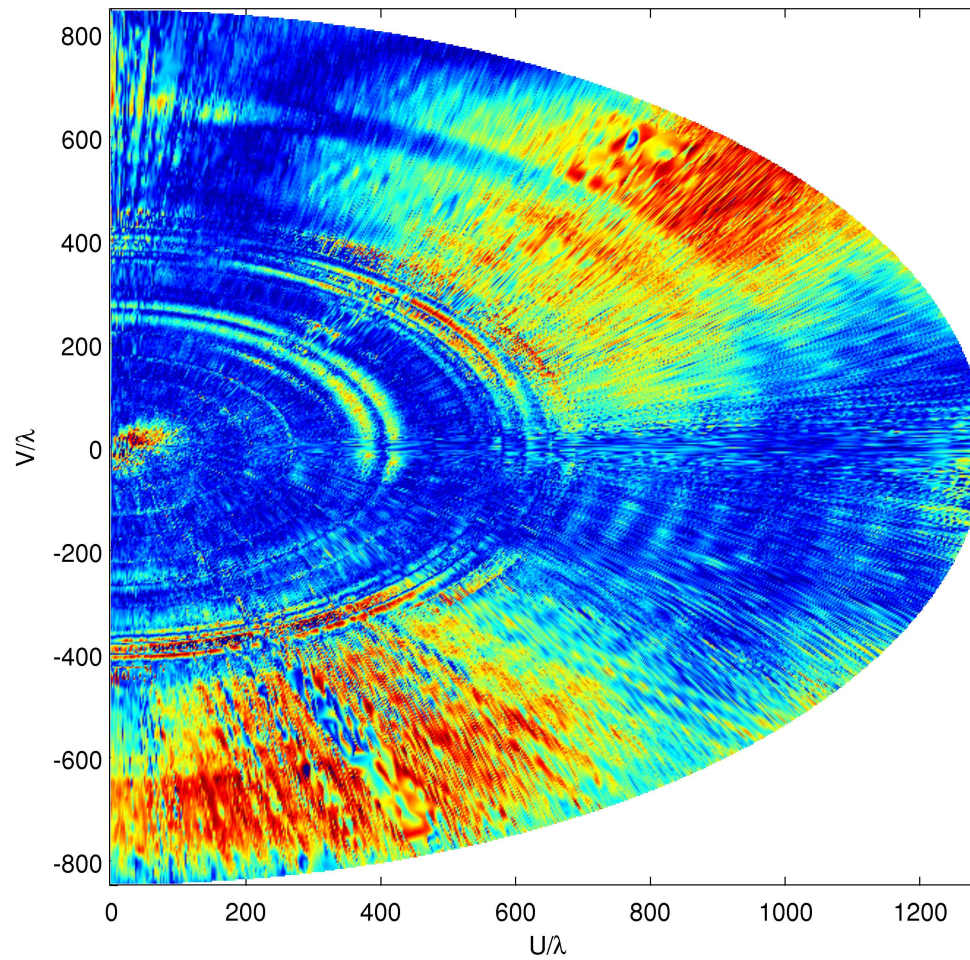
Model



Modes

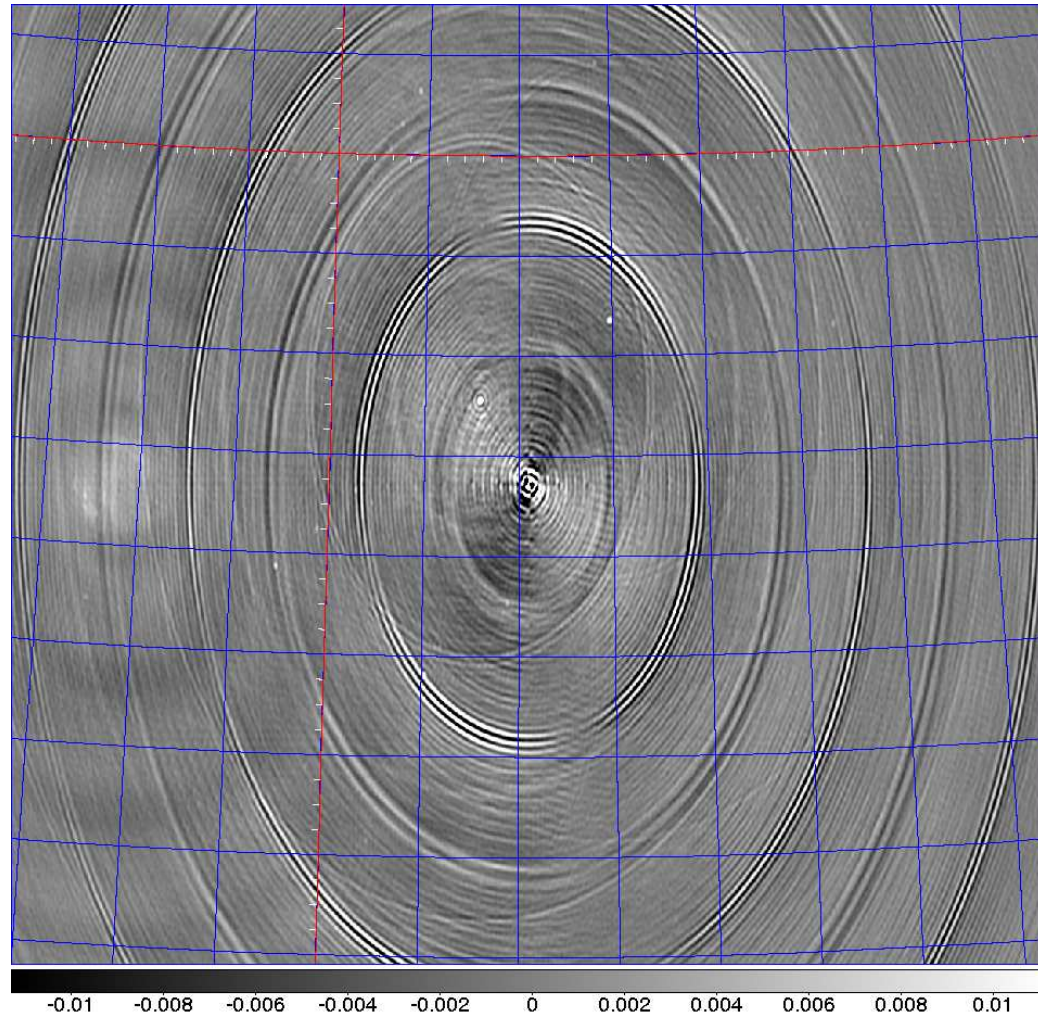
Error

# Residual



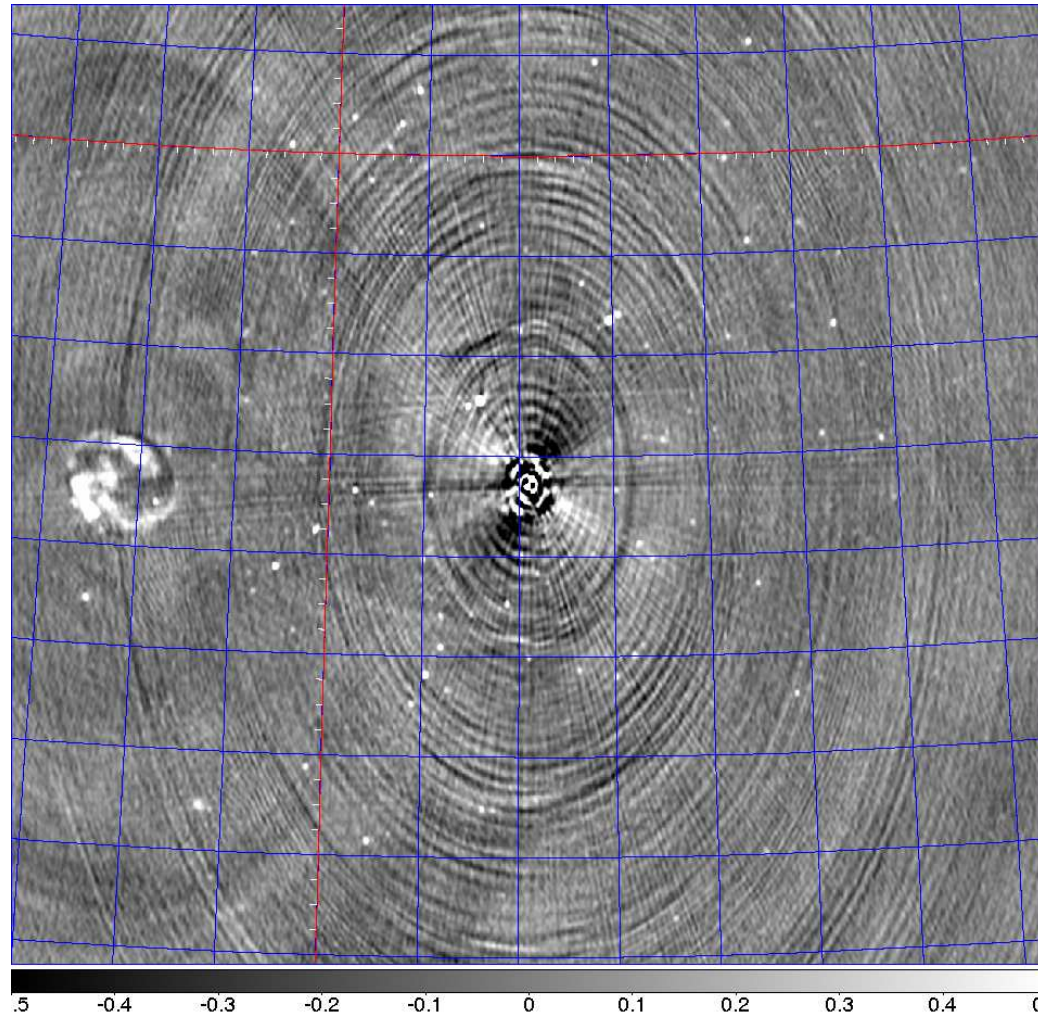
Residual  $|I|$

# Example



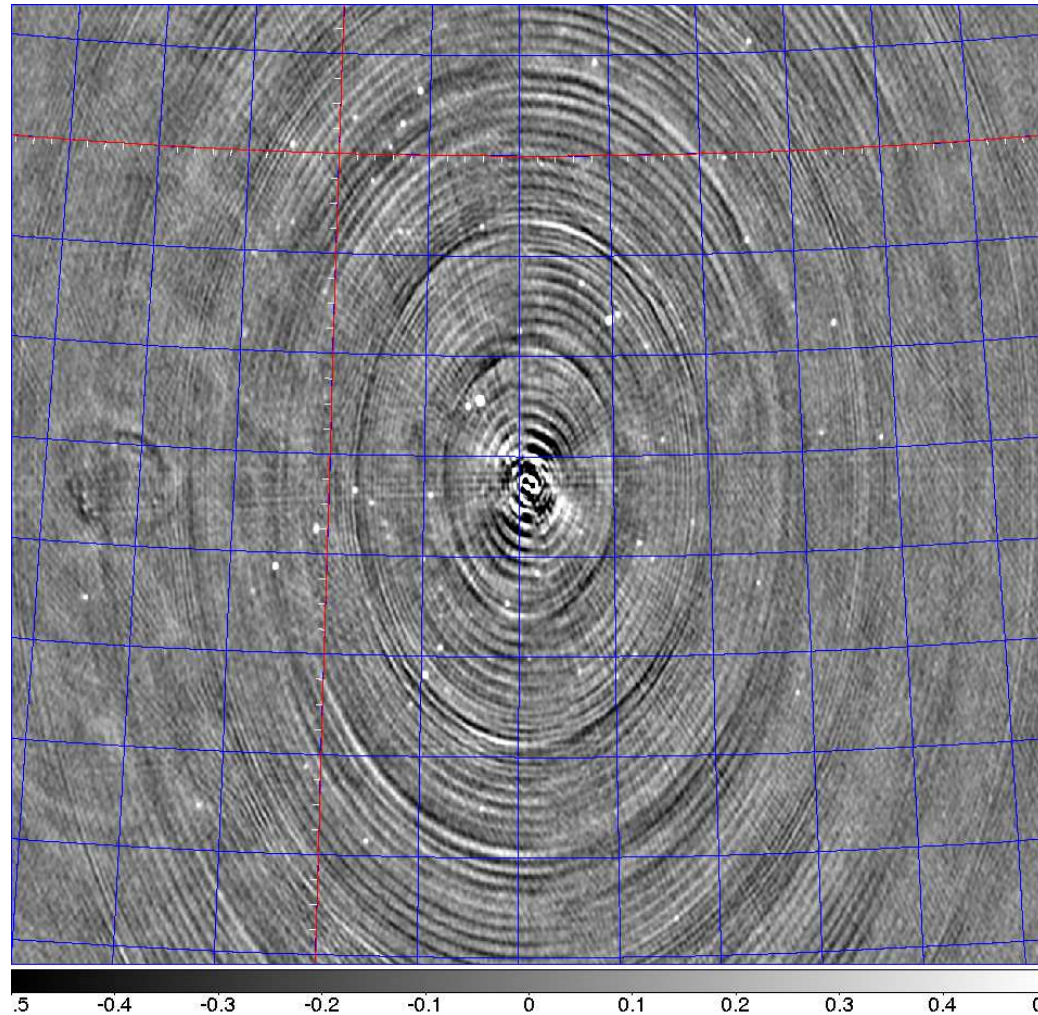
Peak  $\approx$  10000 Jy, Noise I 500 mJy, Q 225 mJy, (140 MHz)

# Example



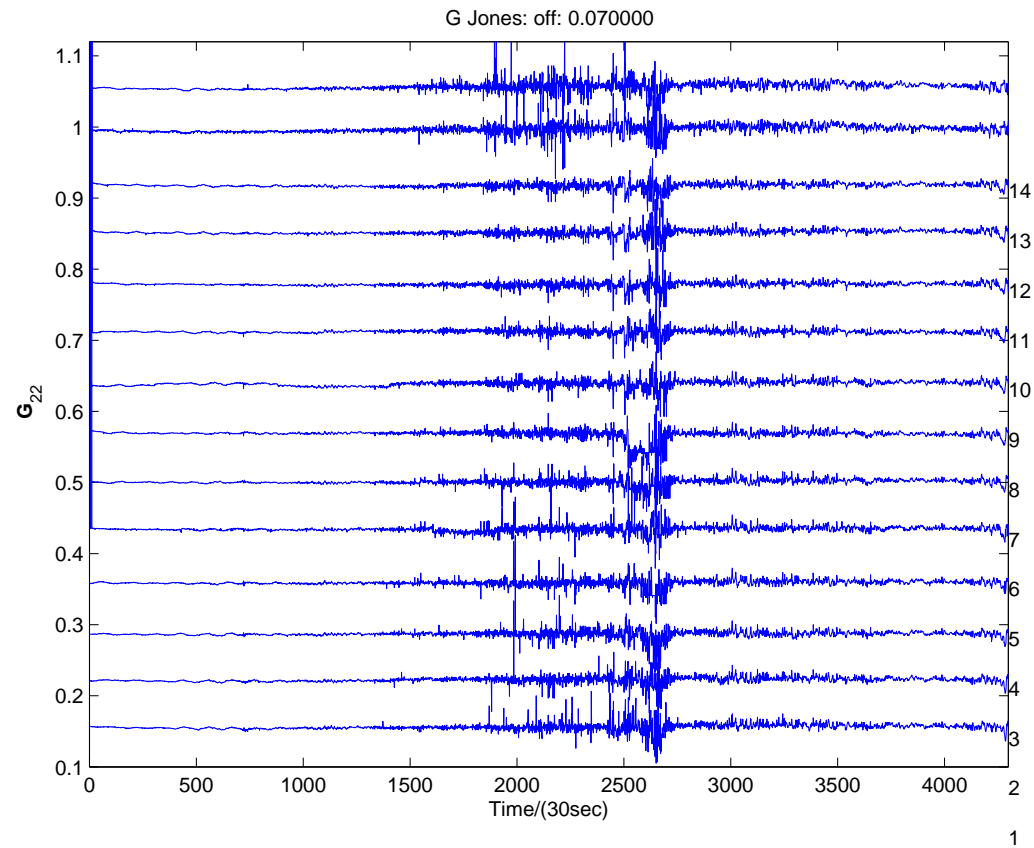
Peak  $\approx$  10000 Jy, Noise I 65 mJy, Q 14 mJy, U,V 10 mJy (116.8-146.7 MHz)

# Example

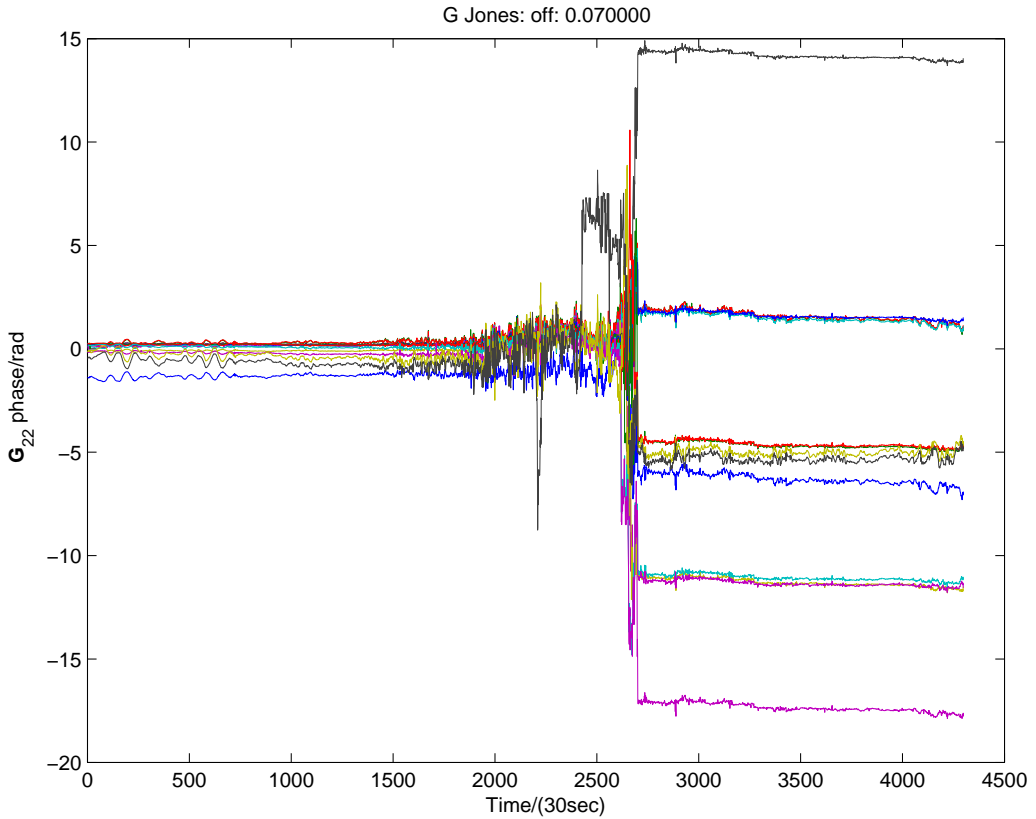


Peak  $\approx 10000$  Jy, Noise I 80 mJy, Q 15 mJy, U,V 12 mJy (138.7-140.7 MHz)

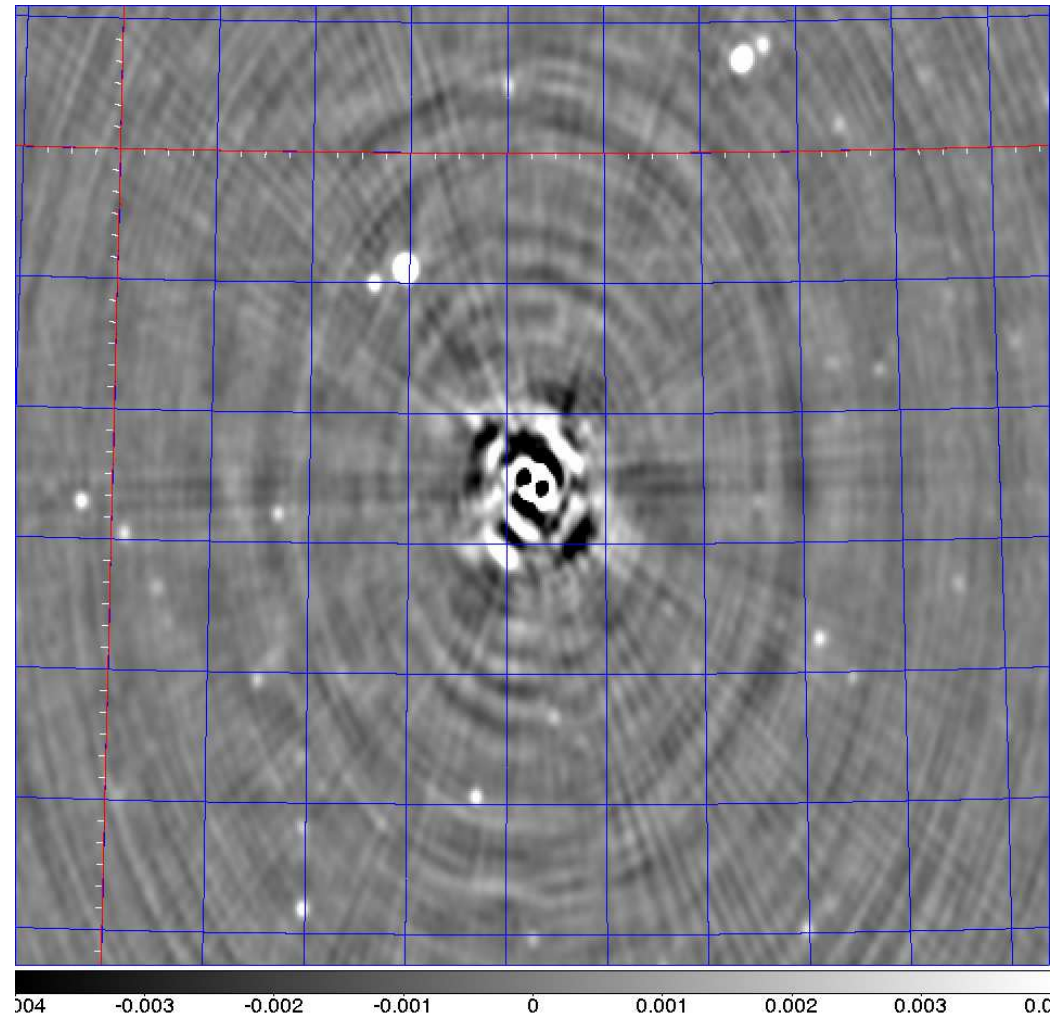
# Calibration



# Calibration

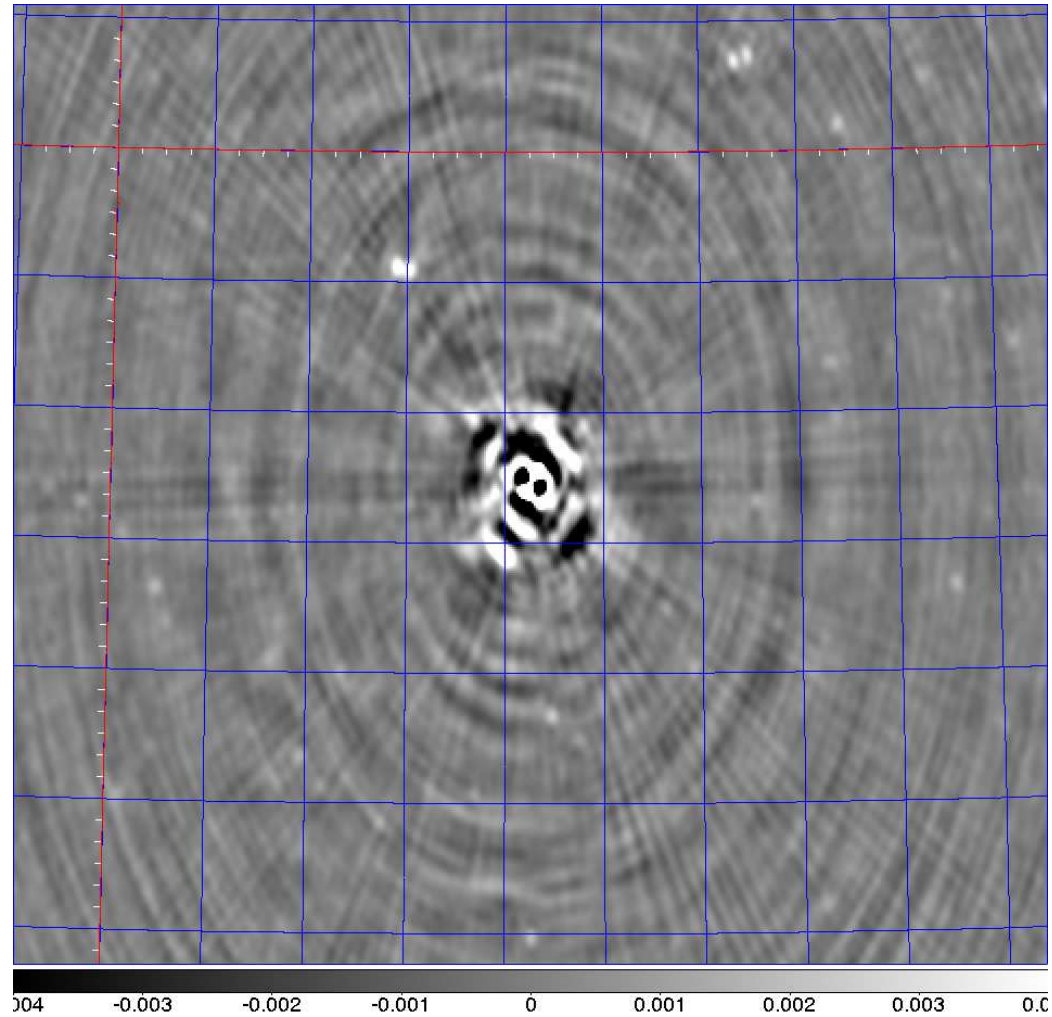


# Example





# Example



About 45 sources subtracted *without* ! direction dependent solving

# Conclusions

- Shapelets provide a way to model extended sources (much larger than the PSF) as well as sources smaller than the PSF.
- Model construction in image plane and  $uv$  plane is possible.
- Hard part is to find the right scale  $\beta$  and the number of modes.
- For the CygA observation a dynamic range  $\approx 150000$  possible. This is still a factor above the noise. Limitations are due to faint RFI, scintillation and the galactic plane.