Non-linear Kalman filters for calibration in radio interferometry

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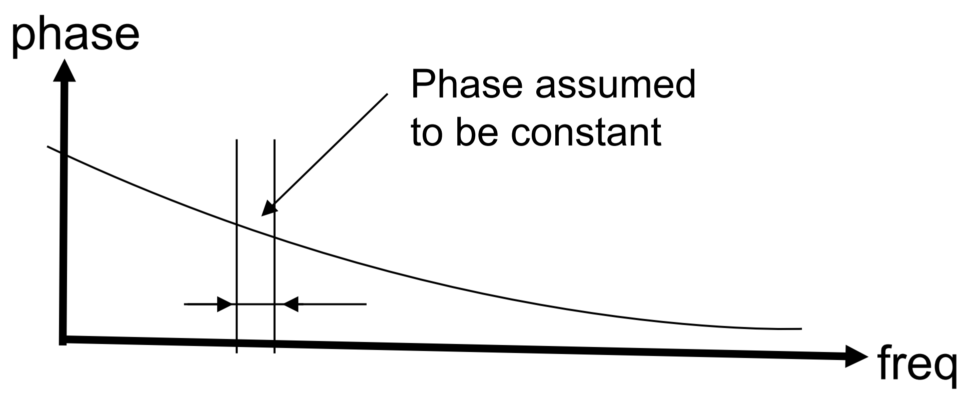
Tasse 2014 [2014A&A...566A.127T]

Main issue: available information, ill conditioning

Traditional Calibration algorithms:

- Fit for Jones matrices
    - But Jones matrices vary over time/freq:
        - Limit usable bandwidth
        - Limit usable time chunk
    - Need for ~10 to ~100 discrete directions
        - /antenna /polarisattion /datachunk
        - tens of thousands of *free* parameters



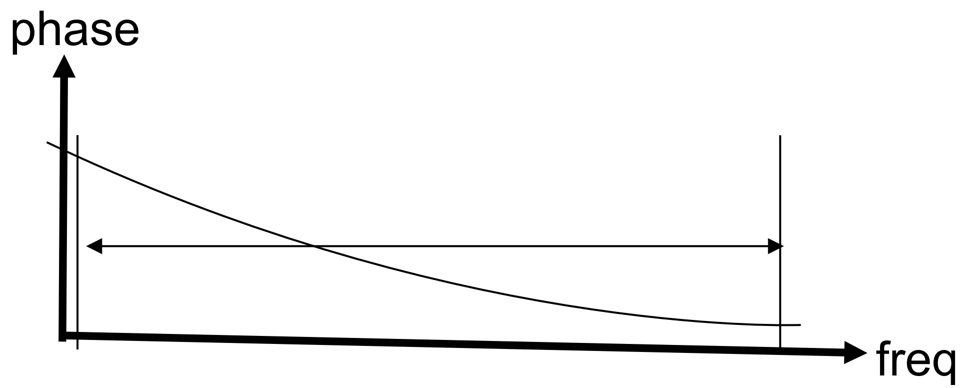
Ill conditioning is dangerous
(source supression, fake high dynamic range, etc)

Our goal: adressing ill-conditioning

Two independent aspects:

- **Physics-based** approach (opposed to **Jones-based** approach)
    - constraining Clocks, TEC-screen – **directly from visibilities**
    - All physical effects have a smooth frequency behaviour
    - We are **not limited in bandpass** anymore!

    - **Much less** degrees of freedom (~100-1000x)
    - **Much more** usable data (~10-100x)



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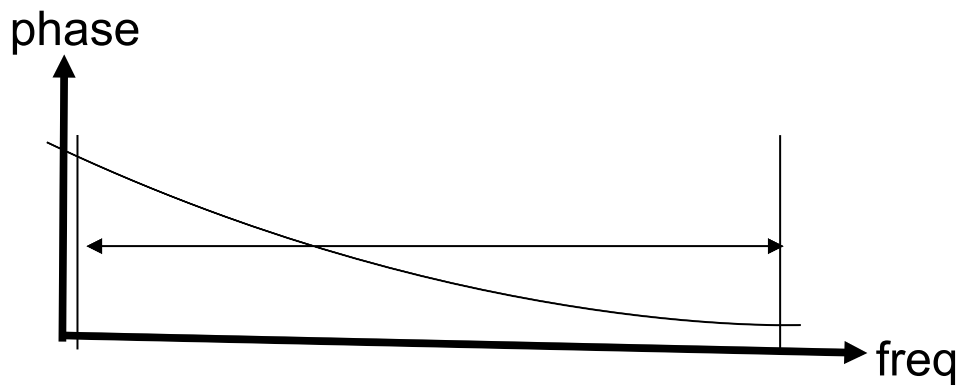


  - **Much less** degrees of freedom (~100-1000x)
  - **Much more** usable data (~10-100x)

- Physical effects are **not stable** in time though....
  - Kalman filter!

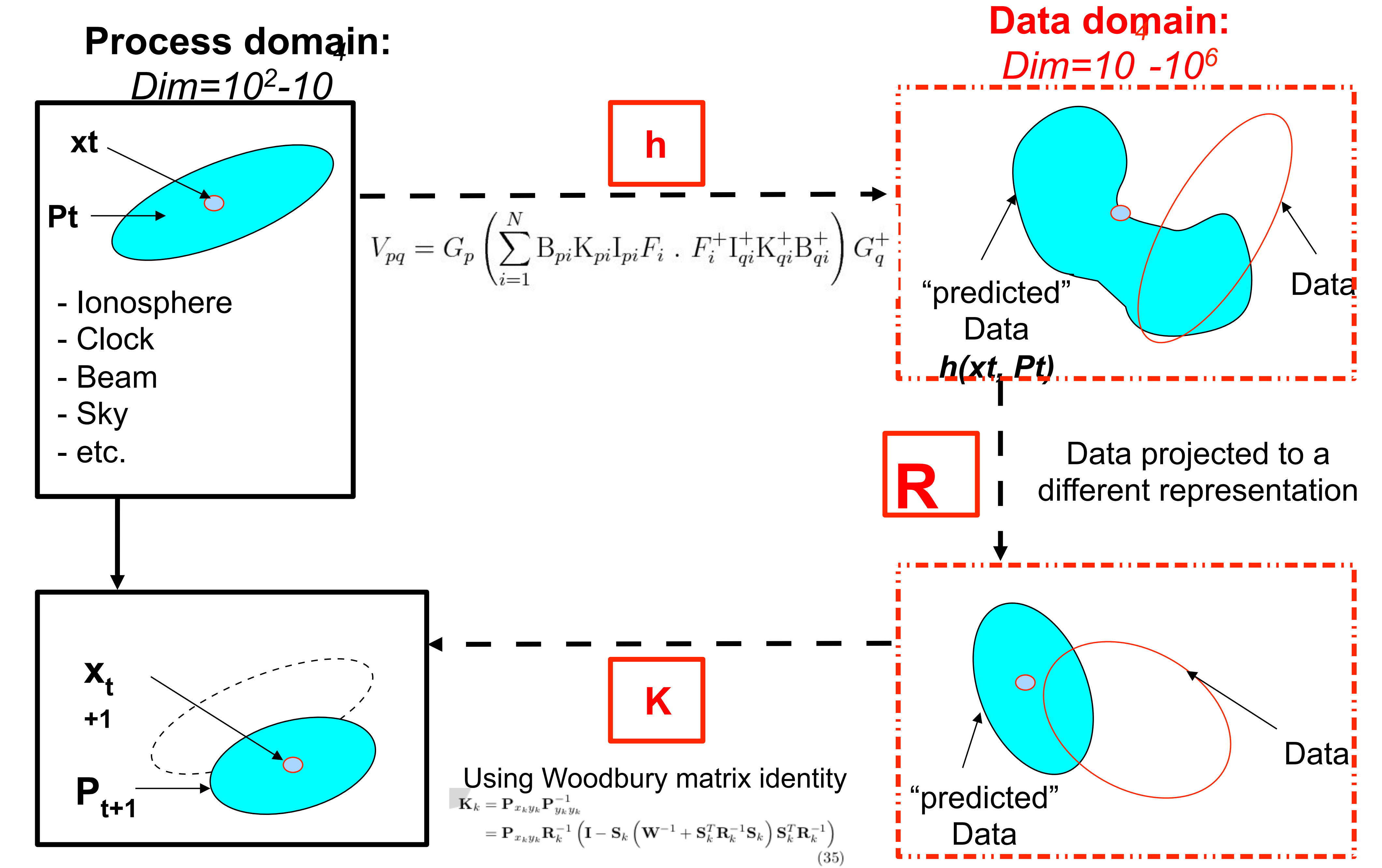
Kalman Filters ? (iterative versus recursive)

- Different from *Levenberg-Maquardt* or *EM*:
  - Kalman filters **do not try** to fit the data at best
  - "Minimum mean square estimators": fit the ***data given information on the expected state***

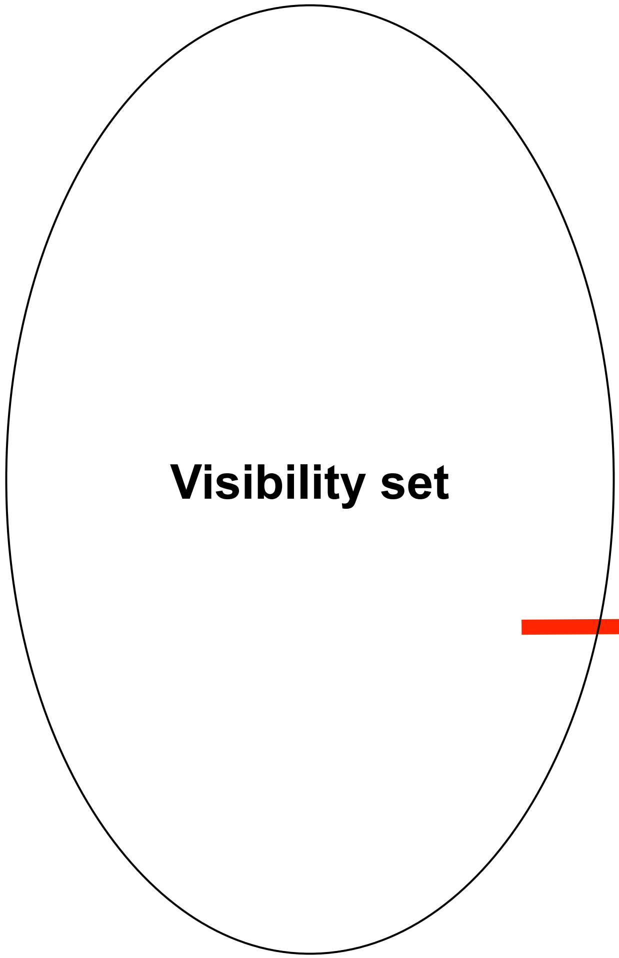
    They "**track** more than they solve"
  - Kalman filters use a **recursive** sequence (as opposed to ***iterative*** for LM, EM etc)

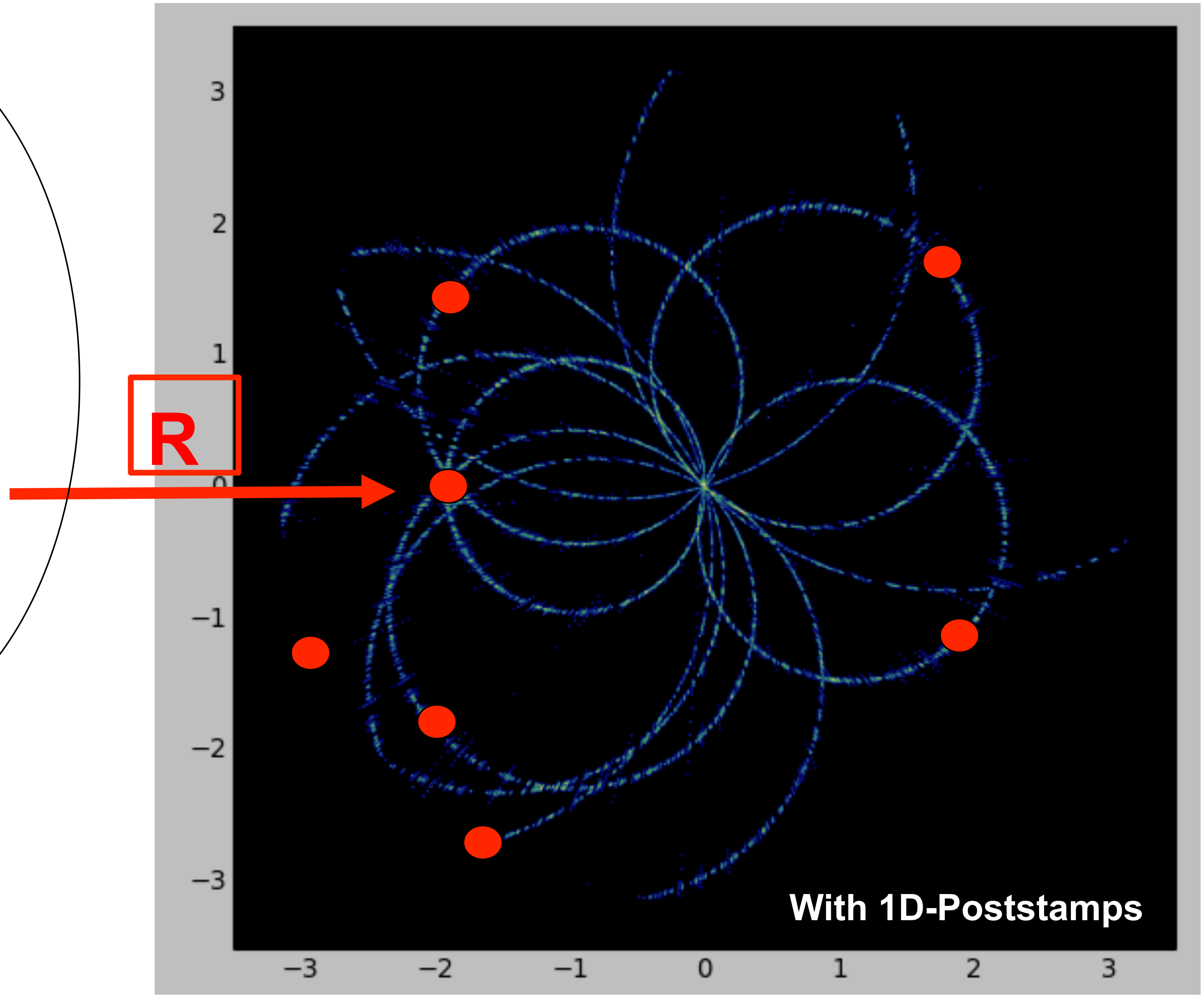
Problem: the **process to measurement** equation is non-linear

Non-linear Kalman Filters....



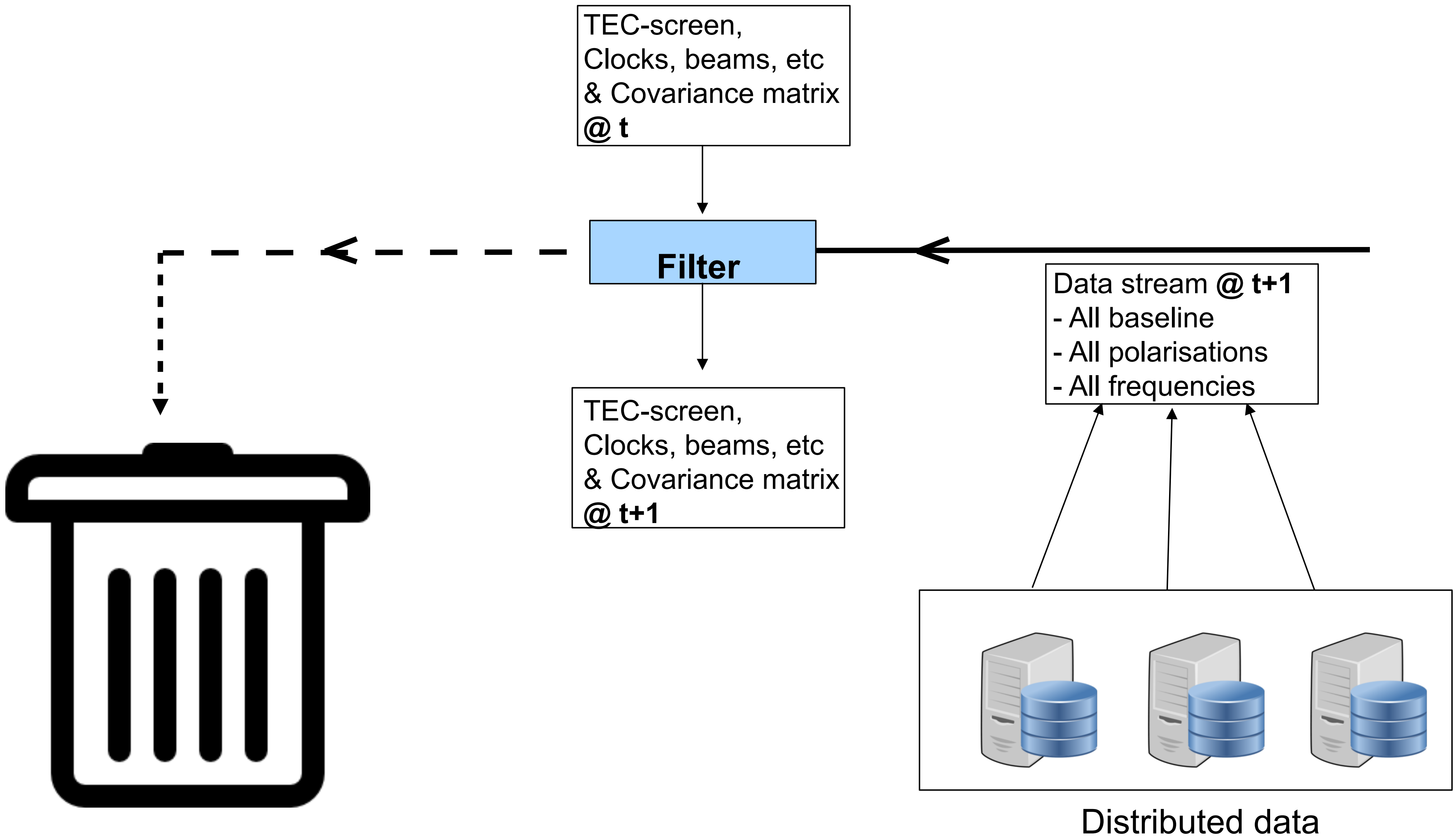
Representation issues....





Equivalent to 1-Dimentional raw-data images

KaFCa (Kalman Filter for Calibration)



Simulated dataset (ionosphere TEC-screen + Clock drift)

- LOFAR HBA (1500 baselines, 4 pols)
- 30 subbands from 100 to 150 MHz
- S/N=5

- variable ionosphere TEC-screen
- Variable clock drift

- 18.000 data points / [time bin (30s)]

Measurement Equation

$$\mathbf{V}_{(pq)t\nu} = \mathbf{h}(\mathbf{x}) = \mathbf{G}_{pt\nu}(\mathbf{x}) \left(\sum_s \mathbf{V}^s_{(pq)t\nu}(\mathbf{x}) k^s_{(pq)t\nu} \right) \mathbf{G}^H_{qt\nu}(\mathbf{x}) \tag{1}$$

$$\mathbf{V}^s_{(pq)t\nu}(\mathbf{x}) = \mathbf{D}_{pst\nu}(\mathbf{x}) \mathbf{X}_s \, \mathbf{D}^H_{qst\nu}(\mathbf{x}) \tag{2}$$

Clocks

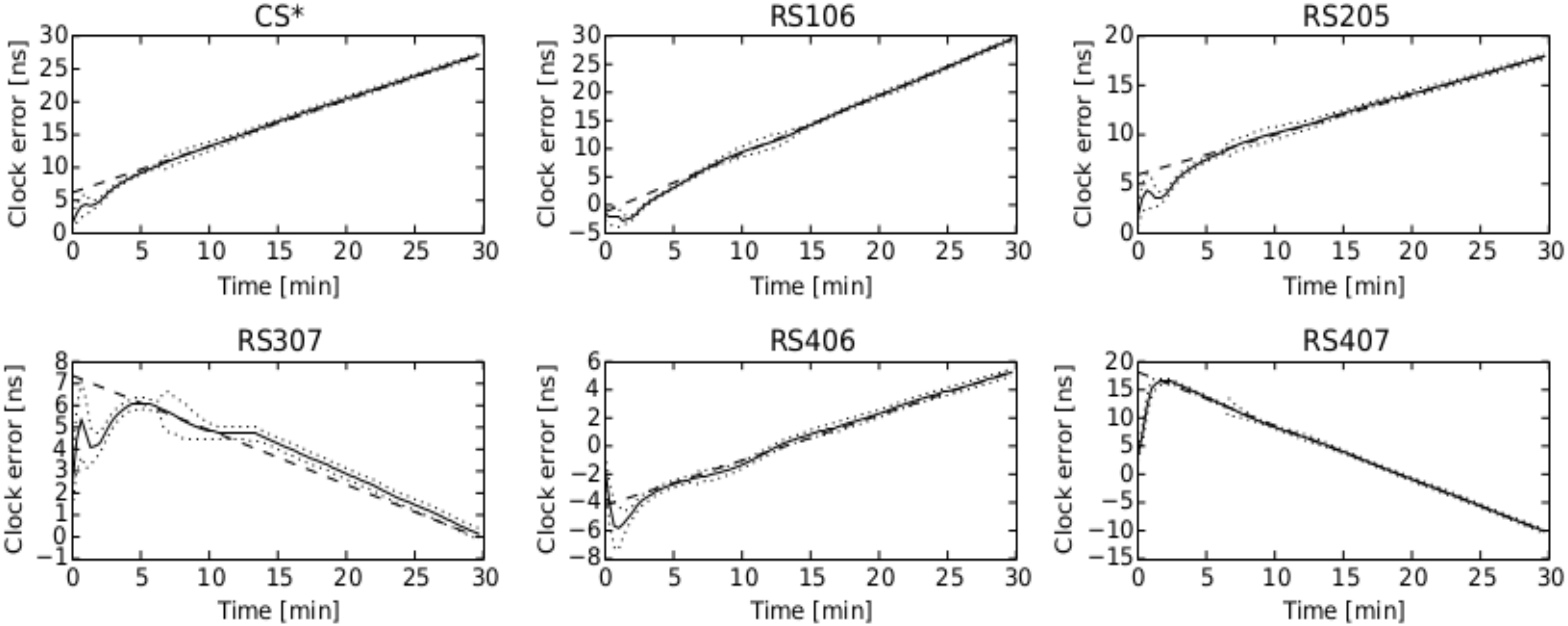
$$\mathbf{G}_{pt\nu}(\mathbf{x}) := \exp\left(2\pi i \nu \; \boldsymbol{\delta}\mathbf{t}_\mathbf{p}(\mathbf{x})\right) \mathbf{I}$$

Ionosphere

$$\mathbf{D}^d_{pt\nu}(\mathbf{x}) := \exp\left(ik\nu^{-1} \; \mathbf{T}^\mathbf{d}_\mathbf{p}(\mathbf{x})\right) \mathbf{I}$$

Simulated dataset (ionosphere TEC-screen + Clock drift)

Clocks solutions



KaFCa on Real data!

- 50 subbands
- ME: TEC-screen, clocks, constant offset