

INTERFEROMETRIC CALIBRATION

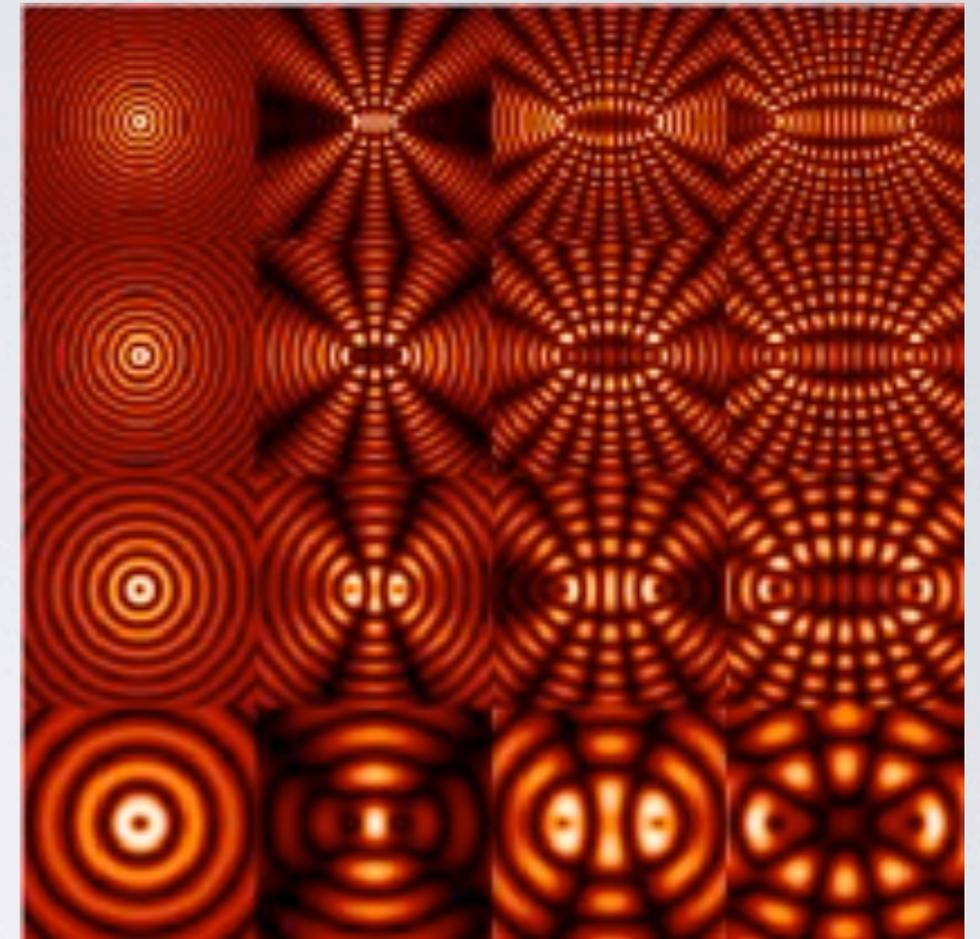
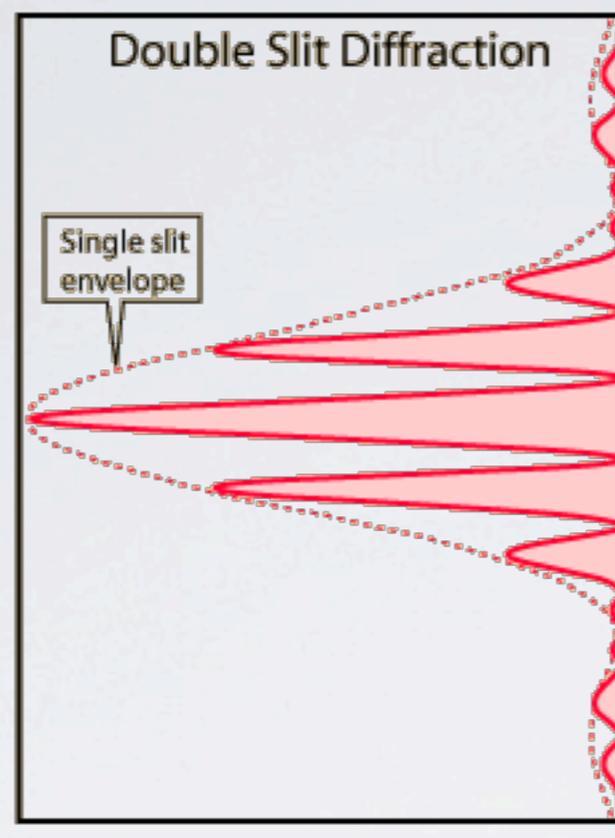
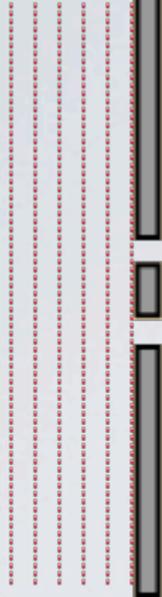
Emanuela Orru'



LOFAR Ionospheric Workshop 2 – 3 JUNE 2016, WARSAW

DIFFRACTION GRATING: LIGHT

Incident plane wave

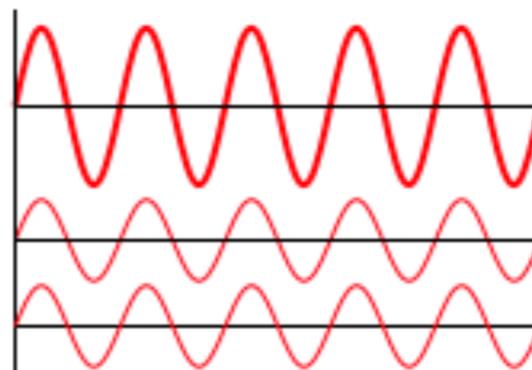


$$d_f = \frac{\lambda}{\sin \theta}$$

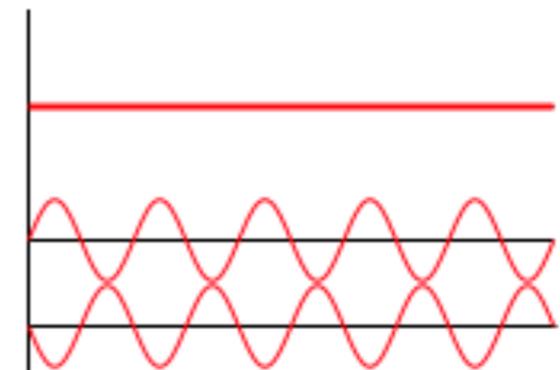
Resultant wave

Wave 1

Wave 2



Constructive interference



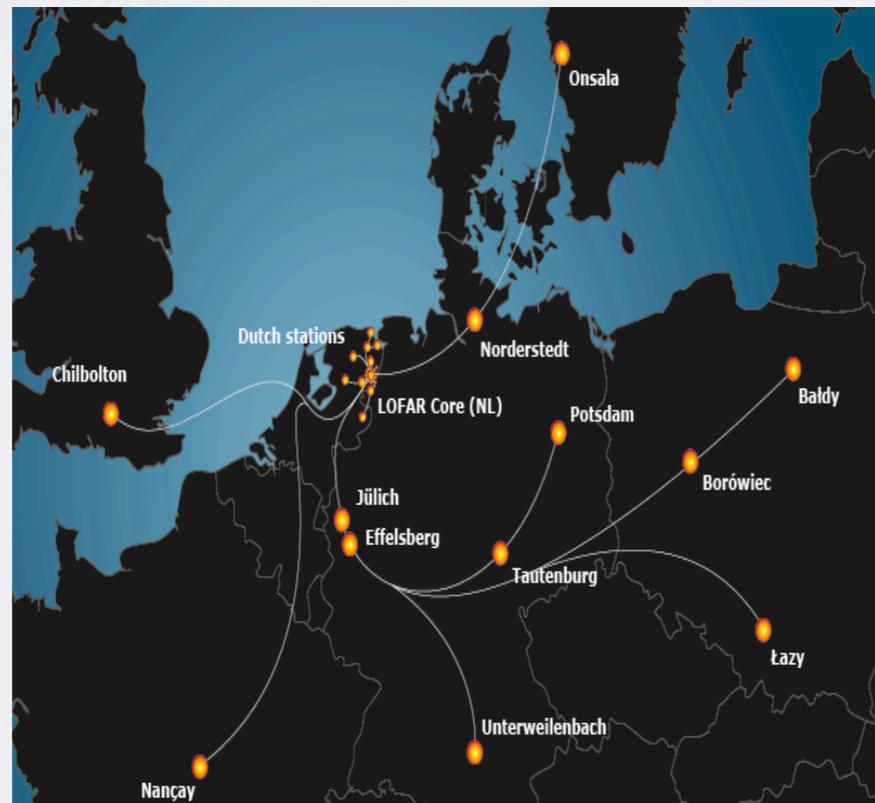
Destructive interference

INTERFEROMETER

$$\theta \propto \frac{\lambda}{D}$$



$D \sim 300 \text{ m}$



$D > 1000 \text{ km}$

INTERFEROMETER



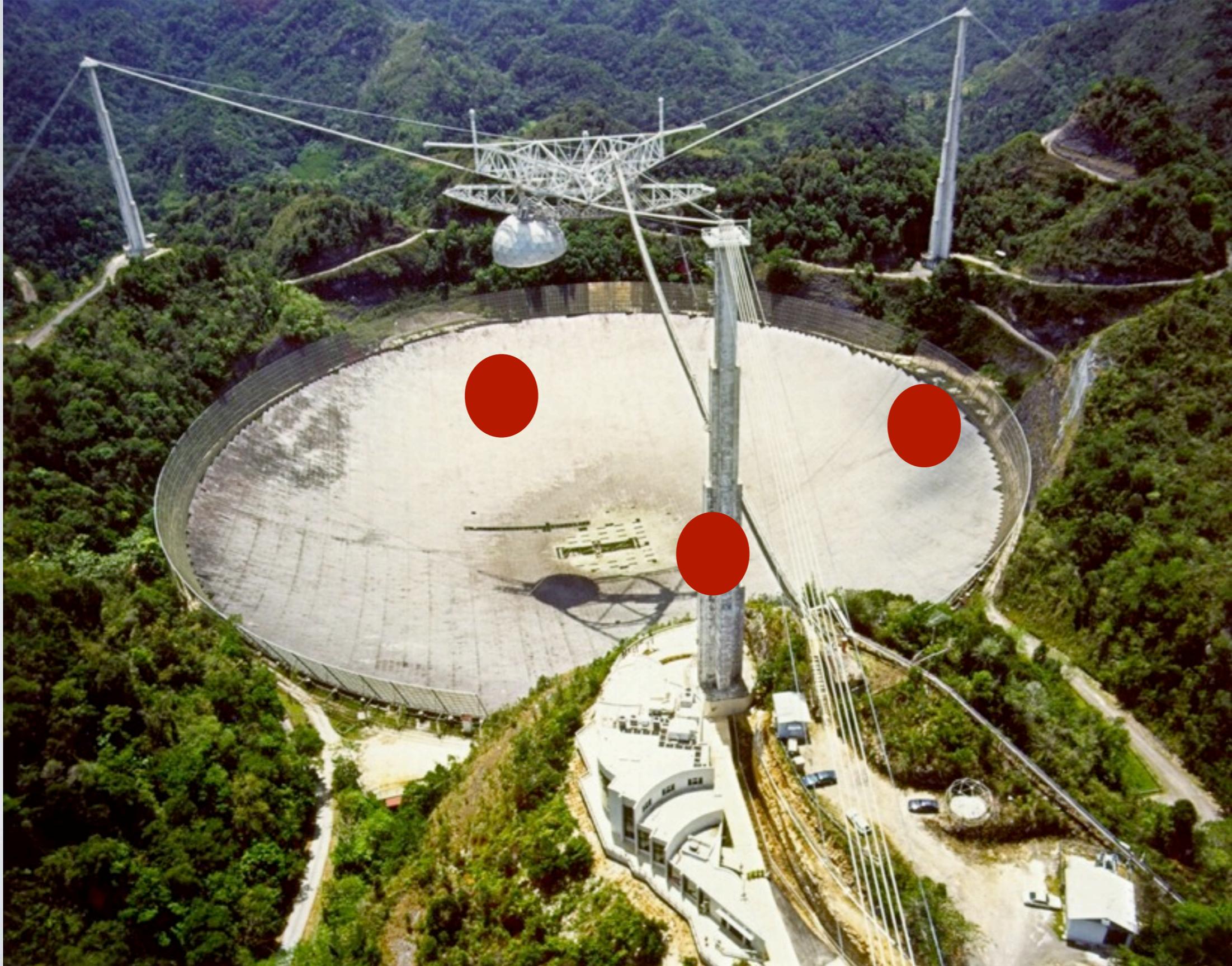
INTERFEROMETER



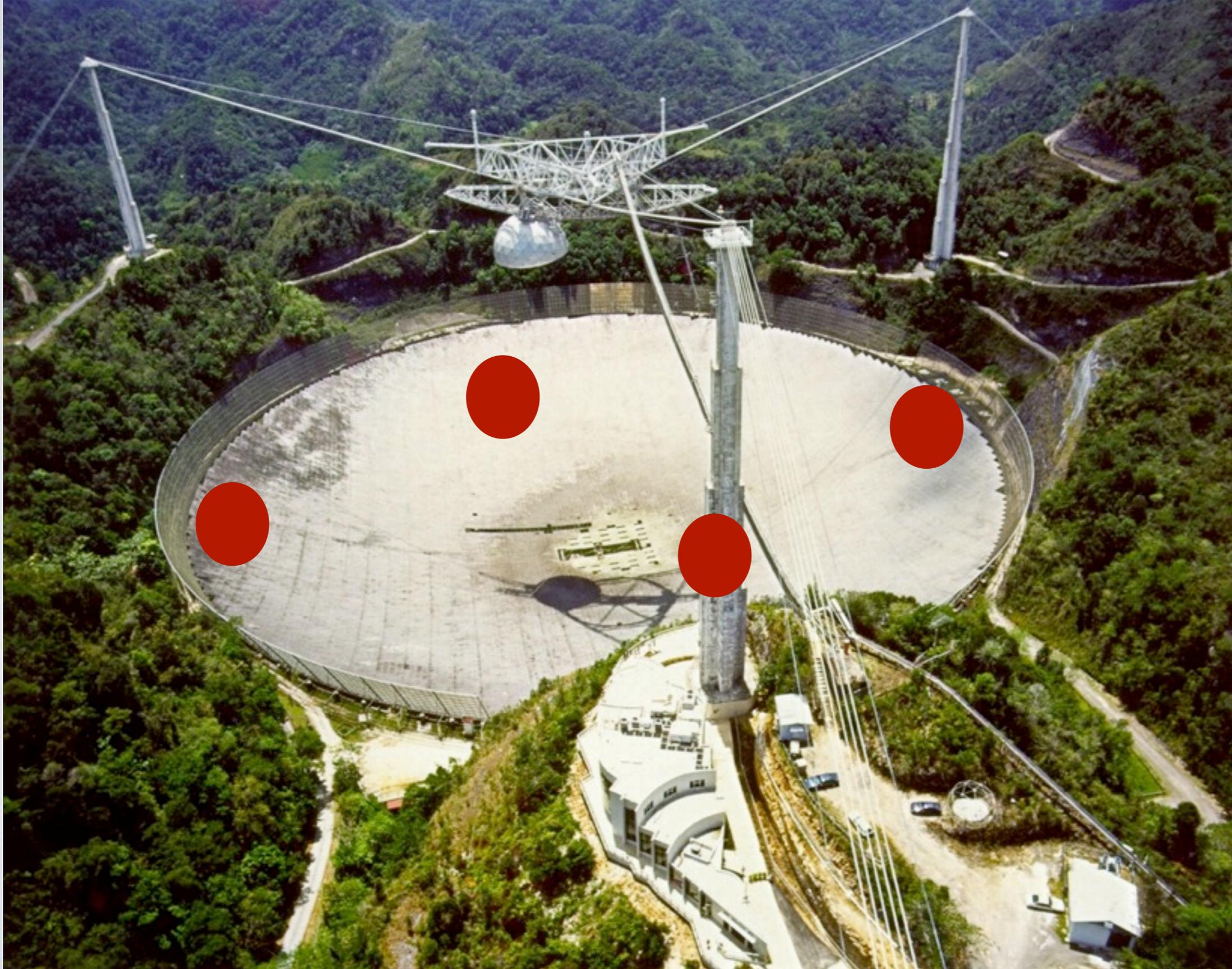
INTERFEROMETER



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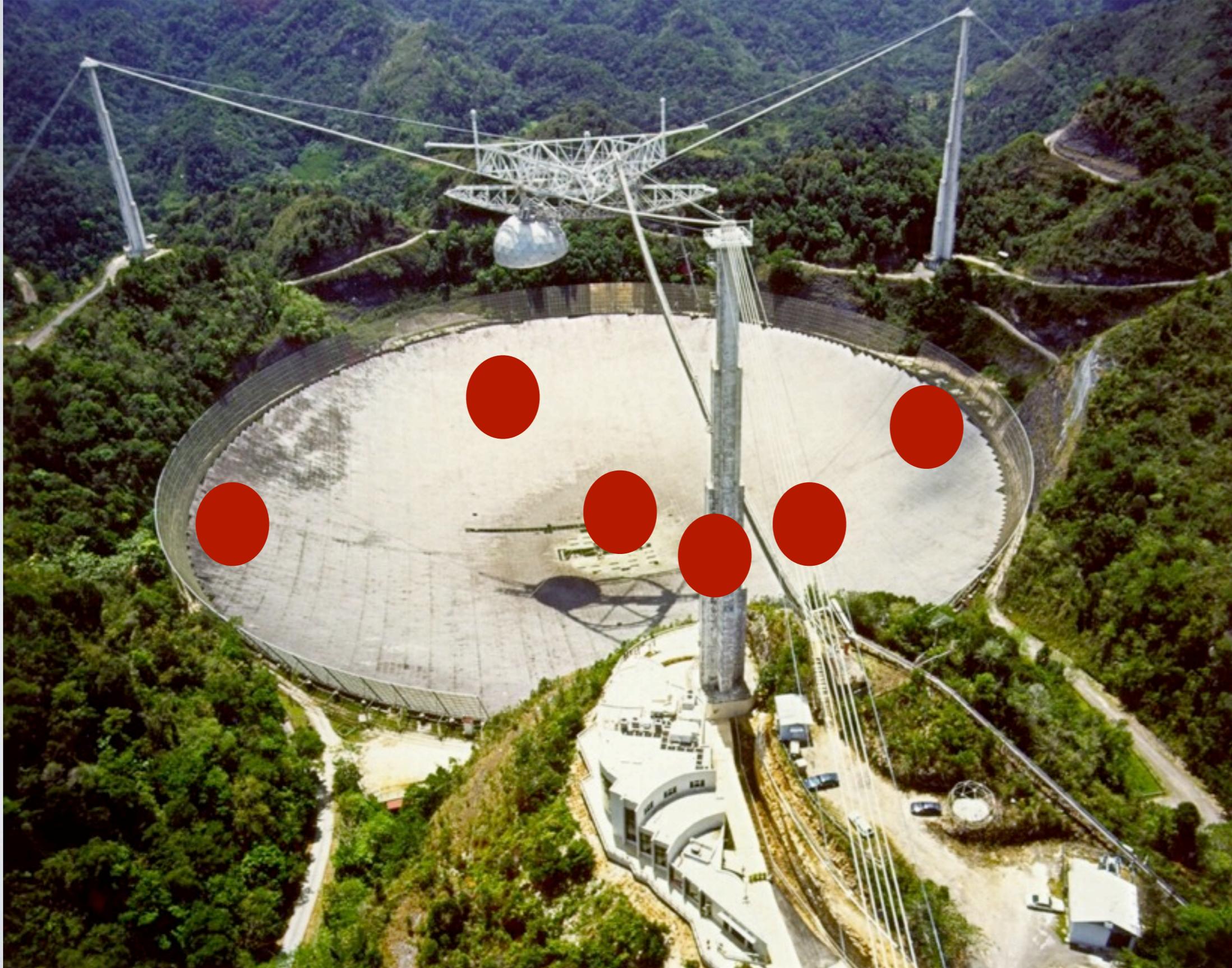
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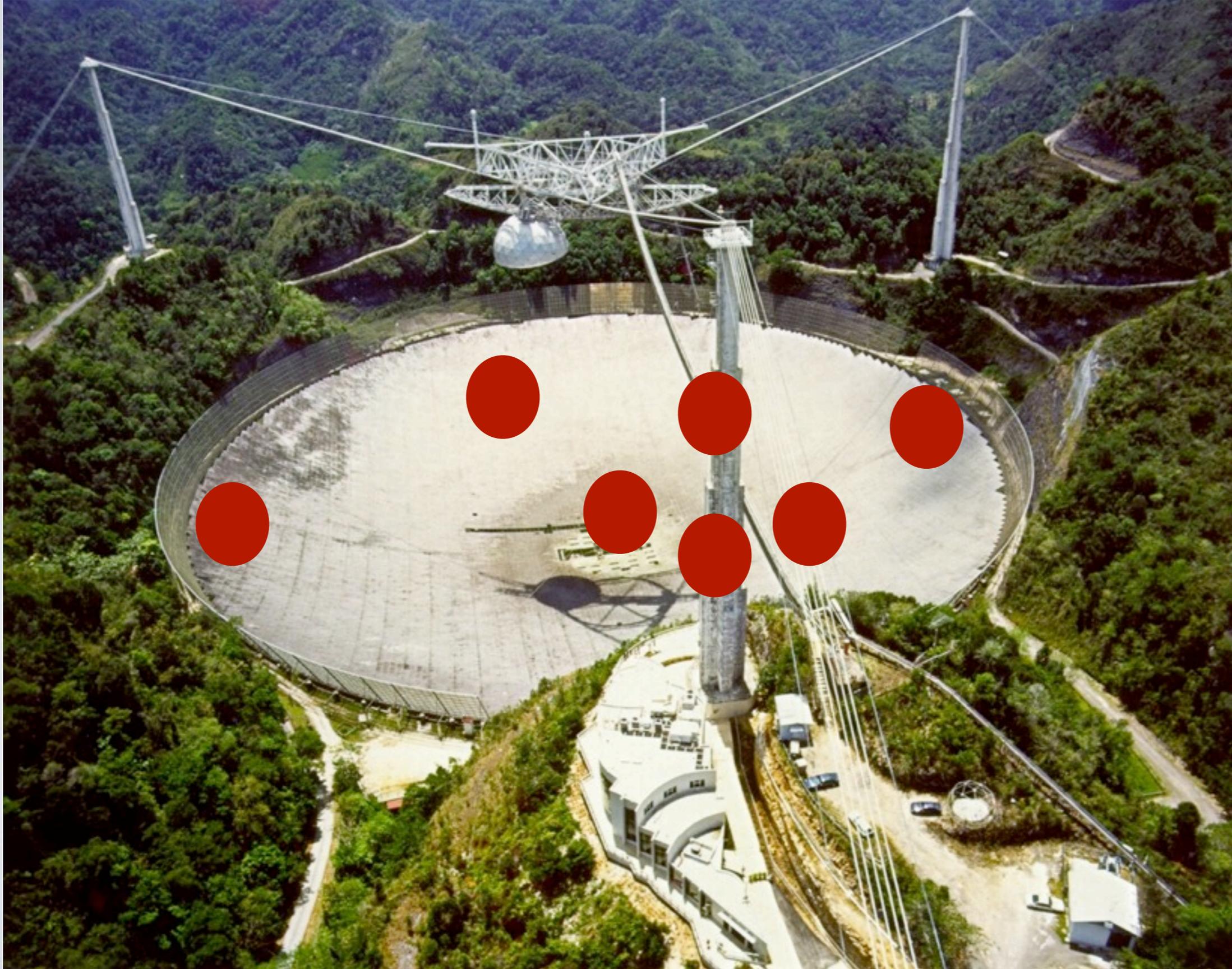
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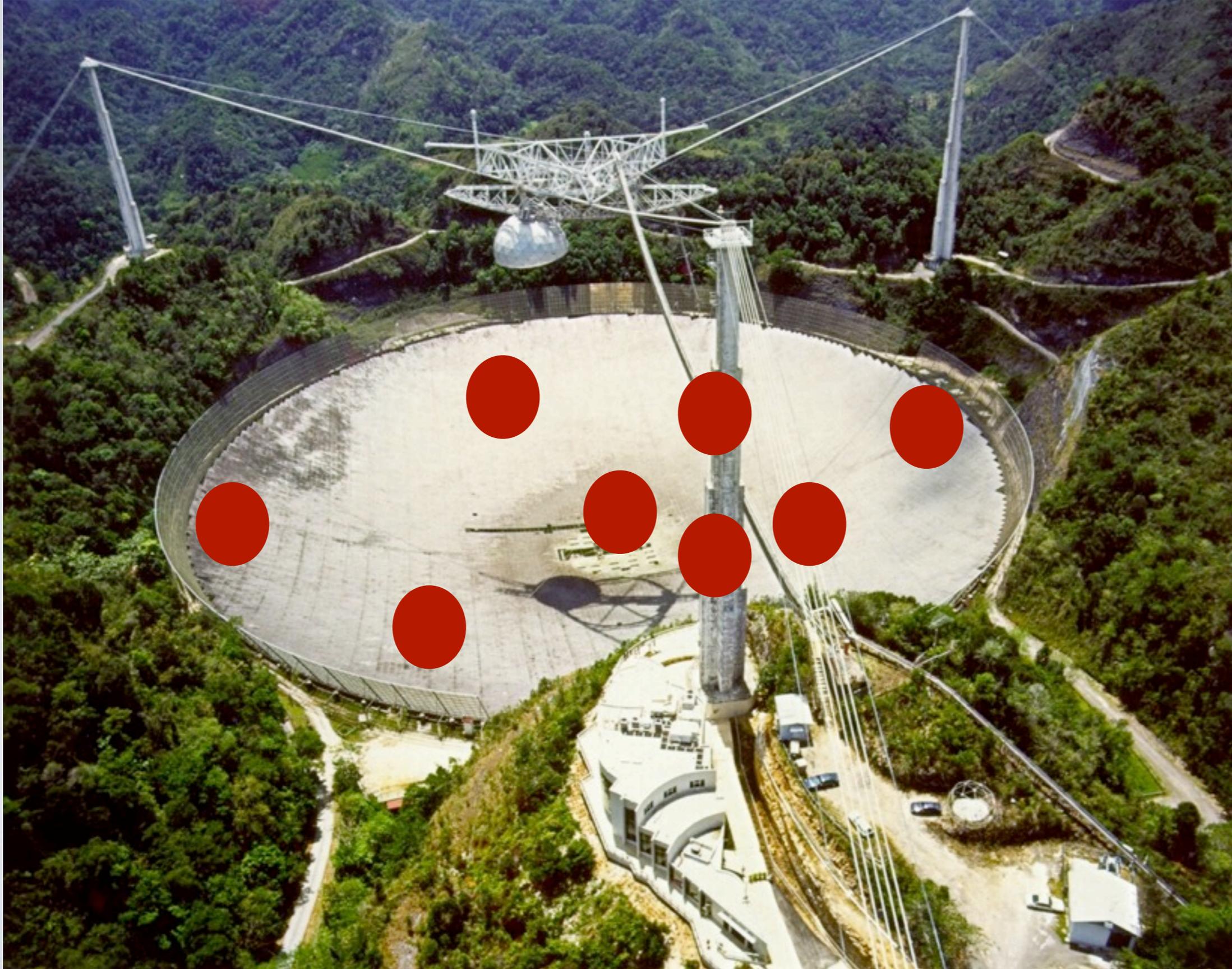
INTERFEROMETER



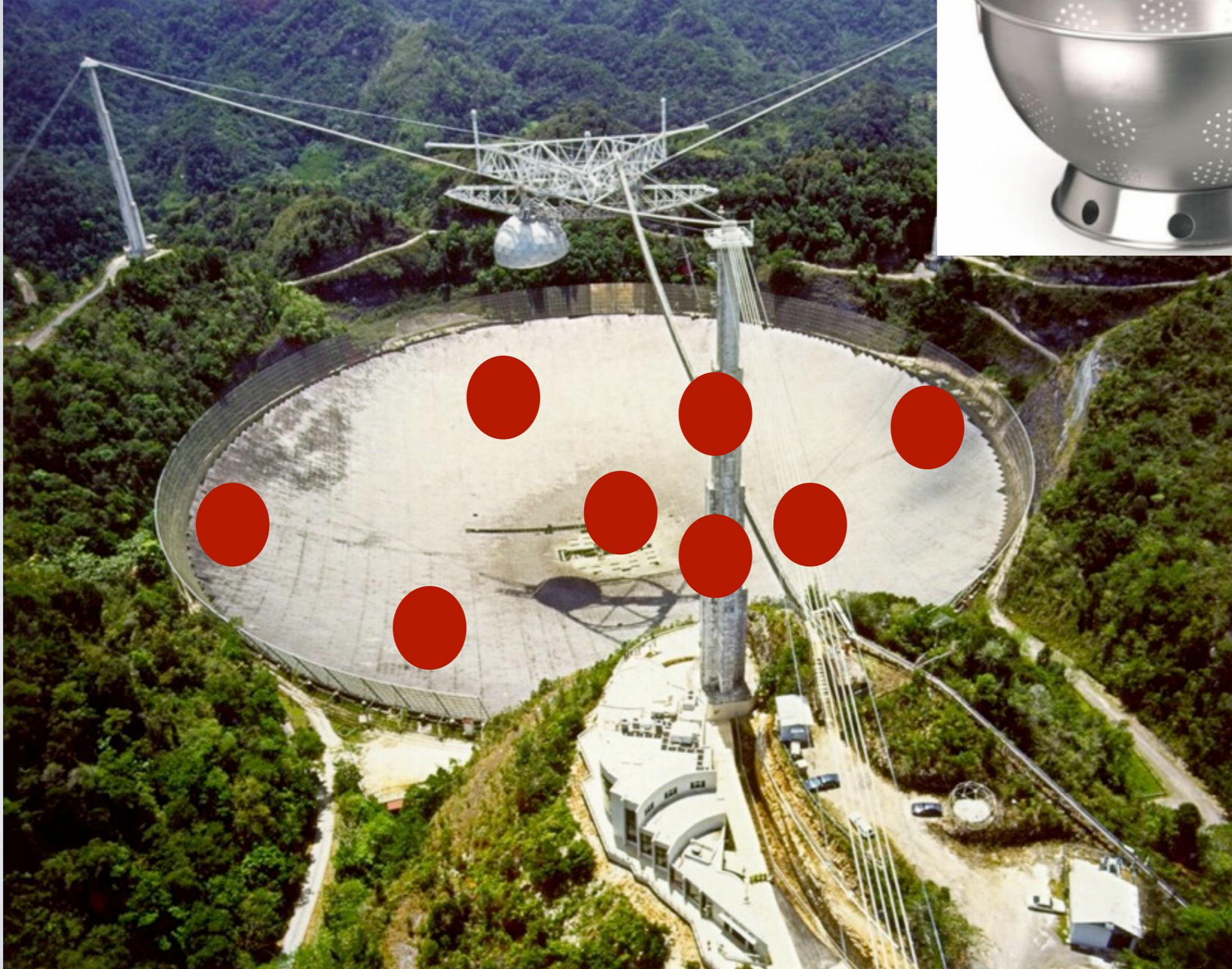
INTERFEROMETER



INTERFEROMETER



INTERFEROMETER



DIFFRACTION GRATING: LIGHT

$$U_1(r, t) = A_1(r) e^{i[\phi_1(r) - \omega t]}$$

$$U_2(r, t) = A_2(r) e^{i[\phi_2(r) - \omega t]}$$

$$I(r) = \int U_1(r, t) + U_2^*(r, t) dt$$

$$I(r) \propto A_1^2 + A_2^2 + b \cos[\phi_1(r) - \phi_2(r)]$$

INTERFEROMETER: RADIO WAVES

geometrical delay

$$\tau_g = \mathbf{b} \cdot \mathbf{s} / c$$

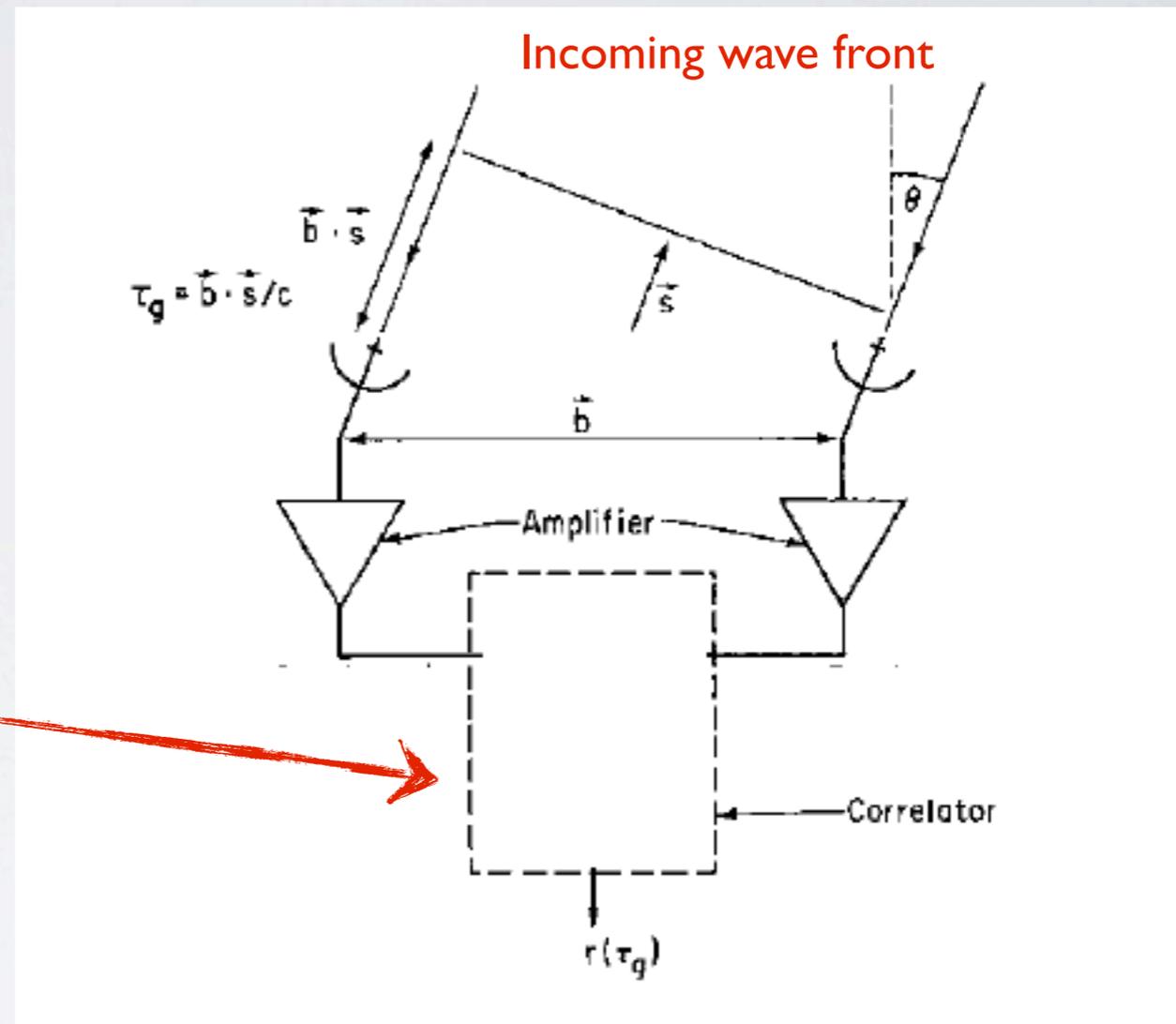
$$E_1(t) = E \sin(2\pi\nu t)$$

$$E_2(t) = E \sin[2\pi\nu(t + \tau_g)]$$

$$V(\mathbf{r1}, \mathbf{r2}) = \langle E(r1)E^*(r2) \rangle$$

van Cittert - Zernike Theorem

$$R(\tau) \propto E^2 \cos(2\pi\nu\tau_g)$$



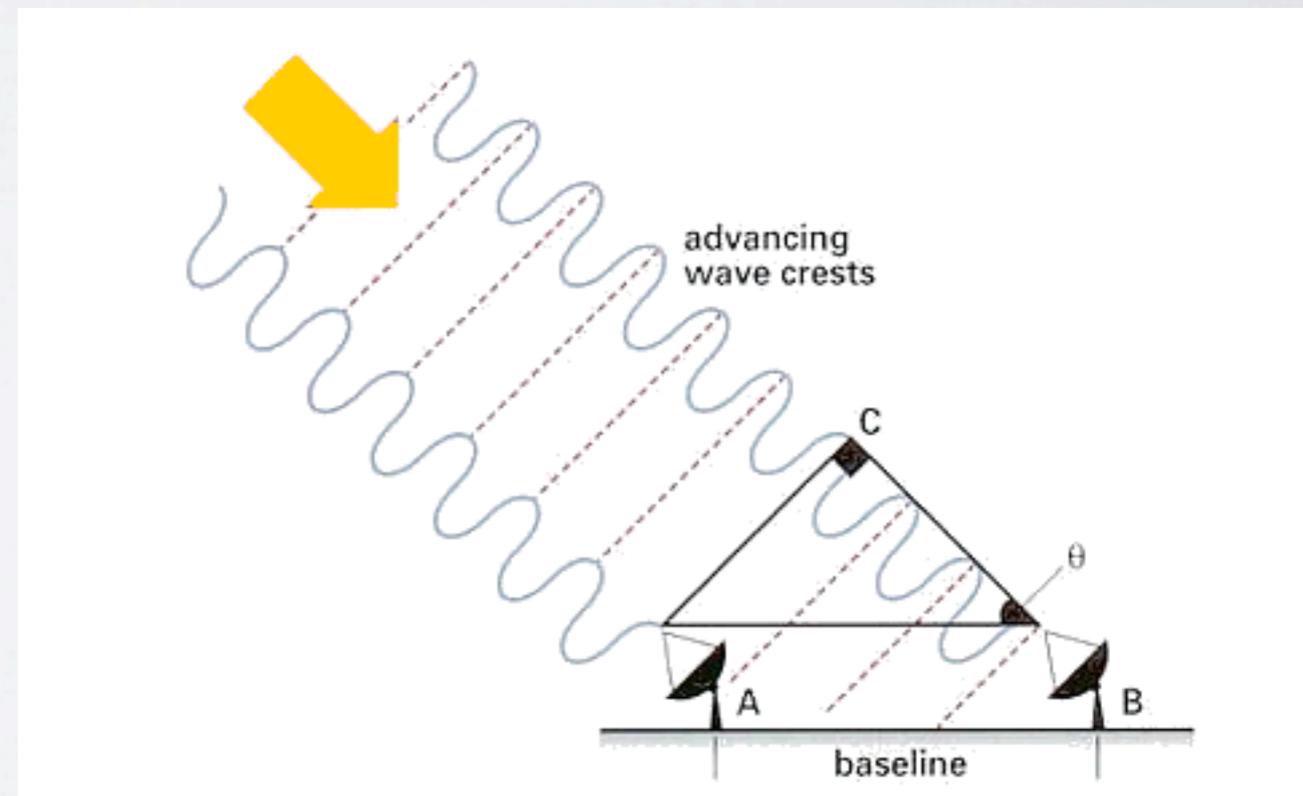
The Fourier transform of the source (sky) intensity is the Visibility function.

INTERFEROMETER: RADIO WAVES

To receive constructive fringes two quantities need to stay
CONSTANT:

- the geometrical delay
- the phase difference of the signal between the two antenna

The radiation is then coherent:
the answer of the interferometer
(visibility function or mutual
coherence function) gives us a
measure of the coherence of the
electromagnetic field

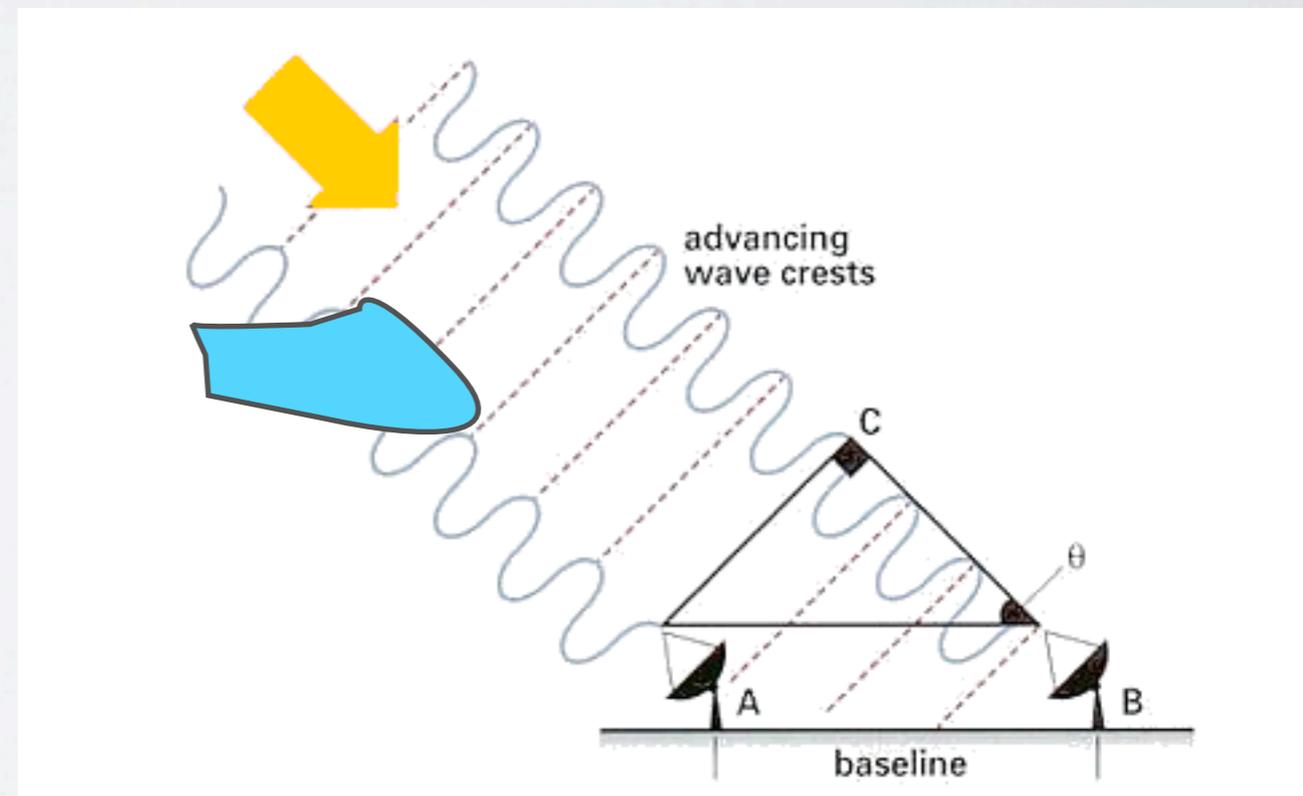


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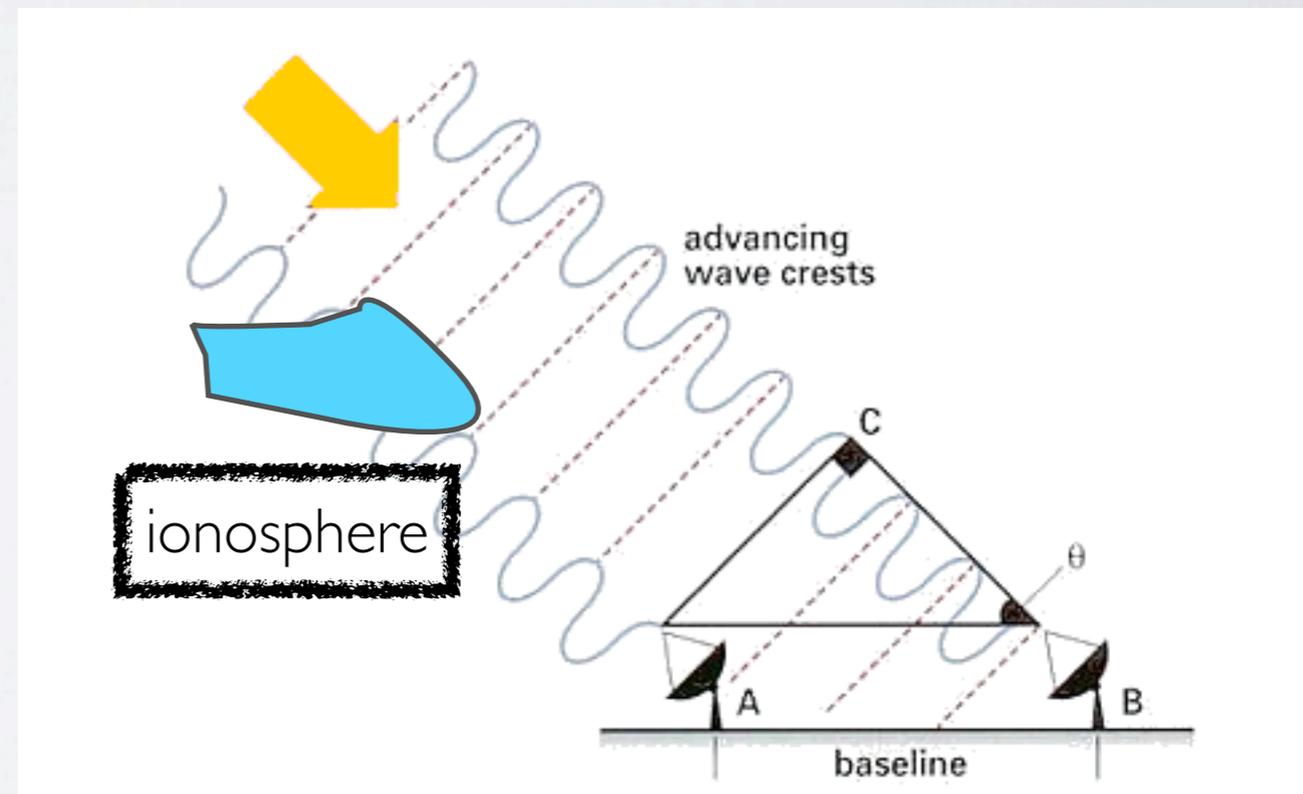


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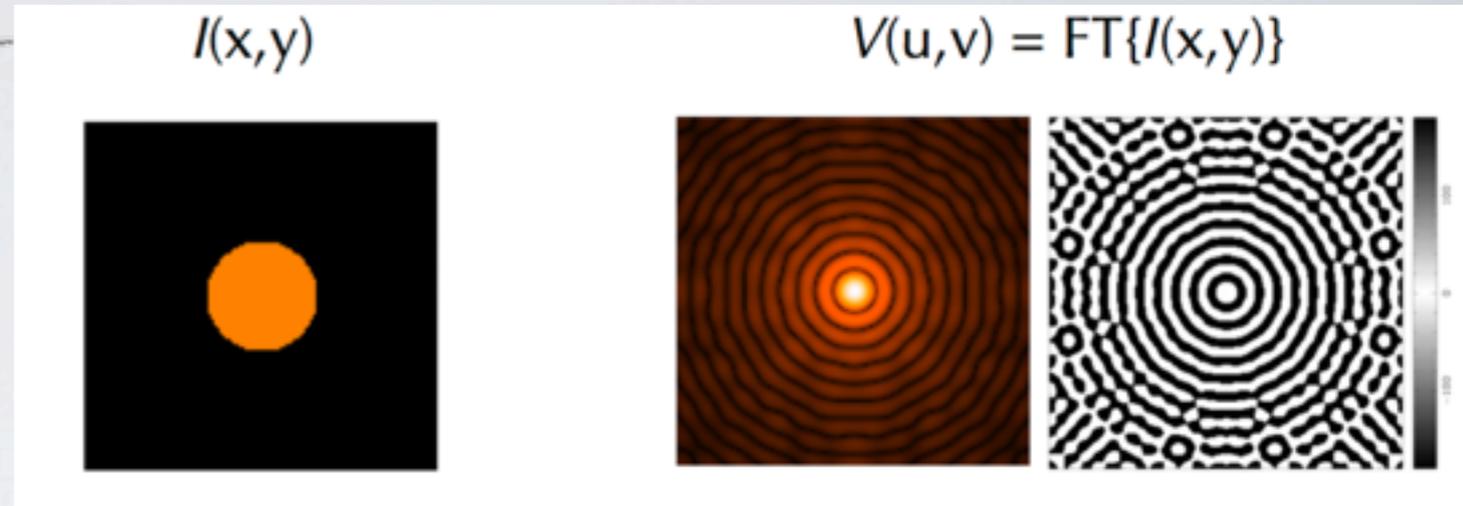
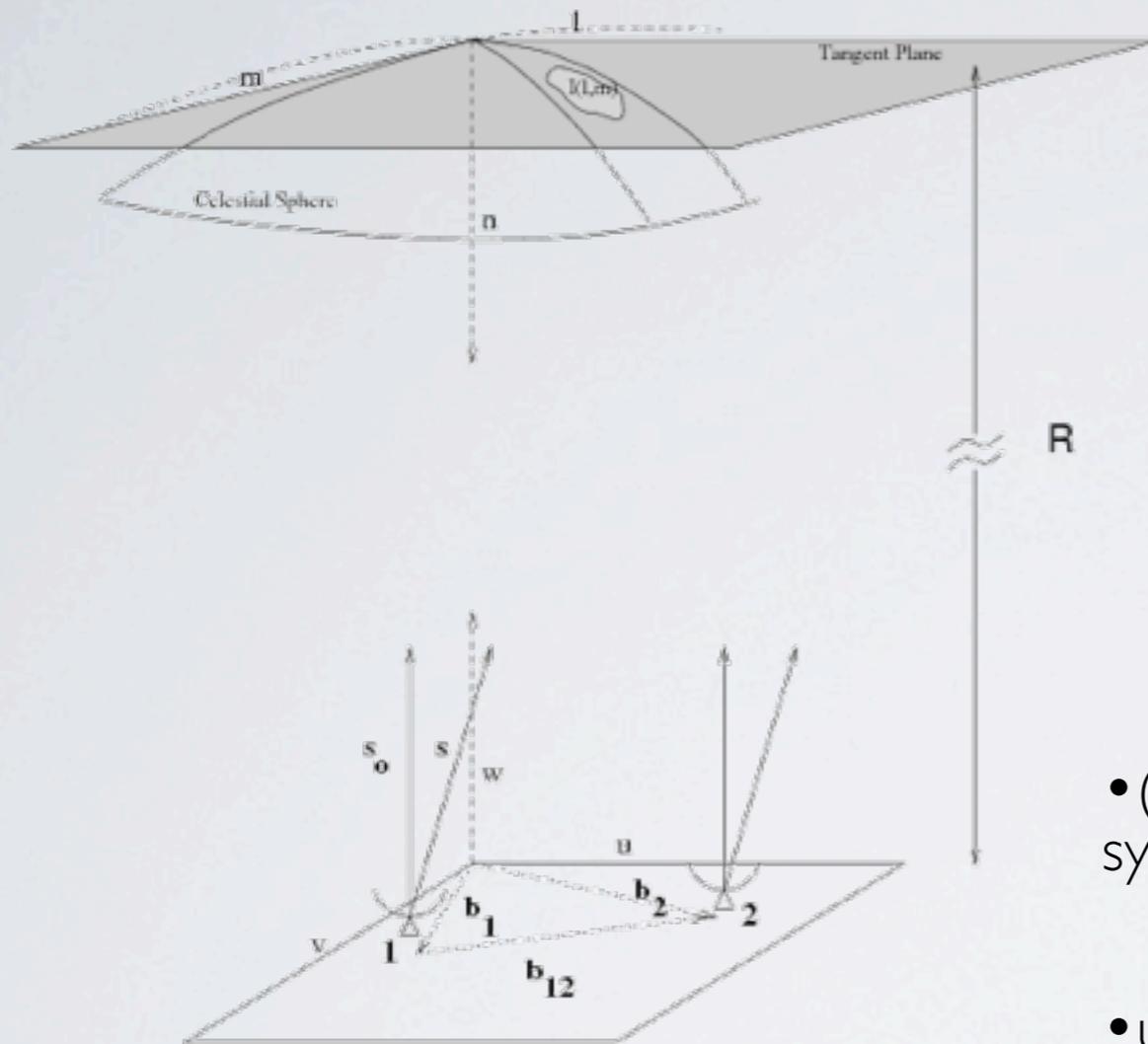
BEAM

- Observations are modified by the presence of the instrument used to do the measure
- the response of a single antenna is given by the cross correlation between the pattern and the intensity distribution of the source

$$\int_{source} A(\theta' - \theta) I(\theta') d\theta'$$

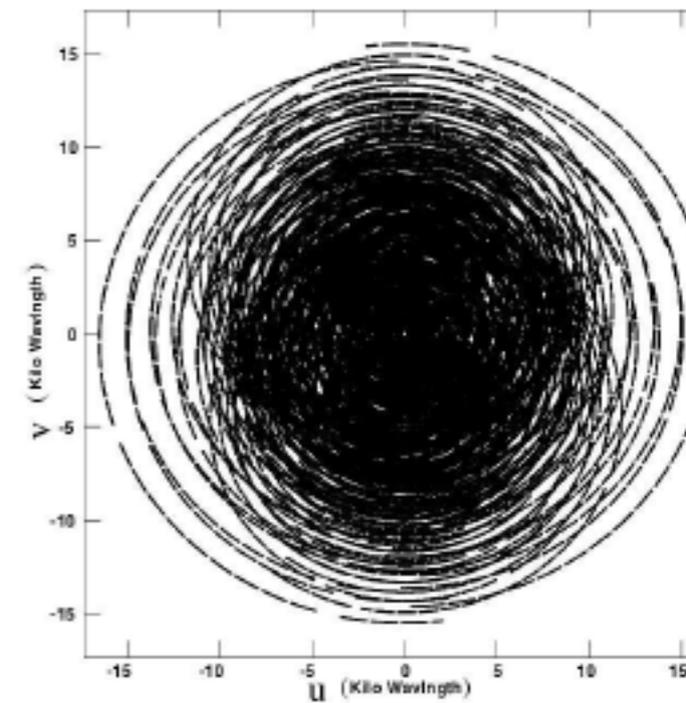
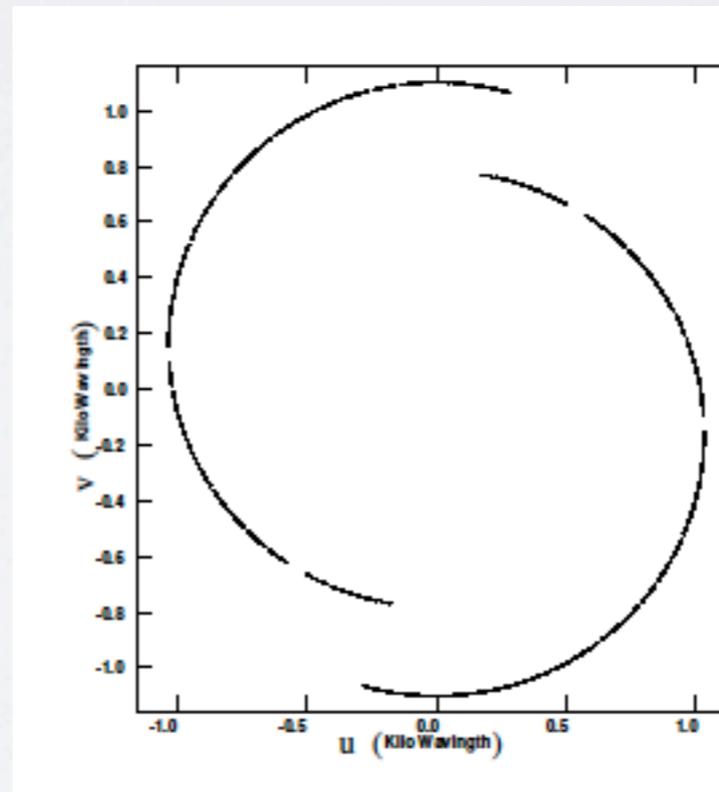
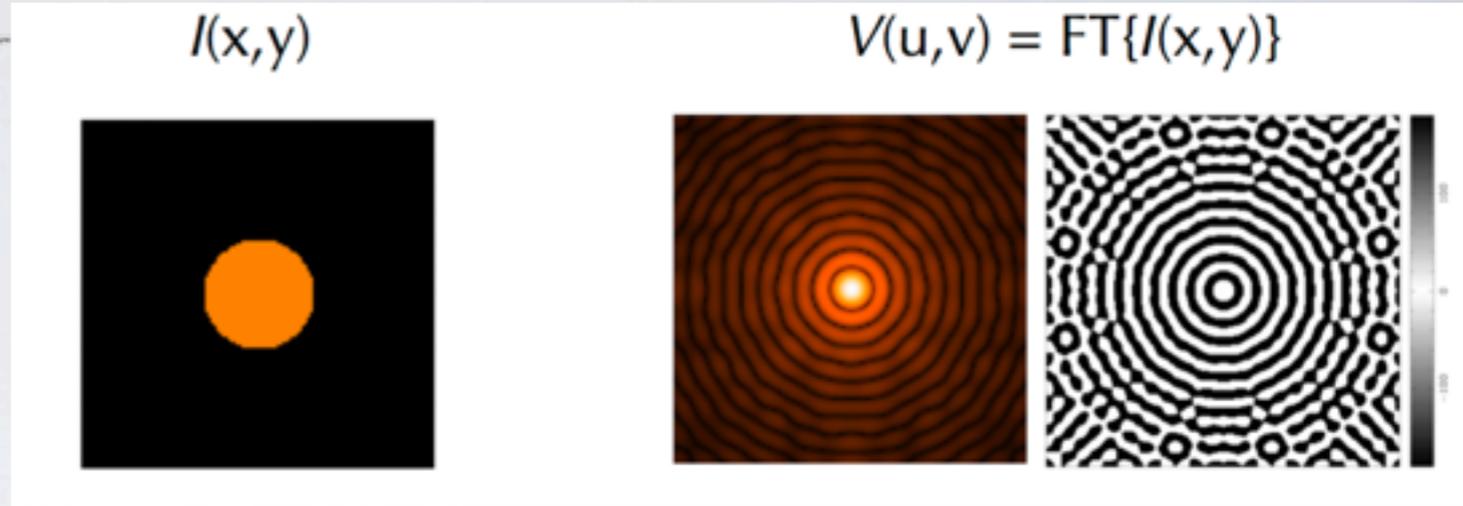
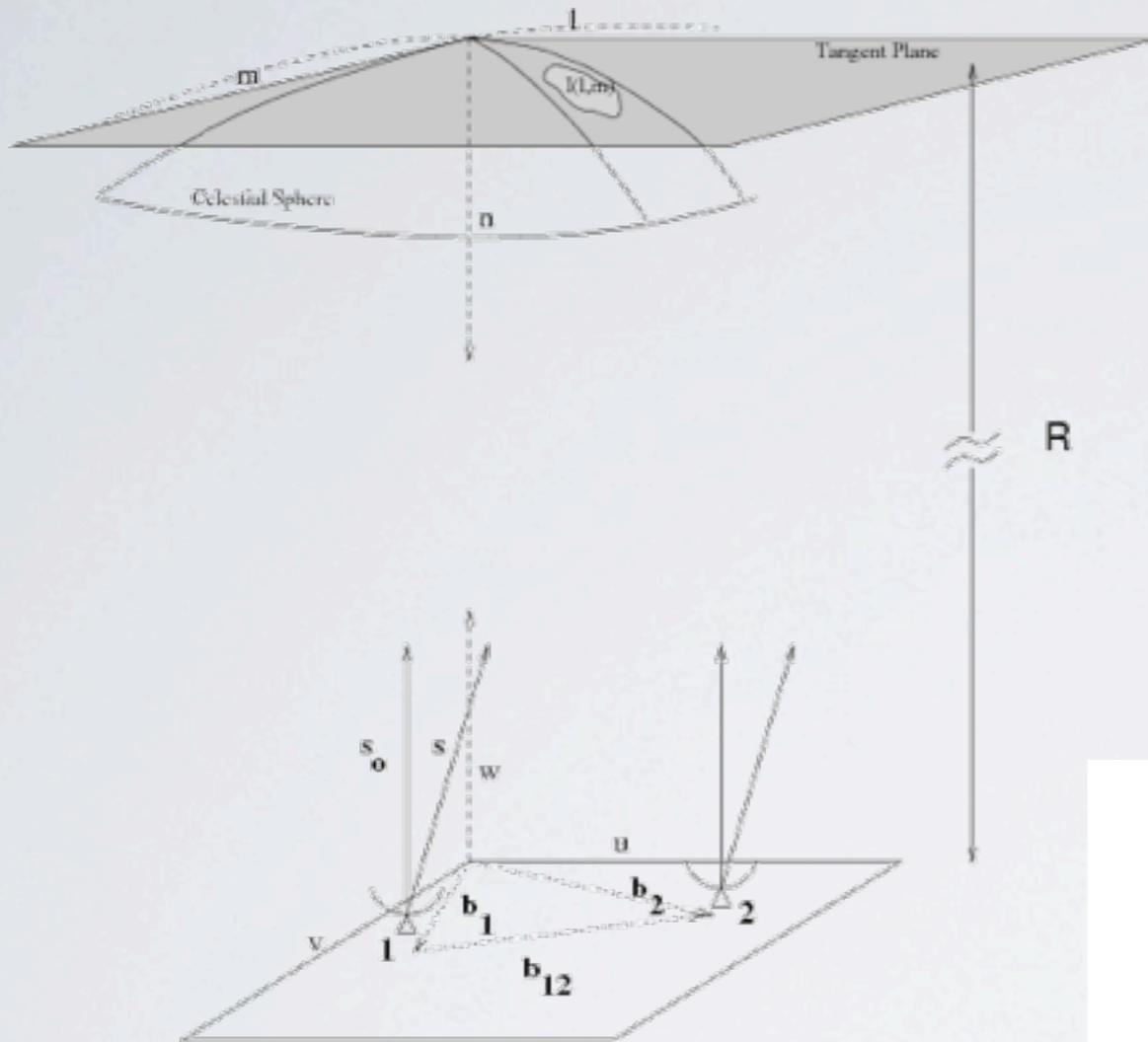


APERTURE SYNTHESIS

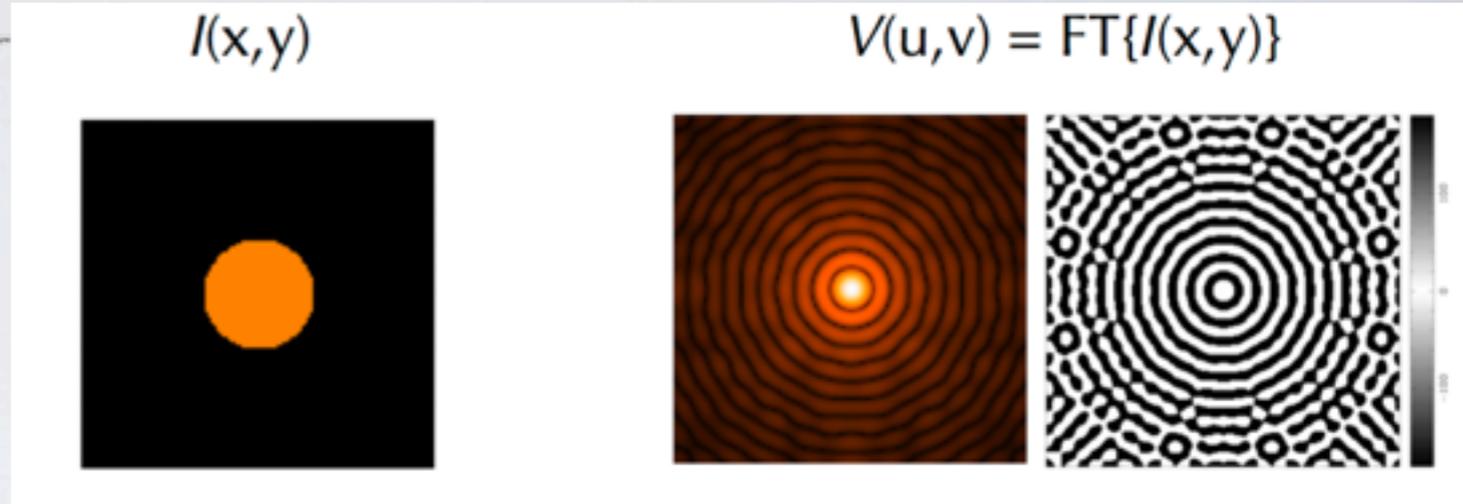
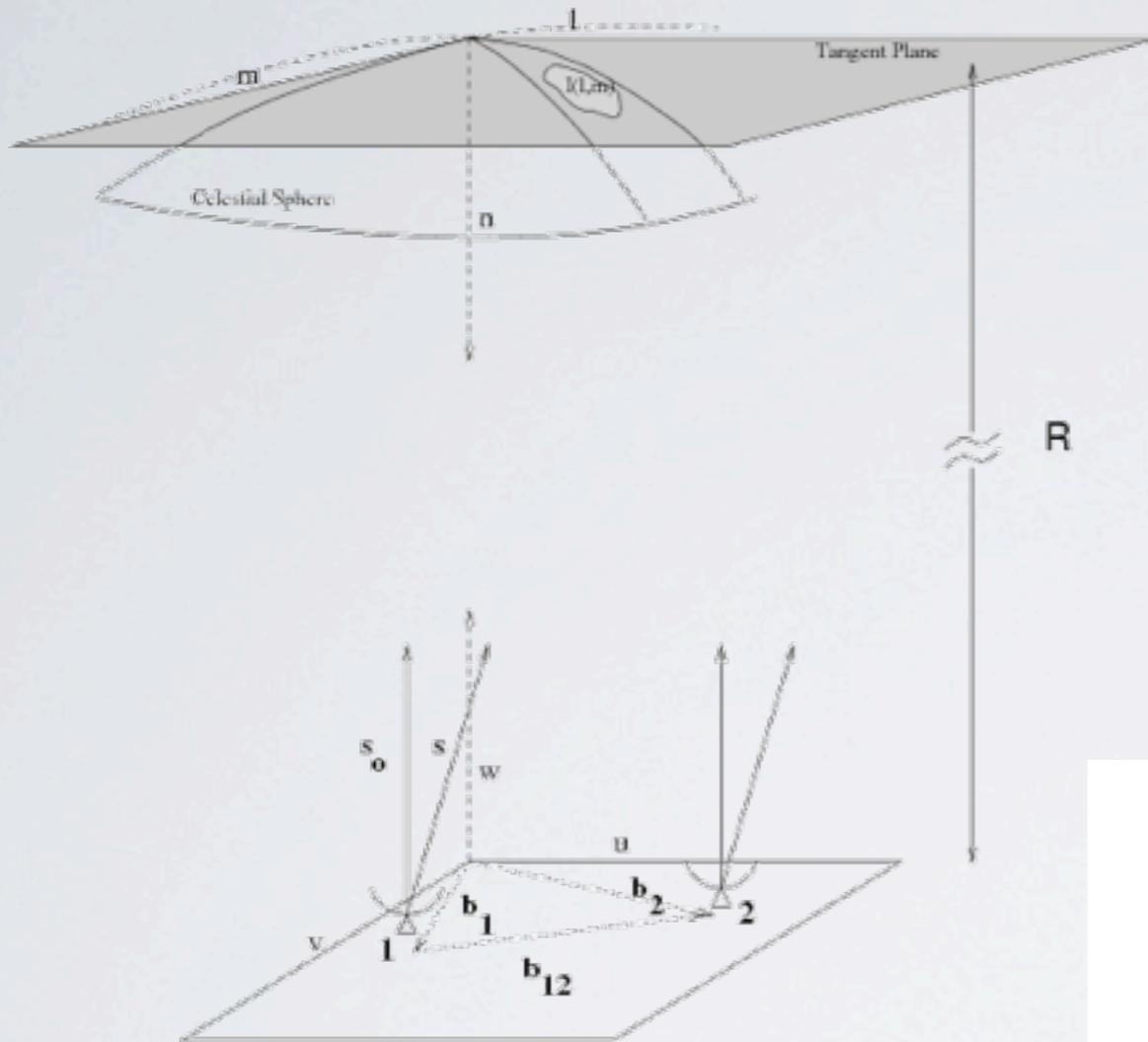


- (x,y) plane and (u,v) plane are conjugate coordinate systems
- uv plane perpendicular to the line of sight
- each uv point univocally represents a baseline wrt a reference position in the array.
- one uv point represents a direction and a scale of a baseline

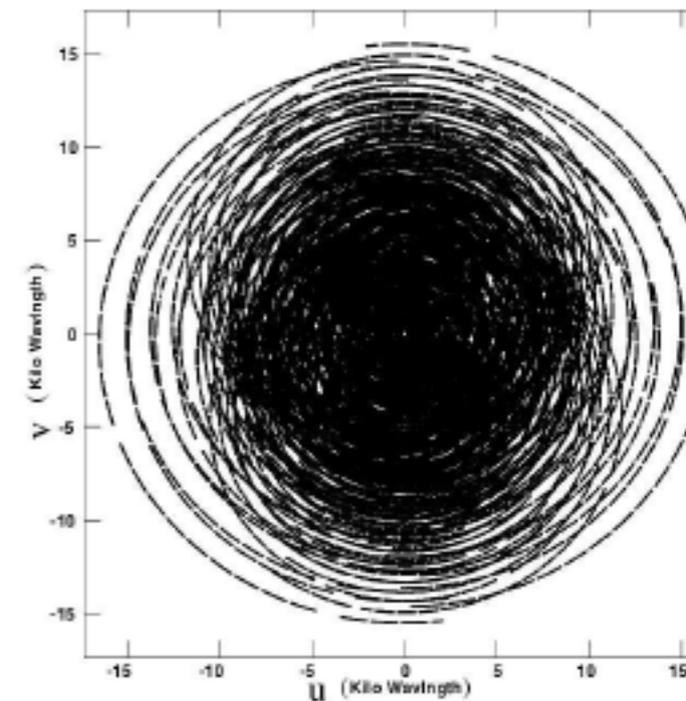
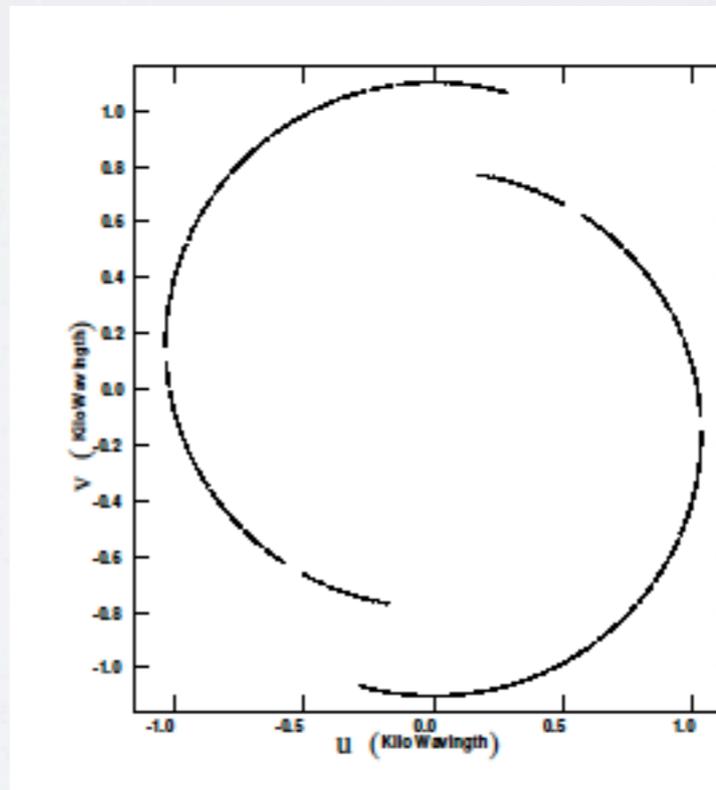
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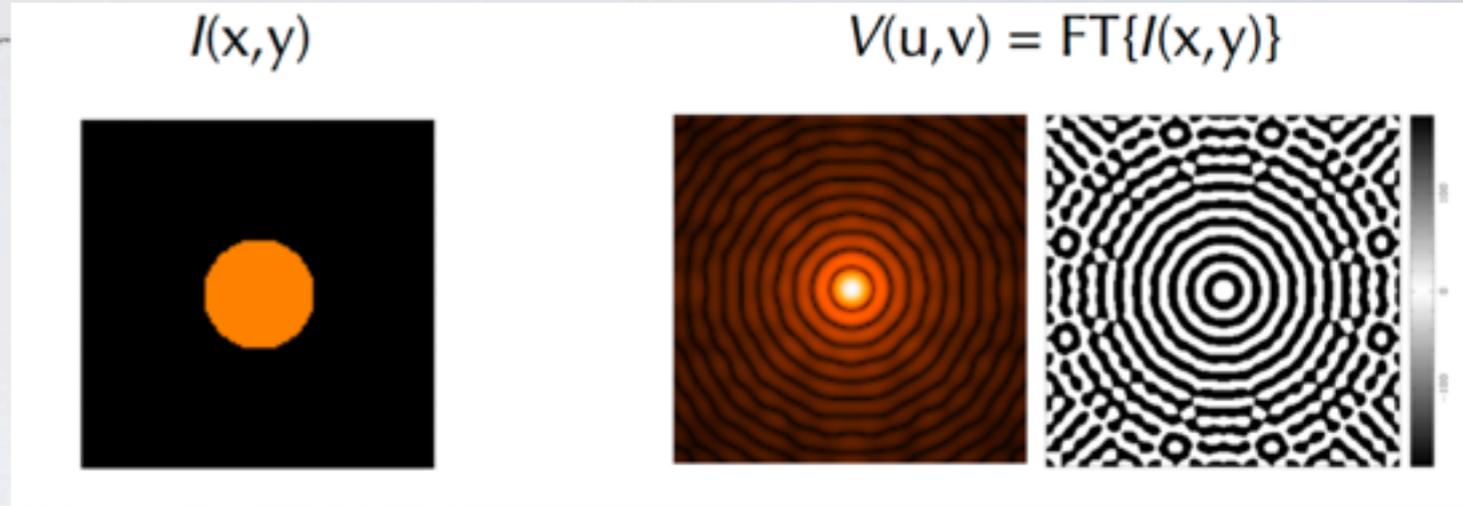
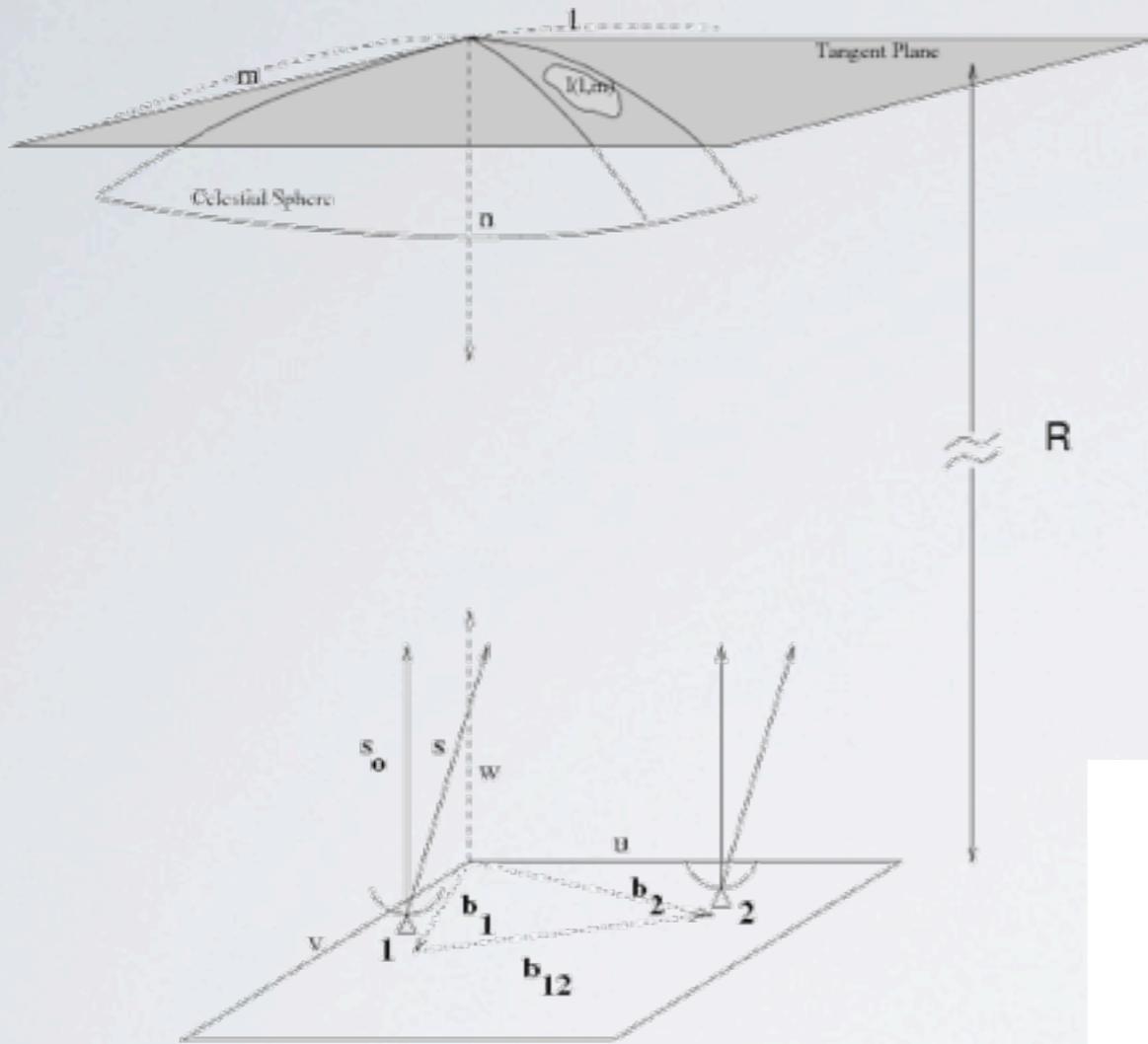
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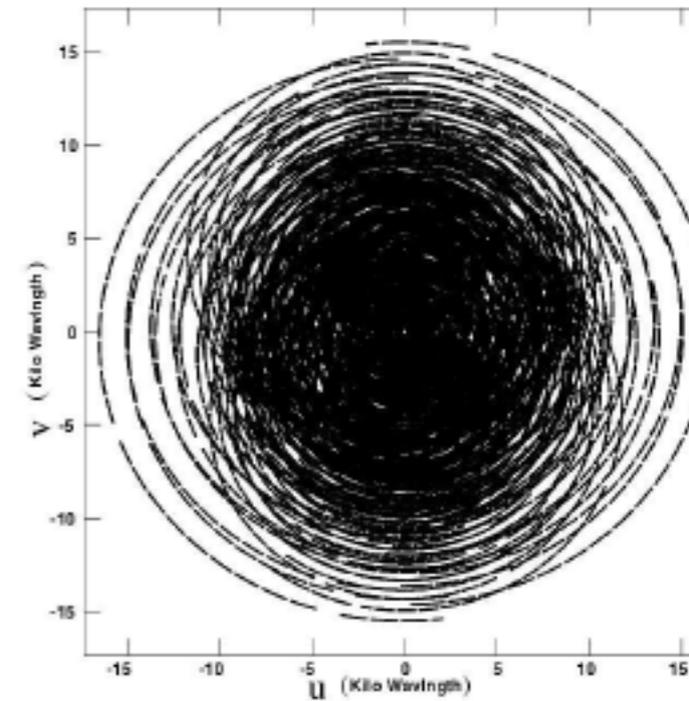
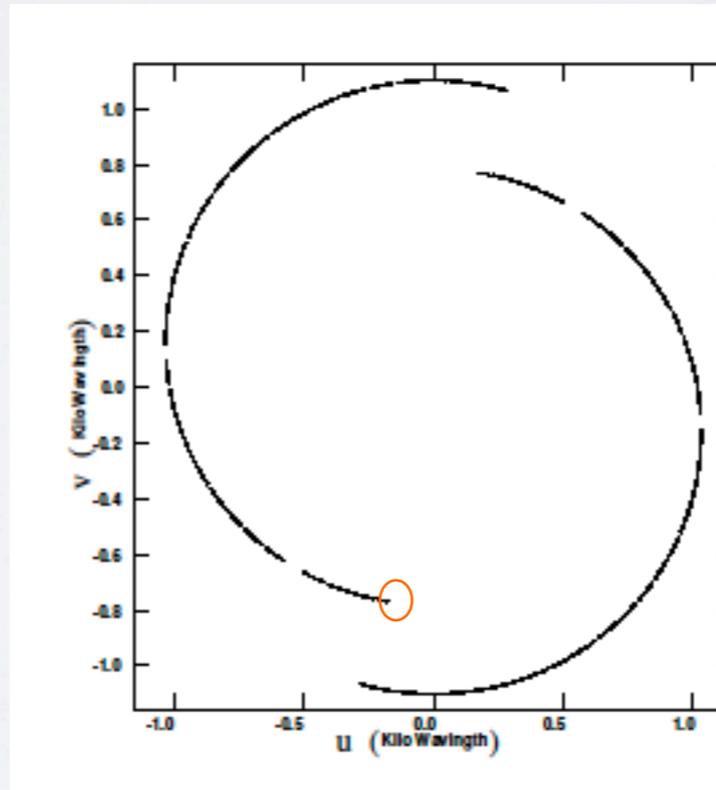
$$t = t_0$$



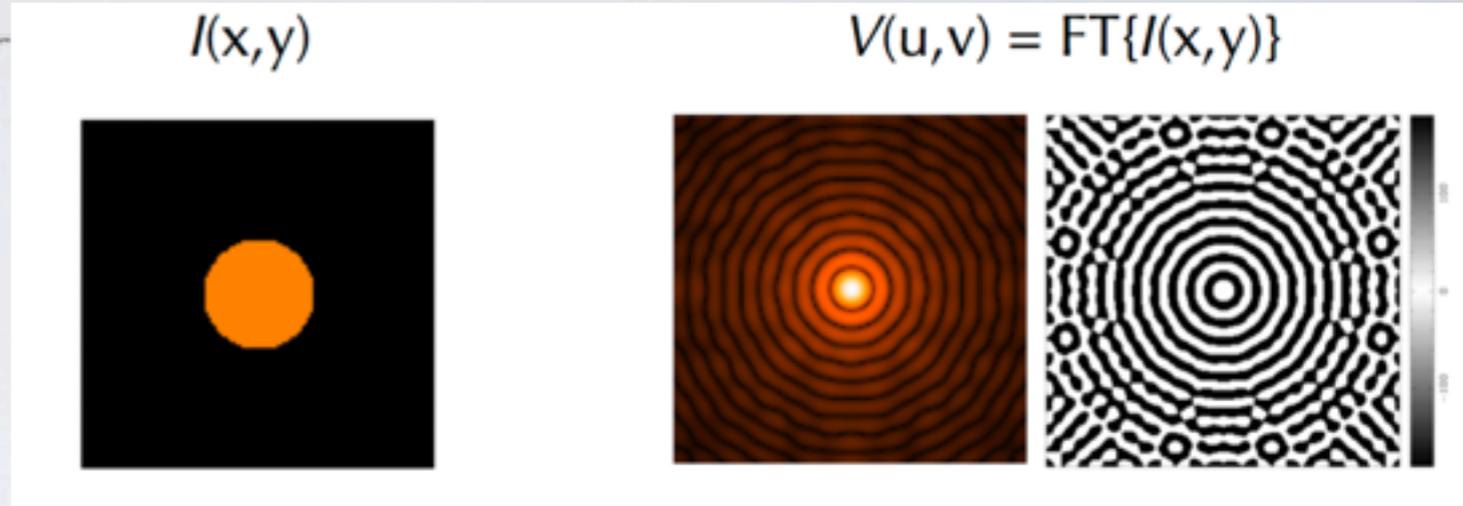
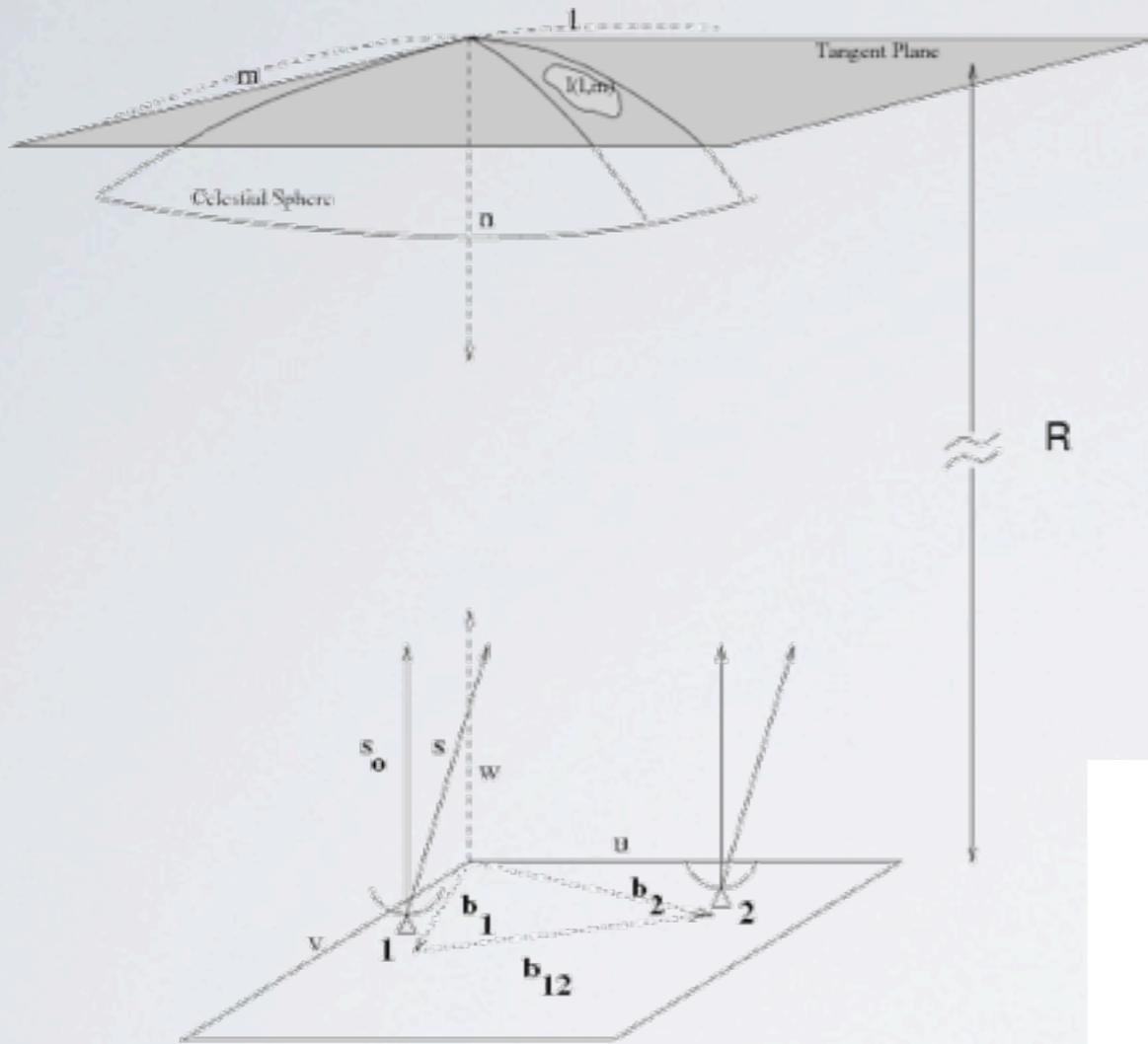
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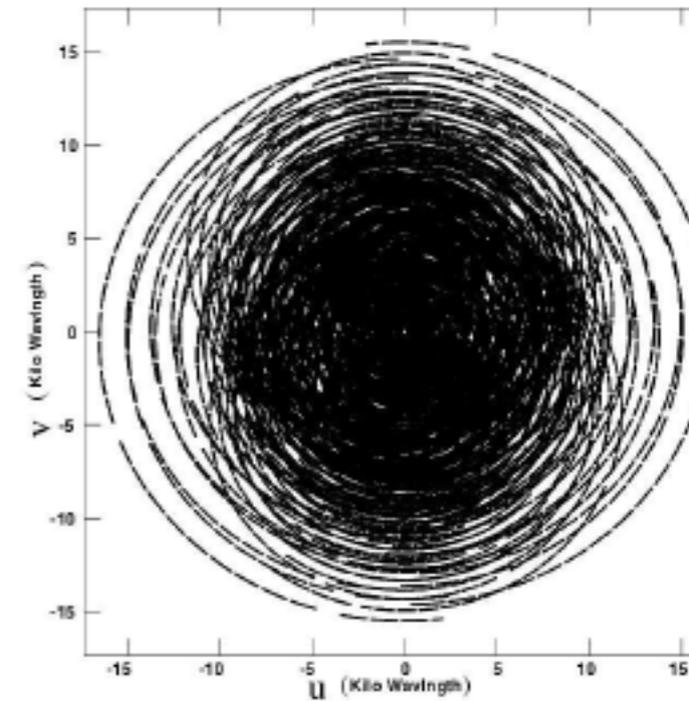
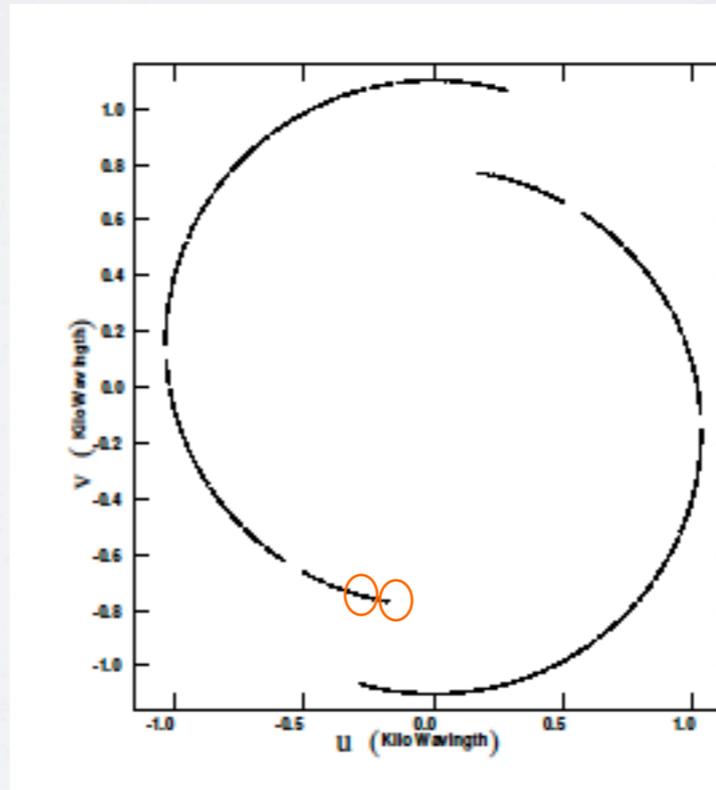
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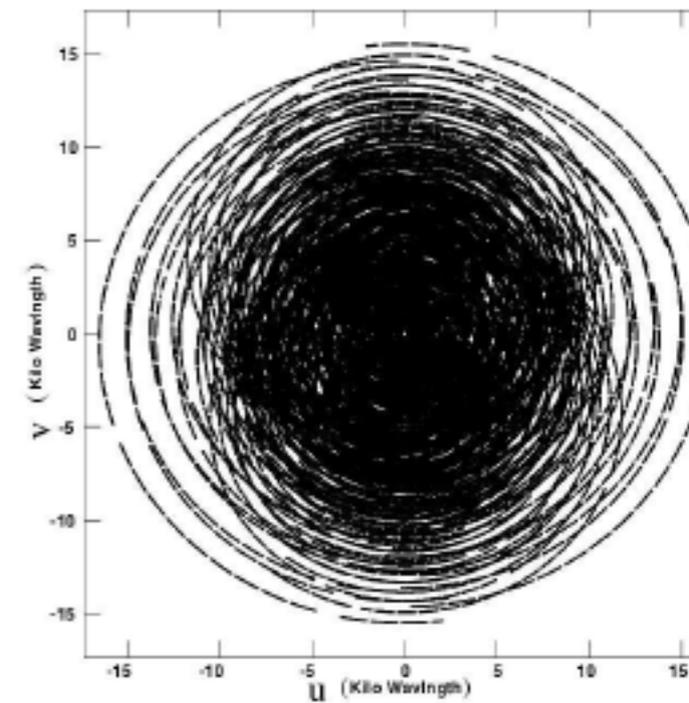
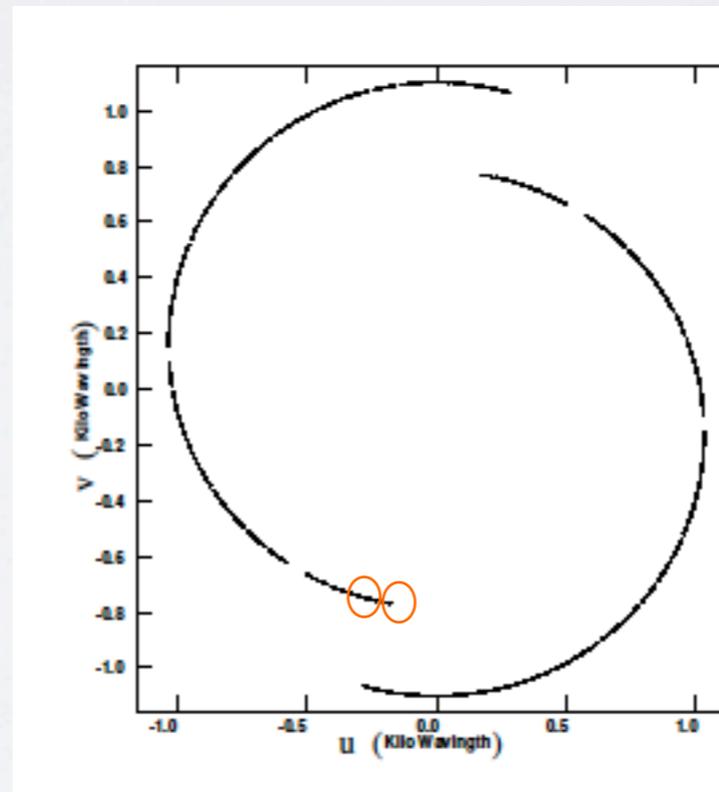
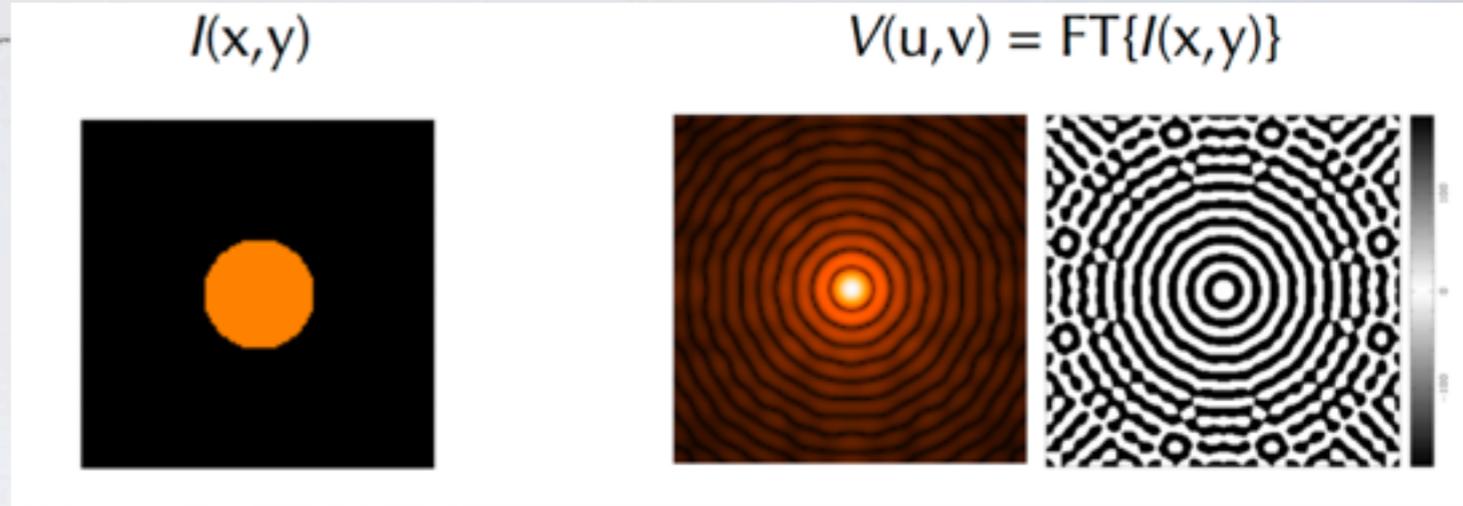
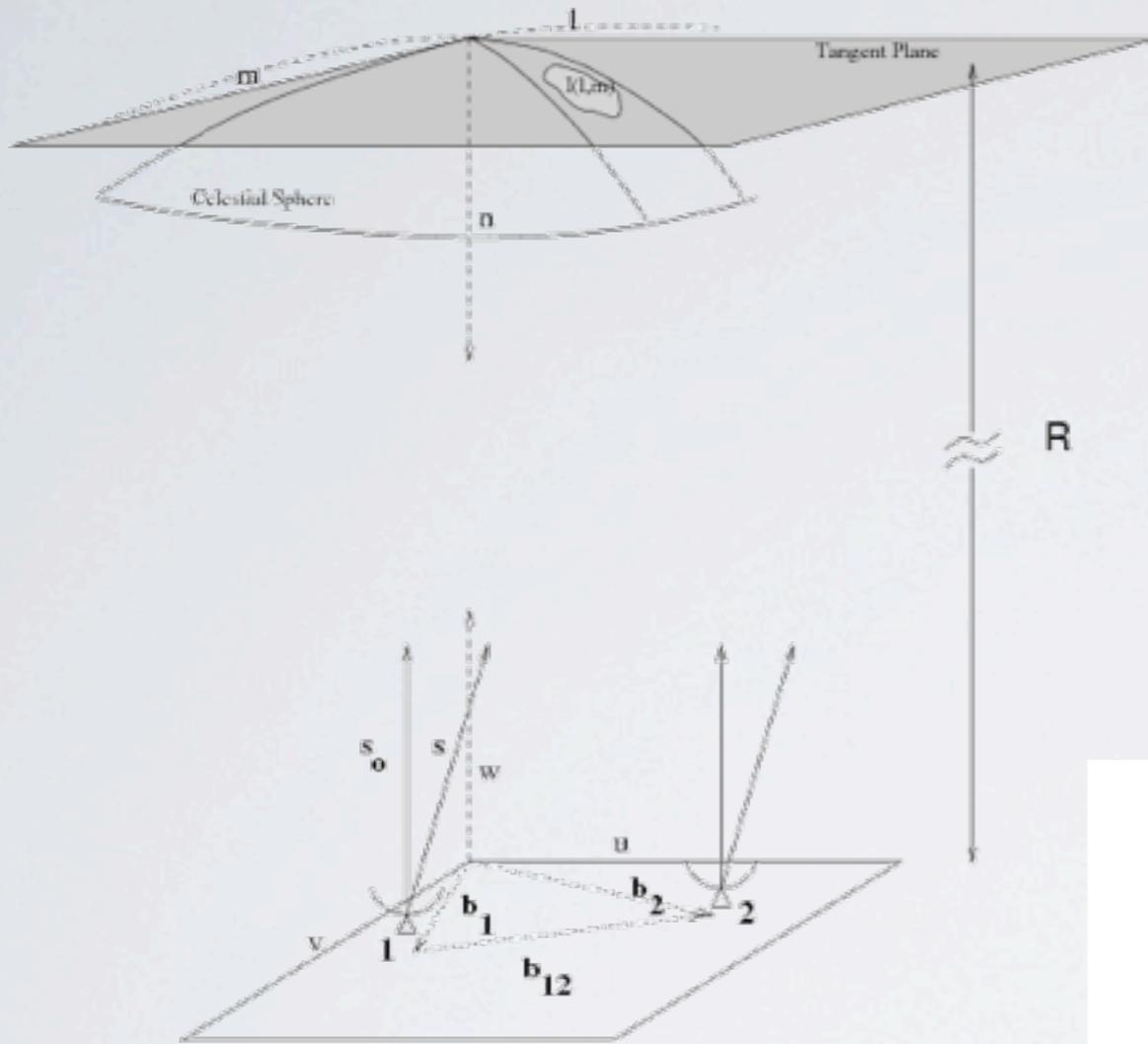
APERTURE SYNTHESIS



$$t = t_0$$



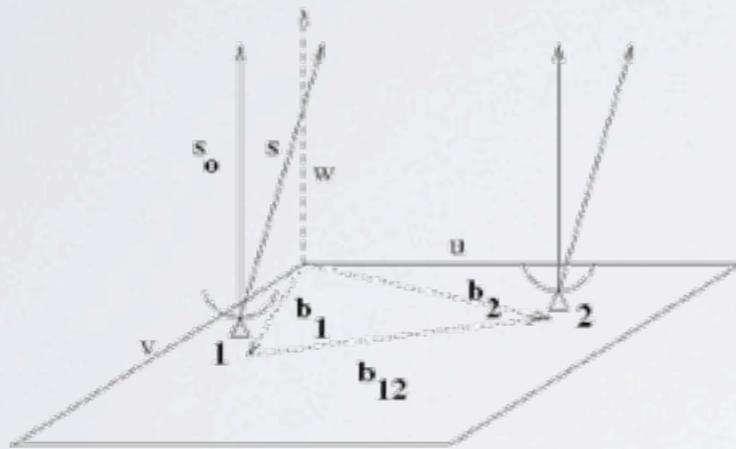
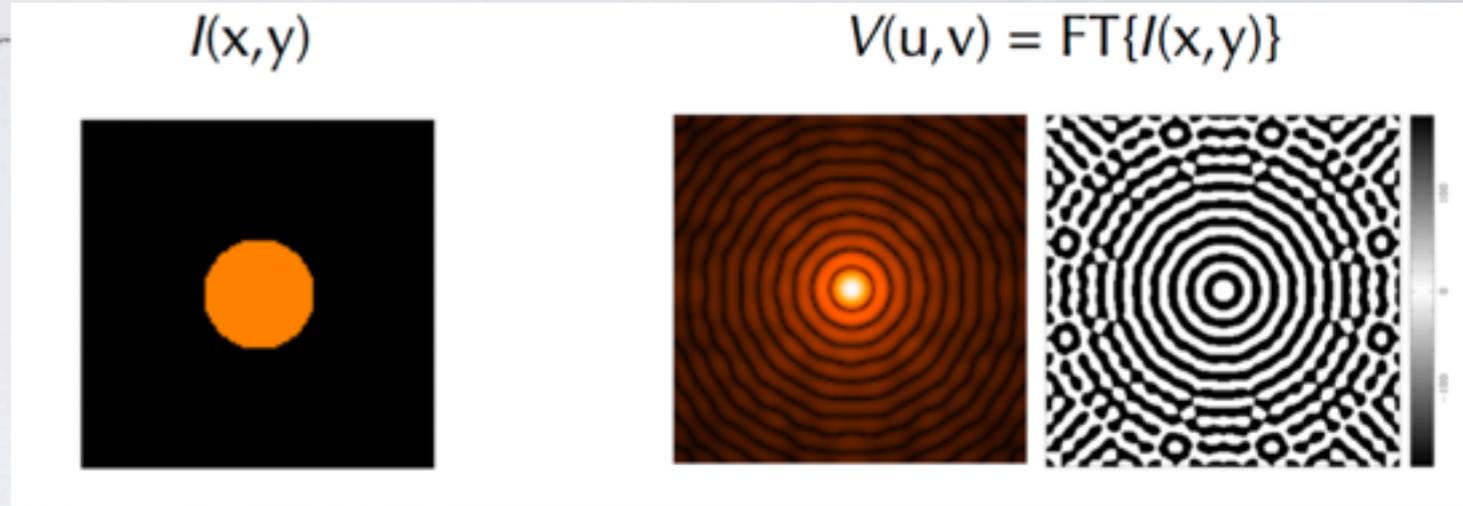
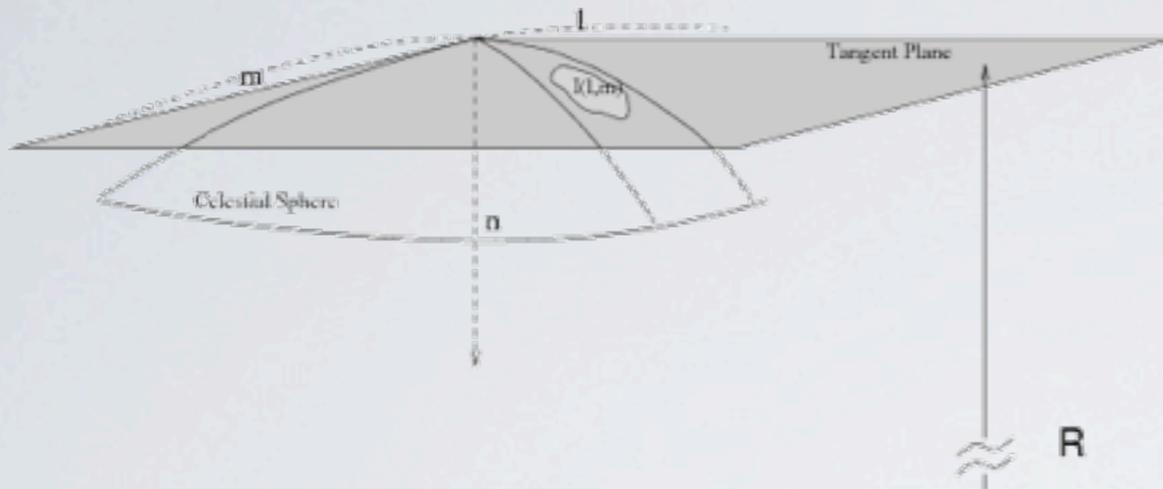
APERTURE SYNTHESIS



$$t = t_1$$

$$t = t_0$$

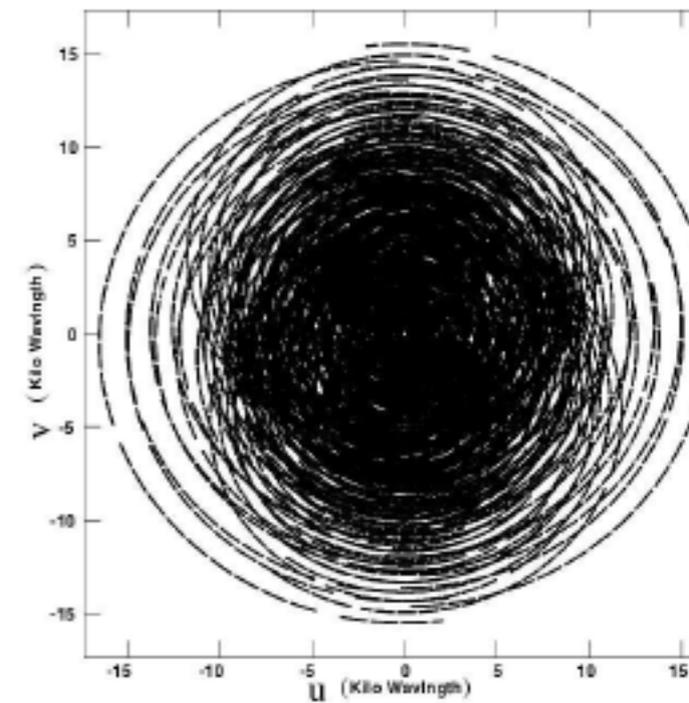
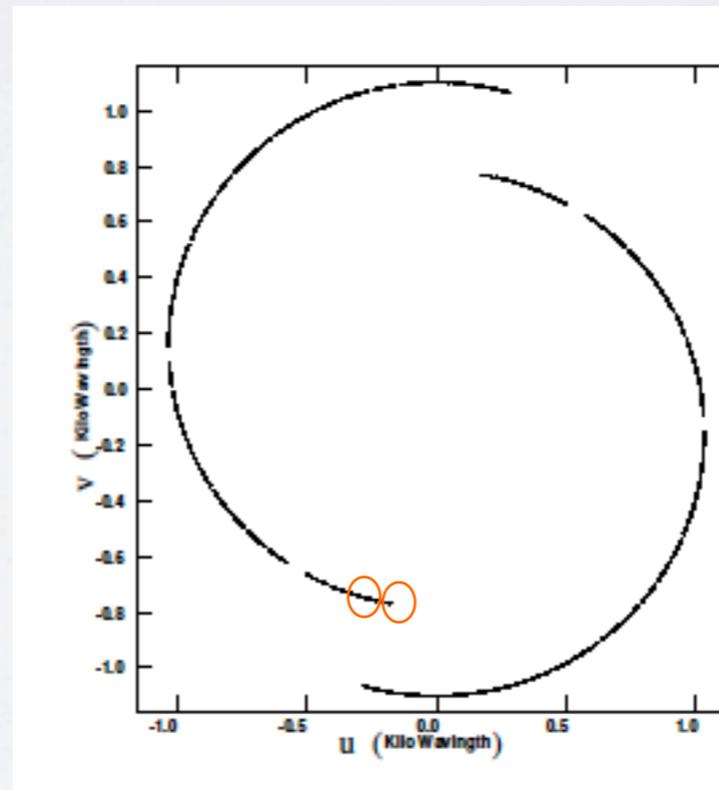
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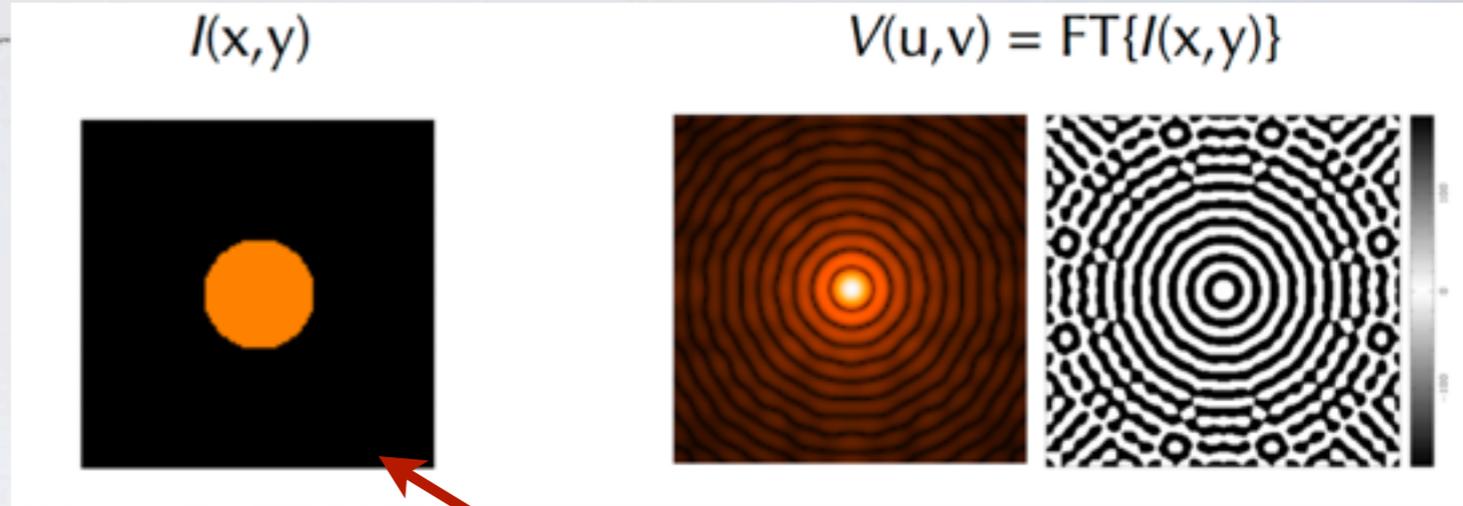
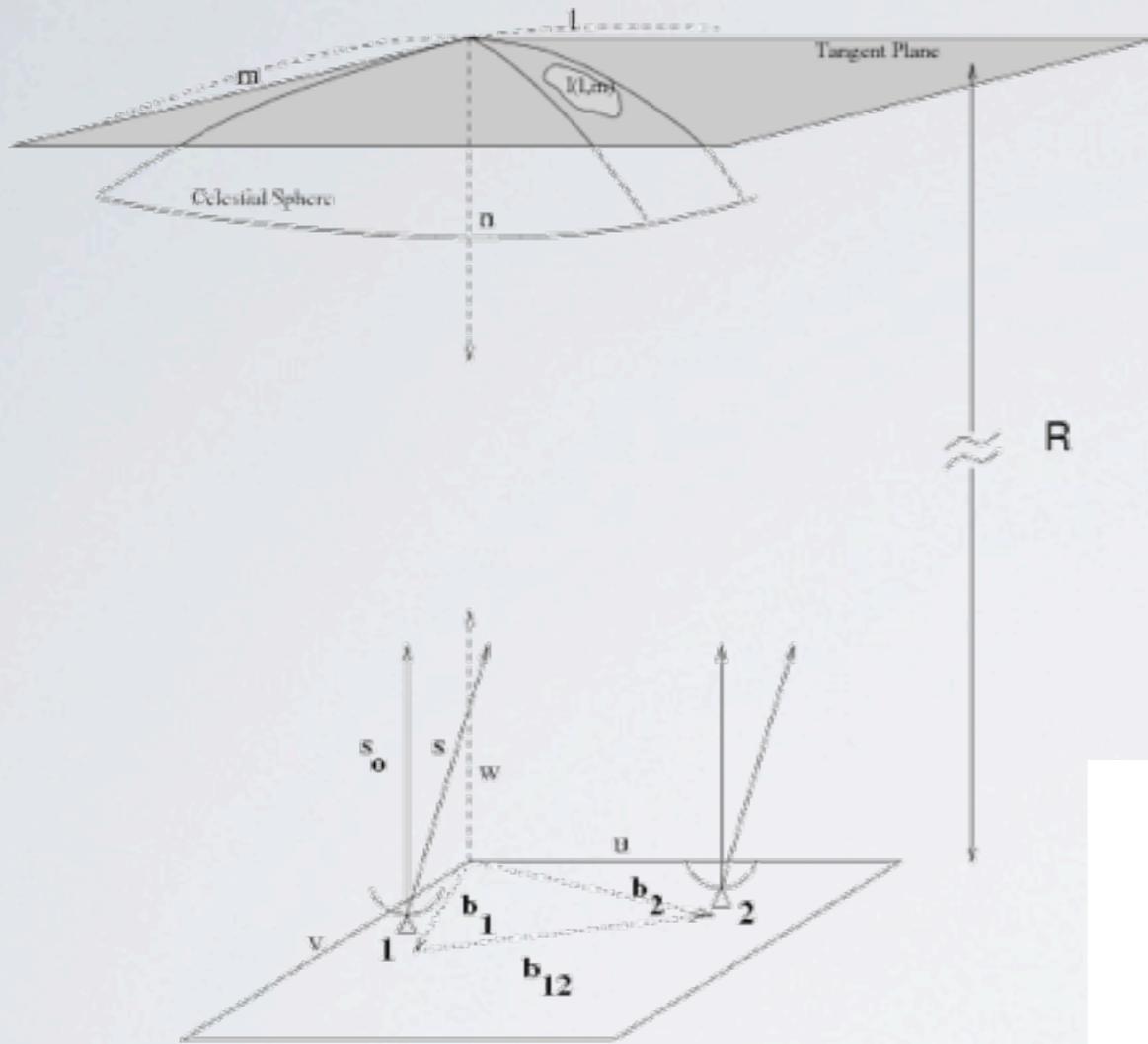
$$t = t_x$$

$$t = t_1$$

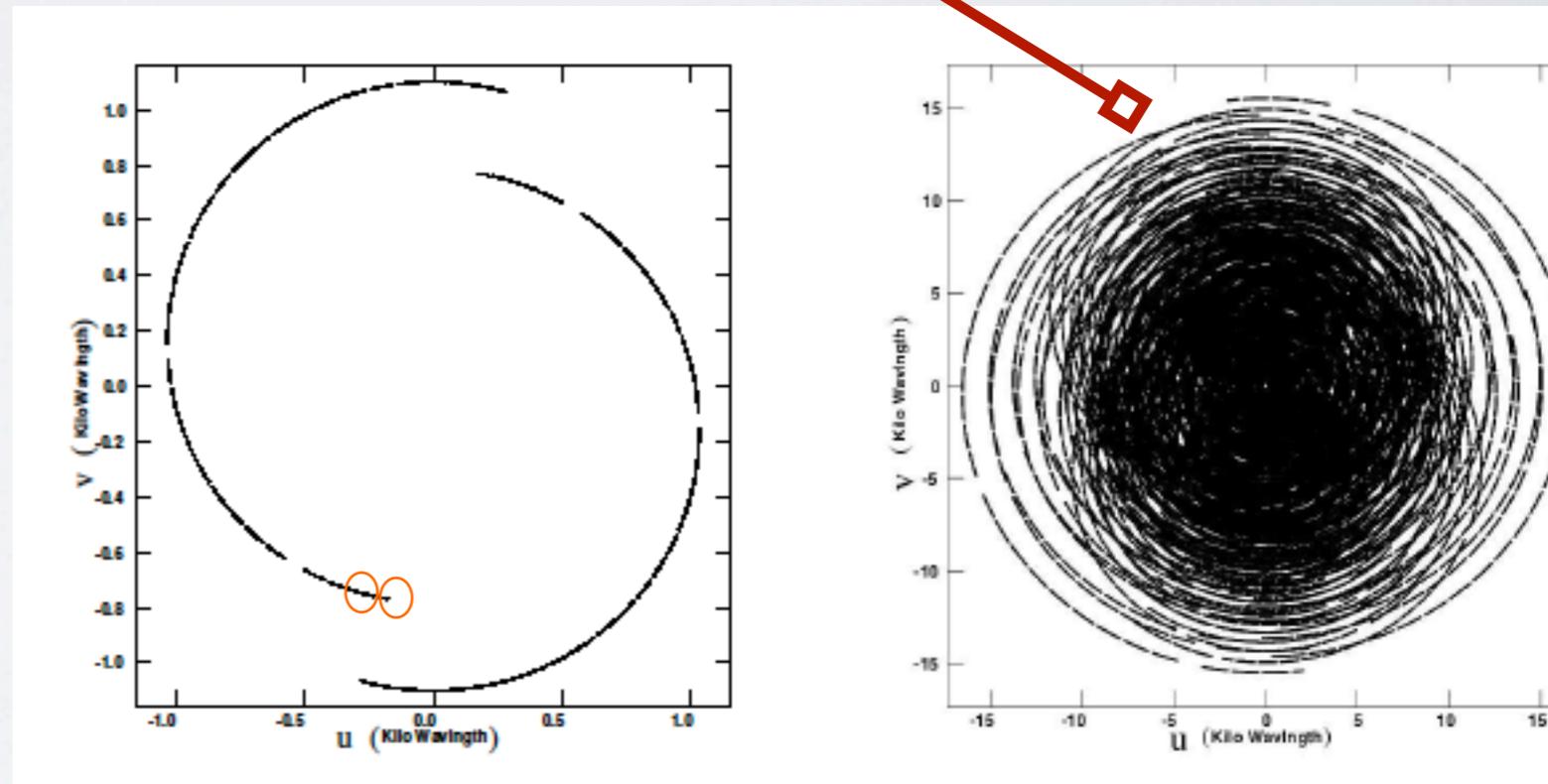
$$t = t_0$$



APERTURE SYNTHESIS



$$t = t_x$$



$$t = t_1$$

$$t = t_0$$

CALIBRATION

- CALIBRATION is the process to determine complex gains G^i aiming at transforming the observed quantities to the proper scale.

$$V_{ij}^{obs} = G_{ij} V_{ij}^{true}$$

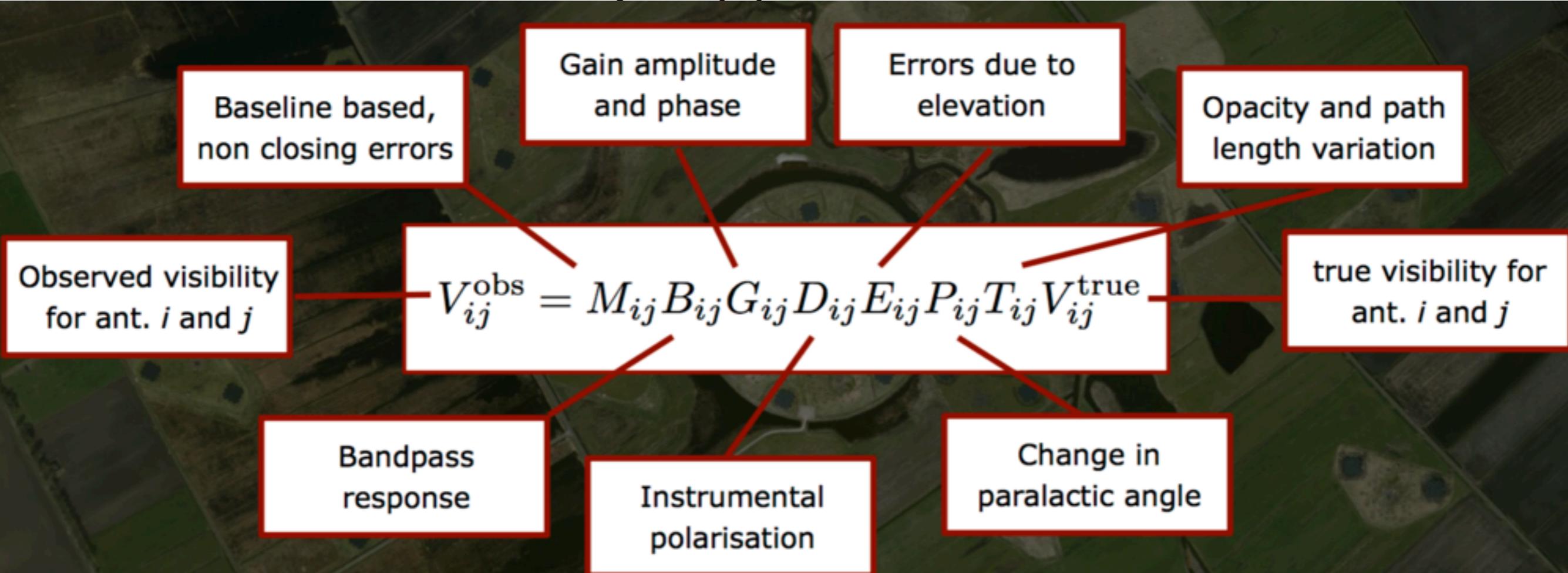
- Standard interferometry $G^i(t)$.
- CALIBRATION involves solving this inverse problem to determine what set of parameters are needed to minimise the difference between the observed visibilities and the model visibilities (our best guess at the true visibilities).
- @ low frequency assumptions are not valid

CALIBRATION

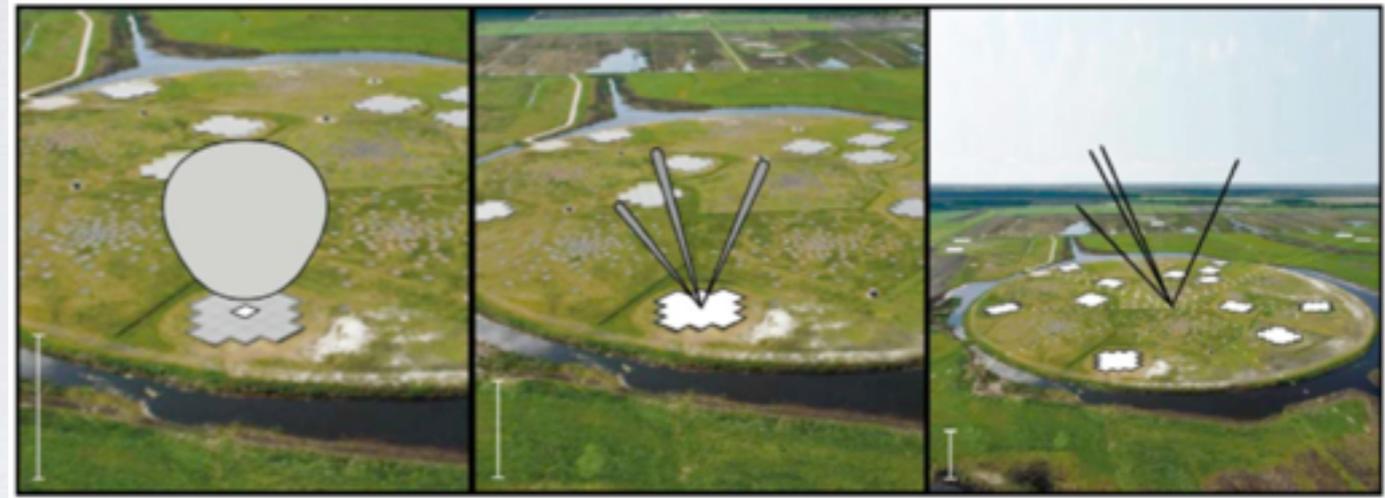
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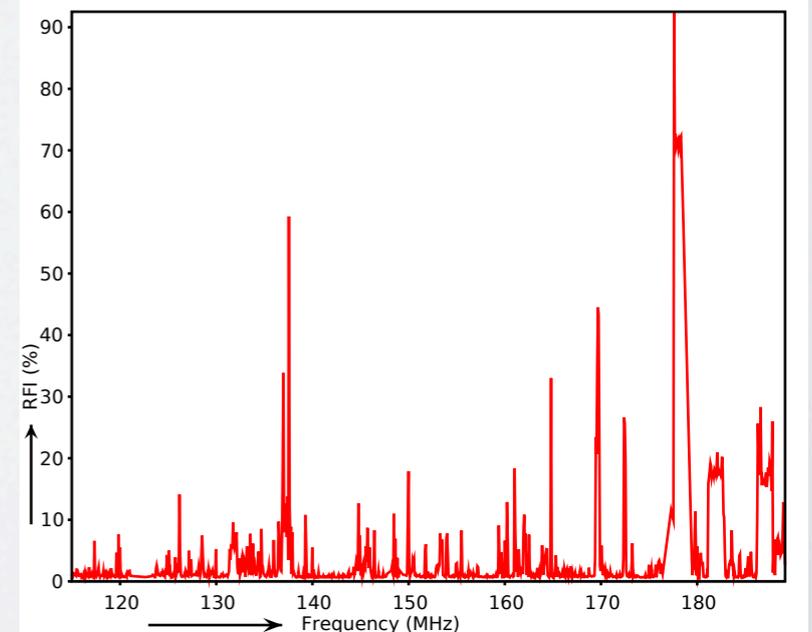
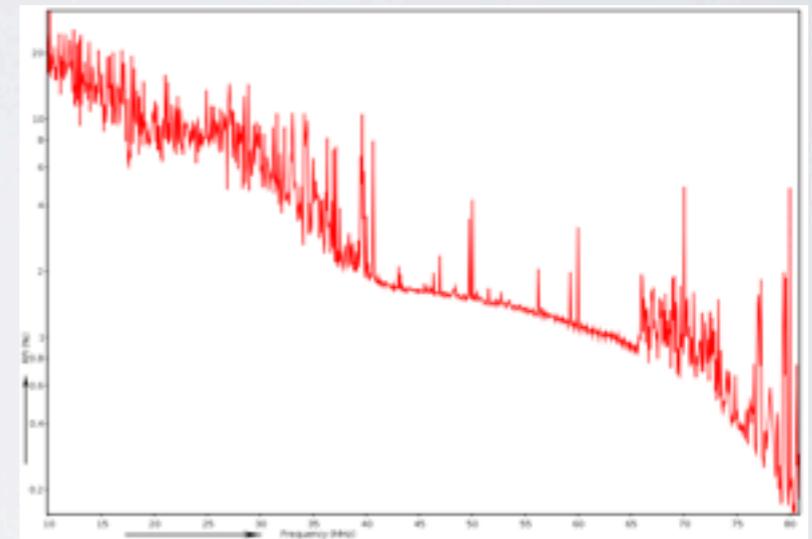
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LOW FREQUENCY AND LOFAR

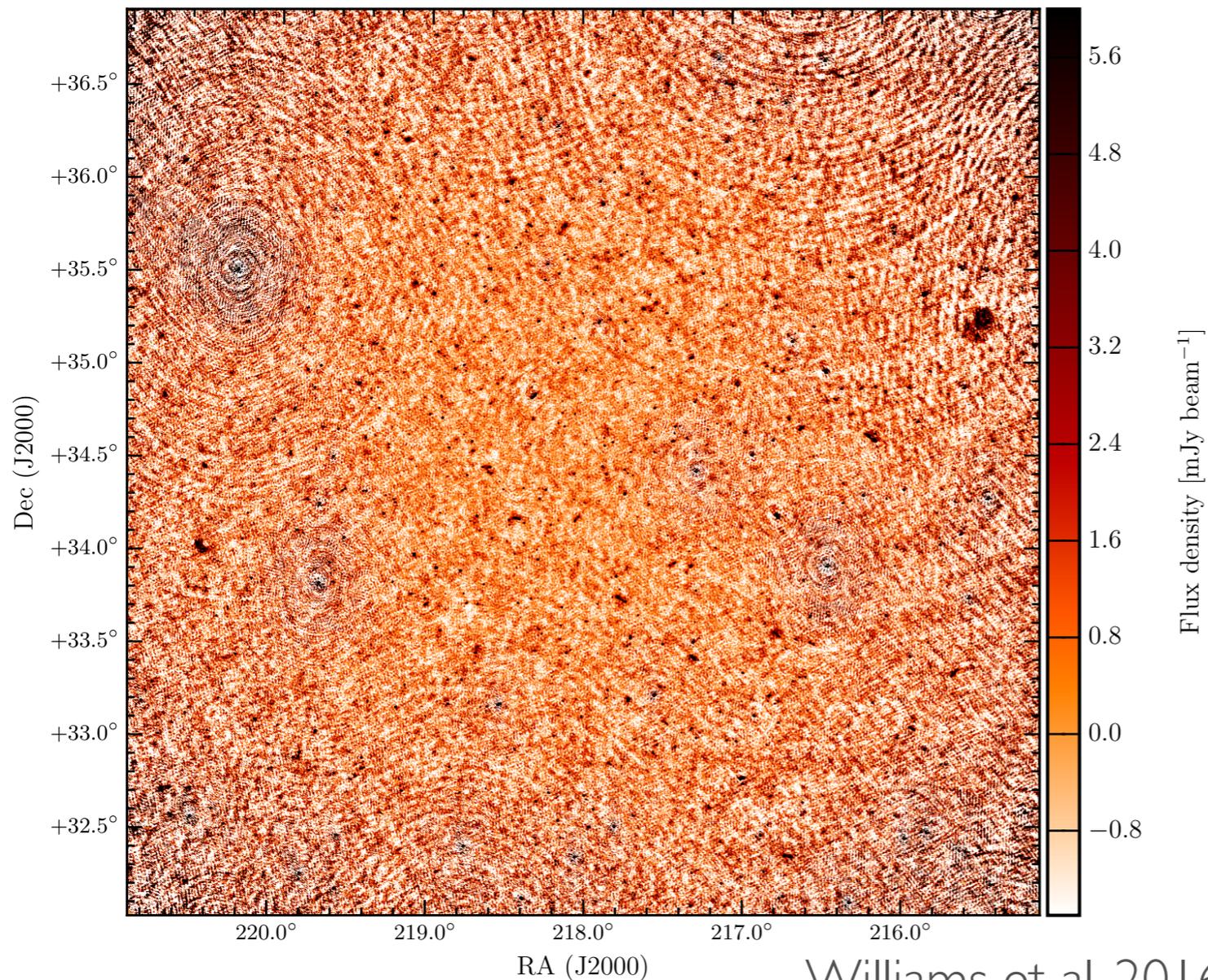


- Multiple beams: need an accurate model for each of the antenna patterns.
- RFIs very severe at low frequency. Now even more due to DAB.



LOW FREQUENCY AND LOFAR

- Wide field of view
- ionosphere and beam vary over the field of view
- Direction dependent effects

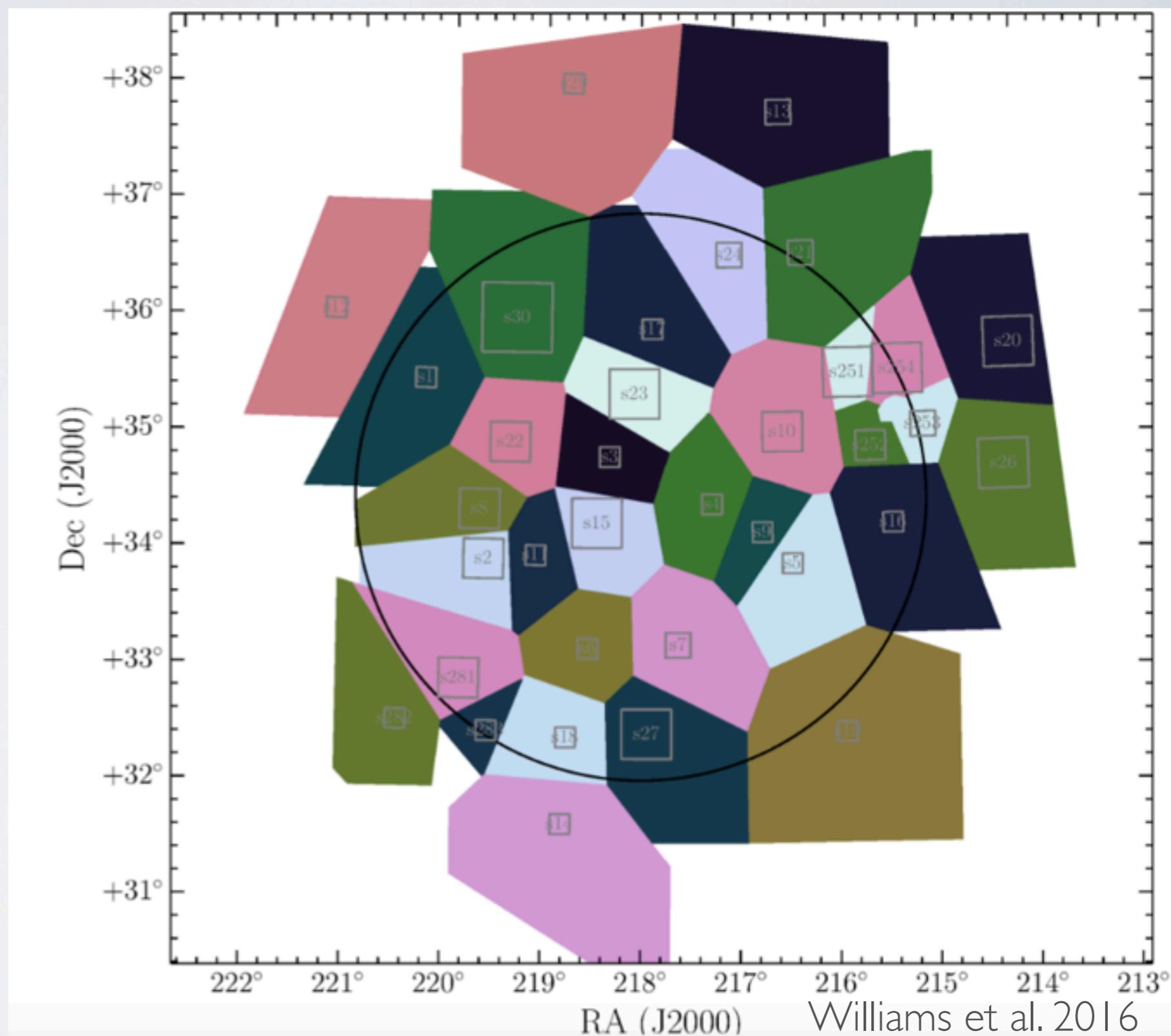


$$G_i(t) \rightarrow G_i(t, \alpha, \delta)$$

$$V_{ij}^{obs} = G_{ij} V_{ij}^{true}$$

Wide field of view

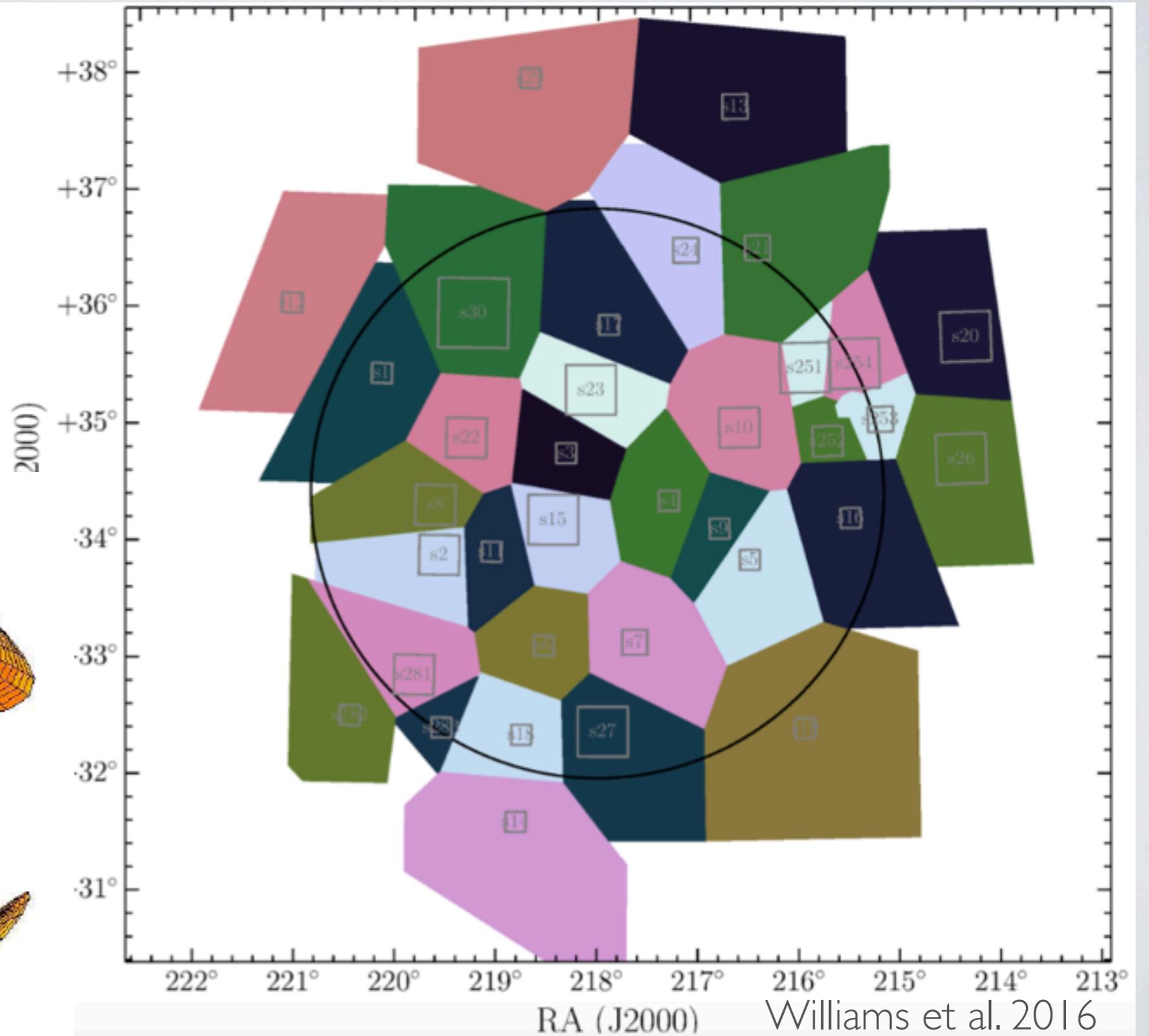
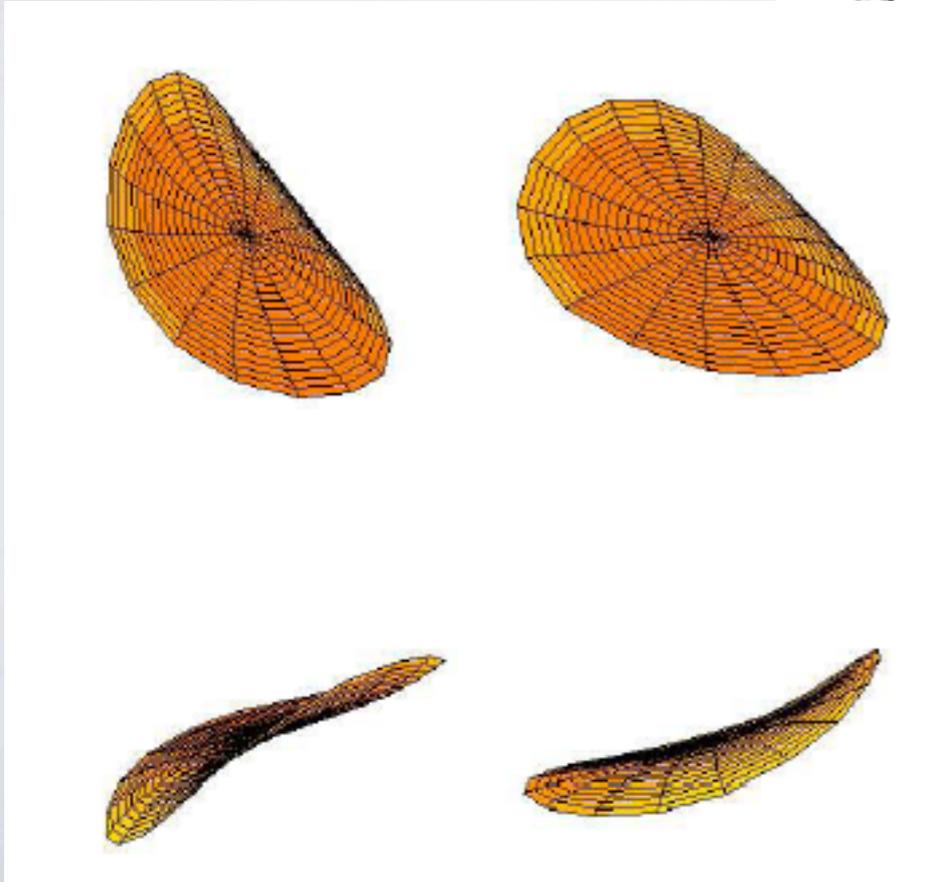
number of
directions in
which the
coherence is
preserved
=
one phase
solution is
enough

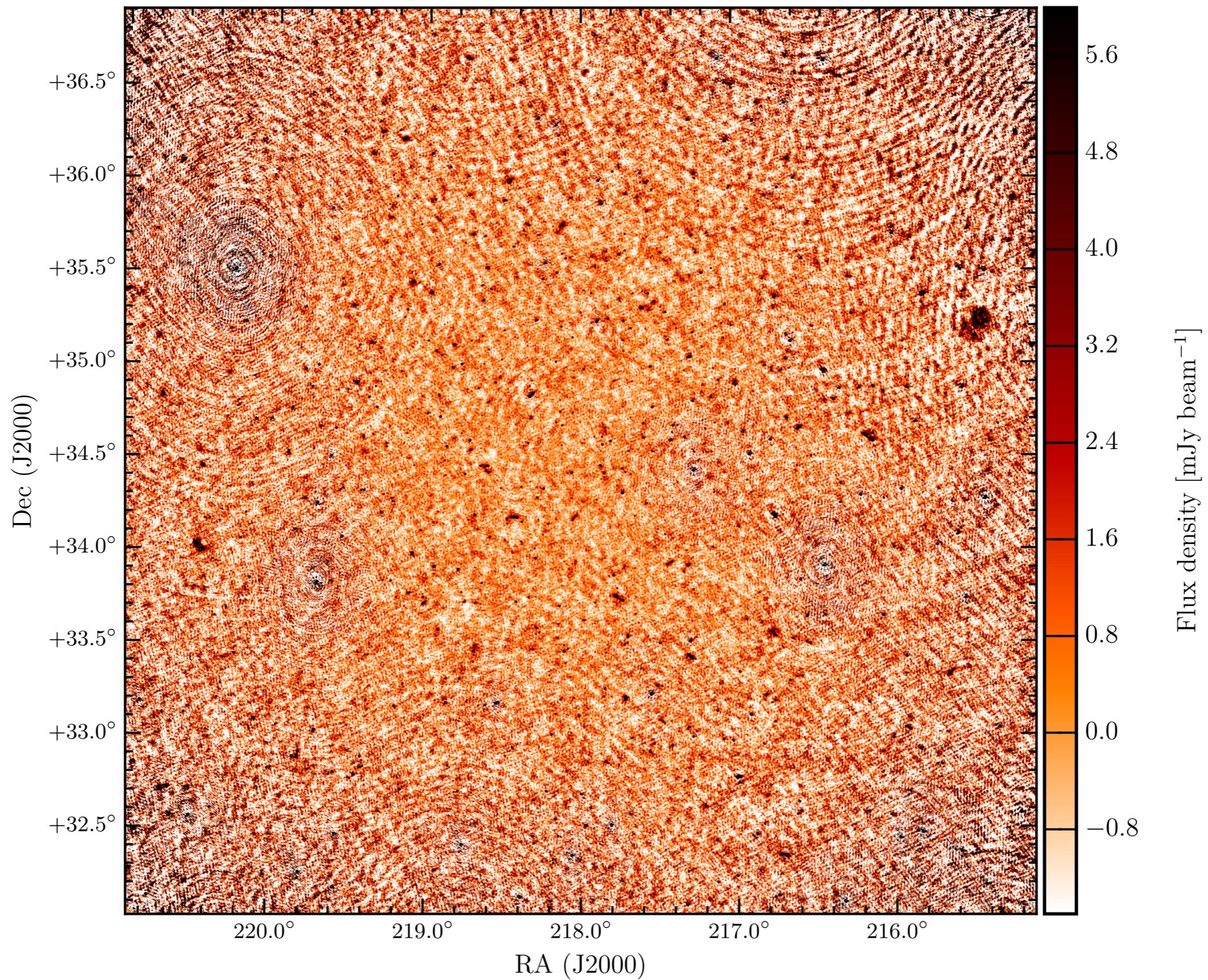


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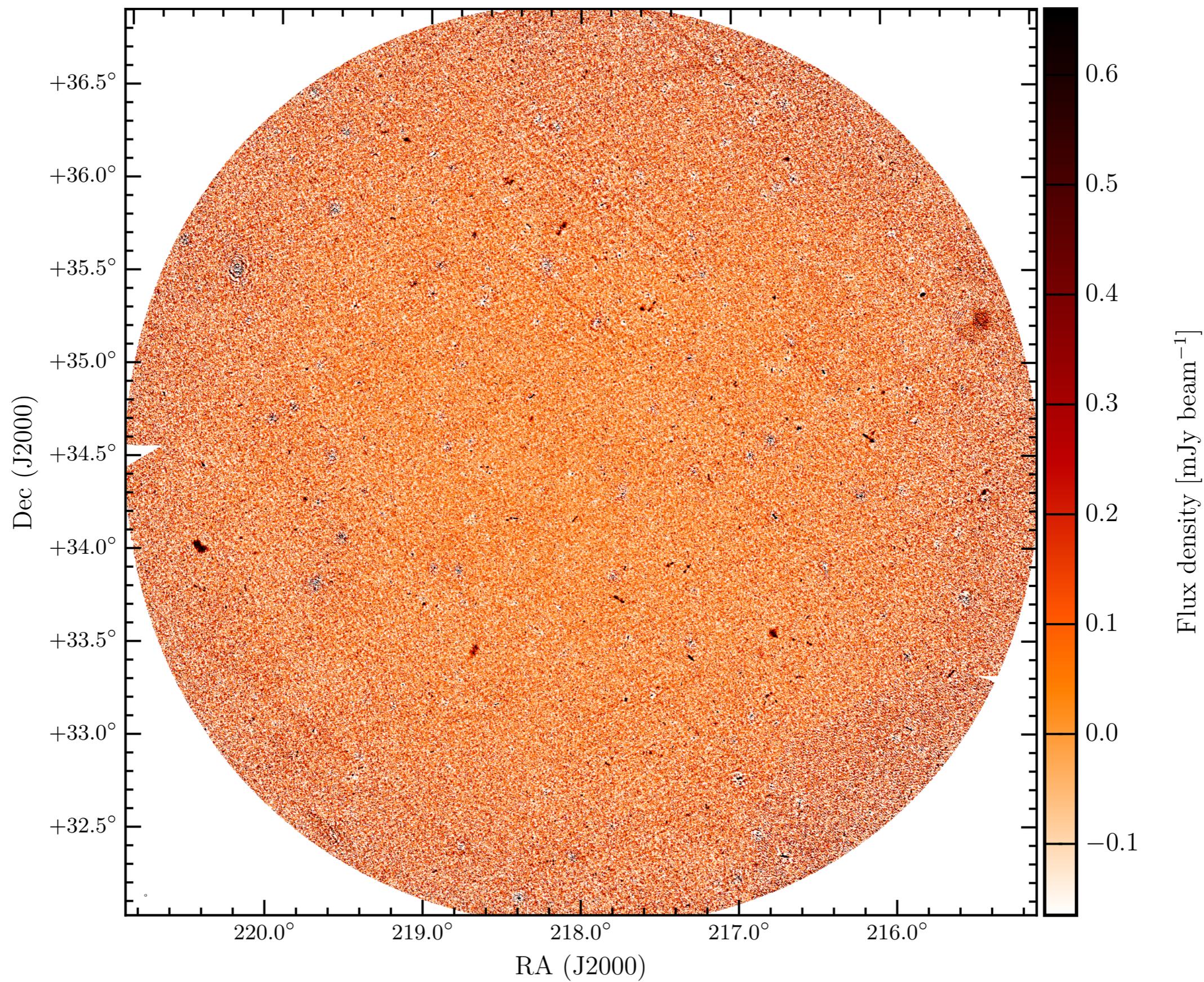
Wide field of view

number of directions in which the coherence is preserved





Williams et al. 2016



Williams et al. 2016

SUMMARY

- Interferometry needed to increase angular resolution
- The answer of the radio interferometer (Visibility) is the anti-FT of the sky brightness
- The interferometer works only if the phase coherence is preserved. E.g. ionospheric effect destroy the phase coherence.
- The calibration of the radio interferometer is meant to solve the Measurement Equation: minimise the differences between observed visibilities and model visibilities
- Direction dependent effects complicate the calibration and increase of orders of magnitude the computing time.