# TEC and scintillation modeling

Marcin Grzesiak

# Terminator

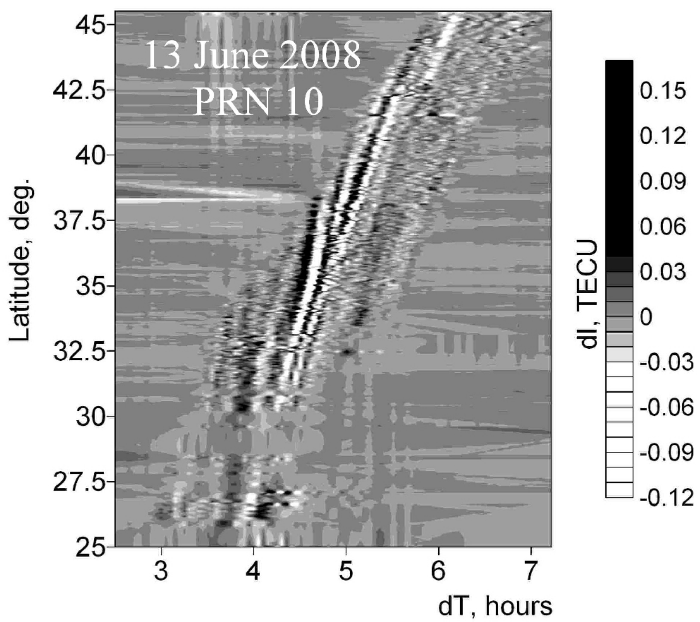
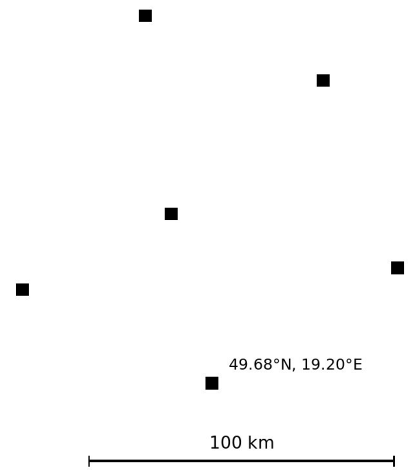
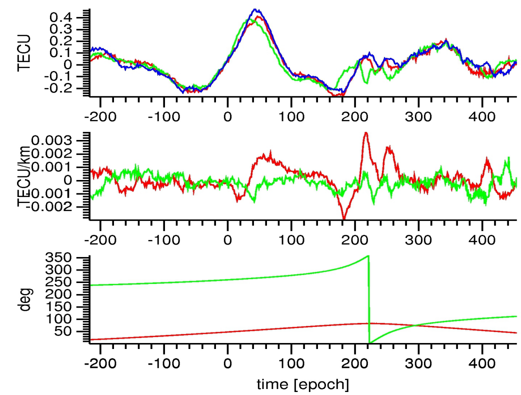



**Fig. 3.** Filtered TEC series ***dI(t)*** for PRN10 and for all GPS sites $S_i$ in dependence on the latitude $\phi$ and the morning terminator local time $dT$.

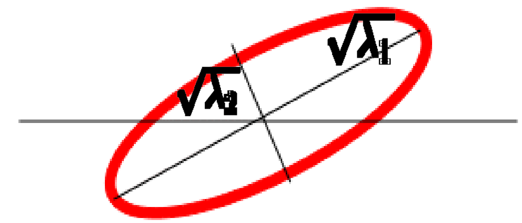
Afraimovich, E. L., Edemskiy, I. K., Voeykov, S. V., Vasyukevich, Yu. V., Zhivetiev, I. V., The first GPS-TEC imaging of space structure of MS wave packets excited by the solar terminator, *Ann. Geophys.*, 27, 1521-1525, 2009

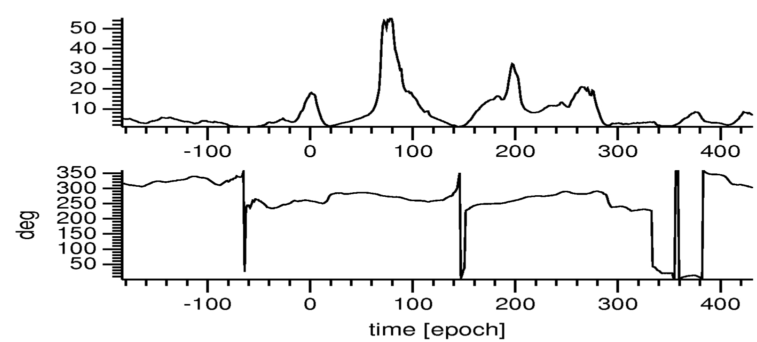



$$C(\boldsymbol{\xi}) = \int d\mathbf{k} P(\mathbf{k}) e^{i\boldsymbol{\xi}\cdot\mathbf{k}}$$

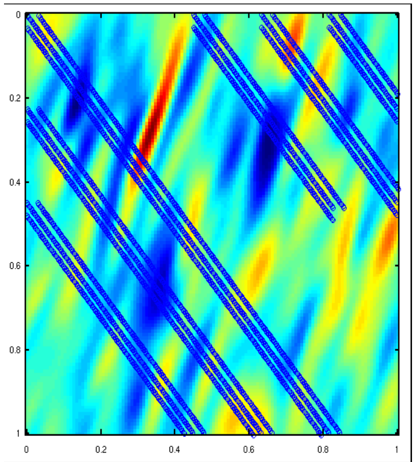
$$g_{kl} = \frac{1}{\mathbf{var}(f)} \left\langle \frac{\partial f}{\partial x_k} \frac{\partial f}{\partial x_l} \right\rangle = \frac{\int d\mathbf{k}\; k_k k_l P(\mathbf{k})}{\int d\mathbf{k} P(\mathbf{k})}$$





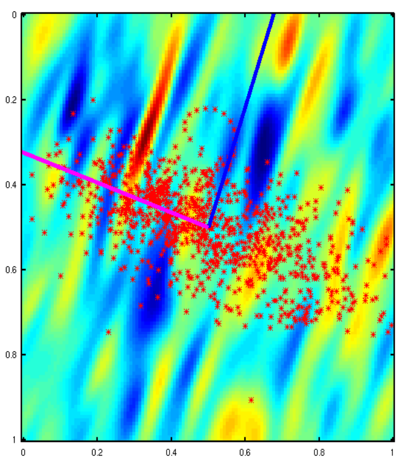

Grzesiak, M., A. Swiatek, Solar terminator-related ionosphere derived from GPS TEC measurements, Acta Geophysica, August 2012, Volume 60, Issue 4, pp 1224-1235

# Structure reconstruction (2D case)

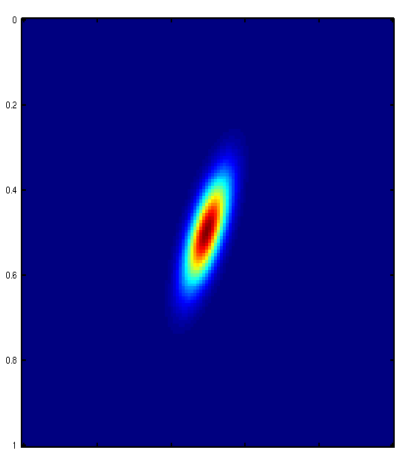


2D test field with satellites traces imposed

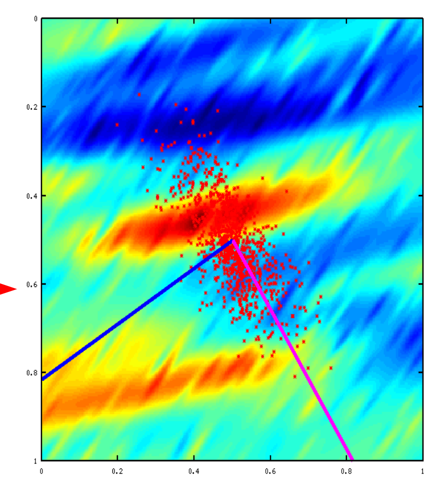
Gradients cloud and eigendirectrions of covariance matrix



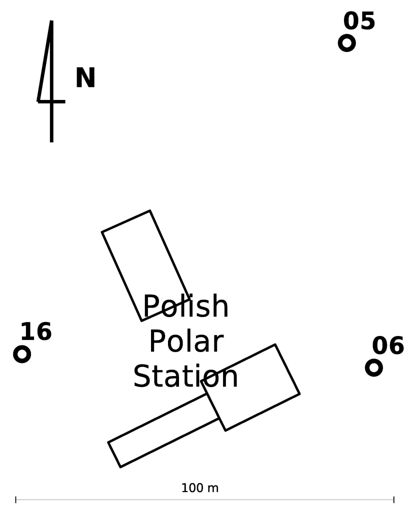
Reconstructed structure



An example of a scalar field not satisfying model assumptions
(there is no “typical” structure here)
and the result of basic analysis.








Established in 1957. Permanent since 1978.

$\phi = 77.00°$ N $\lambda = 15.55°$ E

$\Phi = 74.11°$ N $\Lambda = 109.89°$ (CGM)

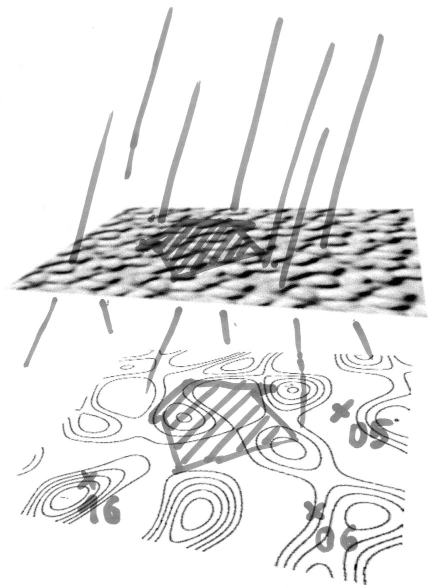
Magnetic midnight at 20:56 UT

Wernik, A. W., M. Pozoga, M. Grzesiak, A. Rokicki, M. Morawski, Monitoring ionospheric scintillation and TEC at the Polish Polar Station on Spitsbergen: instrumentation and preliminary results, Acta Geophysica, 56, 1129-1146, 2008

# Modeling evolution – dispersion analysis

$$\psi(\boldsymbol{r},t) = \int d\boldsymbol{r}' L^t(\boldsymbol{r}-\boldsymbol{r}')\psi(\boldsymbol{r}',0)$$

$$\psi(\boldsymbol{k},t) = \psi(\boldsymbol{k},0)e^{\Omega(\boldsymbol{k})t}$$




$$\mathbb{E}[\psi(\boldsymbol{r}_1,t_1)\psi(\boldsymbol{r}_2,t_2)] = \int d\boldsymbol{k}P(\boldsymbol{k})e^{\Omega(\boldsymbol{k})\tau}e^{i\boldsymbol{k}\cdot\boldsymbol{\zeta}} = C(\boldsymbol{\zeta},\tau),$$

$$\boldsymbol{\zeta} = \boldsymbol{r}_2 - \boldsymbol{r}_1,\ \tau = t_2 - t_1$$

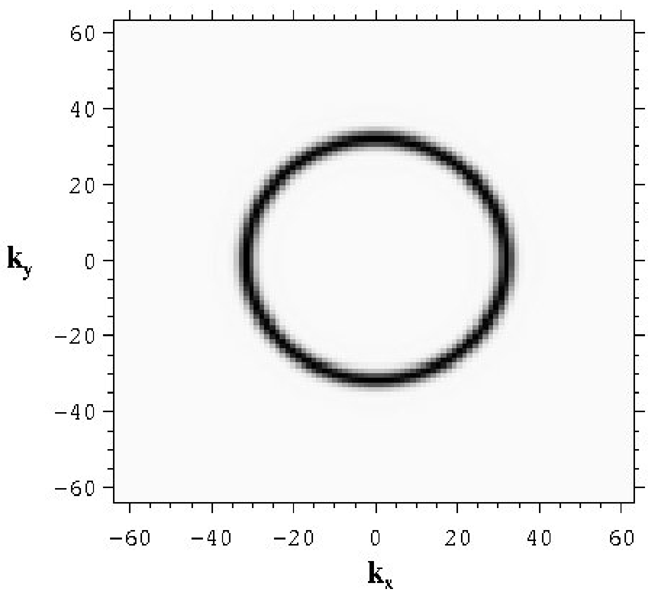
$$P(\boldsymbol{\zeta},\omega) = \int d\tau C(\boldsymbol{\zeta},\tau)e^{i\omega\tau} = \int d\tau e^{-i\omega\tau}\int d\boldsymbol{k}P(\boldsymbol{k})e^{\Omega(\boldsymbol{k})\tau}e^{i\boldsymbol{k}\cdot\boldsymbol{\zeta}} =$$

$$\int d\boldsymbol{k}P(\boldsymbol{k})e^{i\boldsymbol{k}\cdot\boldsymbol{\zeta}}\delta(\omega - \Omega(\boldsymbol{k}))$$

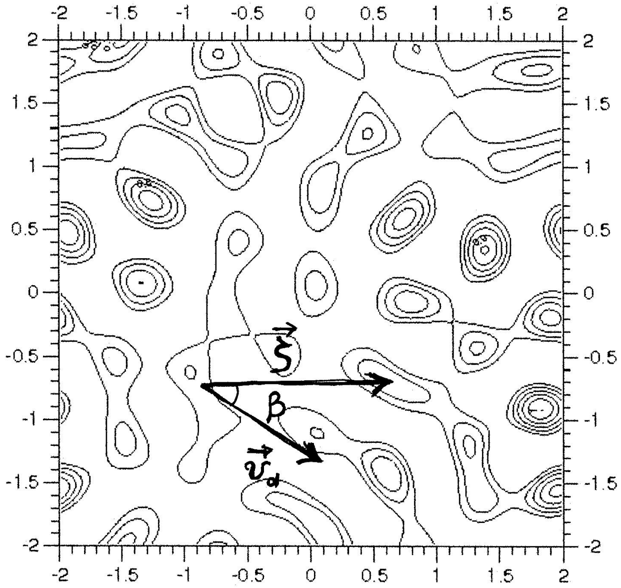
$$\frac{\partial\psi}{\partial t} - \boldsymbol{v}\cdot\nabla\psi = 0$$

$$<\boldsymbol{\zeta}> = \frac{\int d\boldsymbol{\zeta}\boldsymbol{\zeta}C(\boldsymbol{\zeta},\tau)}{\int d\boldsymbol{\zeta}C(\boldsymbol{\zeta},\tau)} \qquad \frac{\partial}{\partial\tau} <\boldsymbol{\zeta}> = \nabla_{\boldsymbol{k}}\Omega(\boldsymbol{k})|_{\boldsymbol{k}=0}$$

# Simulations - example




$$P(\boldsymbol{k}) = \delta(k - k_0)$$

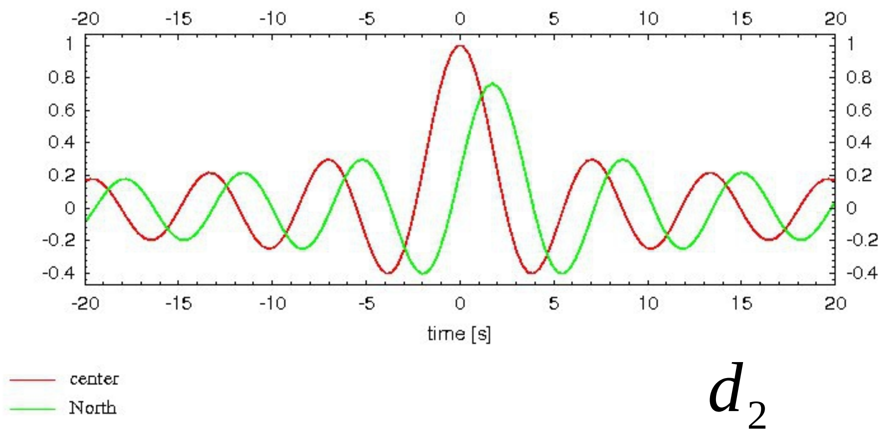


$$C(\boldsymbol{\zeta} + \boldsymbol{v}_d\tau) = \int dkk \int d\alpha \delta(k - k_0) \exp(ik(\zeta\cos(\alpha+\beta) - v_d\tau\cos\alpha)) =$$

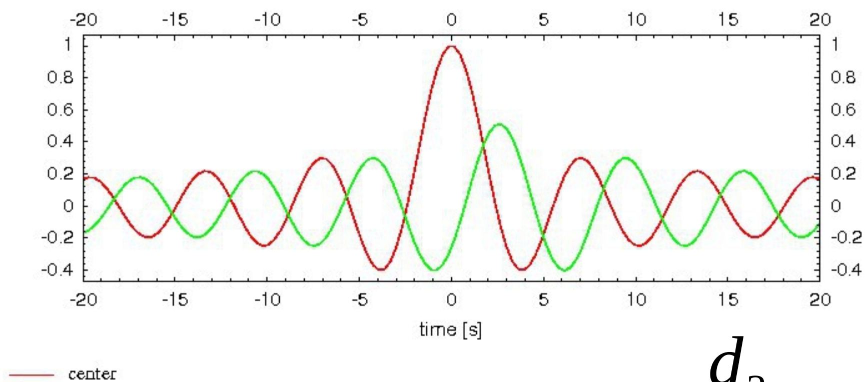
$$k_0 \int d\alpha \exp(ik_0(\zeta\cos(\alpha+\beta) - v_d\tau\cos\alpha)) = 2\pi k_0 J_0\left(k_0\sqrt{\zeta^2 + v_d^2\tau^2 - 2\zeta v_d\tau\cos\beta}\right)$$

# Dependence on separation

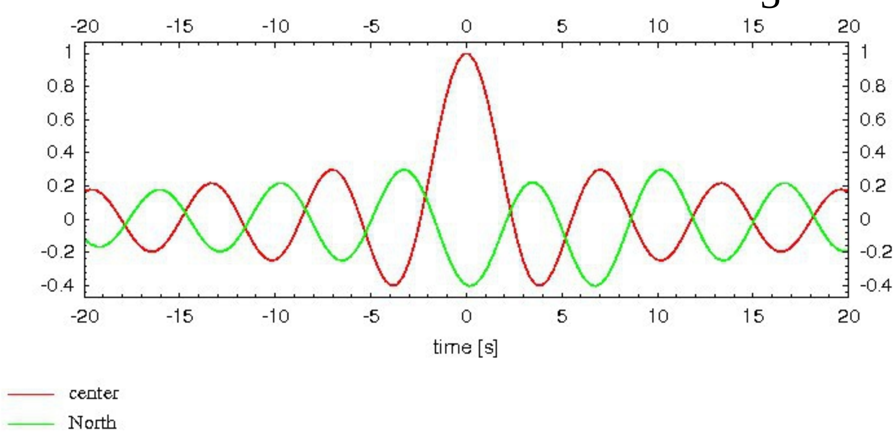
$d_1$




$d_2$




$d_3$



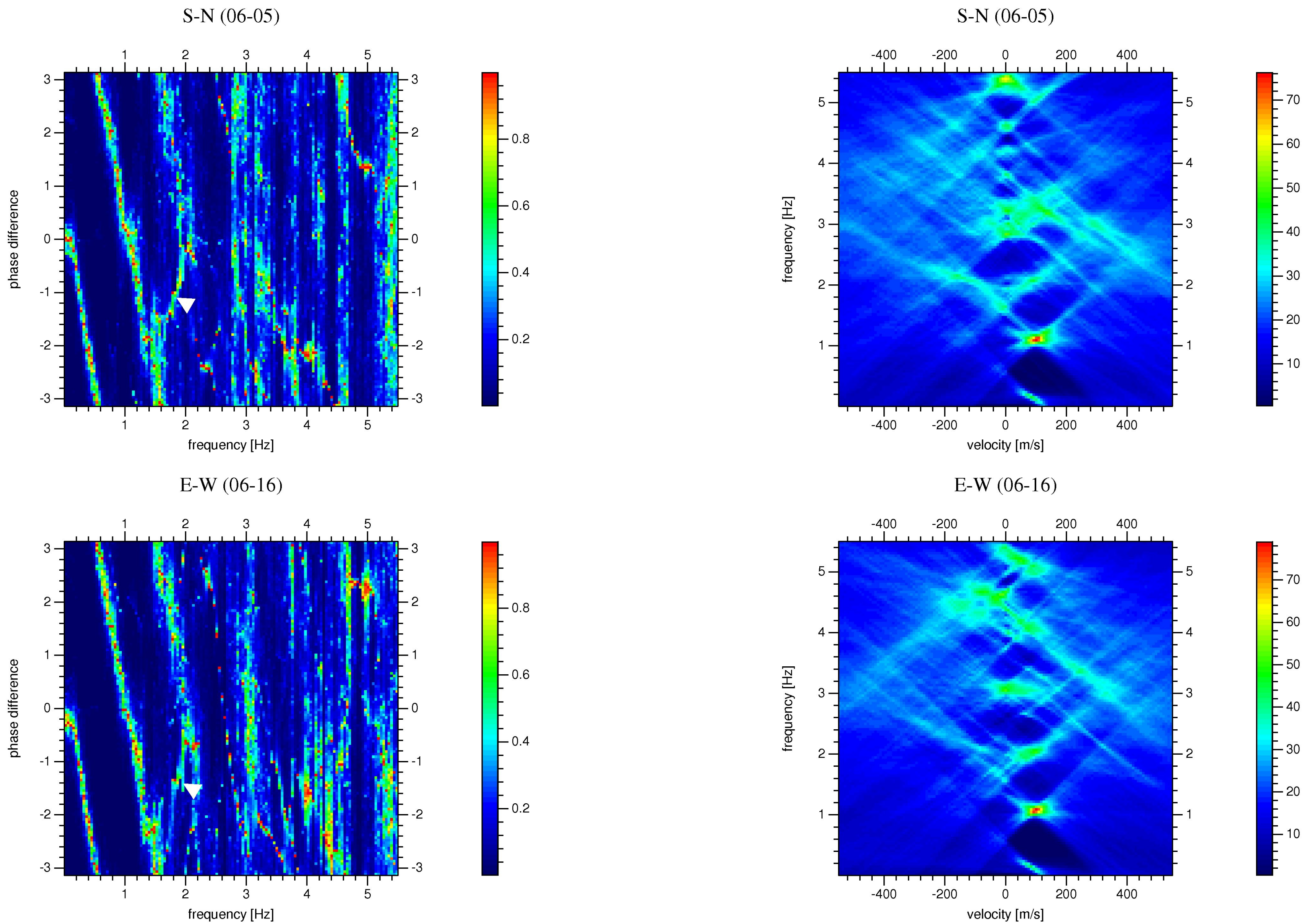

$$\frac{d}{\Delta\tilde{\tau}\cos\beta}$$

$$P(\vec{\xi},\omega) = \int d\tilde{\tau}\; C(\vec{\xi}+\vec{v}_d\tilde{\tau})\, e^{-i\omega\tilde{\tau}}$$

$$v_d = 1,\; k_0 = 1 \longrightarrow \frac{\partial\Delta\phi}{\partial\omega} = \frac{\omega\sin\beta}{\sqrt{1-\omega^2}} + \cos\beta$$

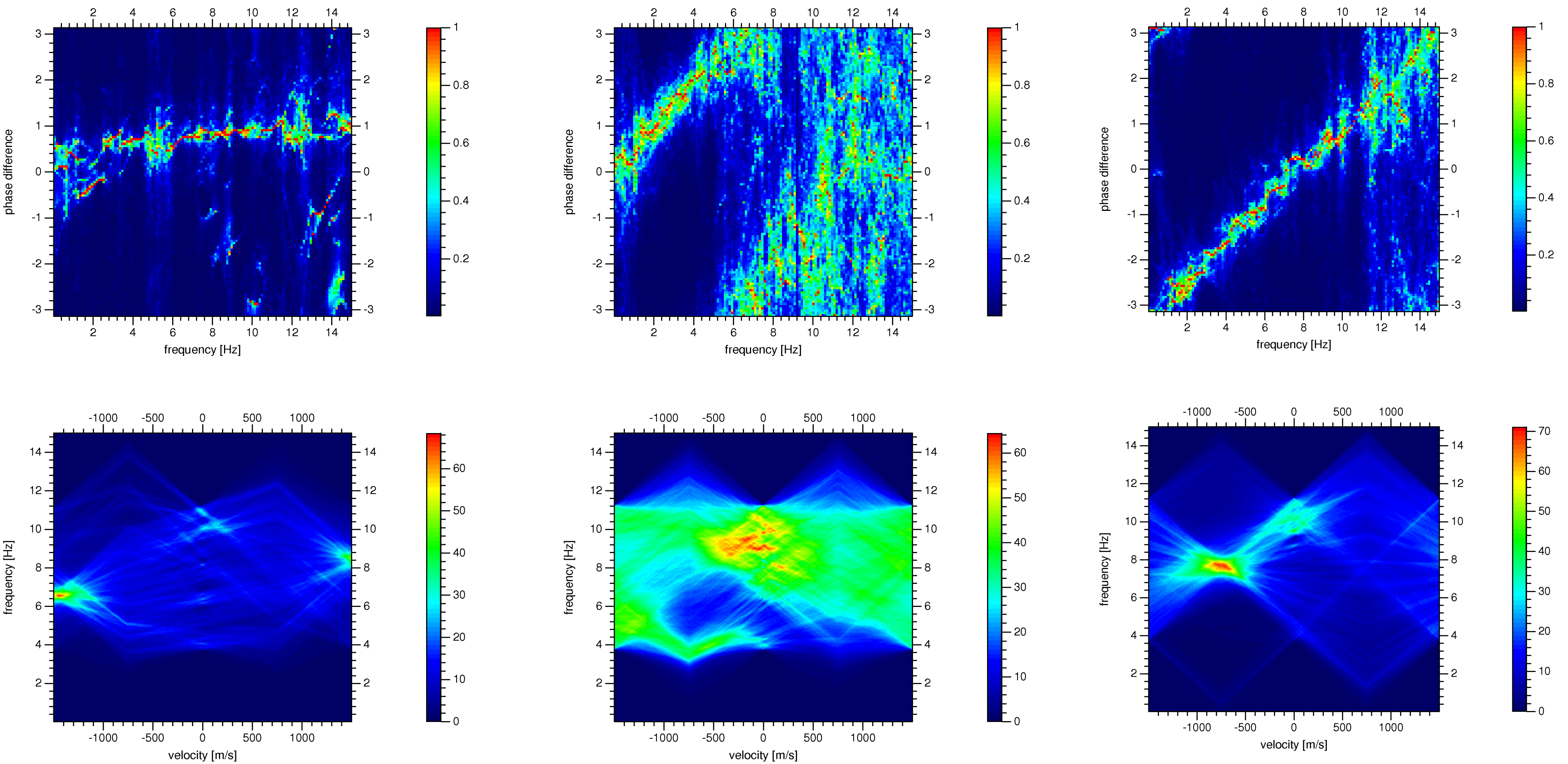
$$d_1 < d_2 < d_3$$

# Drift dispersion



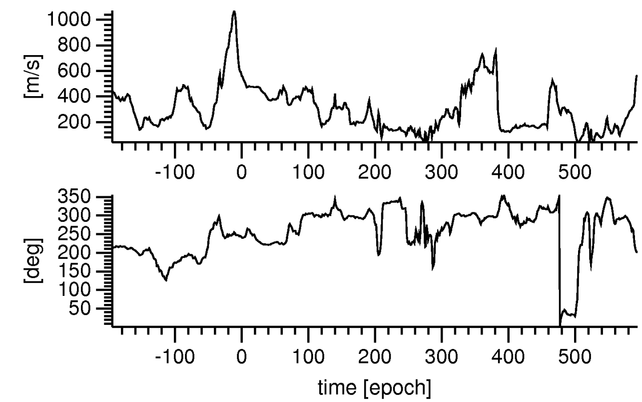

Grzesiak, M., A. W. Wernik, Dispersion analysis of spaced antenna measurements, Annales Geophysicae, 27, 2843-2849, 2009

# 5-6.04.2010 magnetic storm



# Spatio-temporal analysis

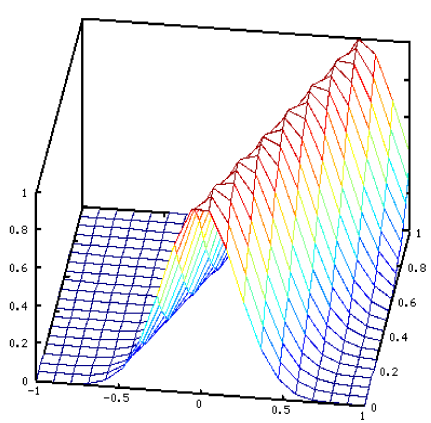
Taylor hypothesis imply that spatio-temporal correlation function “drifts” in delay coordinates, which means that there is a “singular” (with zero eigenvalue of the hessian) direction.



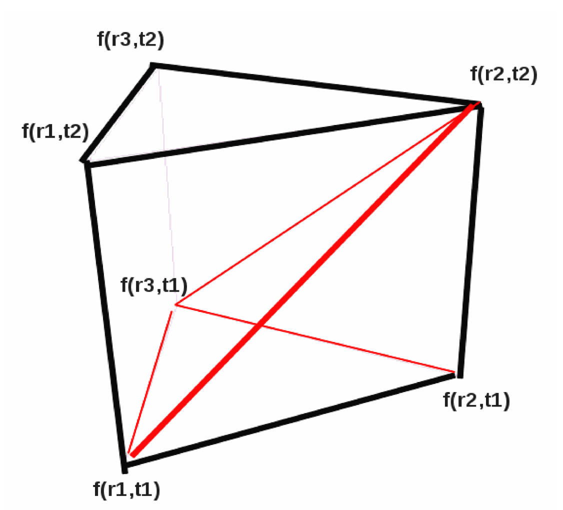

$$\mathbf{r} \to \mathbf{r} - \mathbf{v}t,\ C(\boldsymbol{\zeta}) \to C(\boldsymbol{\zeta} - \mathbf{v}\tau),\ \tau = t_1 - t_2$$

$$\frac{\partial C}{\partial \tau} + \mathbf{v} \cdot \nabla C = 0$$

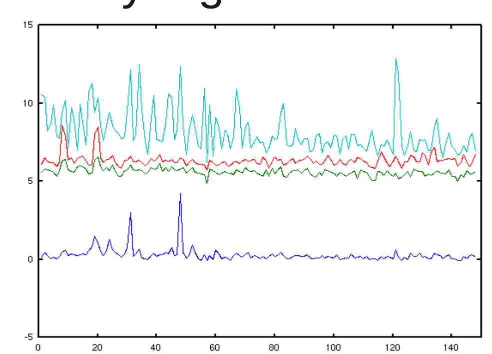
Drifting of spatio-temporal autocorrelation function (1D + time)



Space-time triangulation (2D + time)




Validation of Taylor hypothesis for Cluster data – one of the Eigenvalues of spatio-temporal covariance matrix is very small (figure in y-log scale.



# Experimental TEC/scintillation modeling (climatology)

Scintillation Characteristic (e.g. S4) /TEC = F(MLAT, MLT, Kp, ….)

Where F is a suitable function.

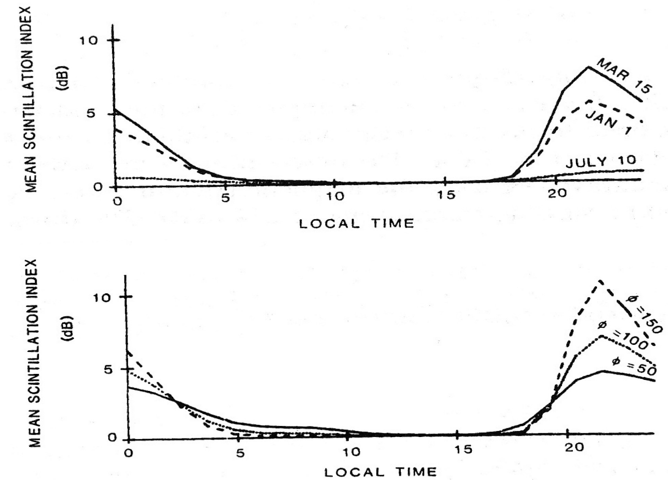
Its choice depends on experiment – a set of observations taken for "many" n-tuples (MLATs, MLTs, Kps,...)

An example: Low-latitude model (Aarons et al., 1985

$$SI(dB) = 2^{(q+r)}$$

q = FA + FB + (−1.5 FA+0.8 FB)·cos[(π/12)(H − 0.2 − 0.25Kp
r = FC{cos[(π/6)(H + 3.3)] − 0.4 cos[(π/4)(H + 1.5)]}
FA = (− 2.7 − 0.3 FD)(S/100)
FB = 0.2 + FD + (0.1 − 0.1 FD)Kp
FC = (1.6 + 0.7 FD)(S/100) + 0.1Kp
FD = cos (2π/365)(D + 1.3) − 0.6 cos (4π/365)(D − 4)

and D is the day number, H is the local time in hours,
S − solar flux at 10 cm, Kp− planetary magnetic index.
All angles are in radians.




Upper plot: Scintillation index for 3 days of the year, with solar flux of 100 and Kp=2. Lower plot: Mean scintillation index for February15 with 10-cm solar flux of 50, 100, and 150 units; Kp=2 (after Aarons, 1985).

Aarons, J., E. MacKenzie, and K. Bhavnani, High-latitude analytical formulas for scintillation levels, Radio Sci., 15, 115-127, 1980.

TEC models based on IRI

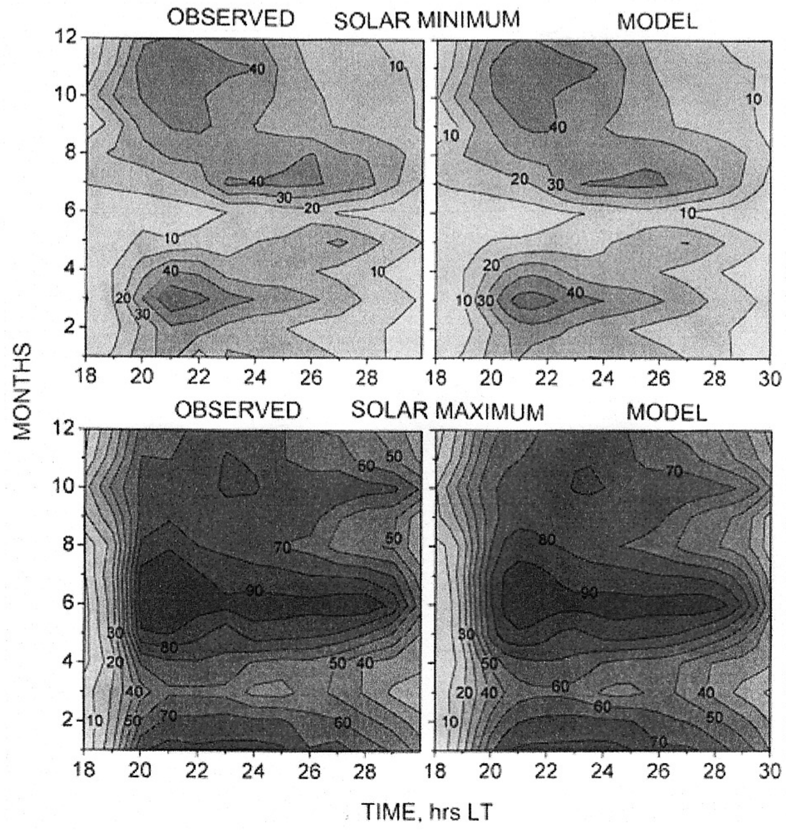
# The choice: **F** interpolating polynomial

An example: Indian model (Iyer et al. 2006)

$$SO(t,d,F,\theta)=\sum\sum\sum\sum a_{i,j,k,l} N_{i,4}(t) N_{j,2}(d) N_{k,2}(F) N_{l,2}(\theta)$$

The scintillation occurrence SO dependence on local time t, day of the year d, solar flux F, and latitude θ is expressed as a simultaneous product of univariate normalized cubic-B splines

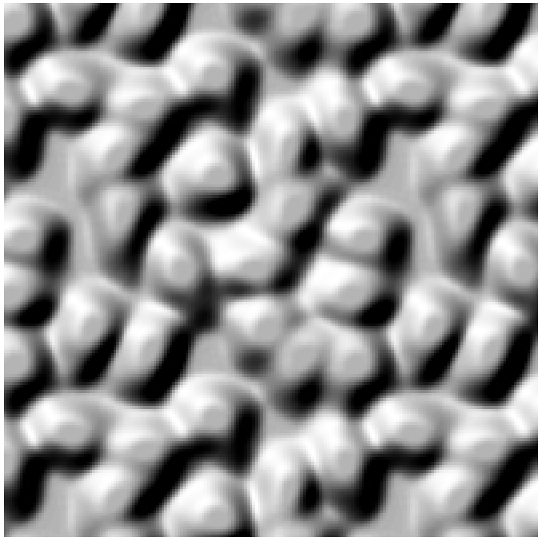
Scintillation occurrence over Trivandrum for solar minimum (upper panels) and maximum (lower panels) (after Iyer et al., 2006).





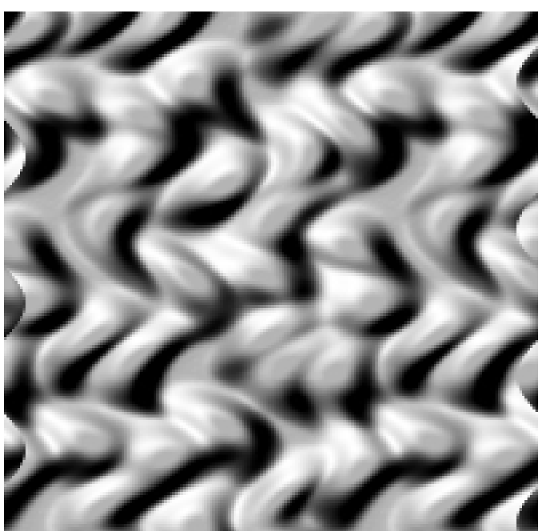
Iyer, K. N., J. R. Souza, B. M. Pathan, M. A. Abdu, M. N., Jivani, and H. P; Joshi, A model of equatorial and low latitude VHF scintillation in India, Indian J. Radio & Space Phys., 35, 98-104, 2005.

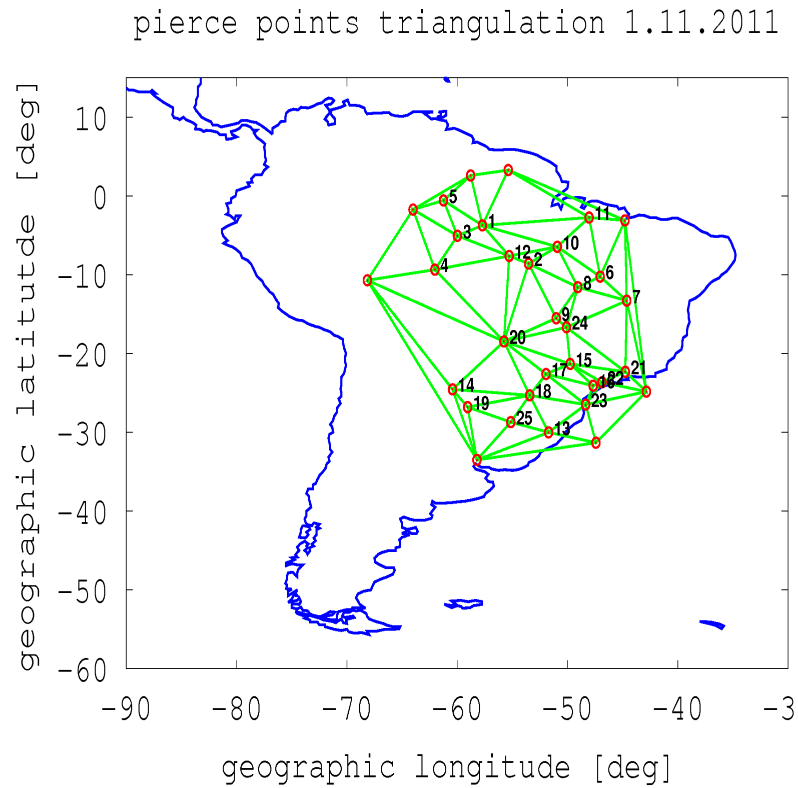
# Deterministic approach



$$K_t$$

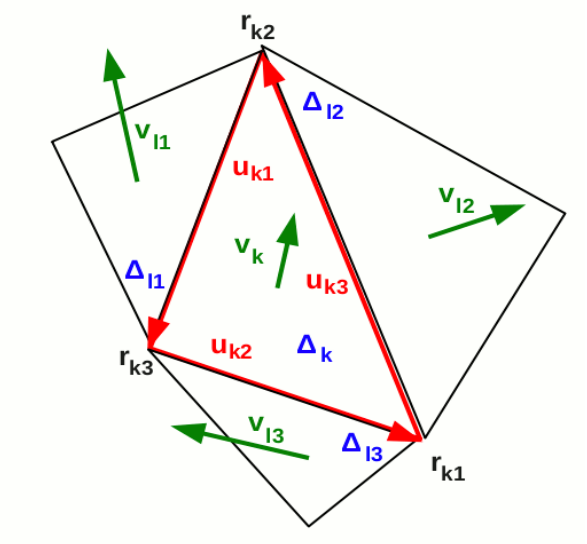
$$\downarrow$$






$$\frac{\partial}{\partial t}\int_{\Delta_k} dVf = -\int_{\partial\Delta_k} d\mathbf{s}\cdot(f\mathbf{v}_k)$$

# Finite volume formulation, identification of parameters (TEC)




$$\mathbf{Mv} = \mathbf{df}$$

$$\mathbf{M} = \mathbf{U\Sigma V}^*.$$

$$\mathbf{v} = \mathbf{V\Sigma}^+\mathbf{U}^*\mathbf{df}$$

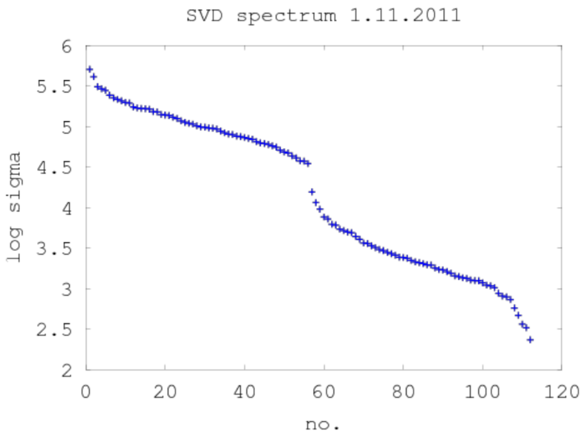



Figure 7: Spectrum of the SVD singular values (from 1 to 112) plotted in decreasing magnitude.

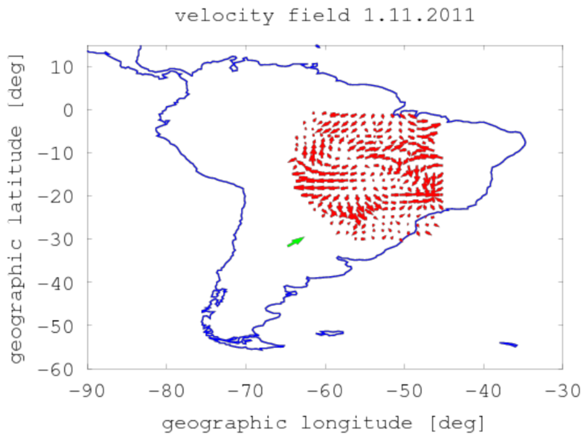
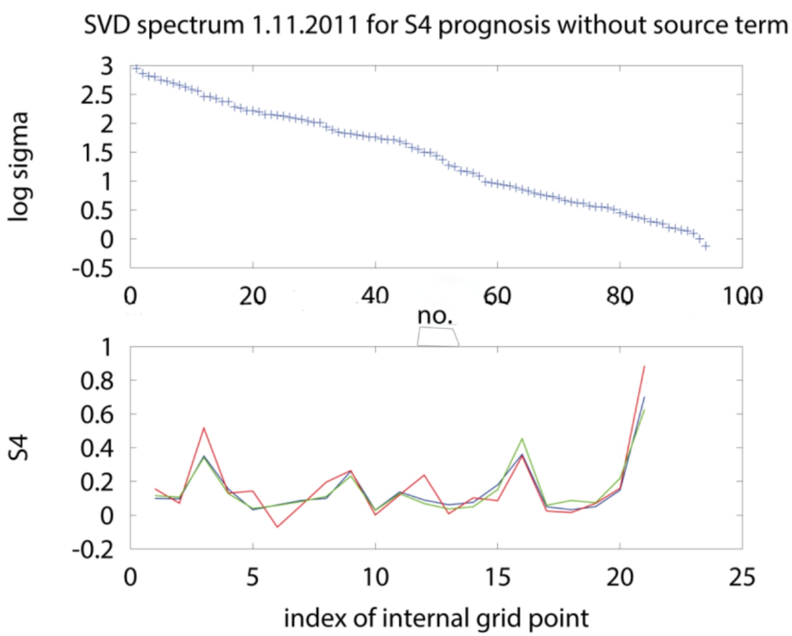


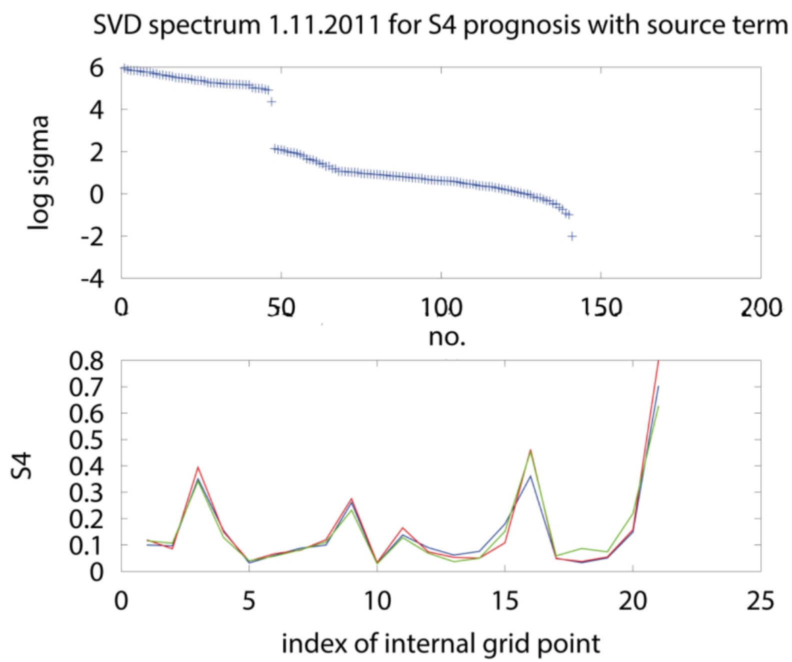

Figure 8: Reconstructed velocity field; the green arrow shows velocity vector with magnitude of 100 m/s

# Scintillation parameters prediction

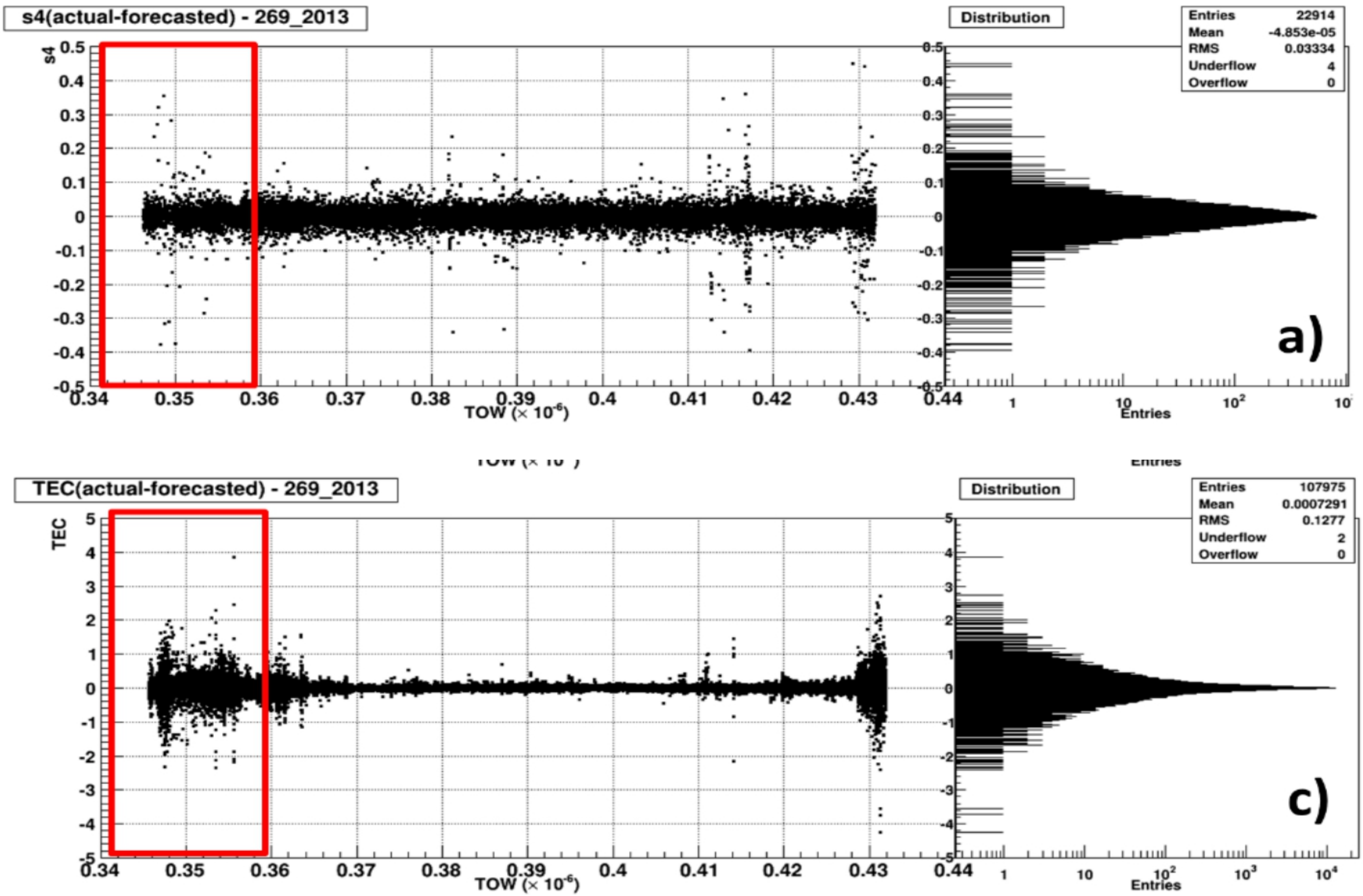
$$\frac{\partial}{\partial t}\int_{\Delta_k} dVf = -\int_{\partial\Delta_k} d\mathbf{s}\cdot(f\mathbf{v}_k)$$

$$\frac{\partial}{\partial t}\int_{\Delta_k} dVf = -\int_{\partial\Delta_k} d\mathbf{s}\cdot(f\mathbf{v}_k) + \int_{\Delta_k} dV\pi_k,$$



# Tests



s4(actual-forecasted) - 269_2013
s4
TOW (× 10⁻⁶)
Distribution
Entries 22914
Mean -4.853e-05
RMS 0.03334
Underflow 4
Overflow 0
Entries
a)
TEC(actual-forecasted) - 269_2013
TEC
TOW (× 10⁻⁶)
Distribution
Entries 107975
Mean 0.0007291
RMS 0.1277
Underflow 2
Overflow 0
Entries
c)