Implementation of trigger for Ultra High Energy ($>10^{21}$ eV) Cosmic Rays Detection

LOFAR (DCLA) Meeting

ASTRON, Dwingeloo

26th June 07

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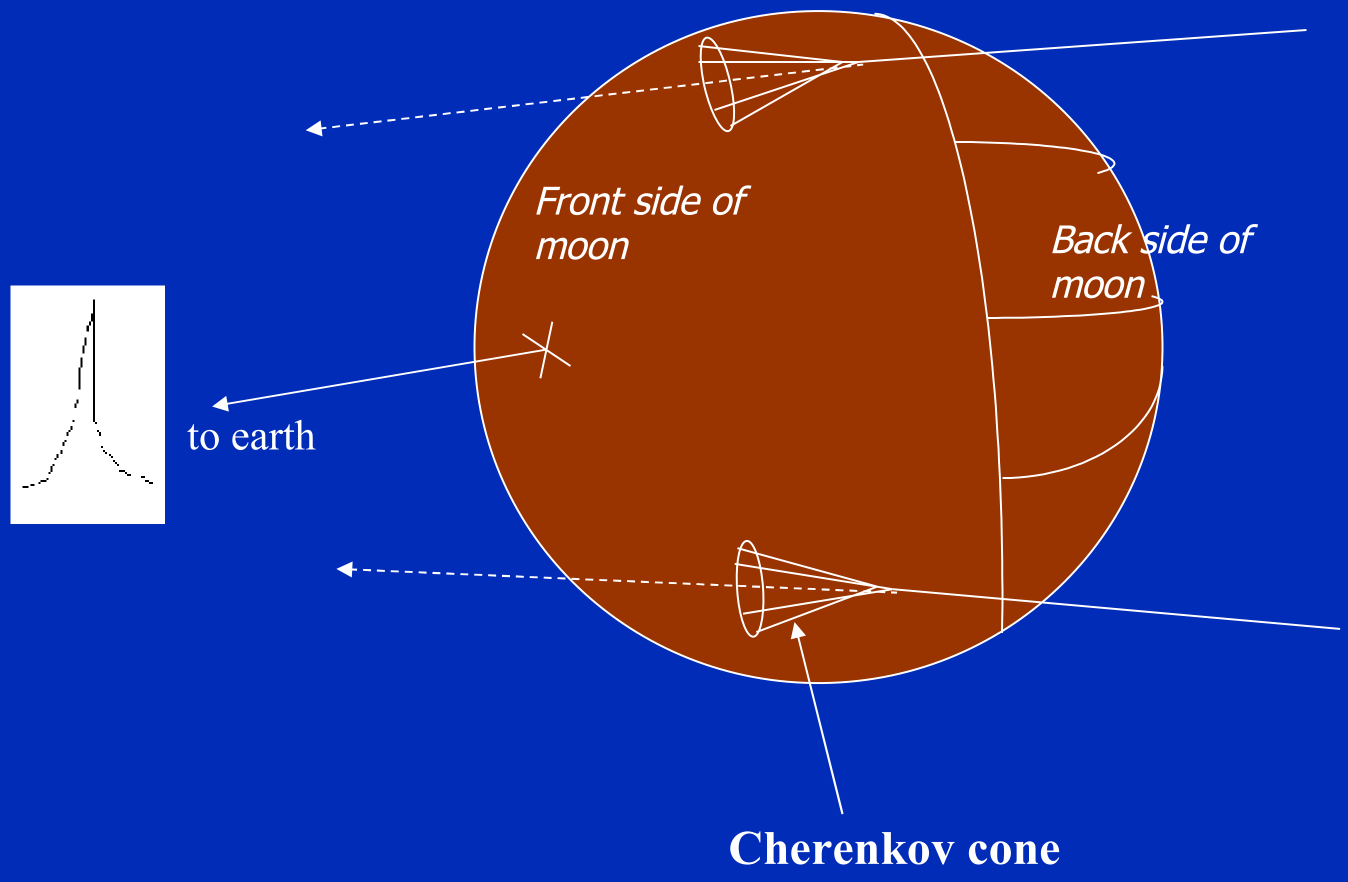
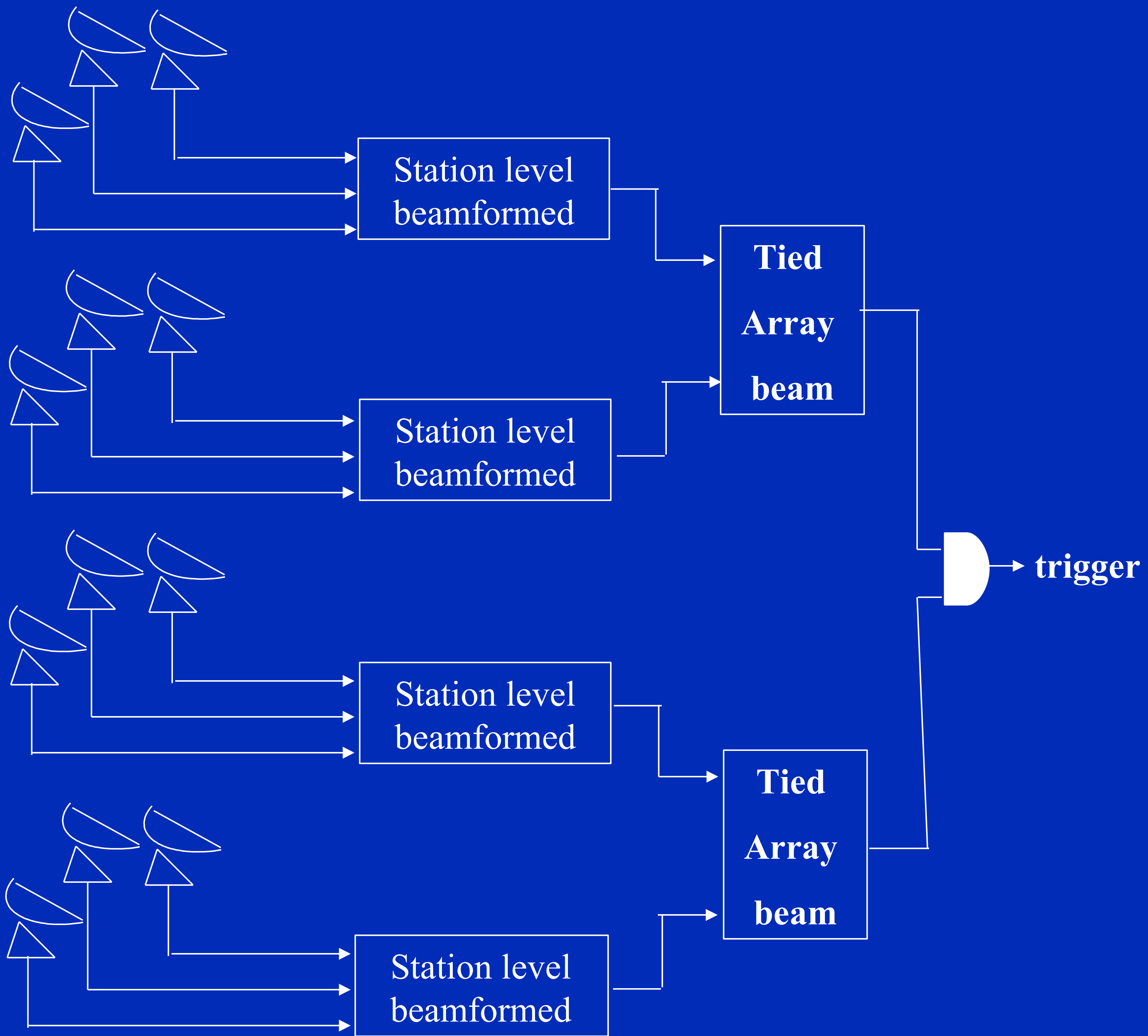
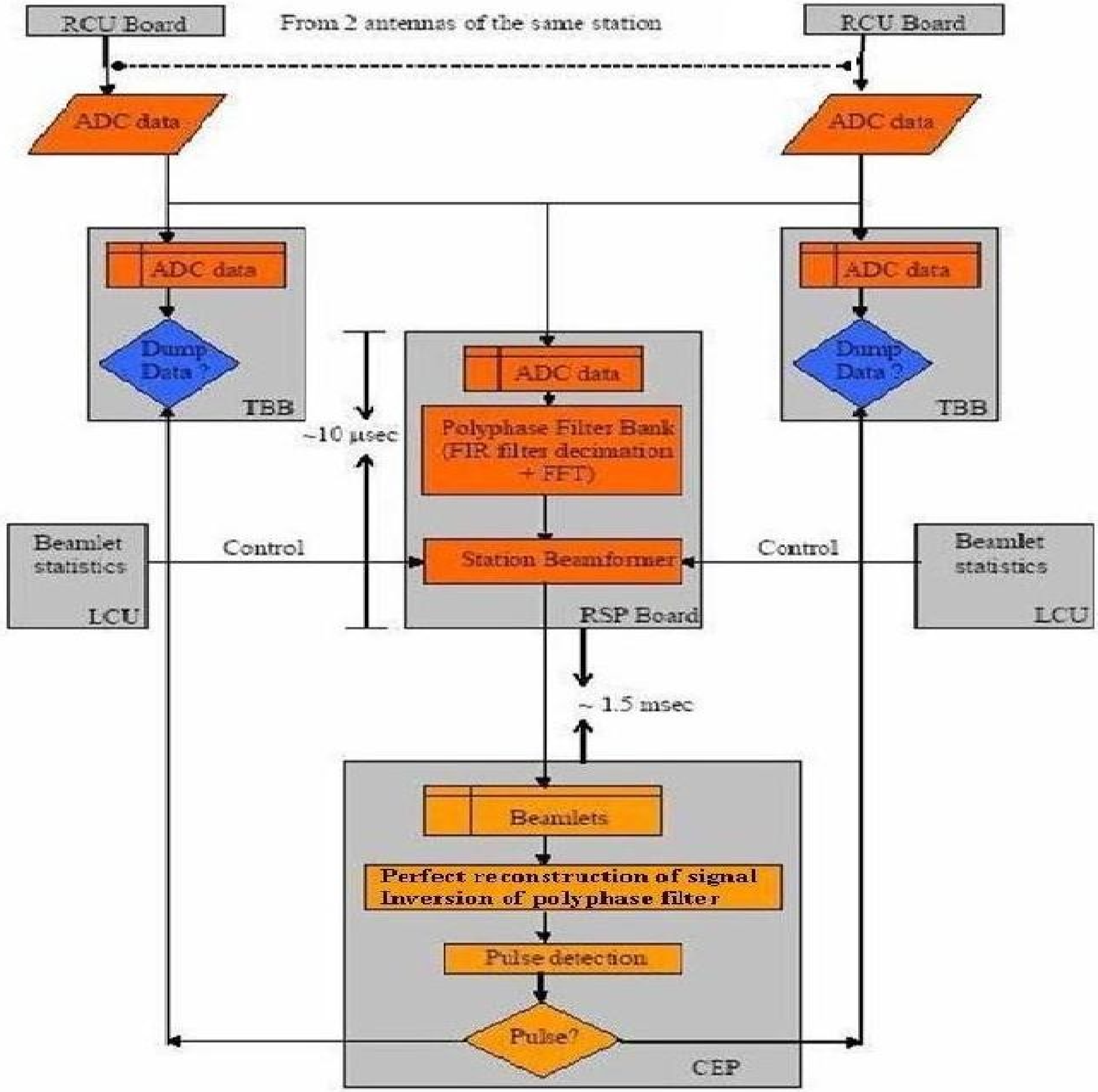
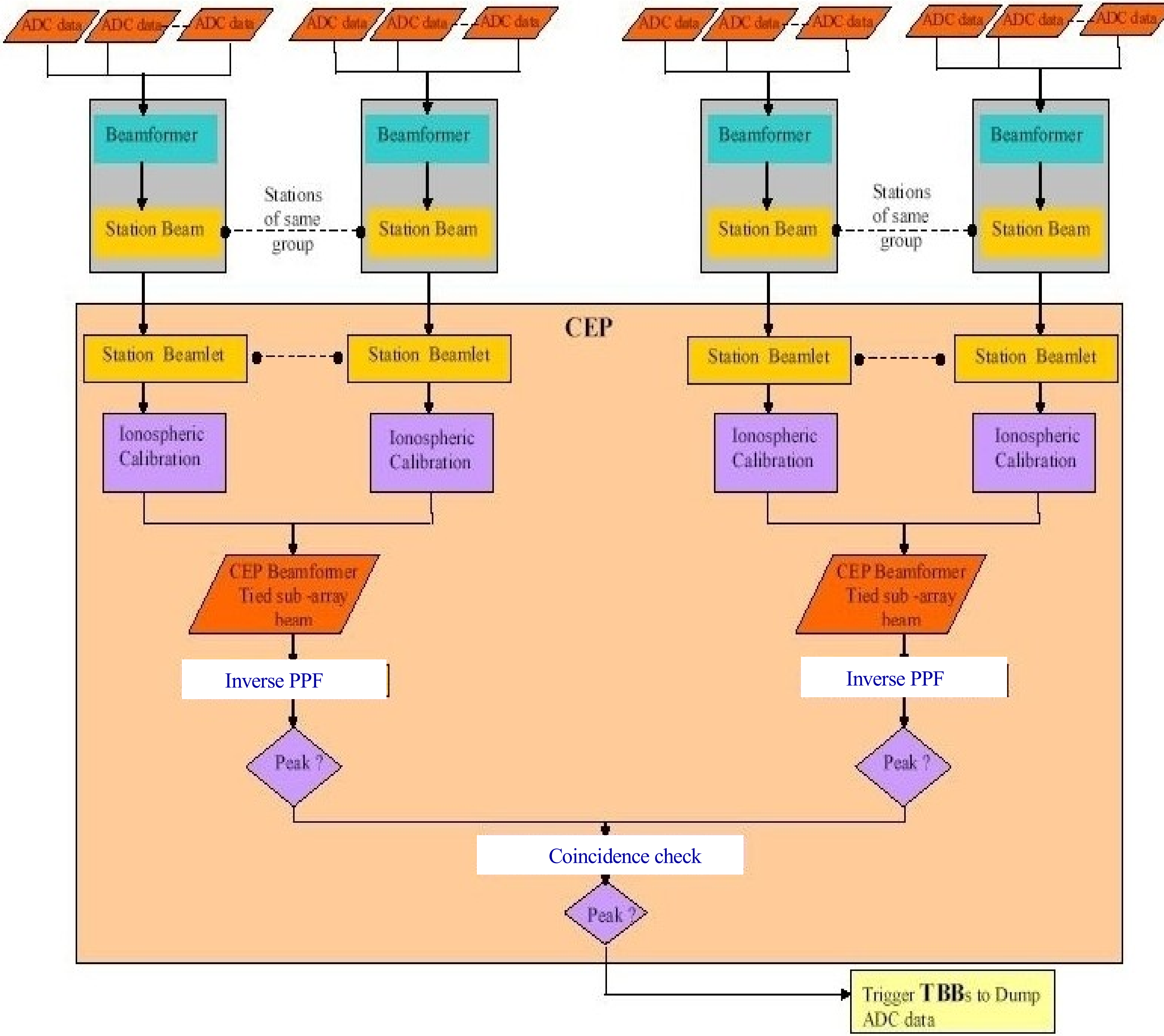
Nijmegen


Front side of moon
Back side of moon
to earth
Cherenkov cone


Station level beamformed
Station level beamformed
Station level beamformed
Station level beamformed
Tied Array beam
Tied Array beam
trigger


RCU Board
From 2 antennas of the same station
RCU Board
ADC data
ADC data
ADC data
Dump Data ?
TBB
ADC data
Dump Data ?
TBB
~10 µsec
ADC data
Polyphase Filter Bank (FIR filter decimation + FFT)
Beamlet statistics
LCU
Control
Station Beamformer
Control
Beamlet statistics
LCU
RSP Board
~ 1.5 msec
Beamlets
Perfect reconstruction of signal
Inversion of polyphase filter
Pulse detection
Pulse?
CEP


ADC data
ADC data
ADC data
Beamformer
Station Beam
Stations of same group
CEP
Station Beamlet
Ionospheric Calibration
CEP Beamformer Tied sub -array beam
Inverse PPF
Peak ?
Coincidence check
Peak ?
Trigger TBBs to Dump ADC data

DISCRETE FOURIER TRANSFORM

$$X[N] = \begin{bmatrix} 1 & 1 & 1 & \cdots & \cdots & 1 \\ 1 & W_N^{1} & W_N^{2} & \cdots & \cdots & W_N^{N-1} \\ 1 & W_N^{2} & W_N^{4} & \cdots & \cdots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & \cdots & W_N^{(N-1)\times(N-1)} \end{bmatrix} x[N]$$

where $W_N = e^{-j2\pi/N}$

for IDFT relations $x[N] = D_n^{-1} X[N]$

where D_N^{-1} is the N × N matrix given by $D_n^{-1} = \frac{1}{N} D_N^{*}$

(* denotes complex conjugate).

Basic IIR digital filter structures

$$y[n] = b_0 x[n] + b_1 x[n-1] + \ldots\ldots\ldots + b_p x[n-p]$$
$$- a_1 y[n-1] - a_2 y[n-2] - \ldots\ldots\ldots\ldots - a_q y[n-q]$$

condensed form of the difference equation is ,

$$y[n] = \sum_{i=0}^{p} b_i x[n-i] - \sum_{j=1}^{q} a_j y[n-j]$$

which, when rearranged becomes :

$$\sum_{j=0}^{q} a_j y[n-j] = \sum_{i=0}^{p} b_i x[n-i] \Rightarrow \sum_{j=0}^{Q} a_j z^{-j} Y(z) = \sum_{i=0}^{P} b_i z^{-i} X(z)$$

we define the transfer function to be :

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{i=0}^{p} b_i z^{-i}}{\sum_{j=0}^{q} a_j z^{-j}}$$

where, z^{-1} is a unit sample delay

Basic FIR digital filter structures

$$H(z) = \sum_{k=0}^{N} h[k] z^{-k}$$

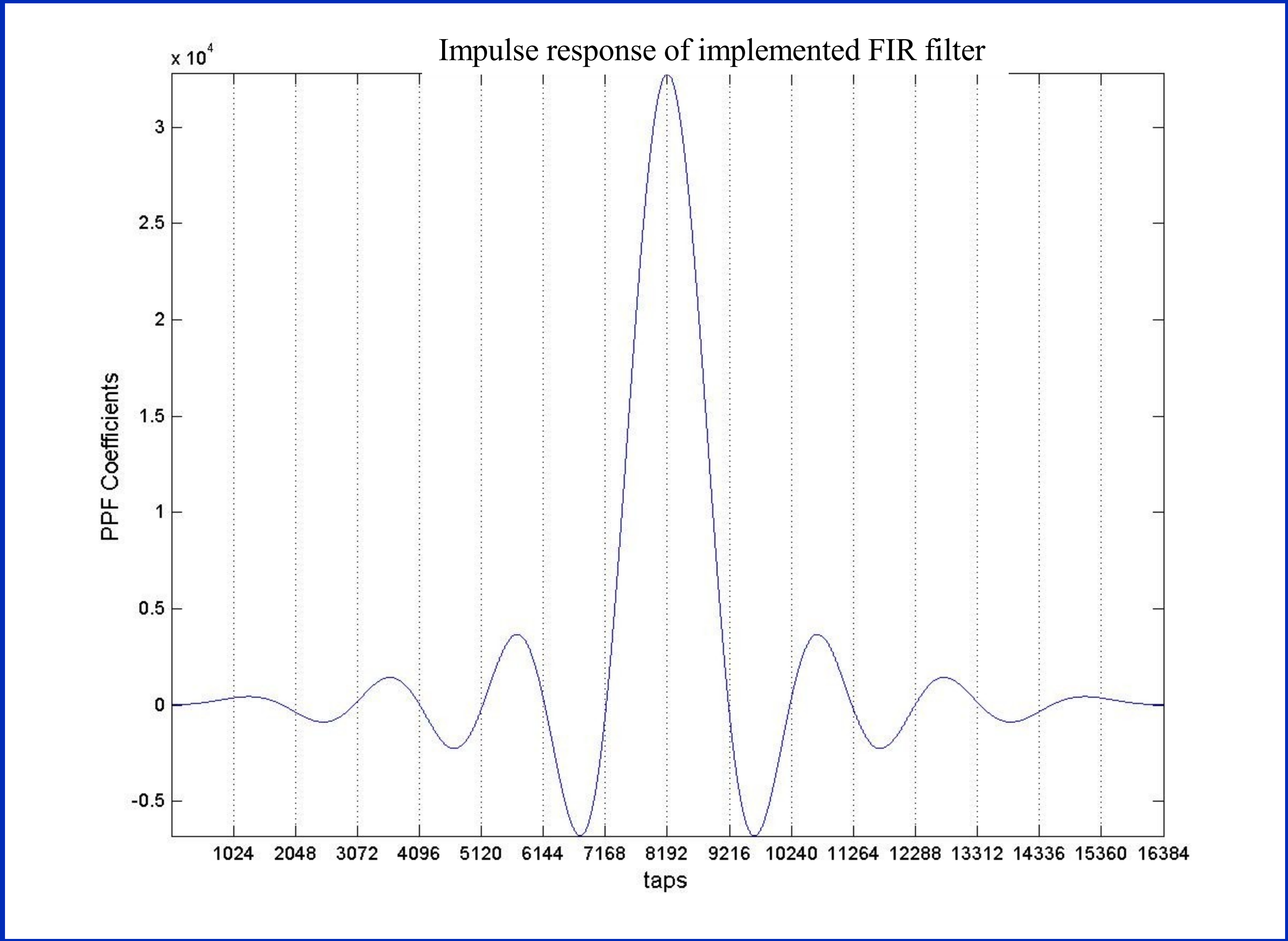
h[k] are b_i's, however a_i's are zero, so no feedback effect

which is a polynomial in z^{-1} of degree N.

In the time domain, the input output relationship can be written as,

$$y[n] = \sum_{k=0}^{N} h[k]x[n-k]$$

where y[n] and x[n] are input and output sequences respectively.



Symmetric Impulse response inherits the linear phase property

Polyphase realization of FIR filters

$$H(z) = h[٠] + h[١]z^{-١} + h[٢]z^{-٢} + h[٣]z^{-٣} + h[٤]z^{-٤} + h[٥]z^{-٥} + h[٦]z^{-٦} + h[٧]z^{-٧} + h[٨]z^{-٨} + h[٩]z^{-٩}$$

$$H(z) = \{h[٠]z^{٠} + h[٢]z^{-٢} + h[٤]z^{-٤} + h[٦]z^{-٦} + h[٨]z^{-٨}\} + \{h[١]z^{-١} + h[٣]z^{-٣} + h[٥]z^{-٥} + h[٧]z^{-٧} + h[٩]z^{-٩}\}$$

$$H(z) = \{h[٠] + h[٢]z^{-٢} + h[٤]z^{-٤} + h[٦]z^{-٦} + h[٨]z^{-٨}\} + z^{-١}\{h[١] + h[٣]z^{-٢} + h[٥]z^{-٤} + h[٧]z^{-٦} + h[٩]z^{-٩}\}$$

$$H(z) = E_٠(z^٢) + z^{-١}E_١(z^٢)$$

where,

$$E_٠(z^٢) = h[٠] + h[٢]z^{-٢} + h[٤]z^{-٤} + h[٦]z^{-٦} + h[٨]z^{-٨}$$

$$E_١(z^٢) = h[١] + h[٣]z^{-٢} + h[٥]z^{-٤} + h[٧]z^{-٦} + h[٩]z^{-٨}$$

$$H(z) = E_0(z^3) + z^{-1}E_1(z^3) + z^{-2}E_2(z^3)$$

L-branch polyphase decomposition of transfer function of order N can be written as

$$H(z) = \sum_{m=0}^{L-1} z^{-m} E_m(z^L)$$

where $E_m(z) = \sum_{n=0}^{[\frac{N+1}{L}]} h[Ln+m]z^{-n}, \qquad 0 \le m \le L-1$

with $h[n] = 0$ for $n > N$

A linear phase FIR filters of order N is either characterized by a symmetric impulse response, i.e., h[n] = h[N-n] or by an antisymmetric impulse response.

$$H(z) = h[0](1 + z^{-7}) + h[1](z^{-1} + z^{-6}) + h[2](z^{-2} + z^{-5}) + h[3](z^{-3} + z^{-4})$$

it requires 4 multipliers, compared to 8 multipliers in the direct form realization of the original length-8 FIR filter.

Uniform Filter Bank

$H_0(z)$ represent a lowpass digital (FIR) filter with an impulse response $h_0[n]$:

$$H_0(z) = \sum_{n=0}^{\infty} h_0[n] z^{-n}$$

The transfer function of k[th] subfilter $H_k(z)$ whose impulse response $h_k[n]$ is defined to be

$$h_k[n] = h_0[n] W_M^{-kn}, \quad 0 \le k \le M-1$$

where $W_M = e^{-j2\pi/M}$

In general,

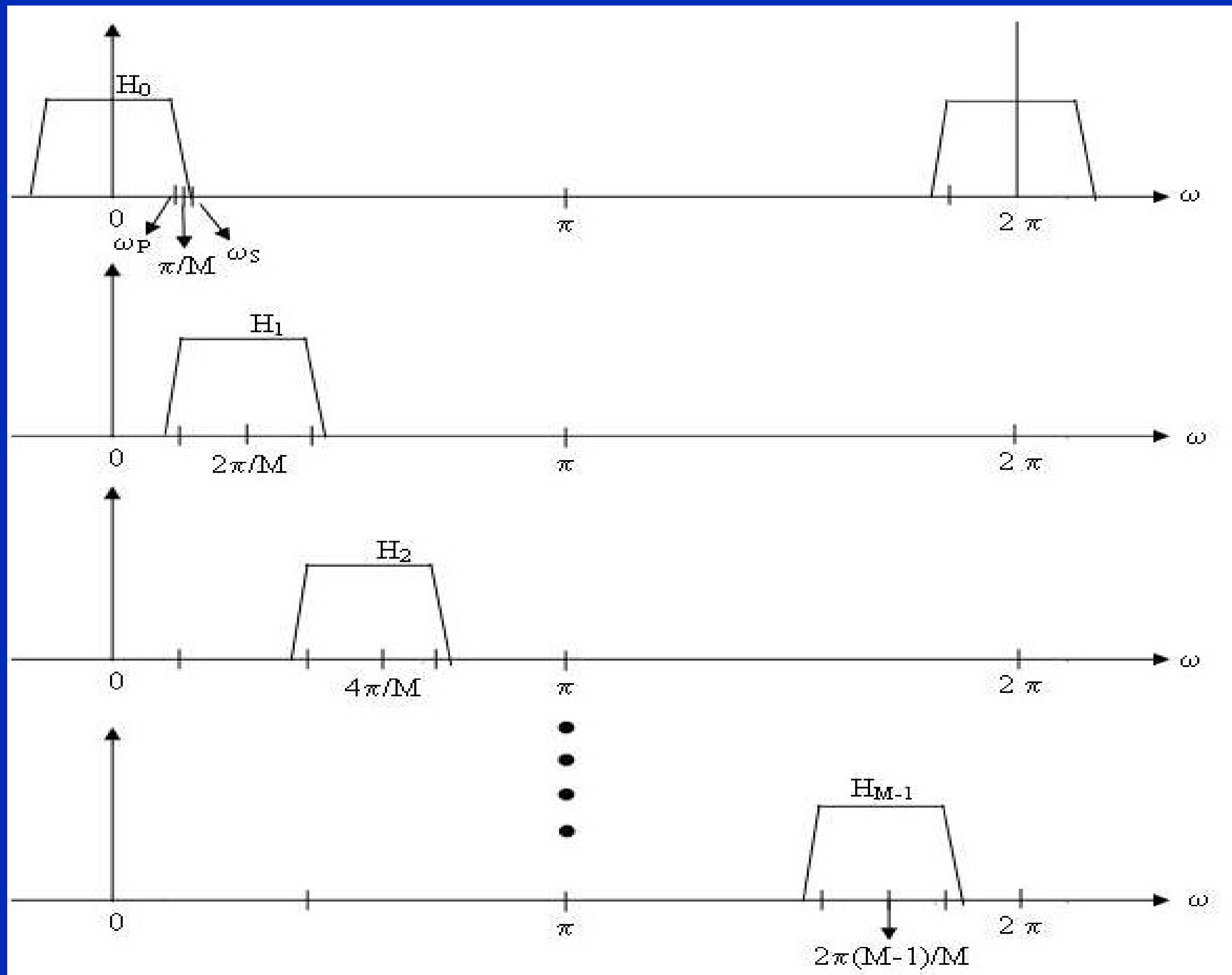
$$H_k(z) = \sum_{n=0}^{\infty} h_k[n] z^{-n} = \sum_{n=0}^{\infty} h_0[n] (z W_M^k)^{-n}, \quad 0 \le k \le M-1$$

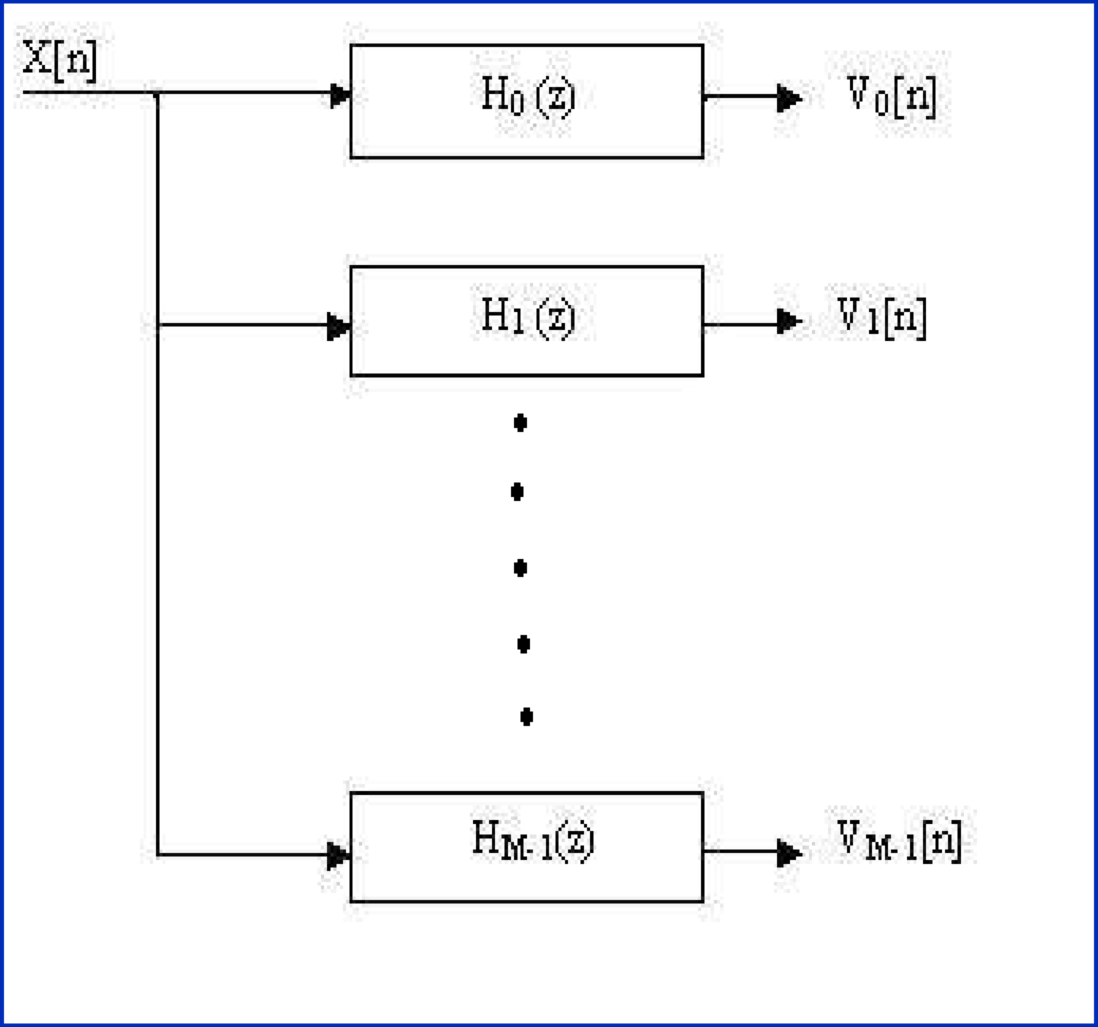
i.e.,

$$H_k(z) = H_0(z W_M^k), \quad 0 \le k \le M-1$$

With corresponding frequency response, shifts by an amount $2\pi k / M$

$$H_k(e^{i\omega}) = H_k(z) = H_0(zW_M^k), \quad 0 \le k \le M-1$$




X[n]
H0 (z)
V0[n]
H1 (z)
V1[n]
HM-1(z)
VM-1[n]

Polyphase implementation of Uniform Filter Bank

M branch polyphase decomposition

$$H_0(z) = \sum_{i=0}^{M-1} z^{-l} E_l(z^M)$$

where $E_l(z)$ is the l th polyphase component of $H_0(z)$:

$$E_l(z) = \sum_{n=0}^{\infty} h_l[n] z^{-n}, \quad 0 \le l \le M-1$$

Substituting z with zW_M^k , we arrive at the M-band polyphase decomposition of $H_k(z)$:

$$H_k(z) = \sum_{l=0}^{M-1} z^{-l} W_M^{-kl} E_l(z^M W_M^{kM}) = \sum_{l=0}^{M-1} W_M^{-kl} z^{-l} E_l(z^M), \quad k = 0,1,\ldots\ldots,M-1$$

In matrix form k^{th} subfilter,

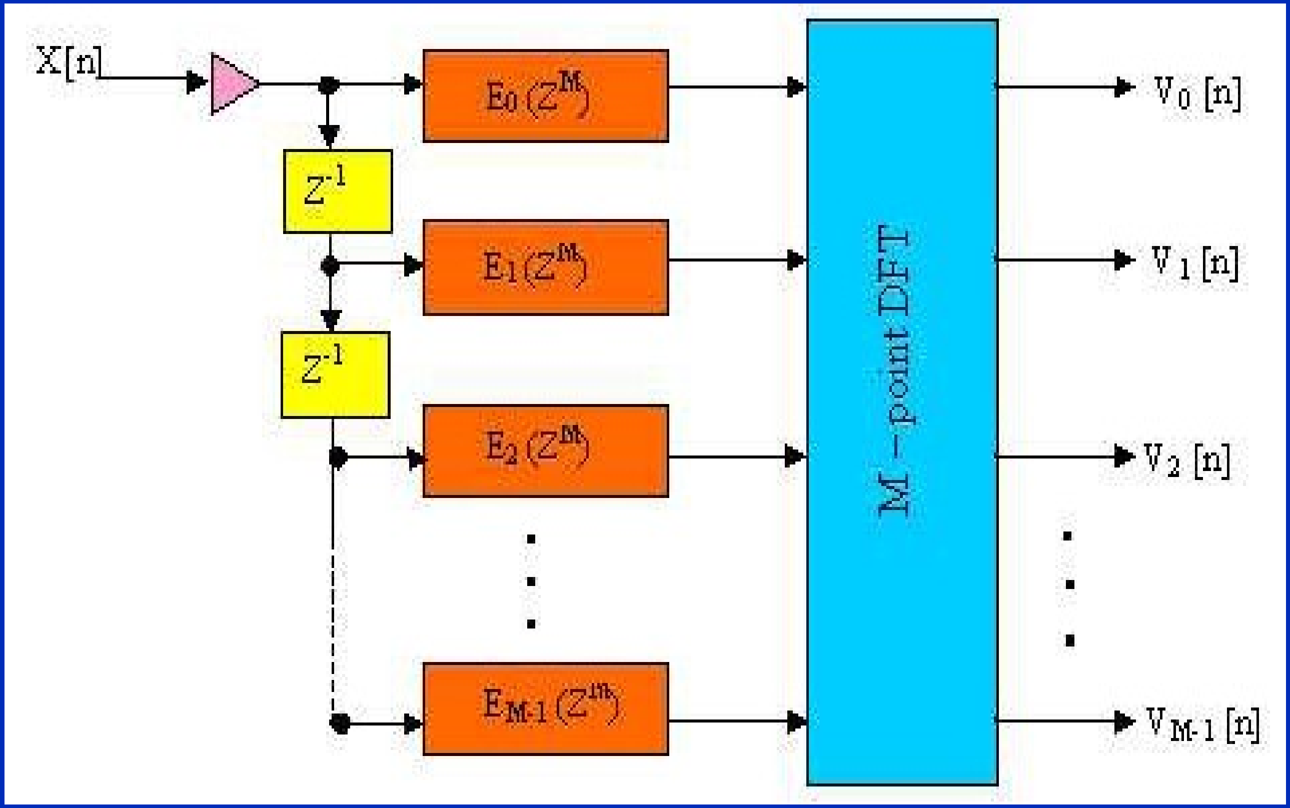
$$H_k(z) = \begin{bmatrix} 1 & W_M^{-k} & W_M^{-2k} & \cdots & W_M^{-(M-1)k} \end{bmatrix} \begin{bmatrix} E_0(z^M) \\ z^{-1}E_1(z^M) \\ z^{-2}E_2(z^M) \\ \vdots \\ z^{-(M-1)}E_{M-1}(z^M) \end{bmatrix}$$

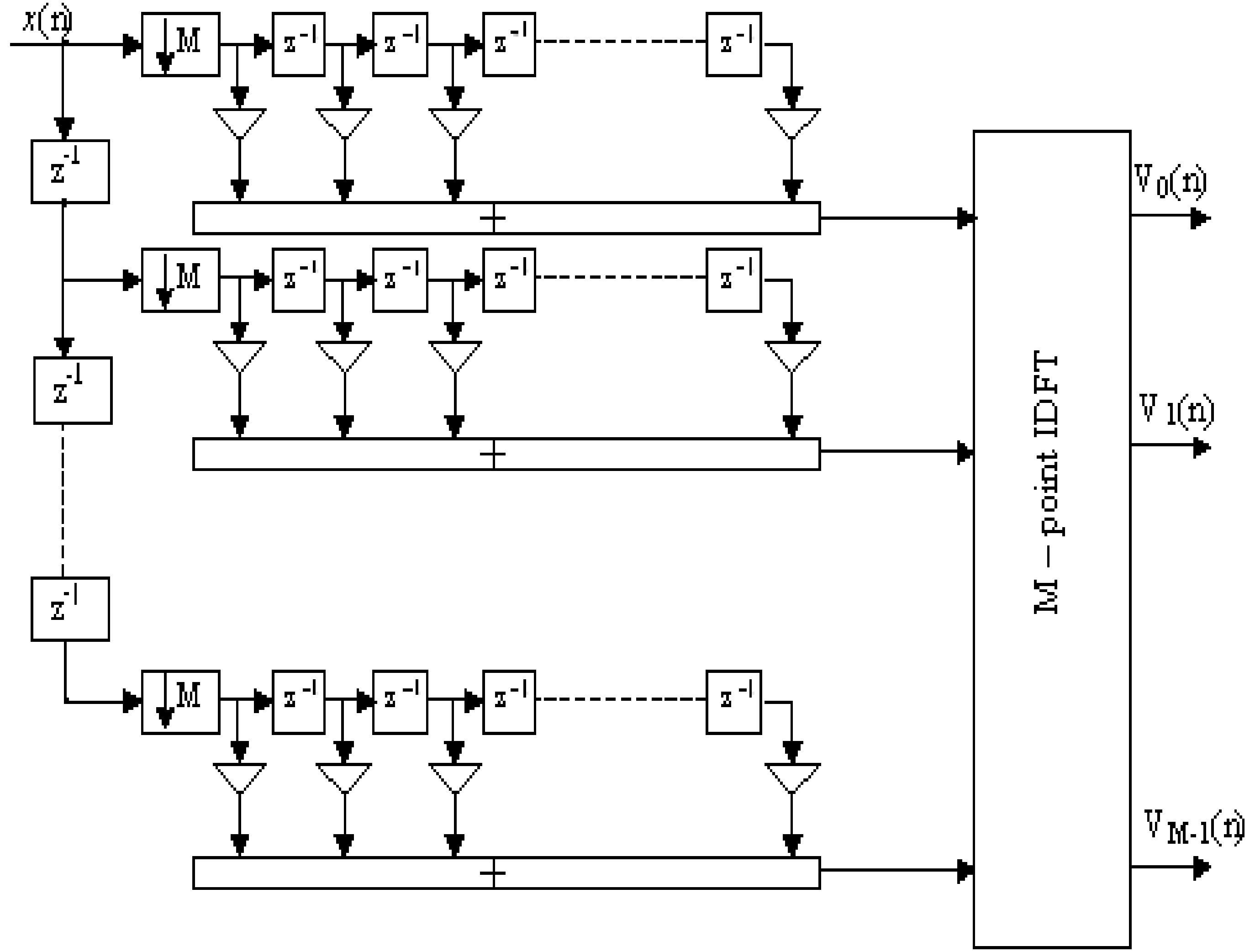
For k=0,1,…., M-1. All there M equations can be combined into a matrix equation as

$$\begin{bmatrix} H_0(z) \\ H_1(z) \\ H_2(z) \\ \vdots \\ H_{M-1}(z) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_M^{-1} & W_M^{-2} & \cdots & W_M^{-(M-1)} \\ 1 & W_M^{-2} & W_M^{-4} & \cdots & W_M^{-2(M-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & W_M^{-(M-1)} & W_M^{-2(M-1)} & W_M^{-2(M-1)} & W_M^{-(M-1)^2} \end{bmatrix} \begin{bmatrix} E_0(z^M) \\ z^{-1}E_1(z^M) \\ z^{-2}E_2(z^M) \\ \vdots \\ z^{-(M-1)}E_{M-1}(z^M) \end{bmatrix}$$

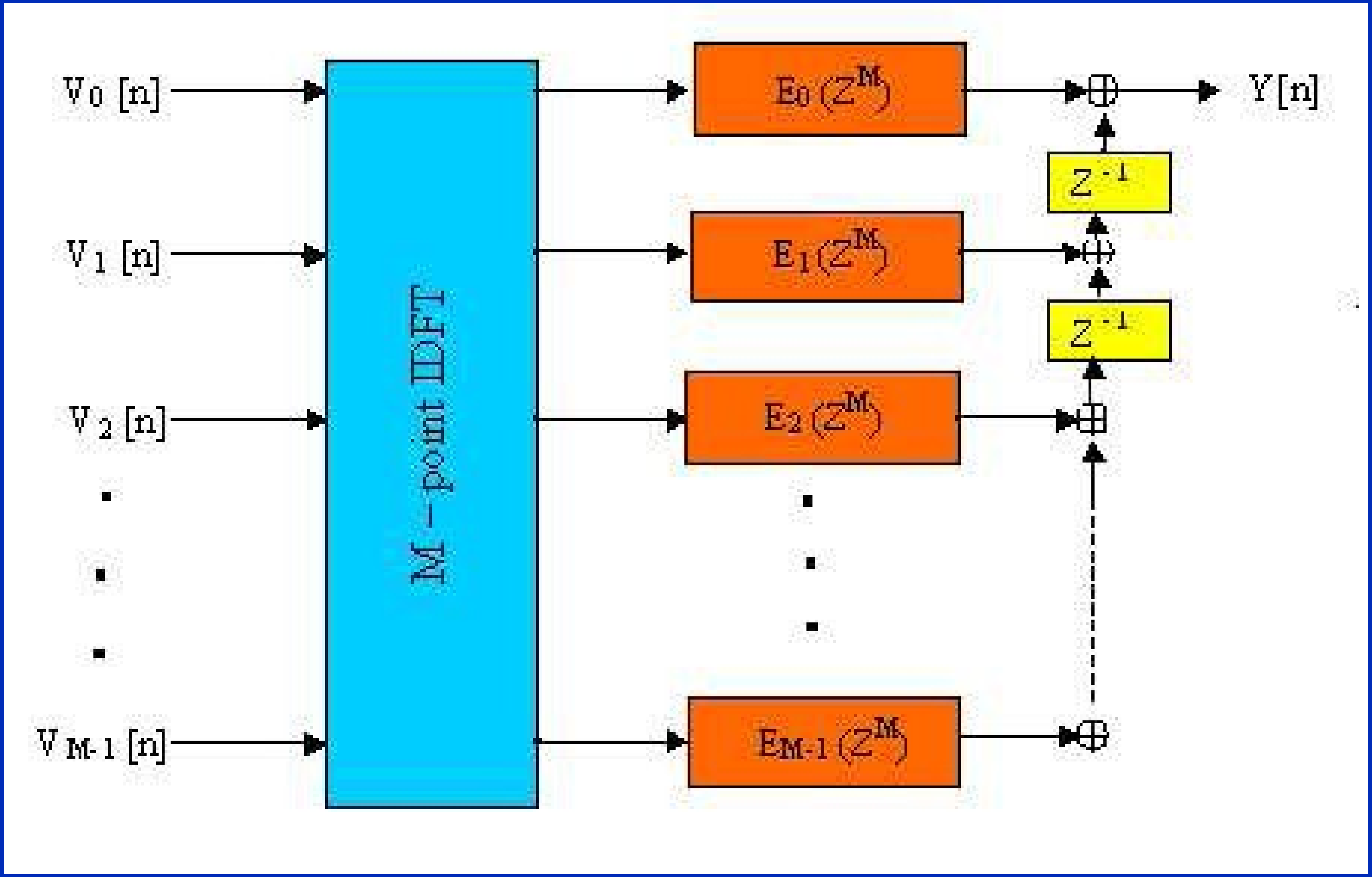
Which is equivalent to

$$\begin{bmatrix} H_0(z) \\ H_1(z) \\ H_2(z) \\ \vdots \\ H_{M-1}(z) \end{bmatrix} = M \cdot D^{-1} \begin{bmatrix} E_0(z) & 0 & 0 & 0 & 0 \\ 0 & E_1(z) & 0 & 0 & 0 \\ 0 & 0 & E_2(z) & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & E_{M-1}(z) \end{bmatrix}$$




x(n)
M
z^-1
+
M – point IDFT
V0(n)
V1(n)
VM-1(n)

Perfect Reconstruction of Signal (Inversion of Polyphase Filter Bank)



$$\tilde{E} = \begin{bmatrix} E_0(z) & 0 & 0 & 0 & 0 \\ 0 & E_1(z) & 0 & 0 & 0 \\ 0 & 0 & E_2(z) & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & E_{M-1}(z) \end{bmatrix}$$

To invert the effect of FIR filters, we need to invert this diagonal matrix

$$\tilde{E}^{-1} = \begin{bmatrix} E_0^{-1}(z) & 0 & 0 & 0 & 0 \\ 0 & E_1^{-1}(z) & 0 & 0 & 0 \\ 0 & 0 & E_2^{-1}(z) & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & E_{M-1}^{-1}(z) \end{bmatrix}$$

where $E_N^{-1}(z) = \dfrac{1}{E_N(z)}$

$$\hat{X}(n) = \tilde{E}^{-1} D \, D^{-1} \tilde{E} \propto x(n)$$

In general,

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{i=0}^{p} b_i z^{-i}}{\sum_{j=0}^{q} a_j z^{-j}}$$

$$a_0 y[n] = b_0 x[n] + b_1 x[n-1] + \ldots\ldots + b_p x[n-p]$$
$$- a_1 y[n-1] - a_2 y[n-2] - \ldots\ldots\ldots - a_q x[n-q]$$

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{i=0}^{p} b_i z^{-i}$$

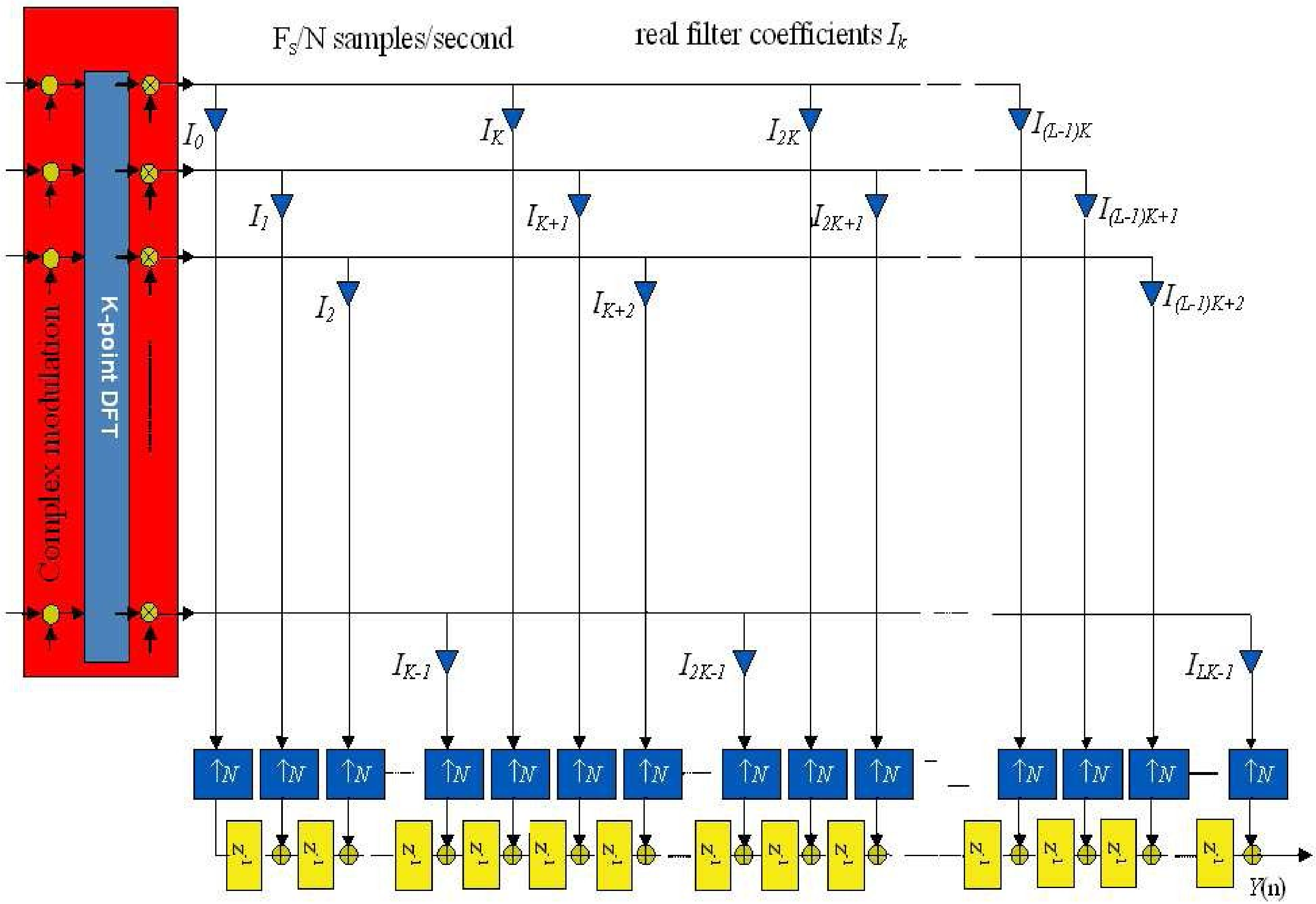
Inversion of PPF

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{\sum_{i=0}^{p} b_i z^{-i}}$$

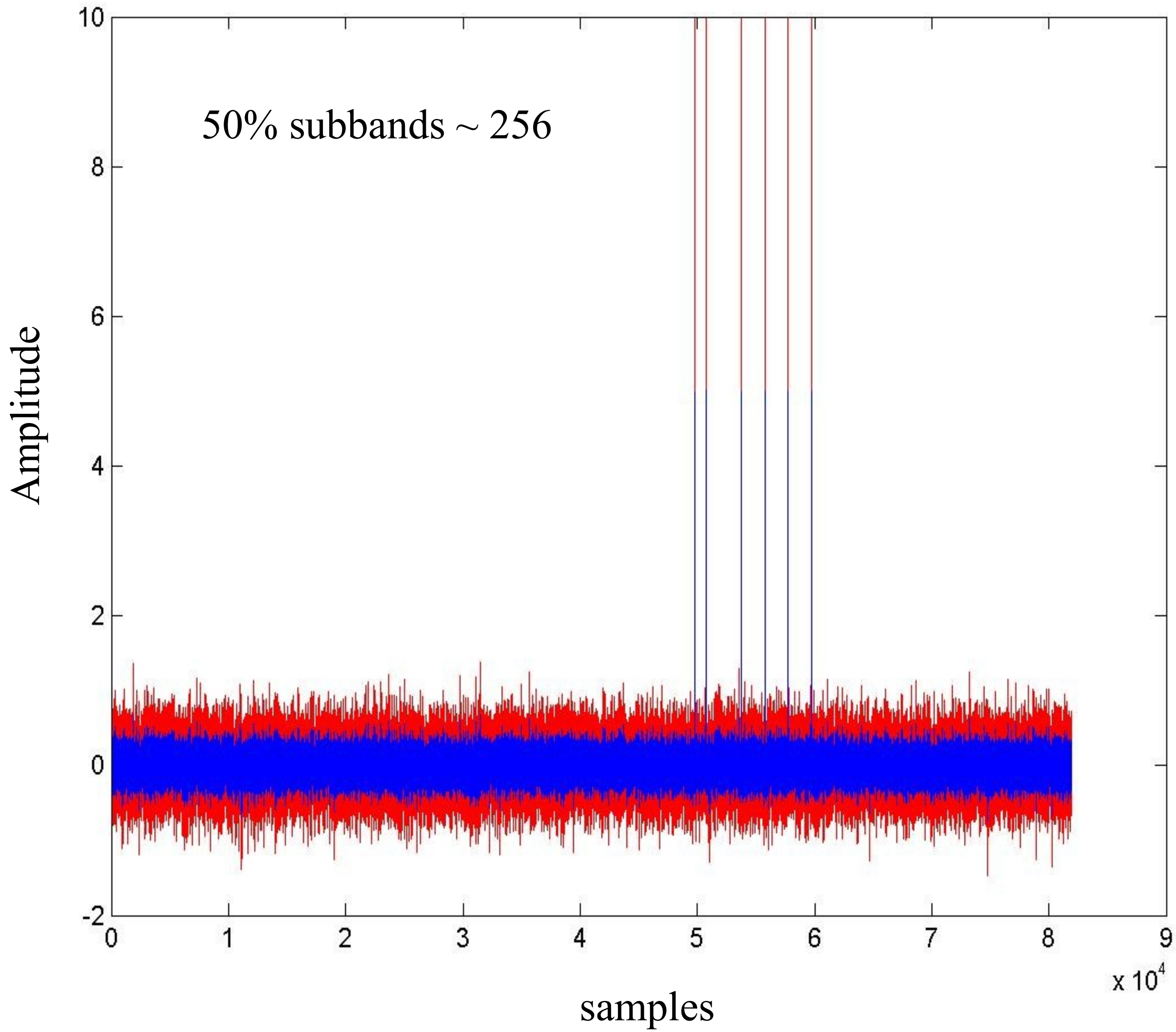
$$\sum_{j=0}^{q} b_j y[n-j] = \sum_{i=0}^{p} a_i x[n-i] = a_0 x[n]; \quad a_0 = 1$$

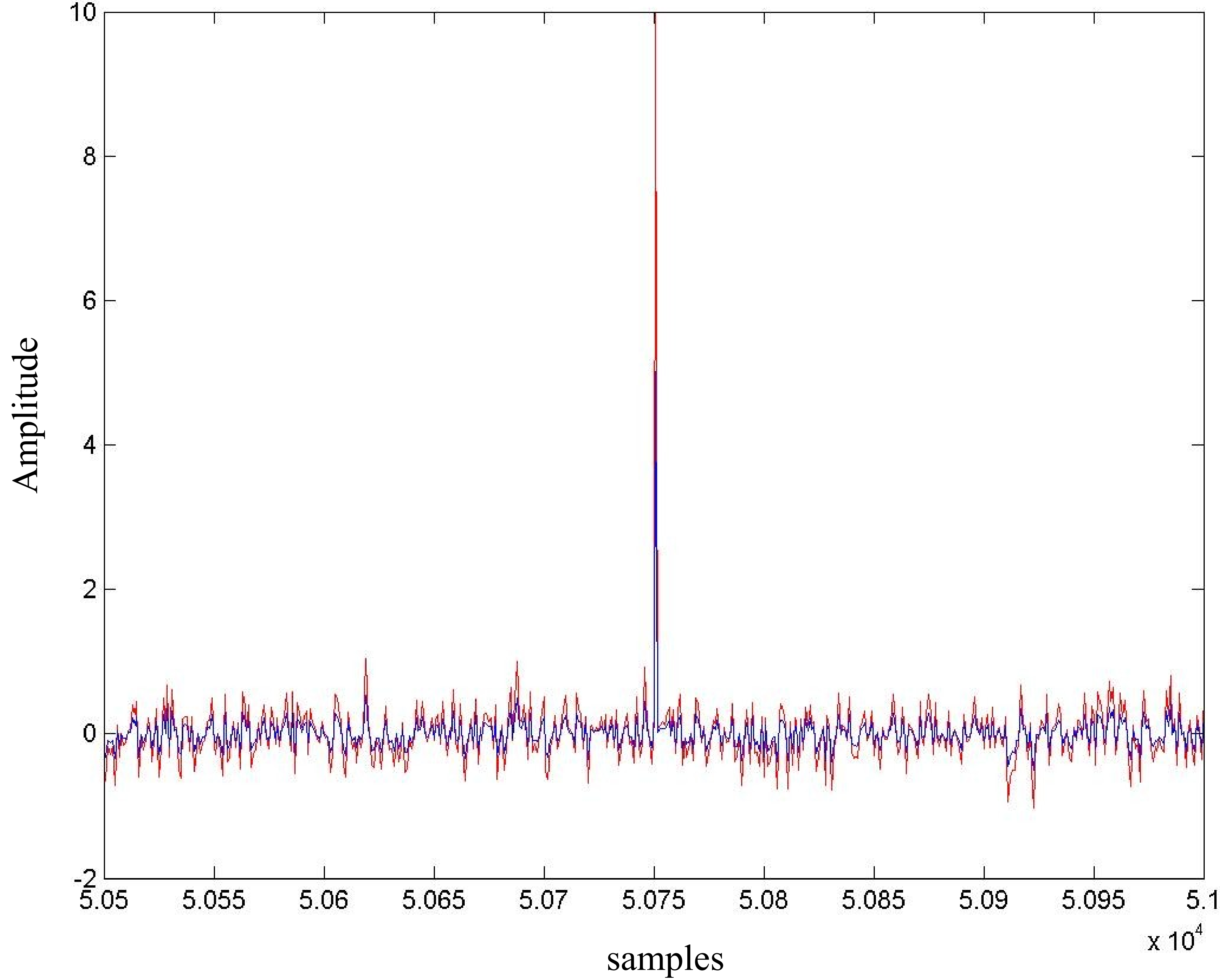
Impulse response

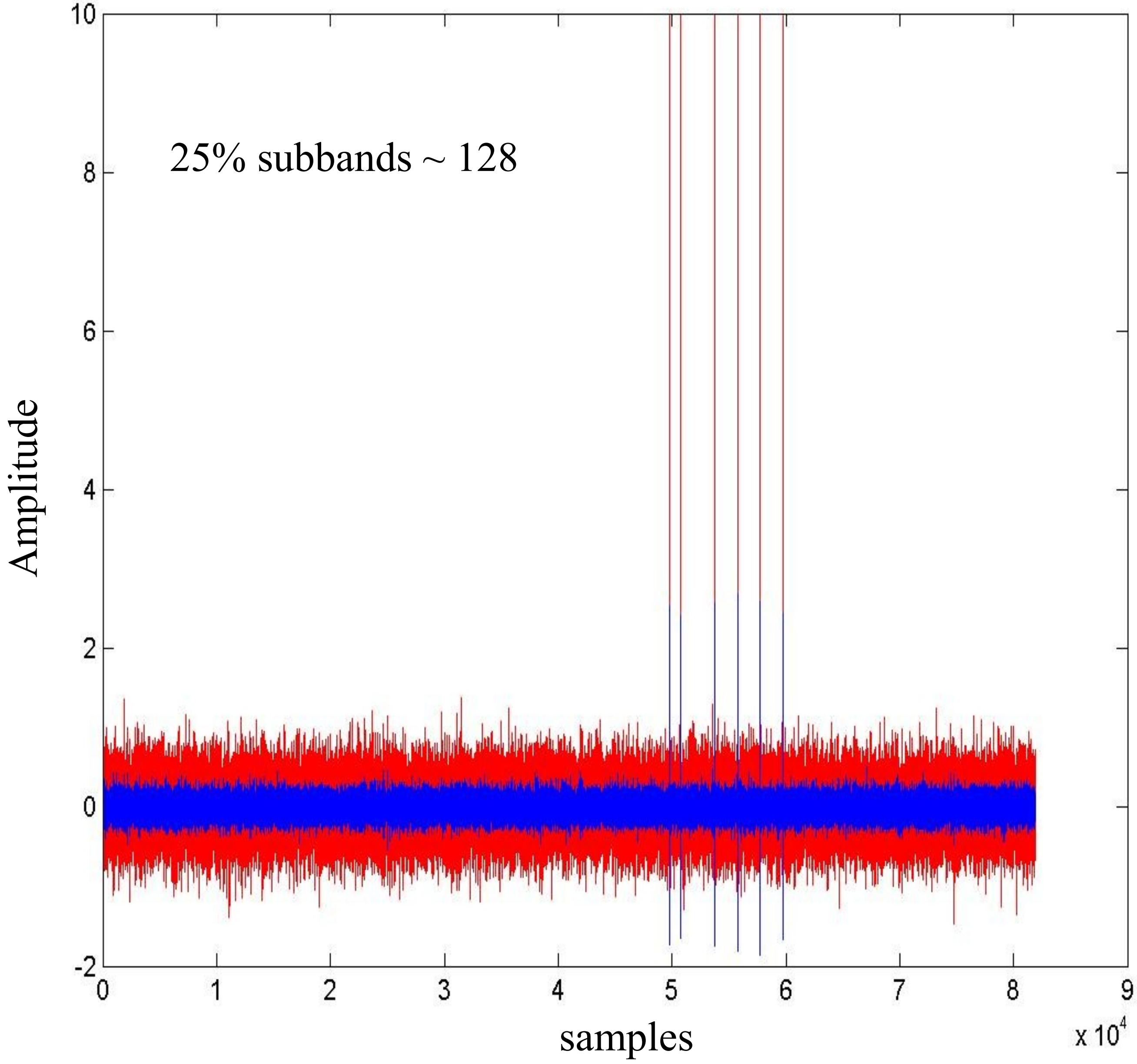
$$y[n] = \frac{1}{b_0}\{a_0 x[n] - b_1 y[n-1] - b_2 y[n-2] - \ldots\ldots\ldots\ldots - b_q x[n-q]\}$$

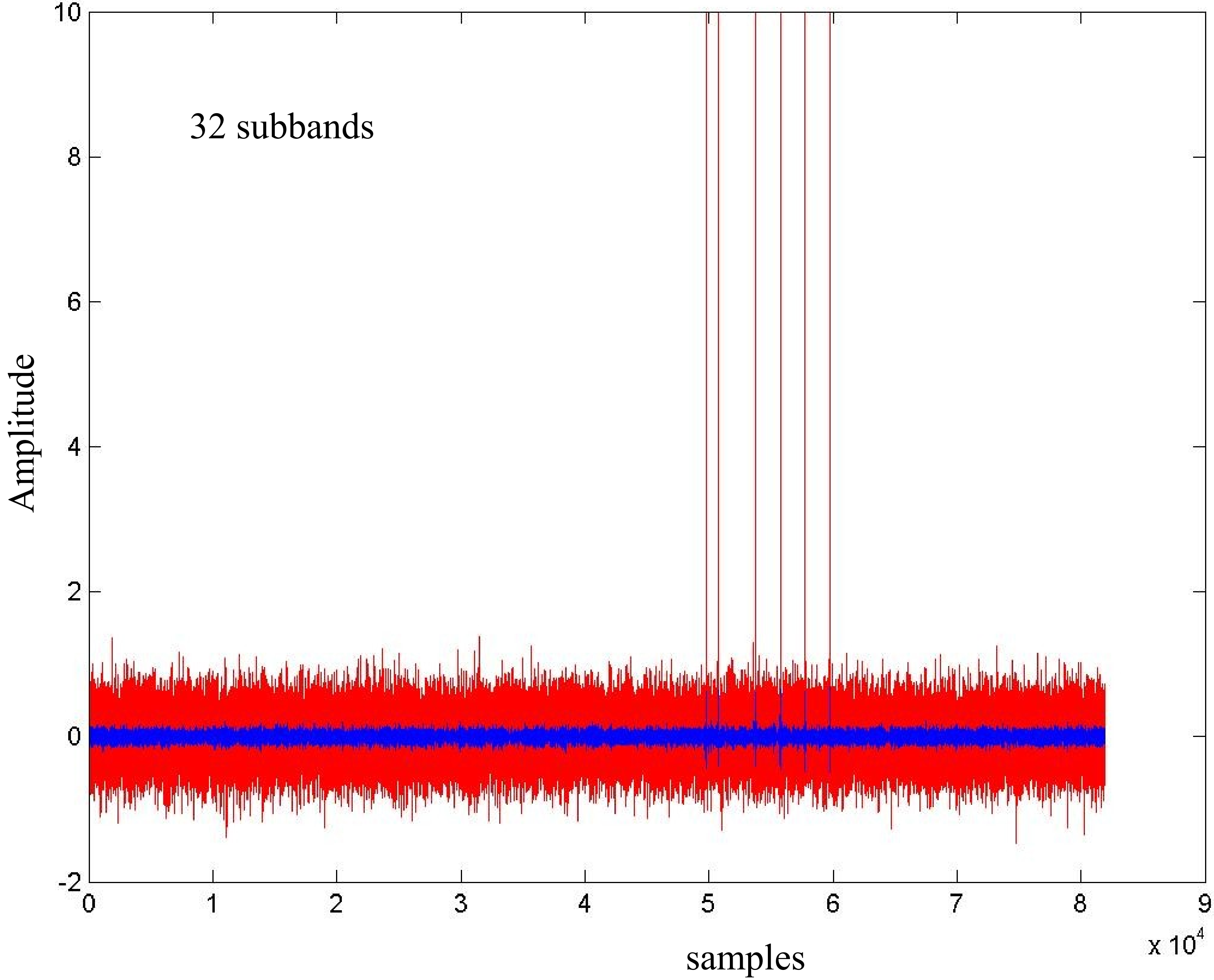


F_s/N samples/second
real filter coefficients I_k
Complex modulation
K-point DFT
I_0
I_K
I_2K
I_(L-1)K
I_1
I_K+1
I_2K+1
I_(L-1)K+1
I_2
I_K+2
I_(L-1)K+2
I_K-1
I_2K-1
I_LK-1
↑N
Z^-1
Y(n)


50% subbands ~ 256
Amplitude
samples
x 10^4
10
8
6
4
2
0
-2
0
1
2
3
4
5
6
7
8
9


10
8
6
4
2
0
-2
Amplitude
5.05
5.055
5.06
5.065
5.07
5.075
5.08
5.085
5.09
5.095
5.1
x 10^4
samples


25% subbands ~ 128
Amplitude
samples
x 10^4
10
8
6
4
2
0
-2
0
1
2
3
4
5
6
7
8
9


32 subbands
Amplitude
samples
x 10^4
10
8
6
4
2
0
-2
0
1
2
3
4
5
6
7
8
9

Ionospheric Calibration

$$n(f) = \sqrt{1 - \frac{f_p^2}{f^2}}$$

***n*(*f*)** **Refractive index**

f_p **plasma frequency**

$$k(f) = \frac{2\pi \, f \sqrt{1 - \frac{f_p^2}{f^2}}}{c}$$

$$f_p = \sqrt{\frac{n_e e^2}{\pi \, m_e}} \cong 9\sqrt{n_e}\, kHz$$

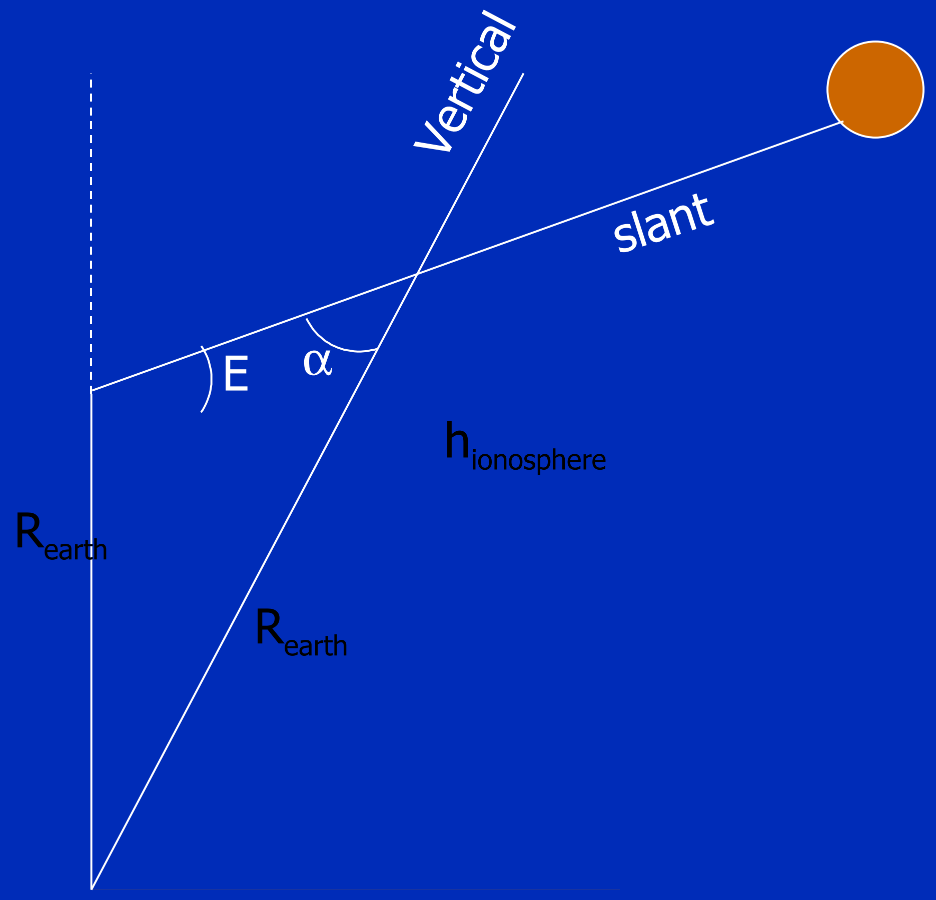
n_e **plasma density**

Corresponding phase delay,

$$\Delta \phi (f) = \frac{2\pi}{f} \left\{ \frac{e^2}{8\pi^2 m_e \varepsilon_0 c} \right\} STEC$$

time delay between adjacent subbands,

$$\Delta T = \frac{\Delta \phi}{2\pi f}$$



$$m = \frac{STEC}{VTEC} = \frac{1}{\cos\alpha};$$

$$m = \frac{1}{\sqrt{1 - \left(\frac{R_{earth}}{R_{earth} + h_{ionosphere}}\right)^2 \cos^2 E}}$$

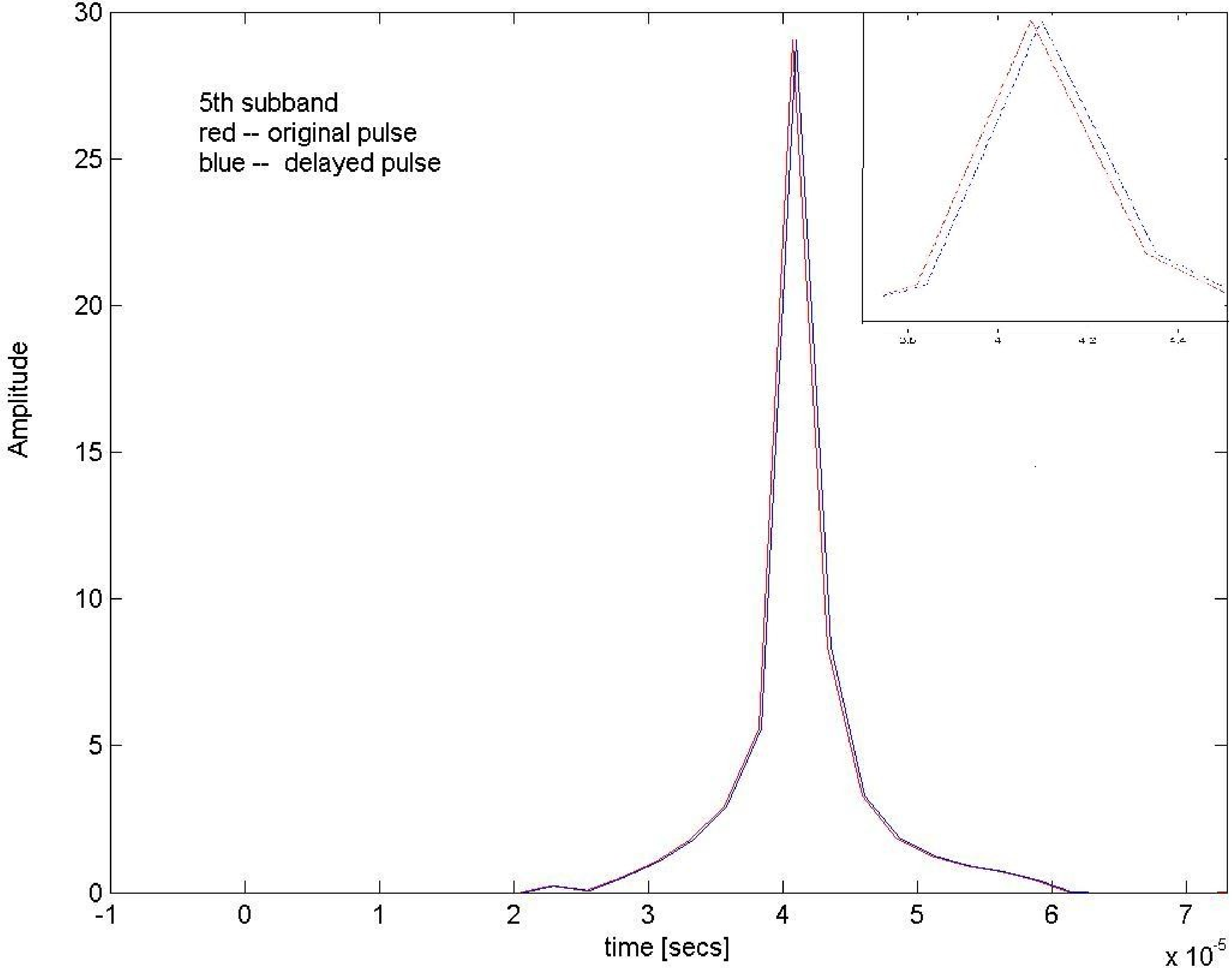
$$\sin E = \sin\phi \cdot \sin\delta + \cos\phi \cdot \cos\delta \cdot \cos H$$

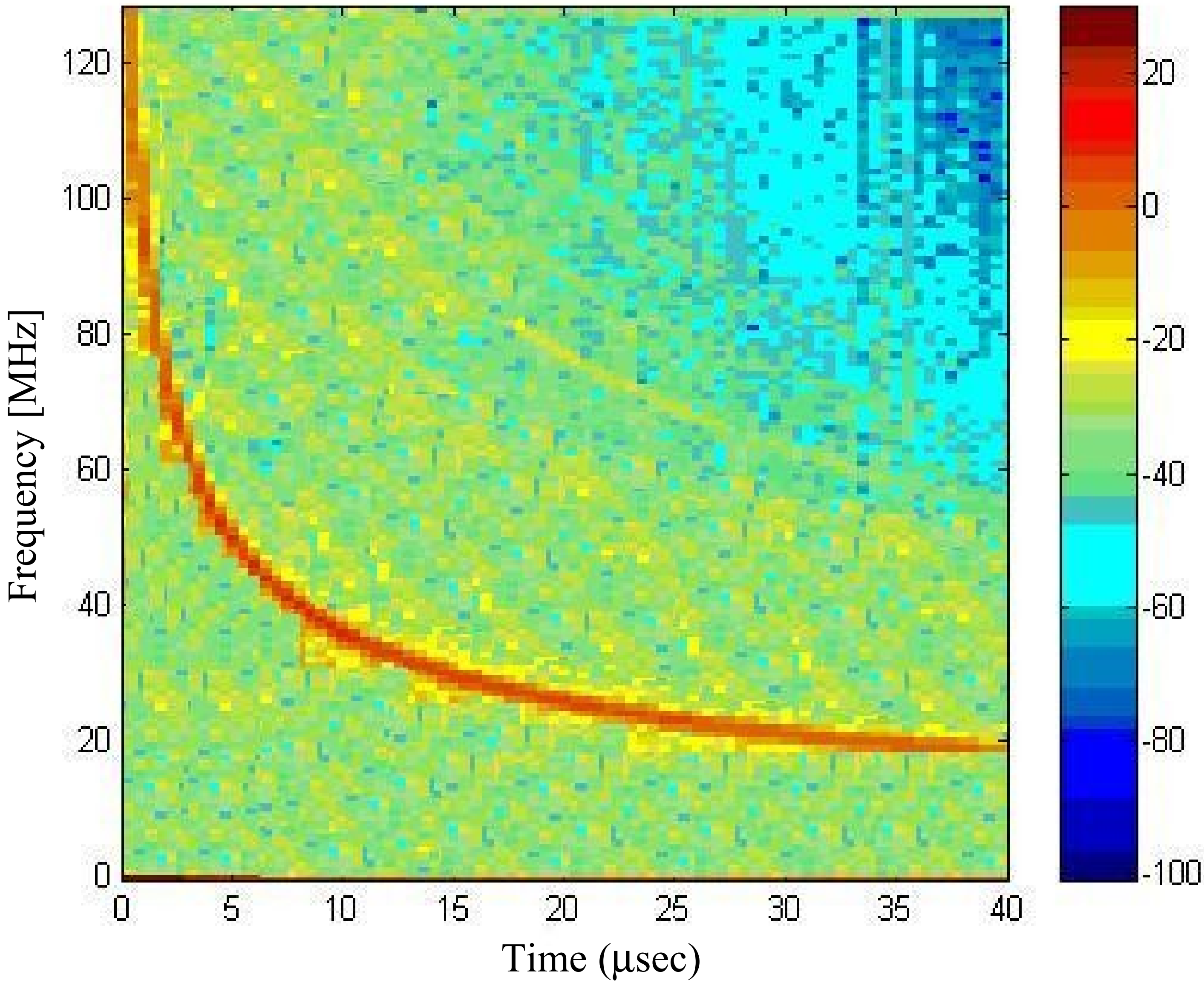
where,

ϕ = geomagnetic latitude of observer

δ = equatorial coordinate declination

H = hour angle


5th subband
red -- original pulse
blue -- delayed pulse
Amplitude
time [secs]
x 10^-5
30
25
20
15
10
5
0
-1
0
1
2
3
4
5
6
7



Frequency [MHz]
0
20
40
60
80
100
120
Time (µsec)
0
5
10
15
20
25
30
35
40
20
0
-20
-40
-60
-80
-100

Thank you !!