Beam Calibration Models

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MIDPREP Workshop
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Outline

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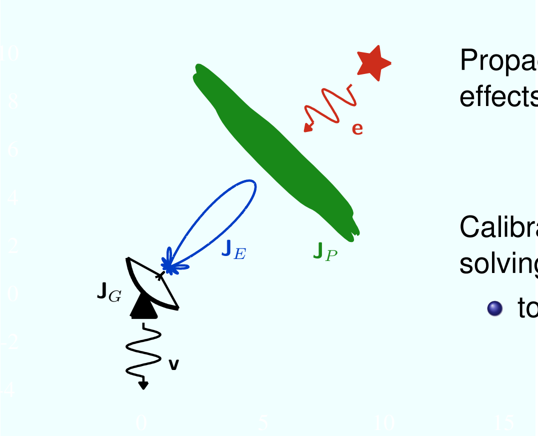
Calibrating for Antenna Radiation Pattern



Propagation and instrumental effects determine **v** for any **e**

$$\mathbf{v} = \mathbf{J}_G\,\mathbf{J}_E\,\mathbf{J}_P\,\mathbf{e}$$

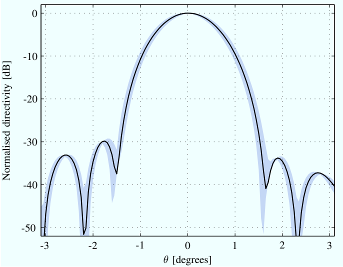
Calibration concerned with solving for **J**-chain

- to determine **e** from **v**

Focus here on primary beam $\mathbf{J}_E$

Why Not Just Measure the Patterns?

Future systems may be limited by accuracy of instrument gain characterisation



Case study: MeerKAT pattern variations due to changing operating conditions (support arm deformation)

- average relative error $> 10\%$

Address this problem through efficient pattern models

Requirements for Efficient Pattern Model

Pattern model in form of summation of basis functions

$$\mathbf{J}_E(\theta, \phi, t, f) = \sum_{k=1}^{K} \mathbf{x}_k(t, f) \circ \mathbf{f}_k(\theta, \phi, t, f)$$

(2×2 matrices, $\circ$ is element-wise product)

- model parameters = weighting coefficients $\{\mathbf{x}_k\}_{k=1}^{K}$ (full-polarisation, time and frequency dependent)
- trade-off: accuracy vs number of terms
- suitable set of basis functions $\{\mathbf{f}_k\}$ that minimises K

In search of the optimal set $\{\mathbf{f}_k\}_{k=1}^{K}$ ...
(for remaining slides, scalar co-pol patterns)

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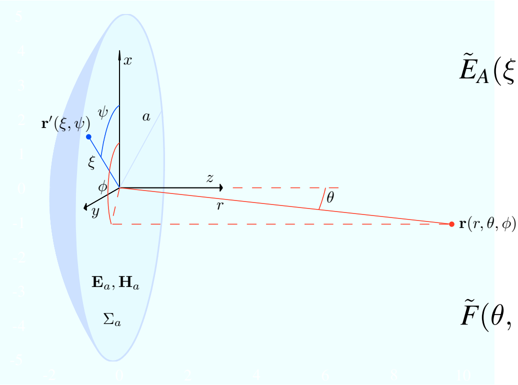
Far Field Model from Aperture Field Model



Aperture field model

$$\tilde{E}_A(\xi,\psi) = \sum_{n,m} a_{n,m} R_{n,m}(\xi)\Psi_{n,m}(\psi)$$

$\downarrow$

$\boxed{\text{AF-to-FF TFM}}$

$\downarrow$

Far field model

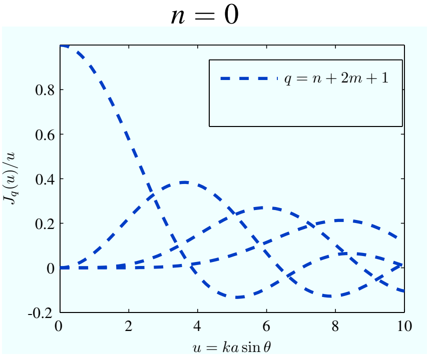
$$\tilde{F}(\theta,\phi) = \sum_{n,m} c_{n,m}\Theta_{n,m}(\theta)\Phi_{n,m}(\phi)$$

Jacobi-Bessel Far Field Model

Zernike polynomial expansion of aperture field yields JB-model

$$\tilde{\mathbf{F}}^{(\text{JB})}(\theta,\phi) \triangleq \sum_{n=0}^{N}\sum_{m=0}^{M}\left(\mathbf{A}_{m,n}\sin n\phi + \mathbf{B}_{m,n}\cos n\phi\right)\frac{J_q(ka\sin\theta)}{ka\sin\theta}$$

Number of terms $= (2N+1)(M+1)$ $\quad q = n + 2m + 1$



Neumann Far Field Model

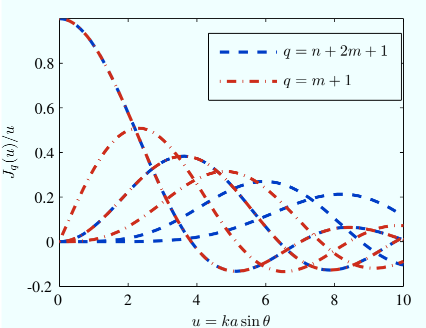
Modify JB-model Bessel function order to obtain N-model

$$\tilde{\mathbf{F}}^{(\mathrm{N})}(\theta,\phi) \triangleq \sum_{n=0}^{N}\sum_{m=0}^{M}\left(\mathbf{A}_{m,n}\sin n\phi + \mathbf{B}_{m,n}\cos n\phi\right)\frac{J_q(ka\sin\theta)}{ka\sin\theta}$$

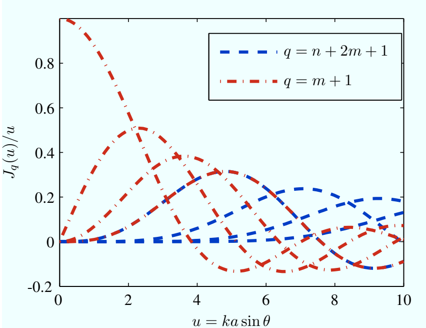
Number of terms $= (2N+1)M+3$

$q = m+1$

$n = 0$



$n = 3$

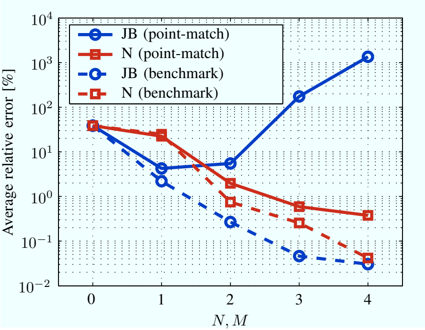


Determining Model Coefficients

Weight difference between model $\tilde{F}$ and pattern F to zero

$$\int_{4\pi} w_l \left(\tilde{F} - F\right) \, \mathrm{d}\Omega = \int_{4\pi} w_l \left(\sum_{k=1}^{K} x_k f_k - F\right) \, \mathrm{d}\Omega = 0$$

Yields linear system $\mathbf{Z}_{L\times K}\mathbf{x}_{K\times 1} = \mathbf{V}_{L\times 1}$
(requires $L \geq K$ for unique solution)



- Galerkin weighting (benchmark solution)
- Dirac-delta weights (point-matching, practical solution)

Constrained Solutions

Assuming we know the ideally expected pattern F_0

$$F \approx F_0 \rightarrow \mathbf{x}_0 \approx \mathbf{x}$$

- obtain $\mathbf{x}_0$ for expected pattern using Galerkin weighting
- solve $\mathbf{x}$ for actual pattern

$$\mathbf{x} = \arg\min_{\mathbf{x}_R} \|\mathbf{x}_R - \mathbf{x}_0\| \quad \text{subject to} \;\; \mathbf{Z}_R\mathbf{x}_R = \mathbf{V}_R$$

$\mathbf{V}_R$ is $L \times 1$ vector of pattern samples (point-matching)
$\mathbf{Z}_R$ is $L \times L$ matrix of basis function samples

Two constrained solution approaches

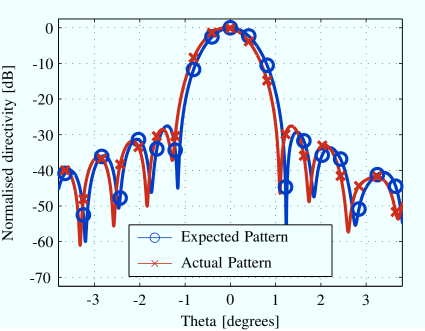
1. Quadratic programming with linear constraints
2. Quadratic penalty function minimisation

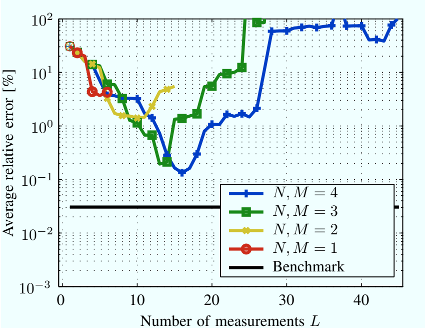
Yields unique solution for $L < K$

Constrained Solution 1

Minimise $\|\mathbf{x} - \mathbf{x}_0\|^2$ subject to $\mathbf{Z}_R\mathbf{x} = \mathbf{V}_R$

$$\begin{bmatrix} \mathbf{1} & \mathbf{Z}_R^H \\ \mathbf{Z}_R & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{V}_R \end{bmatrix}$$

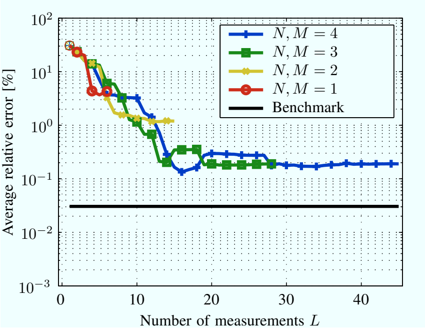


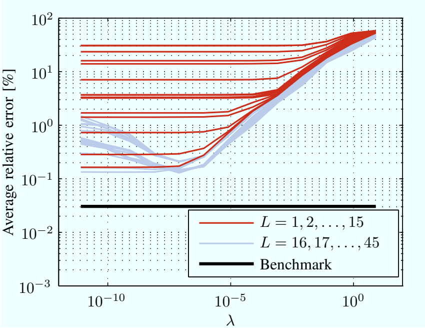


Constrained Solution 2

Minimise penalty function $\|\mathbf{Z}_R\mathbf{x} - \mathbf{V}_R\|^2 + \lambda\|\mathbf{x} - \mathbf{x}_0\|^2$

$$\left(\mathbf{Z}_R^H\mathbf{Z}_R + \mathbf{1}\lambda\right)\mathbf{x} = \left(\mathbf{Z}_R^H\mathbf{V}_R + \lambda\mathbf{x}_0\right)$$



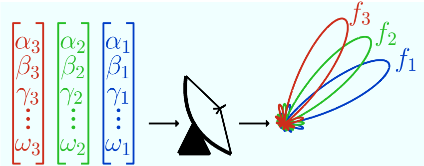


Outline

CBFP Overview

Antenna maps set of geometrical / electrical parameters to particular far field pattern

$Y : \mathcal{D} \to \mathcal{P}$



[1] Use only few elements from $\mathcal{P}$ as basis functions to approximate any $\mathbf{F} \in \mathcal{P}$

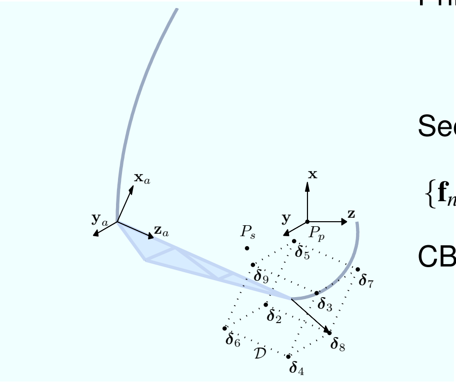
- Primary CBFP = $Y(\boldsymbol{\delta}_1)$
  ideal antenna configuration
- Secondary CBFPs = $\{Y(\boldsymbol{\delta}_n)\}$
  perturbed antenna configurations

[1] Maaskant et al, IEEE TAP, 2012

CBFP Application Example

Feed / subreflector support arm deformation in OG reflector

- position of feed / subreflector
- assumed dominant source of error



Primary CBFP

$$\mathbf{f}_1 = Y(\boldsymbol{\delta}_1) = Y(\mathbf{0})$$

Secondary CBFPs

$$\{\mathbf{f}_n\} = \{Y(\boldsymbol{\delta}_n)\} \quad \text{for } n = 2, 3, \ldots, 9$$

CBFP model for any $\mathbf{F} \in \mathcal{P}$

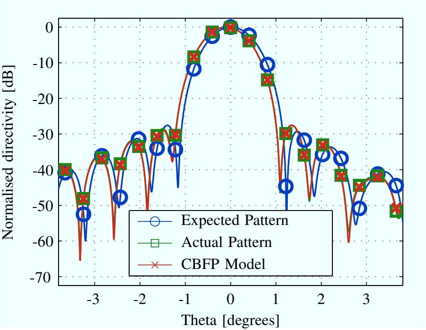
$$\tilde{\mathbf{F}} = \sum_{n=1}^{9} x_n \mathbf{f}_n \approx \mathbf{F}$$

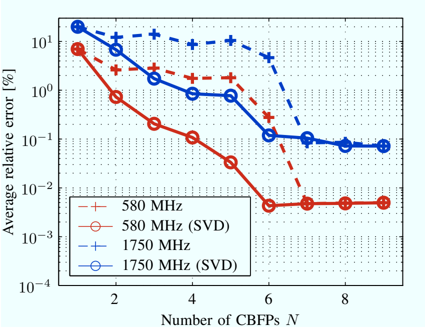
CBFPs Applied to Support Arm Deformation: Results

Use SVD to extract dominant pattern modes

$$\mathbf{M} = \left[\mathbf{f}_1 \quad \mathbf{f}_2 \quad \cdots \quad \mathbf{f}_9\right], \quad \mathbf{USV}^H = \mathbf{M}, \quad \tilde{\mathbf{F}} = \sum_{n=1}^{N} x_n \mathbf{u}_n$$

Solve for coefficients through point-matching





9-term CBFP model
(1750 MHz)

Outline

Summary

Efficient pattern model:

high accuracy and few unknown parameters

Analytic pattern models

- Moderate accuracy
- ✖ Large number of terms
    - Constrained solution to reduce measurements

Characteristic Basis Function Patterns

- ✔ High accuracy
- ✔ Very few terms
- Designed for specific anticipated errors

Thank You

Questions?