Mutual coupling in Lofar LBA & calibrating using characteristic basis functions

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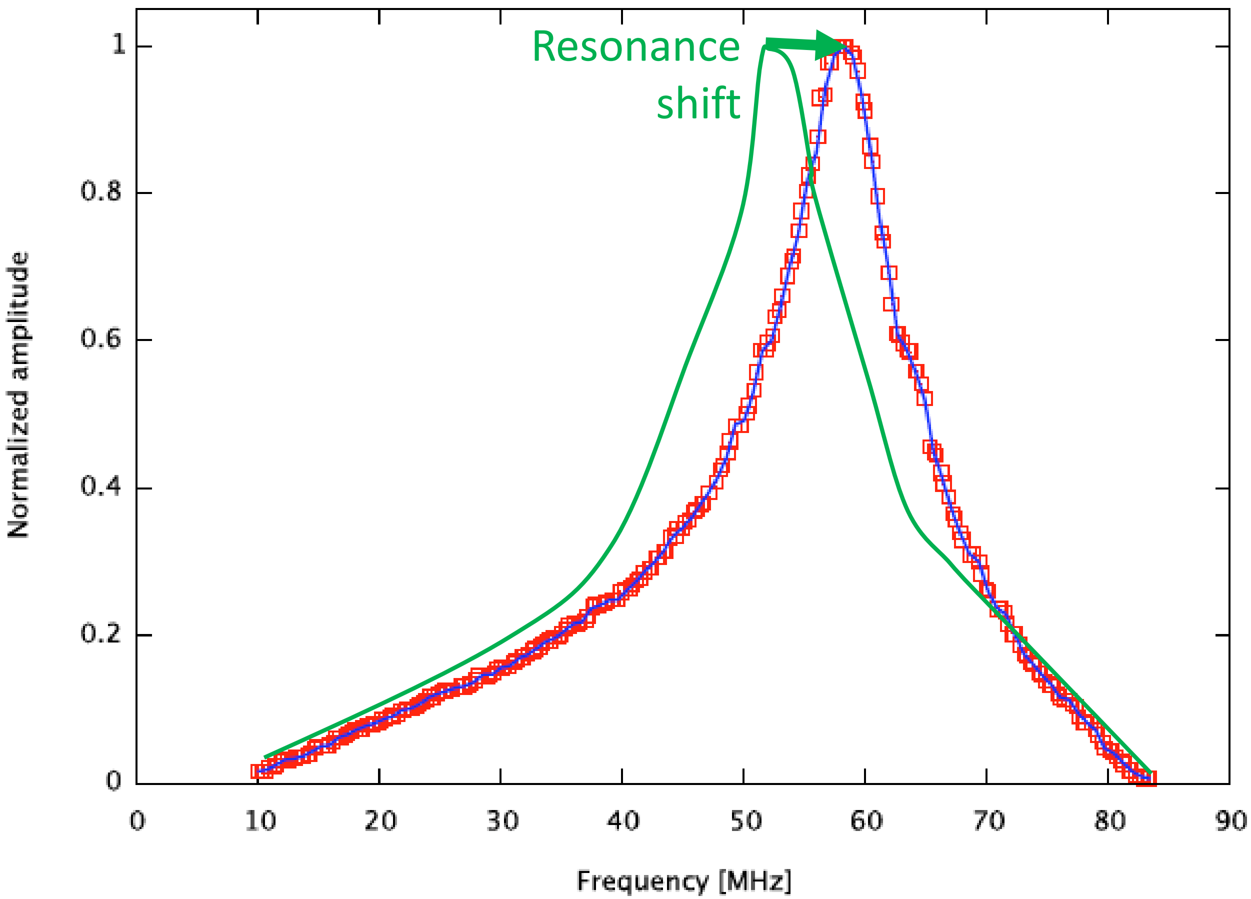
Lofar LBA resonance shift

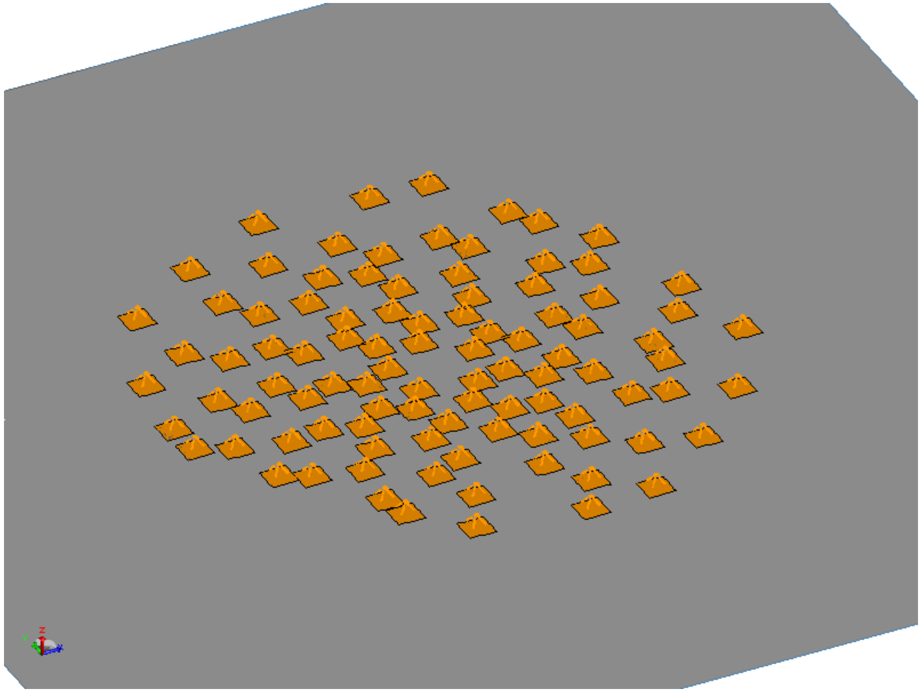


"Truth & Consequences" of finite LBA input impedance

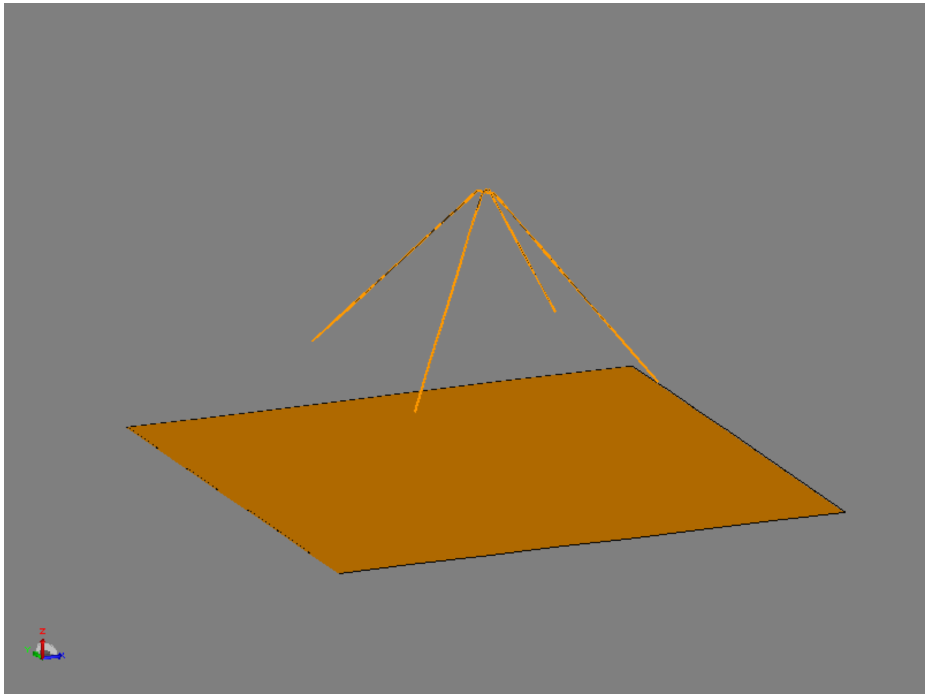
- Is the LBA an *active* antenna?
  - No. It's resonant antenna operating over a 10:1 band
- Does this only mean a *scalar* shift in the gain patterns?
  - No. Finite impedance (instead of open-circuit) means finite current on antenna => scattered radiation
- This leads to mutual-coupling
- Use EM simulation to confirm and quantify

International Lofar (SE-607) LBA

Array

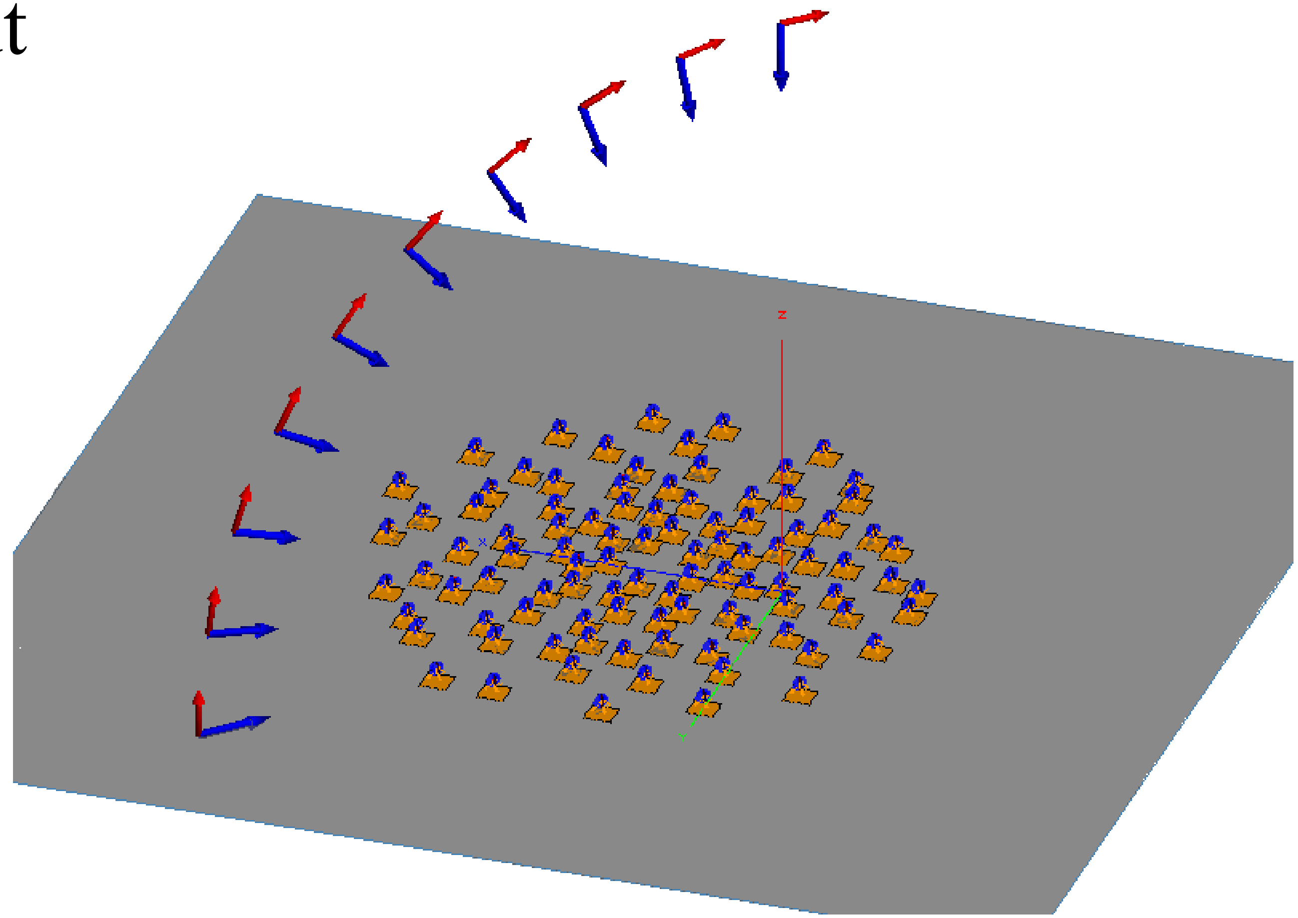


Element

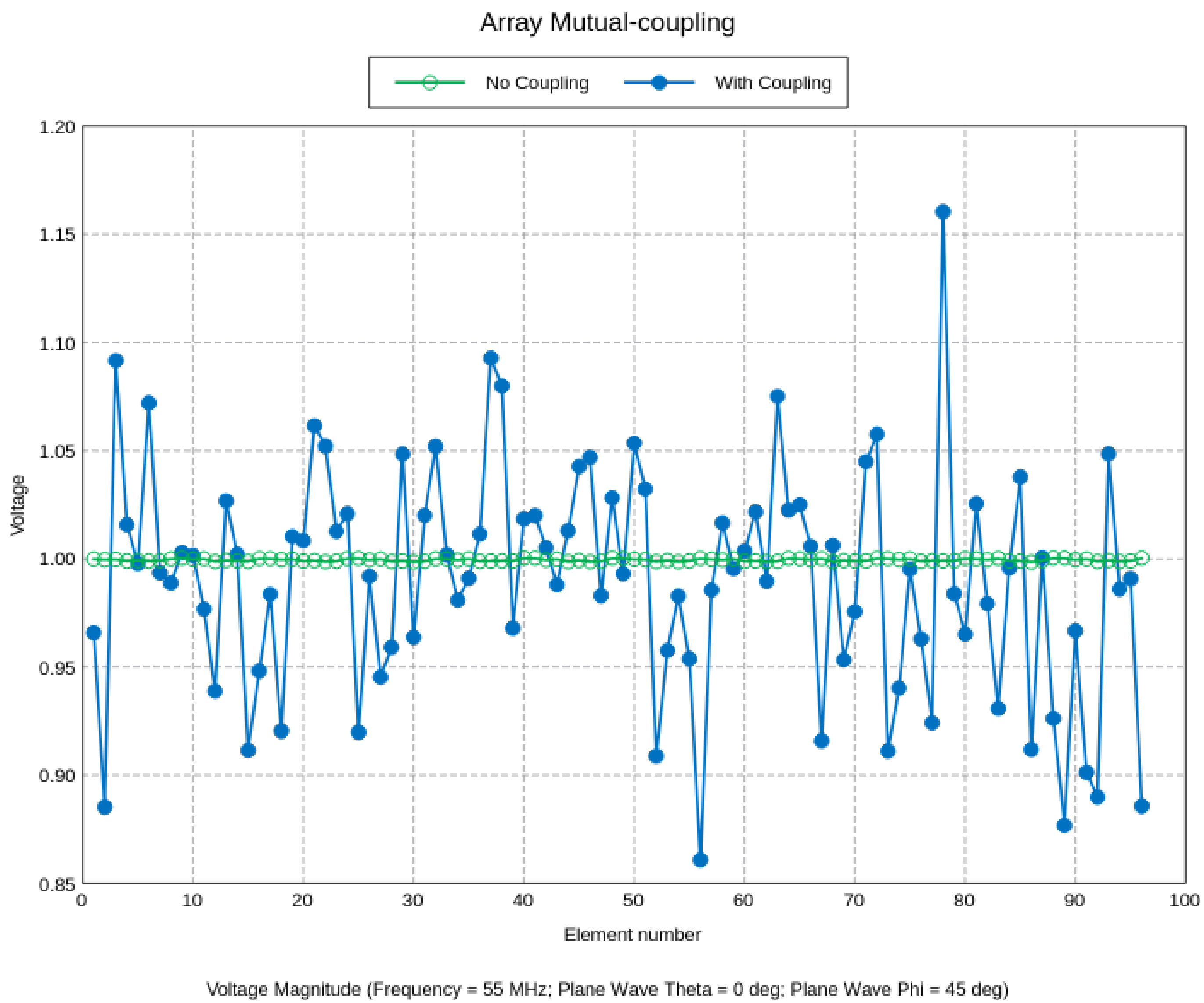


FEKO simulations

- Include finite input impedance LNA
- Run in receiver mode
  - Launch plane waves with given polarization
- Measure voltages on each LNA



Sim Results



Coupling induces 10% deviation from uncoupled voltage

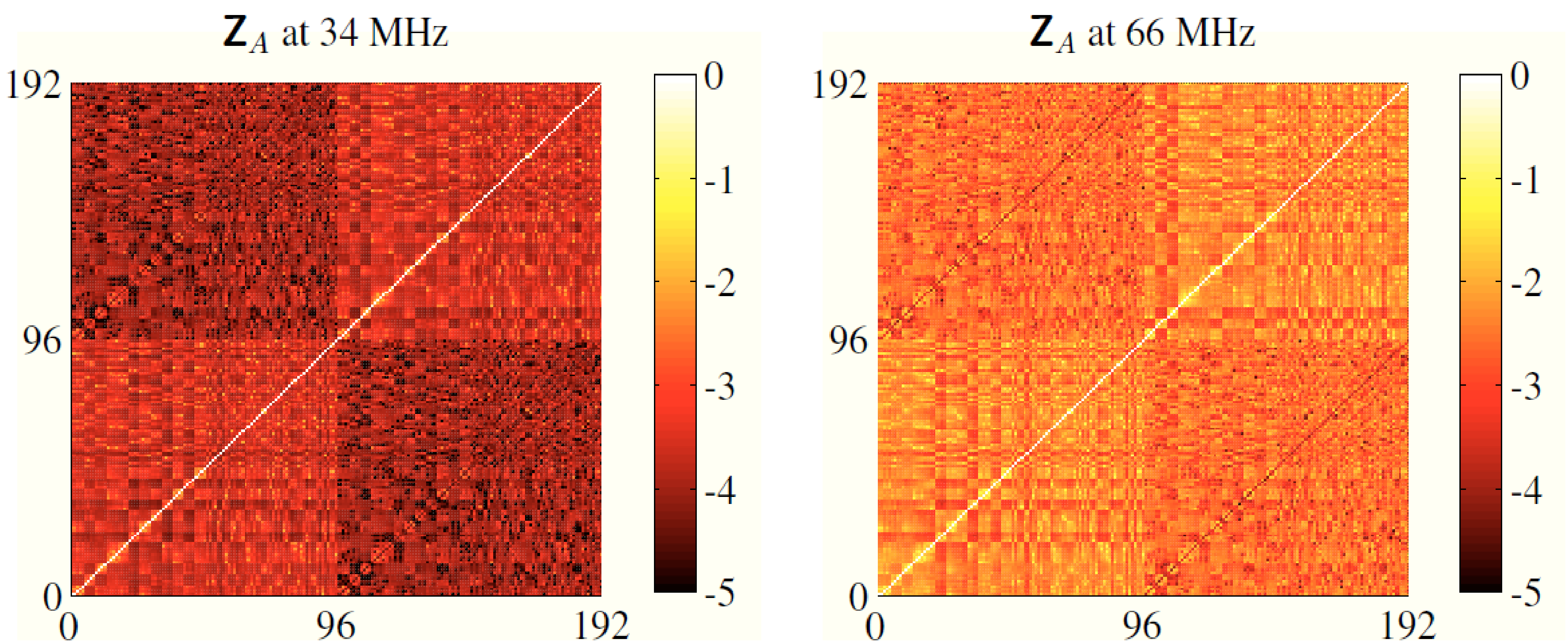
Lofar CBF

André Young

Full-wave simulation of LOFAR LBA in FEKO

Array impedance matrix

- mutual impedances generally higher at higher end of band



(colour scale normalised to maximum diagonal entry)

Full-wave simulation of LOFAR LBA in FEKO

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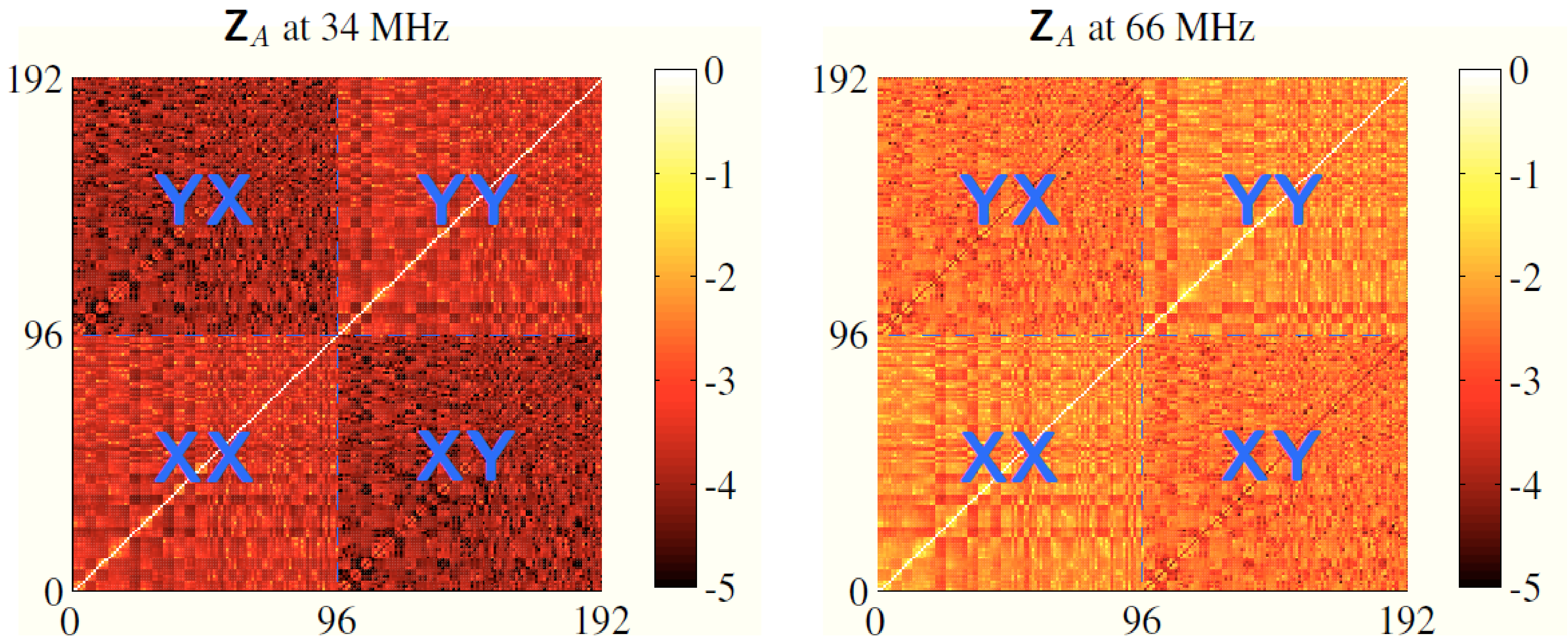
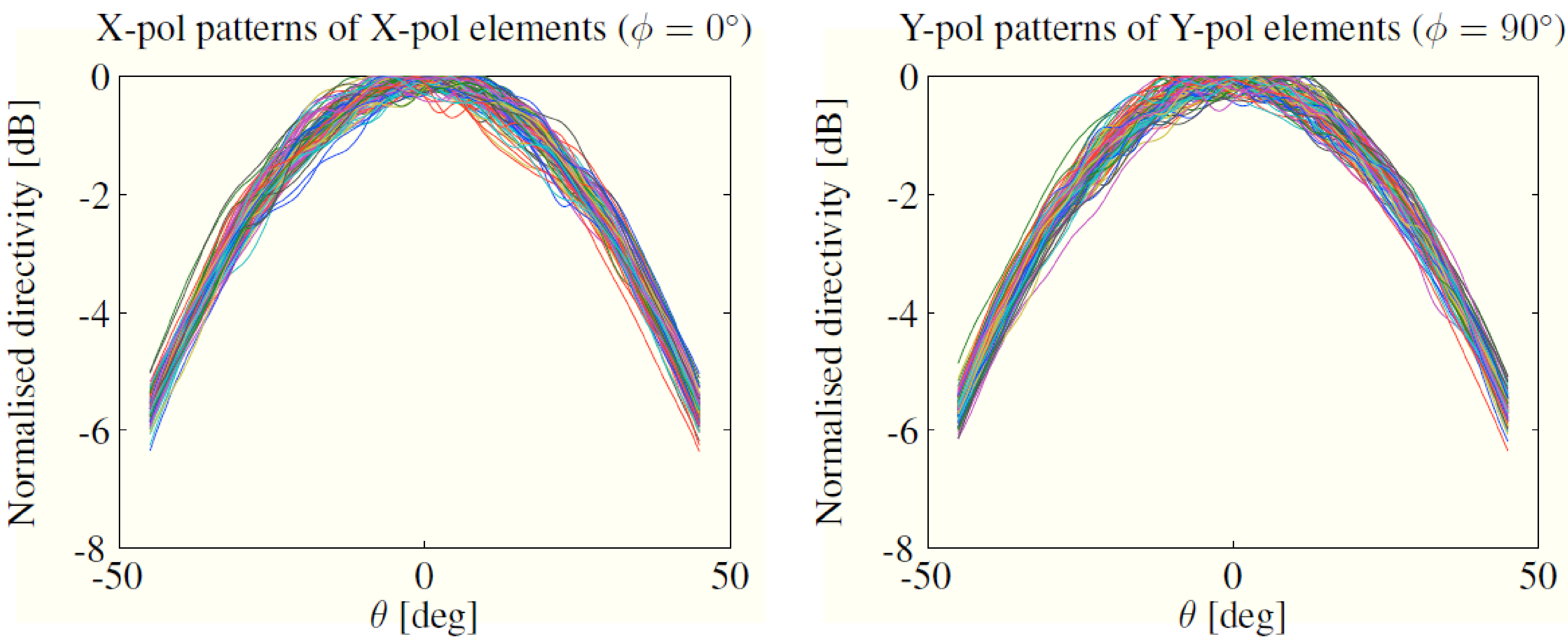


(colour scale normalised to maximum diagonal entry)

Full-wave simulation of LOFAR LBA in FEKO

Open-circuit voltage patterns

- result of mutual coupling clearly visible in variation among element patterns (65 MHz)



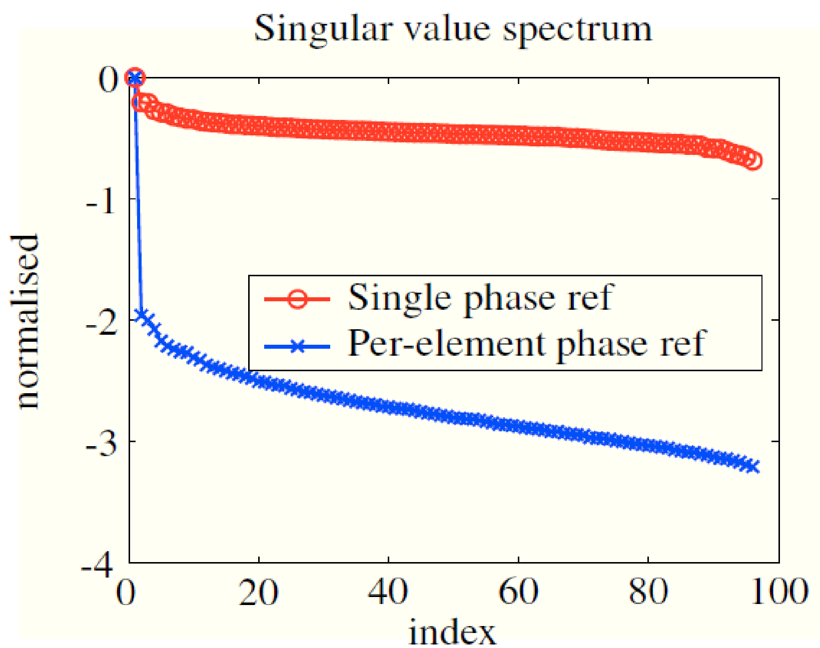
Low-order model for patterns?

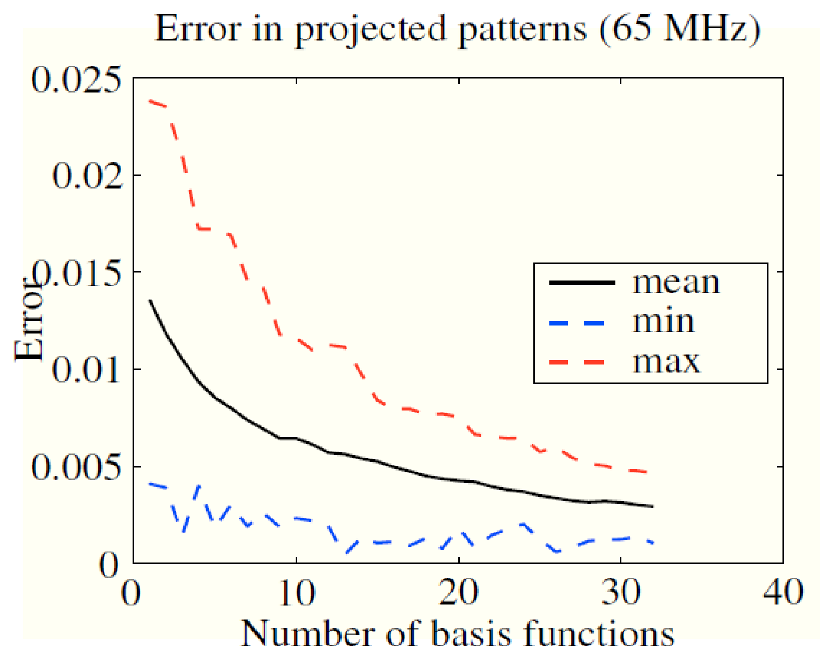
Use SVD to extract dominant modes in all element patterns

$$\mathbf{F}_x^T = \left[\mathbf{f}_x(\Omega_1) \quad \mathbf{f}_x(\Omega_2) \quad \cdots \quad \mathbf{f}_x(\Omega_N)\right]^T, \quad \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H = \mathbf{F}_x^T$$

Project element pattern $\mathbf{p}_{i,x} = [\mathbf{f}_x]_i \left(\{\Omega\}_{n=1}^{N}\right)$ onto span $\{\mathbf{u}_k\}_{k=1}^{N_B}$

$$\hat{\mathbf{p}}_{i,x} = \mathbf{U}_{N_B}\left(\mathbf{U}_{N_B}^H \mathbf{U}_{N_B}\right)^{-1} \mathbf{U}_{N_B}^H \mathbf{p}_{i,x}, \quad \text{Error} = \left\|\hat{\mathbf{p}}_{i,x} - \mathbf{p}_{i,x}\right\| / \left\|\mathbf{p}_{i,x}\right\|$$





(per-element phase ref)

Compensating for Gain / Impedance Variation

Known conditions $\to$ simulate; unknown conditions $\to$ model

Open-circuit voltages at antenna terminals for incident field

$$\mathbf{v}_{\text{oc}} = \sum_{j=1}^{N_j} e_{x,j}\mathbf{f}_x(\Omega_j) + e_{y,j}\mathbf{f}_y(\Omega_j) = \mathbf{F}_x\mathbf{e}_x + \mathbf{F}_y\mathbf{e}_y$$

Voltages appearing across load network

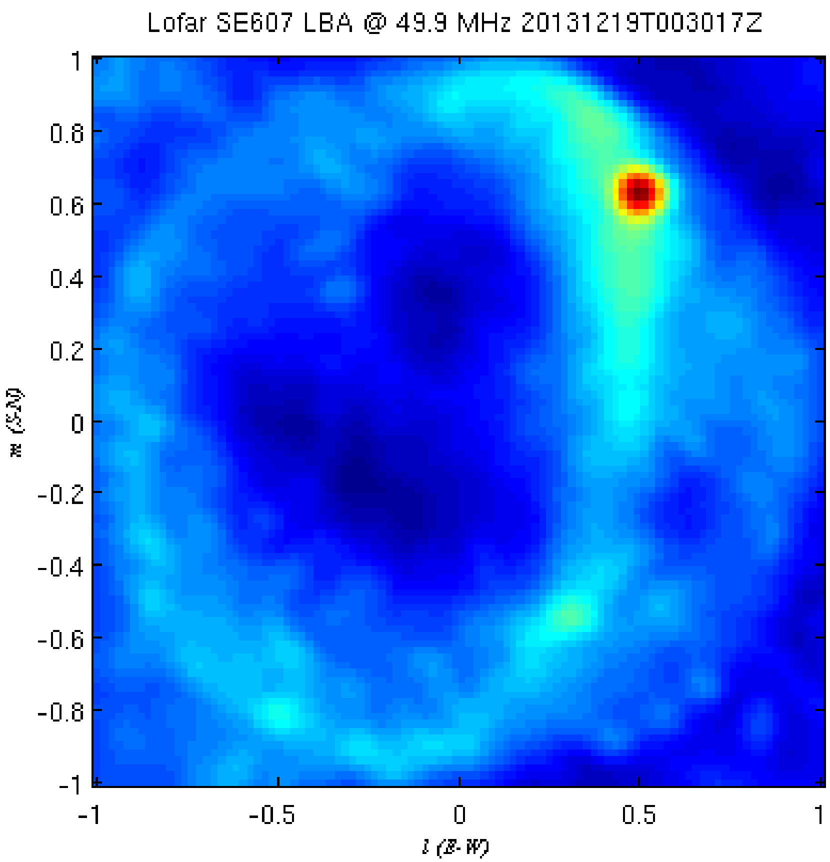
$$\mathbf{v} = \mathbf{Q}\mathbf{v}_{\text{oc}}\,, \qquad \mathbf{Q} = \mathbf{G}\mathbf{Z}_L\left(\mathbf{Z}_L + \mathbf{Z}_A\right)^{-1}$$

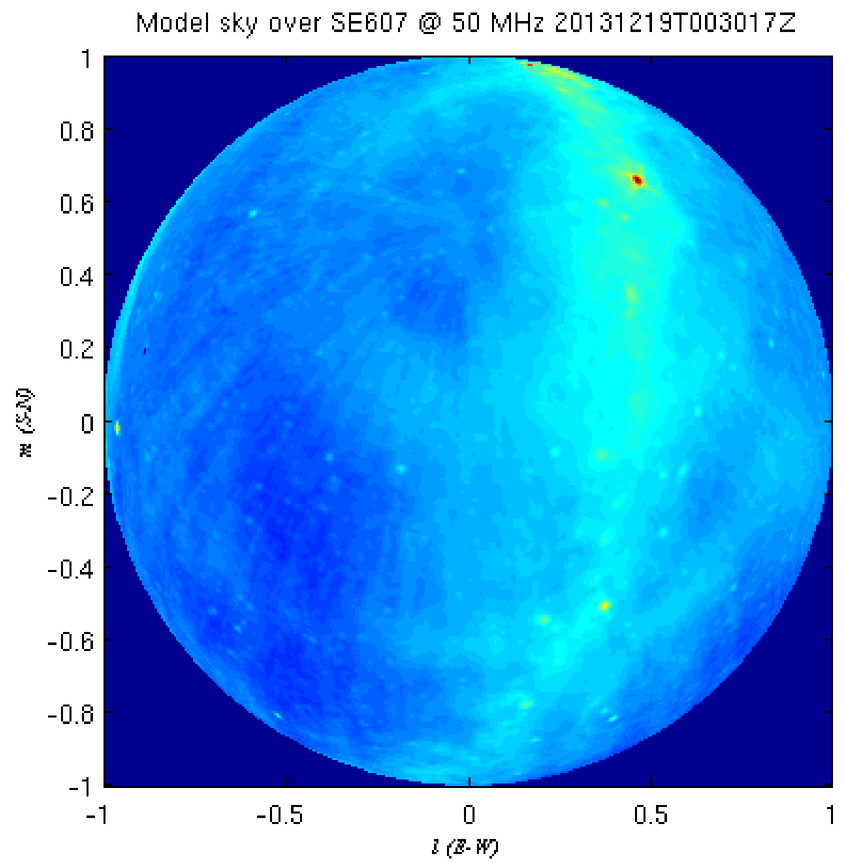
Correlation matrix for entire array

$$\mathbf{R}_{\mathbf{v}} = \mathbf{P}\mathbf{B}\mathbf{P}^H, \quad \mathbf{P} = \mathbf{Q}\mathbf{F}, \quad \mathbf{F} = \begin{bmatrix}\mathbf{F}_x & \mathbf{F}_y\end{bmatrix}, \quad \mathbf{B} = \mathrm{E}\left\{\begin{bmatrix}\mathbf{e}_x\mathbf{e}_x^H & \mathbf{e}_x\mathbf{e}_y^H \\ \mathbf{e}_y\mathbf{e}_x^H & \mathbf{e}_y\mathbf{e}_y^H\end{bmatrix}\right\}$$

Construct basis for $\mathbf{P}$ based on expected variations in $\mathbf{G}$ and $\mathbf{Z}_L$

Calibration & Imaging





Conclusion

- Lofar LBA has mutual-coupling
- Characteristic basis functions may be a useful tool for calibration

Thanks