AA-mid calibratability requirements

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Calibratability

Dynamic range is not a suitable figure-of-merit

- no information on (structure in) the noise floor of the image
- strongly dependent on observed field

Effective noise

- thermal noise
- classical source confusion noise
- psf sidelobe noise (PSN)
- calibration noise (estimation noise + penalty for corrections)
- calibration artefacts

Last 4 factors can be mitigated by **design-for-calibratability**

Impact on system architecture

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Effects of instrument model errors should be less than thermal noise

- Requirement on post-calibration station beam model accuracy
- Requirement on post-calibration BF weight accuracy
- Requirement on receive path stability

Effect of tile size

- Larger (but fewer!) tiles have higher sensitivity but smaller FoV
- What is easiest to calibrate?

Effect of (u,v)-distribution

- Requirement on completeness of (u,v)-plane sampling
- Requirement on minimum number of (remote) stations

Assumed system specifications

- 250 stations
- 53-m diameter
- $T_{sys} = T_{rec} + T_{sky}$ with $T_{rec} = 40$ K
- Frequency range (for analysis): 300 – 1500 MHz
- Antenna pitch: 0.2 m (55k antennas in circular station)
- Source statistics from Ph.D. thesis Bregman
- Instrumental calibration update rate: 10 minutes
- Instrumental calibration resolution: 1 MHz

Model errors versus thermal noise

Wijnholds, AAVP Workshop, Dec. 2011
Wijnholds, SKA-TEL.LFAA.SE.CAL-AADC-TN-001, Jan. 2014

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Imaging can be formulated as a least squares estimation problem:

$$\hat{\boldsymbol{\sigma}} = \underset{\boldsymbol{\sigma}}{argmin} \left\| \hat{\boldsymbol{v}} - \boldsymbol{M}\boldsymbol{\sigma} \right\|^2$$

Generic error propagation formula

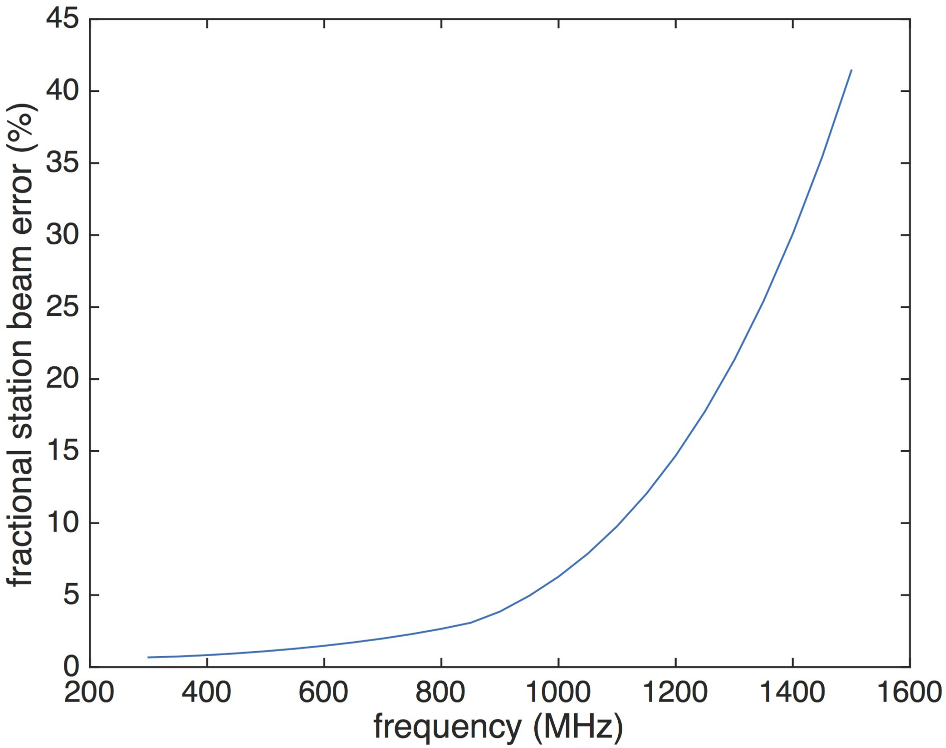
$$\frac{\|\Delta \boldsymbol{\sigma}\|}{\|\boldsymbol{\sigma}\|} = \kappa(\boldsymbol{M}) \left(\frac{\|\Delta \boldsymbol{M}\|_2}{\|\boldsymbol{M}\|_2} + \frac{\|\Delta \boldsymbol{v}\|}{\|\boldsymbol{v}\|} \right)$$

Fractional error on the station beam

$$\epsilon_{stat} = \frac{1}{\sqrt{2}} \frac{\Delta S_0 / \sqrt{B\tau}}{S_{rss}}$$

Required station beam accuracy (1)

- Most stringent below 1 GHz, about 0.5% at 300 MHz
- Rise above 1 GHz due to sparse operating regime
- Very loose above 1 GHz, other constraints may be more stringent



Required station beam accuracy (2)

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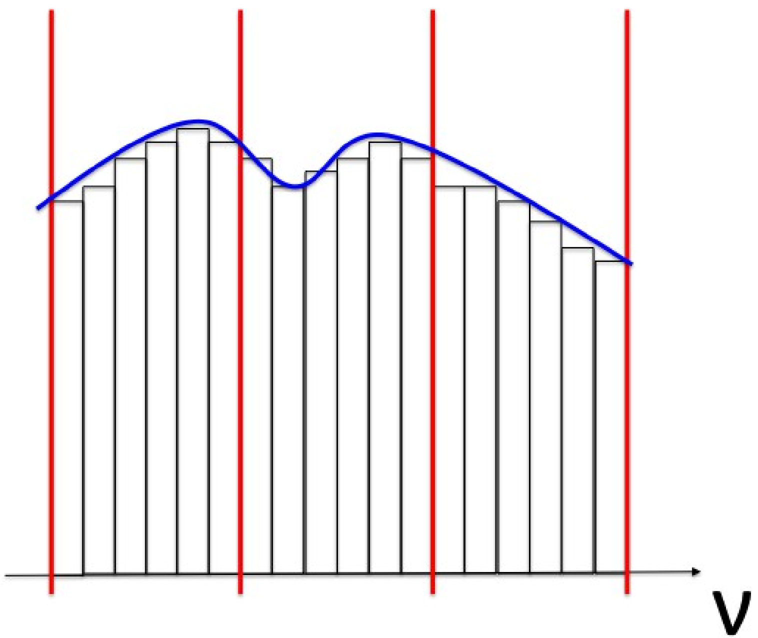
- Analysis by Cathryn Trott and Randall Wayth for SKA-low bandpass
- Assumption: polynomial fit over three coarse channels
- Balancing with thermal noise gives tolerance on station beam
- Consistent (at least for SKA-low) with previous analysis



frequency	rel. error
50 MHz	1.8%
150 MHz	0.54%
200 MHz	0.56%

Required station beam accuracy (3)

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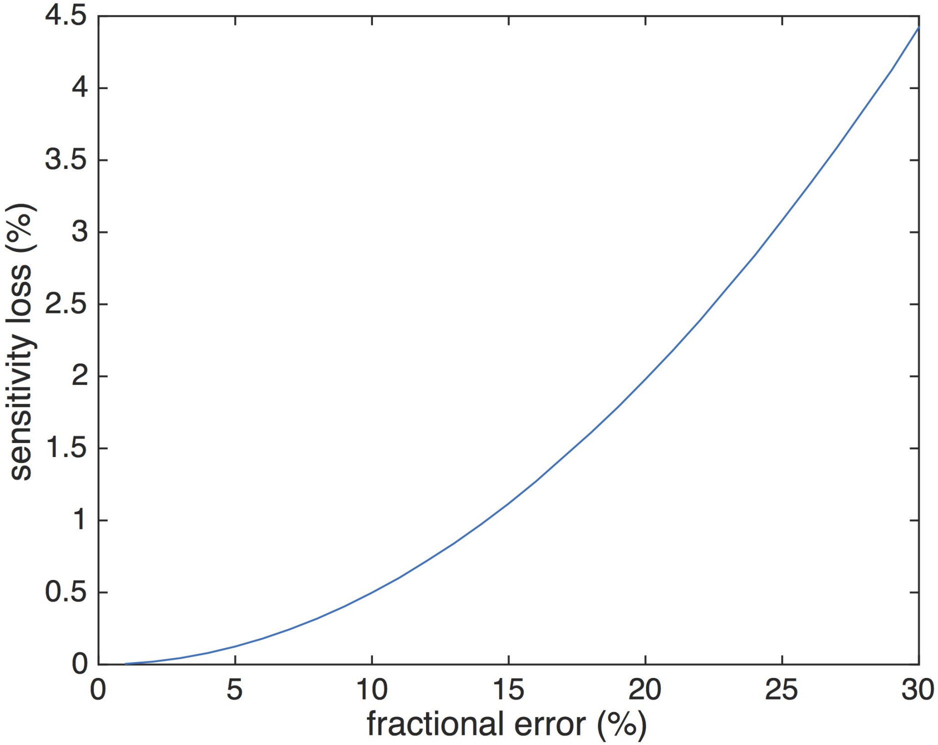
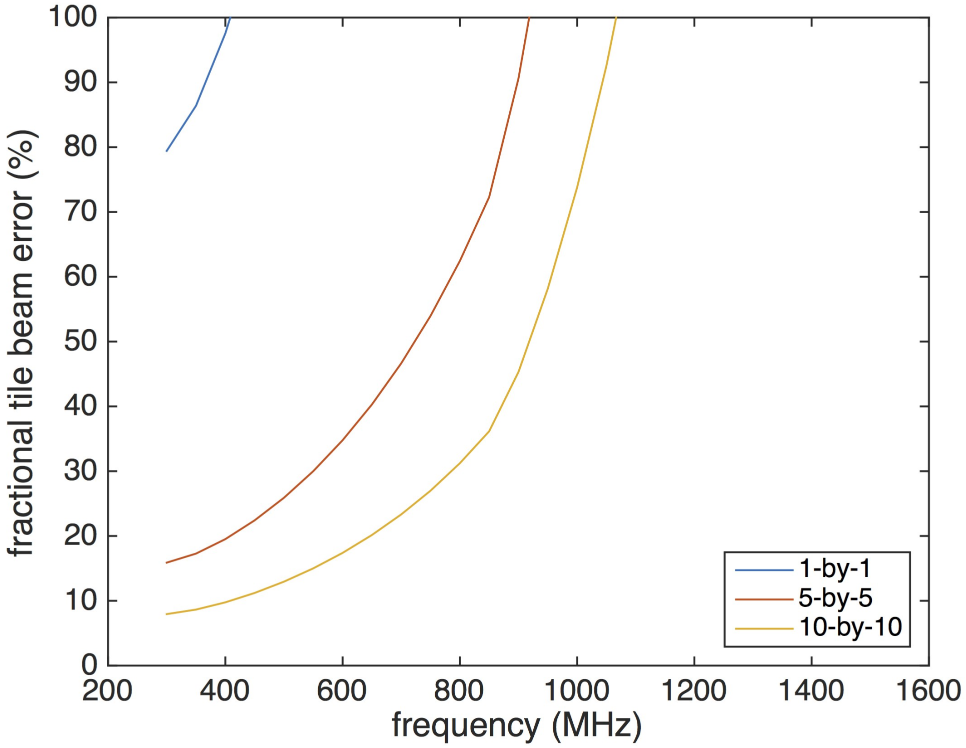
- Large errors lead to coherence loss in imaging process
- SKA accepts 2% coherence loss at edge of FoV on longest baselines
- With BDA, this causes about 0.5% coherence loss in image
- 0.5% coherence loss corresponds to 10% beam error
- Guideline: maximize station beam errors to 10%



Tile beam accuracy

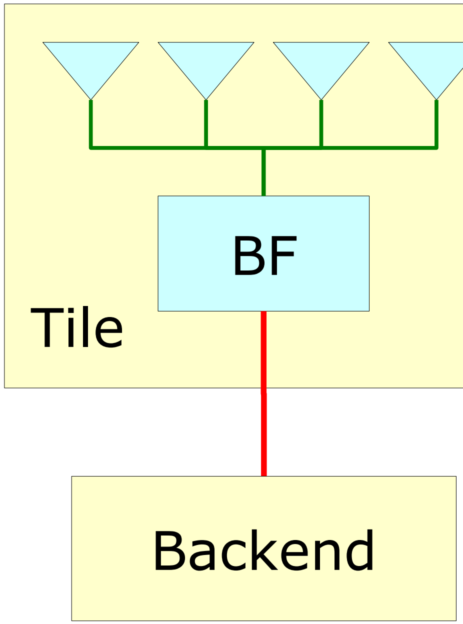
- Element-level accuracy allows 1-bit beamformer
- Other factors may pose more stringent requirements (e.g. app. Eff.)
- Even quite moderate requirement (8%) for 10-by-10 tiles
- Again, sensitivity loss (aperture efficiency) may be larger concern

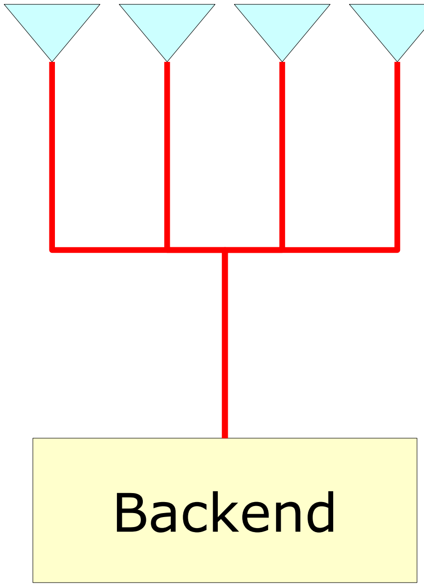


To tile or not to tile

- Short links inside tile:
  - small differential delays
  - no regular calibration needed
- Long tiles-to-backend links
  - potentially large diff. delays
  - regular calibration needed

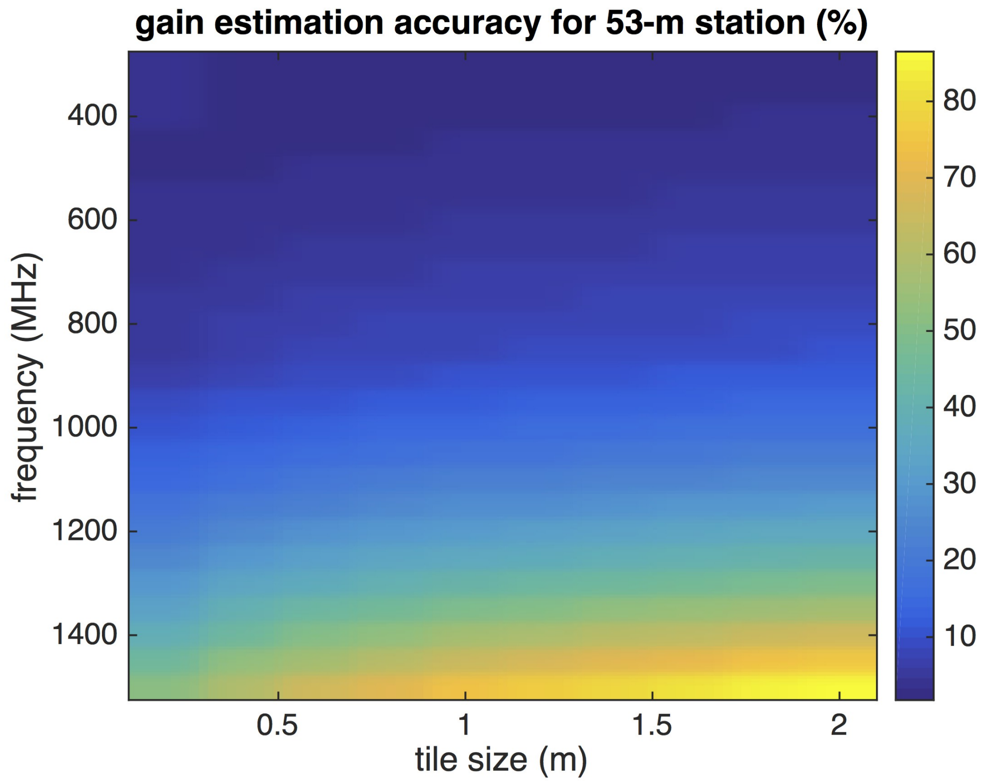
- Long antenna-to-backend links
  - potentially large diff. delays
  - regular calibration needed
- Independent of placement of ADCs due to clock distribution





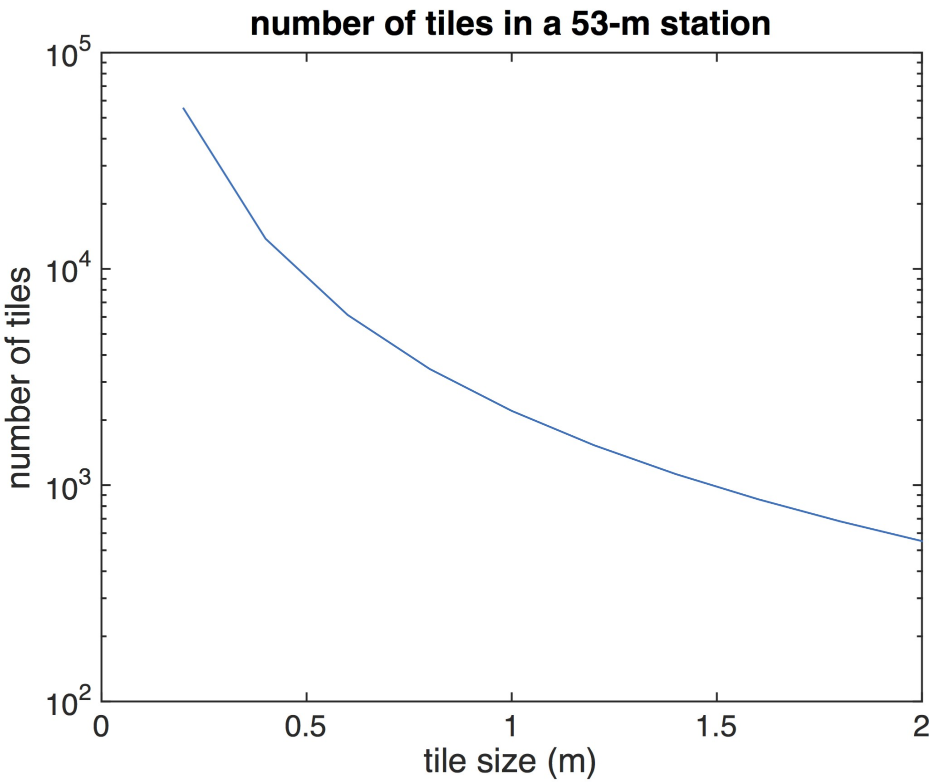
Calibratability at station level

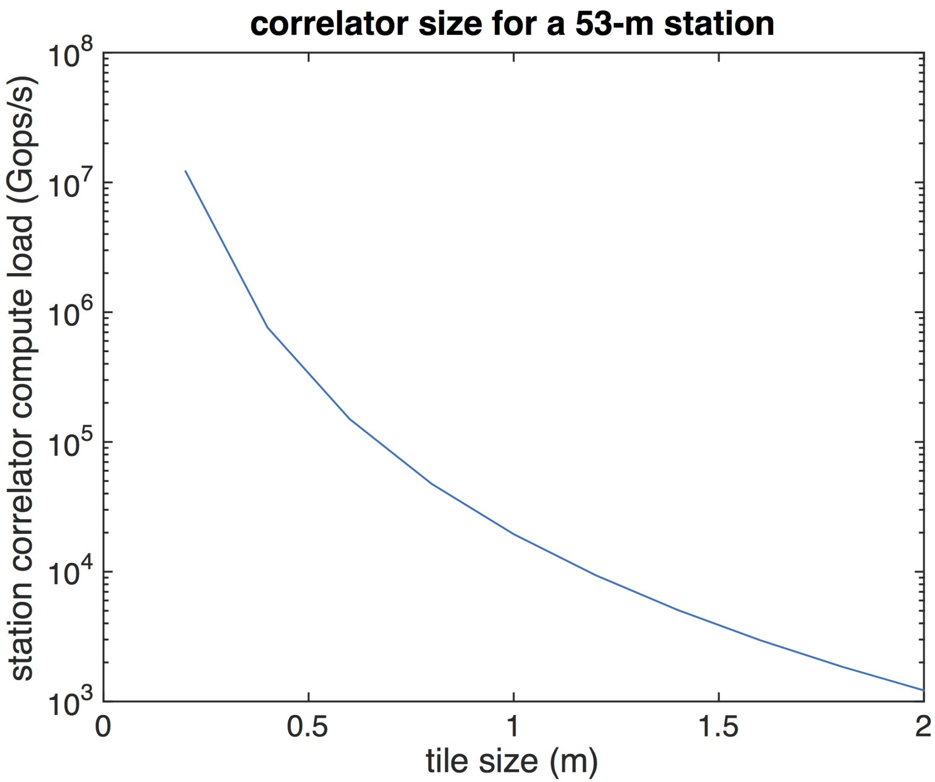
- Smaller tiles have larger FoV but lower sensitivity per tile
- Can we calibrate on the field at which tiles are pointed?
- Answer: yes! (assuming 1 MHz bandwidth, 10 s integration)



Feasibility of online station calibration

- Smaller tiles imply more tiles per station (left)
- Smaller tiles lead to larger station correlator (right, 1 MHz BW)



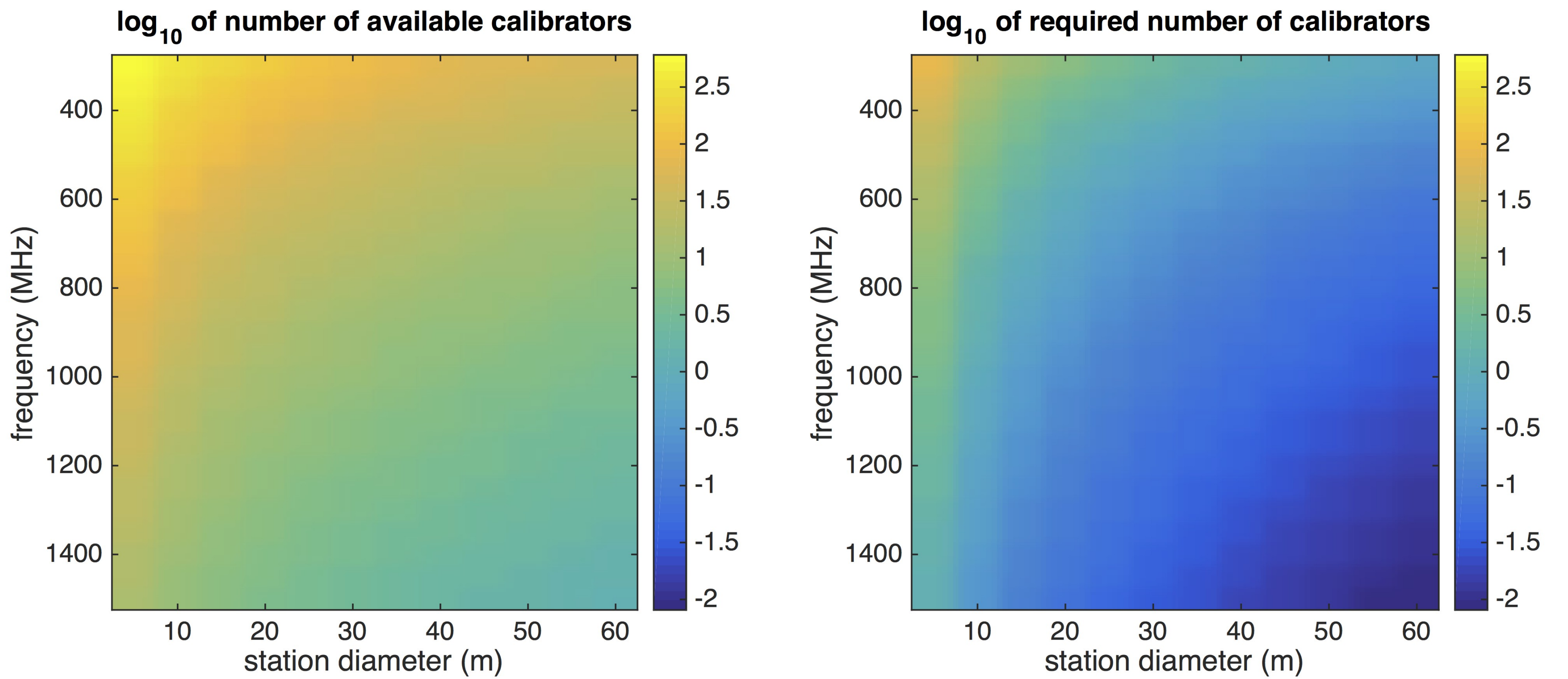


Calibratability of ionosphere

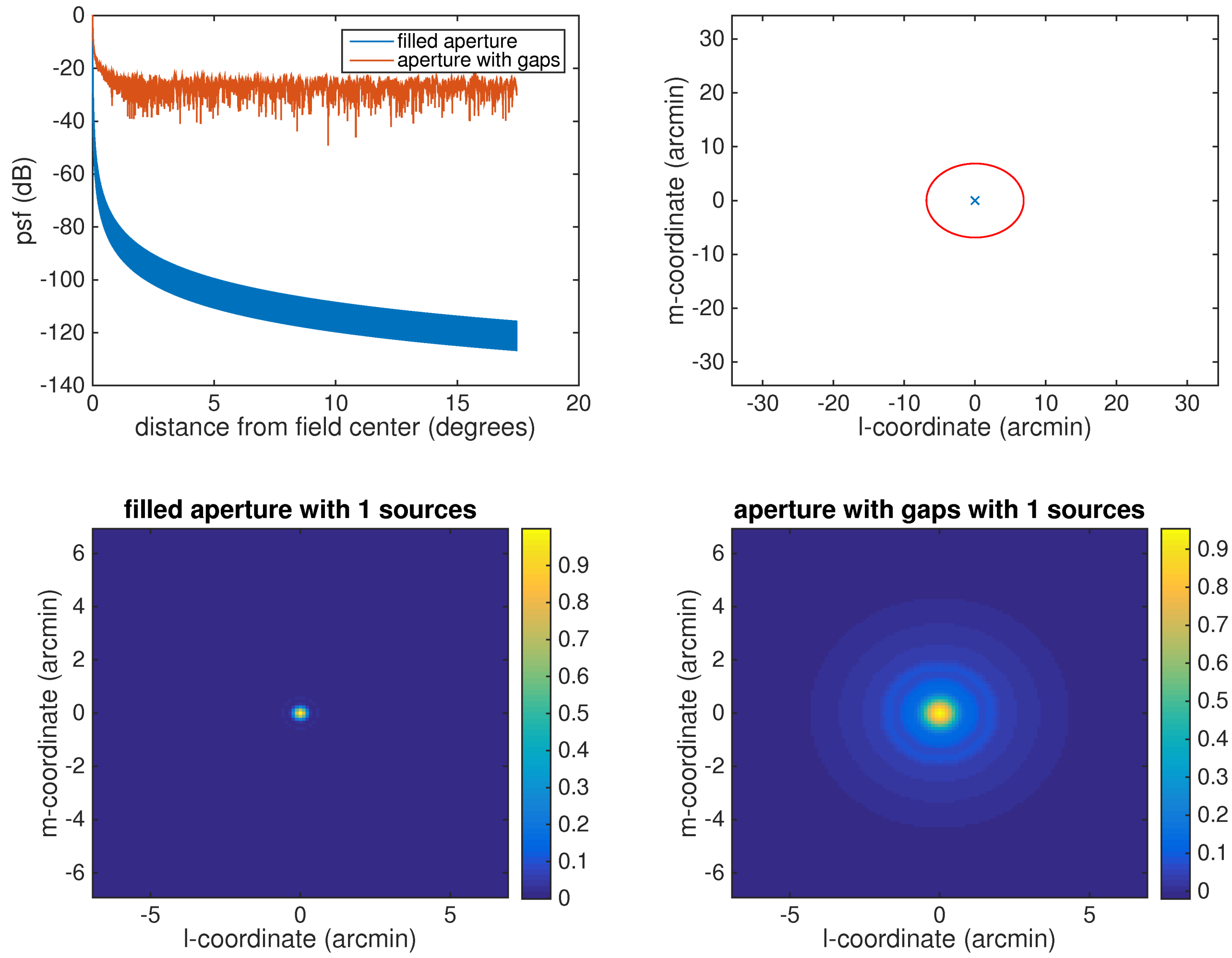
Left: available number of calibrators based on source statistics

Right: required number of calibrators based on ionospheric model

Ionospheric calibratability does not constrain design space



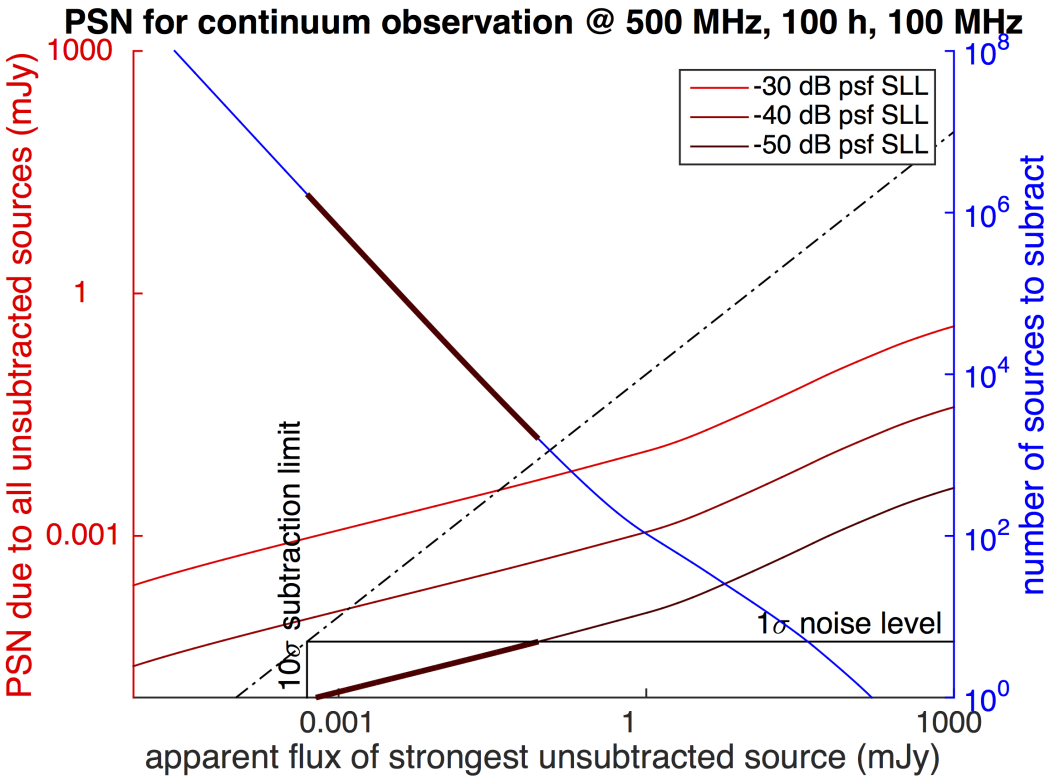
Psf Sidelobe Noise

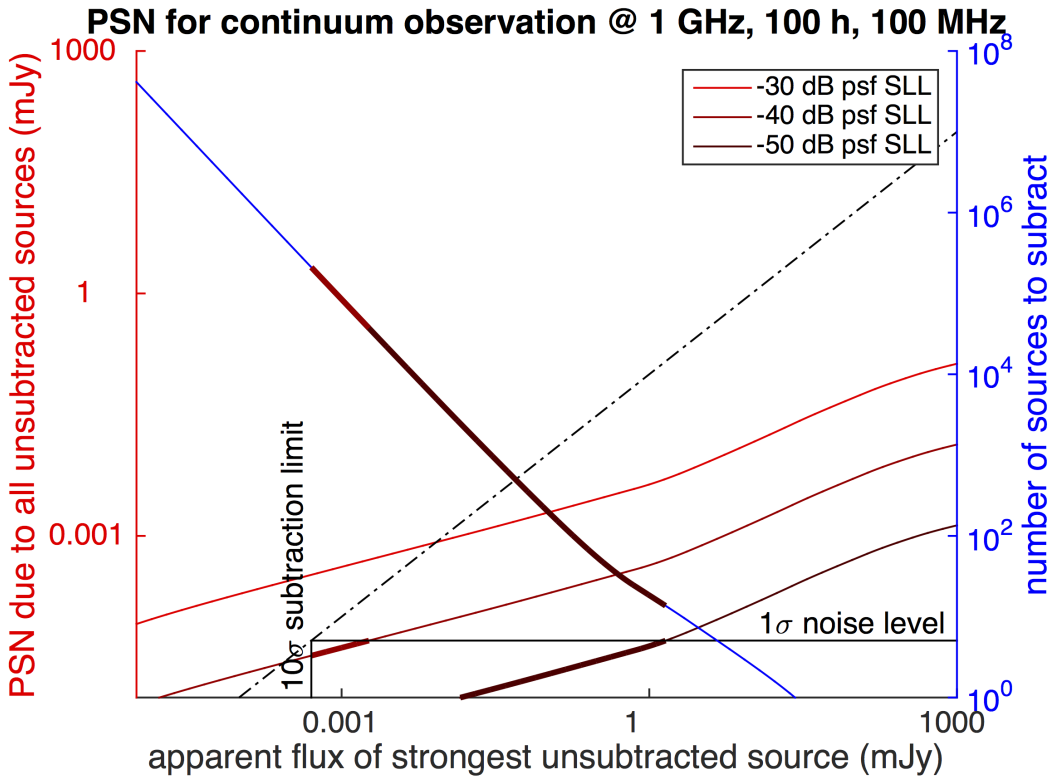


Psf Sidelobe Noise

PSN in continuum observations

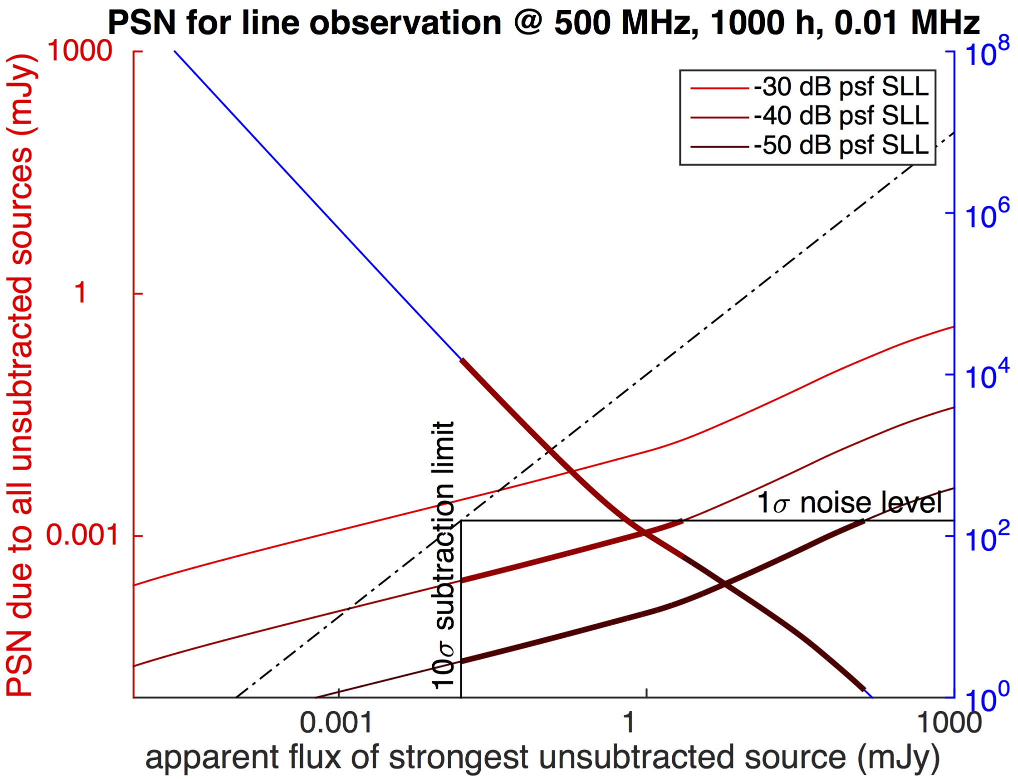
- Lower rms psf sidelobe level (SLL) required at lower frequencies
- Filled (u,v)-plane required in synthesis observations

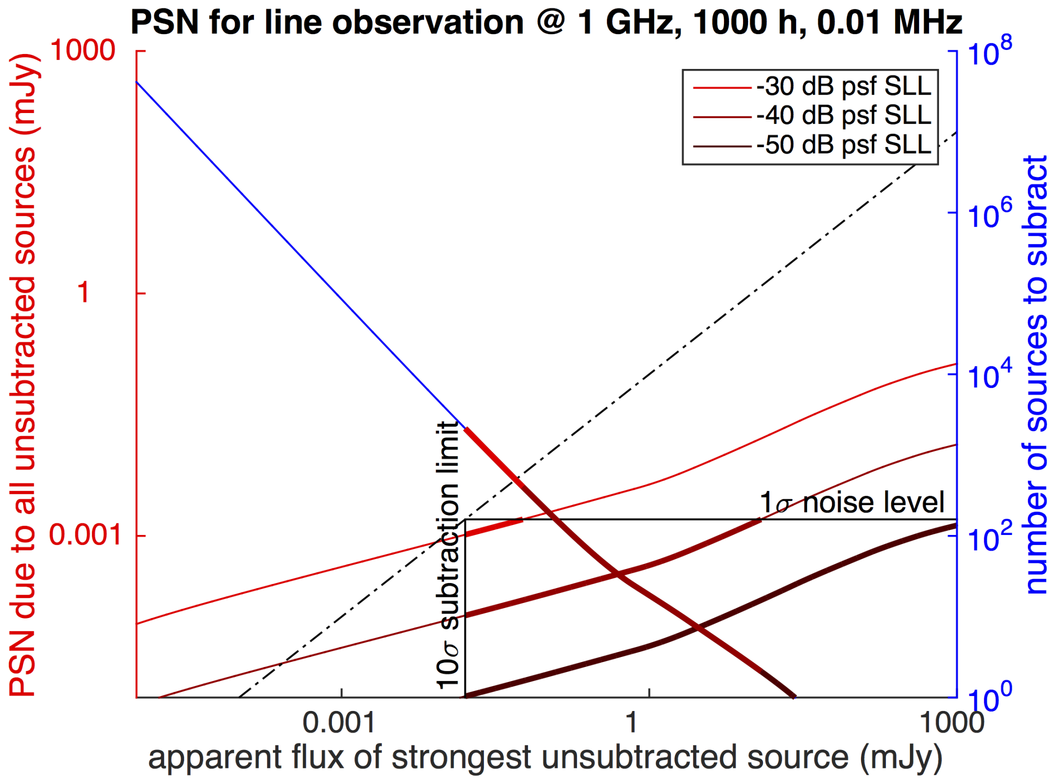




PSN in line observations

- More relaxed than in continuum observations
- Synthesis observations require an almost filled (u,v)-plane





Conclusions

- 53-m stations can be calibrated during observation
  - full sensitivity of station required
  - tile size determines size of station correlator
  - large tolerance on element-level errors
- Ionosphere does not impose constraint on station size
  - station size can be traded for number of stations
- PSN most stringent for continuum
  - @500 MHz: -43 dB (cont.) vs. -33 dB (line) psf rms SLL
  - continuum might be limited by classical confusion instead
  - -33 dB requires an almost filled (u,v)-plane